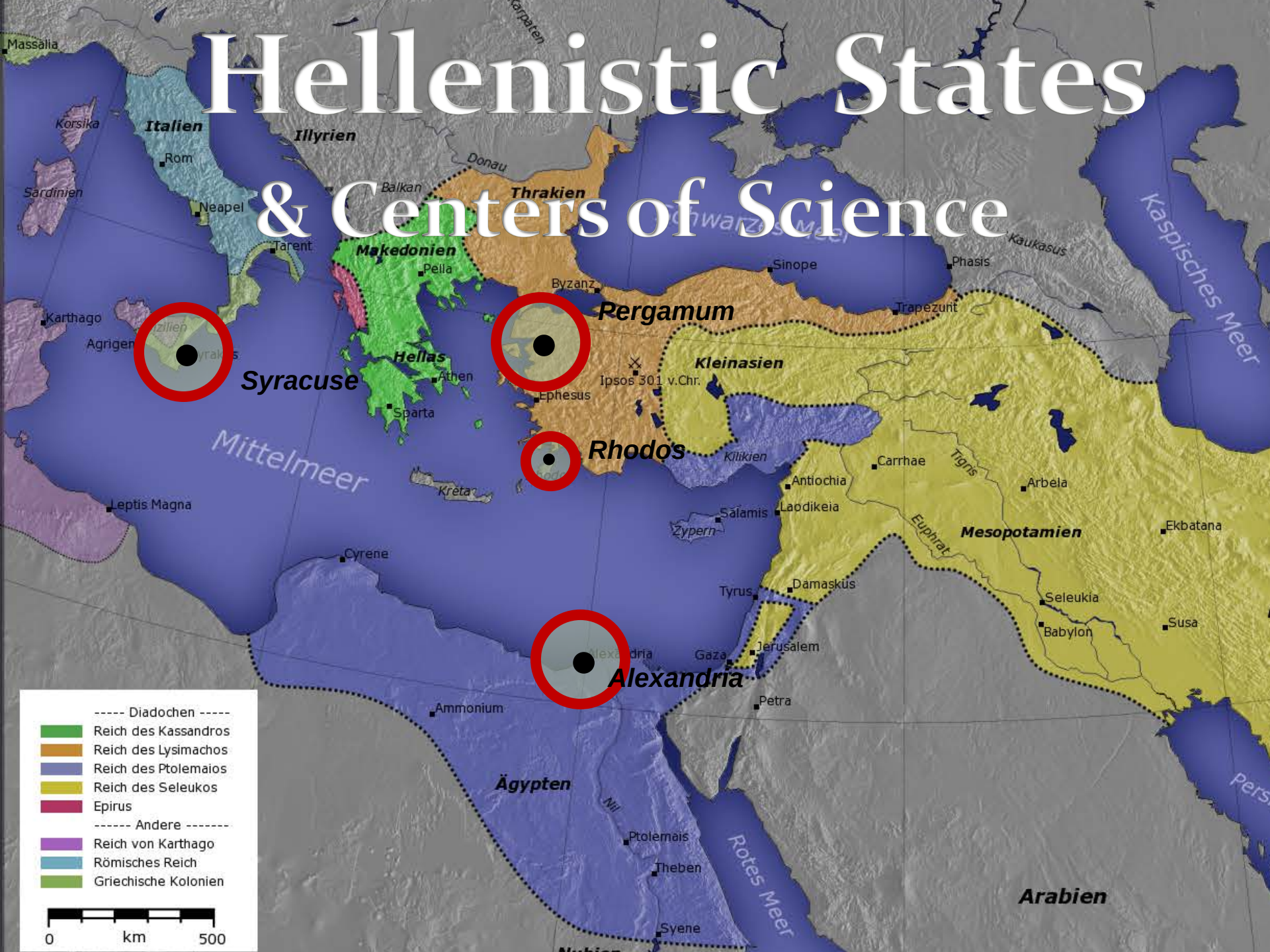
the Universe Mechanized:

Hellenistic Cosmology

the First Scientific Revolution:

Hellenistic World

Hellenistic States & Centers of Science



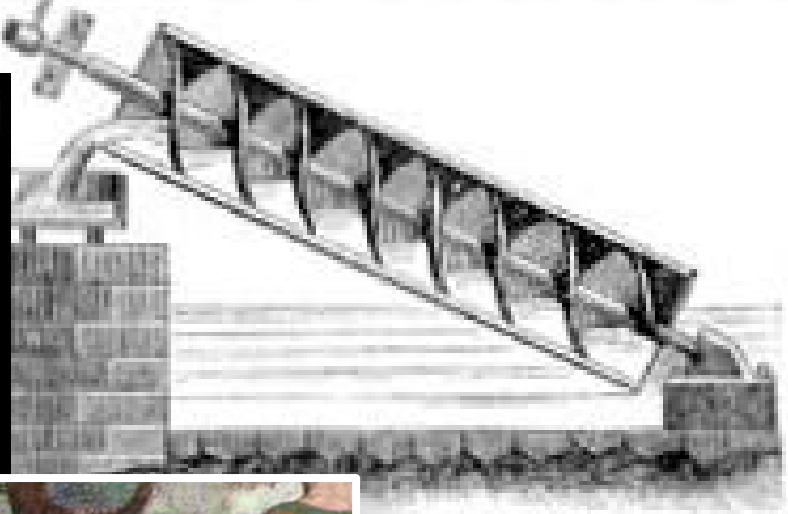
Alexandria

Massalia
Korsika
Italien
Rom
Illyrien
Balkan
Sardinien
Neapel
Tarent
Makedonien
Pella
Karthago
Sizilien
Agrigent
Syrakus
Hellas
Sparta
Mittelmeer
Leptis Magna
Cyrene
Tyrus
Damaskus
Seleukia
Babylon
Susa
Gaza
Jerusalem
Petra
Ammonium
Ägypten
Nil
Ptolemais
Syene
Arabien
Rotes Meer

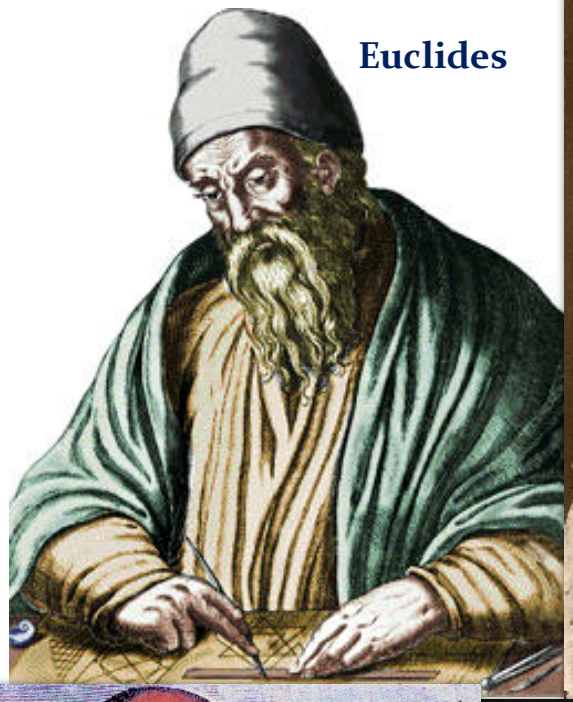
----- Diadochen -----
Reich des Kassandros
Reich des Lysimachos
Reich des Ptolemaios
Reich des Seleukos
Epirus
------ Andere -------
Reich von Karthago
Römisches Reich
Griechische Kolonien

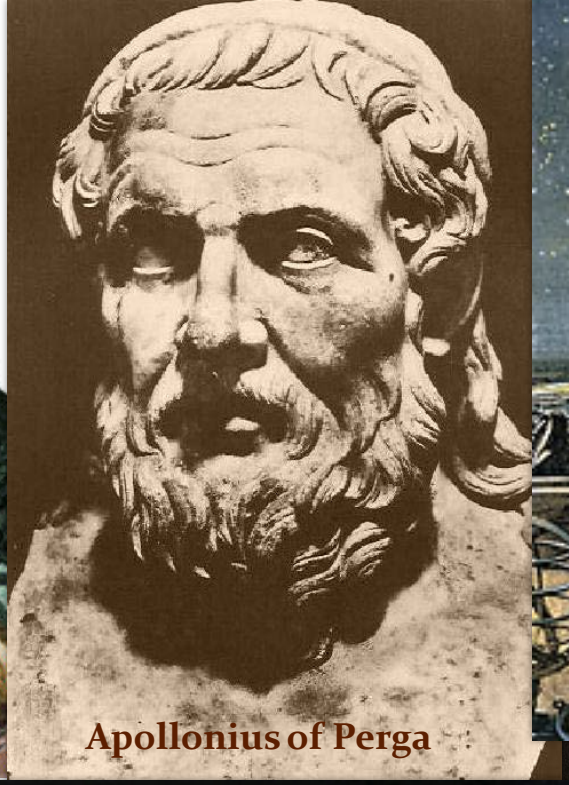
0 km 500

First Scientific Revolution



Euclides
Herophilus
Aristarchus
Ctesibius
Archimedes
Eratosthenes
Apollonius
Hipparchus
Heron of Alexandria
Ptolemaeus

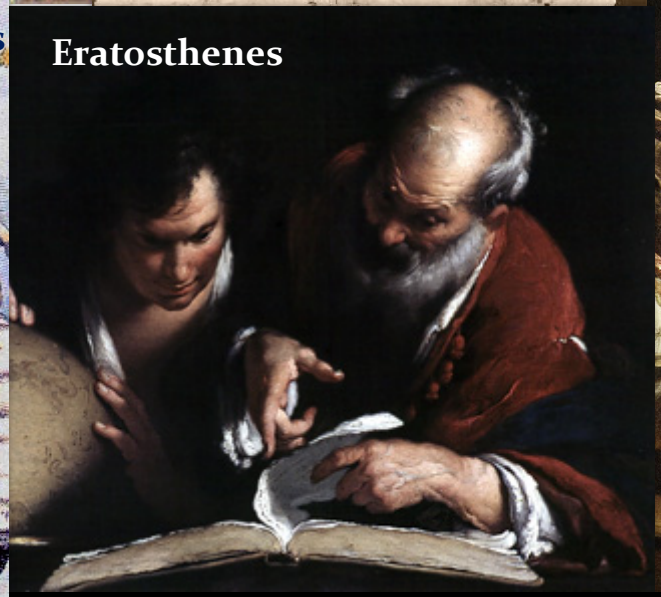

Euclides

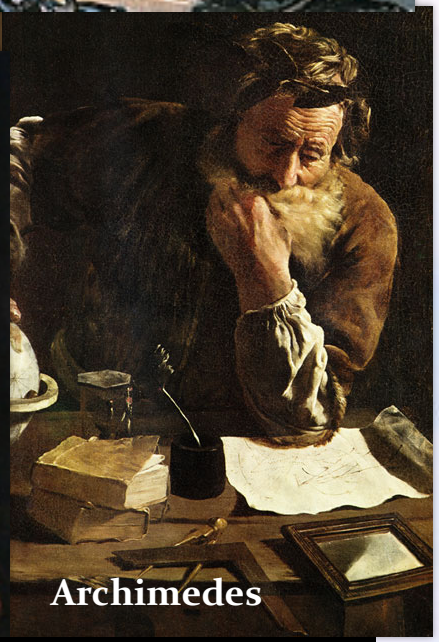

Apollonius of Perga

Hipparchus

Heron

Ptolemaeus


Eratosthenes


Archimedes

Aristarchus

Euclides (~ 300 BC)
Herophilus (335-280 BC)
Aristarchus of Samos (310-230 BC)
Ctesibius (285-222 BC)
Archimedes (287-212 BC)
Eratosthenes (276-194 BC)
Apollonius of Perga (262-190 BC)
Hipparchus of Samos (190-120 BC)
Heron of Alexandria (10-70 AD)
Ptolemaeus (83-168 AD)

Hellenistic Astronomers

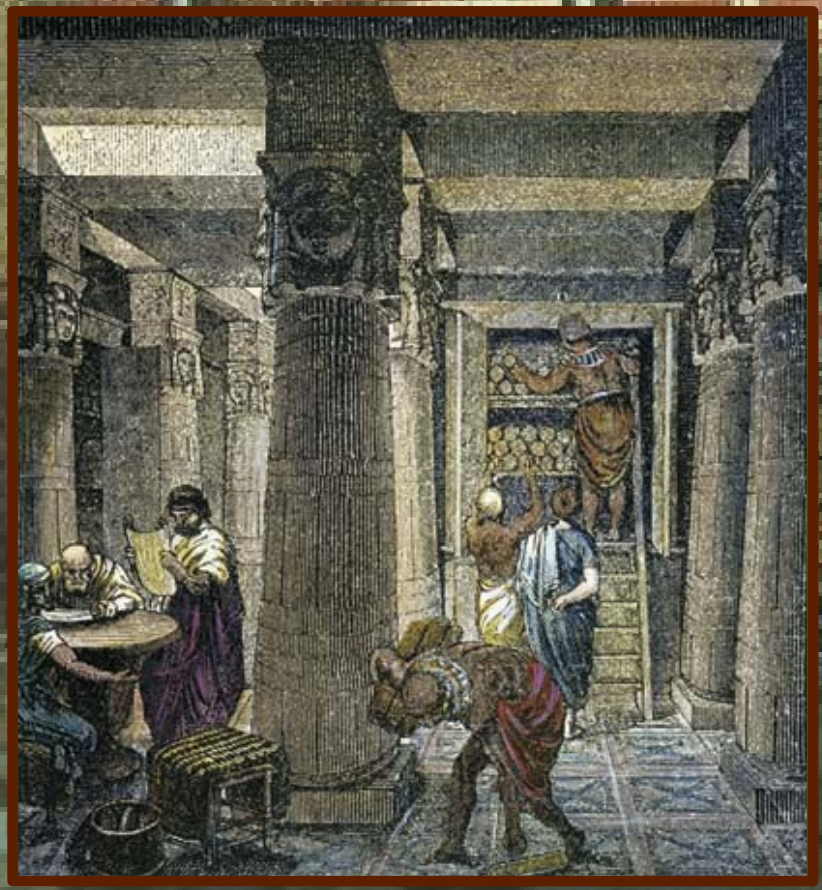
Various astronomers made significant, even amazing, contributions. Noteworthy examples:

- **Aristarchus of Samos** - **Heliocentric Universe**
  - **distance Moon & Sun**
  - **size Sun**
- **Archimedes** - **Planisphere/Planetarium ?**
- **Eratosthenes** - **Diameter Earth**
- **Hipparchus** - **multitude essential contributions**

Problematic is the loss of nearly all, except for a few, of the books and works they have written ...

Mouseion: the Library

Alexandria: Birthplace of the Western Mind



Mouseion: World's first "university"
Library: 700,000 book scrolls
Storage of all knowledge ancient world

Great Library of Alexandria

Aristarchus of Samos
Ἀρίσταρχος

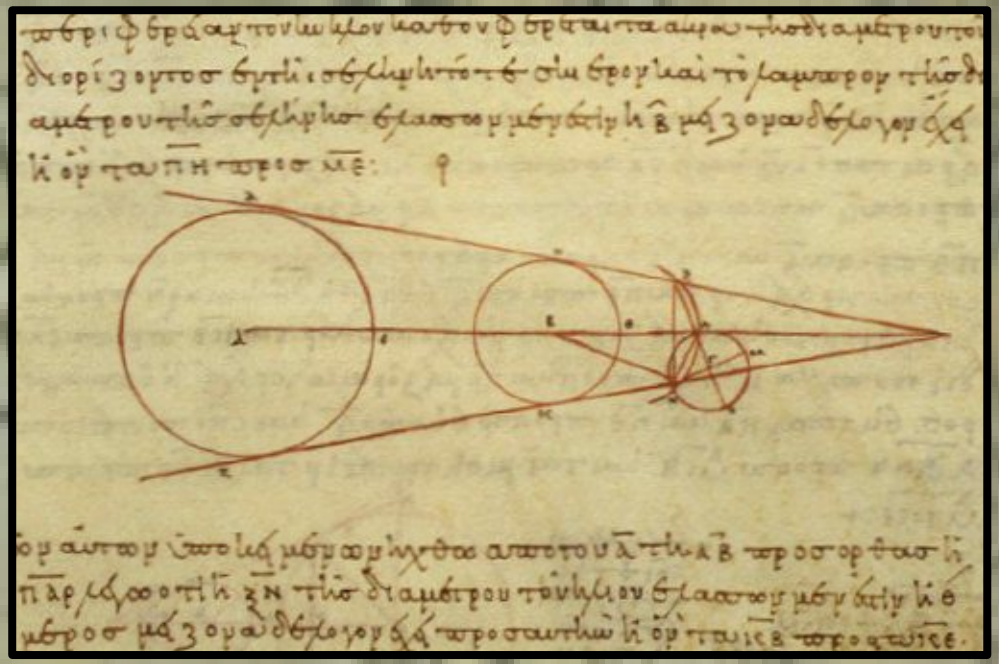
310-230 BCE

Aristarchus of Samos

(Samos, 310-230 BCE)

the ancient Copernicus

"On the Sizes & Distances of the Sun and Moon ":



On the Sizes and Distances

Only one work of Aristarchus survives:

On the Sizes and the Distances of the Sun and Moon

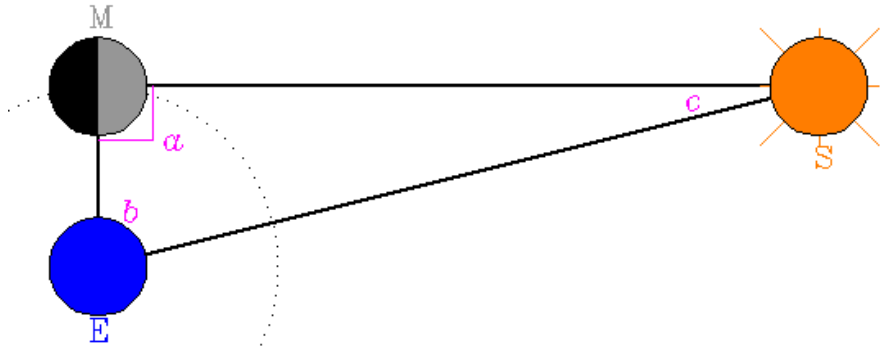
First mathematically based attempt to measure distance Earth-Sun, thus

First attempt to measure scale Universe

Based upon geocentric view of Universe

On the Sizes and the Distances
Greek copy 10th century

On the Sizes and Distances



Aristarchus' geometric construction used to estimate the distance to the Sun.
Earth (E) –Sun (S)-Moon (M) triangle and sizes are not drawn to scale.

