



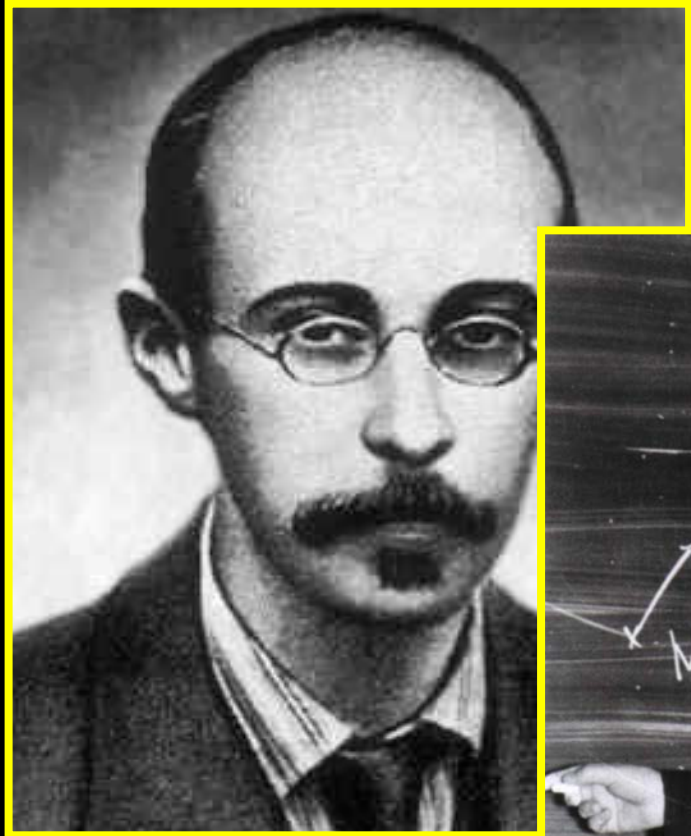
the  
**Big Bang**

**Friedmann, Lemaitre**

**&**

**Cosmic Expansion History**

# Friedmann & Lemaitre



Alexander Friedmann

(1888 -1925)

George Lemaitre

(1894-1966)



They discovered (independently) theoretically the expansion of the Universe as a solution to the Theory of General Relativity.

... and derived the equations that describe the expansion and evolution of the universe,

the foundation for all of modern Cosmology:

**Friedmann-Lemaitre  
Equation**

# Evolving Universe

- Einstein, de Sitter, Friedmann and Lemaitre all realized that in General Relativity, there cannot be a stable and static Universe:
- The Universe either expands, or it contracts ...

- Expansion Universe encapsulated in a

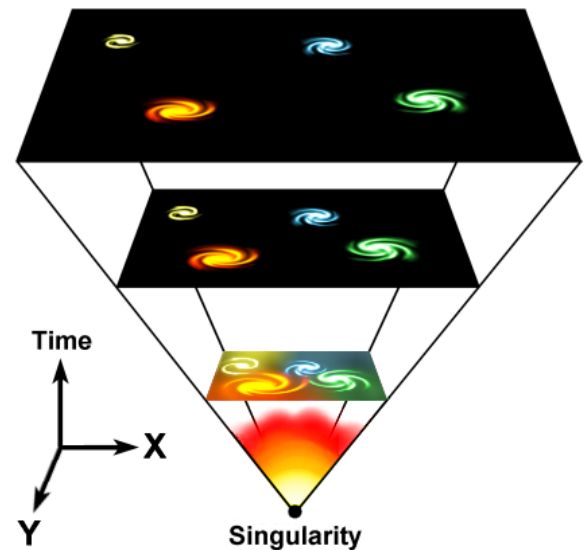
**GLOBAL expansion factor  $a(t)$**

- All distances/dimensions of objects uniformly increase by  $a(t)$ :

at time  $t$ , the distance between two objects  $i$  and  $j$  has increased to

$$\vec{r}_i - \vec{r}_j = a(t) (\vec{r}_{i,0} - \vec{r}_{j,0})$$

- Note: by definition we chose  $a(t_0)=1$ , i.e. the present-day expansion factor

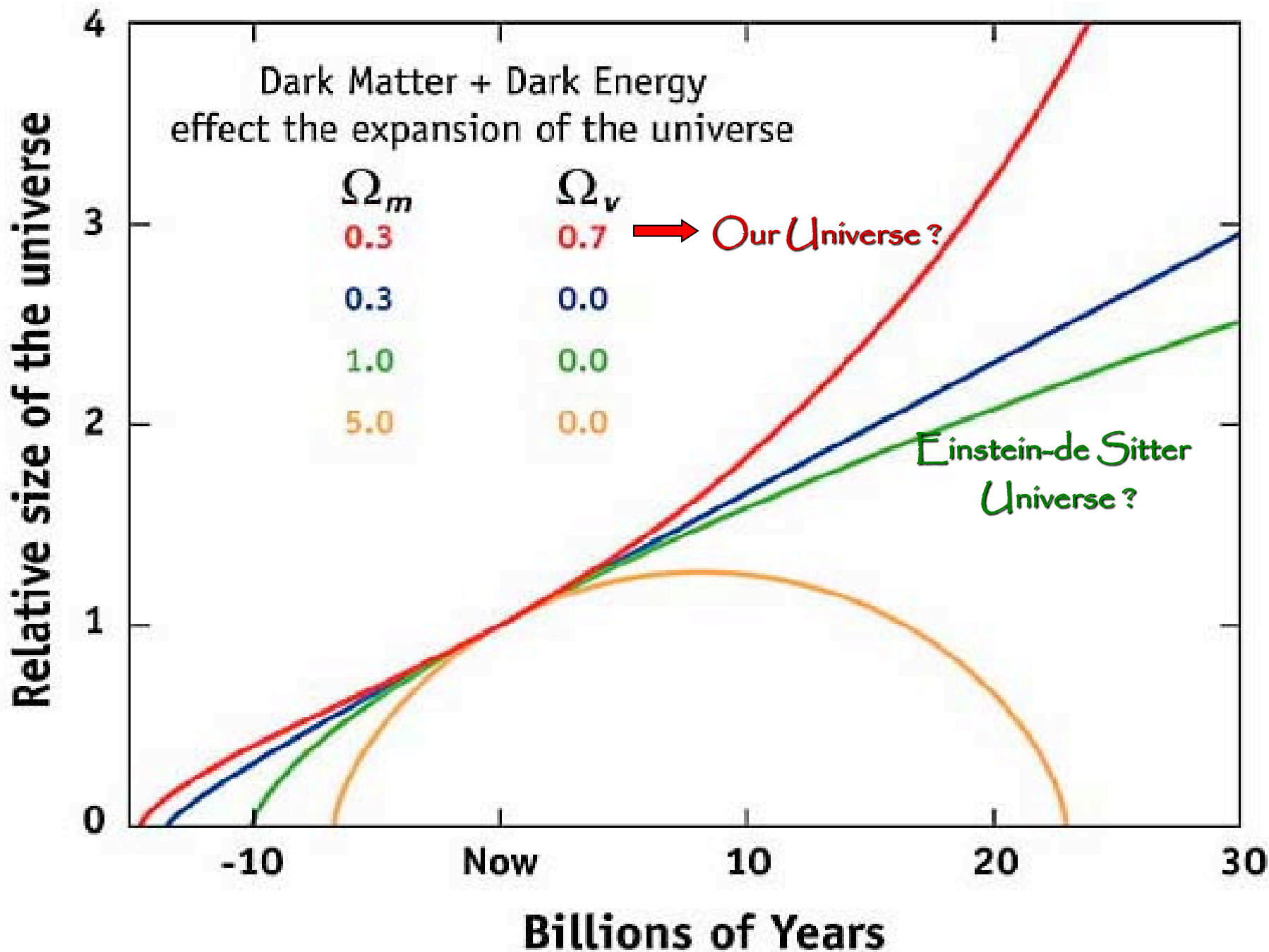


# Evolution & Fate

## Friedmann-Robertson-Walker-Lemaitre Universe

Completely determined by 3 factors:

- energy and matter content  
(density and pressure)
- geometry of the Universe  
(curvature)
- Cosmological Constant



# Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) a + \frac{\Lambda}{3} a$$

$$\dot{a}^2 = \frac{8\pi G}{3} \rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3} a^2$$

# Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is determined by four factors:

- density  $\rho(t)$
- pressure  $p(t)$
- curvature  $kc^2 / R_0^2$   $k = 0, +1, -1$   
 $R_0$ : present curvature radius
- cosmological constant  $\Lambda$

- Density & Pressure:
  - in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
  - pressure = momentum flux
- Curvature:
  - gravity is a manifestation of geometry spacetime
- Cosmological Constant:
  - free parameter in General Relativity
  - Einstein's "biggest blunder"
  - mysteriously, since 1998 we know it dominates the Universe



# FRW Dynamics

In a FRW Universe,  
densities are in the order of the critical density,  
the density at which the Universe has a flat curvature

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 1.8791h^2 \times 10^{-29} \text{ g cm}^{-3}$$

$$\begin{aligned}\rho_0 &= 1.8791 \times 10^{-29} \Omega h^2 \text{ g cm}^{-3} \\ &= 2.78 \times 10^{11} \Omega h^2 M_{\odot} \text{ Mpc}^{-3}\end{aligned}$$

# FRW Dynamics

In a matter-dominated Universe,  
the evolution and fate of the Universe entirely determined  
by the (energy) density in units of critical density:

$$\Omega \equiv \frac{\rho}{\rho_{crit}}$$

Arguably,  $\Omega$  is the most important parameter of cosmology !!!

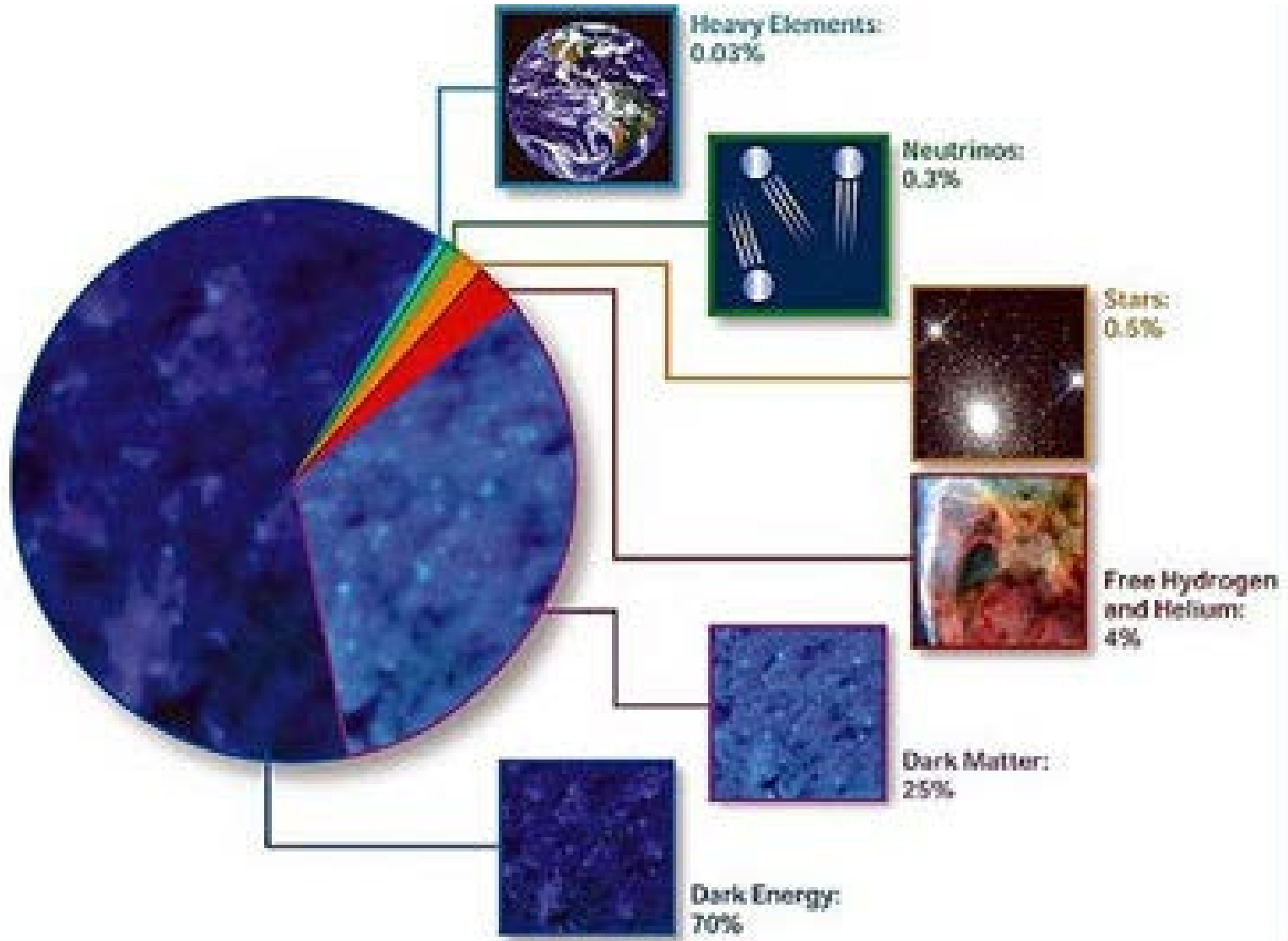
Present-day  
Cosmic Density:

$$\begin{aligned}\rho_0 &= 1.8791 \times 10^{-29} \Omega h^2 \text{ g cm}^{-3} \\ &= 2.78 \times 10^{11} \Omega h^2 M_{\odot} \text{ Mpc}^{-3}\end{aligned}$$

**what the Universe exists of:**

**Cosmic Constituents**

# Cosmic Components



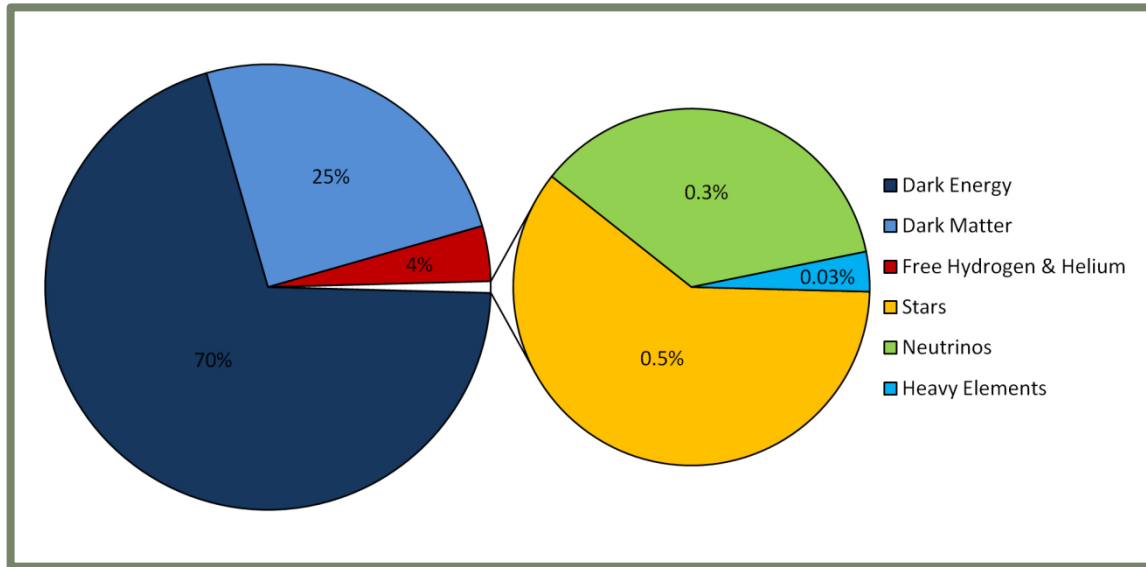
# Cosmic Energy Inventarisatie

1	dark sector		$0.954 \pm 0.003$
1.1	dark energy		$0.72 \pm 0.03$
1.2	dark matter		$0.23 \pm 0.03$
1.3	primeval gravitational waves		$\lesssim 10^{-10}$
2	primeval thermal remnants		$0.0010 \pm 0.0005$
2.1	electromagnetic radiation		$10^{-4.3 \pm 0.0}$
2.2	neutrinos		$10^{-2.9 \pm 0.1}$
2.3	prestellar nuclear binding energy		$-10^{-4.1 \pm 0.0}$
3	baryon rest mass		$0.045 \pm 0.003$
3.1	warm intergalactic plasma		$0.040 \pm 0.003$
3.1a	virialized regions of galaxies	$0.024 \pm 0.005$	
3.1b	intergalactic	$0.016 \pm 0.005$	
3.2	intracluster plasma		$0.0018 \pm 0.0007$
3.3	main sequence stars	spheroids and bulges	$0.0015 \pm 0.0004$
3.4		disks and irregulars	$0.00055 \pm 0.00014$
3.5	white dwarfs		$0.00030 \pm 0.00008$
3.6	neutron stars		$0.00005 \pm 0.00002$
3.7	black holes		$0.00007 \pm 0.00002$
3.8	substellar objects		$0.00014 \pm 0.00007$
3.9	HI + HeI		$0.00062 \pm 0.00010$
3.10	molecular gas		$0.00016 \pm 0.00006$
3.11	planets		$10^{-6}$
3.12	condensed matter		$10^{-5.6 \pm 0.3}$
3.13	sequestered in massive black holes		$10^{-5.4}(1 + \epsilon_n)$
4	primeval gravitational binding energy		$-10^{-6.1 \pm 0.1}$
4.1	virialized halos of galaxies		$-10^{-7.2}$
4.2	clusters		$-10^{-6.9}$
4.3	large-scale structure		$-10^{-6.2}$



