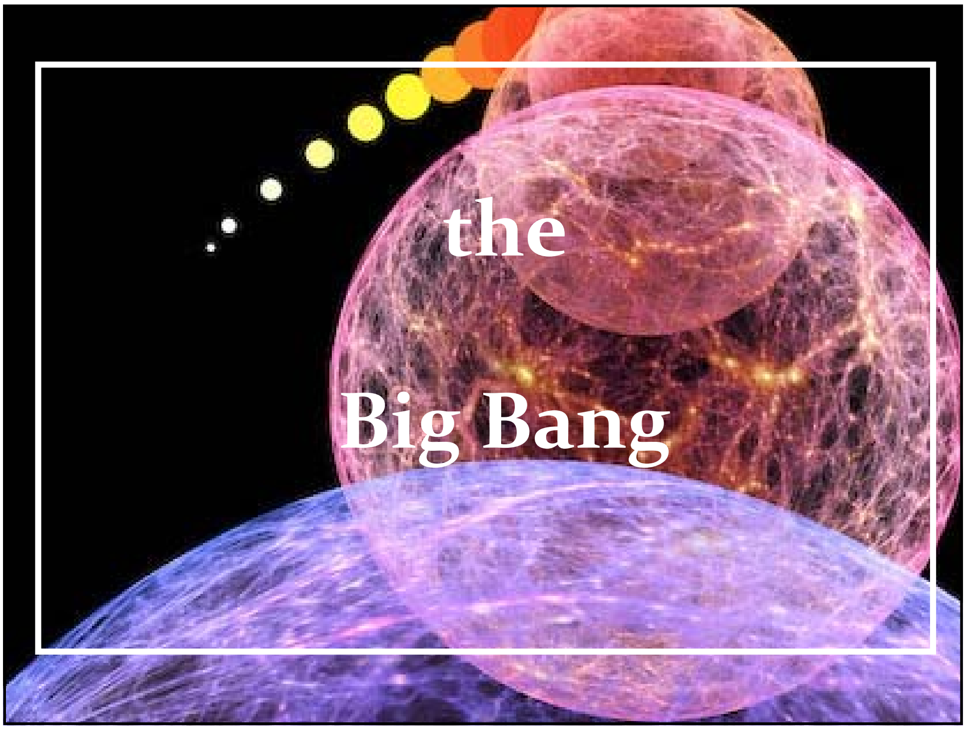
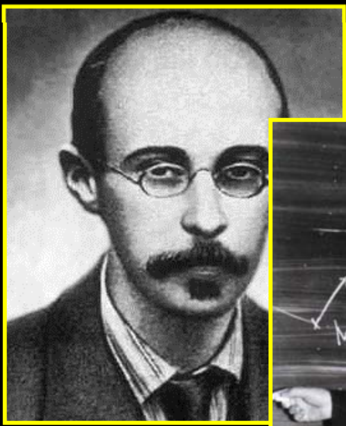


Friedmann, Lemaitre

&

Cosmic Expansion History

Friedmann & Lemaitre



Alexander Friedmann (1888 -1925)
George Lemaitre (1894-1966)

They discovered (independently) theoretically the expansion of the Universe as a solution to the Theory of General Relativity.

... and derived the equations that describe the expansion and evolution of the universe,

the foundation for all of modern Cosmology:

Friedmann-Lemaitre Equation

Evolving Universe

- Einstein, de Sitter, Friedmann and Lemaitre all realized that in General Relativity, there cannot be a stable and static Universe:
- The Universe either expands, or it contracts ...

- Expansion Universe encapsulated in a

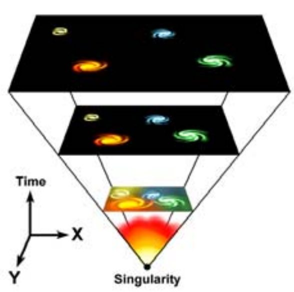
GLOBAL expansion factor a(t)

- All distances/dimensions of objects uniformly increase by a(t):

at time t, the distance between two objects i and j has increased to

$$\vec{r}_i - \vec{r}_j = a(t)\left(\vec{r}_{i,0} - \vec{r}_{j,0}\right)$$

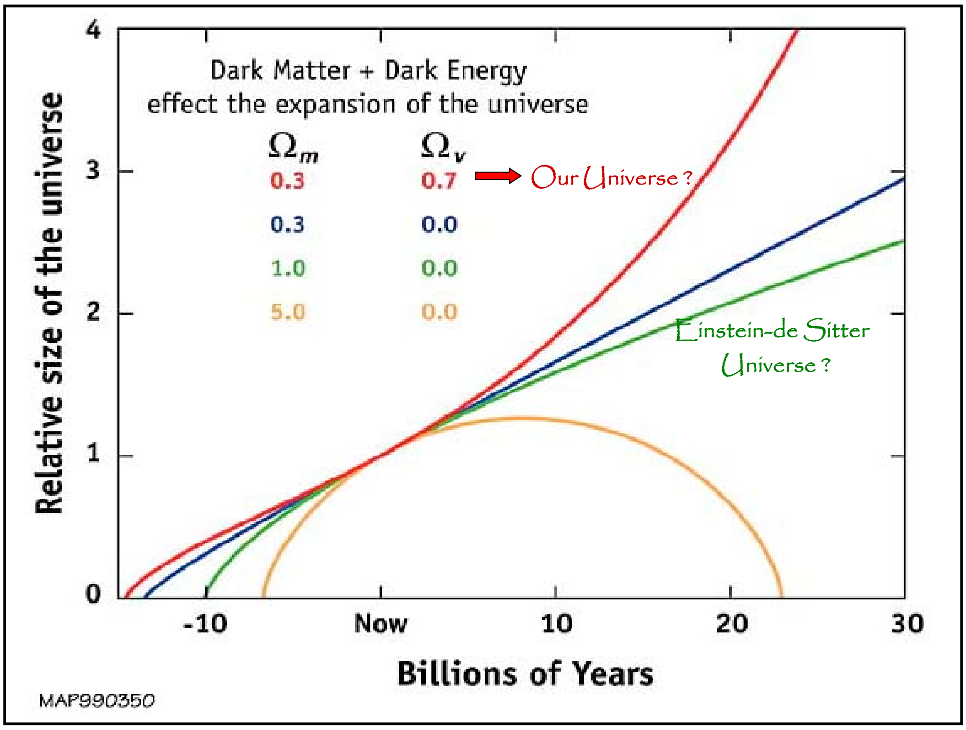
- Note: by definition we chose $a(t_o)=1$, i.e. the present-day expansion factor



Evolution & Fate
Friedmann-Robertson-Walker-Lemaitre Universe

Completely determined by 3 factors:

- energy and matter content
  (density and pressure)
- geometry of the Universe
  (curvature)
- Cosmological Constant



Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^{\ 2}} + \frac{\Lambda}{3}a^2$$

Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is determined by four factors:

- density $\rho(t)$
- pressure $p(t)$
- curvature $kc^2 / R_0^{\ 2}$ $k = 0, +1, -1$
  R_0: present curvature radius
- cosmological constant Λ

• Density & Pressure: - in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
- pressure = momentum flux

• Curvature: - gravity is a manifestation of geometry spacetime

• Cosmological Constant: - free parameter in General Relativity
- Einstein's "biggest blunder"
- mysteriously, since 1998 we know it dominates the Universe

FRW Dynamics

In a FRW Universe,
densities are in the order of the critical density,
the density at which the Universe has a flat curvature

$$\rho_{crit} = \frac{3H_0^{\;2}}{8\pi G} = 1.8791 h^2 \times 10^{-29}\, g\, cm^{-3}$$

$$\rho_0 = 1.8791 \times 10^{-29}\, \Omega h^2 \;\; g\, cm^{-3}$$
$$= 2.78 \times 10^{11}\, \Omega h^2 \quad M_\odot Mpc^{-3}$$

FRW Dynamics

In a matter-dominated Universe,
the evolution and fate of the Universe entirely determined
by the (energy) density in units of critical density:

$$\Omega \equiv \frac{\rho}{\rho_{crit}}$$

Arguably, Ω is the most important parameter of cosmology !!!

