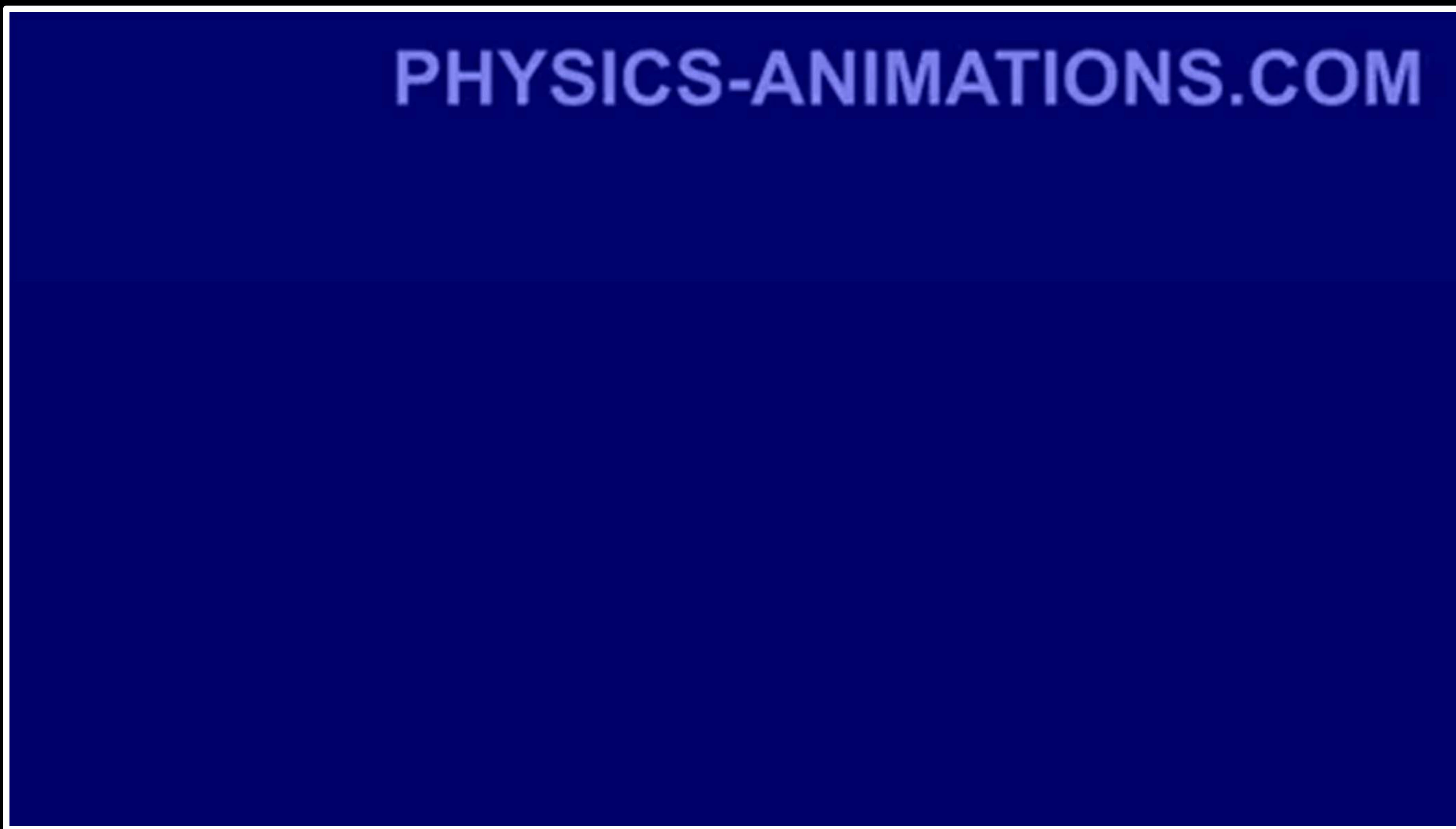
Shock Waves


PHYSICS-ANIMATIONS.COM

Shocks

1. Shocks are sudden transitions in flow properties such as density, velocity and pressure;

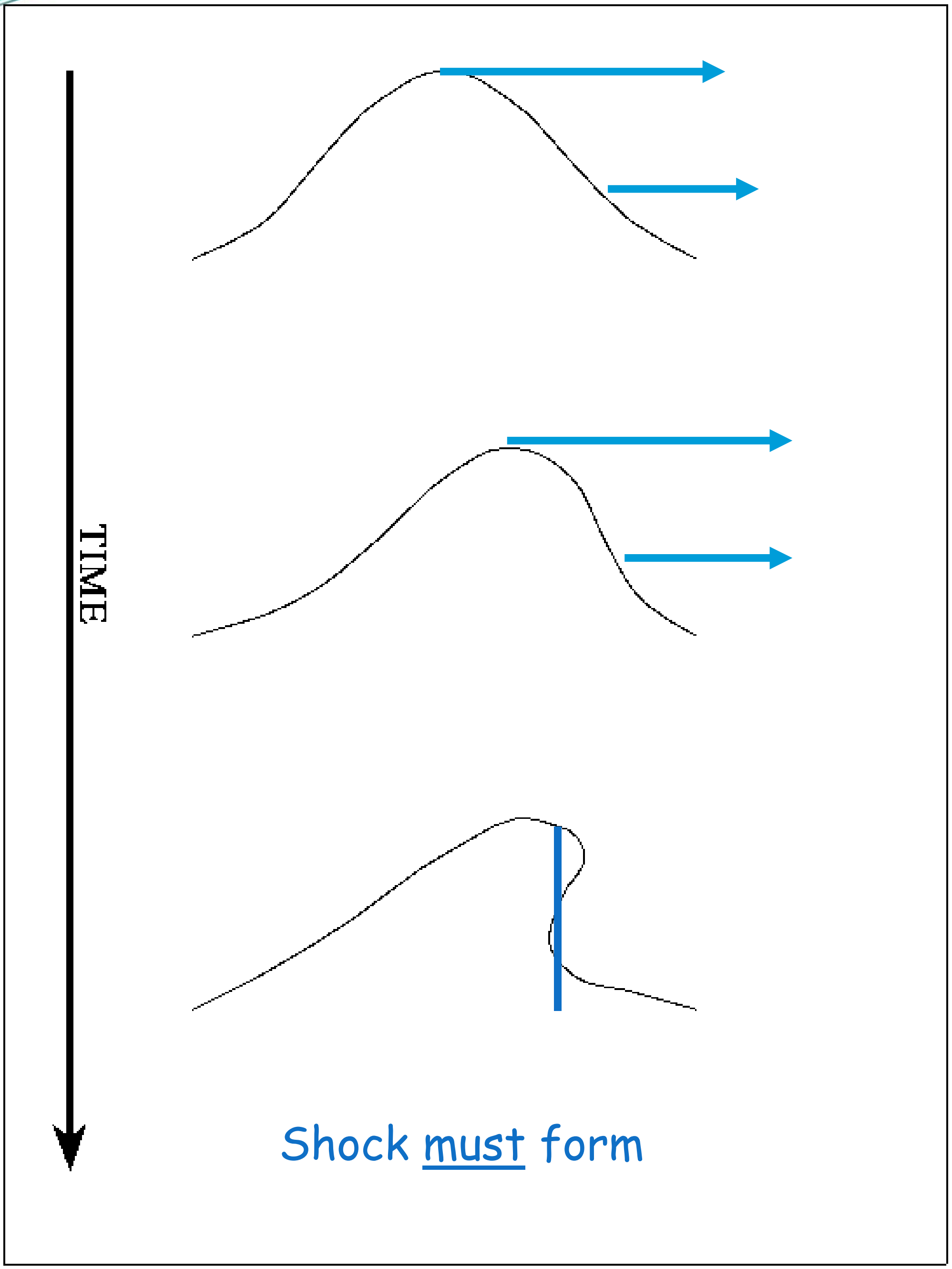
2. In shocks the kinetic energy of the flow is converted into heat, (pressure);

3. Shocks are inevitable if sound waves propagate over long distances;

4. Shocks always occur when a flow hits an obstacle supersonically

5. In shocks, the flow speed along the shock normal changes from supersonic to subsonic

Wave Breaking



High-pressure/density regions move faster

$$u = \frac{2c_{s0}}{\gamma - 1}\left[\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right]$$