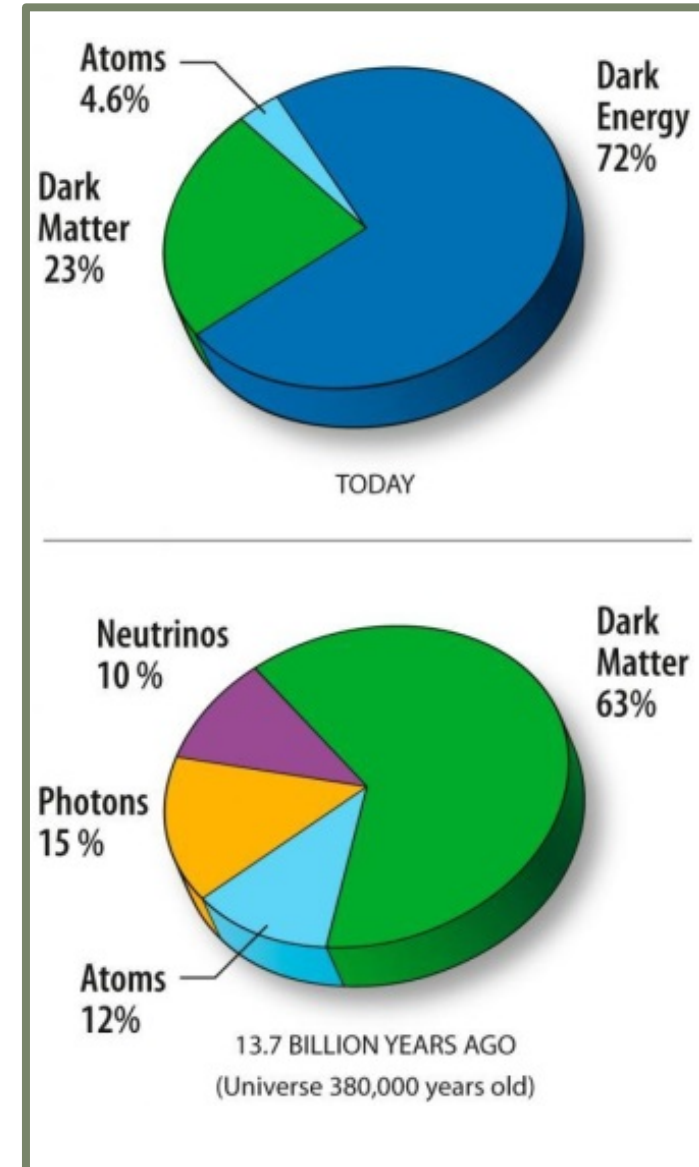
sterren slechts  
~0.1% energie  
Heelal

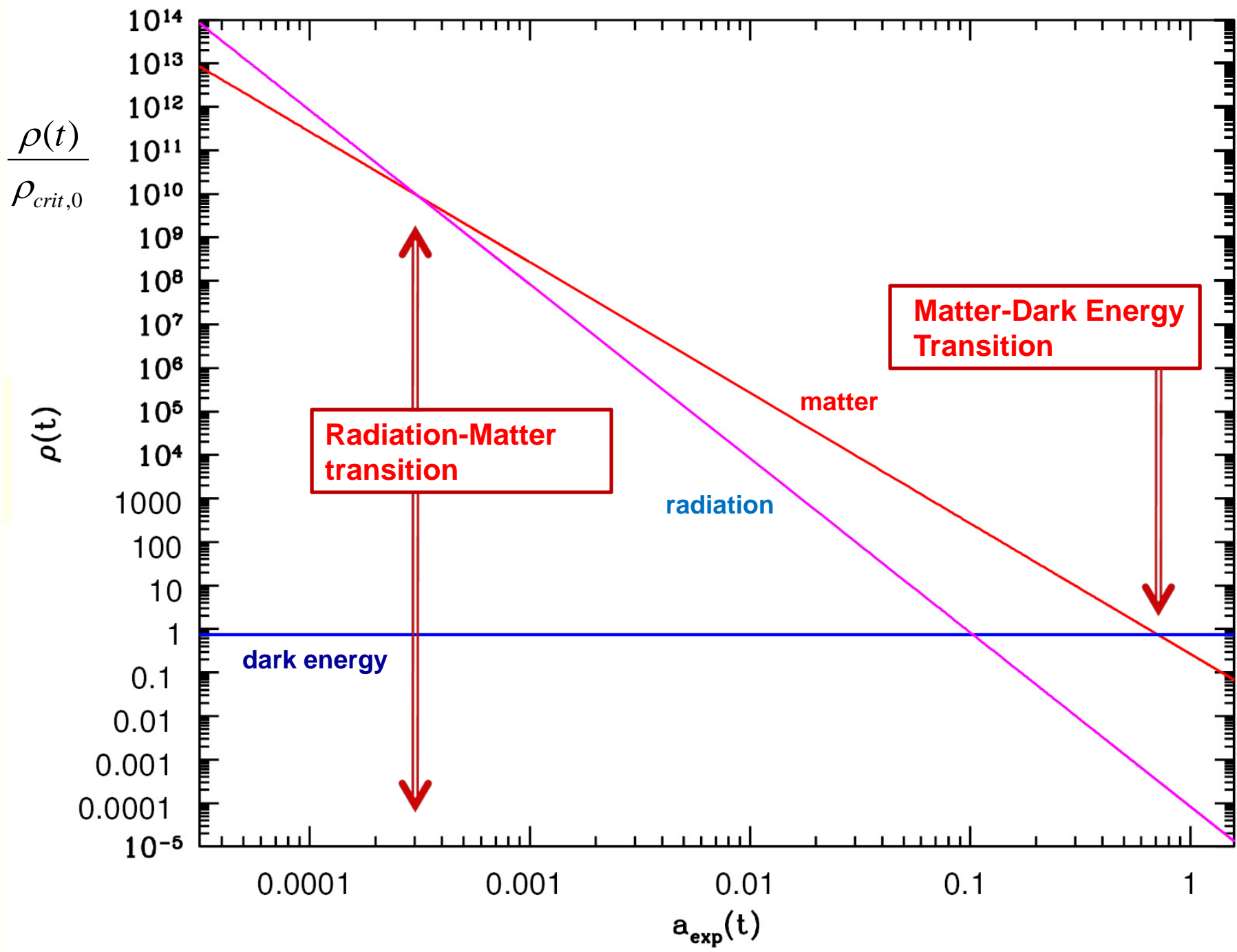
# Cosmic Constitution

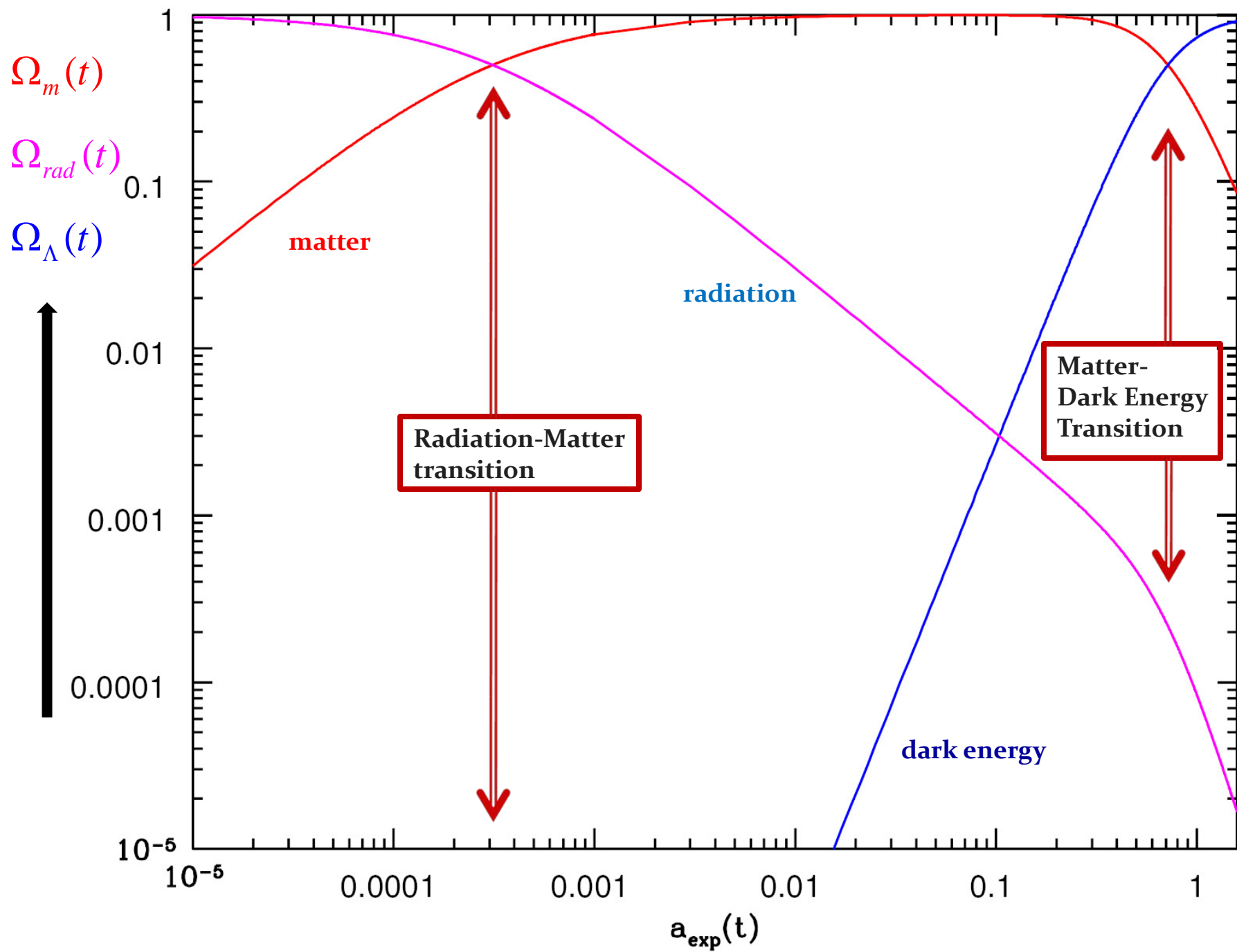


Cosmic Pie Diagram

Changes in Time:









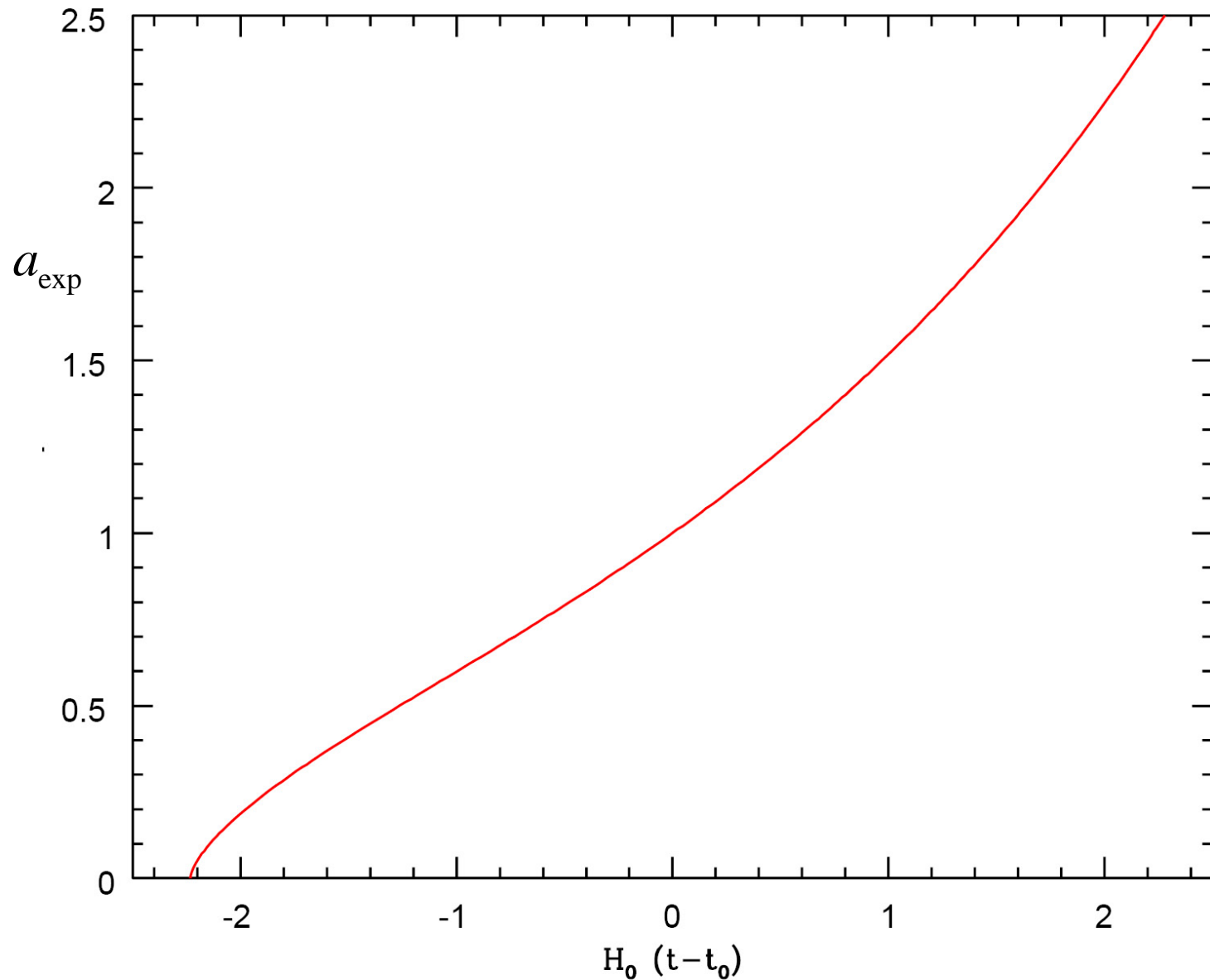
**Our Universe:**

**the Concordance Cosmos**

# Concordance Universe Parameters

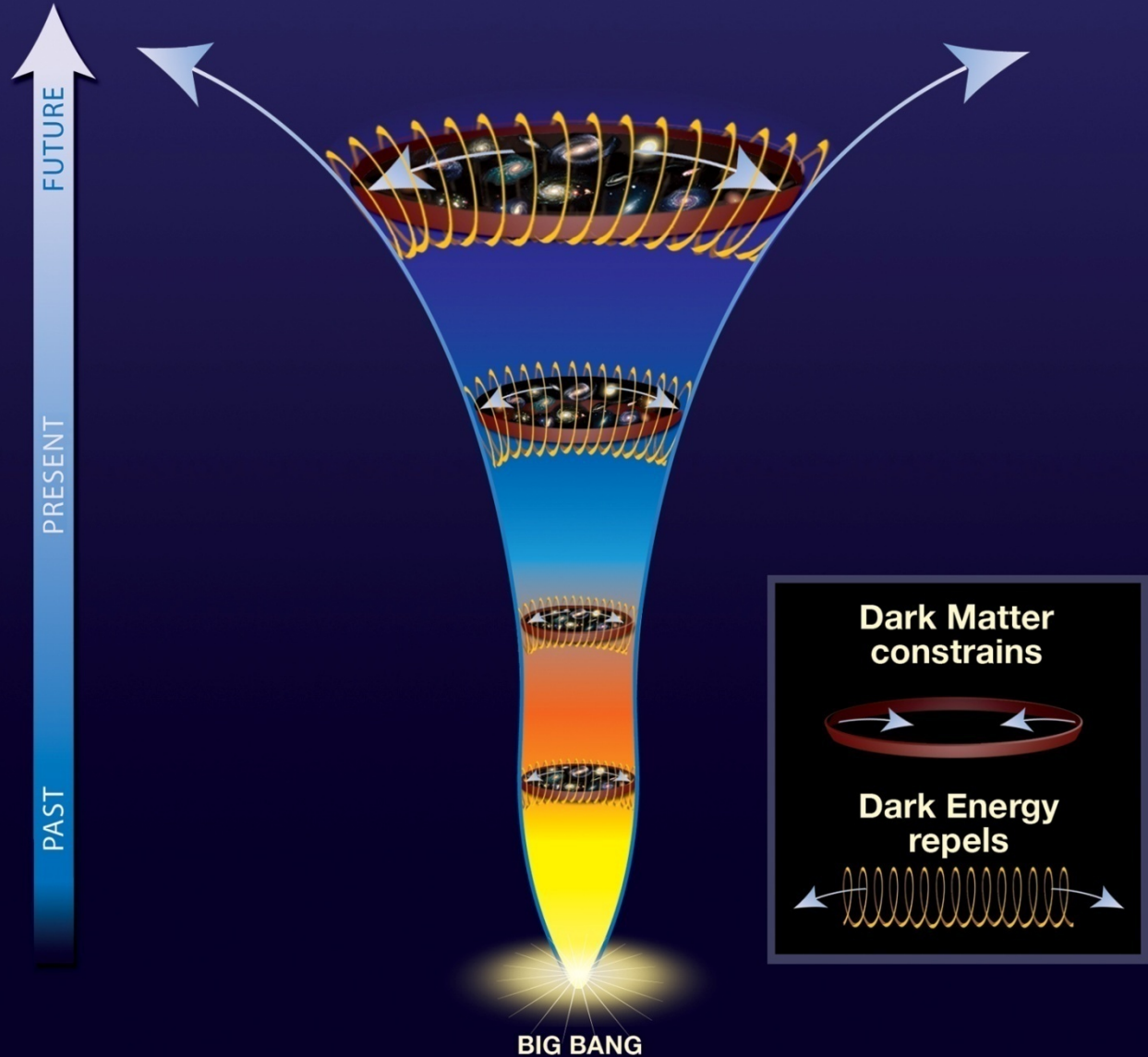
Hubble Parameter		$H_0 = 71.9 \pm 2.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$	
Age of the Universe		$t_0 = 13.8 \pm 0.1 \text{ Gyr}$	
Temperature CMB		$T_0 = 2.725 \pm 0.001 \text{ K}$	
Matter	Baryonic Matter Dark Matter	$\Omega_m = 0.27$	$\Omega_b = 0.0456 \pm 0.0015$ $\Omega_{dm} = 0.228 \pm 0.013$
Radiation	Photons (CMB) Neutrinos (Cosmic)	$\Omega_{rad} = 8.4 \times 10^{-5}$	$\Omega_\gamma = 5 \times 10^{-5}$ $\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy		$\Omega_\Lambda = 0.726 \pm 0.015$	
Total		$\Omega_{tot} = 1.0050 \pm 0.0061$	

# Concordance Expansion



# Cosmic tug of war

The force of dark energy surpasses that of dark matter as time progresses.



Heden & Toekomst:

VERSNELLING



Vroeger:

VERTRAGING



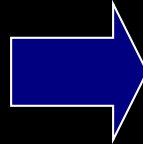
# Age of the Universe

# Hubble Time

- The repercussions of Hubble's discovery are truly tremendous: the inescapable conclusion is that the universe has a finite age !
- Just by simple extrapolation back in time we find that at some instant the objects will have touched upon each other, i.e.  $r(t_H)=0$ . If we assume for simplicity that the expansion rate did remain constant (which it did not !), we find a direct measure for the age of the universe, the

**Hubble Time:**

$$t_H = \frac{1}{H}$$



$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$



$$t_0 = 9.78h^{-1} \text{ Gyr}$$

The Hubble parameter is usually stated in units of km/s/Mpc.