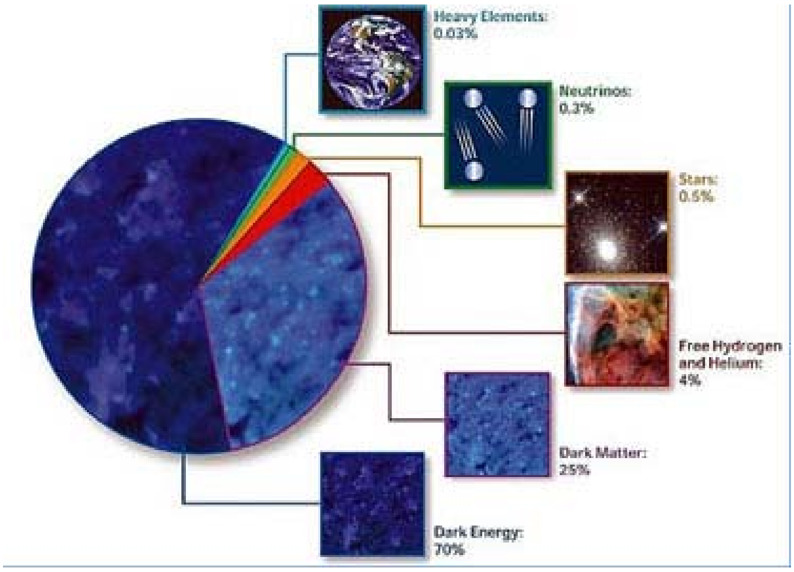
Present-day
Cosmic Density:

$$\rho_0 = 1.8791 \times 10^{-29}\, \Omega h^2 \;\; g\, cm^{-3}$$
$$= 2.78 \times 10^{11}\, \Omega h^2 \quad M_\odot Mpc^{-3}$$

what the Universe exists of:

Cosmic Constituents

Cosmic Components



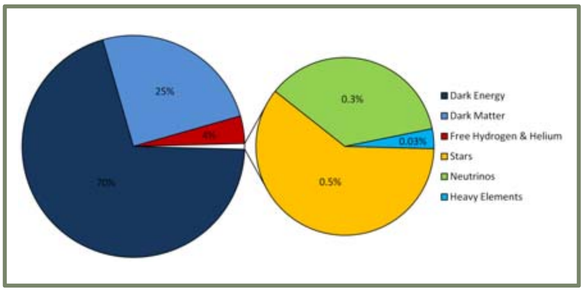
Cosmic Energy Inventarisation

1	dark sector				0.954 ± 0.003
1.1	dark energy			0.72 ± 0.03	
1.2	dark matter			0.23 ± 0.03	
1.3	primeval gravitational waves			$\lesssim 10^{-10}$	
2	primeval thermal remnants				0.0010 ± 0.0005
2.1	electromagnetic radiation			$10^{-4.3\pm0.0}$	
2.2	neutrinos			$10^{-2.9\pm0.1}$	
2.3	prestellar nuclear binding energy			$-10^{-4.1\pm0.0}$	
3	baryon rest mass				0.045 ± 0.003
3.1	warm intergalactic plasma			0.040 ± 0.003	
3.1a	virialized regions of galaxies		0.024 ± 0.005		
3.1b	intergalactic		0.016 ± 0.005		
3.2	intracluster plasma			0.0018 ± 0.0007	
3.3	main sequence stars	spheroids and bulges		0.0015 ± 0.0004	
3.4		disks and irregulars		0.00055 ± 0.00014	
3.5	white dwarfs			0.00036 ± 0.00008	
3.6	neutron stars			0.00005 ± 0.00002	
3.7	black holes			0.00007 ± 0.00002	
3.8	substellar objects			0.00014 ± 0.00007	
3.9	HI + HeI			0.00062 ± 0.00010	
3.10	molecular gas			0.00016 ± 0.00006	
3.11	planets			10^{-6}	
3.12	condensed matter			$10^{-5.6\pm0.3}$	
3.13	sequestered in massive black holes			$10^{-5.4}(1+\epsilon_n)$	
4	primeval gravitational binding energy				$-10^{-6.1\pm0.1}$
4.1	virialized halos of galaxies			$-10^{-7.2}$	
4.2	clusters			$-10^{-6.9}$	
4.3	large-scale structure			$-10^{-6.2}$	

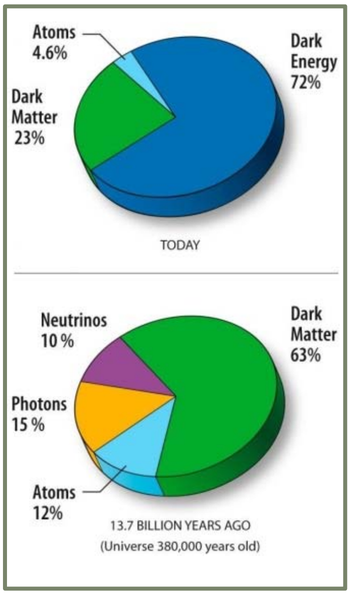
sterren slechts ~0.1% energie Heelal

Fukugita & Peebles 2004

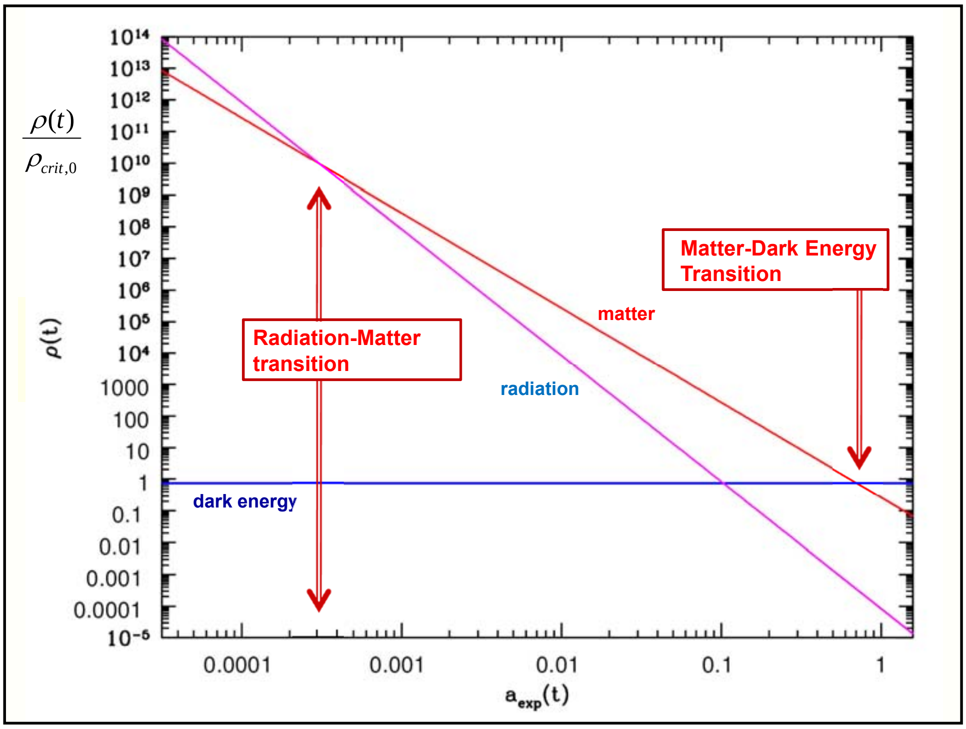
Cosmic Constitution

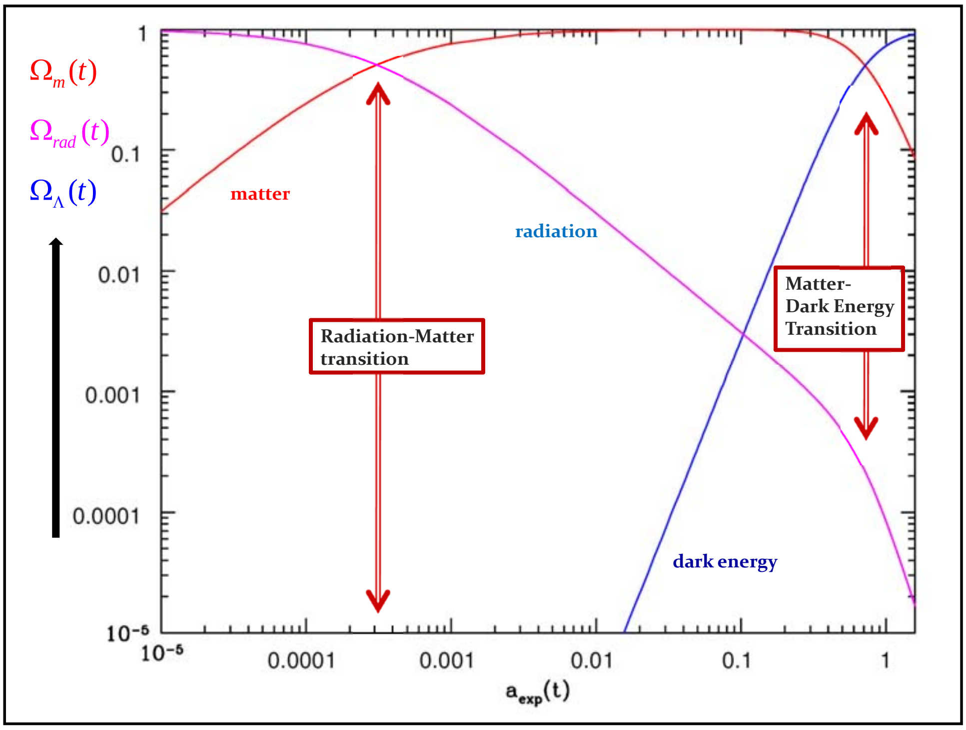


Cosmic Pie Diagram



Changes in Time:


ρ(t) / ρ_crit,0
ρ(t)
Radiation-Matter transition
Matter-Dark Energy Transition
matter
radiation
dark energy
a_exp(t)


Ω_m(t)
Ω_rad(t)
Ω_Λ(t)
matter
radiation
dark energy
Radiation-Matter transition
Matter-Dark Energy Transition
a_exp(t)