$$\approx c_{s0}\left(\frac{\Delta\rho}{\rho}\right)$$

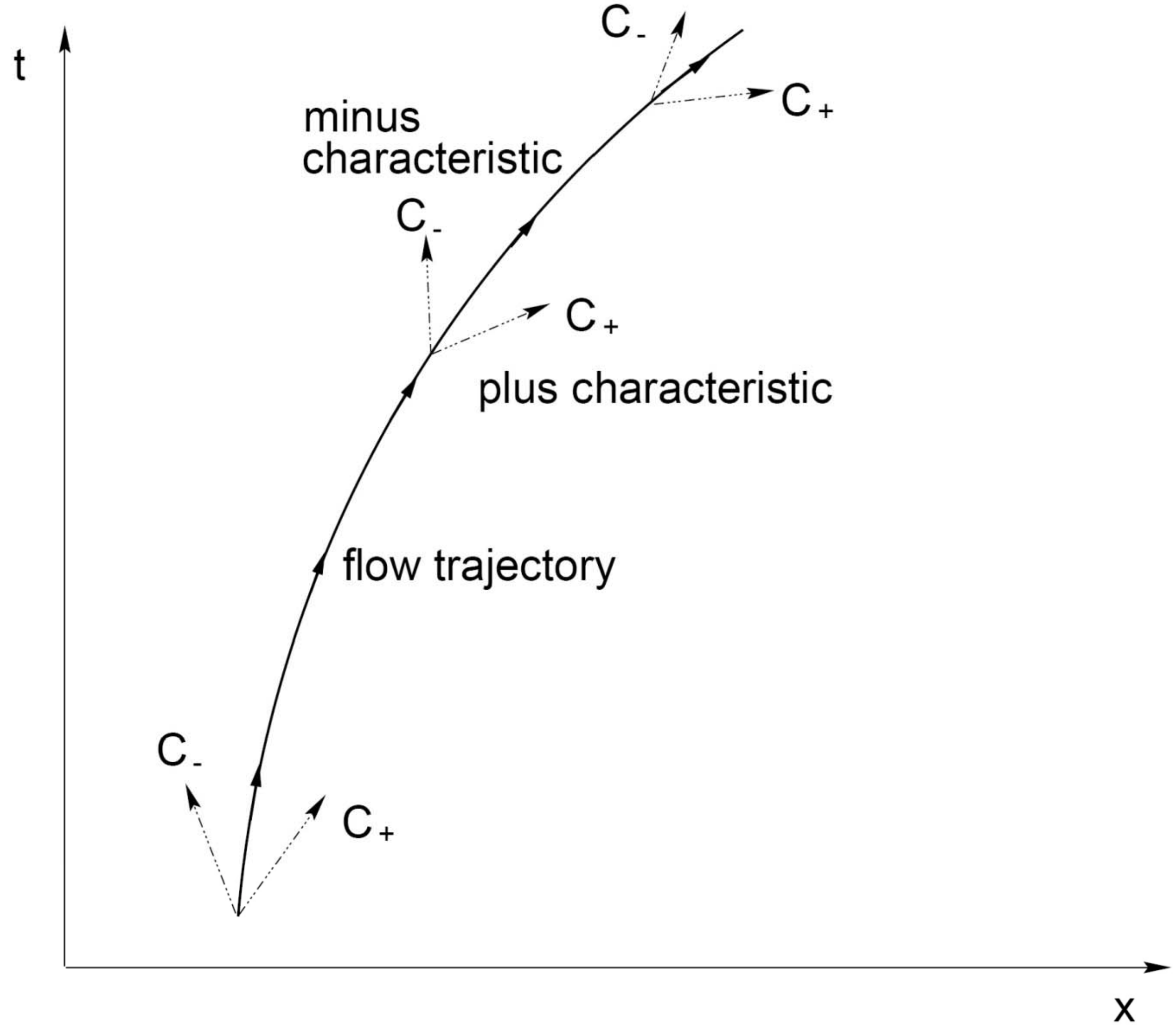
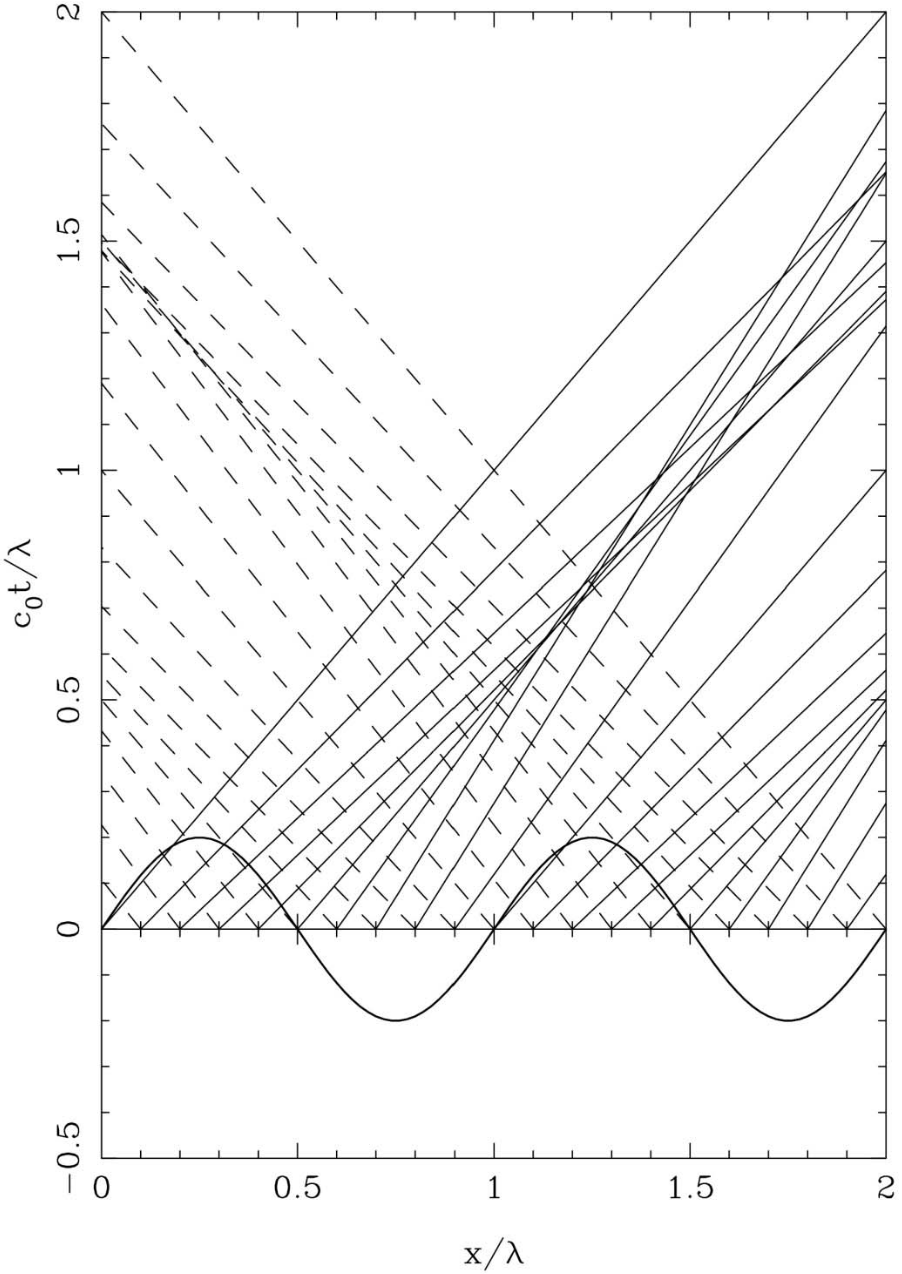
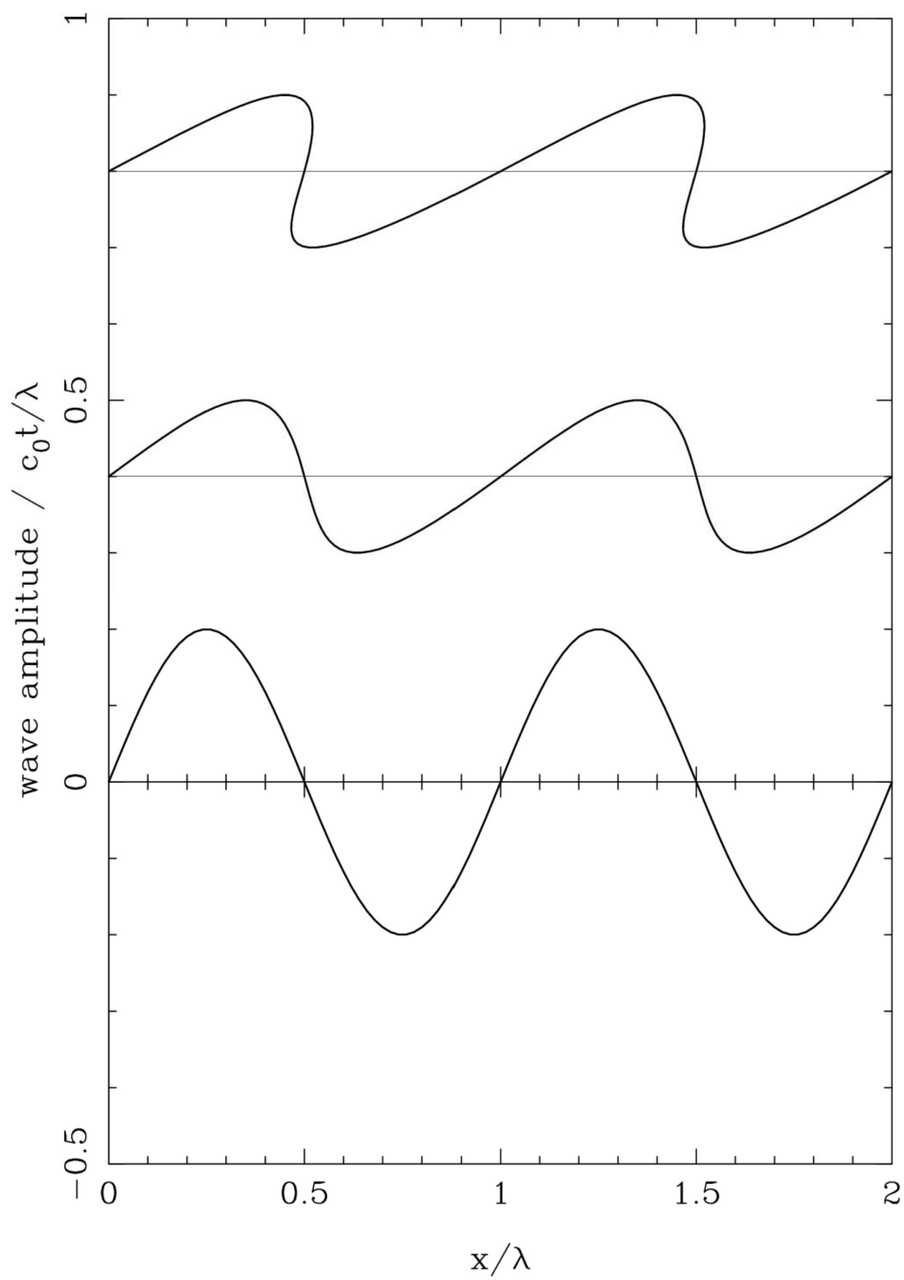