It's customary to express it in units of 100 km/s/Mpc, expressing the real value in terms of the dimensionless value  $h=H_0/[100 \text{ km/s/Mpc}]$ .

The best current estimate is  $H_0=72 \text{ km/s/Mpc}$ . This sets  $t_0 \sim 10 \text{ Gyr}$ .

# Hubble Parameter

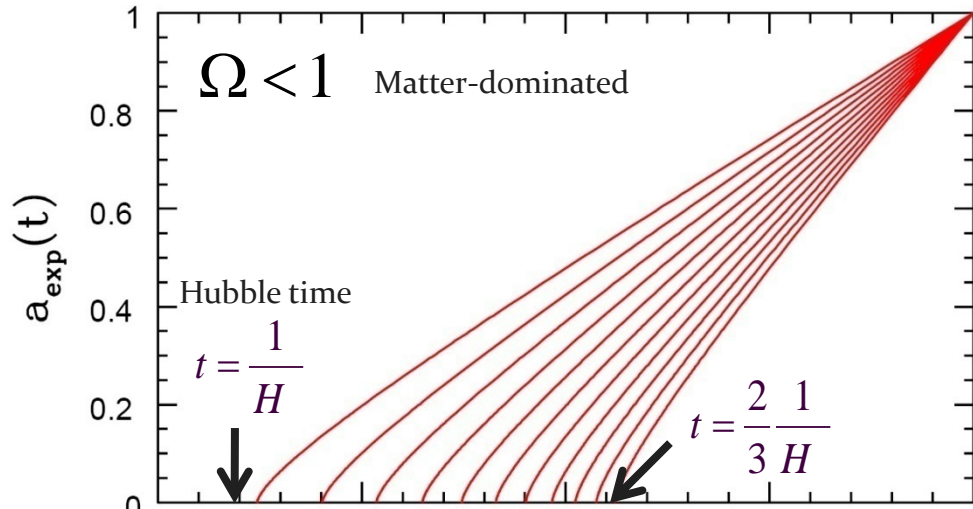
- For a long time, the correct value of the Hubble constant  $H_0$  was a major unsettled issue:

$$H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \longleftrightarrow H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

- This meant distances and timescales in the Universe had to deal with uncertainties of a factor 2 !!!
- Following major programs, such as Hubble Key Project, the Supernova key projects and the WMAP CMB measurements,

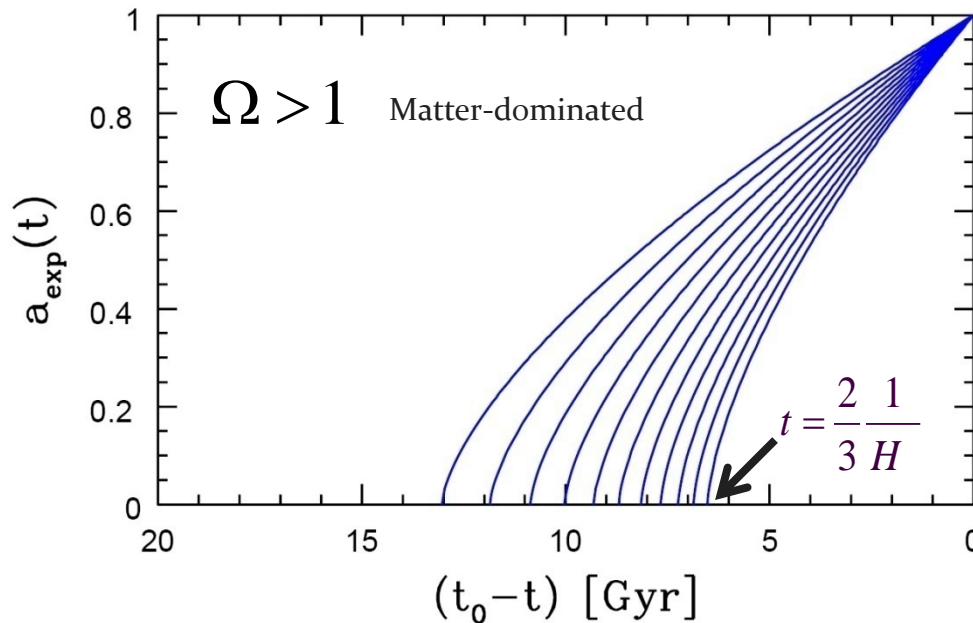
$$H_0 = 71.9^{+2.6}_{-2.7} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

# Age of the Universe



Age of a FRW universe at Expansion factor  $a(t)$

$$H t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{\text{rad}}}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2 + (1 - \Omega)}}$$





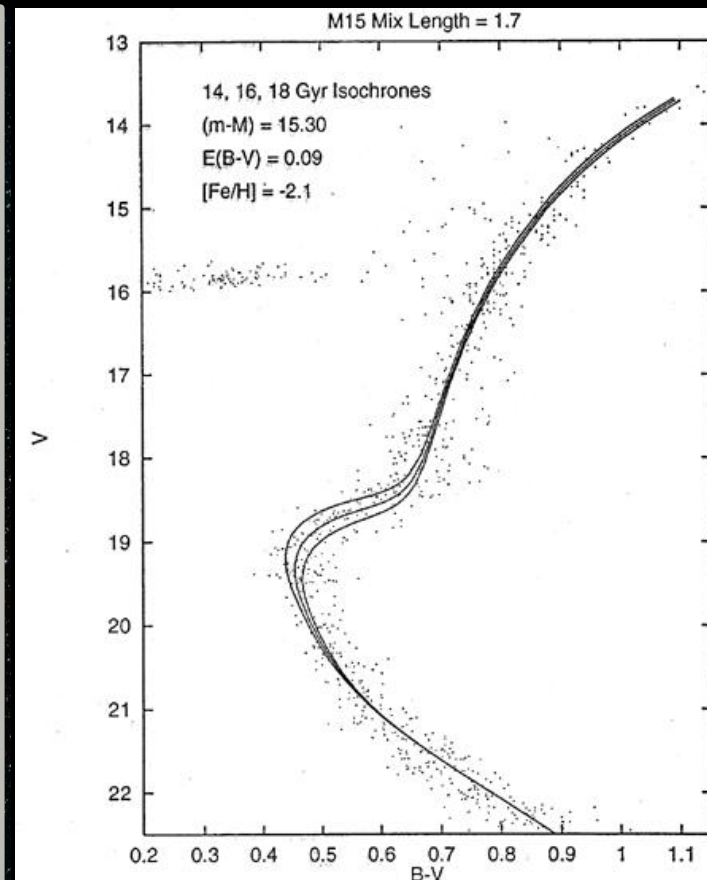
# Cosmic Age

estimated age of the oldest stars in Universe  
far in excess of estimated  
age of matter-dominated FRW Universe:

**Globular cluster stars:** 13-15 Gyr  
**Universe:** 10-12 Gyr

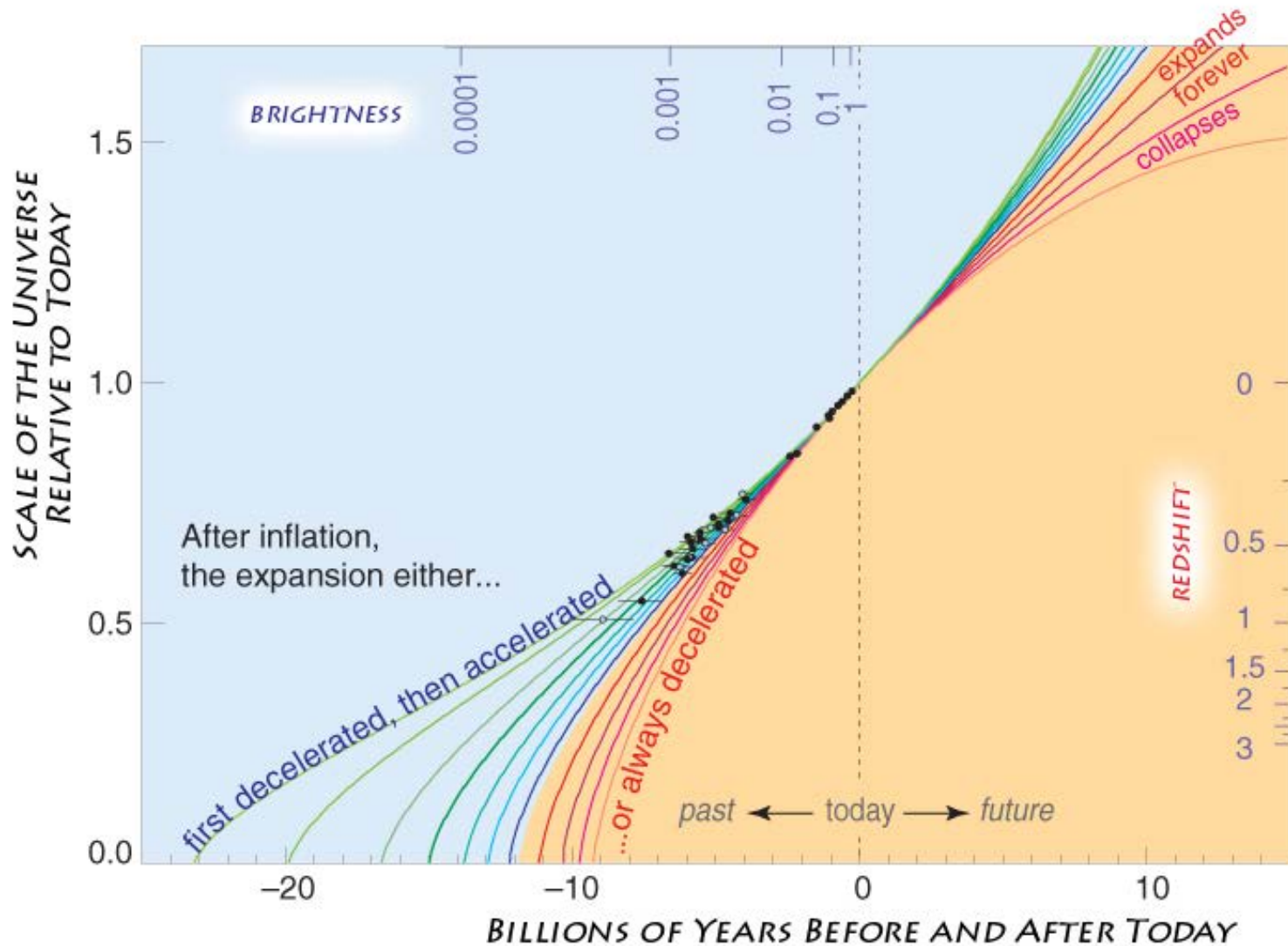
## Globular Clusters

- Roughly spherical assemblies of 100,000-200,000 stars
- Radius ~ 20-50 pc: extremely high star density
- Globulars are very old, amongst oldest objects in local Universe
- Stars formed around same time: old, red, population
- Colour-magnitude diagram characteristic:  
accurate age determination on the basis of stellar evolution theories.



Typical  
1980-1990s  
isochrone fit

# Concordance Expansion



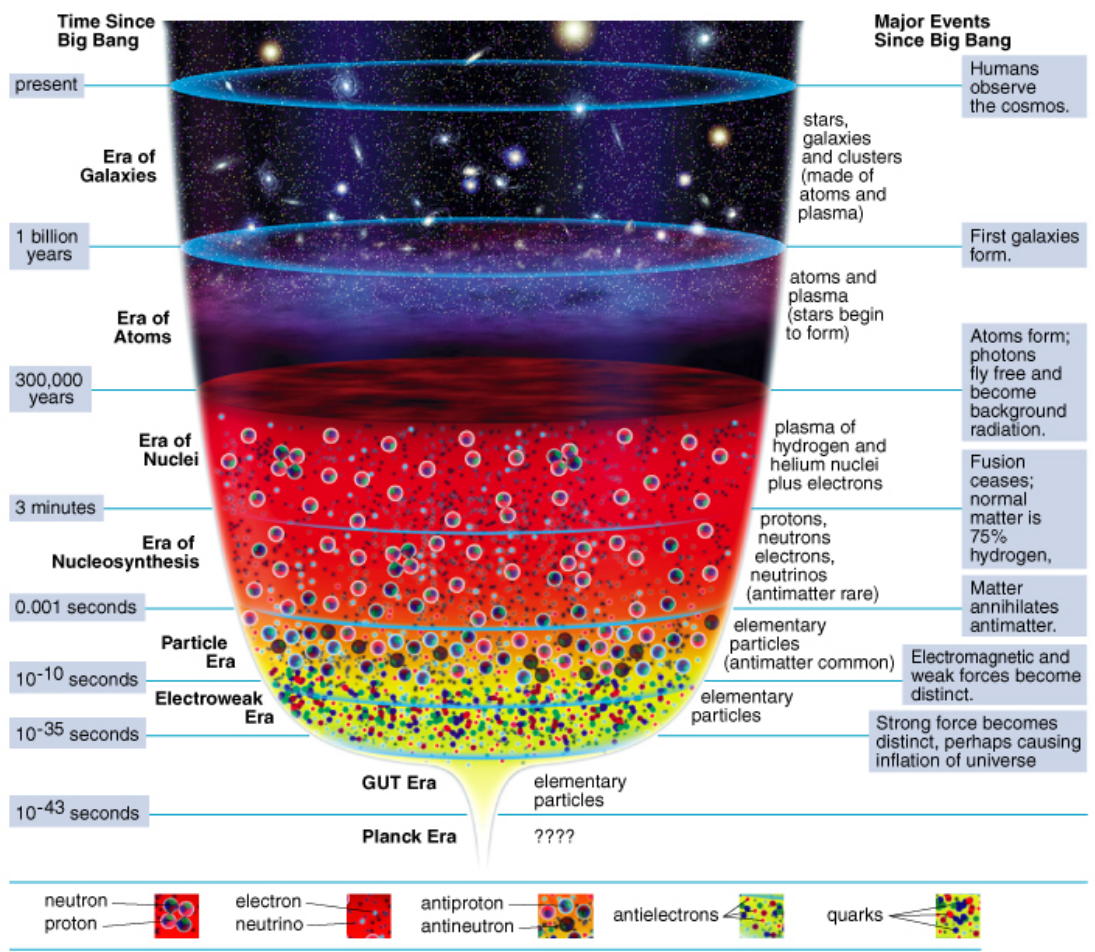
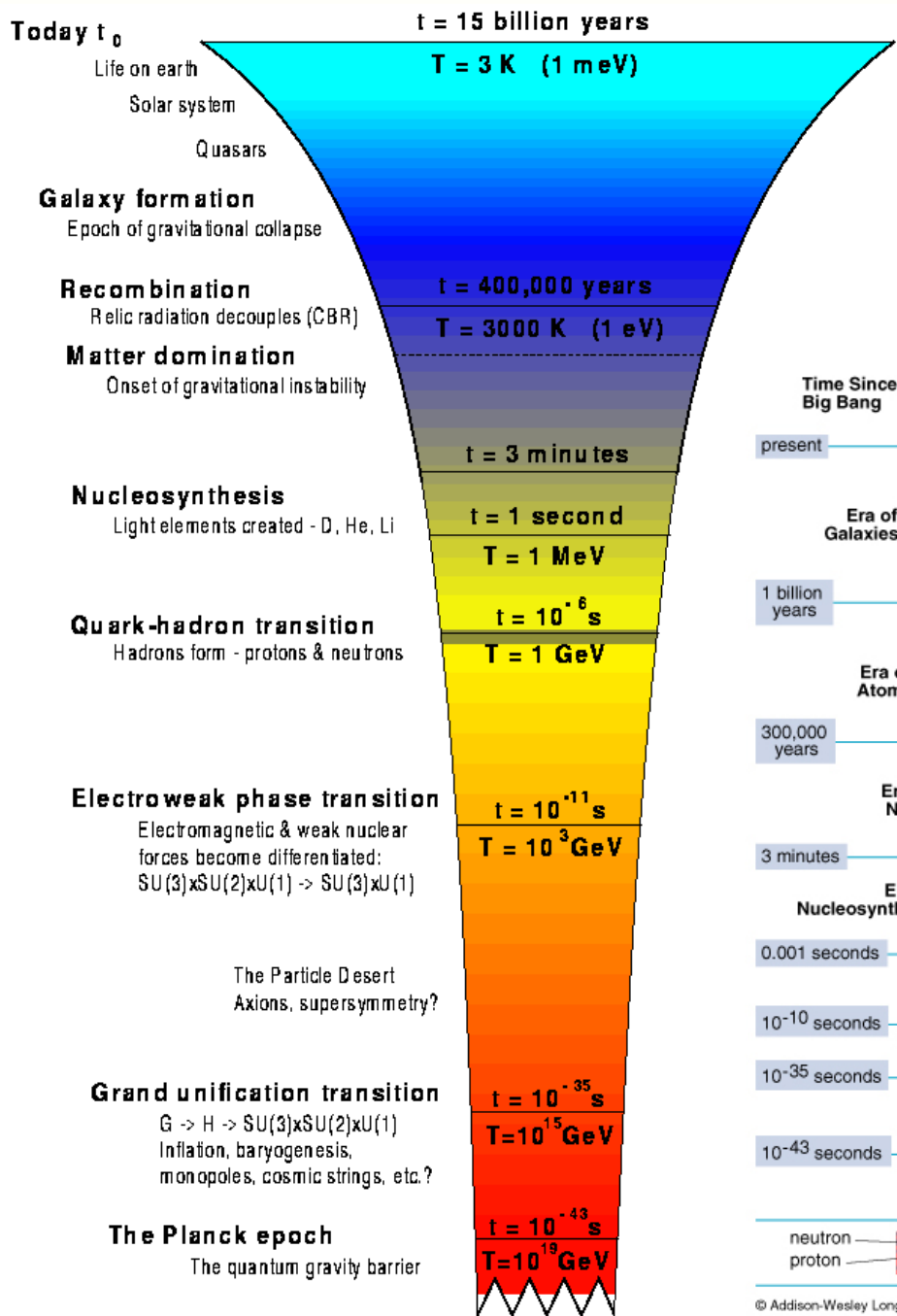
# Adiabatic Expansion

# Adiabatic Expansion

- The Universe of Einstein, Friedmann & Lemaitre expands *adiabatically*
- Energy of the expansion of the Universe corresponds to the decrease in the energy of its constituents
- *The Universe COOLS as a result of its expansion !*

$$T(t) \propto 1 / a(t)$$

# Adiabatic Expansion reconstruction Thermal History of the Universe



# Cosmic Epochs

Planck Epoch

Phase Transition Era

Hadron Era

Lepton Era

Radiation Era

Post-Recombination Era

GUT transition  
electroweak transition  
quark-hadron transition

muon annihilation  
neutrino decoupling  
electron-positron annihilation  
primordial nucleosynthesis

radiation-matter equivalence  
recombination & decoupling

Structure & Galaxy formation  
Dark Ages  
Reionization  
Matter-Dark Energy transition