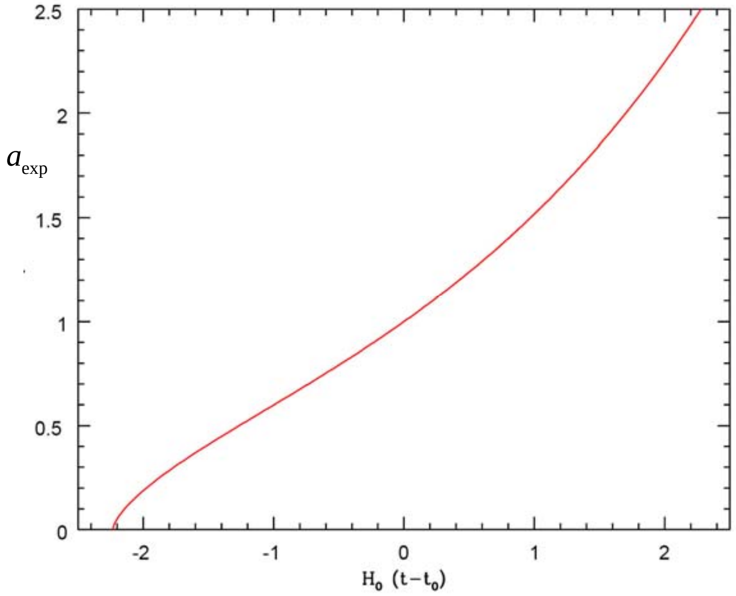
Our Universe:

the Concordance Cosmos

Concordance Universe Parameters

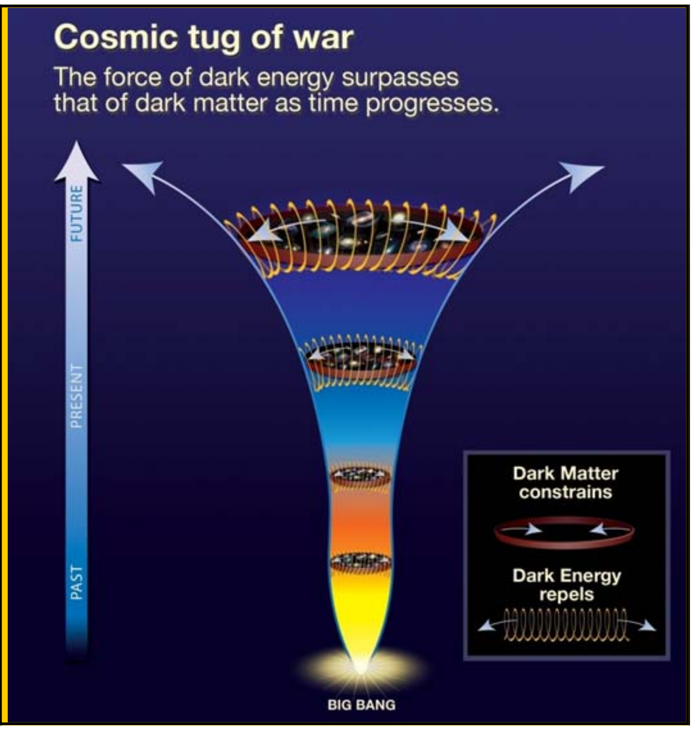
Hubble Parameter		$H_0 = 71.9 \pm 2.6\ km\ s^{-1}\ Mpc^{-1}$	
Age of the Universe		$t_0 = 13.8 \pm 0.1\ Gyr$	
Temperature CMB		$T_0 = 2.725 \pm 0.001 K$	
Matter	Baryonic Matter Dark Matter	$\Omega_m = 0.27$	$\Omega_b = 0.0456 \pm 0.0015$ $\Omega_{dm} = 0.228 \pm 0.013$
Radiation	Photons (CMB) Neutrinos (Cosmic)	$\Omega_{rad} = 8.4 \times 10^{-5}$	$\Omega_\gamma = 5 \times 10^{-5}$ $\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy		$\Omega_\Lambda = 0.726 \pm 0.015$	
Total		$\Omega_{tot} = 1.0050 \pm 0.0061$	

Concordance Expansion



Heden & Toekomst:
VERSNELLING

Vroeger:
VERTRAGING



Age of the Universe

Hubble Time

- The repercussions of Hubble's discovery are truly tremendous: the inescapable conclusion is that the universe has a finite age !
- Just by simple extrapolation back in time we find that at some instant the objects will have touched upon each other, i.e. $r(t_H)=0$. If we assume for simplicity that the expansion rate did remain constant (which it did not !), we find a direct measure for the age of the universe, the

Hubble Time:

$$t_H = \frac{1}{H}$$

$$H_0 = 100h \; km\, s^{-1} Mpc^{-1}$$

$$\Downarrow$$

$$t_0 = 9.78 h^{-1} \; Gyr$$

The Hubble parameter is usually stated in units of km/s/Mpc.
It's customary to express it in units of 100 km/s/Mpc, expressing the real value in terms of the dimensionless value $h=H_o/[100\ km/s/Mpc]$.
The best current estimate is $H_o=72$ km/s/Mpc. This sets t_o~10 Gyr.

Hubble Parameter

- For a long time, the correct value of the Hubble constant H_o was a major unsettled issue:

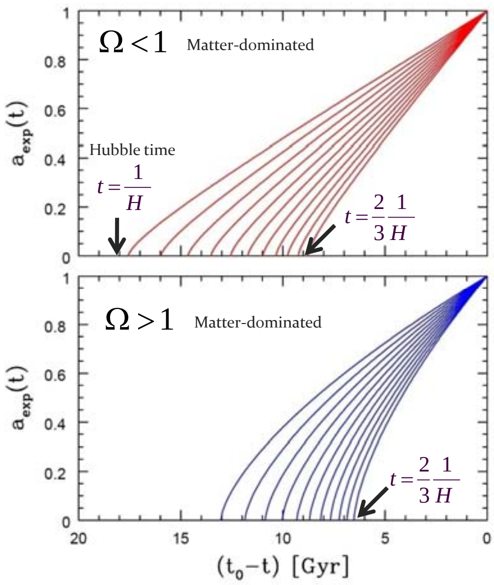
H_o = 50 km s^{-1} Mpc^{-1} ⟷ H_o = 100 km s^{-1} Mpc^{-1}

- This meant distances and timescales in the Universe had to deal with uncertainties of a factor 2 !!!

- Following major programs, such as Hubble Key Project, the Supernova key projects and the WMAP CMB measurements,

$$H_0 = 71.9^{+2.6}_{-2.7}\ km\, s^{-1} Mpc^{-1}$$

Age of the Universe



Age of a FRW universe at Expansion factor a(t)

$$H\,t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad}}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2 + (1-\Omega\ )}}$$

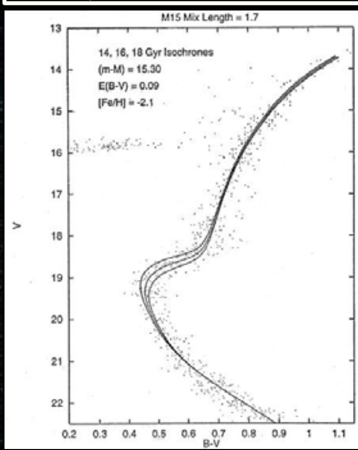
Cosmic Age

estimated age of the oldest stars in Universe far in excess of estimated age of matter-dominated FRW Universe:

Globular cluster stars:	**13-15 Gyr**
Universe:	**10-12 Gyr**

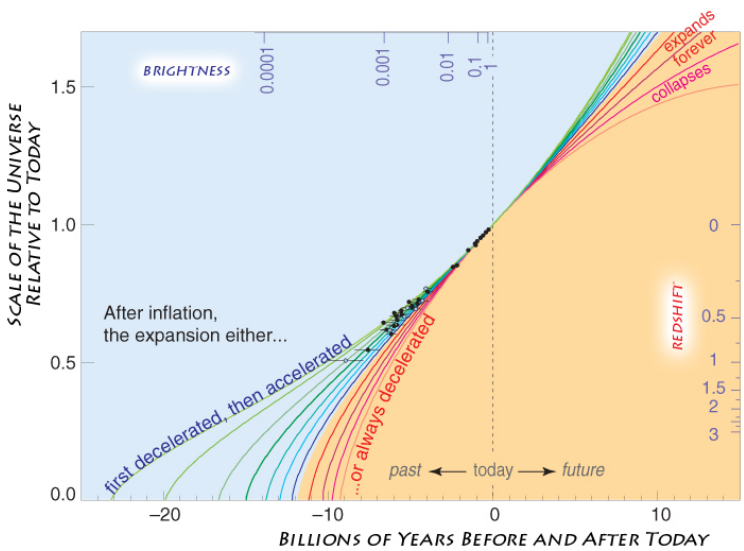
Globular Clusters

- Roughly spherical assemblies of 100,000-200,000 stars
- Radius ~ 20-50 pc: extremely high star density
- Globulars are very old, amongst oldest objects in local Universe
- Stars formed around same time: old, red, population
- Colour-magnitude diagram characteristic: accurate age determination on the basis of stellar evolution theories.