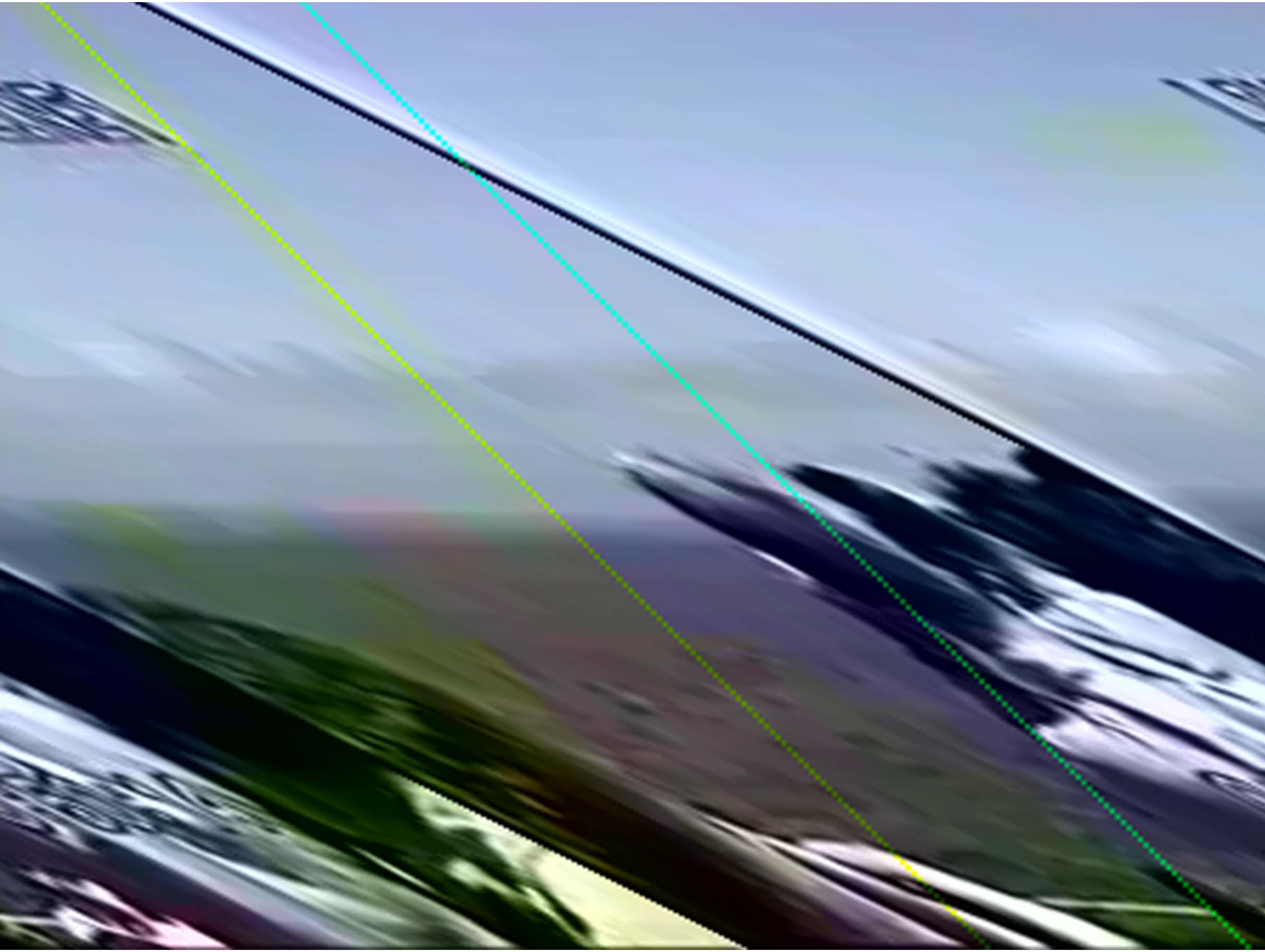
Figure 7.1: *Diagram showing the space-time flow line, defined by* $\mathrm{d}x = u\,\mathrm{d}t$*, and the two characteristics* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *defined by* $\mathrm{d}x = (u + c_\mathrm{s})\,\mathrm{d}t$ *and* $\mathrm{d}x = (u - c_\mathrm{s})\,\mathrm{d}t$*. From each point in the flow two characteristics originate along which* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are constant respectively. Note that the value of* $\mathcal{C}_\pm$ *can be different on the different characteristics so that the characteristic variables* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are* **not** *global constants!*