Measure angle b:

$c = 90° - b$ $\quad EM/ES = \sin(c)$

Aristarchus:

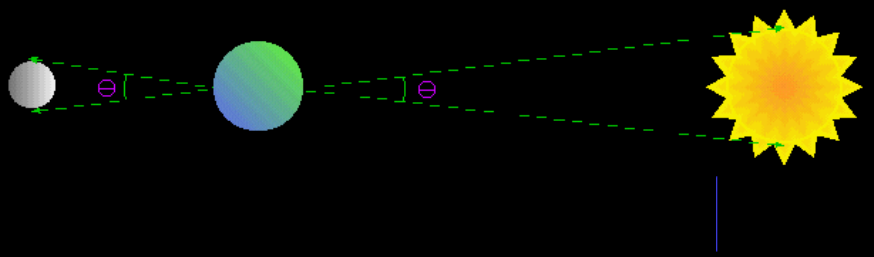
$b = 87°$ real value: $b = 89°50$

$ES = 19\ EM$ real value: $ES = 397\ EM$

Numerically, very unstable procedure, reason for huge error. Nonetheless,

On the Sizes and the Distances
Greek copy 10th century

On the Sizes and Distances



Aristarchus' estimate of size Sun:

angular diameter Sun ~
angular diameter Moon

Dist. Earth-Sun = 19 Dist. Earth-Moon

⇒ size Sun = 19 x size Moon

⇒ size Sun > size Earth

On the Sizes and the Distances
Greek copy 10th century

Aristarchus: Heliocentric Universe

Archimedes, "the Sand Reckoner" (~200 BCE):

You King Gelon are aware the 'universe' is the name given by most astronomers to the sphere the center of which is the center of the Earth, while its radius is equal to the straight line between the center of the Sun and the center of the Earth. This is the common account as you have heard from astronomers.

But Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the 'universe' just mentioned.

His hypotheses are that the fixed stars and the Sun remain unmoved, that

the Earth revolves about the Sun

on the circumference of a circle, the Sun lying in the middle of the orbit, and that the sphere of fixed stars, situated about the same center as the Sun, is so great that the circle in which he supposes the Earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface.

Aristarchus: Heliocentric Universe

Aristarchus' idea of Heliocentric Universe encountered sceptical, even hostile, reactions:

- Could not explain the absence of parallax of fixed stars (or they should be very, very far away …)
- Impiety … (even for those "rational" Greeks …)

"Cleanthes thought it was the duty of the Greeks to indict Aristarchus of Samos on the charge of impiety for putting in motion the Hearth of the universe [i.e. the earth], . . . supposing the heaven to remain at rest and the earth to revolve in an oblique circle, while it rotates, at the same time, about its own axis"

Plutarchus, "On the Apparent Face in the Orb of the Moon"

Eratosthenes of Cyrene
Ἐρατοσθένης

276 BC - 194 BC

Eratosthenes

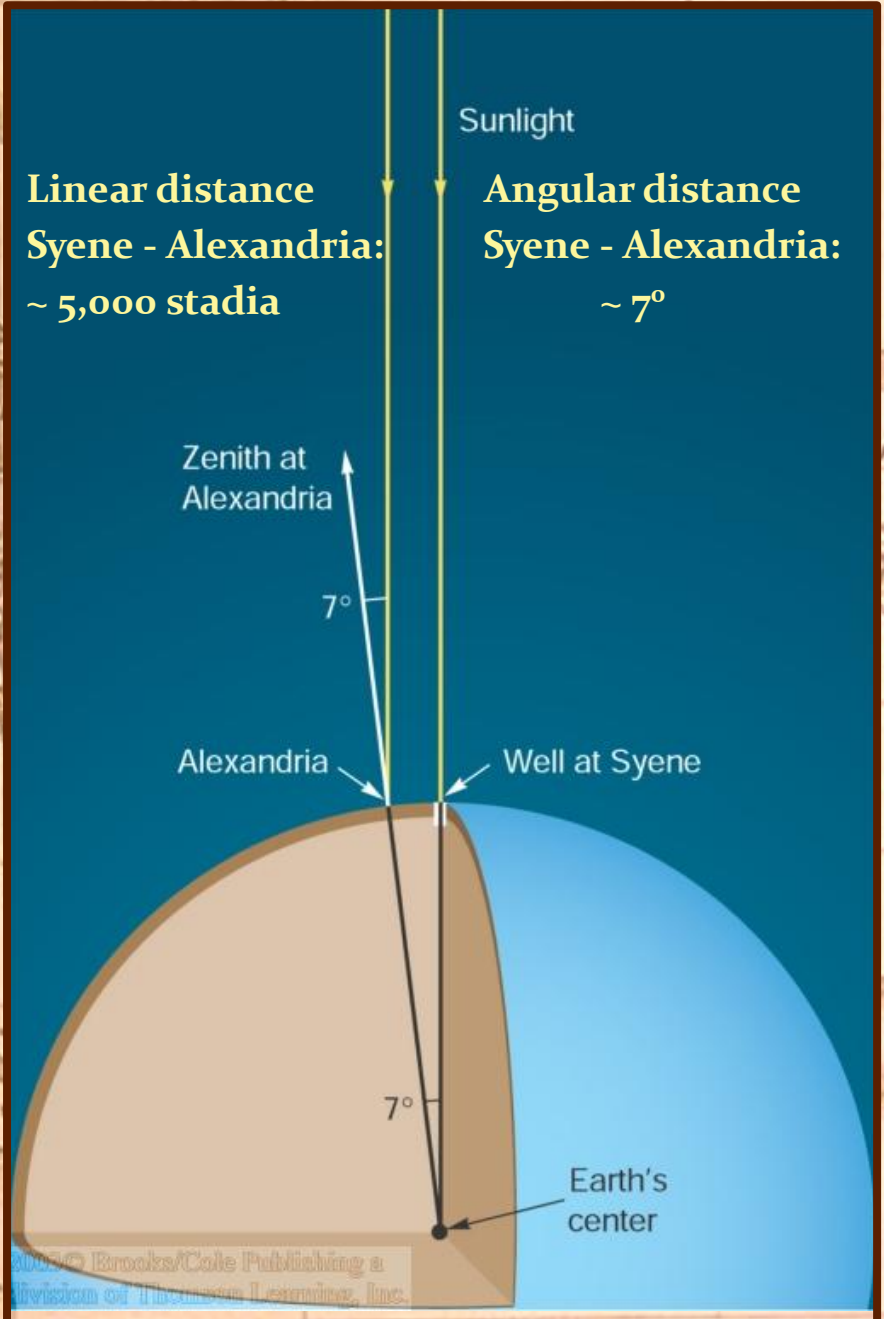
of Cyrene
(276 -194 B.C.E.)

- **Studied in Alexandria & Athens**
  - **Mathematician**
  - **Astronomer**
  - **Geographer**
  - **Poet**
  - **Athlete**
- **2nd Chief librarian Great Library of Alexandria**
- **Friend of Archimedes**
- **Invented armillary sphere (240 BC)**
- **Calculated Earth's Circumference**
- **Became blind in 194 BC, starved himself to death**

Earth Circumference:
Eratosthenes' measurement: 39,690 km - within 1%
(if Egyptian stadia)

Linear distance
Syene - Alexandria:
~ 5,000 stadia

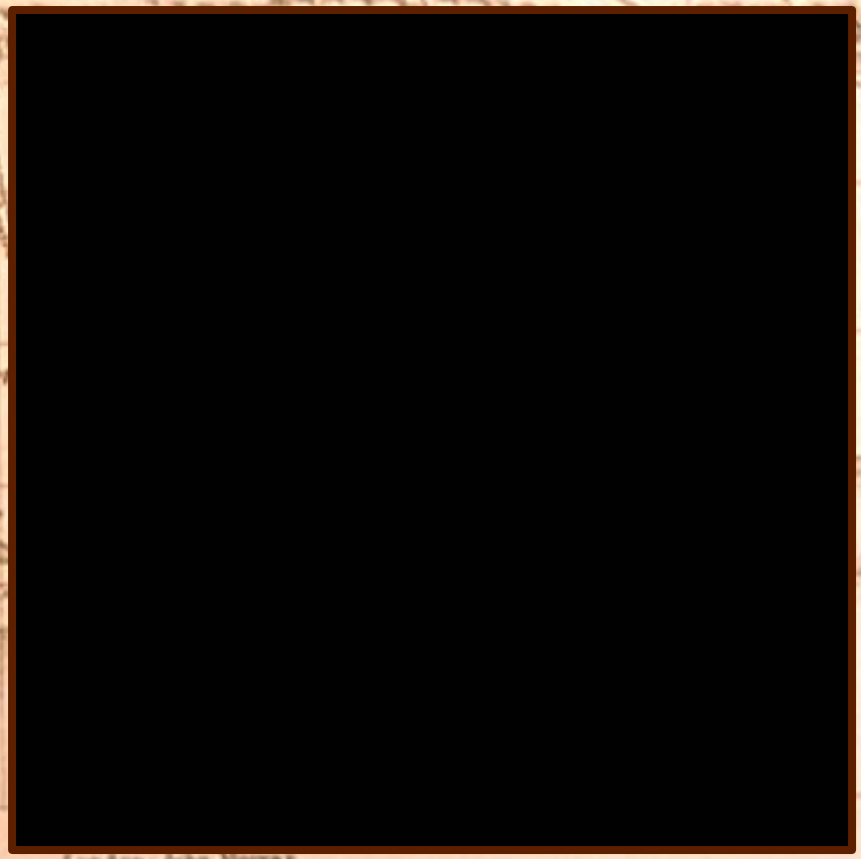
Angular distance
Syene - Alexandria:
~ 7°



Eratosthenes' measurement Earth diameter

Eratosthenes

Earth Circumference Measurement



Archimedes of Syracuse
Αρχιμηδης

c. 287 – 212 BCE

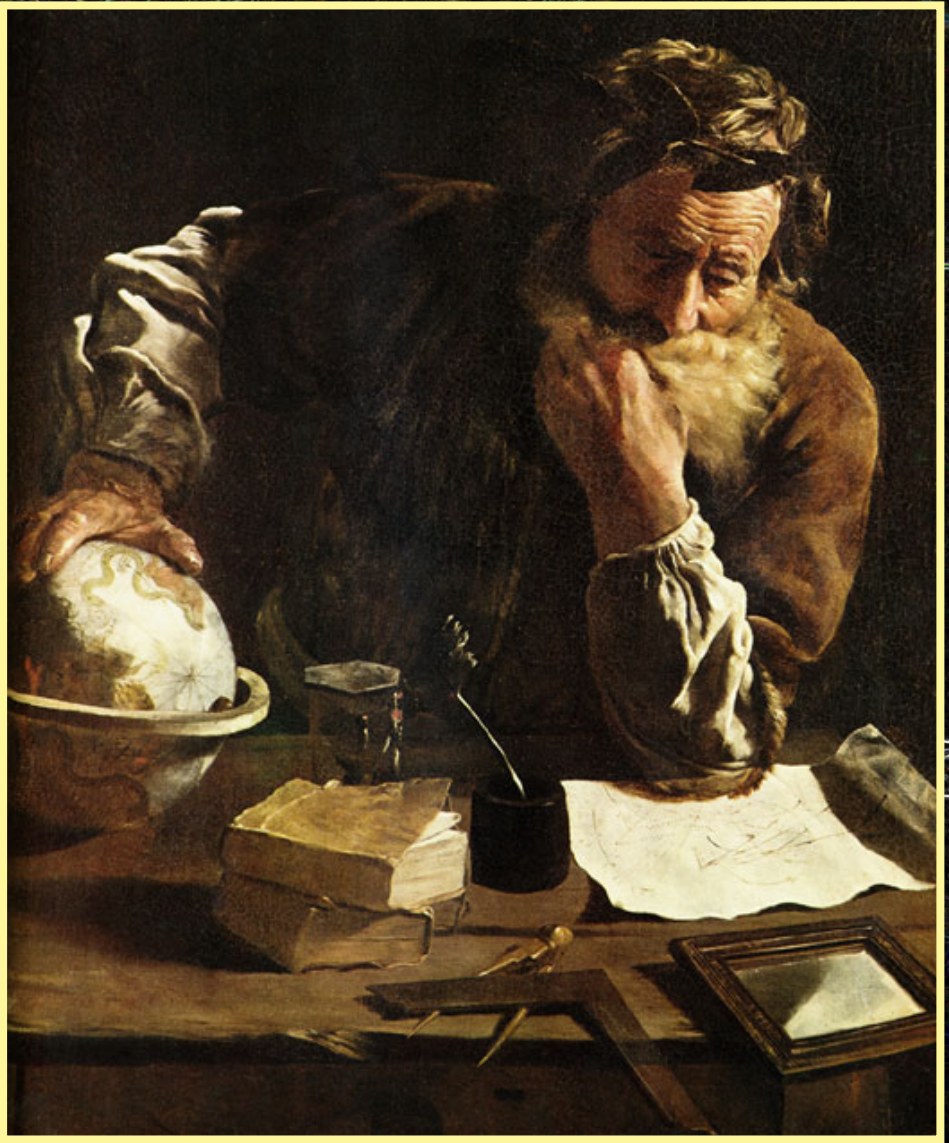
Archimedes

Syracuse, 287-211/212 BC,

Greatest mathematician & scientist of antiquity (all time ?):

- Probably studied in Alexandria, under followers Euclides
- Killed by Roman soldier, upon Roman conquest Syracuse
- Family Hieron II, king Syracuse ?

- Inventions:
  - war machines ...
  - water screw
  - water organ (?)
  - burning mirrors (???)
  - planetarium !!!!!!



Cicero mentions two planetarium like machines...

"For when Archimedes fastened on a globe the movements of moon, sun and five wandering stars, he, just like Plato's God who built the world in the "Timaeus", made one revolution of the sphere control several movements utterly unlike in slowness and speed. Now if in this world of ours phenomena cannot take place without the act of God, neither could Archimedes have reproduced the same movements upon a globe without divine genius"

Cicero, Tusculan Disputations, Book I, Section XXV

Archimedes

- Pappus of Alexandria:
  Archimedes wrote book
  "On Sphere-Making"

  … is this Antikythera … ?

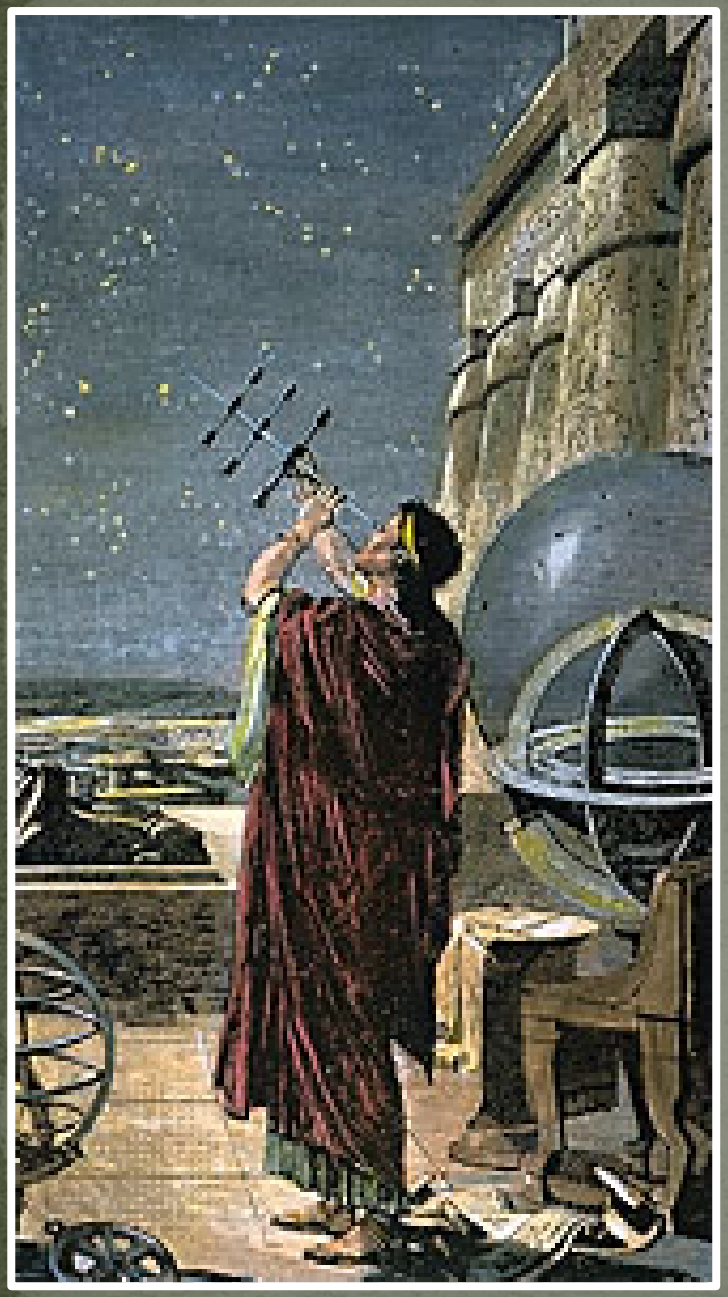
- Compare with

  Archimedes Palimpsest:
  … "On the Method" …

  Fundamentals Calculus,
  Integral calculus …

Hipparcus
Ιππαρχος

c. 190 – 120 BCE



Hipparchus of Nicaea (190-120 BC)

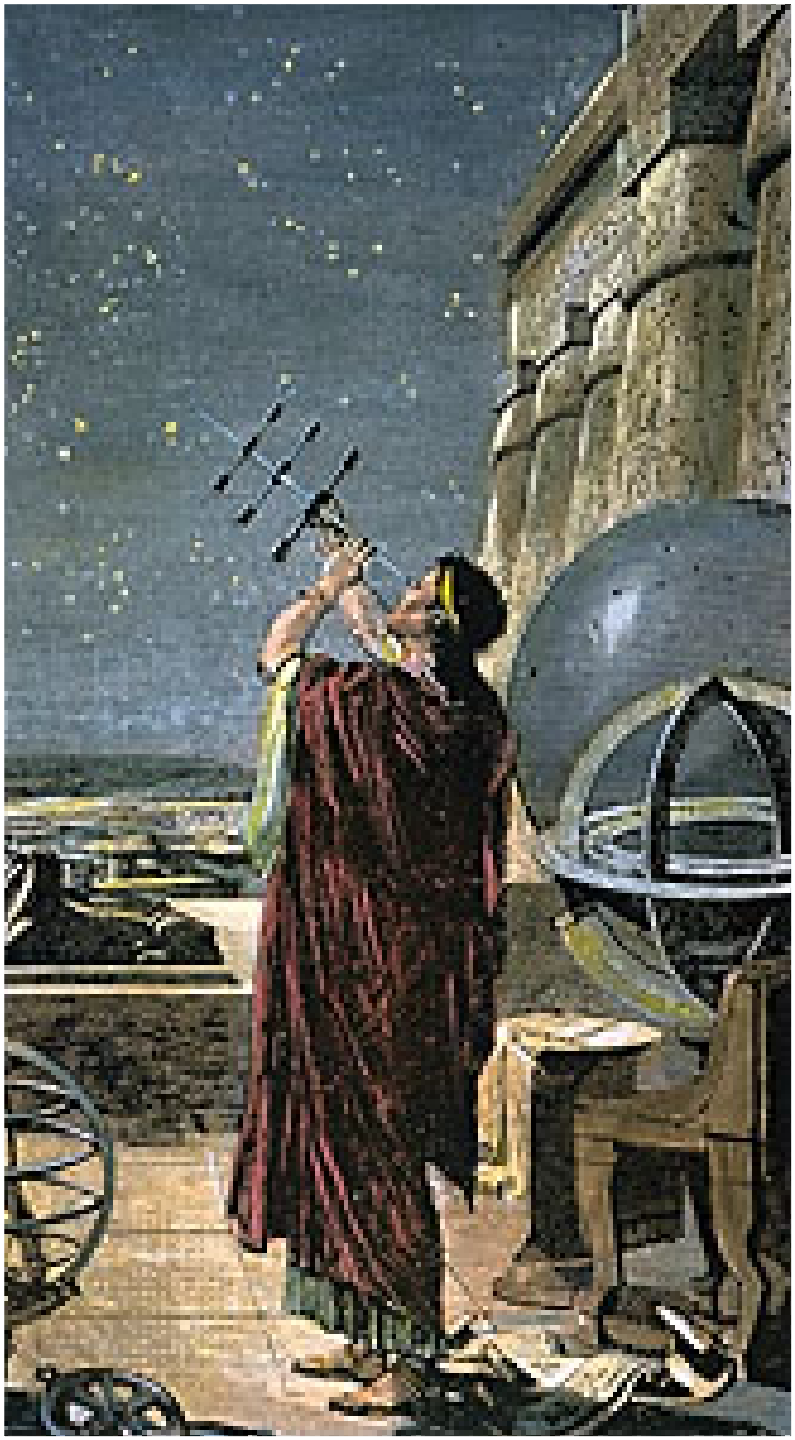
- Antiquities' Greatest Astronomer
- Responsible for the true Revolution in Astronomy
- Synthesis of
  Babylonian Observational Astronomy
  Greek Theoretical/Geometric Models
- Astronomy as true Modern Science:
  Experiment & Theory