Typical 1980-1990s isochrone fit

Concordance Expansion

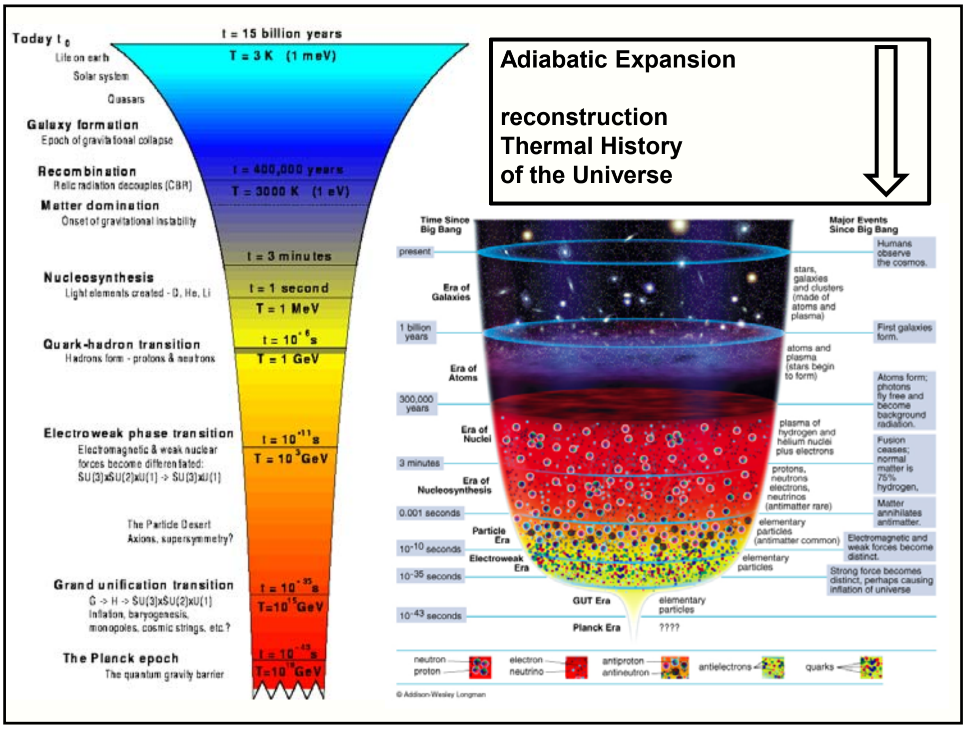


Adiabatic Expansion

Adiabatic Expansion

- The Universe of Einstein, Friedmann & Lemaitre expands *adiabacally*
- Energy of the expansion of the Universe corresponds to the decrease in the energy of its constituents
- *The Universe COOLS as a result of its expansion !*

$$T(t) \propto 1/a(t)$$



Cosmic Epochs

Planck Epoch		$t < 10^{-43}$ sec
Phase Transition Era	GUT transition electroweak transition quark-hadron transition	10^{-43} sec < t < 10^{5}sec
Hadron Era		t ~ 10^{-5} sec
Lepton Era	muon annihilation neutrino decoupling electron-positron annihilation primordial nucleosynthesis	10^{-5} sec < t < 1 min
Radiation Era	radiation-matter equivalence recombination & decoupling	1 min < t <379,000 yrs
Post-Recombination Era	Structure & Galaxy formation Dark Ages Reionization Matter-Dark Energy transition	t > 379,000 yrs

Big Bang:

the Evidence

Big Bang Evidence

- Olber's paradox:
  the night sky is dark
  ⇒ finite age Universe (13.7 Gyr)

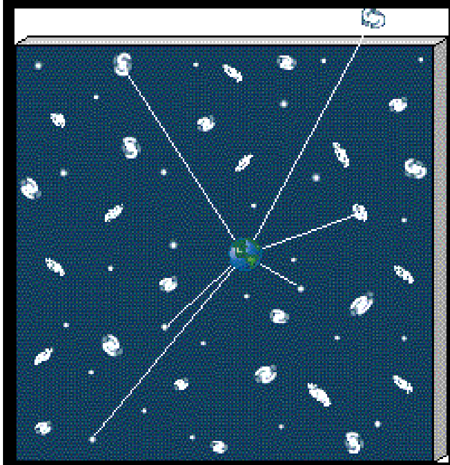
- Hubble Expansion
  uniform expansion, with
  expansion velocity ~ distance: v = H r

- Explanation Helium Abundance 24%:
  light chemical elements formed (H, He, Li, ...)
  after ~3 minutes ...

- The Cosmic Microwave Background Radiation:
  the 2.725K radiation blanket, remnant left over
  hot ionized plasma ⇒ neutral universe
  (379,000 years after Big Bang)

- Distant, deep Universe indeed looks different ...

1. Olber's Paradox



In an infinitely large, old and unchanging Universe each line of sight would hit a star:

→

Sky would be as bright as surface of star:

1. Olber's Paradox

In an infinitely large, old and unchanging Universe each line of sight would hit a star:

→

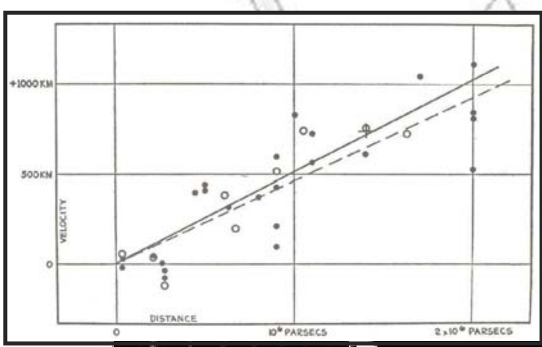
Sky would be as bright as surface of star:

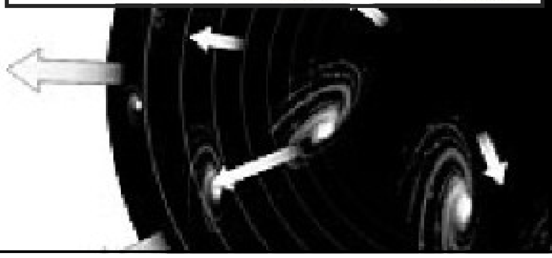
Night sky as bright as Solar Surface, yet the night sky is dark

→

finite age of Universe (13.8 Gyr)