Characteristics of a sound wave



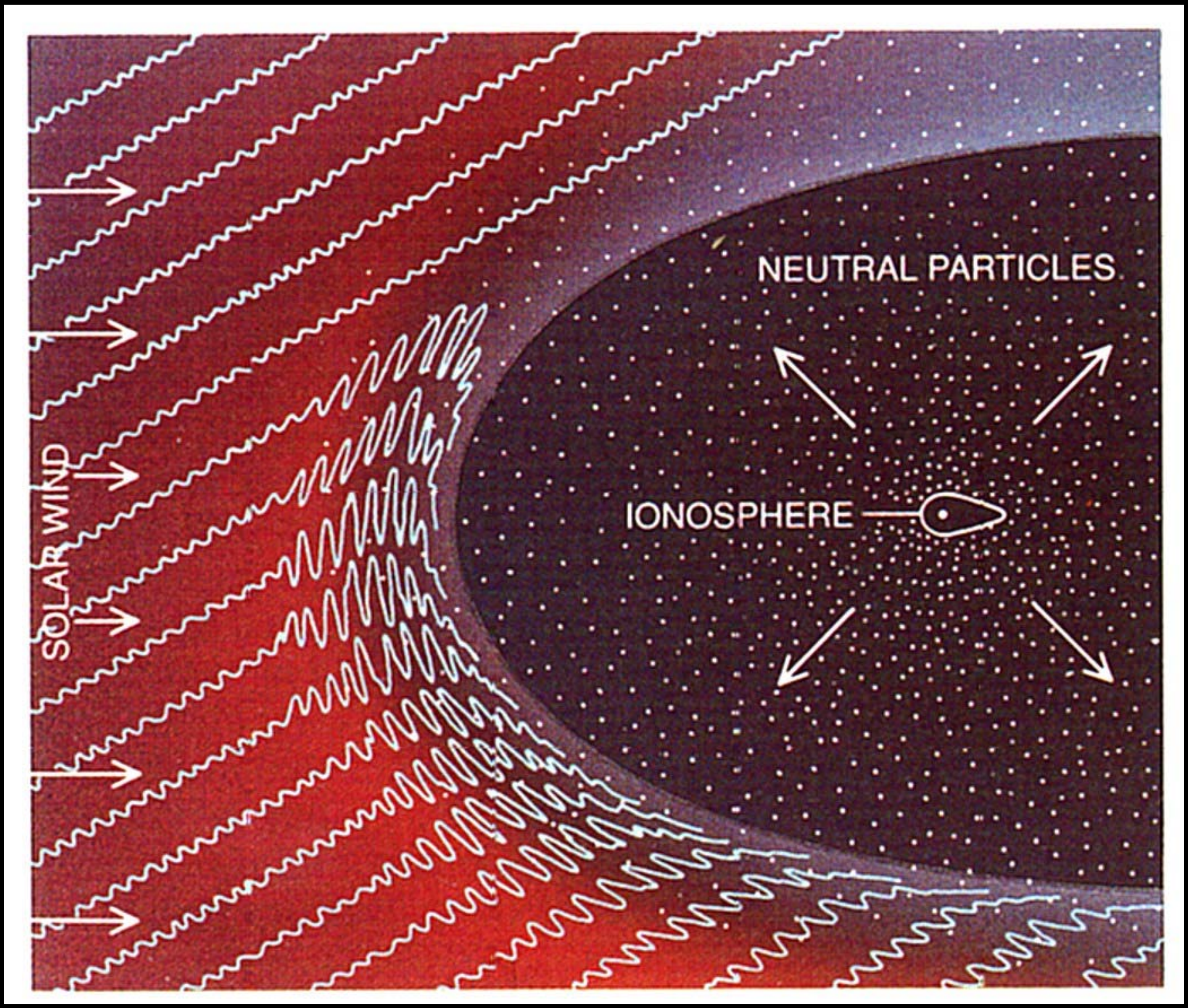
Sound wave steepening

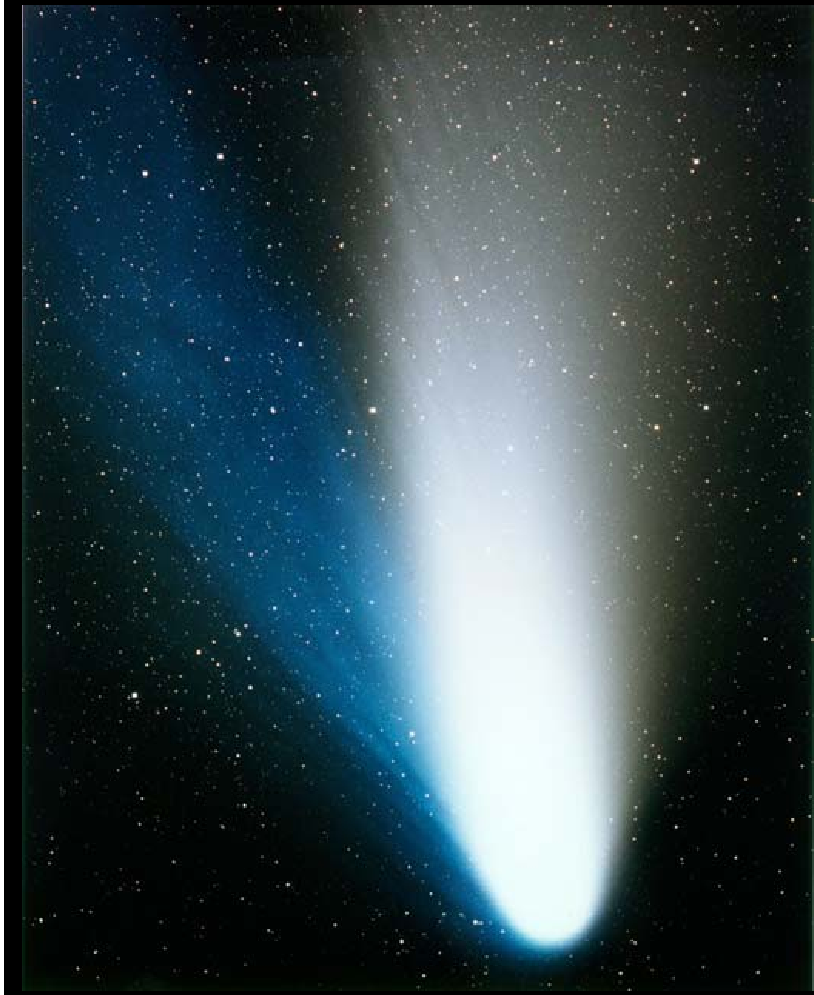




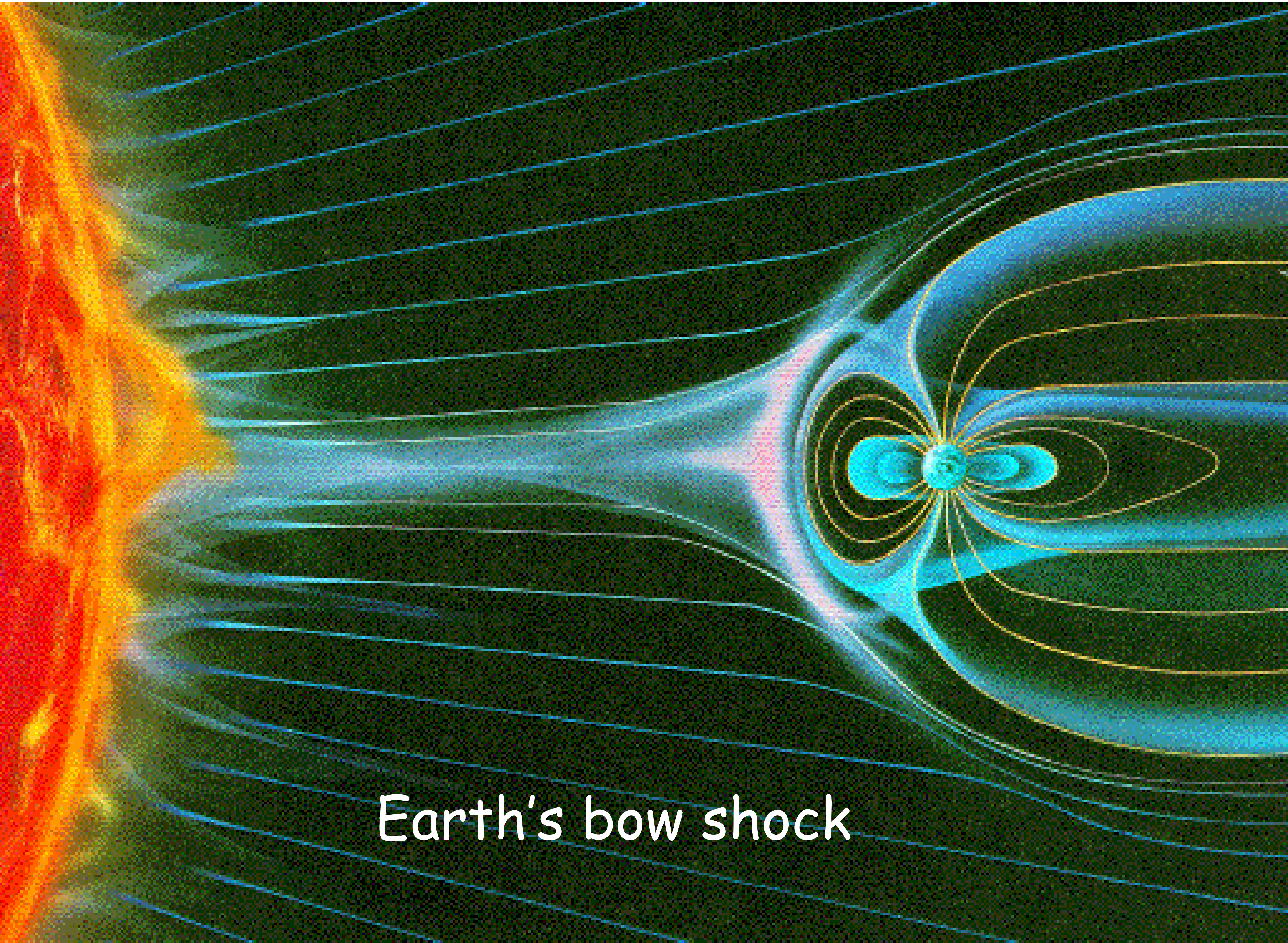
Chelyabinsk Meteorite (Feb. 2013): Sonic Boom