Hipparcus

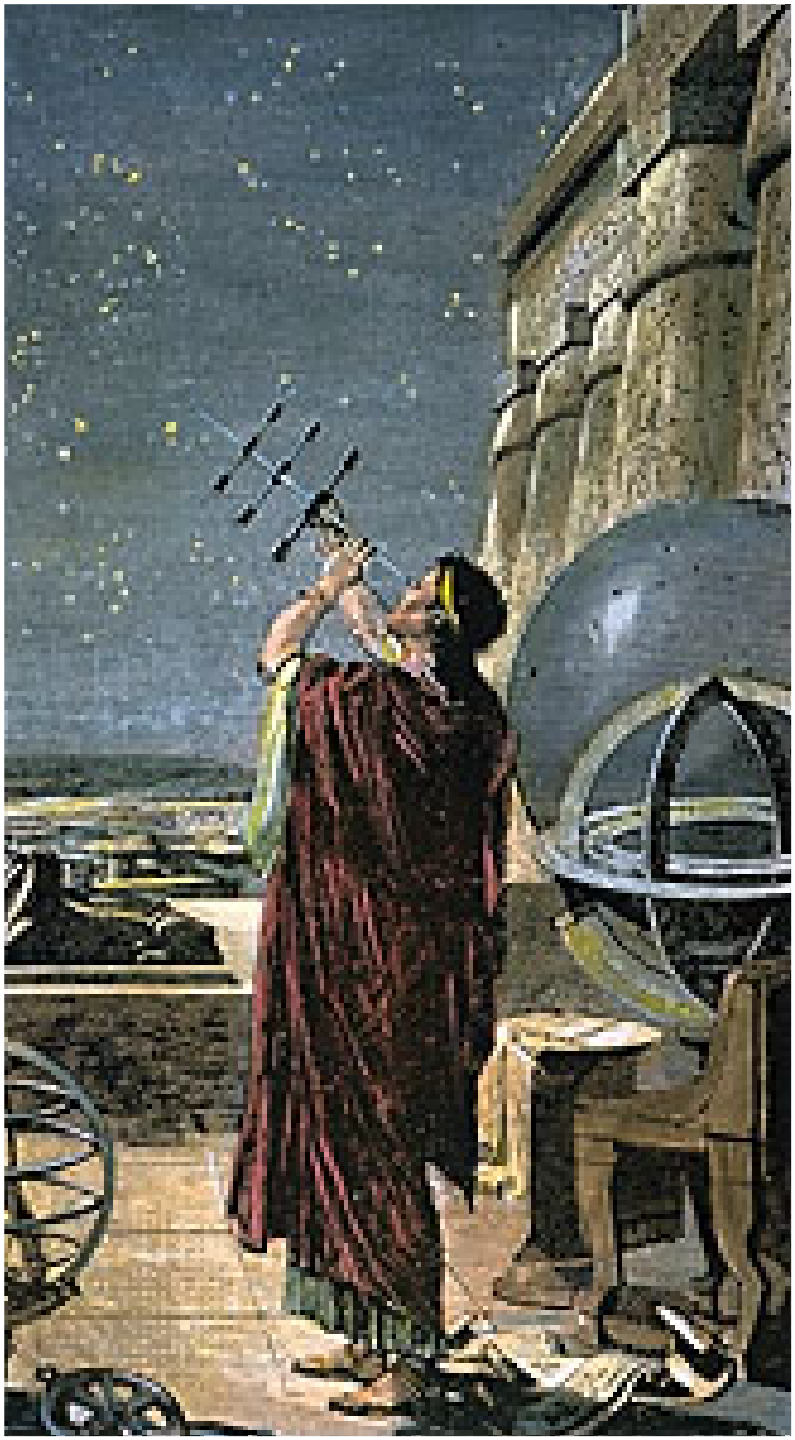
(Nicaea-Rhodos 190-120 BCE)

Greatest astronomer Greek antiquity

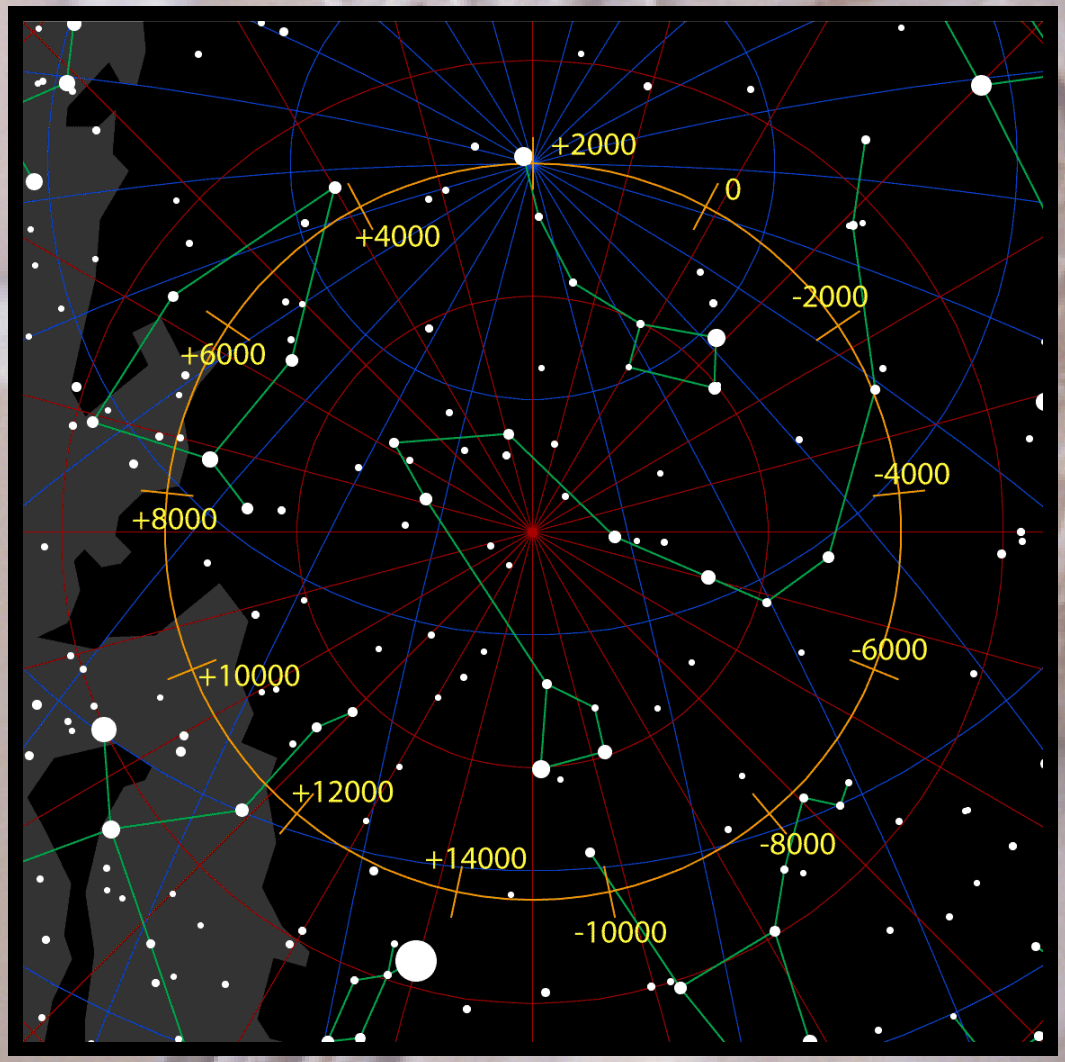
- Trigonometric Tables
- Precession of the Equinoxes
- Motion moon:
  synodic, anomalistic, … month
- Solar & Lunar eclipses
- Orbit of the Moon:
  epicyclic theory
- Distance Moon
- Star catalogue & Celestial Globe
  Lost, yet … Farnese Atlas ?
- Defined Magnitude Scale
- Invented the Astrolabe



Star Magnitudes



Precession


+2000
0
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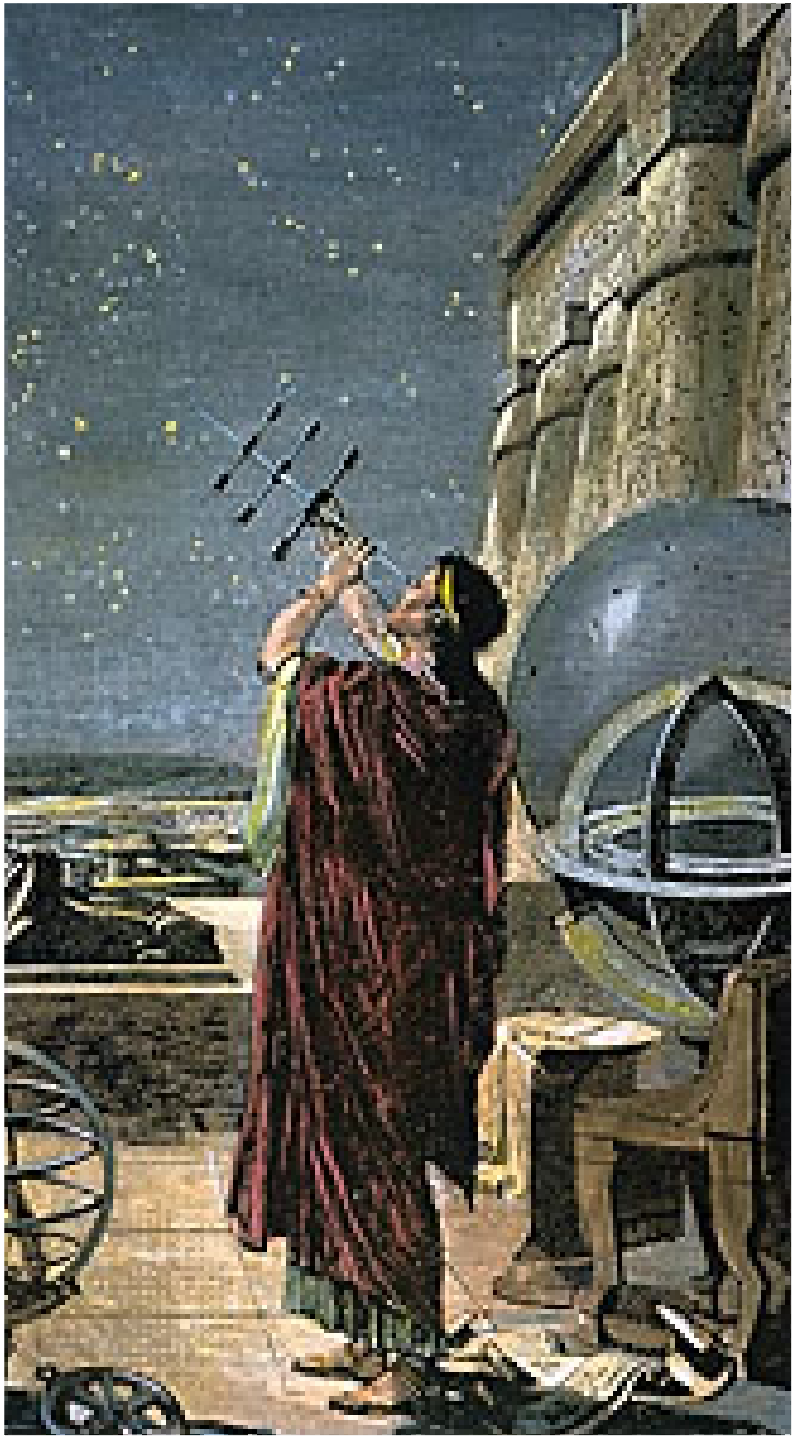
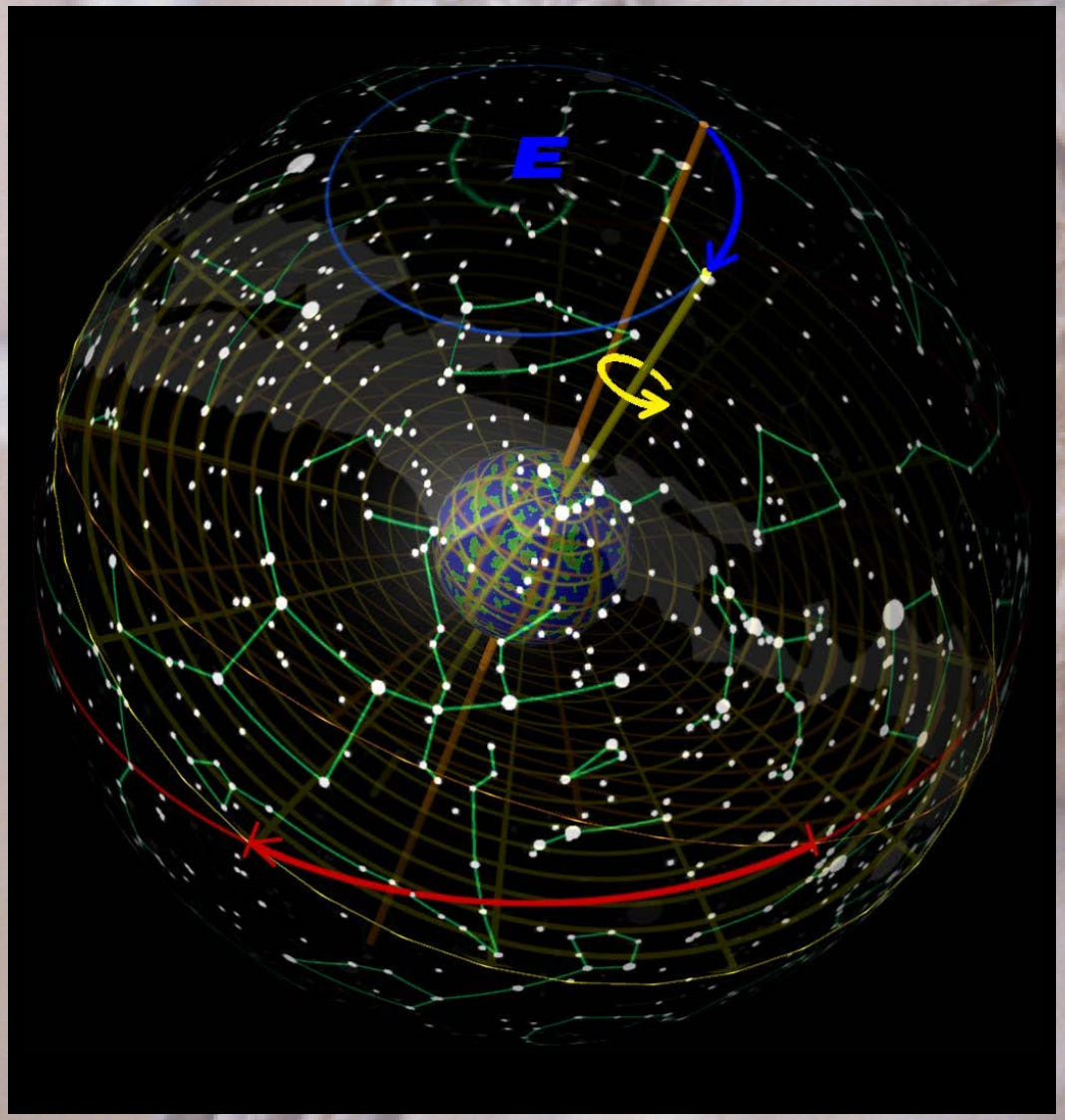
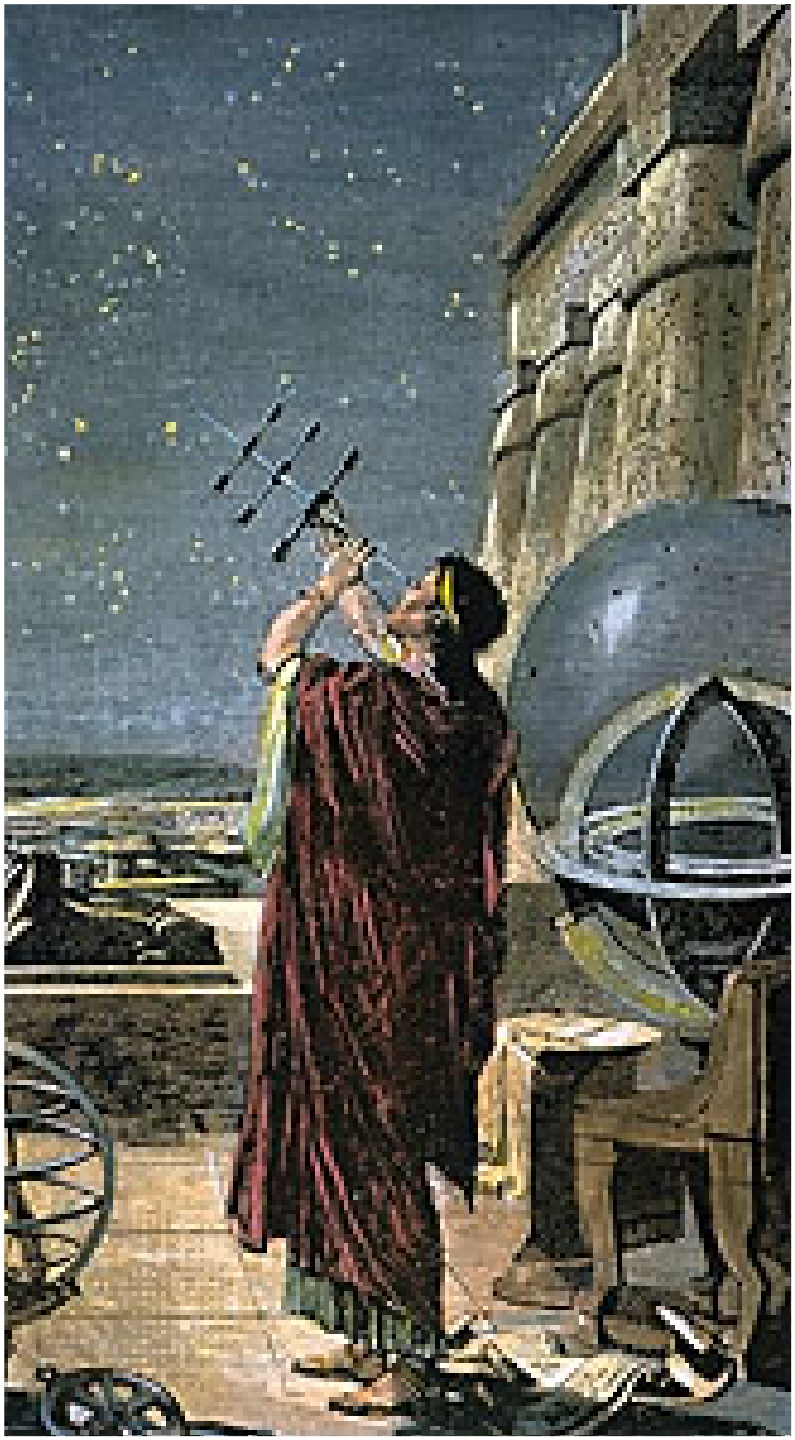
+12000
-8000
+14000
-10000