$t < 10^{-43}$  sec

$10^{-43}$  sec  $< t < 10^5$  sec

$t \sim 10^{-5}$  sec

$10^{-5}$  sec  $< t < 1$  min

1 min  $< t < 379,000$  yrs

$t > 379,000$  yrs

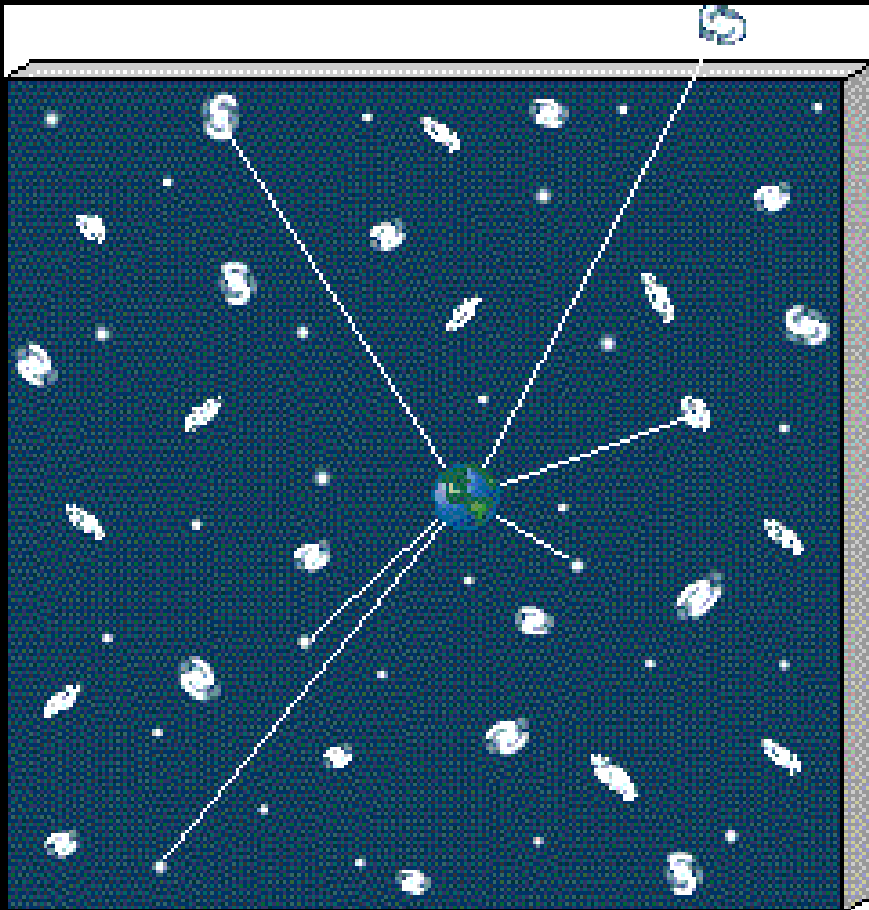
# **Big Bang: the Evidence**

# Big Bang Evidence

- Olber's paradox:  
the night sky is dark  
||→ finite age Universe (13.7 Gyr)
- Hubble Expansion  
uniform expansion, with  
expansion velocity ~ distance:  $v = H r$
- Explanation Helium Abundance 24%:  
light chemical elements formed (H, He, Li, ...)  
after ~3 minutes ...
- The Cosmic Microwave Background Radiation:  
the 2.725K radiation blanket, remnant left over  
hot ionized plasma ||→ neutral universe  
(379,000 years after Big Bang)
- Distant, deep Universe indeed looks different ...



# 1. Olber's Paradox

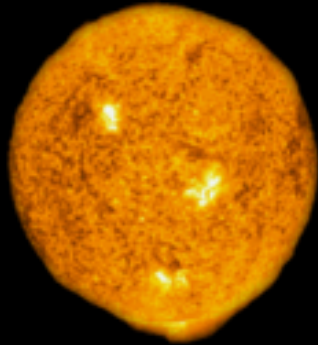


In an infinitely large, old and unchanging Universe each line of sight would hit a star:



Sky would be as bright as surface of star:

# 1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:



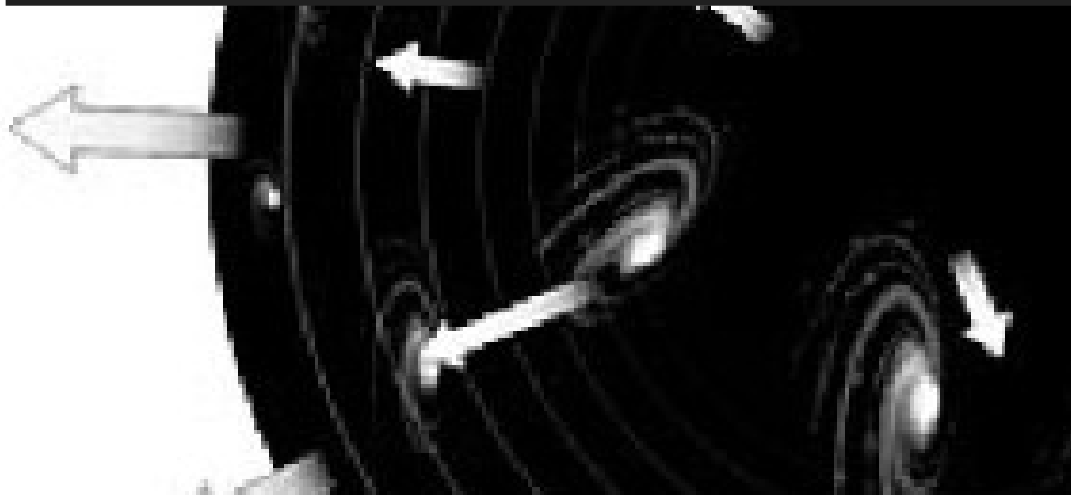
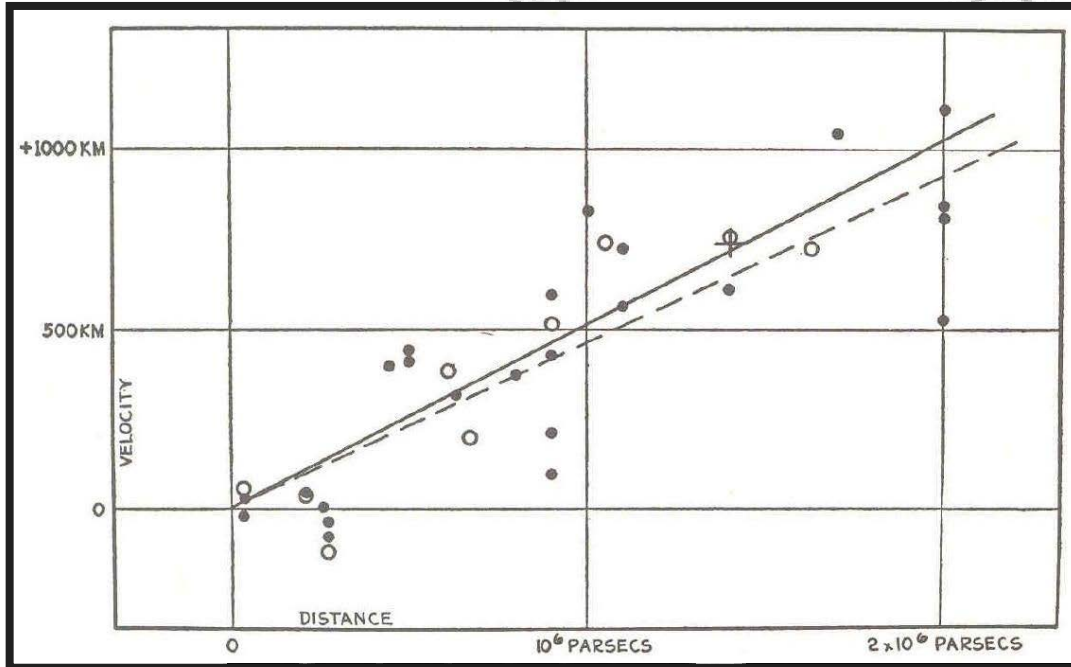
Sky would be as bright as surface of star:

Night sky as bright as  
Solar Surface, yet  
the night sky is dark



finite age of Universe (13.8 Gyr)

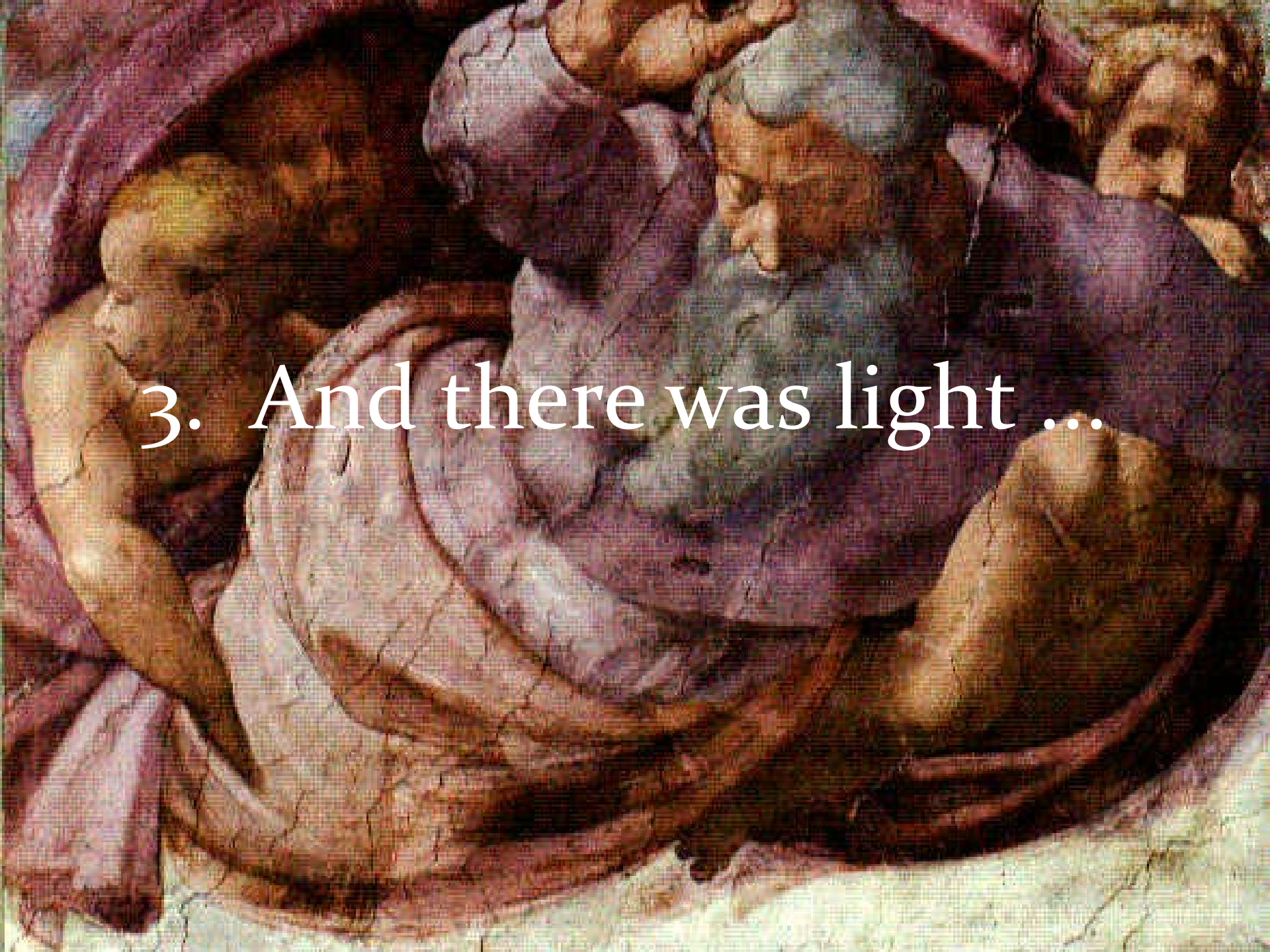
# 2. Hubble Expansion



$$v_{rad} = cz = H_0 r$$

$H_0$  : Hubble constant

specifies expansion rate  
of the Universe

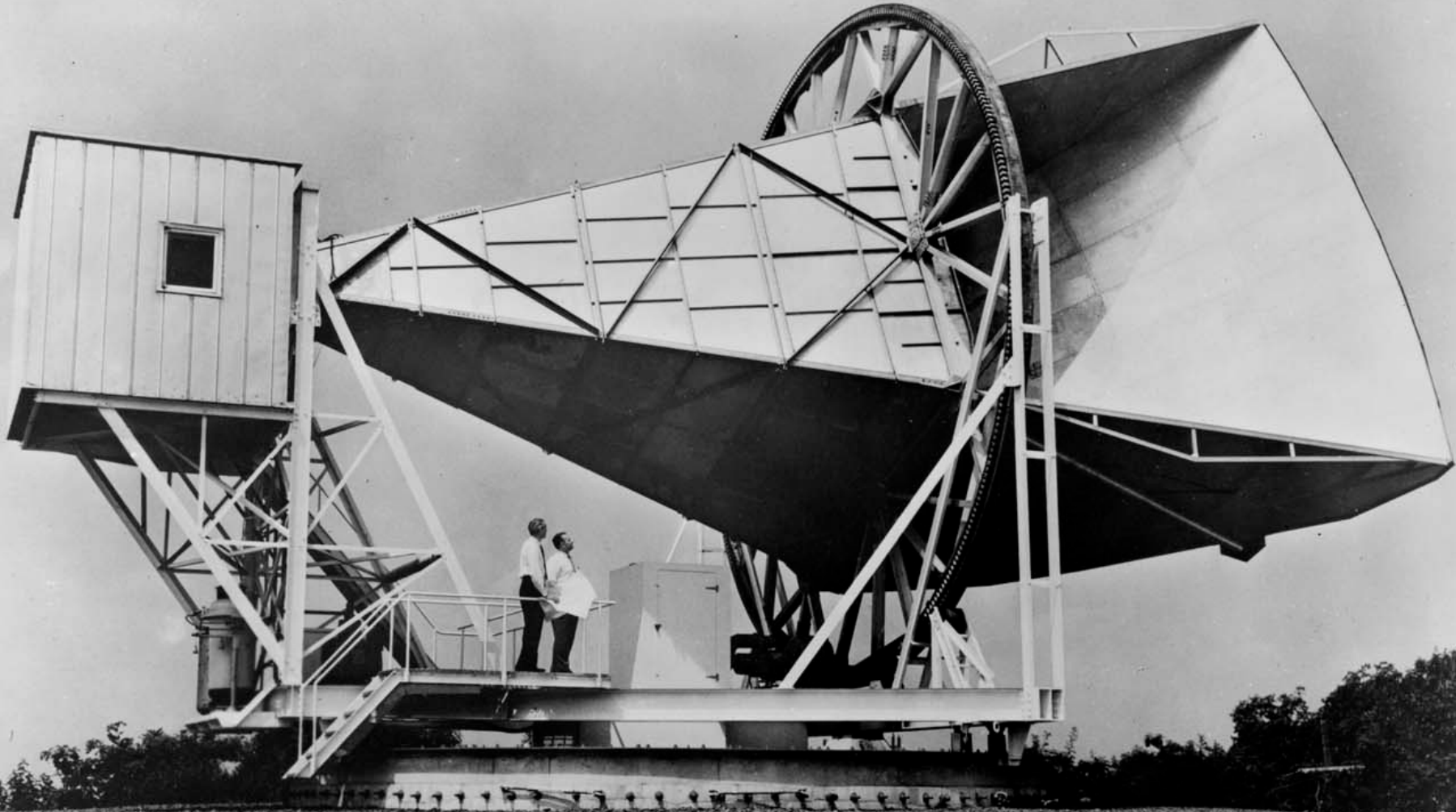


3. *And there was light ...*

... and there was light ...