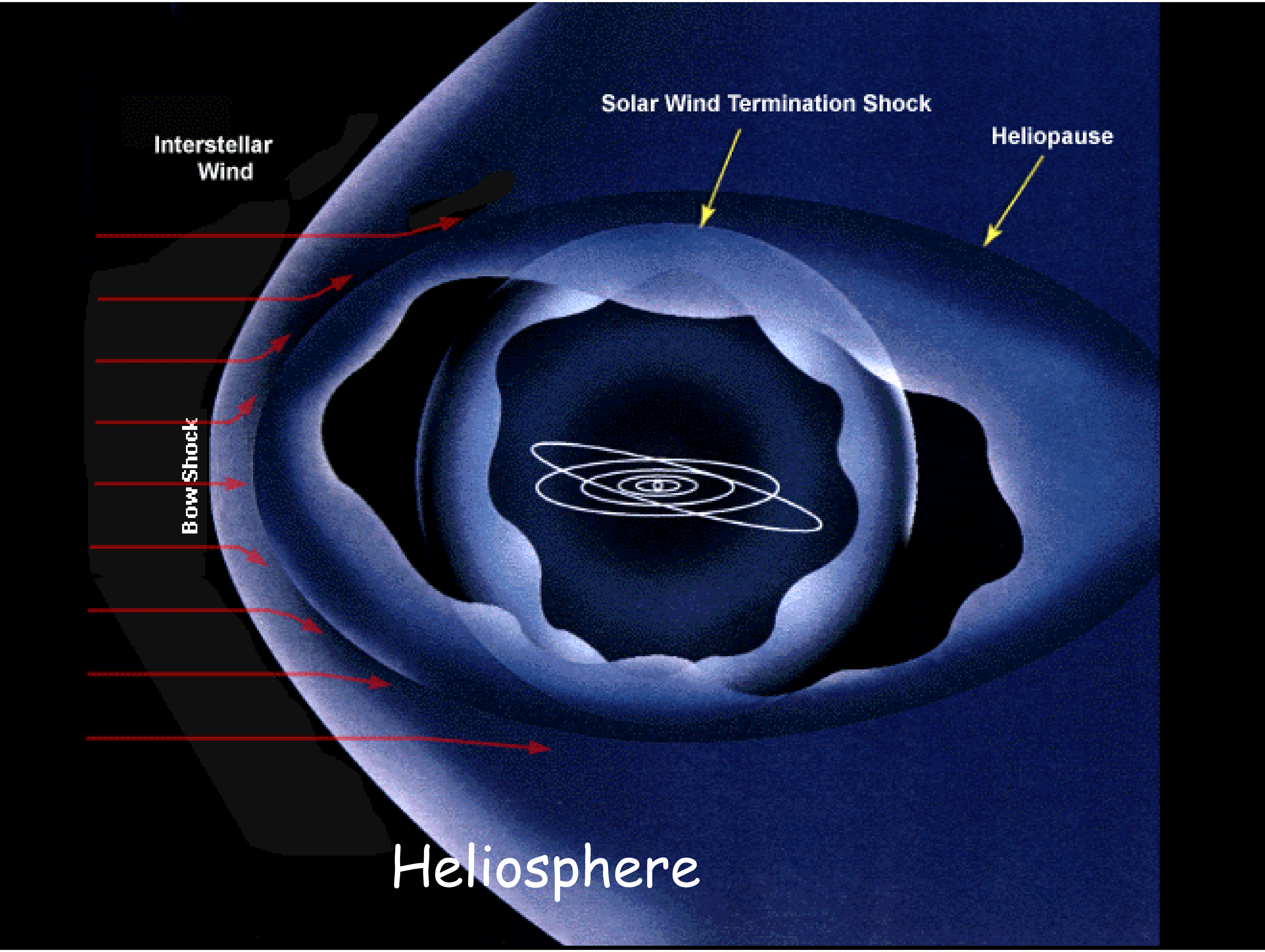
Examples of Astrophysical shocks



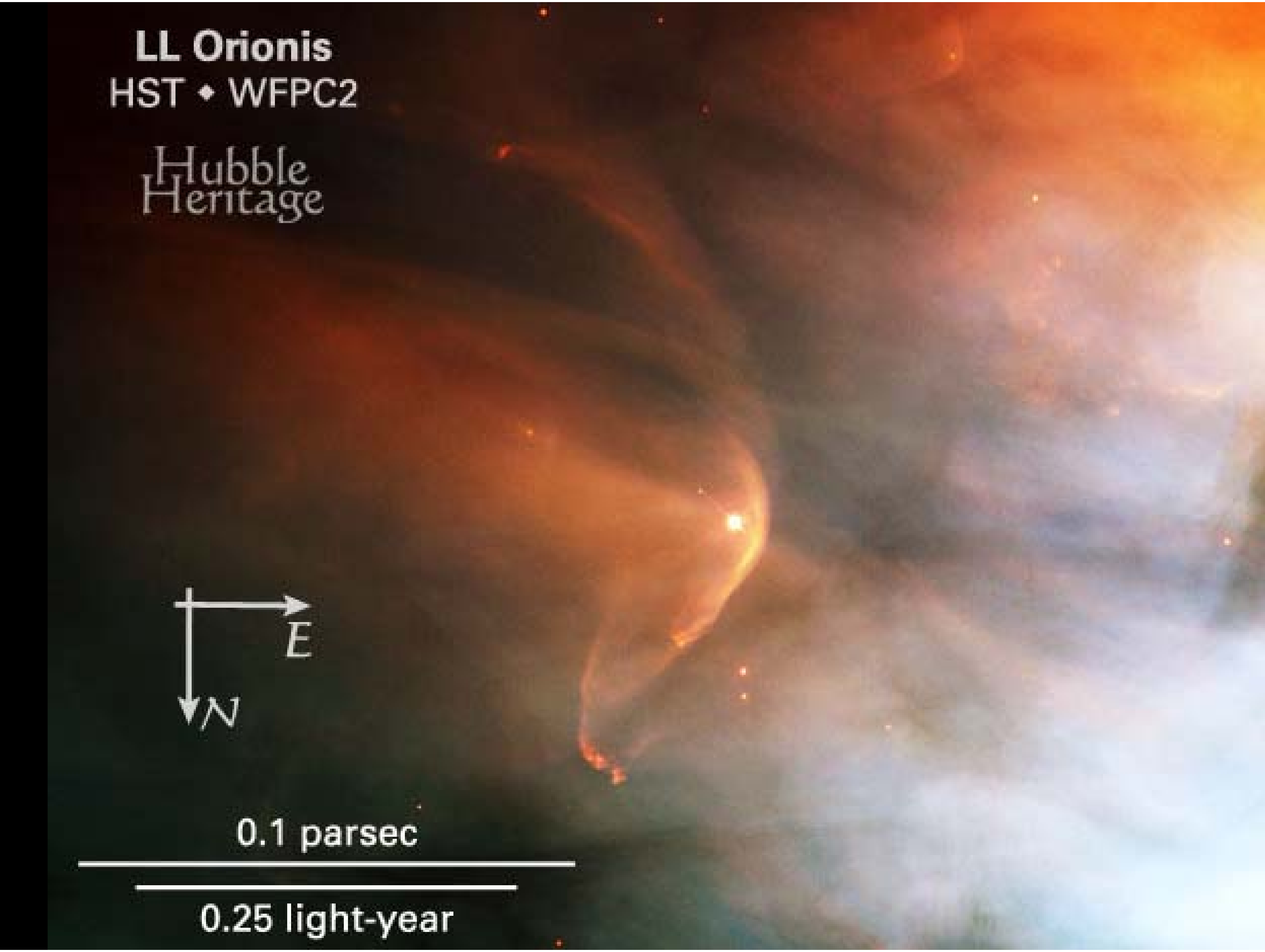


Cometary bow-shocks


Earth's bow shock

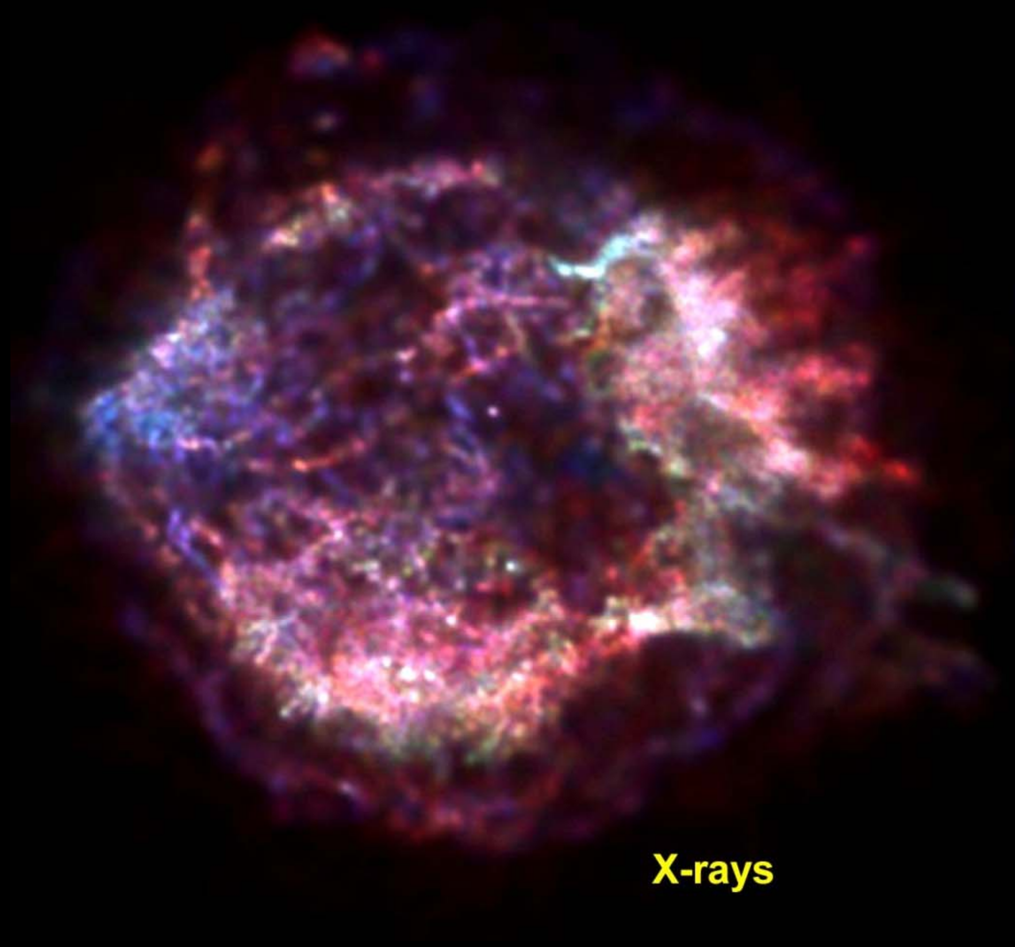


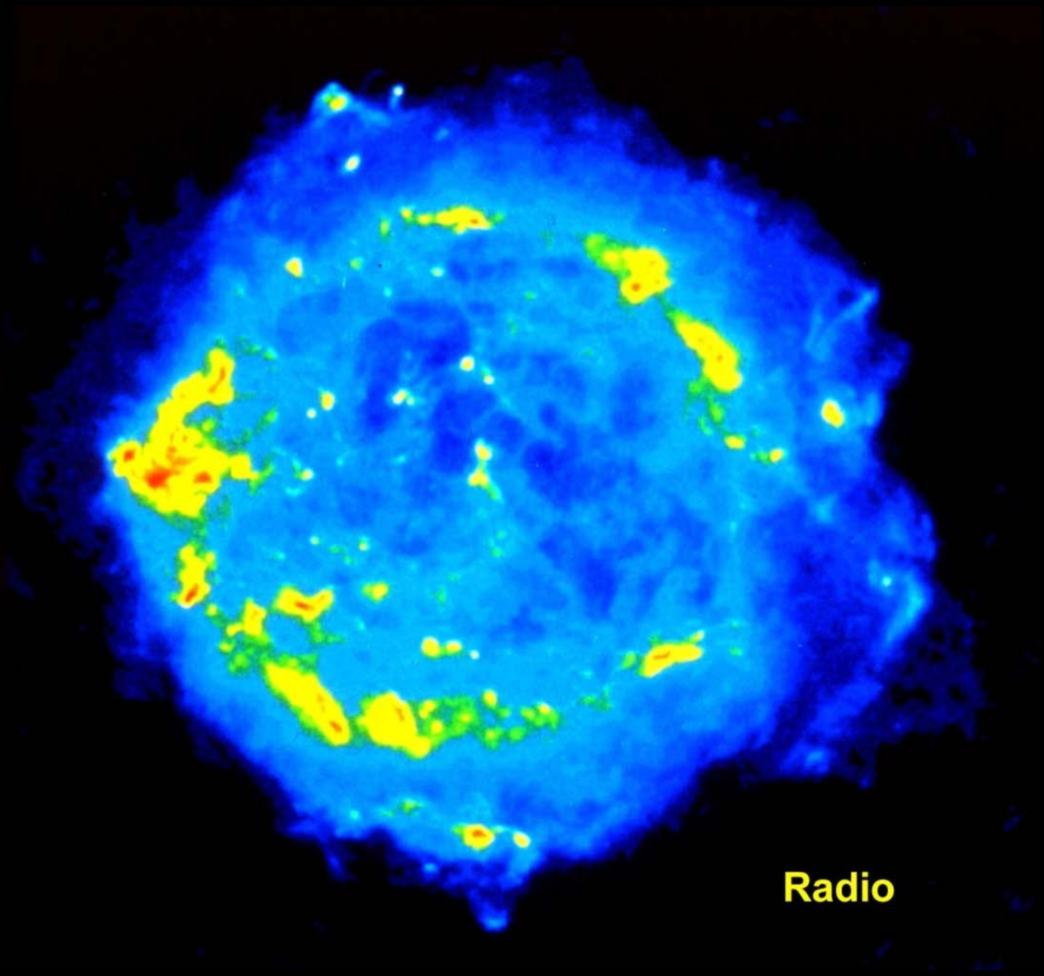
Heliosphere



LL Orionis
HST • WFPC2