2. Hubble Expansion

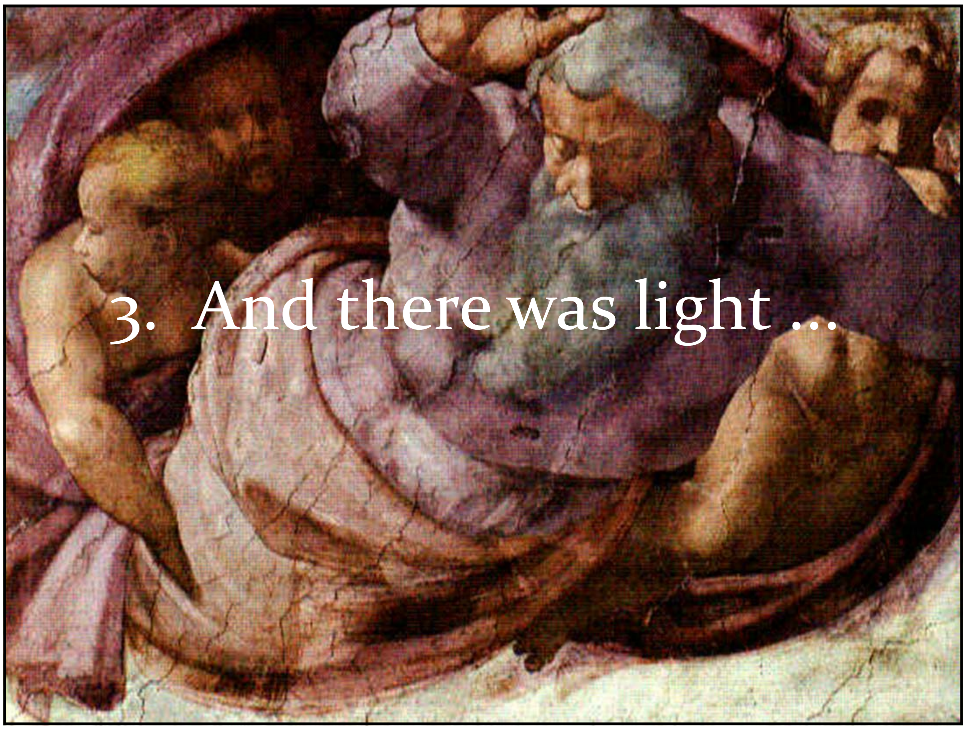




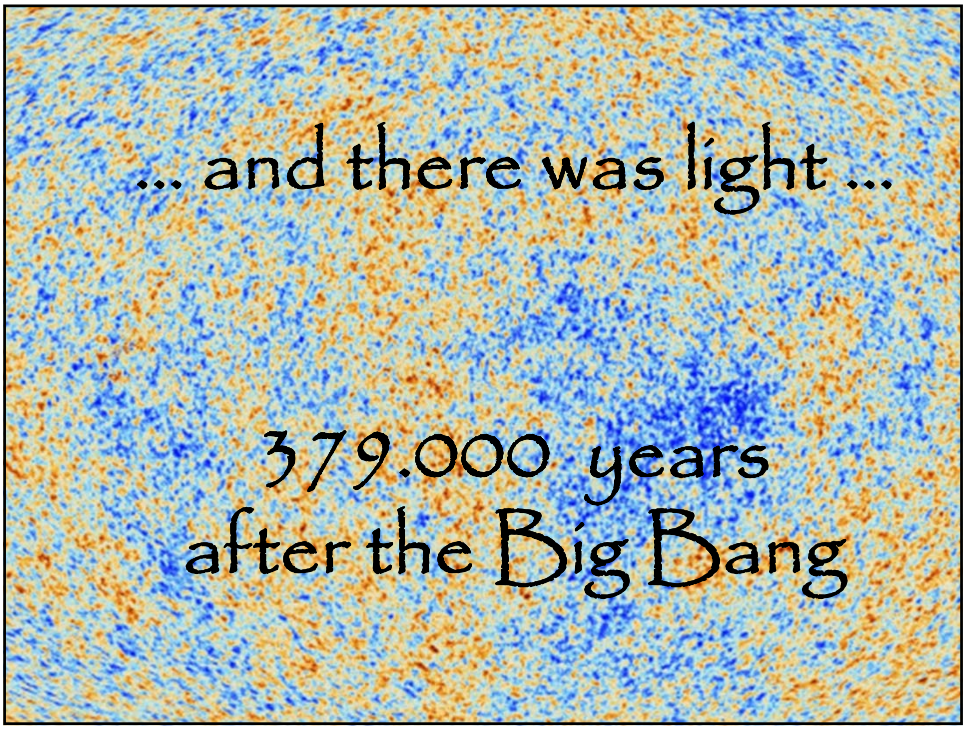
$$v_{rad} = cz = H_0 r$$

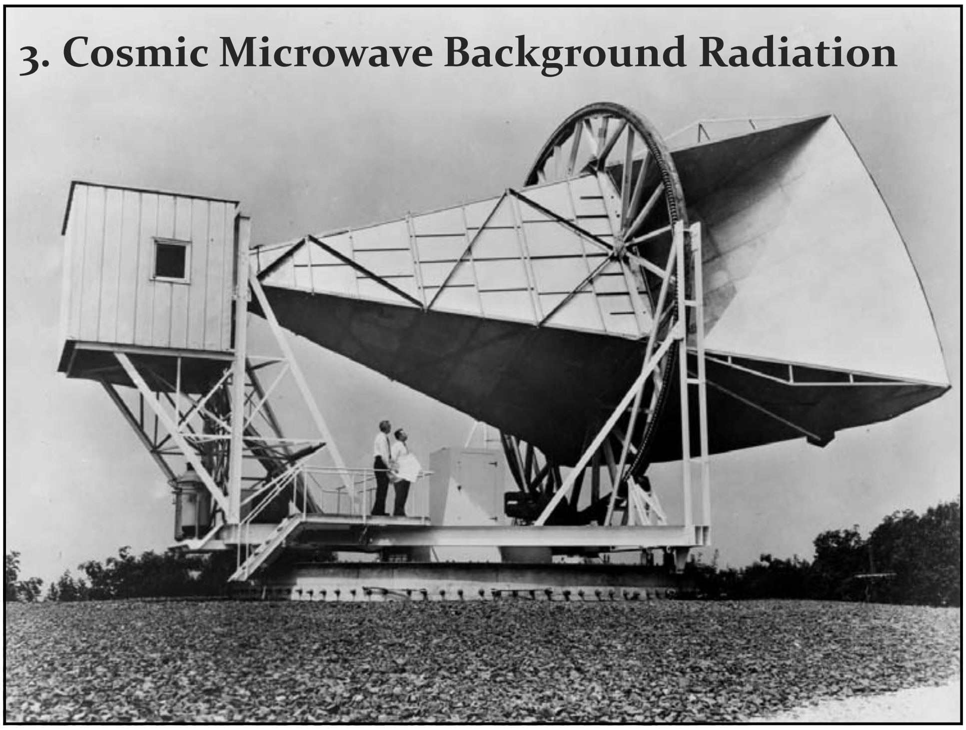
H_0 : **Hubble constant**

specifies expanssion rate of the Universe



3. And there was light …

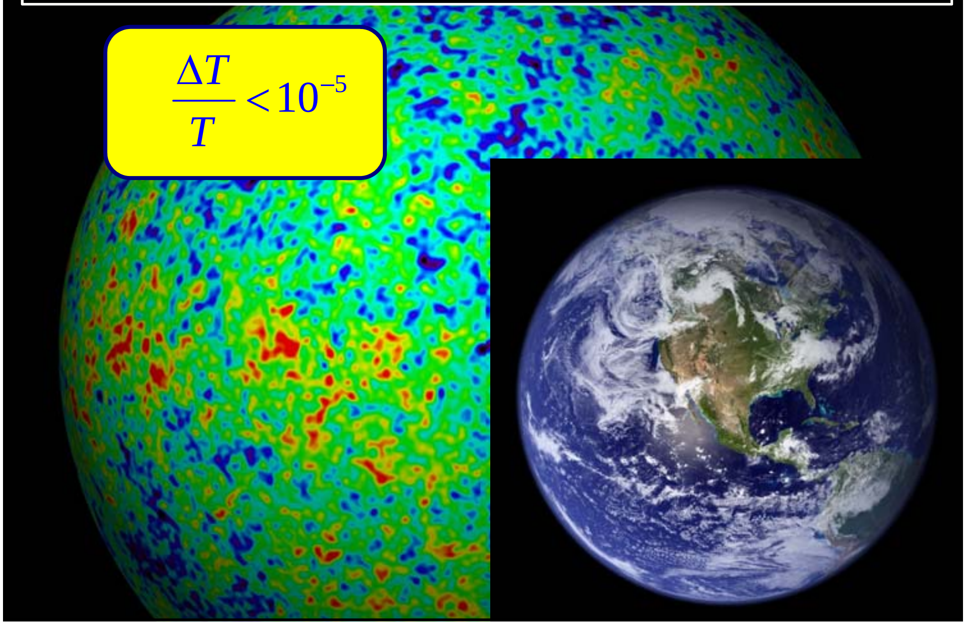

… and there was light …
379.000 years
after the Big Bang

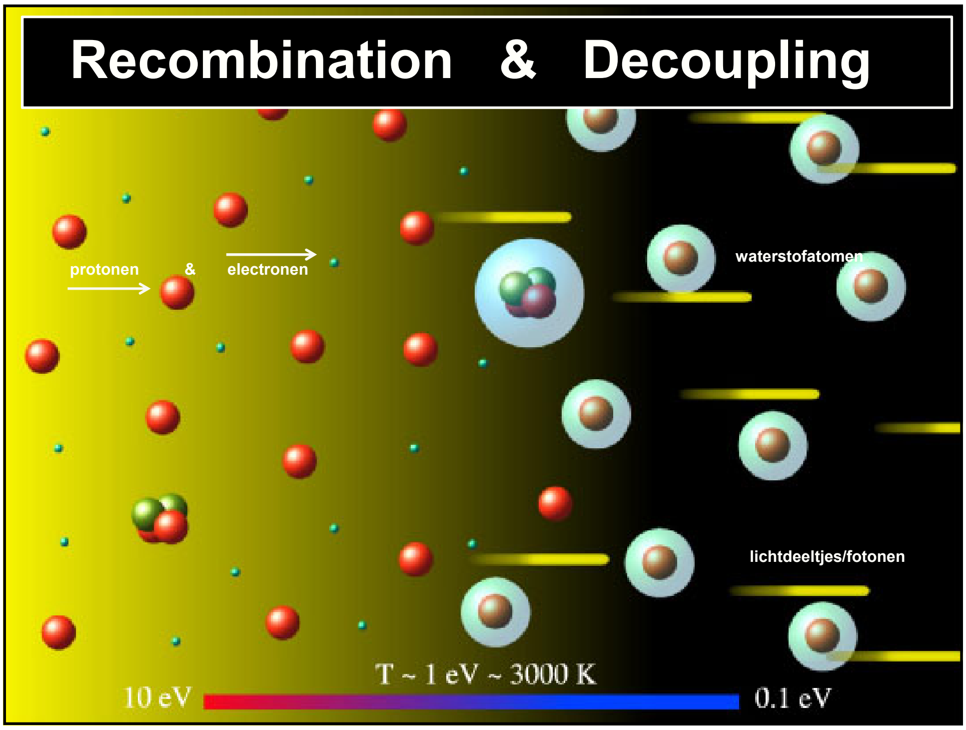

3. Cosmic Microwave Background Radiation

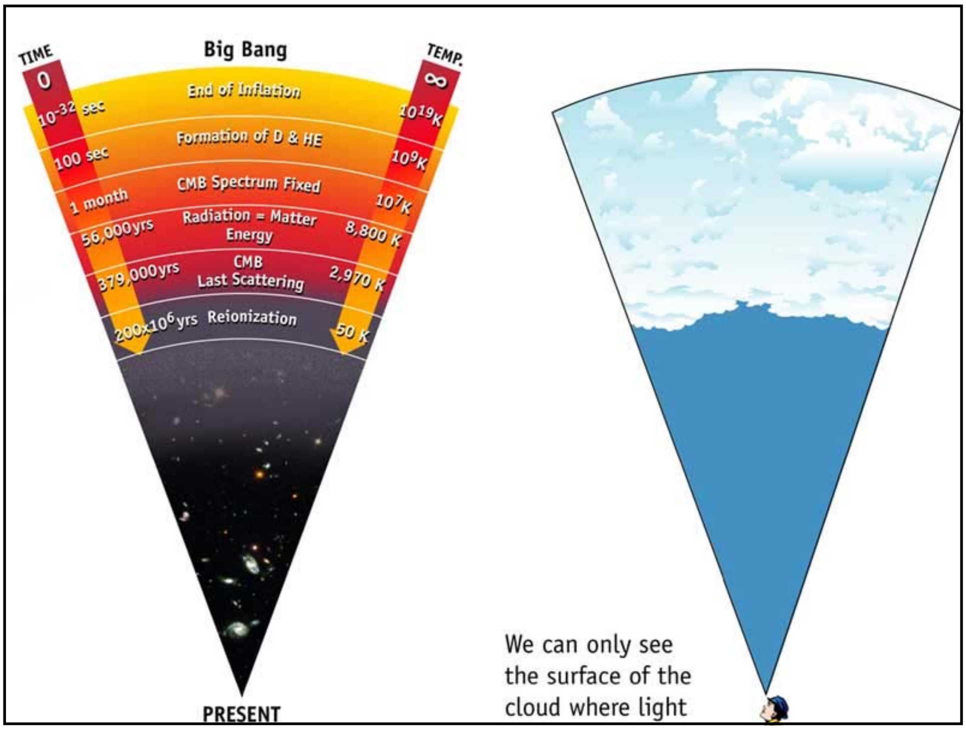
Cosmic Light (CMB): the facts

- Discovered serendipitously in 1965 — Penzias & Wilson, Nobelprize 1978 !!!!!
- Cosmic Licht that fills up the Universe uniformly
- Temperature: $T_\gamma = 2.725$ K
- (CMB) photons most abundant particle in the Universe: $n_\gamma \sim 415\ \mathrm{cm}^{-3}$
- Per atom in the Universe: $n_\gamma/n_B \sim 1.9 \times 10^9$
- Ultimate evidence of the Big Bang !!!!!!!!!!!!!!!!!!

Extremely Smooth Radiation Field




Recombination & Decoupling
protonen
&
electronen
waterstofatomen
lichtdeeltjes/fotonen
T ~ 1 eV ~ 3000 K
10 eV
0.1 eV


TIME
Big Bang
TEMP.
0
∞
End of Inflation
10-32 sec
10¹⁹K
Formation of D & HE
100 sec
10⁹K
CMB Spectrum Fixed
1 month
10⁷K
Radiation = Matter Energy
56,000yrs
8,800 K
CMB Last Scattering
379,000yrs
2,970 K
Reionization