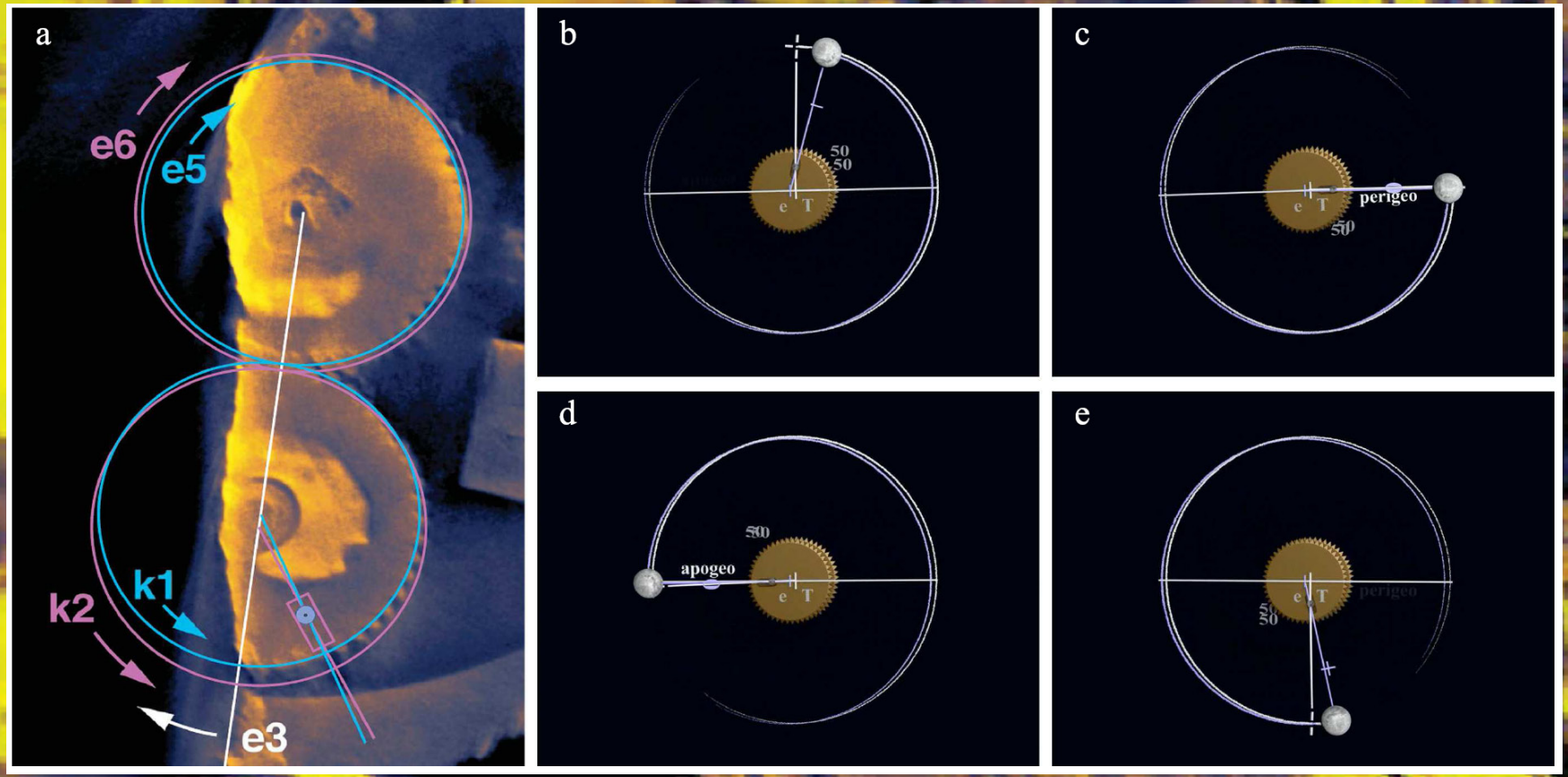


Precession





Greek Miracle ?



Hipparcus' Moon Orbit Computer

Farnese Atlas:

Hipparchus' star catalog ?

Farnese Atlas:

Hipparchus star catalog ?

Farnese Atlas is the oldest surviving pictorial Record of Western constellations

Roman times ~ A.D. 150,
presumed to represent constellations mapped in earlier Greek work

Atlas labors under the weight because he had been sentenced by Zeus to hold up the sky.

The globe shows:
- a depiction of the night sky as seen from outside the outermost celestial sphere
- low reliefs depicting 41 (42) of the 48 classical Greek constellations including:
    - Aries the ram
    - Cygnus the swan
    - Hercules

Epicycle Theory

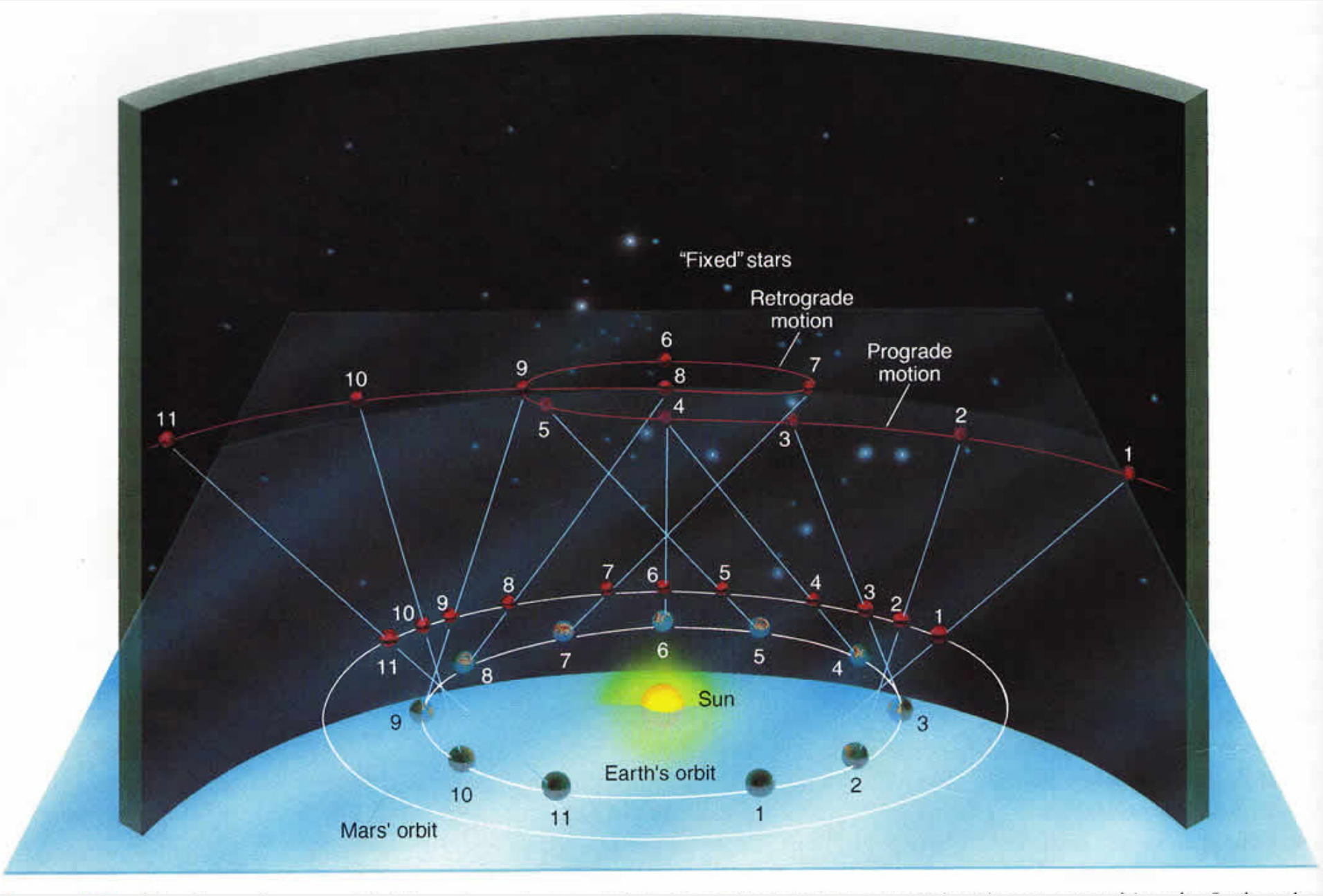
Retrograde Planetary Motion

Superior Planets:

- planet moves through in the night sky slower than the stars.
- each night planet appears to lag a little behind the stars: prograde motion
- near opposition, the planet would appear to reverse and move through
- the night sky faster than the stars for a time in retrograde motion
- before reversing again and resuming prograde.

Inferior planets:

- always observed to be near the sun
- appearing only shortly before sunrise or shortly after sunset
- apparent retrograde motion occurs during
- the transition between evening star into morning star, as they pass between Earth and the sun.


"Fixed" stars
Retrograde motion
Prograde motion
Sun
Earth's orbit
Mars' orbit
1
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10
11

Epicycle Theory

- Apollonius of Perga 262-190 BCE
- Hipparchus (of Rhodos) 190-120 BCE
- Ptolemaeus 100-170 AD

Planetary motion in a Geocentric Universe:

planets move on

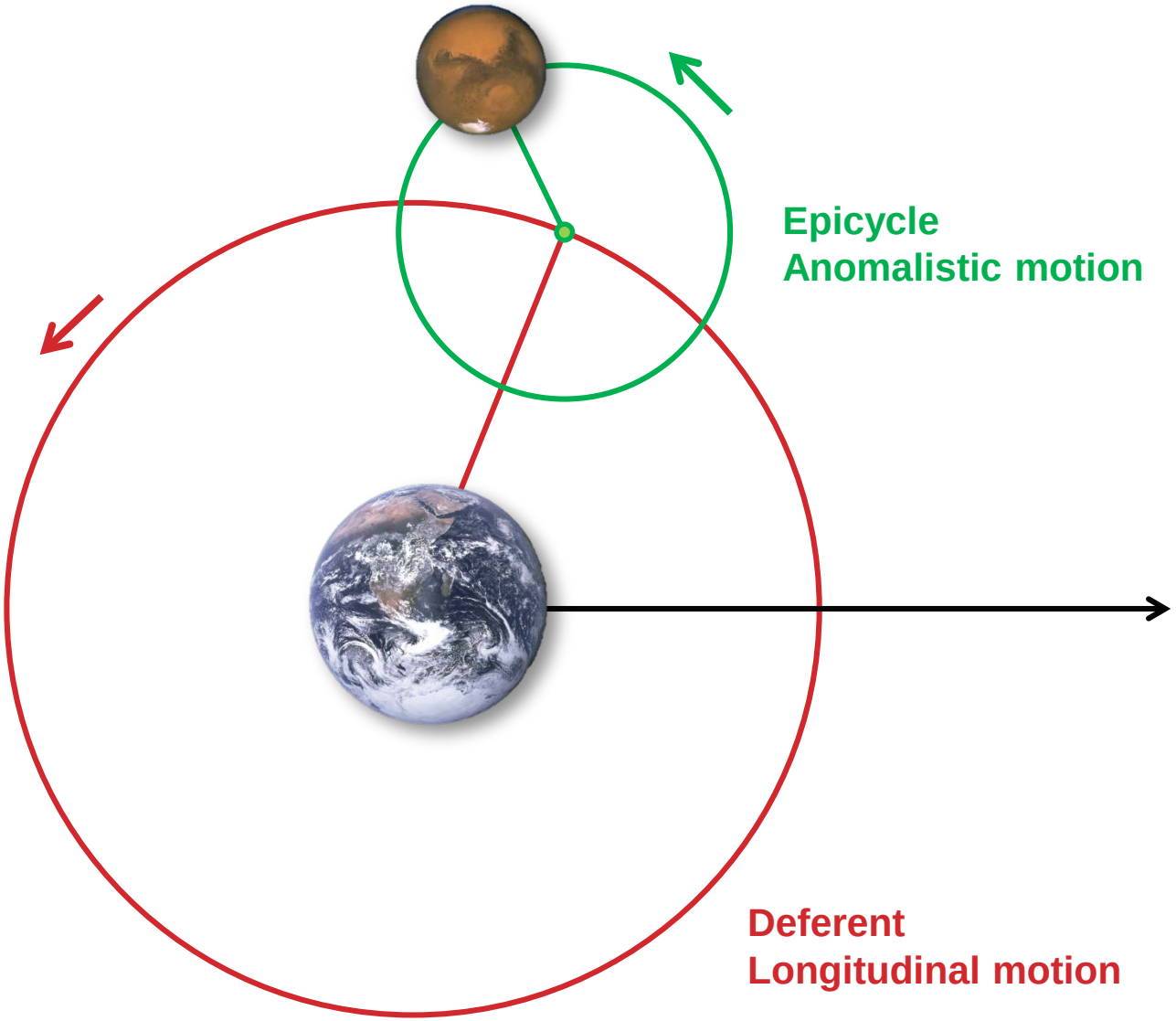
- small circle - epicycle

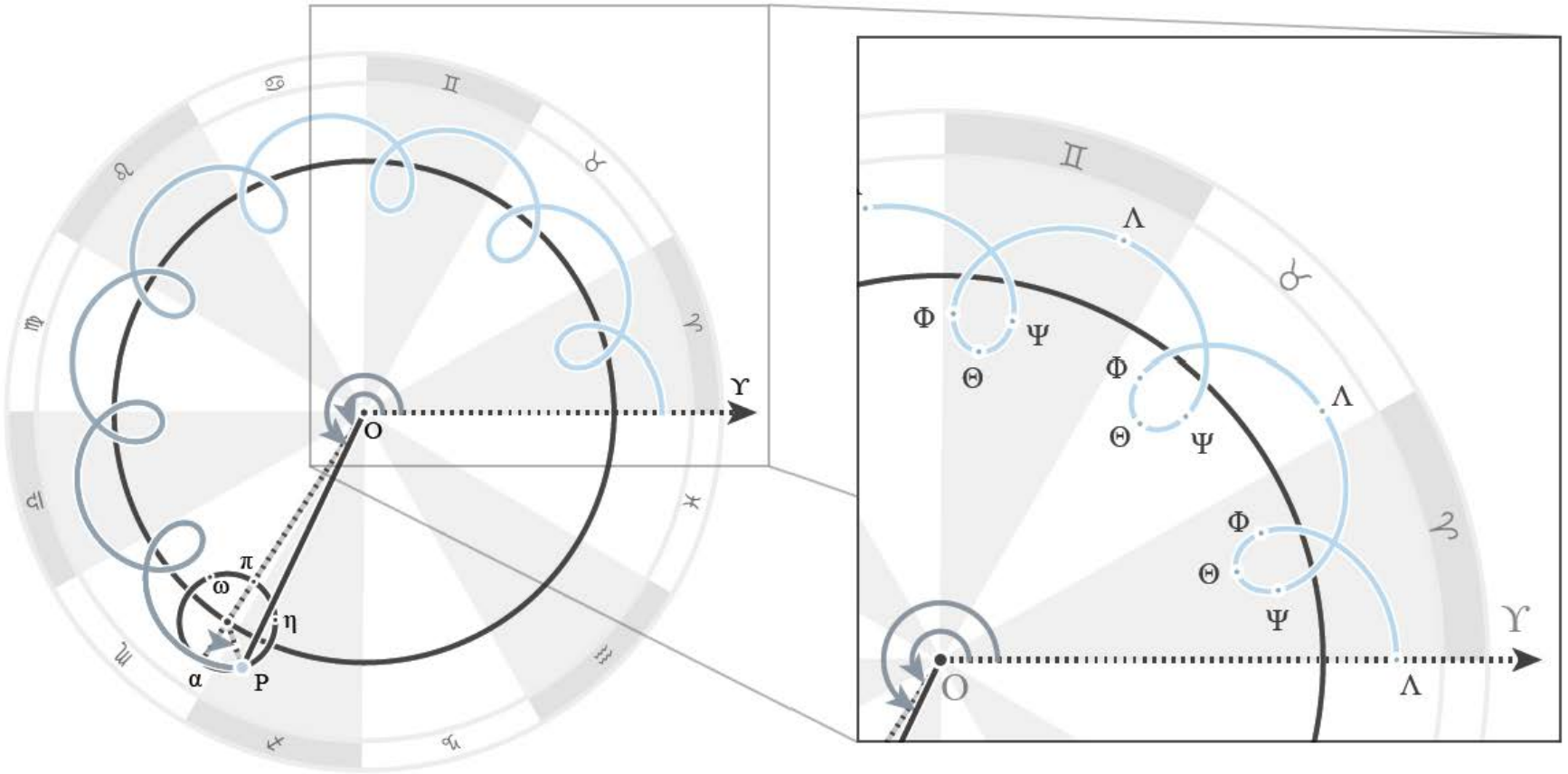
Superimposed and moving on

- large circle - deferent

- both epicycle and deferent rotate clockwise
- roughly parallel to ecliptic
- Explains retrograde motion of planets on the sky