Hubble Heritage
E
N
0.1 parsec
0.25 light-year

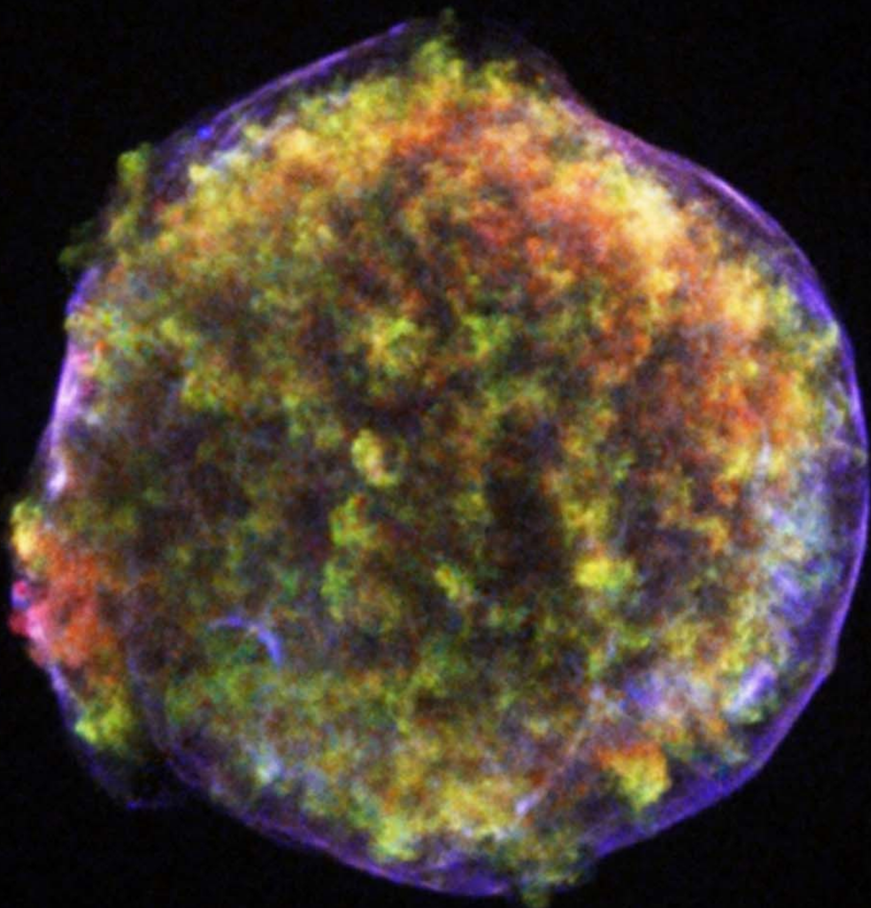
Supernova Remnant Cassiopeia A



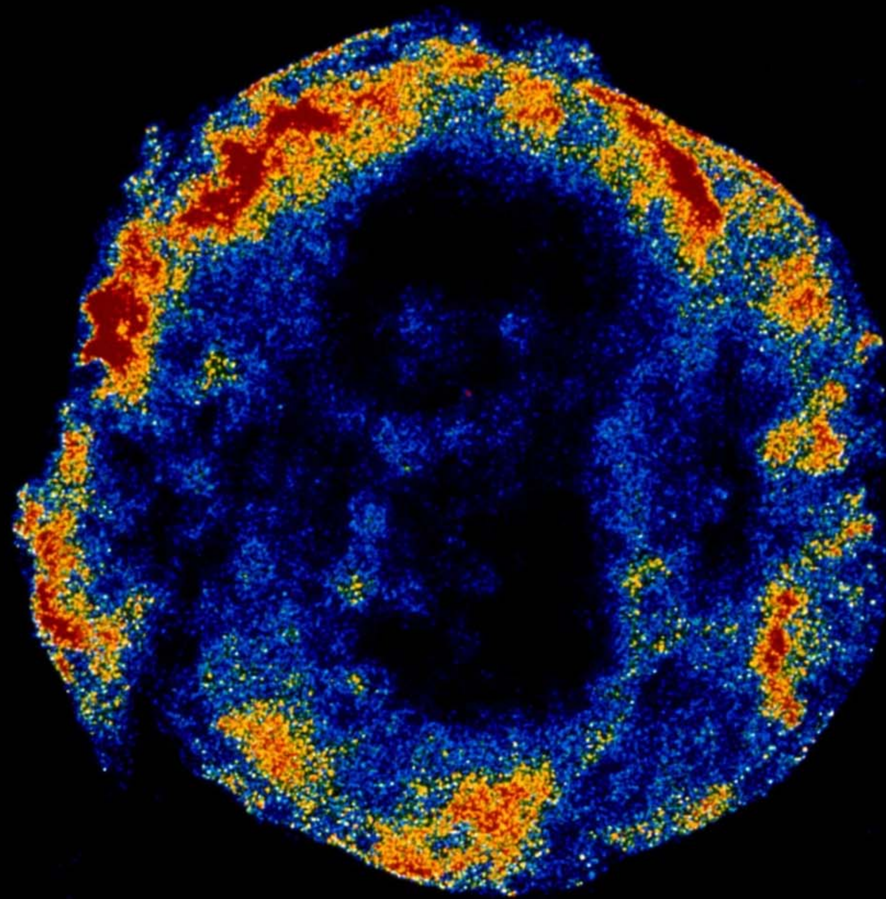


Supernova blast waves

Tycho's Remnant (SN 1572AD)

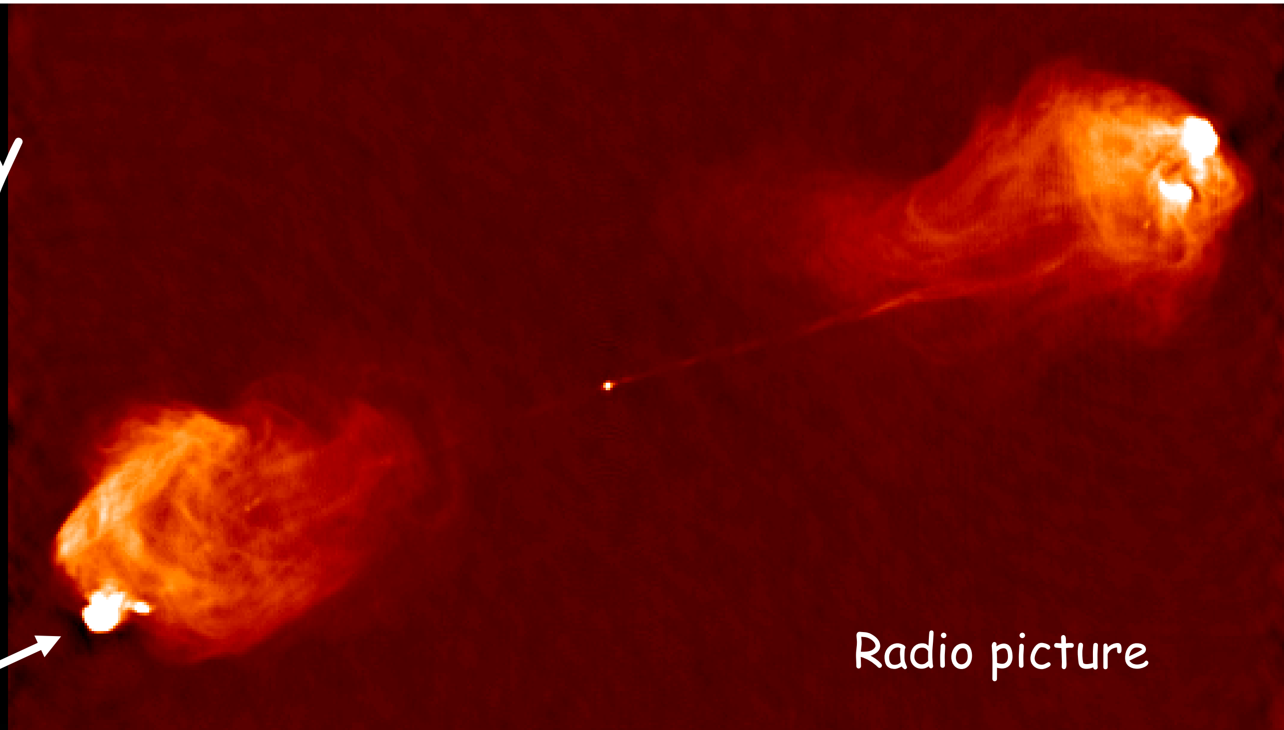


X-Rays (CHANDRA Observatory)