379.000 years  
after the Big Bang

# 3. Cosmic Microwave Background Radiation

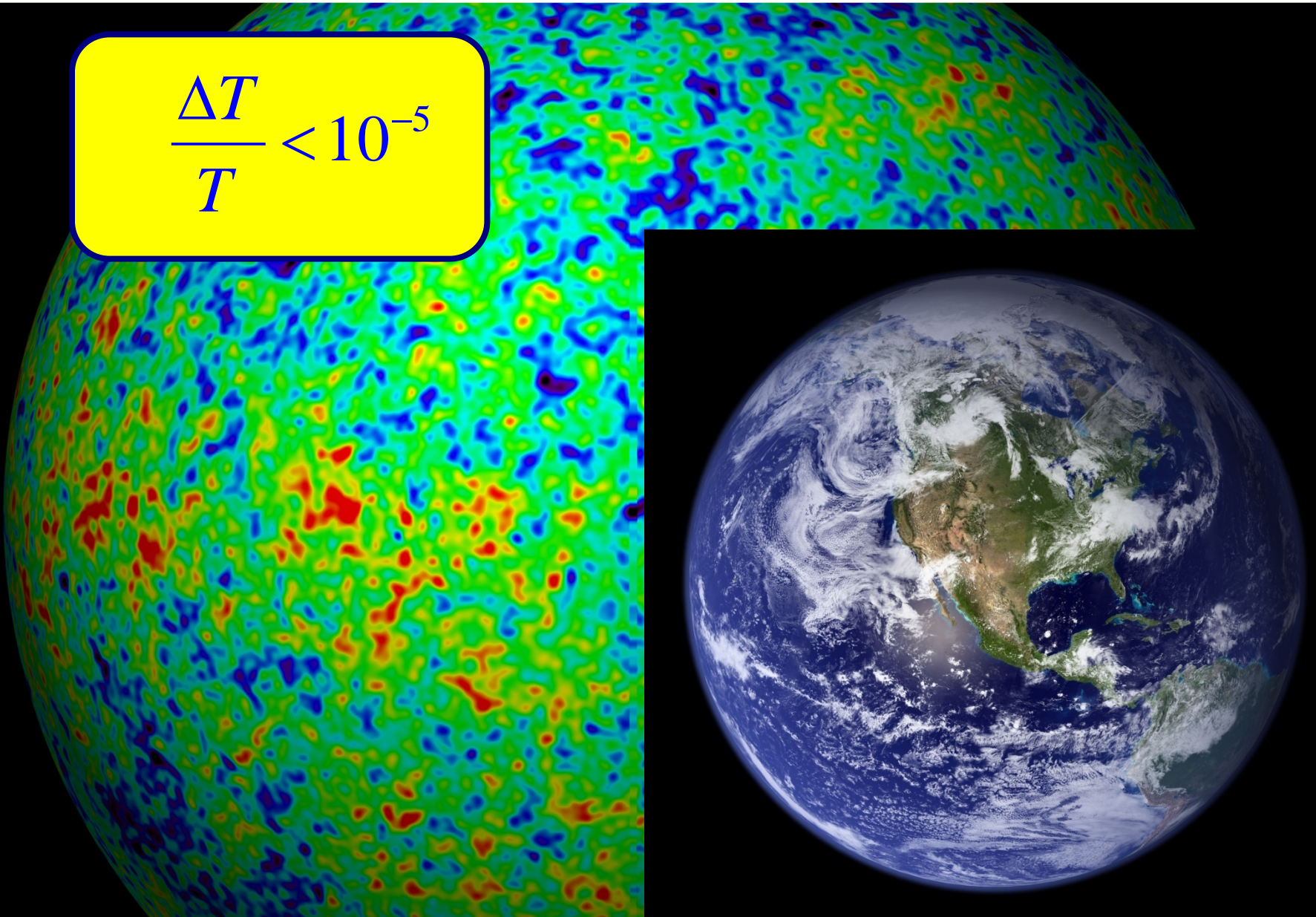


# Cosmic Light (CMB): the facts

- ❑ Discovered serendipitously in 1965  
**Penzias & Wilson,  
Nobelprize 1978 !!!!!**
- ❑ Cosmic Light that fills up the Universe uniformly
- ❑ Temperature:  **$T_\gamma = 2.725 \text{ K}$**
- ❑ (CMB) photons most abundant particle in the Universe:  
 **$n_\gamma \sim 415 \text{ cm}^{-3}$**
- ❑ Per atom in the Universe:  **$n_\gamma/n_B \sim 1.9 \times 10^9$**
- ❑ **Ultimate evidence of the Big Bang !!!!!!!!!!!!!!!!!!!!!!!**

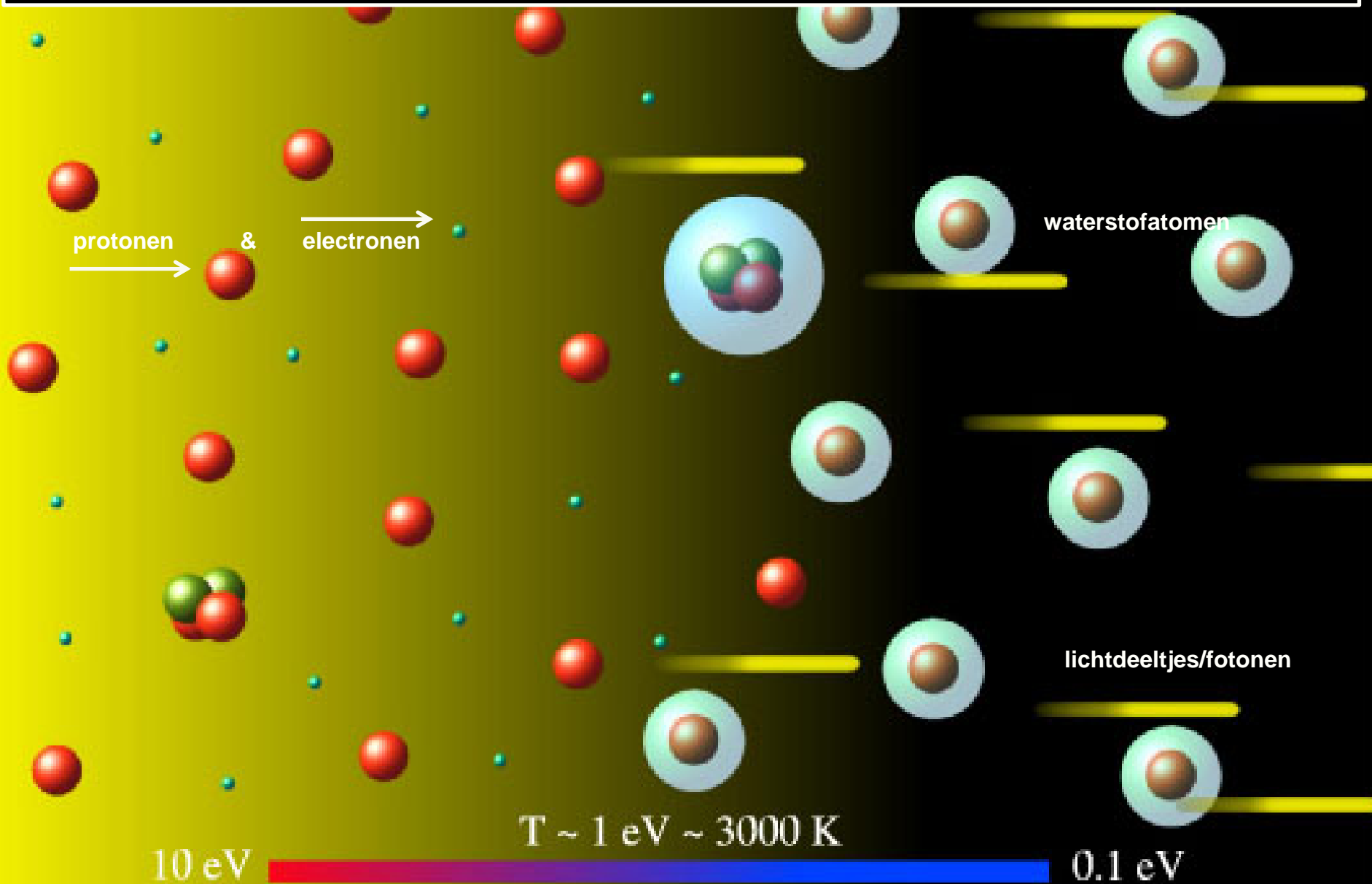
# Extremely Smooth Radiation Field

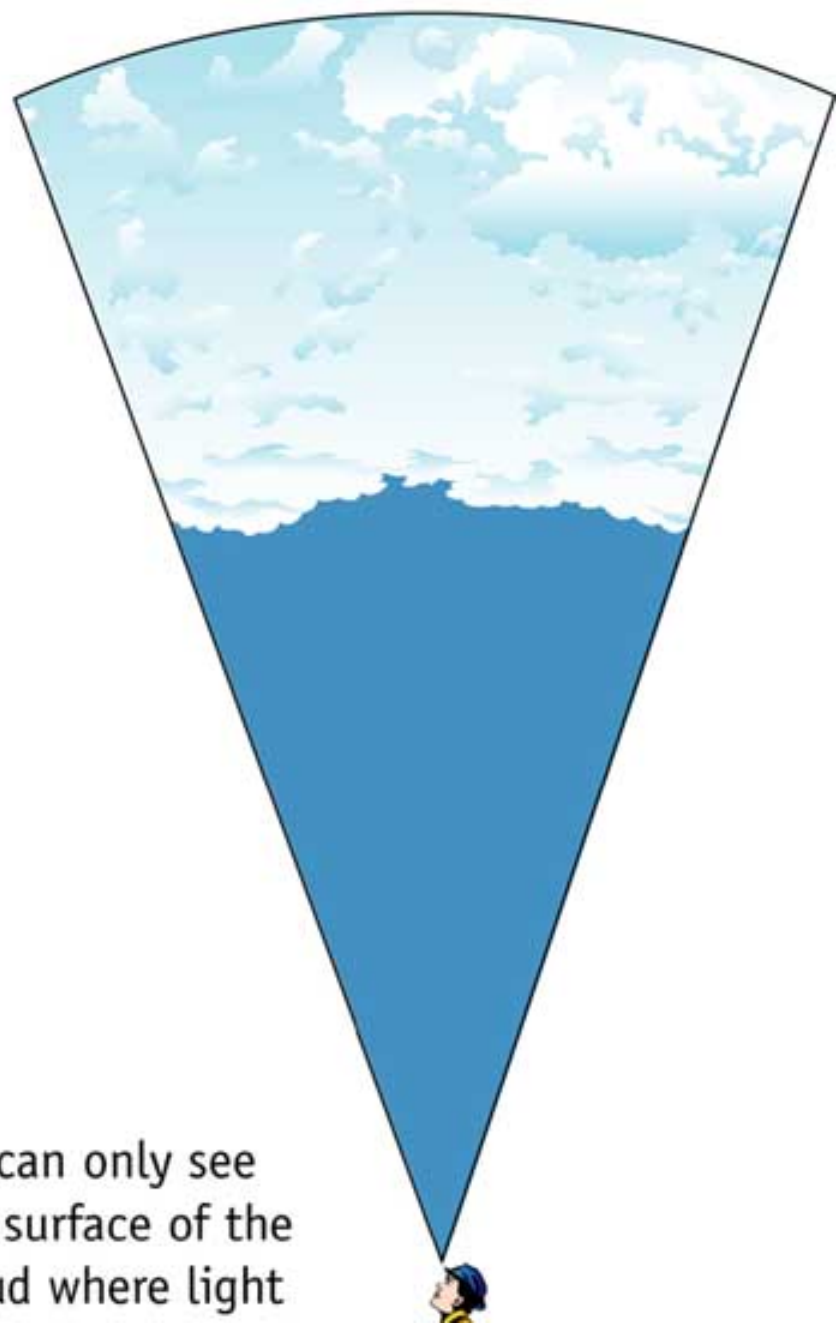
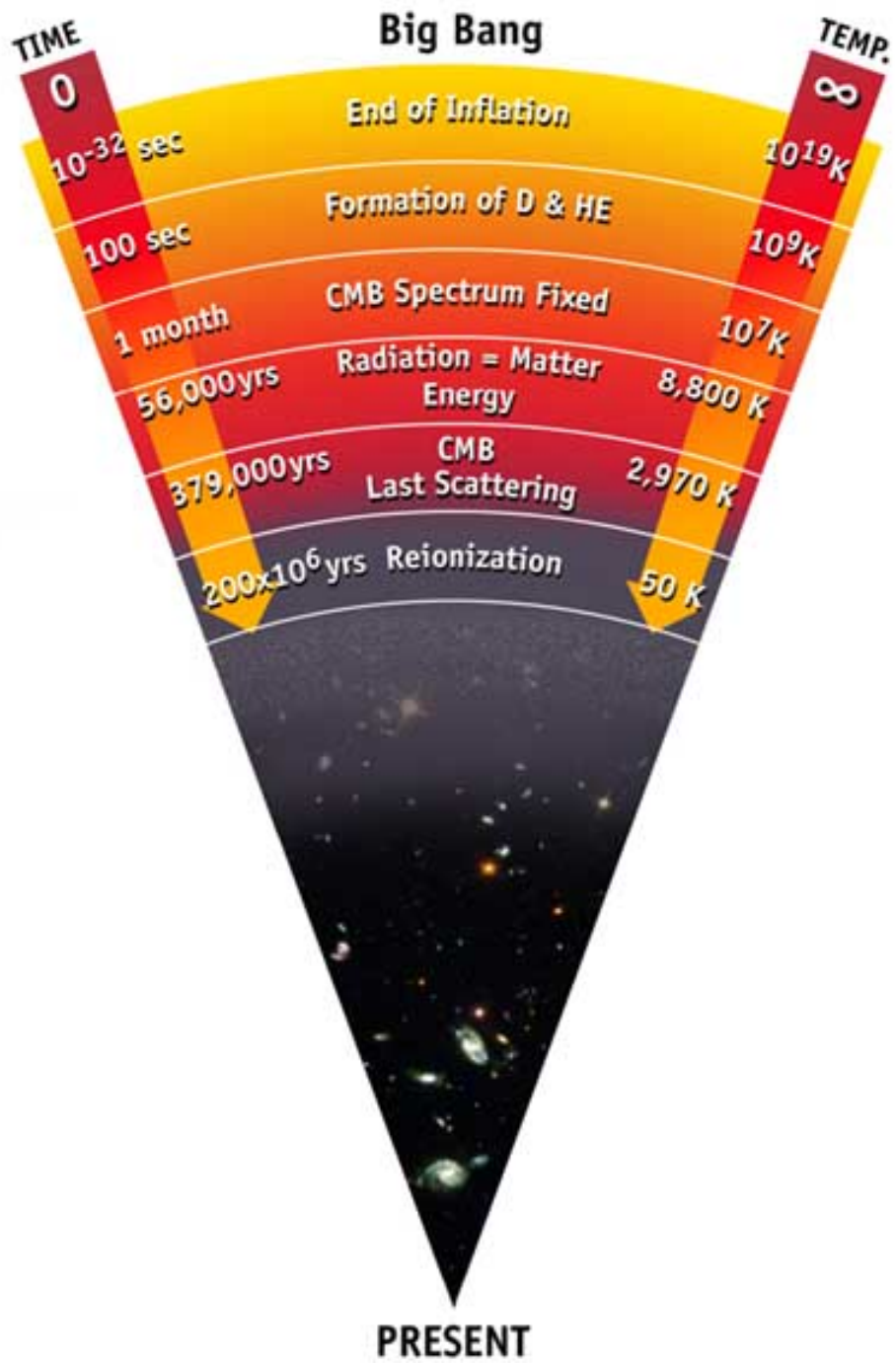
$$\frac{\Delta T}{T} < 10^{-5}$$





# Recombination & Decoupling





We can only see the surface of the cloud where light

# the Cosmic TV Show



## Note:

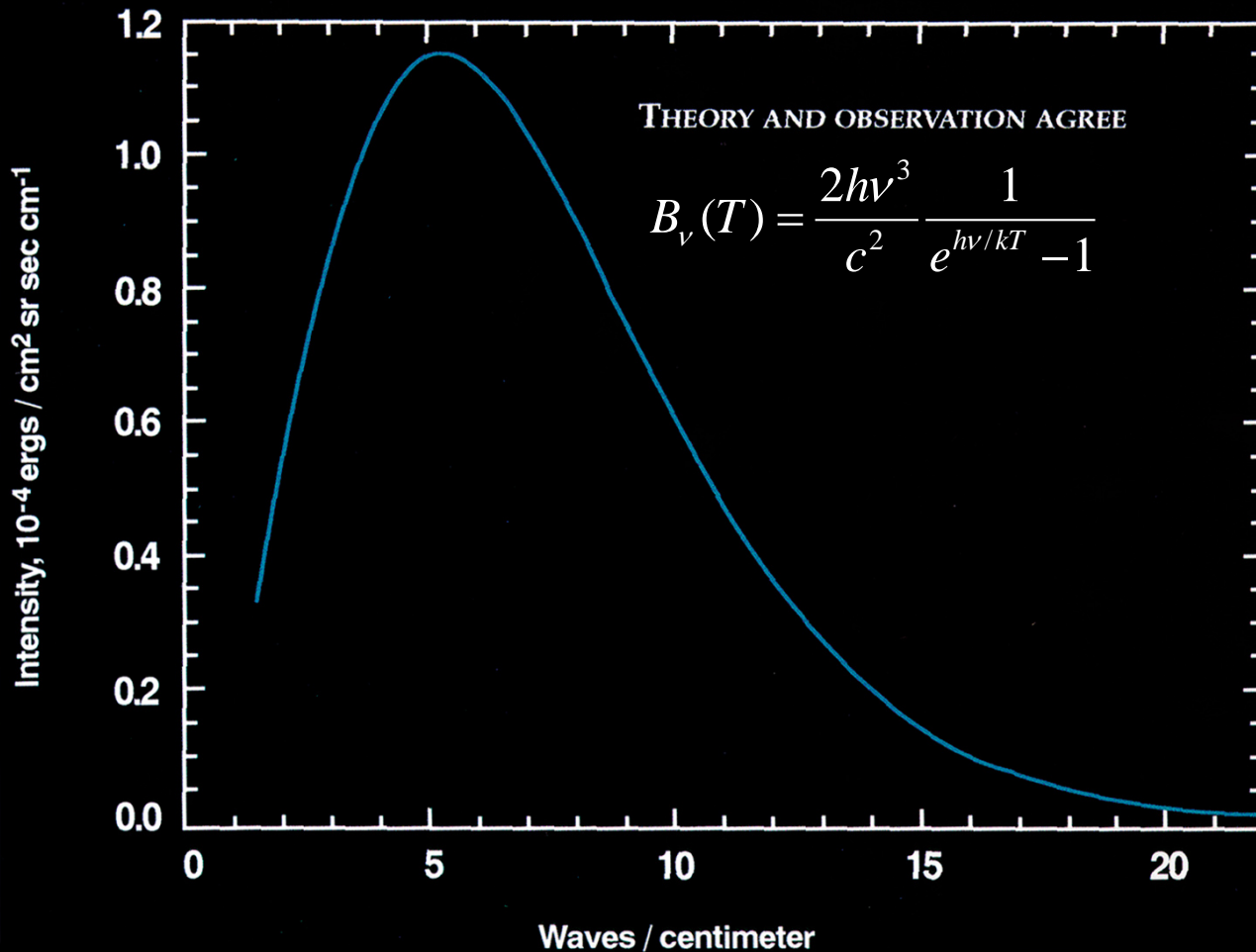
far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your (camping) tv set ...

!!!! Live broadcast from the Big Bang !!!!