Epicycle Model: essentials

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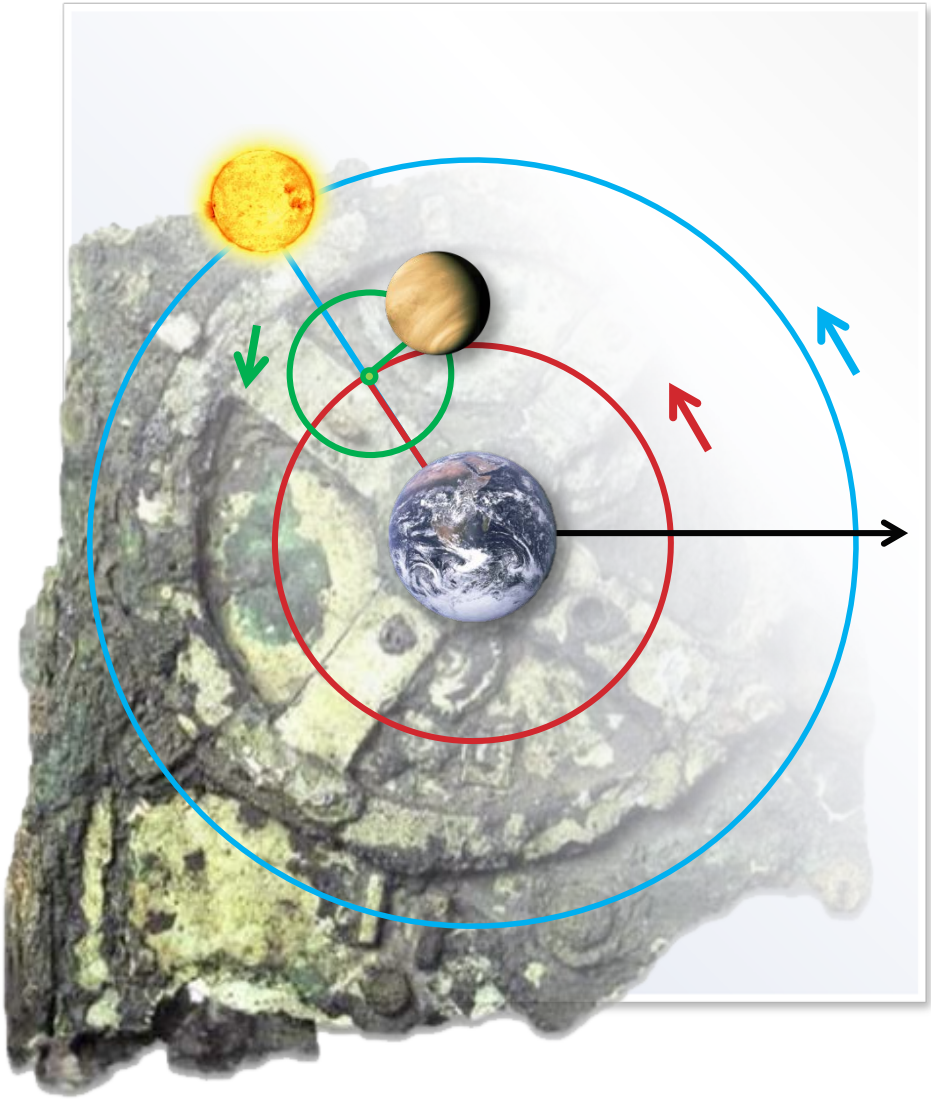
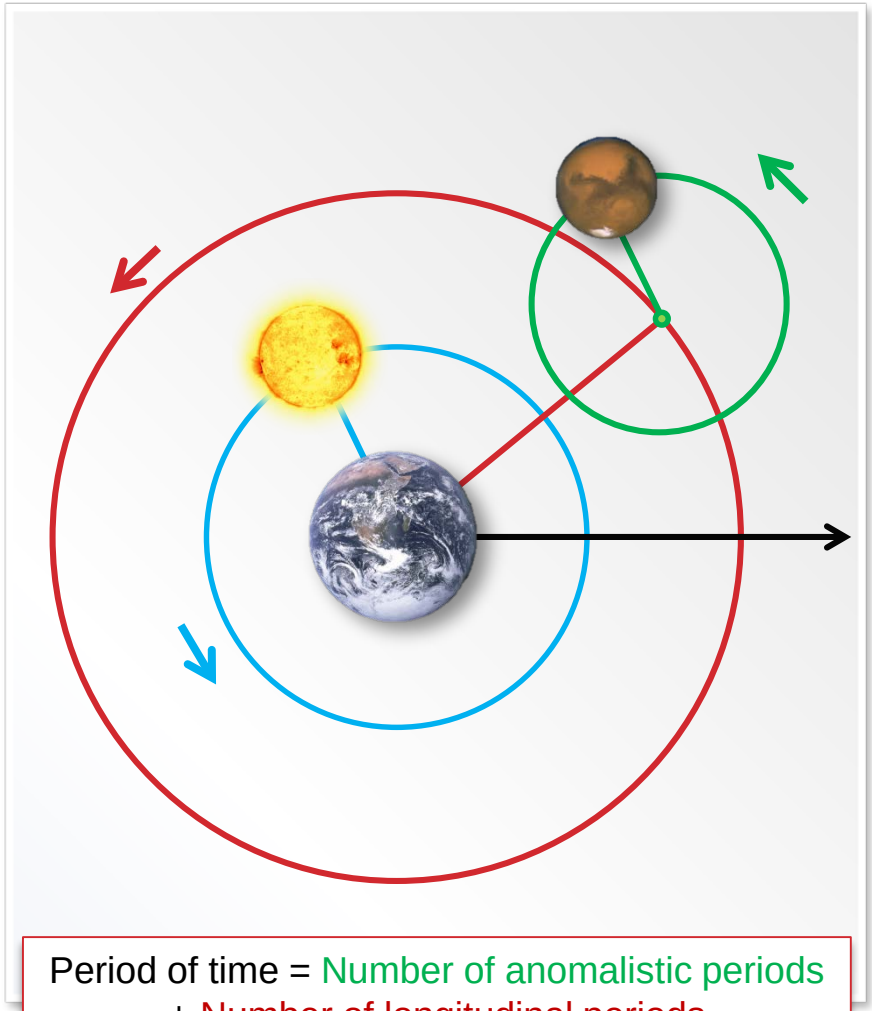
Inferior and Superior planets



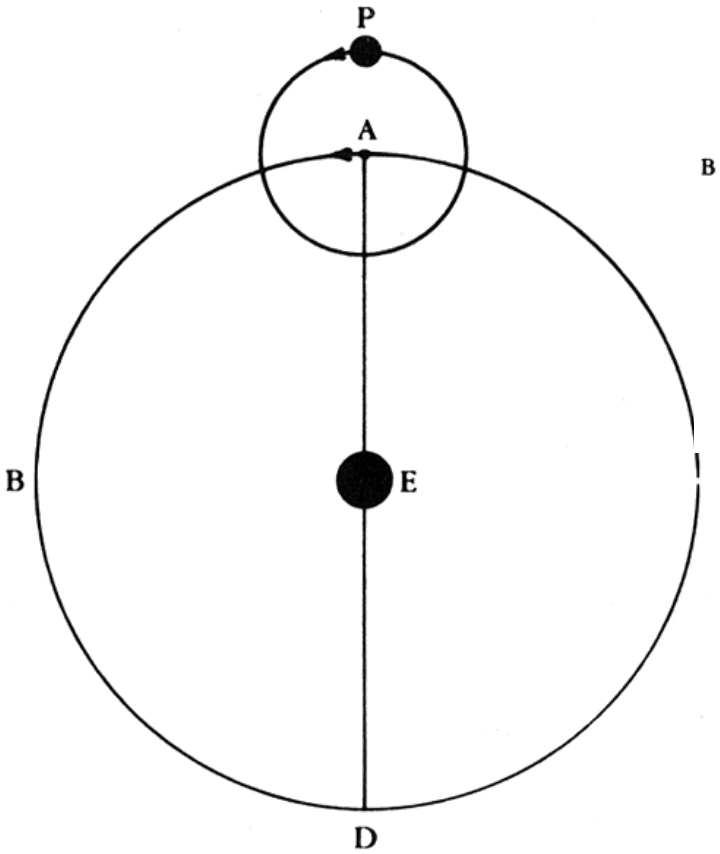


Period of time = Number of anomalistic periods
+ Number of longitudinal periods

Ptolemaeus Epicycle Theory: Equant

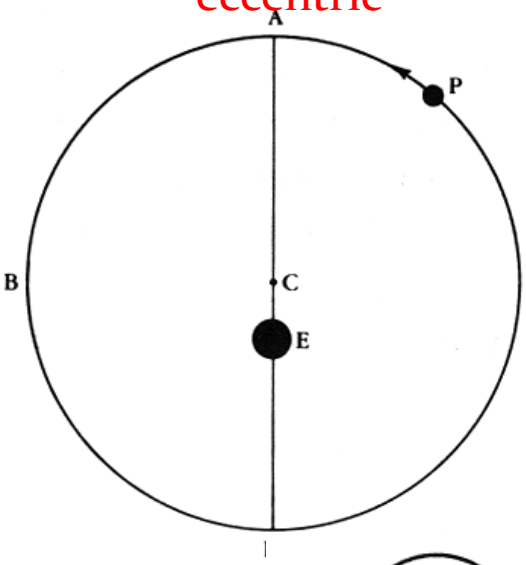
- The angular rate at which the epicycle traveled was not constant unless he measured it from another point which he called the *equant*.
- It was the angular rate at which the deferent moved around the point midway between the equant and the Earth (the eccentric) that was constant;
- the epicycle center swept out equal angles over equal times only when viewed from the equant.
- It was the use of equants to decouple uniform motion from the center of the circular deferents that distinguished the Ptolemaic system.

Ptolemaeus Epicycle Theory

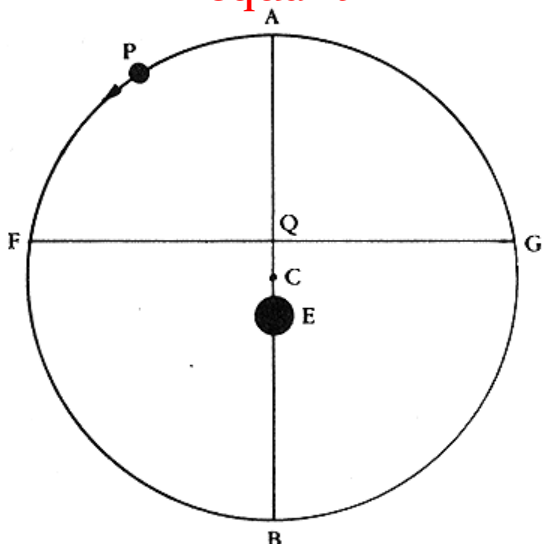


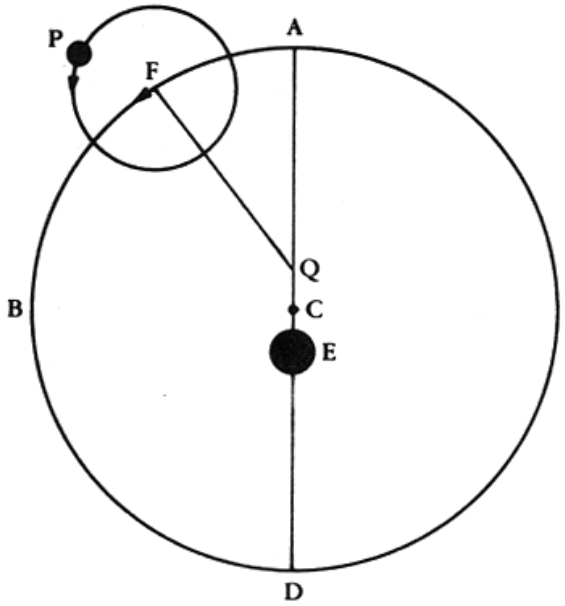
epicycle (basic)

eccentric



equant

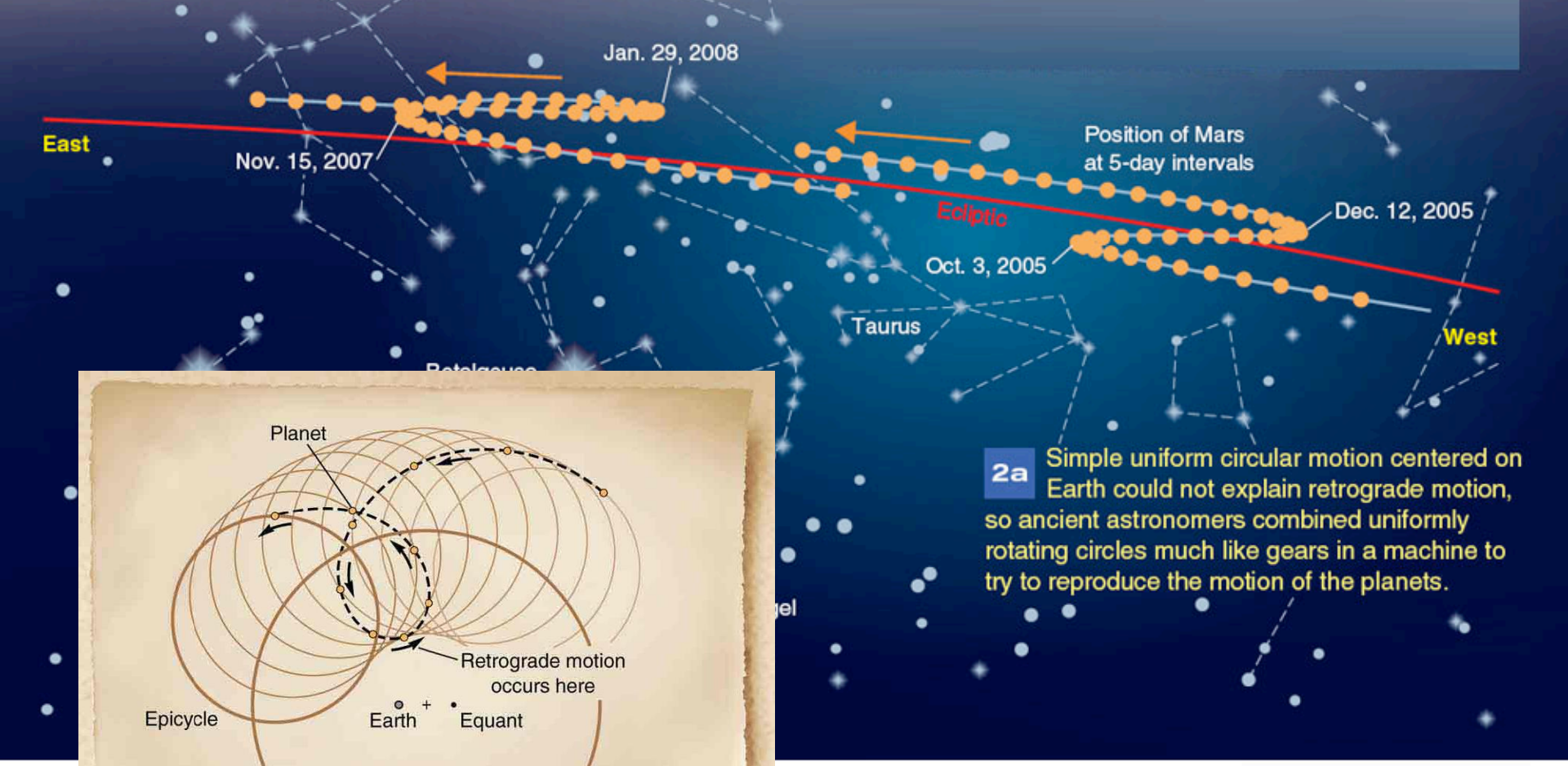


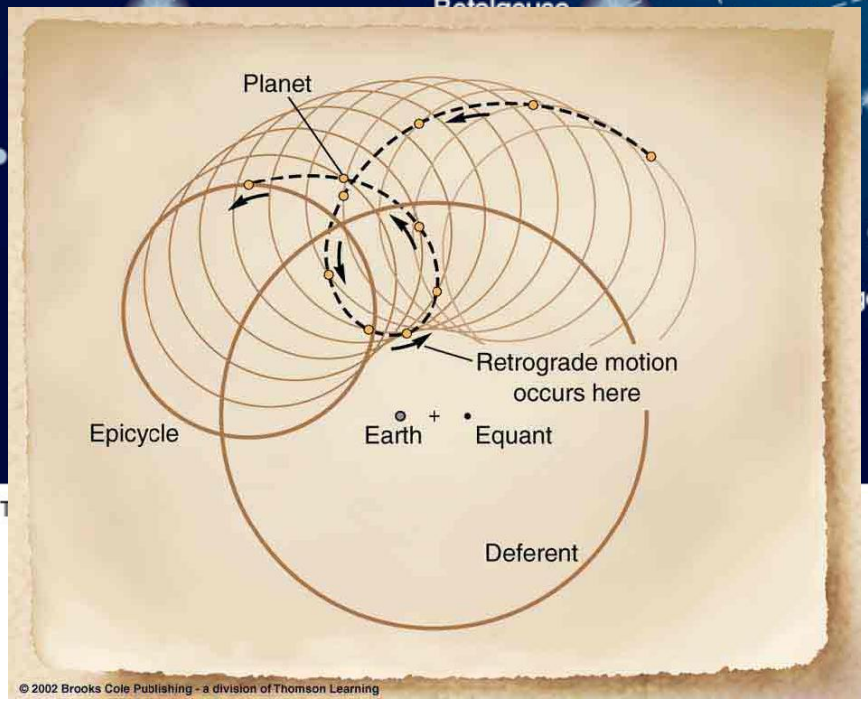


Ptolemaeus Planet Orbits

Ptolemaeus estimates of orbit sizes

Body	Mean size (in Earth radii)	Modern value (semi-major axis, in Earth radii)	Ratio (modern/Ptolemy)	Ratio (modern/Ptolemy, normalized to sun = 1)
Moon	48	60.3	1.26	0.065
Mercury	115	9090	79.0	4.1
Venus	622.5	16,980	27.3	1.4
Sun	1210	23,480	19.4	1
Mars	5040	35,780	7.10	0.37
Jupiter	11,504	122,200	10.6	0.55
Saturn	17,026	225,000	13.2	0.68
Star shell	20,000	----	----	----





Ptolemaeus Epicycle Theory

Geocentric Universe

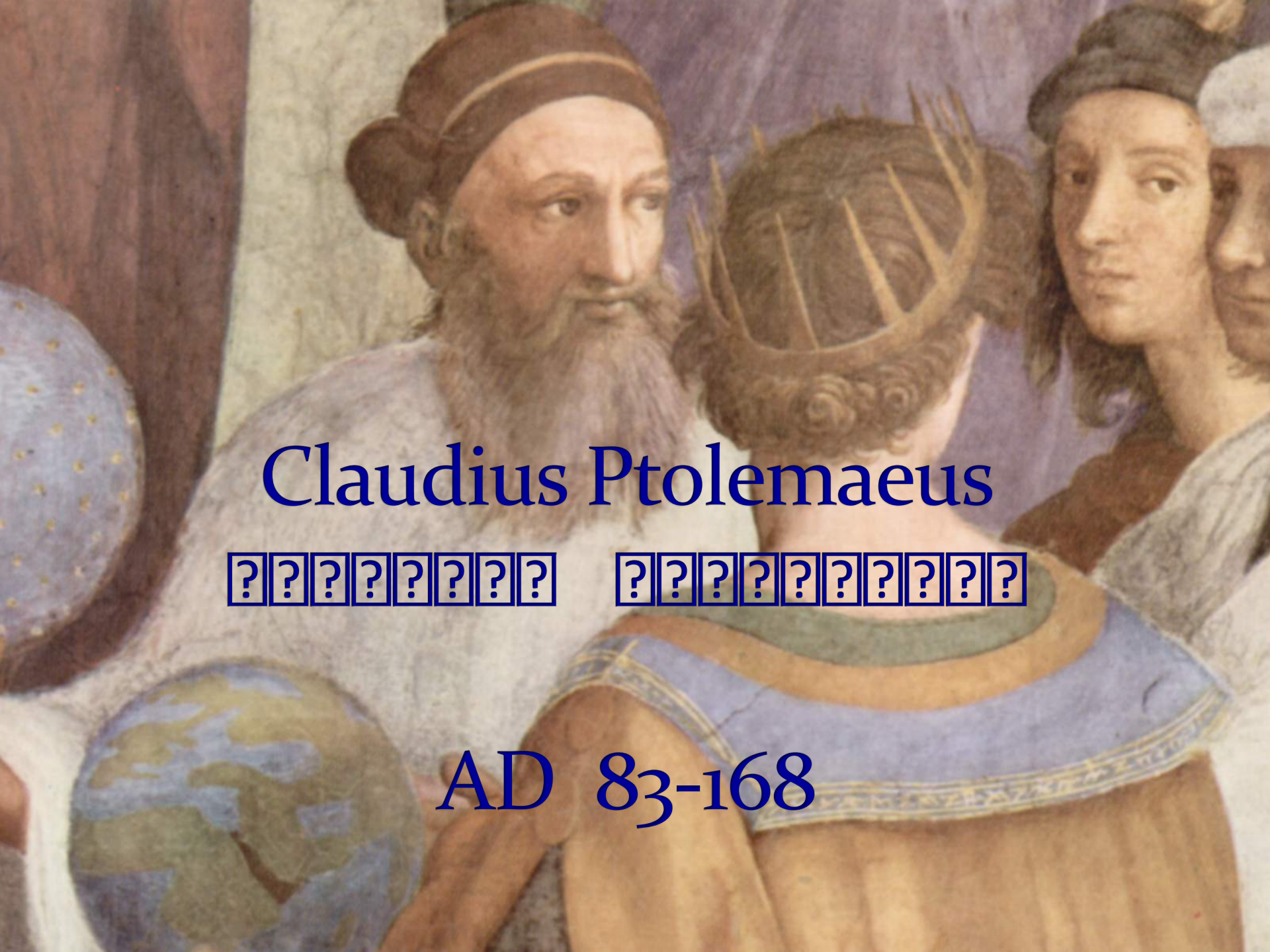
Ptolemaeus' Geocentric Universe is one of the 2 world world systems under discussion in Galileo Galilei's book (1632)

Dialogue Concerning the Two Chief World System

It had to give way to the heliocentric Universe, 1500 yrs after the Almagest

Claudius Ptolemaeus
Κλαυδιος Πτολεμαιος

AD 83-168



Claudius Ptolemaeus

□□□□□□□□ □□□□□□□□□

AD 83-168

Claudius Ptolemaeus

Thebaid/Ptolemais Hermiou-Alexandria 83-168 A.D.