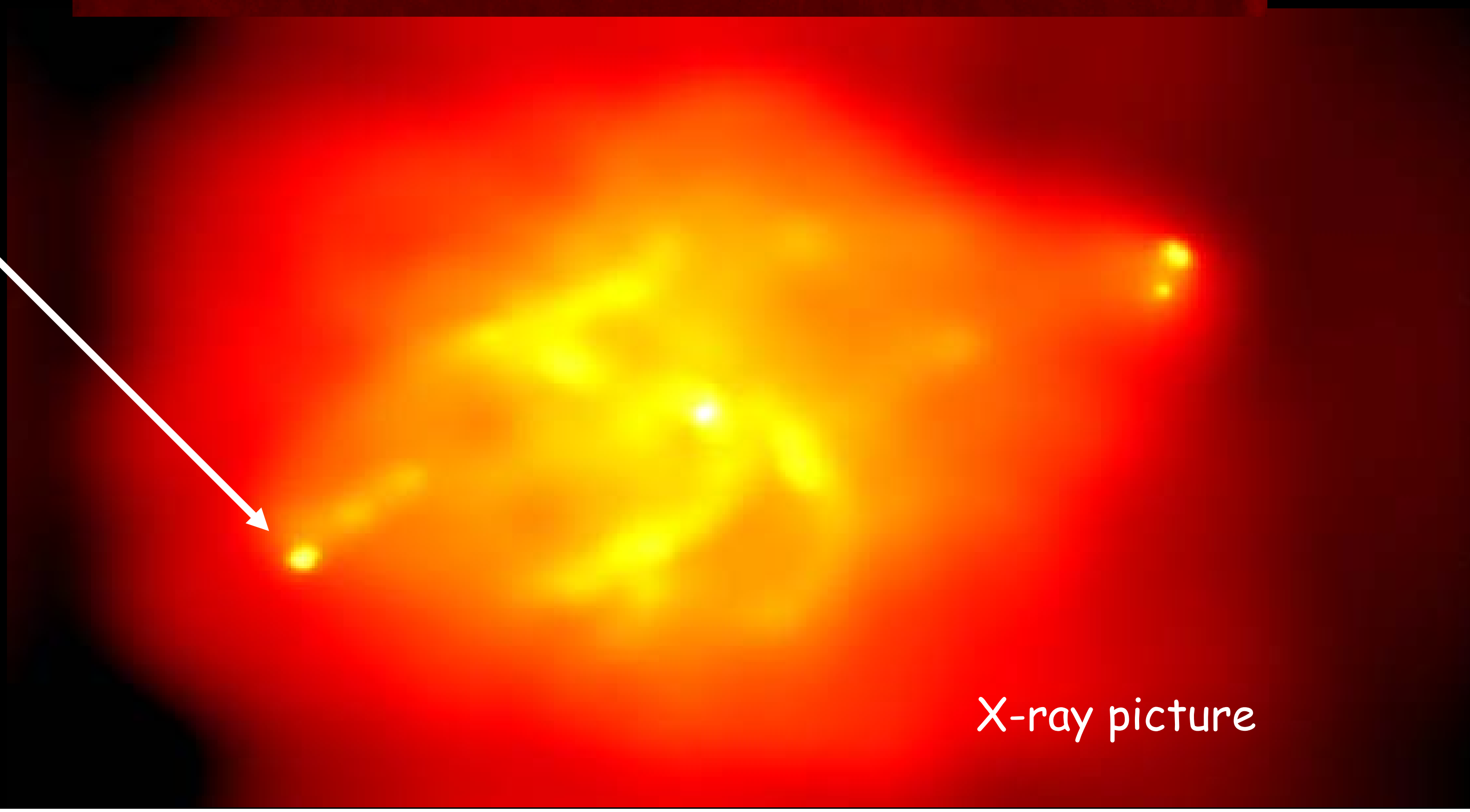


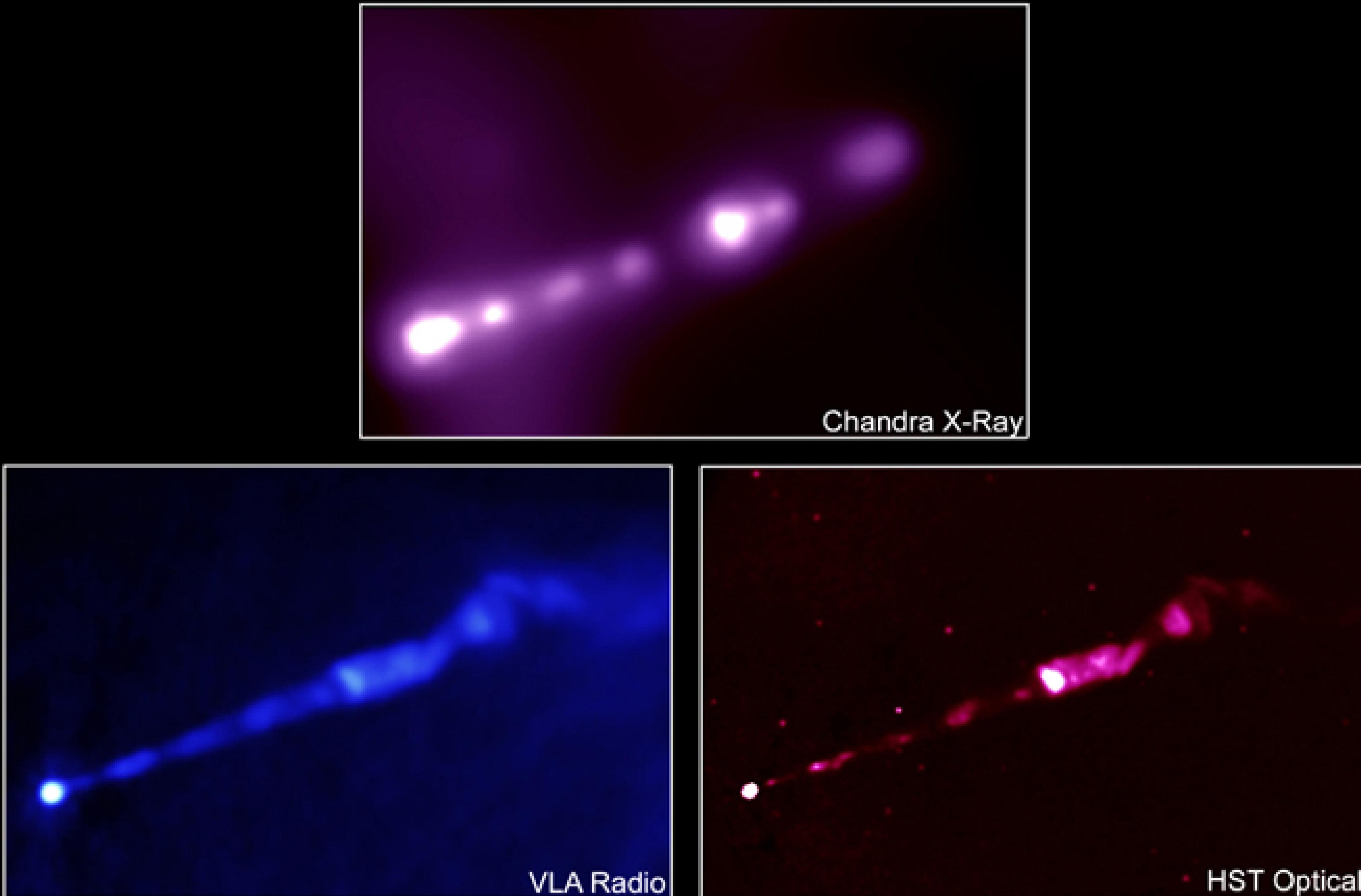
Radio (21cm)

Radio galaxy Cygnus A

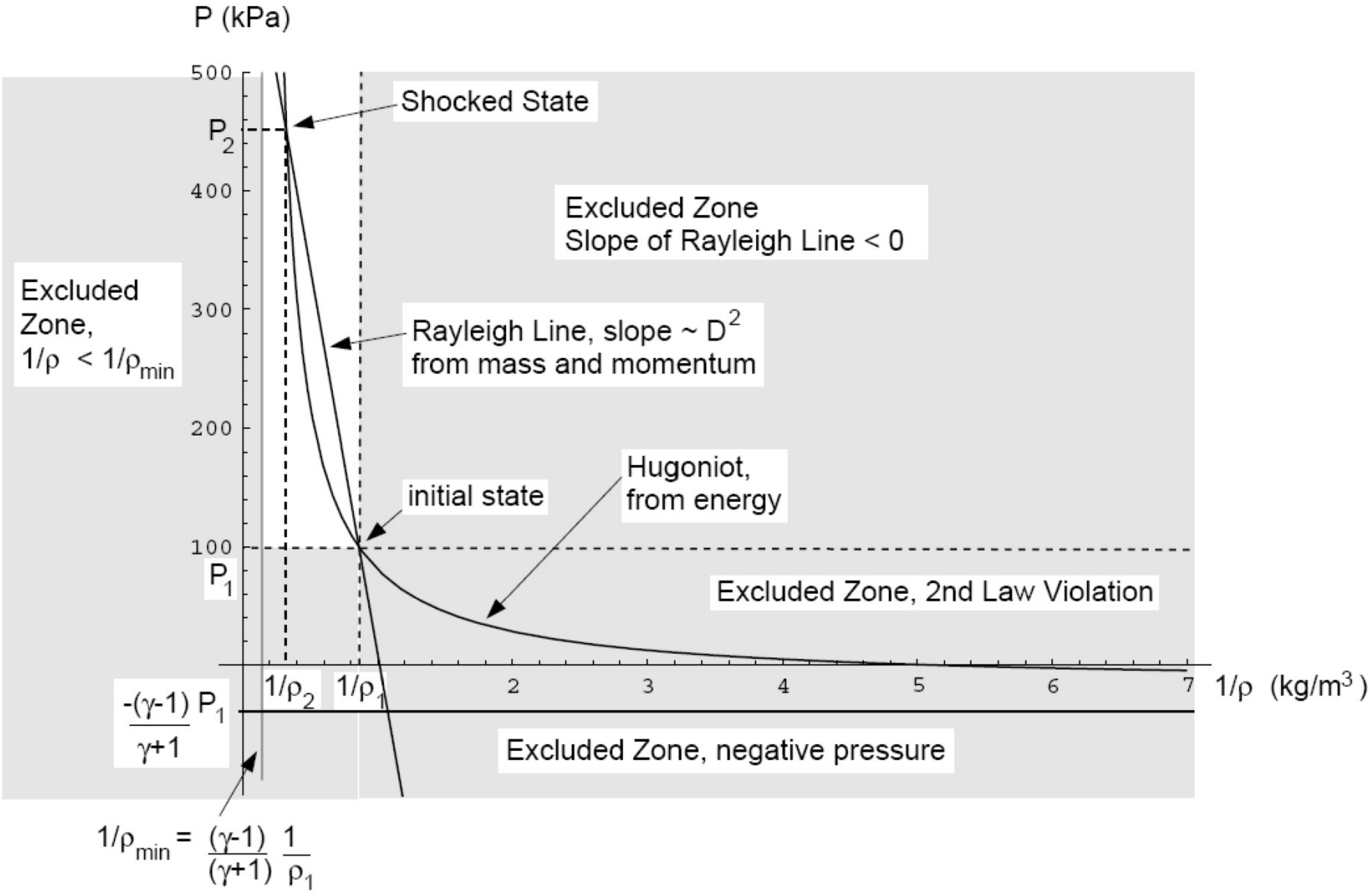


Hot spots are shocks!