Courtesy: W. Hu

# Energy Spectrum Cosmic Light

## COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE



☐ COBE-DIRBE:

temperature  $T = 2.725$  K

• John Mather

Nobel prize physics

2006

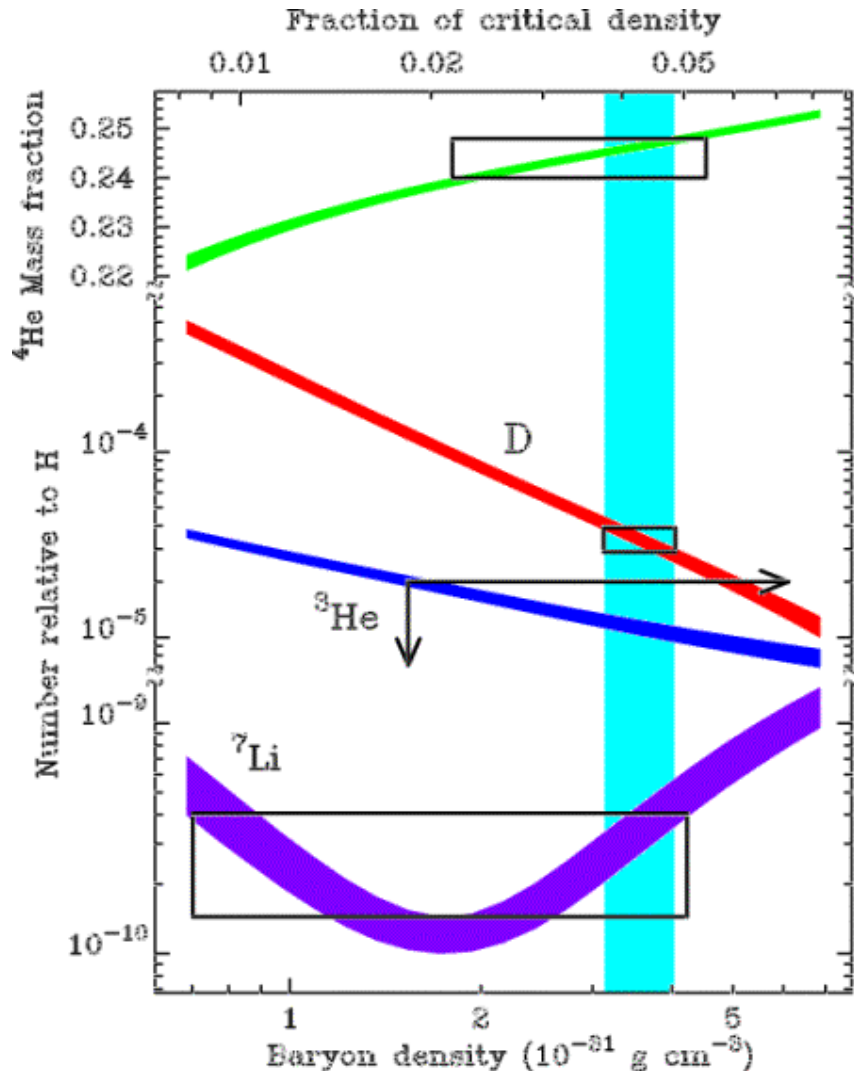
☐ Most perfect

Black Body

Spectrum ever seen !!!!

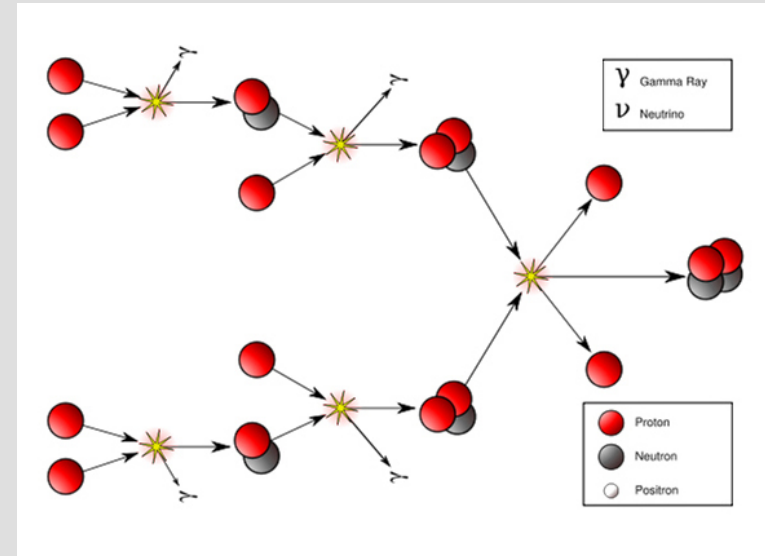
# 4. Proton-Neutron & Helium

p/n ~1/7: 1 min na BB



## Mass Fraction Light Elements

24%  $^4\text{He}$  nuclei  
traces D,  $^3\text{He}$ ,  $^7\text{Li}$  nuclei  
75% H nuclei (protons)

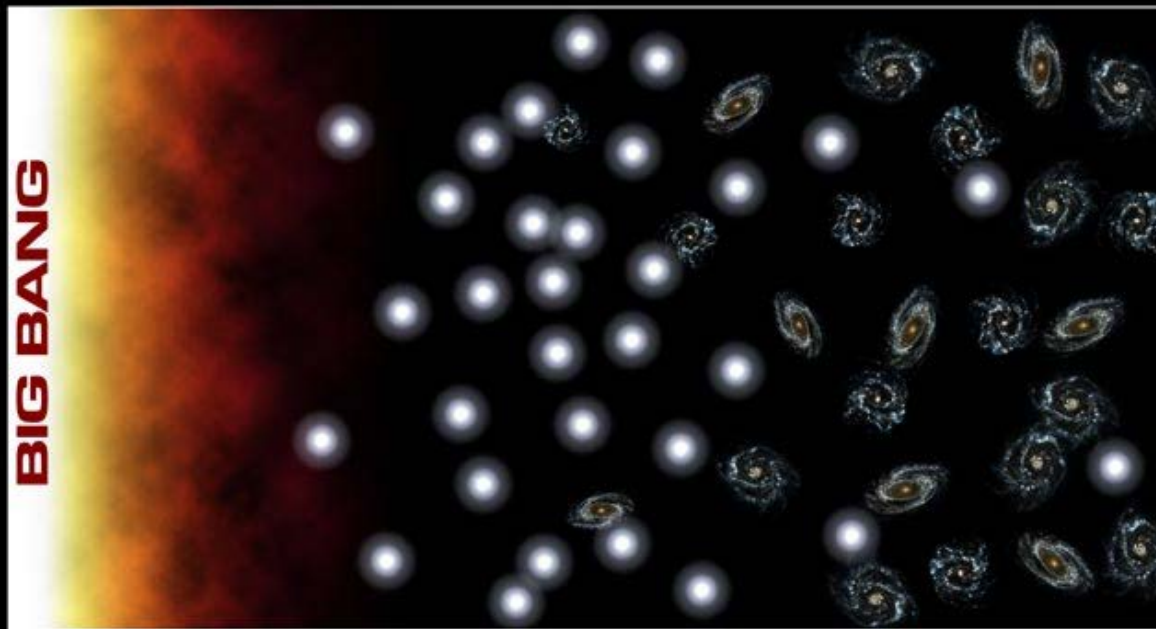


Between 1-200 seconds after Big Bang, temperature dropped to  $10^9$  K:

Fusion protons & neutrons into light atomic nuclei

# 5. the Changing Universe

Timeline of the Universe



Early Universe

Galaxy Formation

Today

Newborn Galaxies

Normal Galaxies

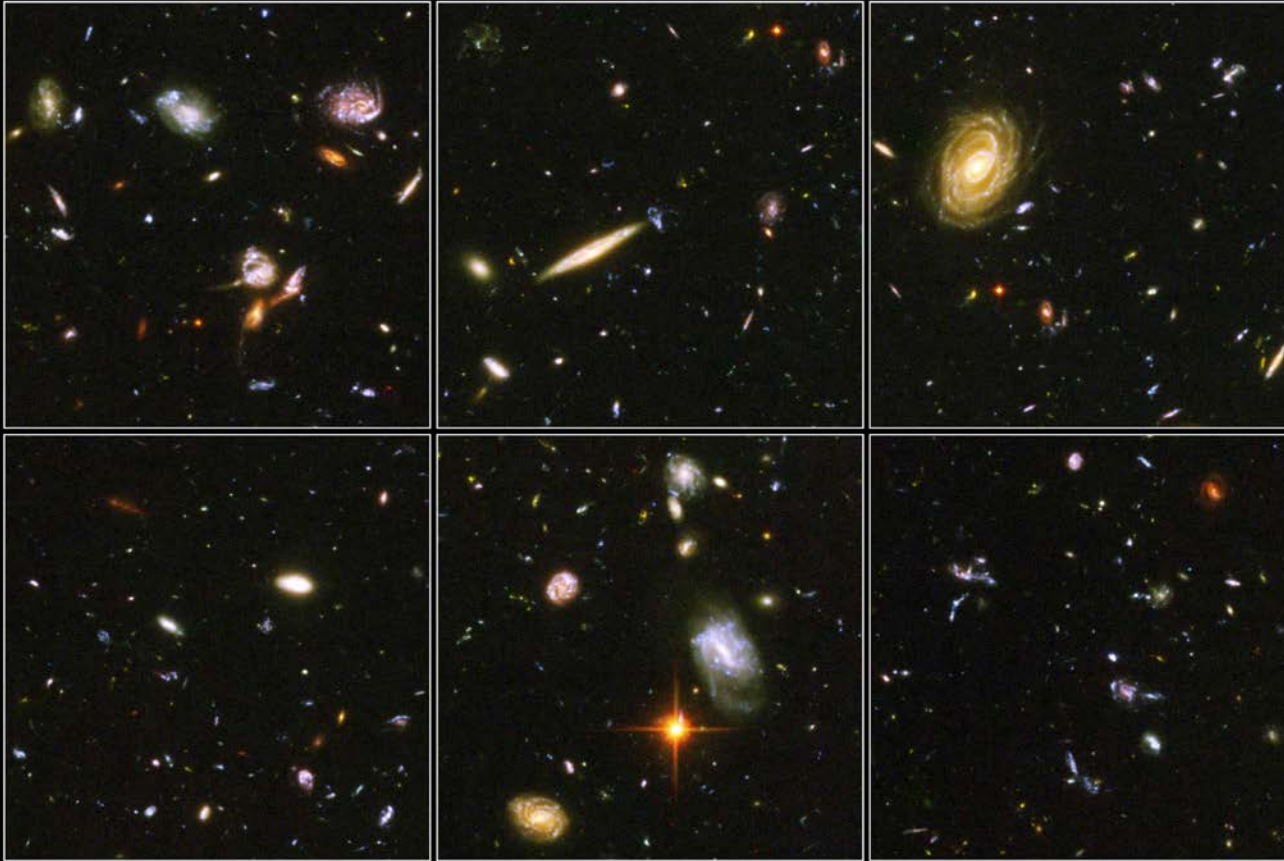
At great depths  
the Universe  
looks completely  
different

- and thus  
long ago :  
Depth= Time



Galaxies in  
Hubble Ultra  
Deep Field

# 5. the Changing Universe



**Hubble Ultra Deep Field Details**  
Hubble Space Telescope • Advanced Camera for Surveys

At great depths  
the Universe  
looks completely  
different

- and thus  
long ago :  
Depth= Time



Galaxies in  
Hubble Ultra  
Deep Field

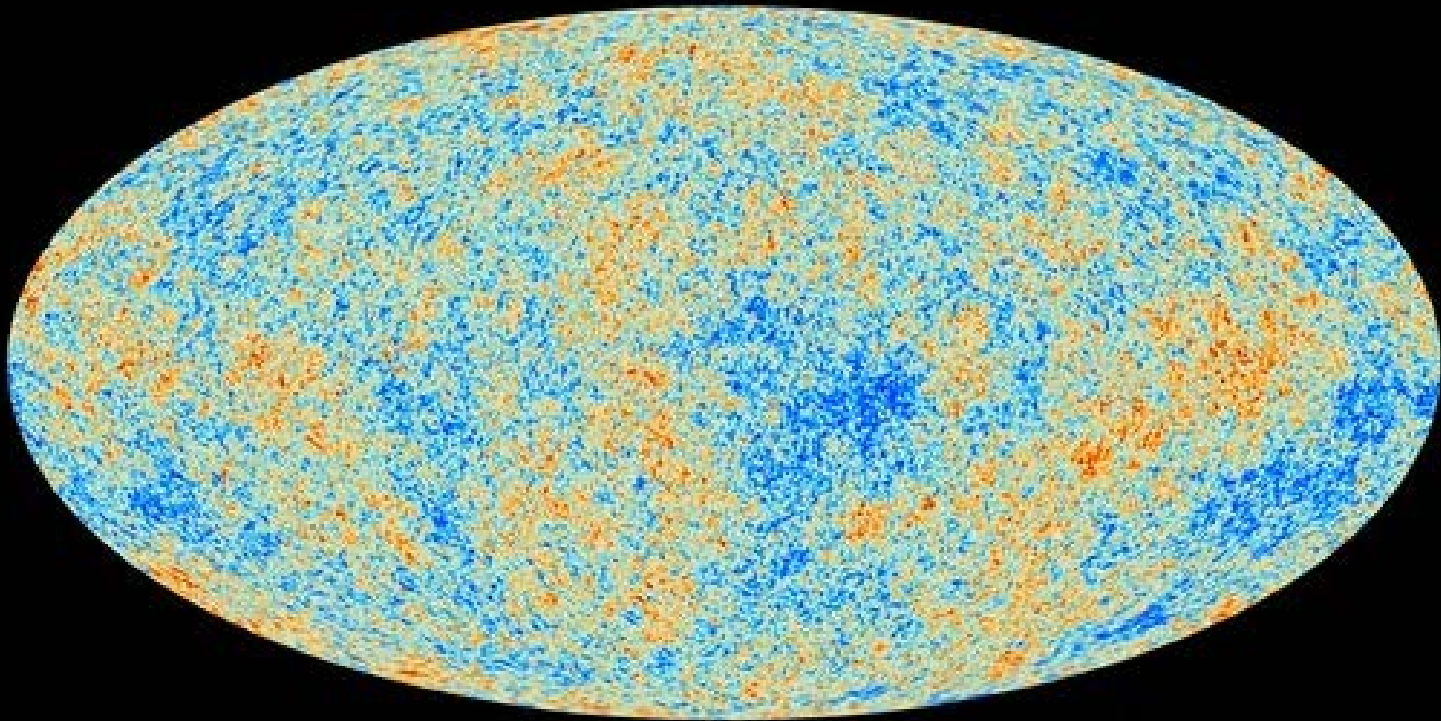
# Cosmic Curvature



**How Much ?**

**Cosmic Curvature**

# Cosmic Microwave Background



**Map of the Universe at Recombination Epoch (Planck, 2013):**

☐ **379,000 years after Big Bang**

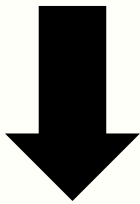
☐ **Subhorizon perturbations: primordial sound waves**

☐  **$\Delta T/T < 10^{-5}$**

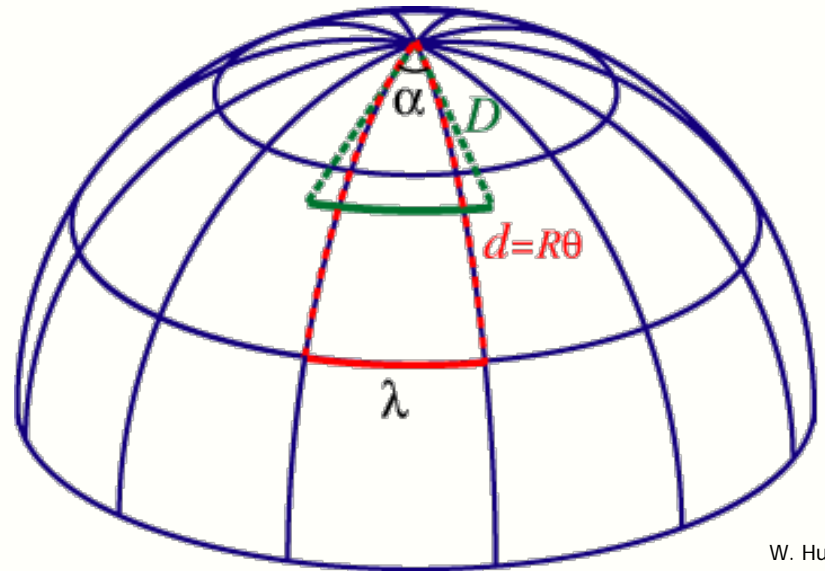
# Measuring Curvature

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space



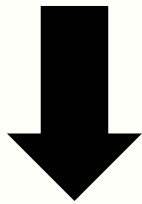
**Geometry of Space**



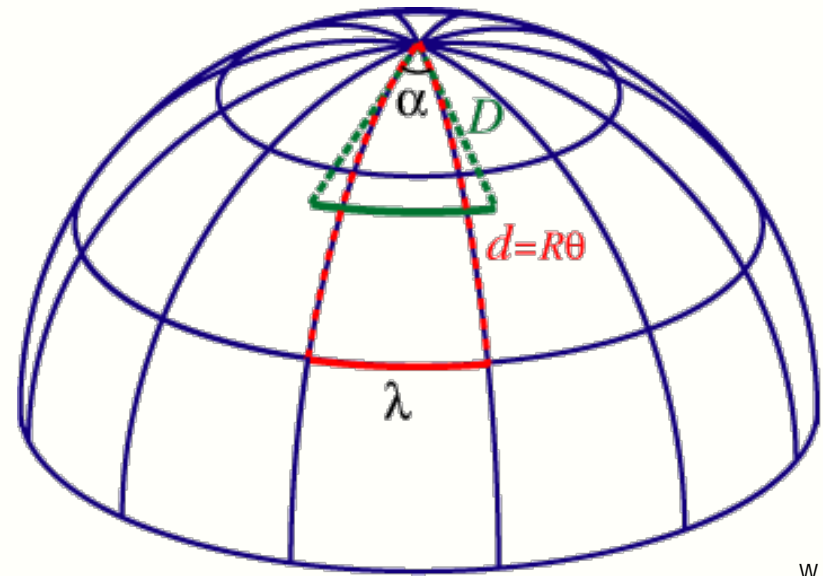
W. Hu

# Measuring Curvature

- Object with known physical size, at large cosmological distance:
- Sound Waves in the Early Universe !!!!