- Mathematician
- Astronomer
- Geographer
- Astrologer

additional interests in

- Optics
- Music
- Philosophy

Claudius Ptolemaeus

- Culmination & Synthesis
  Hellenistic Astronomy
  Geography in Classical World

- Lasting and dominant influence,
  > 1500 yrs,
  European & Islamic science

Ptolemy's Bibliography

- *Almagest* (13 books) — astronomy
- *Geography* (8 books) — geography
- *Optics* (5 books) — physics
- *Tetrabiblos* (4 books) — astrology
- *Harmonics* (3 books)
- *Planetary Hypotheses* (2 books) — astronomy
- *Analemma*
- *Canobic Inscription*
- *Planispherium* — astronomy
- Other astronomical works
- Lost works

Almagest

Geografia

Tetrabiblos

Geografia

Geografia

- Standard of geographical theory until the 1500's, until age of exploration•
  Only surviving geographical treatise from antiquity

- Represents whole corpus of geographical knowledge acquired in Graeco-Roman antiquity

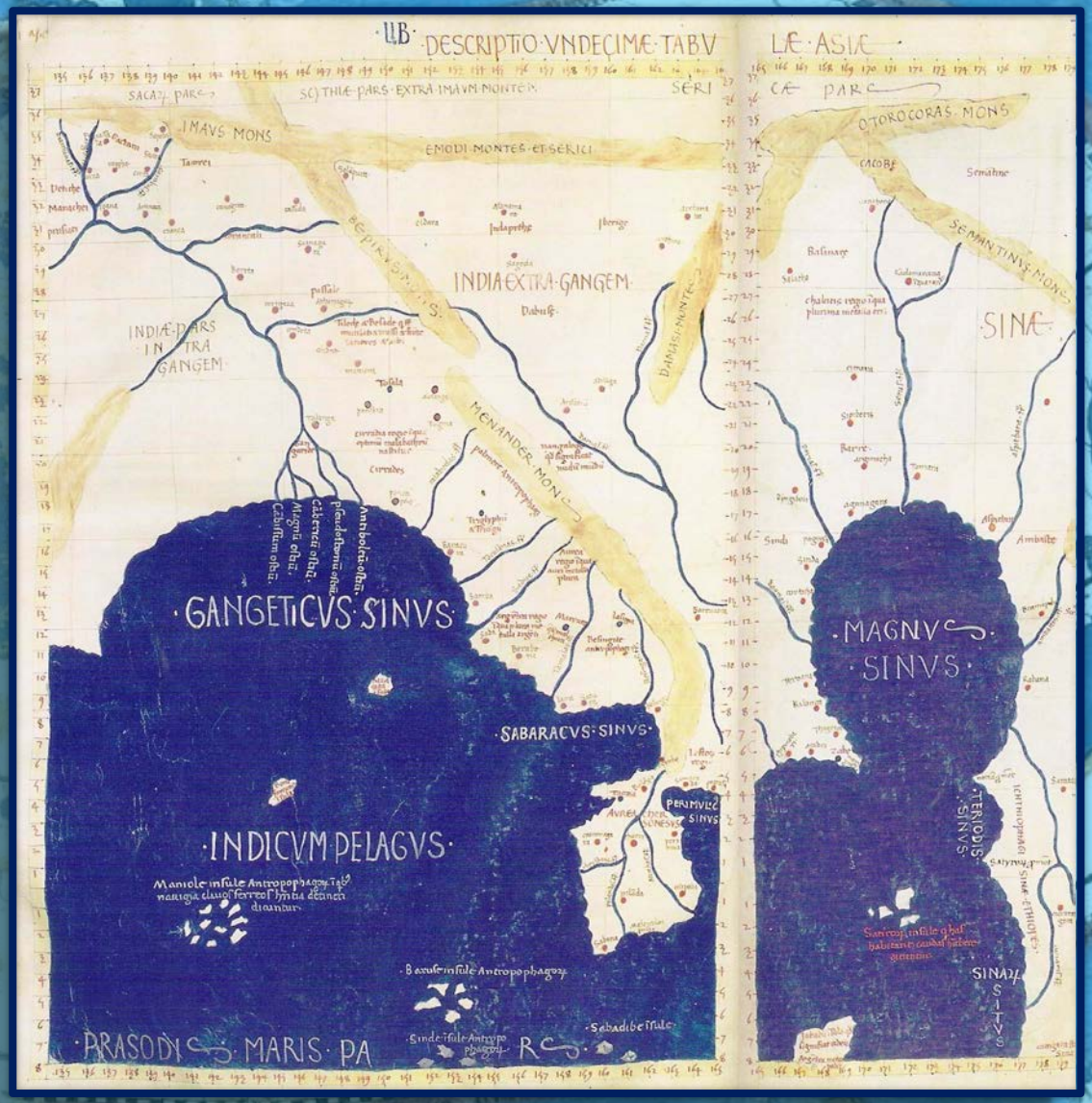
- Mapping earth with mathematical procedures from astronomy: coordinates, parallels, meridians

- Divided into 8 books:
  - Book 1: Introduction and directions to recreate Ptolemy's Map
  - Books 2-7: Latitudinal and longitudinal data for ~ 8,000 cities
  - Book 8: Description of 28 regional maps

Map of Ancient India:

crucial importance for development Roman trade network with India

Geografia

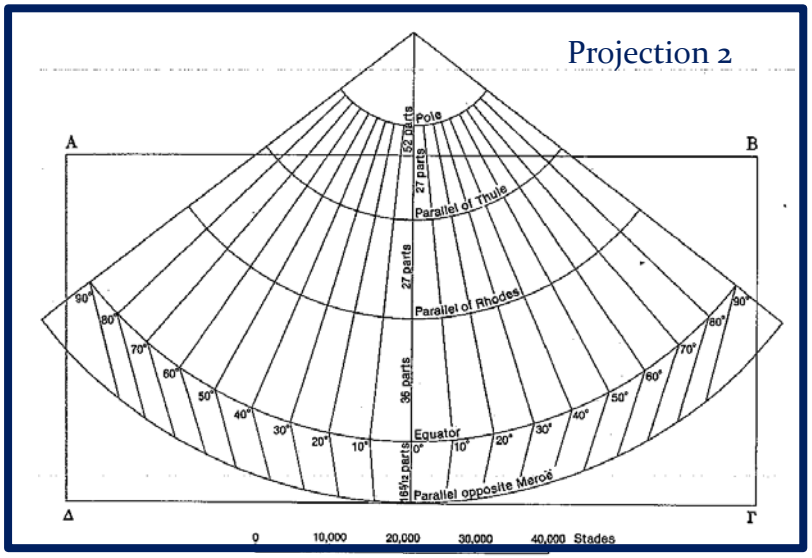


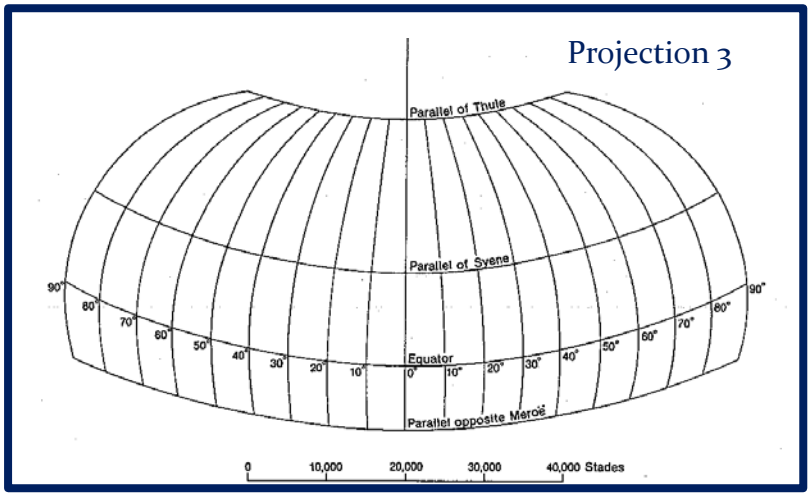
Geografia - Oikoumene/□□□□□□□□

- Based on the work of his immediate predecessor,
  Marinus of Tyre (80-130 A.D.)
- Combined data from a variety of sources
- Ptolemy was the first geographer to use longitude and latitude to create coordinates
- Derived 21 latitude lines – fairly accurate
- Described 4 different projections
- Believed the *oikoumene* (inhabited world) to span
  180 degrees (longitude) of the earth's 360
- Limits of the *oikoumene*
  - Northern bound: 63°N (the Thule parallel)
  - Southern bound: 16°25'S
    (the parallel opposite the equator from the one running through Meroë)
  - *Oikoumene* stretches from
    the Canary Islands in the west to China in the east

Geografia: four Map Projections

- Projection 1
  - Straight meridians & Straight parallels
  - Very similar to Marinus' map
- Projection 2
  - Straight meridians & Curved parallels
  - Preferred method of Ptolemy's successors
  - Constant scale in relation to Rhodes parallel
  - 36+1 parallel meridians, each 5 degrees apart
- Projection 3
  - Curved meridians & Straight parallels
  - made extreme parallels more accurate
- Projection 4
  - View of globe from distance
  - External rings represent latitude lines

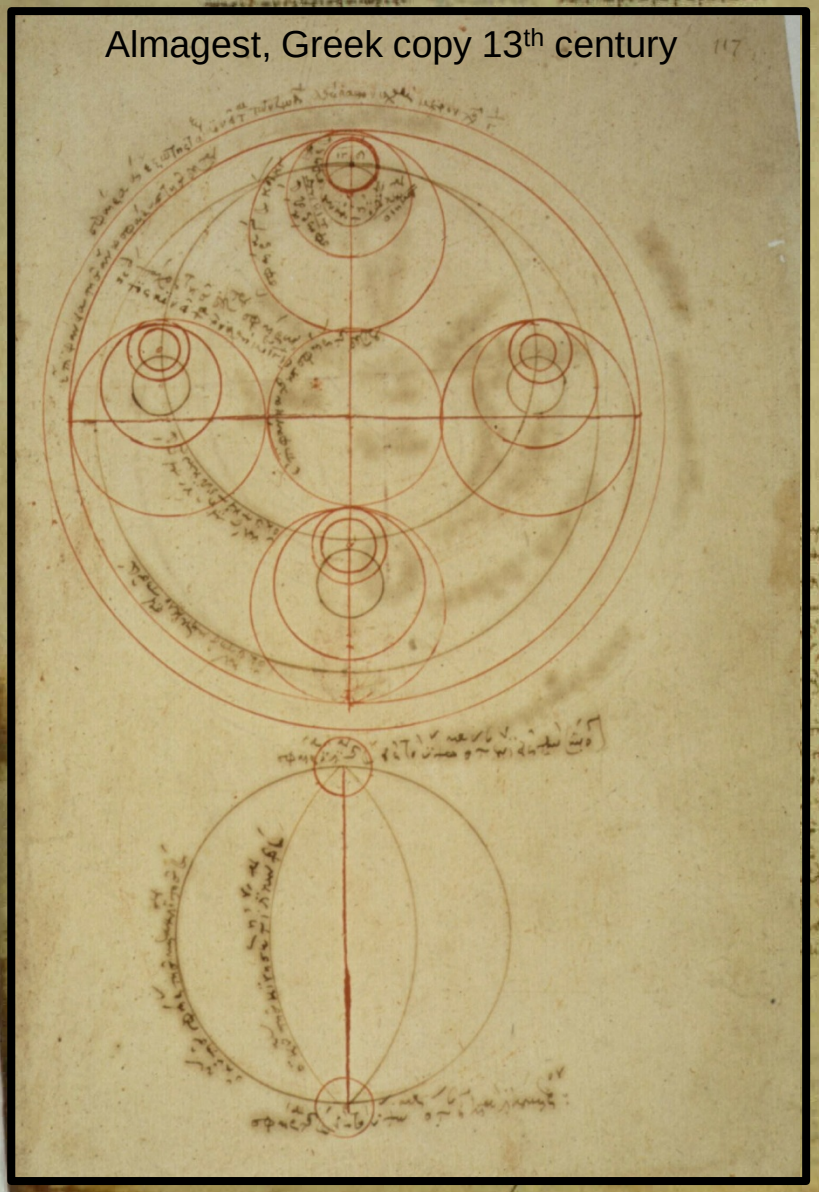




Almagest

Ι Μεγαλι Συνταξιζ
Μαθηματικη Συνταξιζ

?????????? ??????? - Almagest


Almagest, Greek copy 13th century

"The Great Book"

Most Important & Influential Astronomical Work of Antiquity

- Mathematical and Astronomical treatise proposing the complex motions of stars and planetary paths

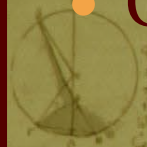
- Written in 147 / 148 A.D.:
  inscription in Canopus, by Ptolemaeus

- Thirteen Books

- Original in Greek:
  Mathematike Syntaxis - Mathematical Treatise
  He Megale Syntaxis - "The Great Treatise"

- Best known by its Arab name:
  Almagest - "The Great Book"

Almagest the Greatest



- One of the most influential scientific works in history

  (along with Euclid's "Elements", Copernicus' "Revolutionibus",
  Galilei's "Dialogues" Newton's "Principia",
  Darwin's "Origin of Species")

- One of most influential books of all time

  (perhaps only after Bible, Qur'an,
  along with Euclid's "Elements", ...)

Almagest the Greatest

- Systematic methodology

Observational Data

⟺

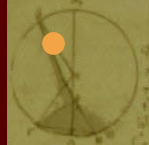
Numerical parameters planetary models

⟺

Construction tables celestial phenomena

(solar, lunar & planetary positions; solar & lunar eclipses, ...)

Almagest the Greatest



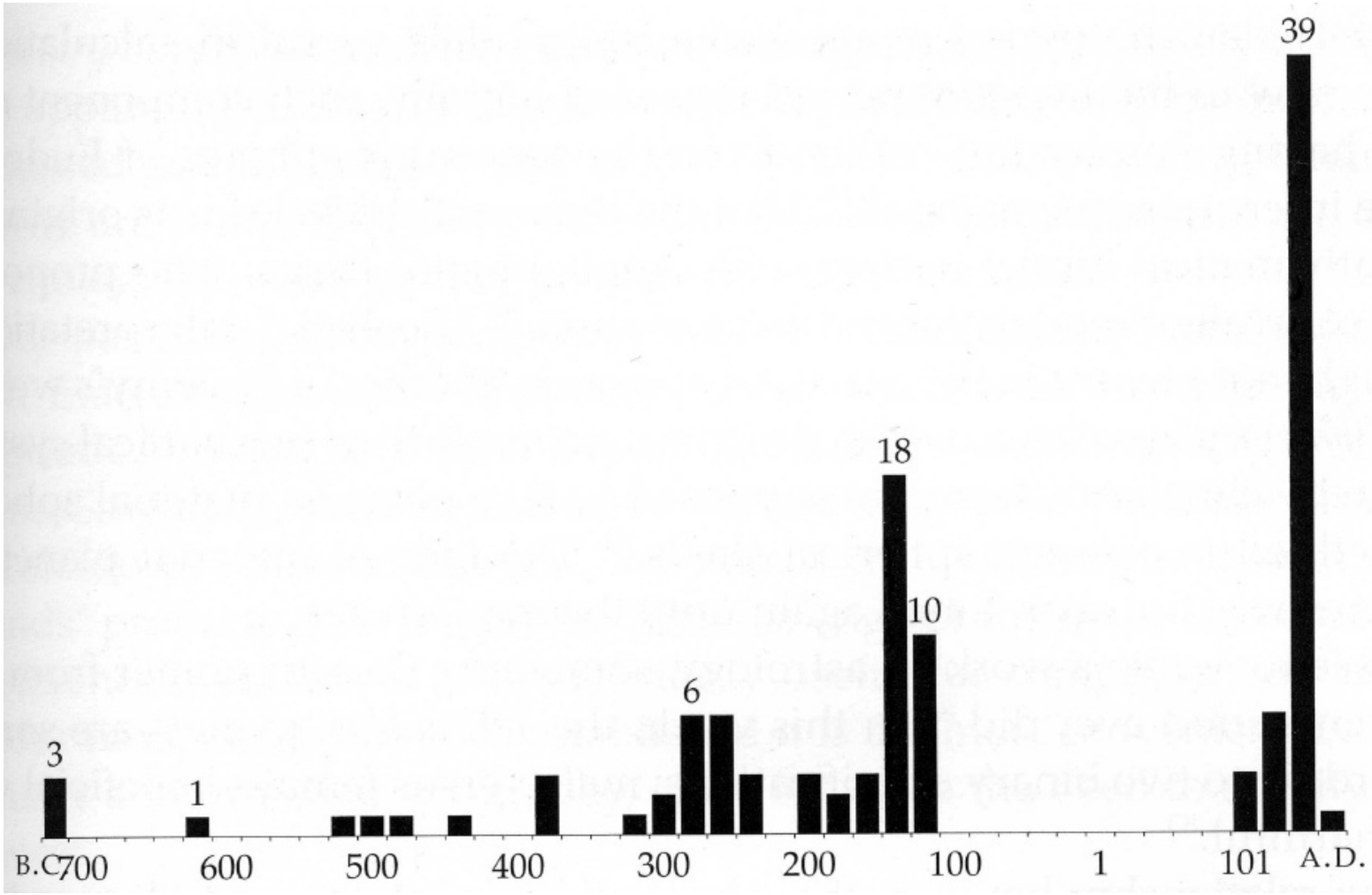
Success Almagest:

Loss of most of work scientific predecessors:

being obsolete, they ceased to be copied

Is this true ?