200x10⁶yrs
50 K
PRESENT
We can only see
the surface of the
cloud where light

the Cosmic TV Show

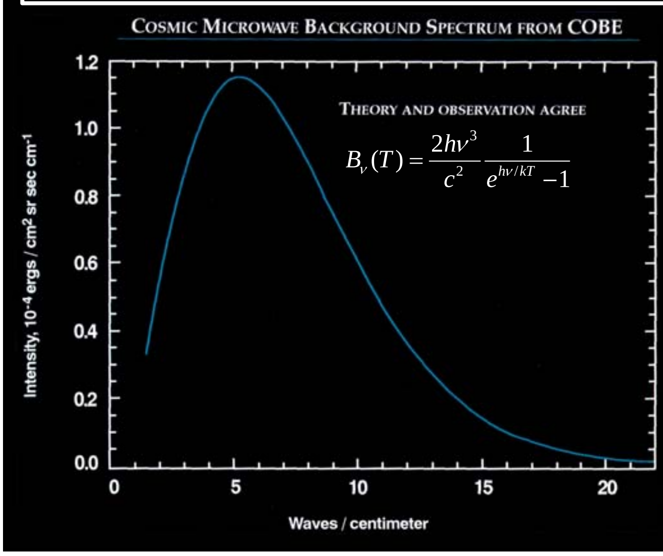
Note:

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your (camping) tv set …

!!!! Live broadcast from the Big Bang !!!!

Courtesy: W. Hu

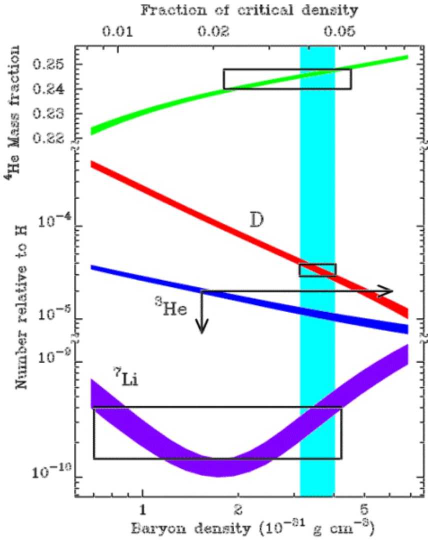
Energy Spectrum Cosmic Light



- COBE-DIRBE: temperatureT = 2.725 K
- John Mather Nobelprize physics 2006
- Most perfect Black Body Spectrum ever seen !!!!

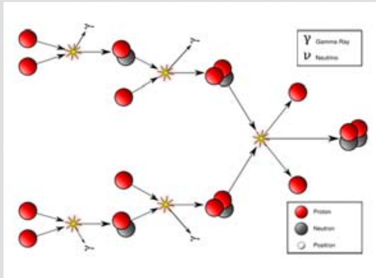
4. Proton-Neutron & Helium

p/n ~1/7: 1 min na BB



Mass Fraction Light Elements

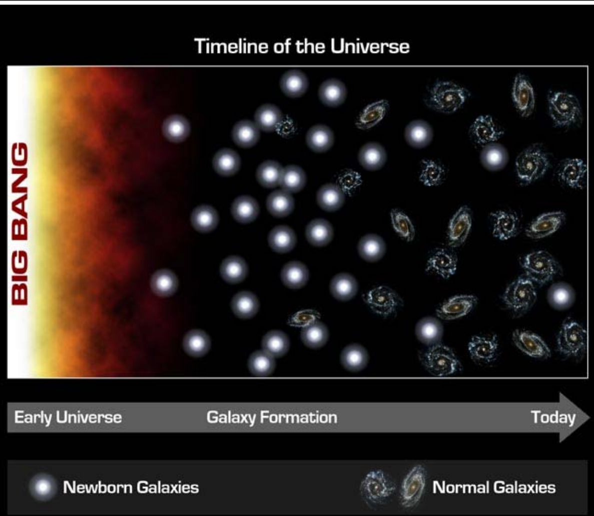
24%	^{4}He nuclei
traces	D, ^{3}He, ^{7}Li nuclei
75%	H nuclei (protons)



Between 1-200 seconds after Big Bang, temperature dropped to 10^9 K:

Fusion protons & neutrons into light atomic nuclei

5. the Changing Universe



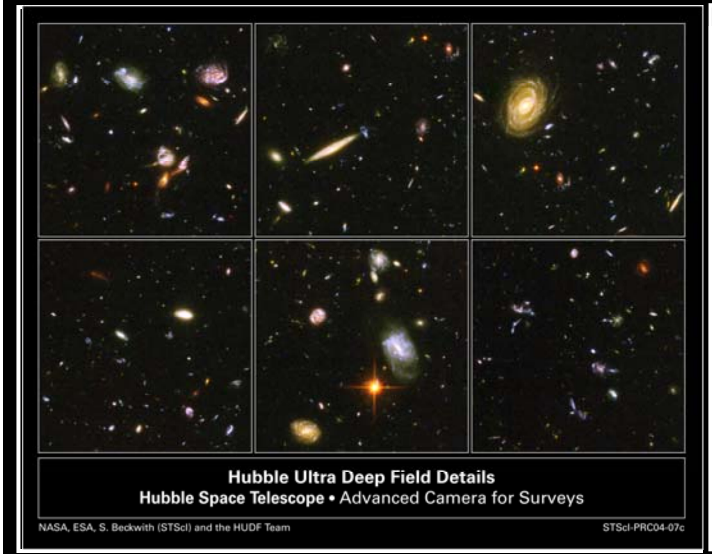
At great depths the Universe looks completely different

- and thus long ago : Depth= Time

→

Galaxies in Hubble Ultra Deep Field

5. the Changing Universe



At great depths the Universe looks completely different

- and thus long ago : Depth= Time

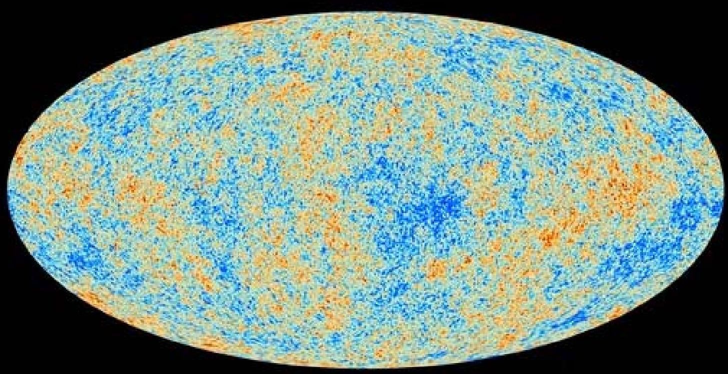
Galaxies in Hubble Ultra Deep Field

Cosmic Curvature

How Much ?