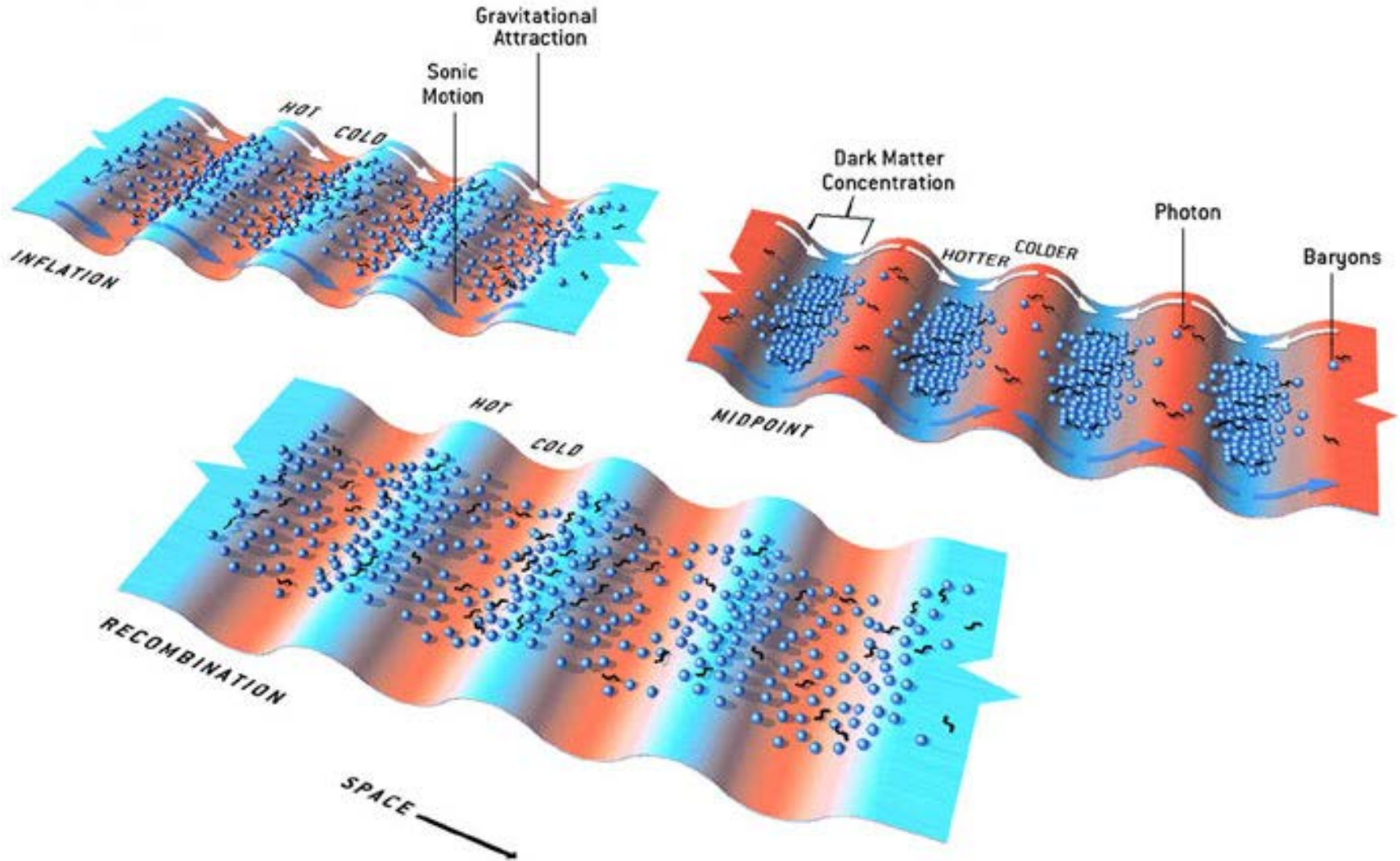


Temperature Fluctuations  
CMB



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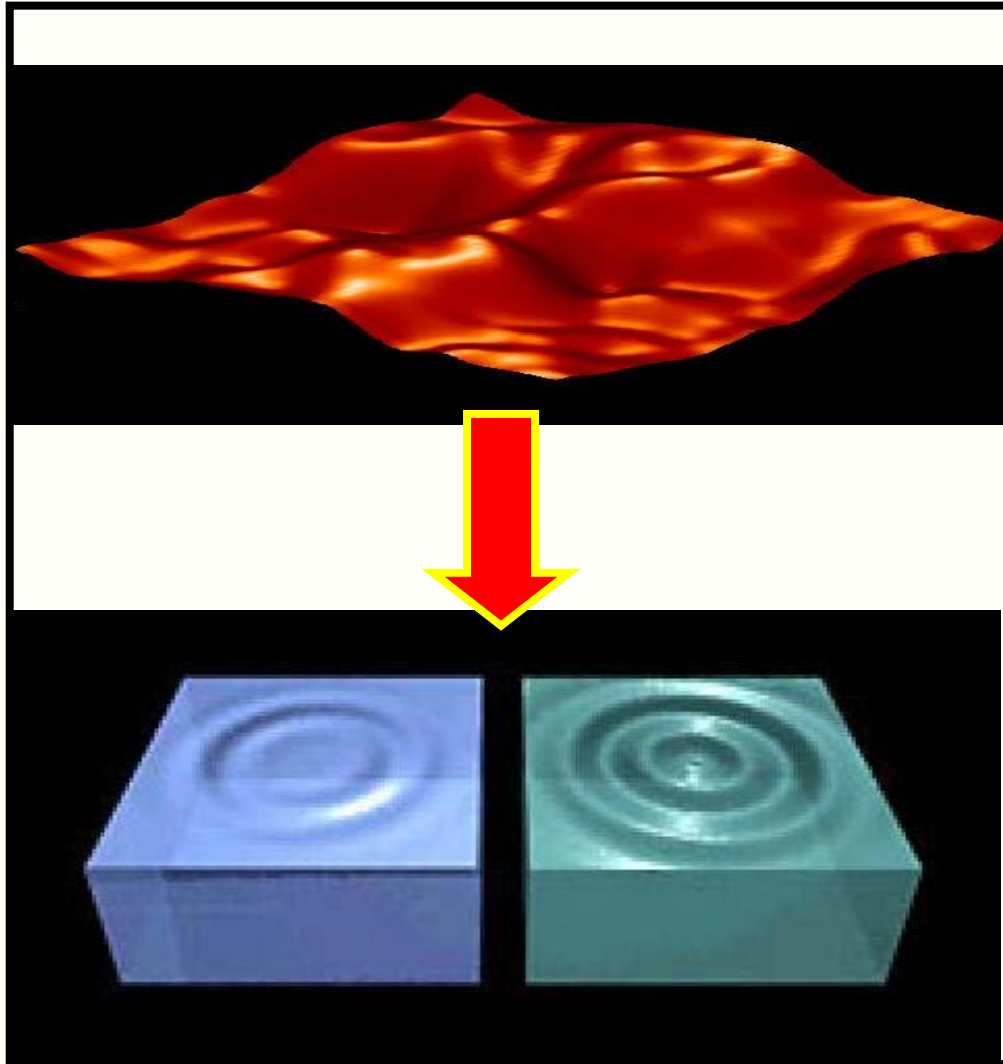
# Fluctuations-Origin



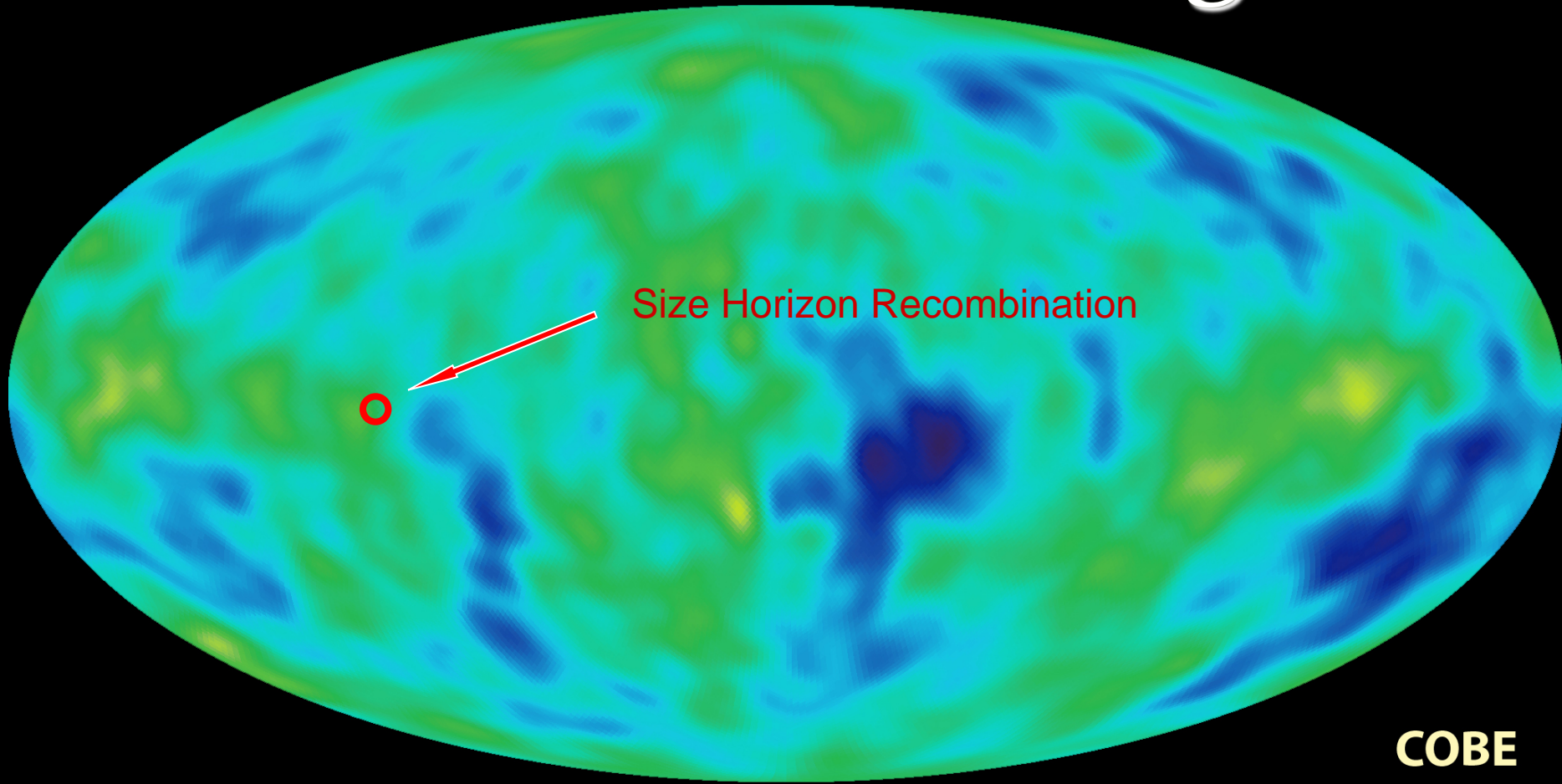
# Music of the Spheres

- small ripples in primordial matter & photon distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists
- compressions and rarefactions in photon gas: sound waves
- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of “instrument”:
  - (sound) horizon size last scattering
- Observed, angular size:  $\theta \sim 1^\circ$ 
  - exact scale maximum compression, the “cosmic fundamental mode of music”

W. Hu



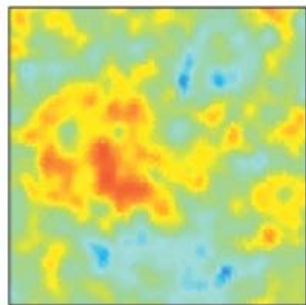
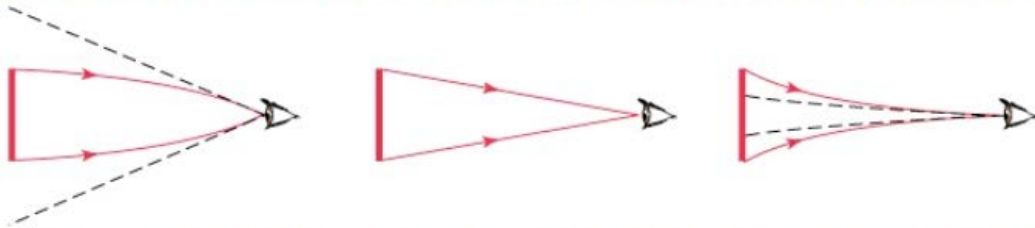
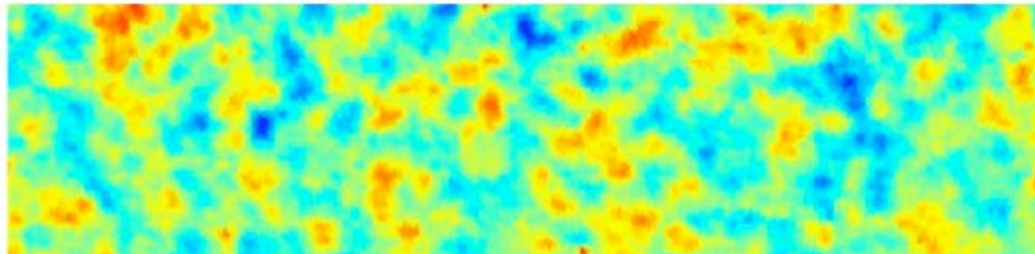
# Cosmic Microwave Background



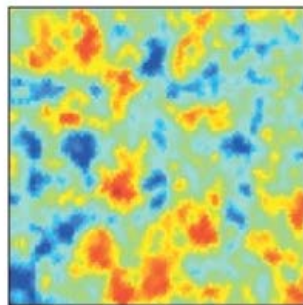
COBE measured fluctuations:  $> 7^\circ$   
Size Horizon at Recombination spans angle  $\sim 1^\circ$

# Flat universe from CMB

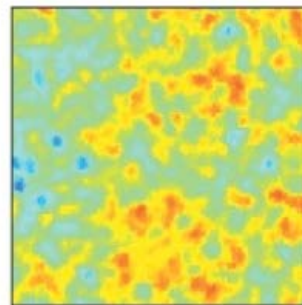
- **First peak: flat universe**



Closed:  
hot spots  
appear larger



Flat:  
appear as big  
as they are



Open:  
spots appear  
smaller

We know the redshift and the time it took for the light to reach us:

from this we know the

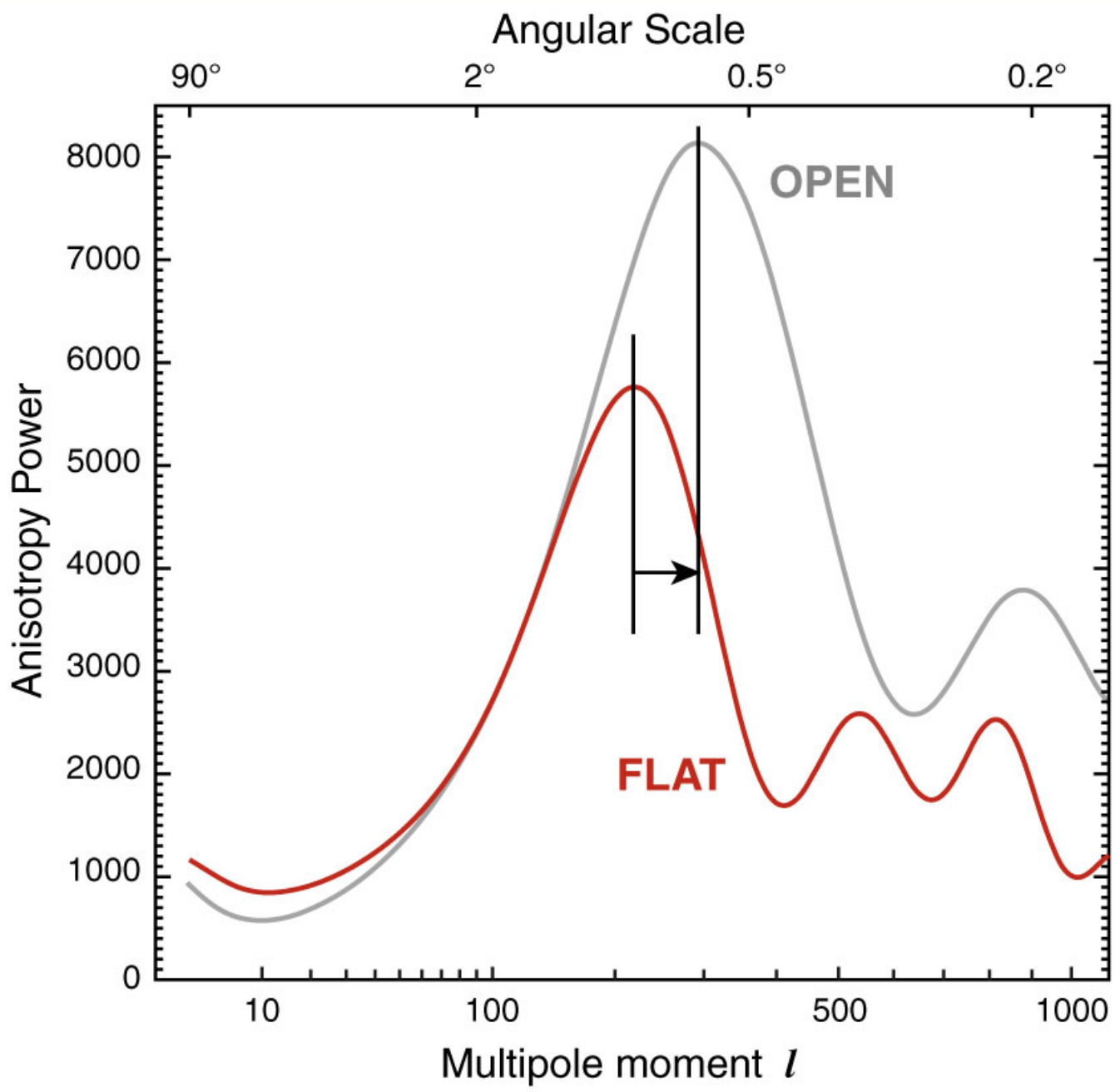
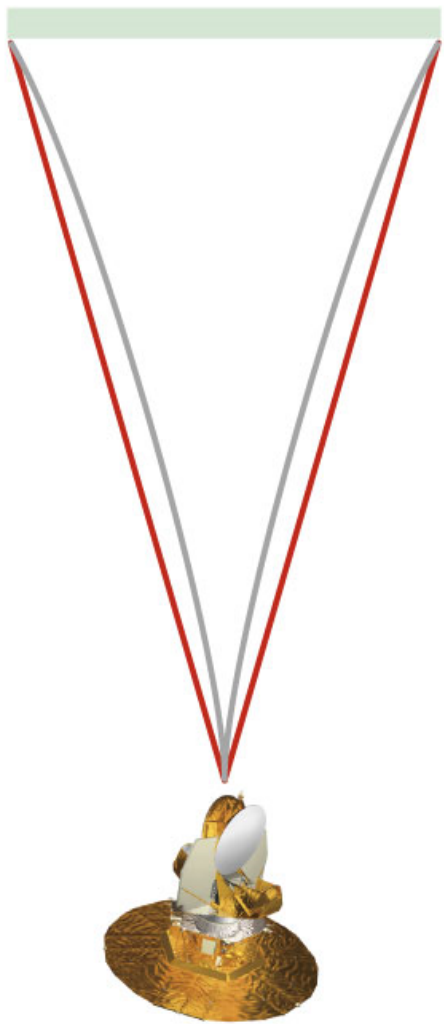
- length of the legs of the triangle
- the angle at which we are measuring the sound horizon.