Almagest the Greatest

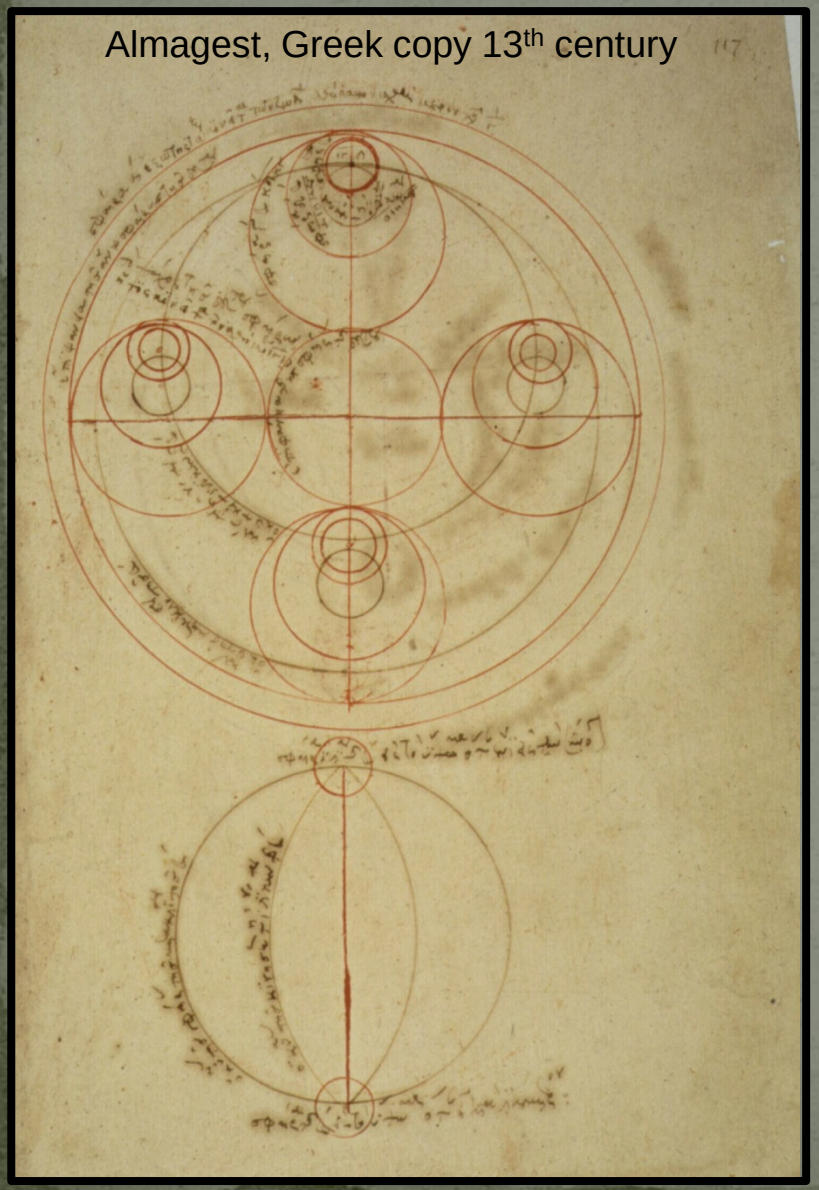


Μαθηματικη Συνταξιζ

Almagest - the Greatest

Almagest, Greek copy 9th century

Syntaxis - Almagest


Almagest, Greek copy 13th century

"The Great Book"

most Important & Influential Astronomical Work of Antiquity

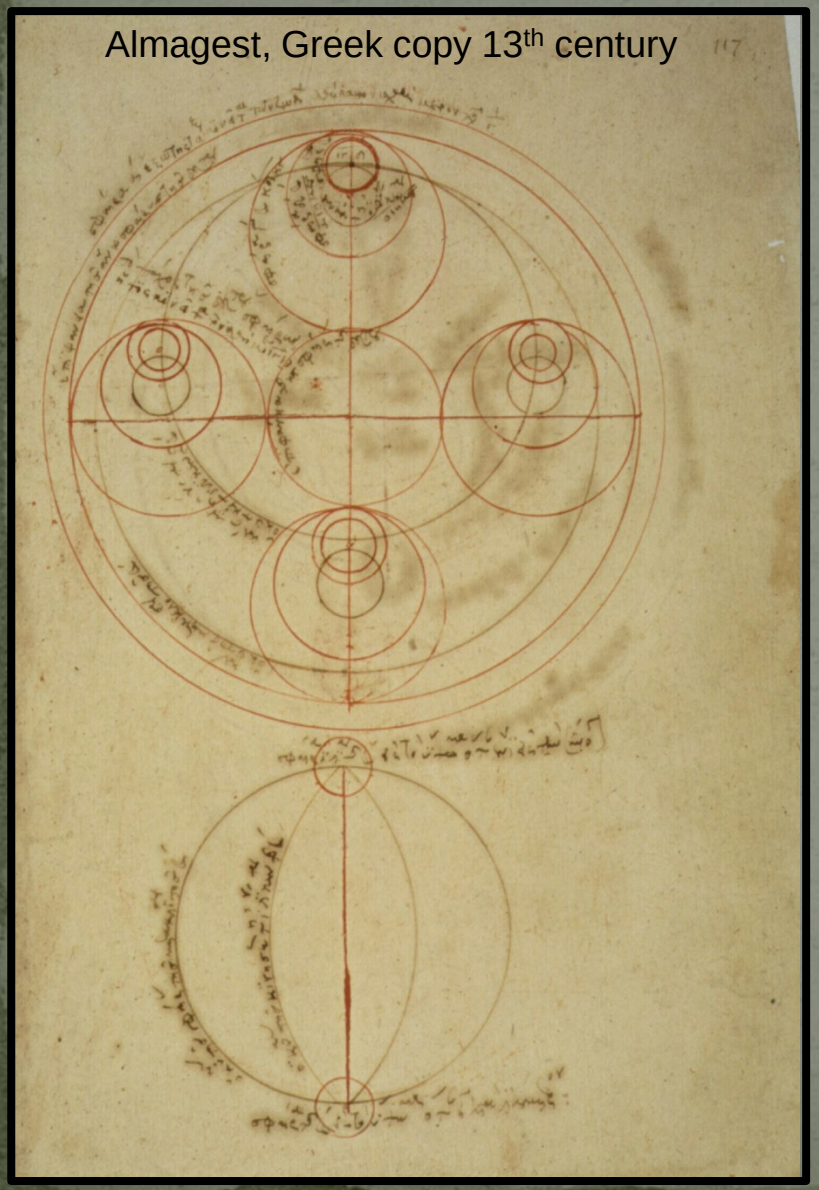
Ptolemy first scientist to spell out inductive method:
- models framed from preliminary facts
- expand models by logical induction
- testing hypothesis against reality

Only surviving comprehensive ancient treatise on astronomy:
- most important source of information on ancient Greek astronomy

Geocentric Model
Epicycle Theory

Dominated astronomy for > 13 centuries
- Roman (Byzantine, Western) world
  Arab world

Syntaxis - Almagest


Almagest, Greek copy 13th century

"The Great Book"

most Important & Influential Astronomical Work of Antiquity

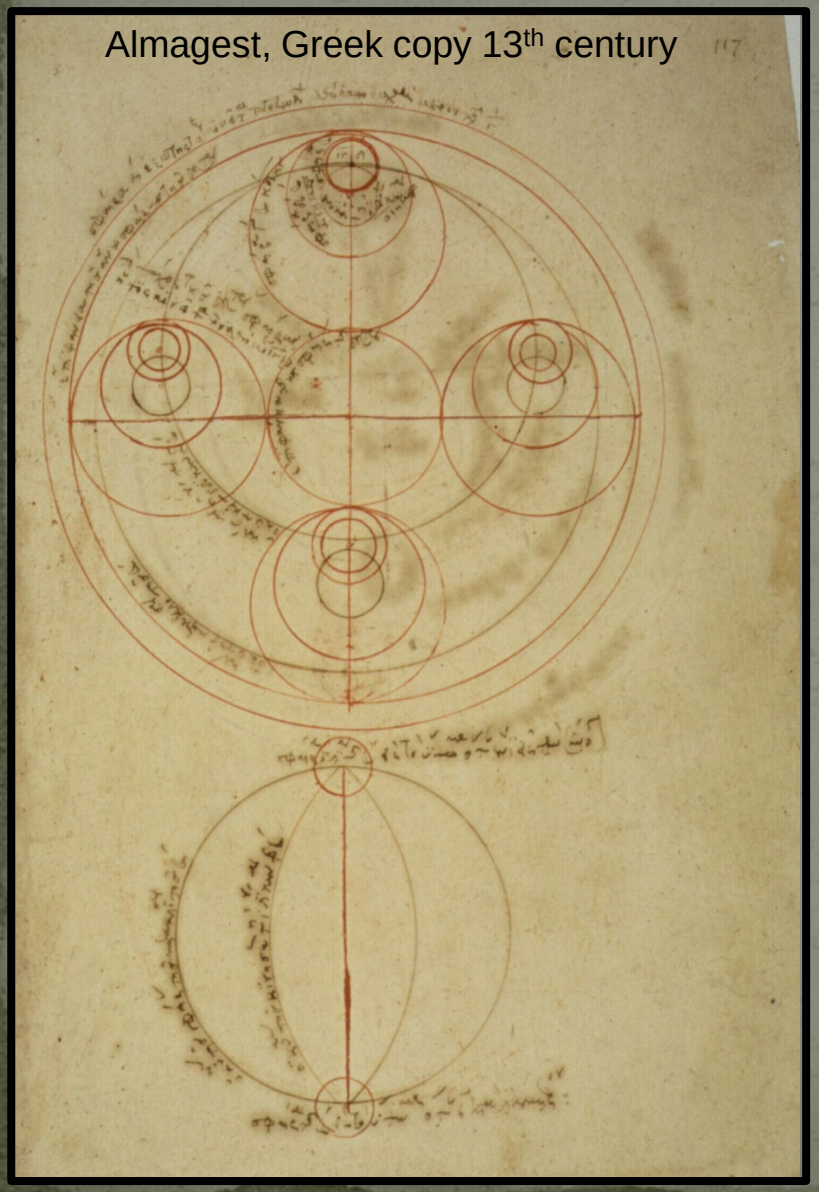
Geometrical models based on 800 yrs observations
(Babylonians, Hipparchus, …)

Models presented in convenient tables

Calculations fairly accurate for prediction
solar and lunar eclipses

Almagest also contains star catalogue
- appropriated version Hipparchus' catalogue
- 48 constellations: modern ones, not full sky

Syntaxis - Almagest

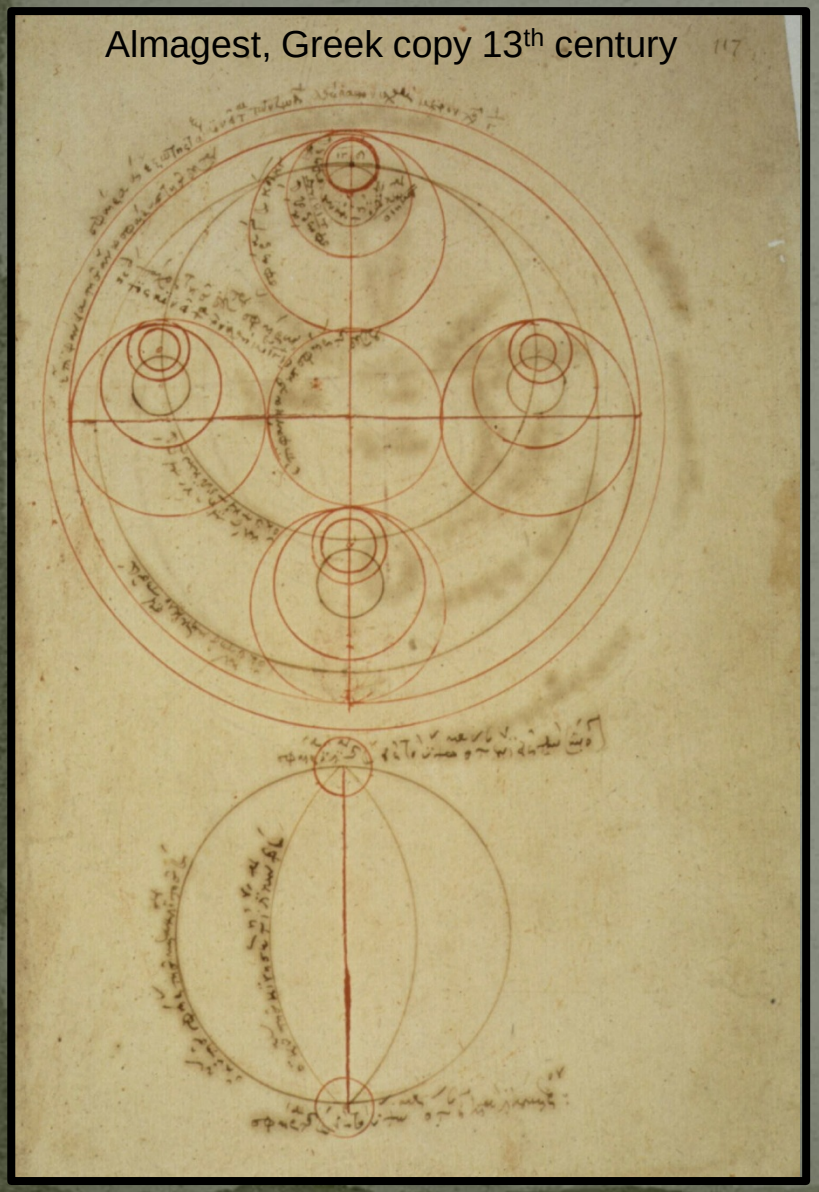

Almagest, Greek copy 13th century

Ptolemaeus' Cosmos

The cosmology of the *Almagest:*
five main points
each subject of a chapter Book I.

- The celestial realm is spherical,
  and moves as a sphere.
- The earth is a sphere.
- The earth is at the center of the cosmos.
- The earth,
  in relation to the distance of the fixed stars,
  has no appreciable size,
  must be treated as a mathematical point
- The earth does not move.

Syntaxis - Almagest



Ptolemaeus' Planetary Models

Order of planetary spheres:

- Moon
- Mercury
- Venus
- Sun
- Mars
- Jupiter
- Saturn
- Sphere fixed stars

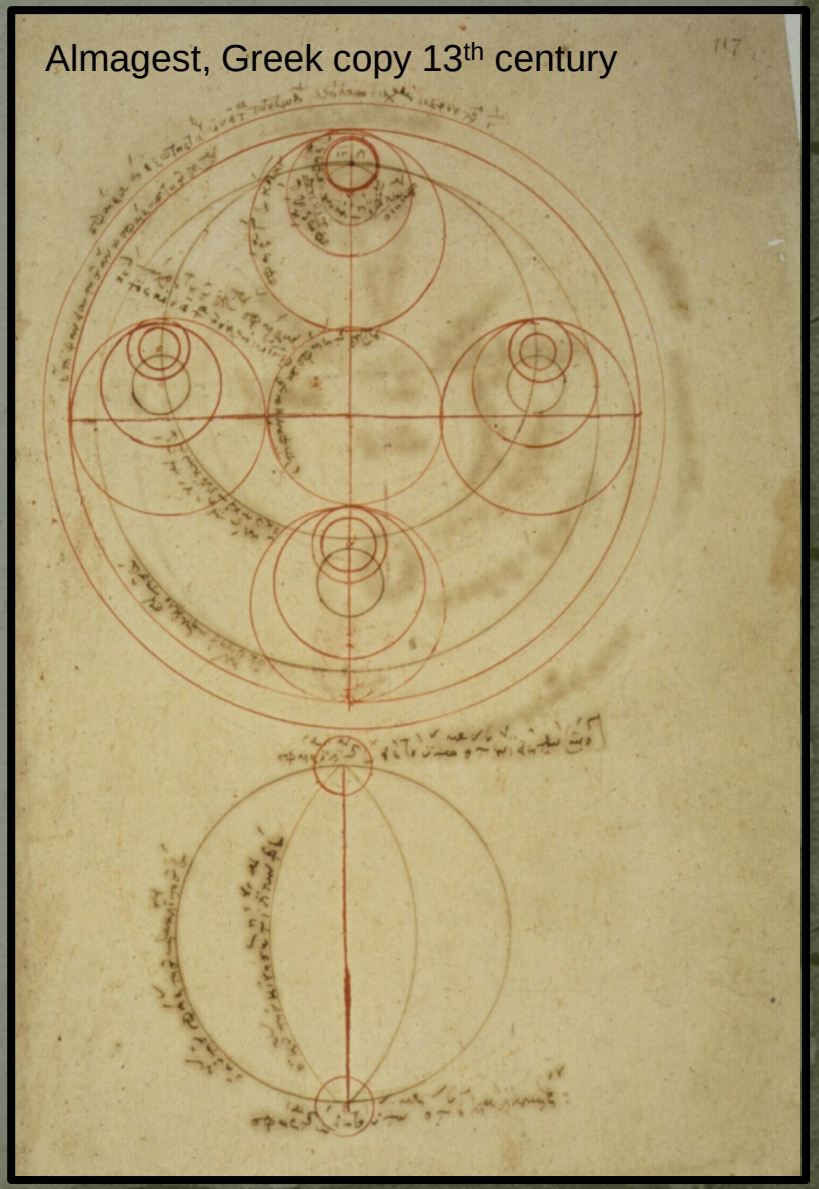
Almagest:

Detailed Inventory

Syntaxis - Almagest

Almagest: 13 books

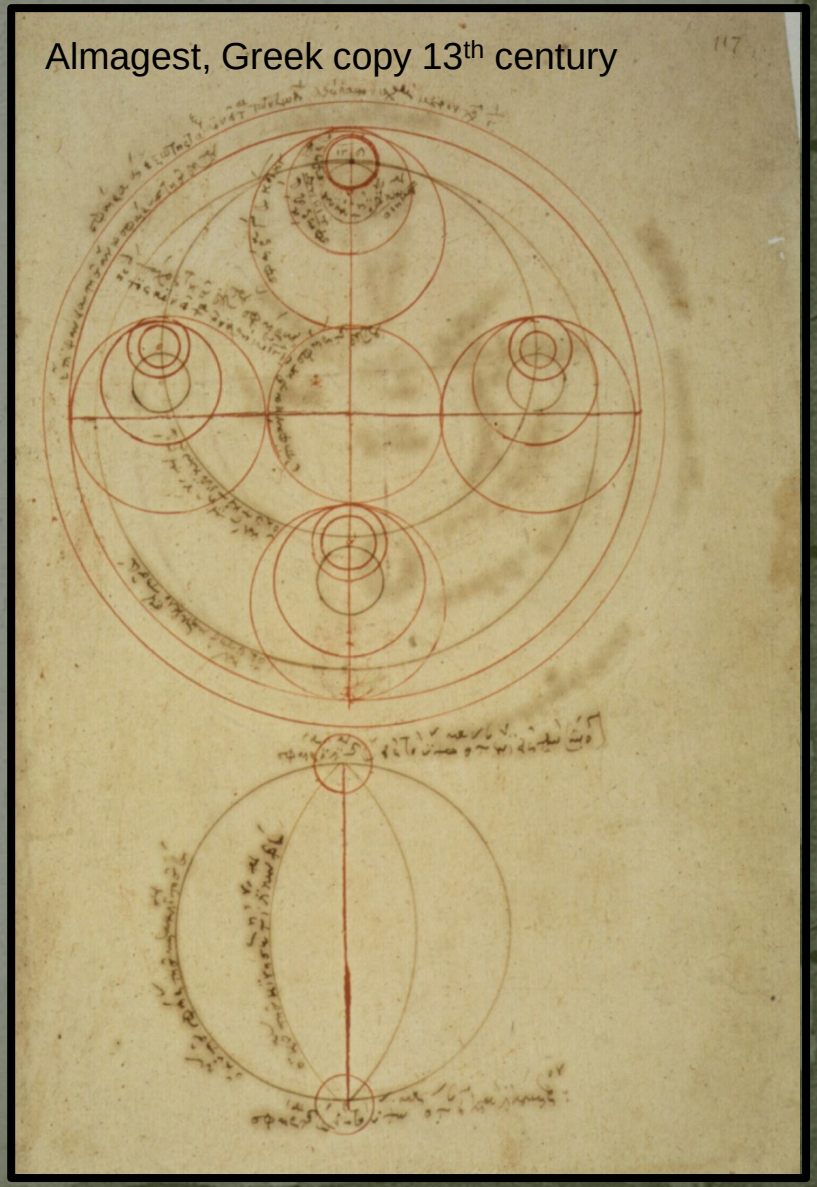
- Book III:
  - length of the year, and the motion of the Sun
  - explains Hipparchus' discovery of the precession of the equinoxes
  - begin explanation epicycles
- Books IV & V:
  the motion of the Moon:
  - lunar parallax
  - motion of the lunar apogee
  - sizes and distances of the Sun and Moon relative to Earth
- Book VI:
  solar and lunar eclipses



Syntaxis - Almagest

Almagest: 13 books

- Book I:
  outline of Aristotelian cosmology:
  - on the spherical form of the heavens,
  - the (spherical) Earth lying motionless at centre
  - the fixed stars and the various planets revolving around the earth
  - followed by explanation of chords with a set of chord tables
  - observations of the obliquity of the ecliptic
  - introduction to spherical trigonometry
- Book II:
  problems associated with the daily motion attributed to the heavens:
  - risings and settings of celestial object
  - length of daylight
  - determination of latitude
  - points at which the Sun is vertical
  - shadows of the gnomon at the equinoxes and solstices
  - other things which change with the spectator's position. There is also
  - a study of the angles made by the ecliptic with vertical, with tables.