Cosmic Curvature

Cosmic Microwave Background



Map of the Universe at Recombination Epoch (Planck, 2013):

- **379,000 years after Big Bang**
- **Subhorizon perturbations: primordial sound waves**
- **$\Delta T/T$ < 10-5**

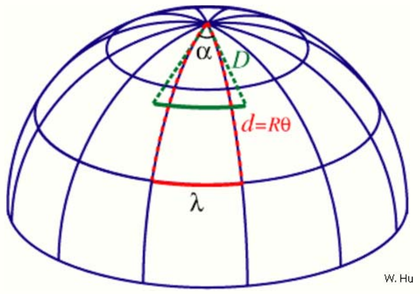
Measuring Curvature

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space

⬇

Geometry of Space

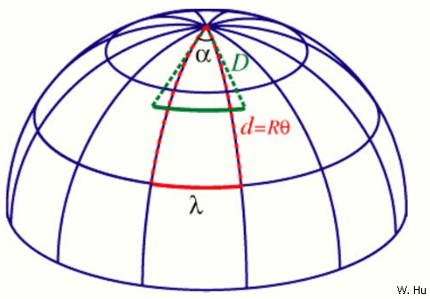


Measuring Curvature

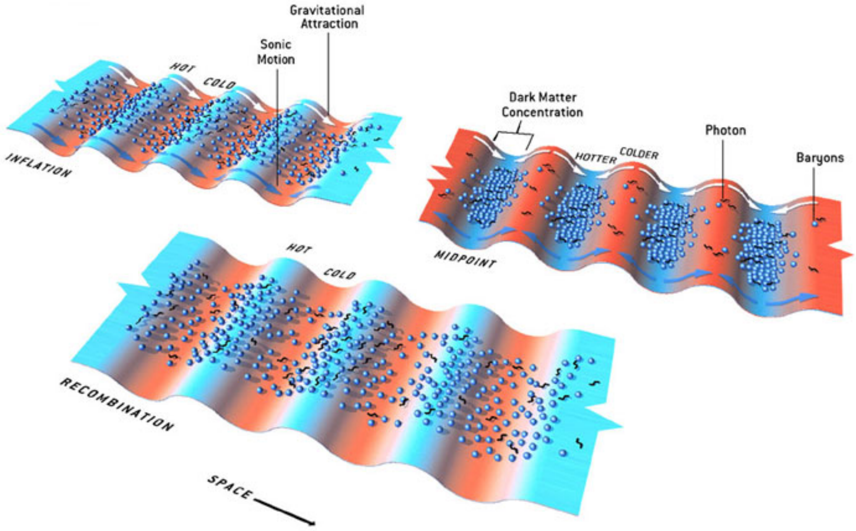
- Object with known physical size, at large cosmological distance:
- Sound Waves in the Early Universe !!!!

⬇

Temperature Fluctuations CMB



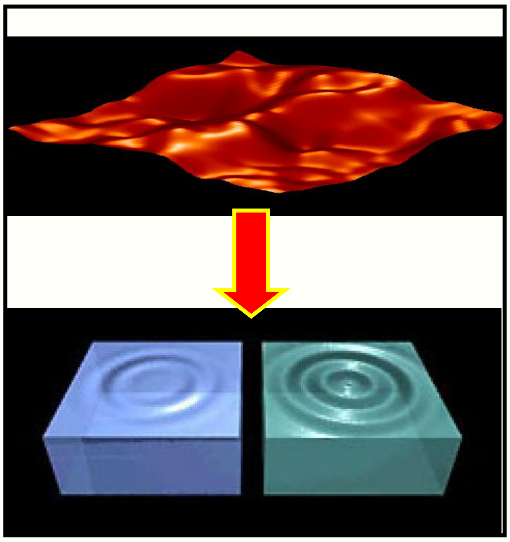
Fluctuations-Origin



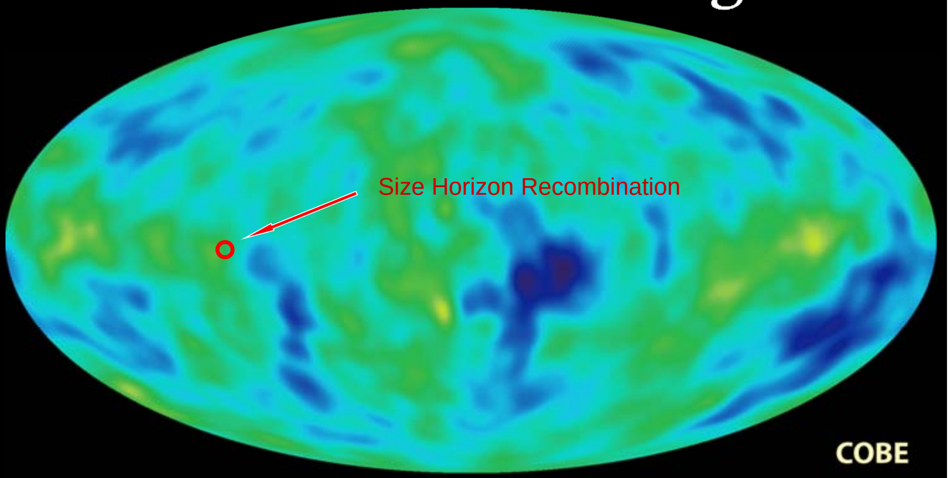
Music of the Spheres

- small ripples in primordial matter & photon distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists
- compressions and rarefactions in photon gas: sound waves
- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of "instrument":
  - (sound) horizon size last scattering
- Observed, angular size: $\theta \sim 1^o$
  - exact scale maximum compression, the "cosmic fundamental mode of music"

W. Hu



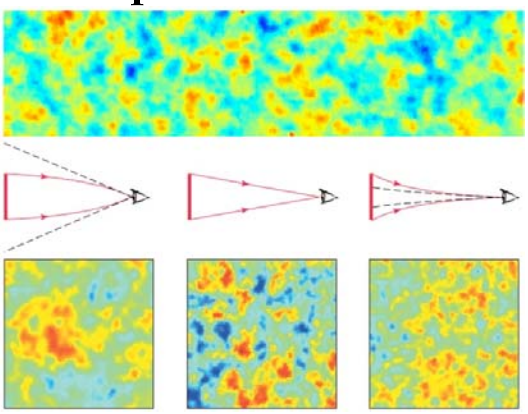
Cosmic Microwave Background



COBE measured fluctuations: $> 7^\circ$
Size Horizon at Recombination spans angle $\sim 1^\circ$

Flat universe from CMB

- **First peak: flat universe**



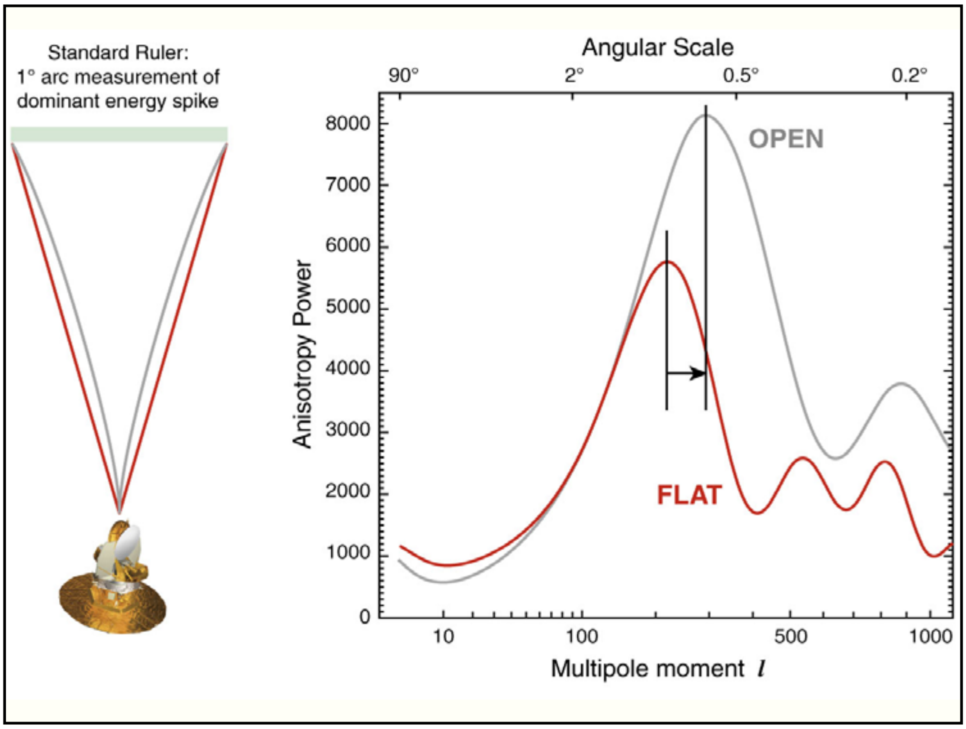
Closed: hot spots appear larger	Flat: appear as big as they are	Open: spots appear smaller