$$v \approx \frac{c}{\sqrt{3}}$$

$$l \approx 200 / \sqrt{1 - \Omega_k}$$

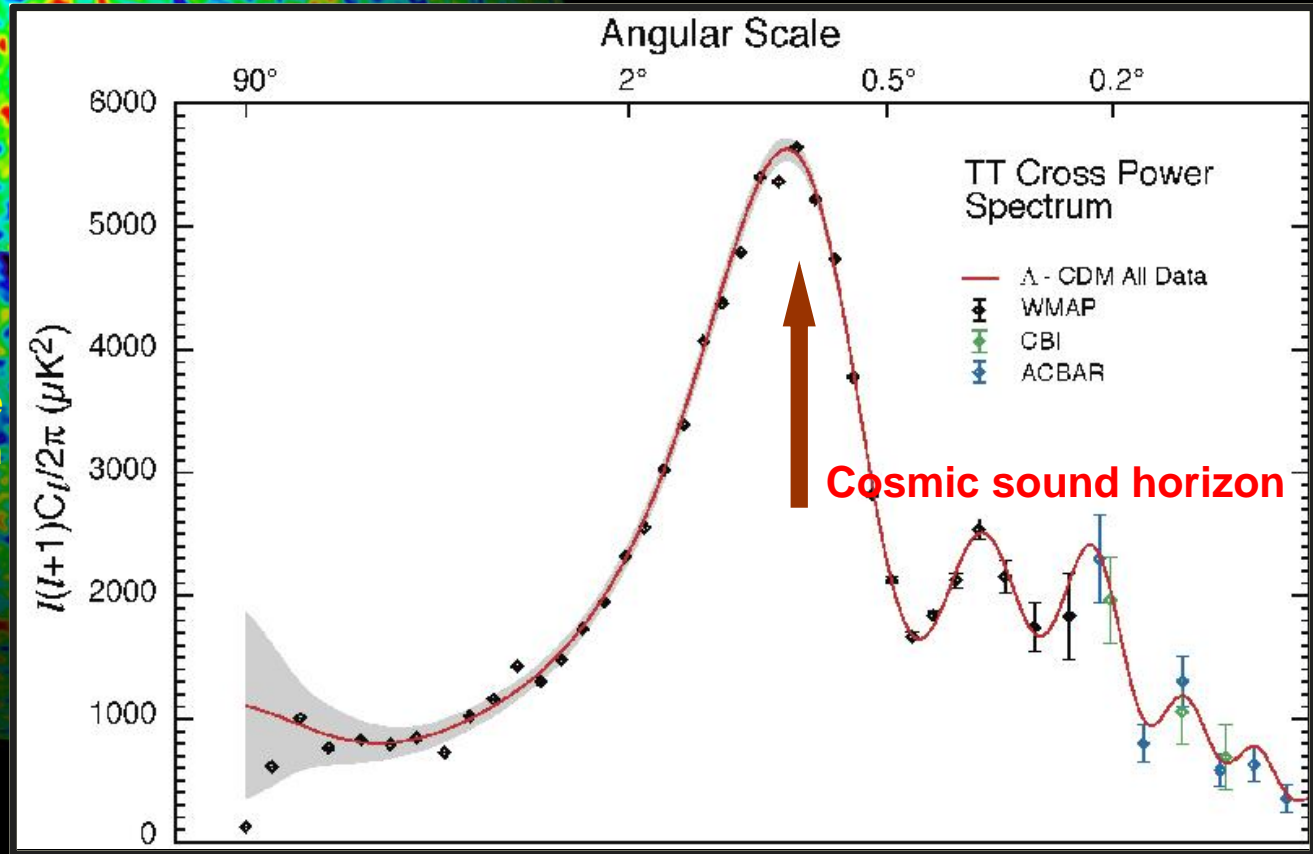


Standard Ruler:  
1° arc measurement of  
dominant energy spike



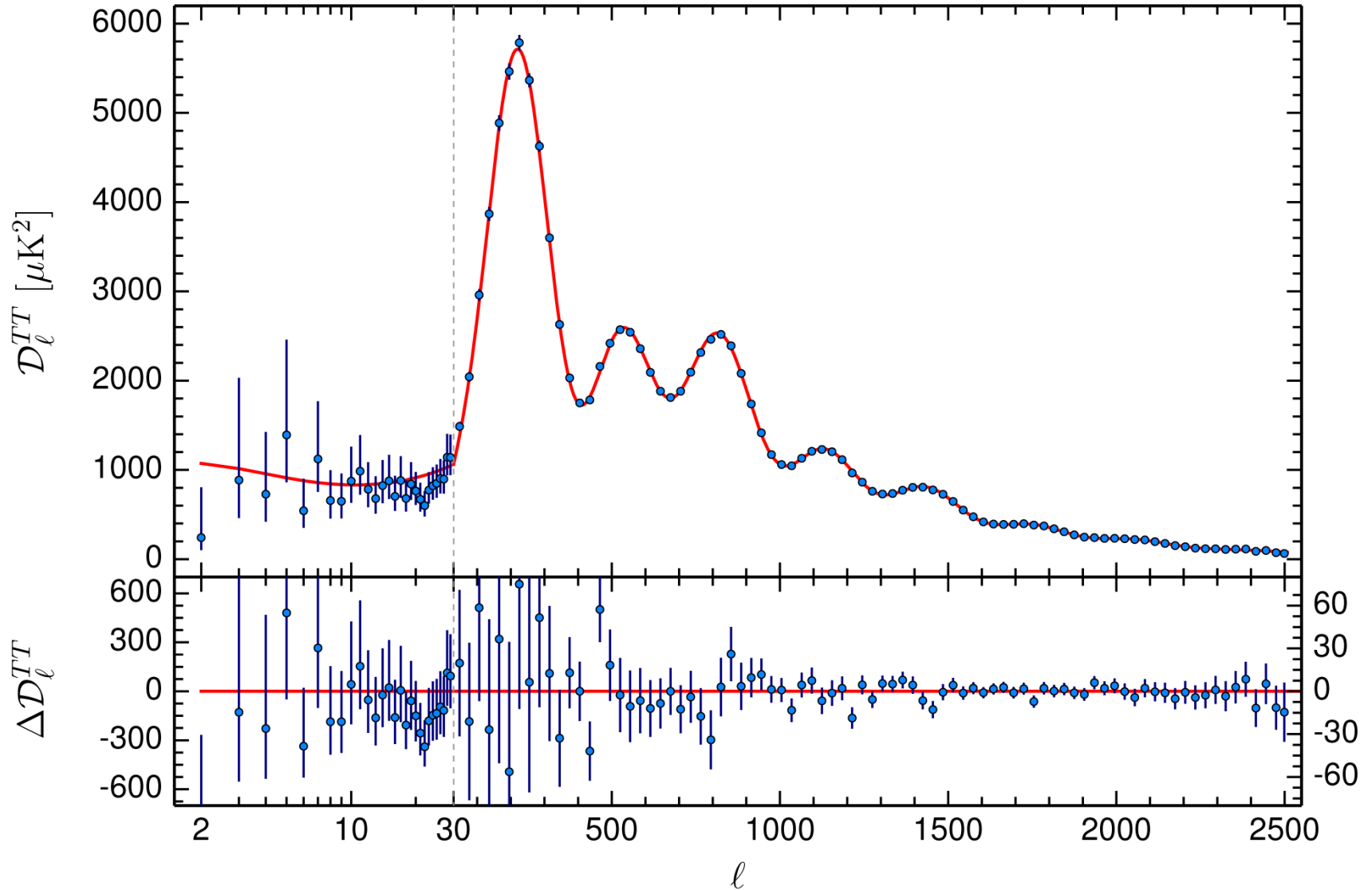
# The Cosmic Tonal Ladder

The WMAP CMB temperature power spectrum



The Cosmic Microwave Background Temperature Anisotropies:  
Universe is almost perfectly FLAT !!!!

# CMB - Fluctuations



# Cosmic Horizons

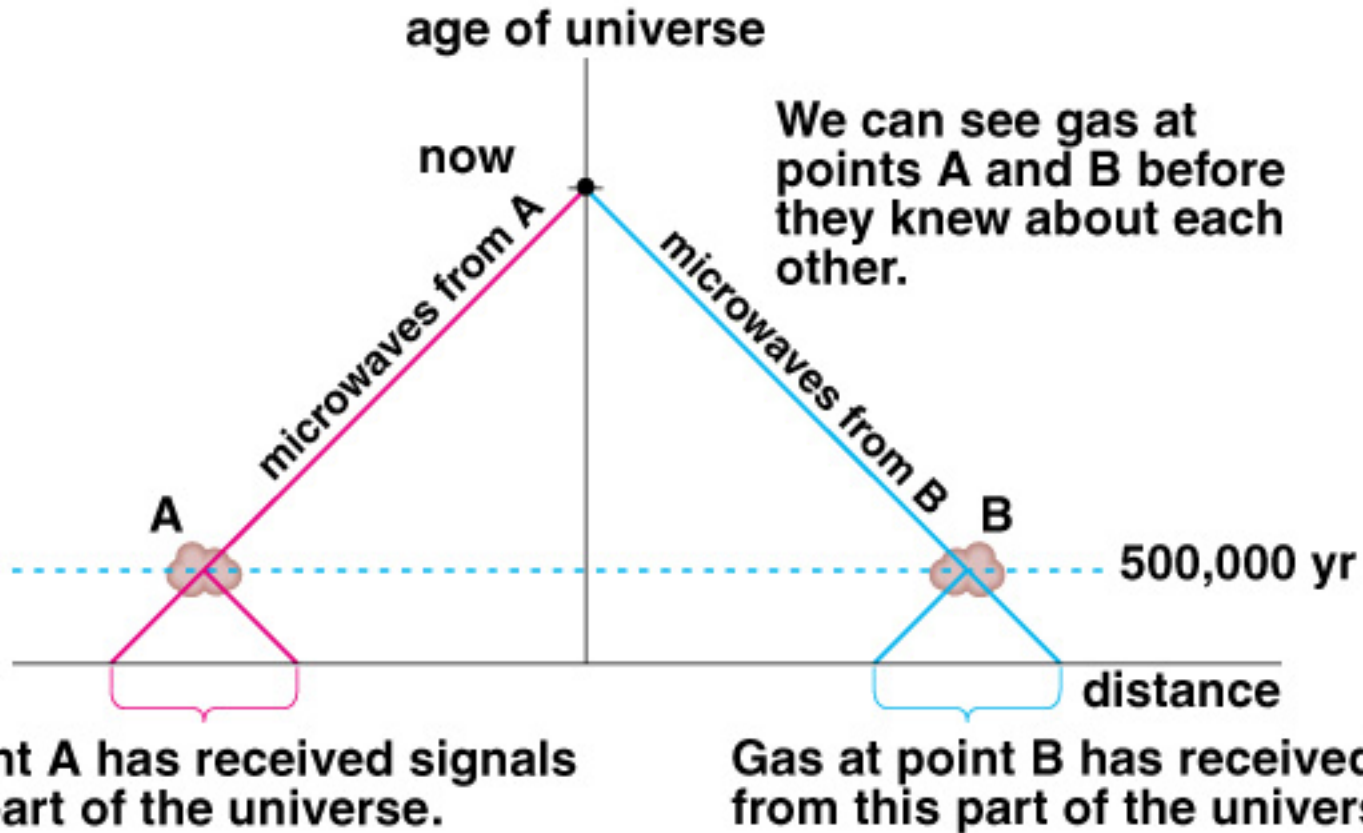
# Cosmic Horizons

Fundamental Concept for our understanding of the physics of the Universe:

- Physical processes are limited to the region of space with which we are or have ever been in physical contact.
- What is the region of space with which we are in contact ?  
Region with whom we have been able to exchange photons  
(photons: fastest moving particles)
- From which distance have we received light.
- Complication: - light is moving in an expanding and curved space  
- fighting its way against an expanding background
- This is called the

Horizon of the Universe

# Cosmic Horizons

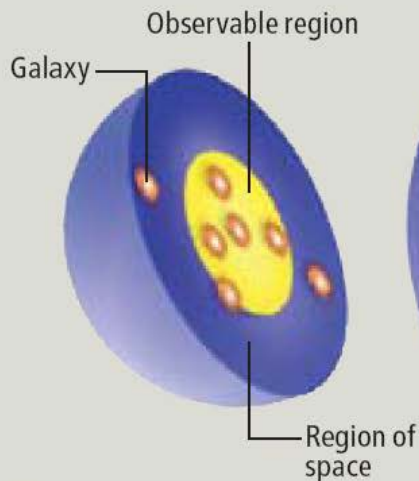


Copyright © Addison Wesley.

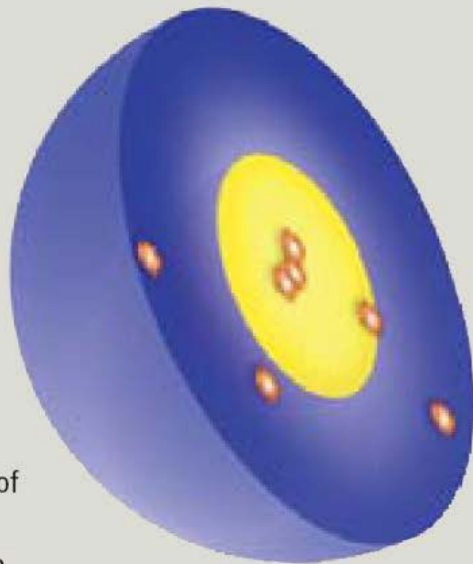
Horizon of the Universe:  
distance that light travelled since the Big Bang

# EXPANDING UNIVERSE, SHRINKING VIEW

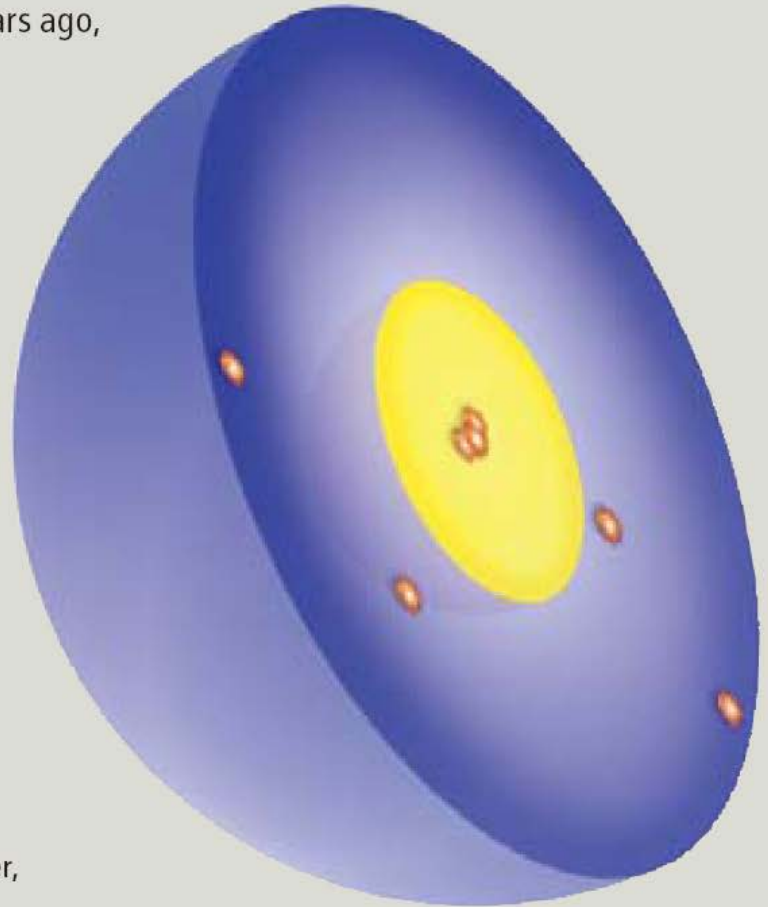
The universe may be infinite, but consider what happens to the patch of space around us (*purple sphere*), of which we see only a part (*yellow inner sphere*). As space expands, galaxies (*orange spots*) spread out. As light has time to propagate, we observers on Earth (or our predecessors or descendants) can see a steadily increasing volume of space. About six billion years ago, the expansion began to accelerate, carrying distant galaxies away from us faster than light.



- 1 At the onset of acceleration, we see the largest number of galaxies that we ever will.



- 2 The visible region grows, but the overall universe grows even faster, so we actually see a smaller fraction of what is out there.



- 3 Distant galaxies (those not bound to us by gravity) move out of our range of view. Meanwhile, gravity pulls nearby galaxies together.

## NOTE:

Because space is expanding uniformly, alien beings in other galaxies see this same pattern.

# Cosmic Future



# Cosmic Fate

## 100 Gigayears: the end of Cosmology

The night sky on Earth (assuming it survives) will change dramatically as our Milky Way galaxy merges with its neighbors and distant galaxies recede beyond view.



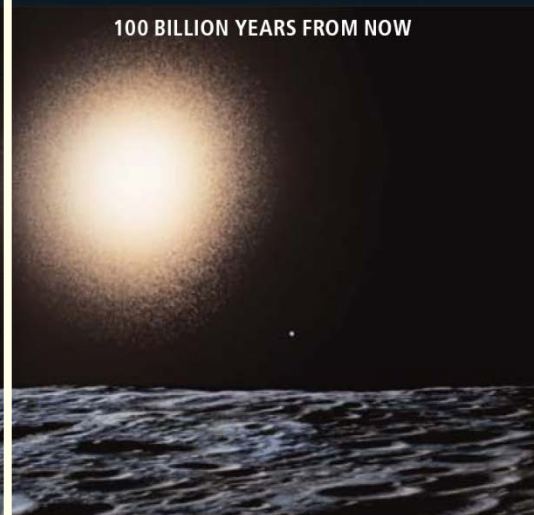
NOW

DIFFUSE BAND stretching across the sky is the disk of the Milky Way. A few nearby galaxies, such as Andromeda and the Magellanic Clouds, are visible to the naked eye. Telescopes reveal billions more.



5 BILLION YEARS FROM NOW

ANDROMEDA has been moving toward us and now nearly fills the sky. The sun swells to red giant size and subsequently burns out, consigning Earth to a bleak existence.



100 BILLION YEARS FROM NOW

SUCCESSOR to the Milky Way is a ball-like supergalaxy, and Earth may float forlornly through its distant outskirts. Other galaxies have disappeared from view.



100 TRILLION YEARS FROM NOW

LIGHTS OUT: The last stars burn out. Apart from dimly glowing black holes and any artificial lighting that civilizations have rigged up, the universe goes black. The galaxy later collapses into a black hole.