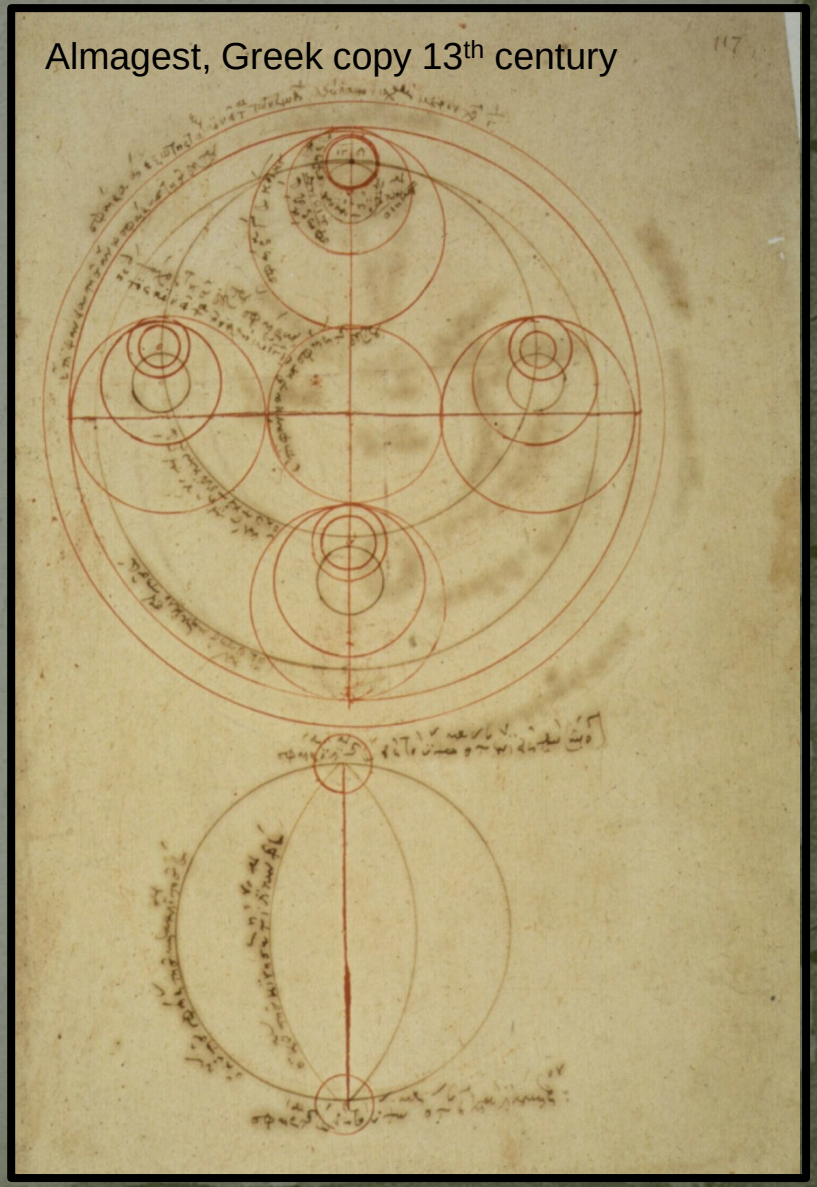

Almagest, Greek copy 13th century

Syntaxis - Almagest

Almagest: 13 books

•Books VII & VIII:

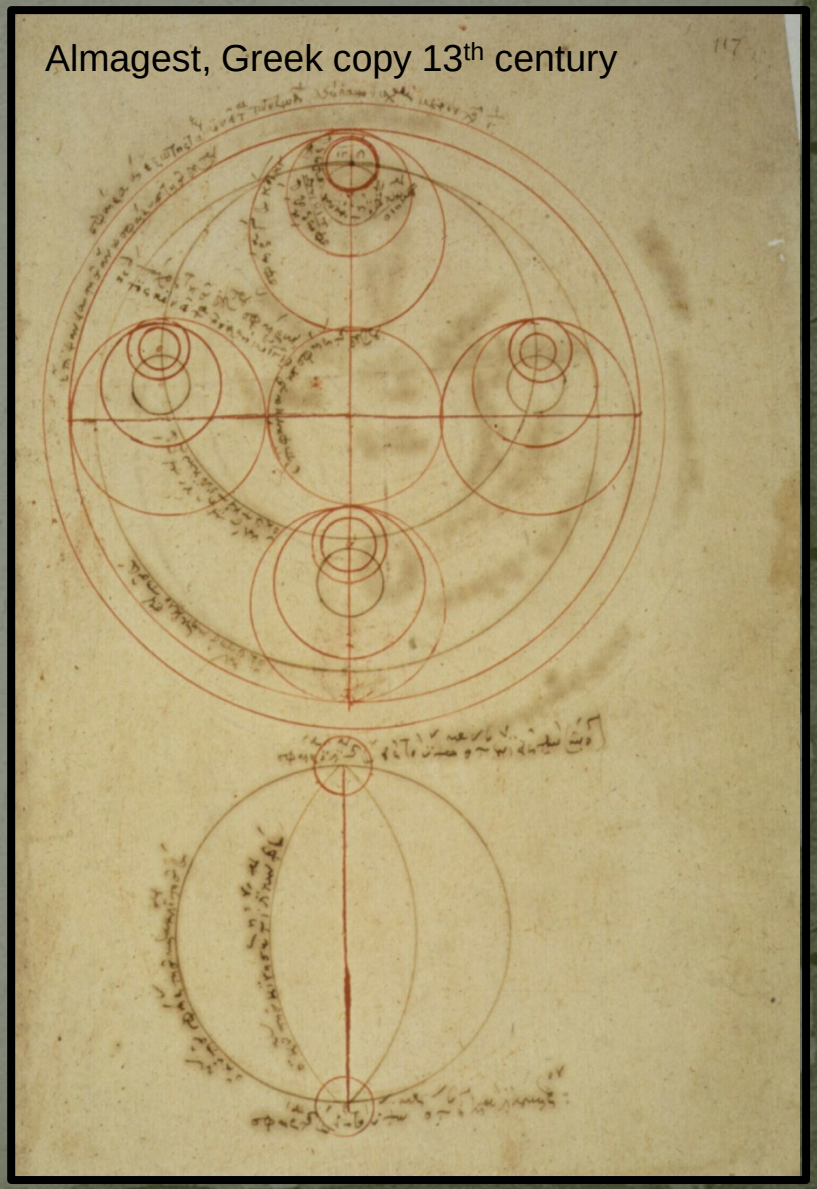
- motions of the fixed stars:
  - includes precession of the equinoxes
- star catalogue of 1022 stars:
  - described by positions in the constellations
  - magnitude scale for brightness:
    + brightness brightest stars marked of the 1st magnitude (m = 1),
    + faintest 6th magnitude (m = 6), limit human visual perception
    + each grade of magnitude considered twice the brightness of the following grade (log. scale).
    + system believed to have originated with Hipparchus
    + Stellar positions: Hipparchan origin (despite Ptolemy's claim to the contrary)
-



Syntaxis - Almagest

Almagest: 13 books

- Book IX:
  - general issues associated with creating models for the five (naked eye) planets
  - motion of Mercury
- Book X:
  motions of Venus and Mars
- Book XI:
  motions of Jupiter and Saturn
- Book XII:
  stations and retrogradations,
  - occurring when planets appear to pause, then briefly reverse their motion against the background of the zodiac.
  - Ptolemy understood these terms to apply to Mercury and Venus as well as the outer planets
- Book XIII:
  motion in latitude:
  the deviation of planets from the ecliptic



Epicycle Theory

Middle Ages

Ptolemaeus to Copernicus

One school in history of astronomy:

minor imperfections in the original Ptolemaic system were discovered through observations accumulated over time.

It was mistakenly believed that more levels of epicycles (circles within circles) were added to the models to match more accurately the observed planetary motions.

The multiplication of epicycles is believed to have led to a nearly unworkable system by the 16th century.

Copernicus created his heliocentric system in order to simplify the Ptolemaic astronomy of his day, thus succeeding in drastically reducing the number of circles.

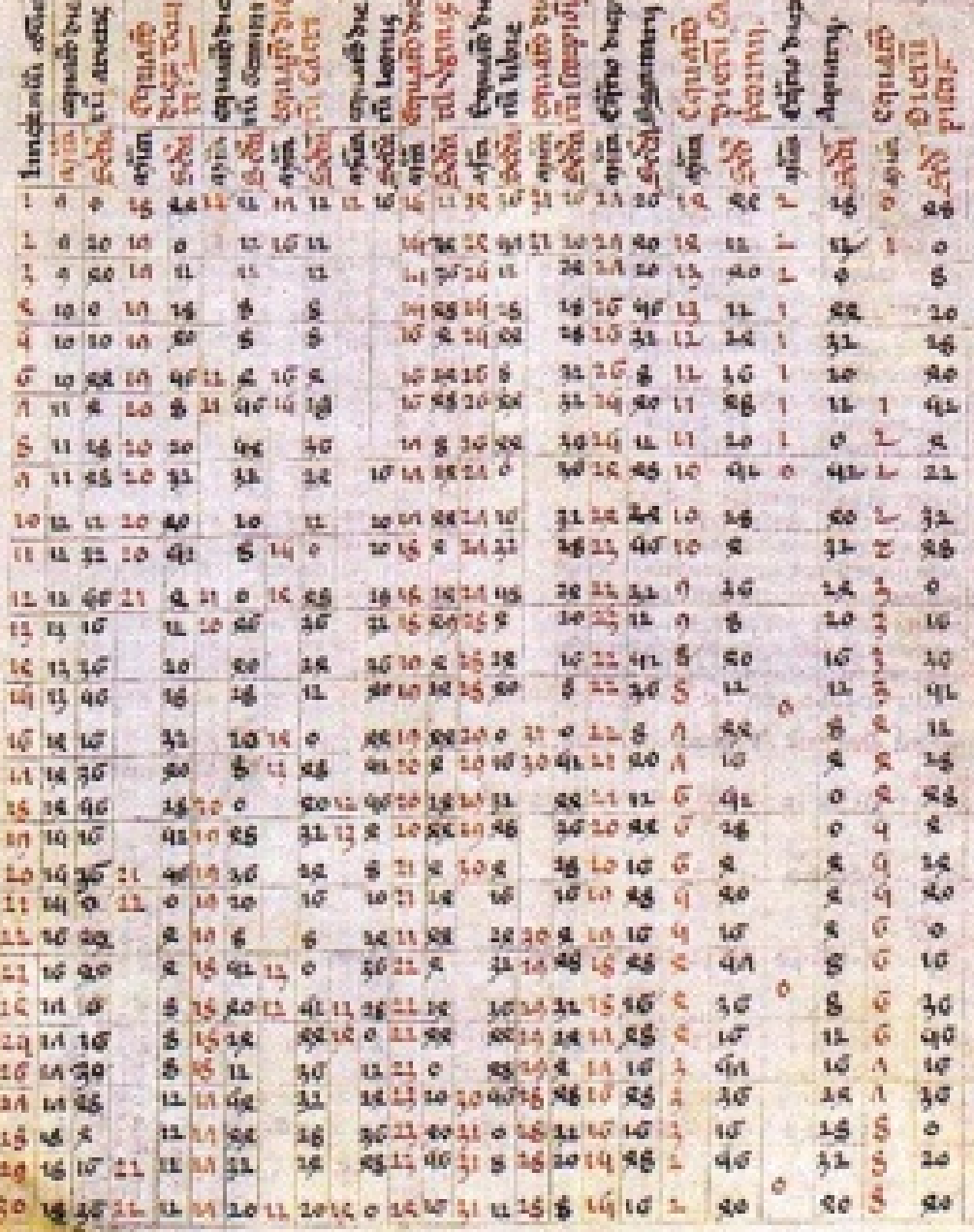
With better observations additional epicycles and eccentrics were used to represent the newly observed phenomena till in the later Middle Ages the universe became a 'Sphere/With Centric and Eccentric scribbled o'er,/Cycle and Epicycle, Orb in Orb'

Alfonsine Tables

- Tables of solar, lunar and planetary positions wrt. fixed stars
- Called after Alfons X of Castile the Wise (el Sabio) (1221-1284)
- composed in 1252, in Toledo
- assembled by translators of Toledo
- based on observations an studies of Islamic scholars

- most popular astronomical tables for 300 years (incl. Copernicus)

Tabula equationis dierum cum nocibus suis



Linea	Equatio dierum arietis		Equatio dierum tauri		Equatio dierum geminorum		Equatio dierum cancri		Equatio dierum leonis		Equatio dierum virginis		Equatio dierum libre		Equatio dierum scorpionis		Equatio dierum sagittarii		Equatio dierum capricorni		Equatio dierum aquarii		Equatio dierum piscium	
	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta
1	9	0	15	44	12	12	17	12	12	16	14	12	14	36	31	16	27	20	14	44	2	28	0	48
2	9	20	19	0		12	16	12			14	24	14	47	31	20	27	40	14	12	2	12	1	0
3	9	40	19	12		12		12			14	36	14	12		24	27	20	13	40	2	0		8
4	10	0	19	24		8		8			14	48	14	24		28	26	46	13	12	1	44		20
5	10	20	19	40		8		8			16	4	14	44		28	26	32	12	24	1	32		28
6	10	44	19	56	12	4	16	4			16	24	16	8		32	26	8	12	36	1	20		40
7	11	4	20	8	21	56	14	18			16	48	16	44		32	24	40	11	48	1	12	1	52
8	11	28	20	20		44		36			17	8	16	44		36	24	12	11	20	1	0	2	4
9	11	48	20	32		32		24		16	17	24	17	0		36	24	48	10	52	0	52	2	22
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12	12	56	21	4	21	0	14	48		28	18	24	17	48		24	23	32	9	36		24	3	0
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14	13	36		20		40		24		36	19	4	18	24		16	22	52	8	40		16	3	36
15	13	56		28		28		12		40	19	24	18	40		8	22	36	8	12		12	3	52
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19	14	16		52	19	48		32	13	4	20	44	20	48		36	20	44	6	28		0	4	4
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22	16	44		4	19	8		8		24	21	44		24	30	4	19	16	4	16		4	6	0
23	16	40		4	18	52	13	0		36	22	4		32	19	48	18	48	4	57		8	6	16
24	17	0		8	18	40	12	52	13	28	22	24		36	19	32	18	16	4	36	0	8	6	36
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30	18	36	22	12	17	20	12	20	14	0	24	16	31	12	18	8	14	16	2	40	0	40	8	40

Tabula commutationis additionis dierum longioris per altitudinem et commutatio altitudinis axis per medietatem additionis dierum

Altitudo axis		Medietas additionis dierum longioris		Altitudo axis		Medietas additionis dierum longioris		Altitudo axis		Medietas additionis dierum longioris		Altitudo axis	
Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta	Gradus	Minuta
0	30		12	14	30	6	47	30	30	14	44	24	30
1	0	0	16	16	0	7	11	31	0	14	12	26	0
1	30	0	37	16	30		24	31	30	14	31	26	30
2	0		42	17	0		40	32	0	14	40	27	0
2	30	0	6	17	30	7	54	32	30	16	8	27	30
3	0	1	19	18	0	8	9	33	0	16	27	28	0
3	30	1	32	18	30		23	33	30	16	47	28	30
4	0	1	44	19	0		38	34	0	17	7	29	0
4	30	1	48	19	30	8	52	34	30	17	28	29	30
5	0	2	12	20	0	9	8	35	0	17	49	30	0
5	30	2	24	20	30		23	35	30	18	9	30	30
6	0	2	38	21	0		39	36	0	18	30	31	0
6	30	2	52	21	30	9	55	36	30	18	51	31	30
7	0	2	4	22	0	10	11	37	0	19	12	32	0
7	30		38	22	30		27	37	30	19	34	32	30
8	0		52	23	0		43	38	0	19	57	33	0
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11	0	3	52	26	0		24	41	0	22	18	36	0
11	30	4	6	26	30		41	41	30	22	43	36	30
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12	30	4	32	27	30	13	14	42	30	23	34	37	30
13	0	4	4	28	0		30	43	0	23	2	38	0
13	30	6	1	28	30		47	43	30	24	27	38	30
14	0	6	14	29	0	14	4	44	0	24	56	39	0
14	30	6	29	29	30		20	44	30	24	24	39	30
15	0	6	43	30	0	14	37	45	0	24	53	40	0

Ptolemaeus to Copernicus

- As a measure of complexity:

  - the number of circles is given as 80 for Ptolemy,
  - versus a mere 34 for Copernicus

- *By this time each planet had been provided with from 40 to 60 epicycles to represent after a fashion its complex movement among the stars. Amazed at the difficulty of the project, Alfonso is credited with the remark that had he been present at the Creation he might have given excellent advice.*