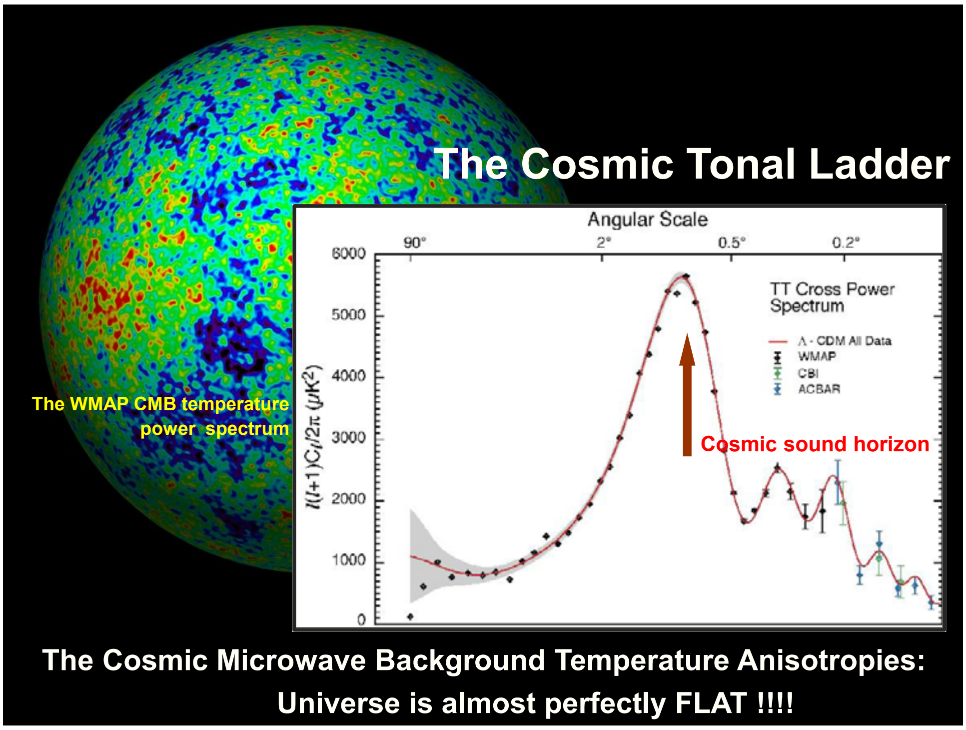
We know the redshift and the time it took for the light to reach us:

from this we know the
- length of the legs of the triangle
- the angle at which we are measuring the sound horizon.

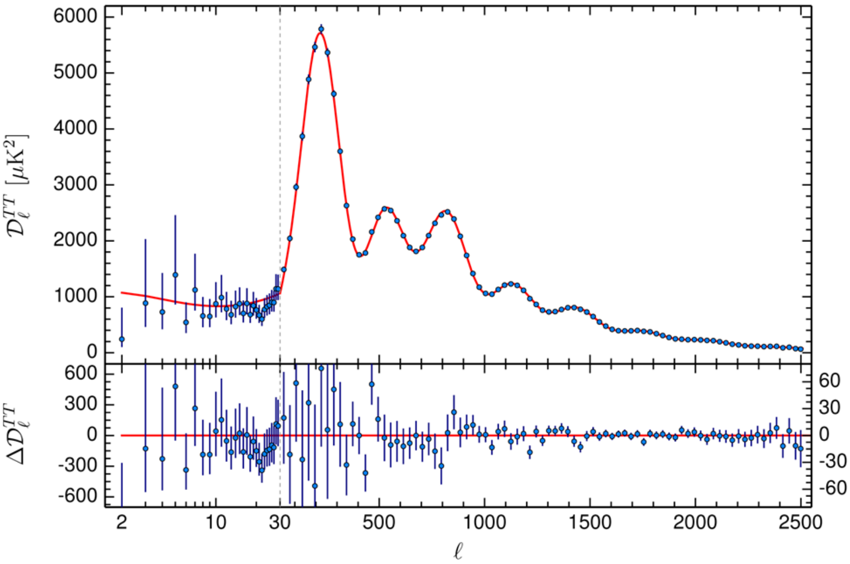
$$v \approx \frac{c}{\sqrt{3}}$$

$$\ell \approx 200/\sqrt{1-\Omega_k}$$


Standard Ruler:
1° arc measurement of dominant energy spike
Angular Scale
90°
2°
0.5°
0.2°
OPEN
FLAT
Anisotropy Power
8000
7000
6000
5000
4000
3000
2000
1000
0
10
100
500
1000
Multipole moment l


The Cosmic Tonal Ladder
The WMAP CMB temperature power spectrum
Angular Scale
90°
2°
0.5°
0.2°
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR
Cosmic sound horizon
l(l+1)Cl/2π (μK²)
6000
5000
4000
3000
2000
1000
0
The Cosmic Microwave Background Temperature Anisotropies:
Universe is almost perfectly FLAT !!!!

CMB - Fluctuations



Cosmic Horizons

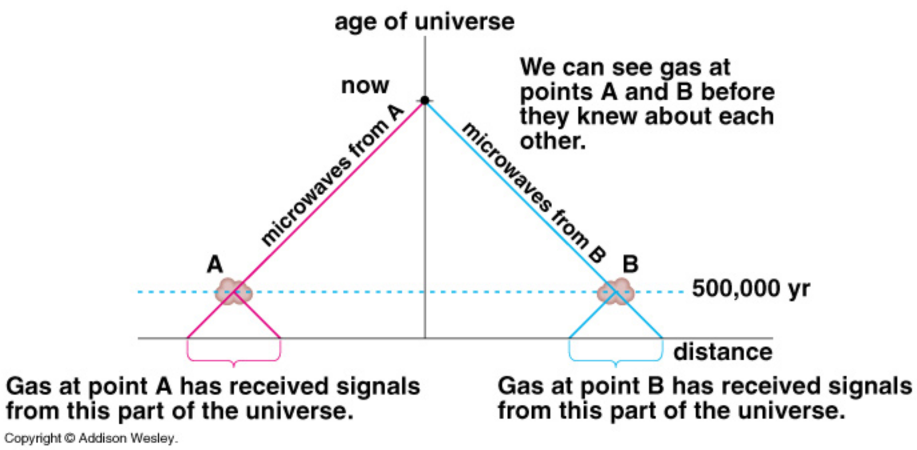
Cosmic Horizons

Fundamental Concept for our understanding of the physics of the Universe:

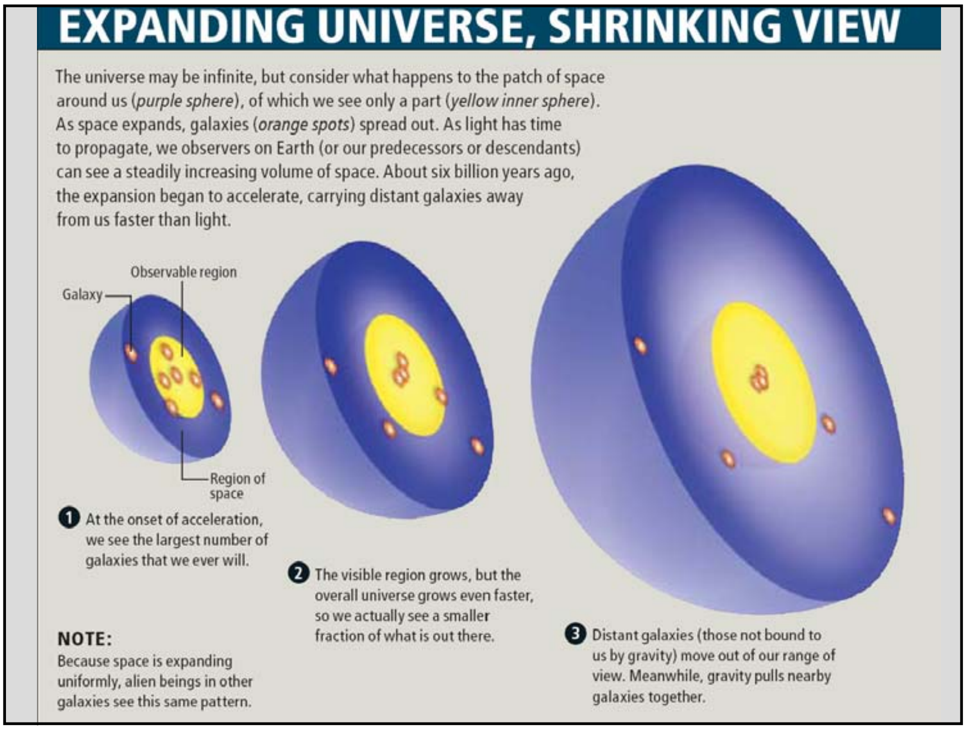
- Physical processes are limited to the region of space with which we are or have ever been in physical contact.
- What is the region of space with which we are in contact ?
  Region with whom we have been able to exchange photons
  (photons: fastest moving particles)
- From which distance have we received light.
- Complication: - light is moving in an expanding and curved space
  - fighting its way against an expanding background
- This is called the

Horizon of the Universe

Cosmic Horizons



Horizon of the Universe:
distance that light travelled since the Big Bang



Cosmic Future

Cosmic Fate

100 Gigayears:
the end of Cosmology