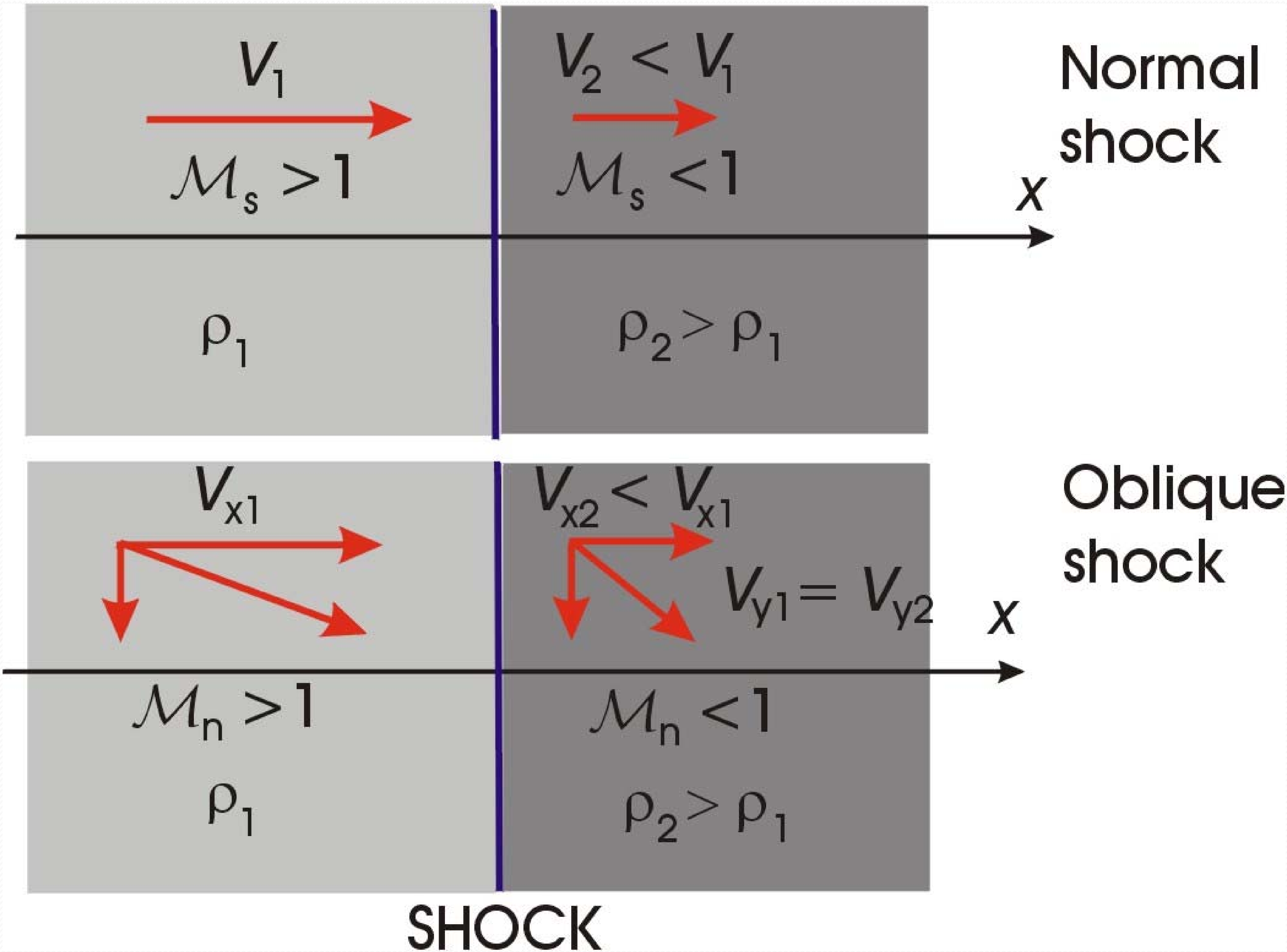




‘Knots’ in jet of Galaxy M87 are shocks!



P (kPa)
500
400
300
200
100
P2
P1
Shocked State
Excluded Zone
Slope of Rayleigh Line < 0
Excluded Zone, 1/ρ < 1/ρmin
Rayleigh Line, slope ~ D2 from mass and momentum
initial state
Hugoniot, from energy
Excluded Zone, 2nd Law Violation
1/ρ2
1/ρ1
2
3
4
5
6
7
1/ρ (kg/m3)
-(γ-1) P1 / γ+1
Excluded Zone, negative pressure
1/ρmin = (γ-1)/(γ+1) 1/ρ1



V1
V2 < V1
Ms >1
Ms <1
Normal shock
x
ρ1
ρ2 > ρ1
Vx1
Vx2 < Vx1
Vy1 = Vy2
Oblique shock
x
Mn >1
Mn <1
ρ1
ρ2 > ρ1
SHOCK

Summary : Shock Physics

Across an infinitely thin steady shock you have, in the shock frame where the shock is at rest, the following Rankine-Hugoniot Jump conditions:

Mass-flux conservation

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

Momentum-flux conservation

$$\rho_1 \left(V_{n1}\right)^2 + P_1 = \rho_2 \left(V_{n2}\right)^2 + P_2$$

$$V_{t1} = V_{t2}$$

Energy-flux conservation

$$\tfrac{1}{2}\left(V_{n1}\right)^2 + \frac{\gamma P_1}{(\gamma-1)\rho_1} = \tfrac{1}{2}\left(V_{n2}\right)^2 + \frac{\gamma P_2}{(\gamma-1)\rho_2}$$

Summary: Rankine-Hugoniot relations (for normal shock)

Fundamental parameter:
Mach Number

$$\mathcal{M}_s \equiv \frac{\text{shock speed}}{\text{sound speed}} = \frac{V_1}{c_{s1}}$$

R-H Jump Conditions
relate the up- and downstream
quantities at the shock:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1}$$

From normal shock to oblique shocks:

All relations remain the same if one makes the replacement:

$$V_1 \Rightarrow V_{n1} = V_1 \cos\theta_1 \ ,$$

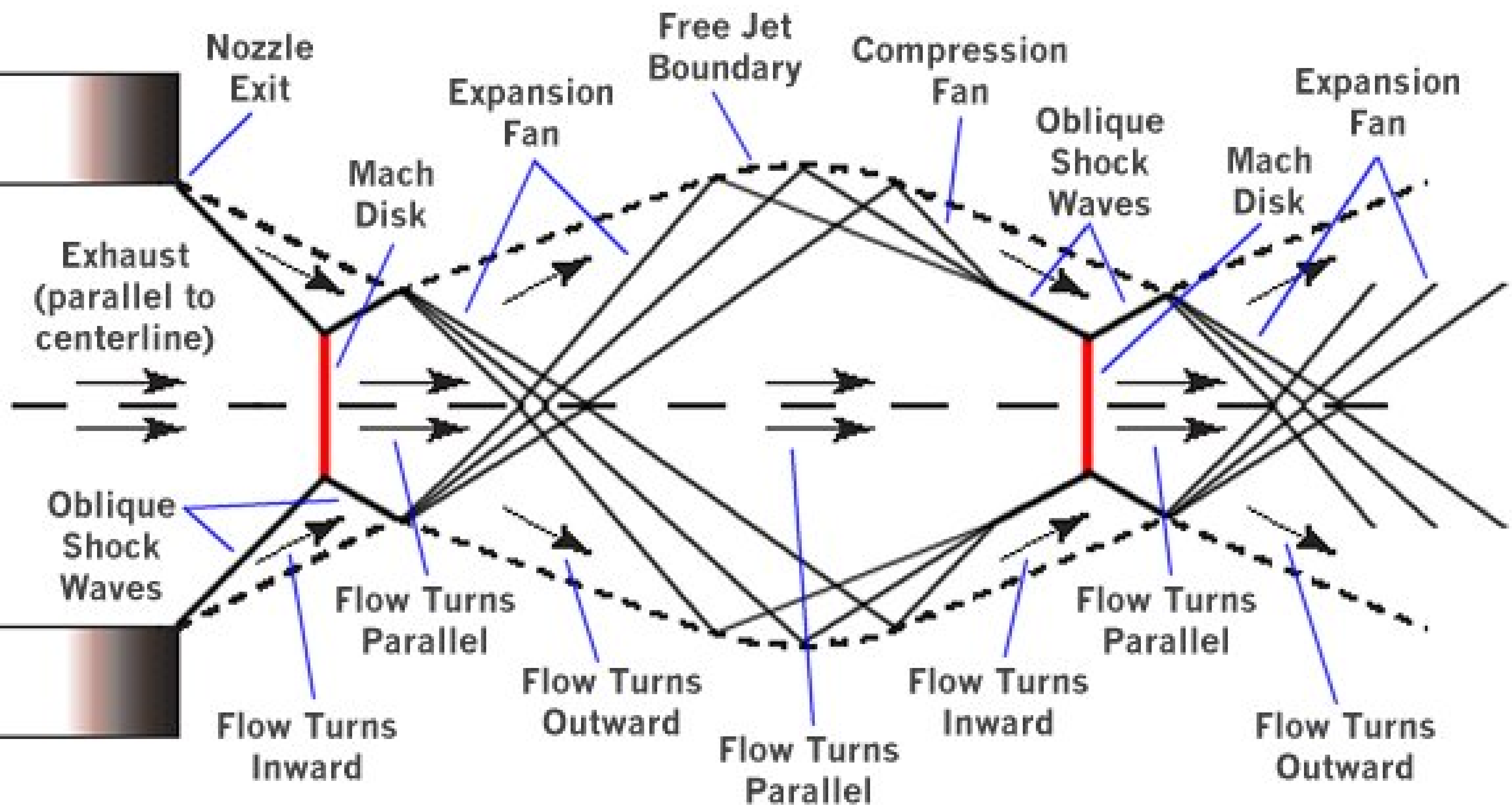
$$\mathcal{M}_S \Rightarrow \mathcal{M}_n = V_{n1} / c_{s1} = \mathcal{M}_S \cos\theta_1$$

θ is the angle between upstream velocity and normal on shock surface

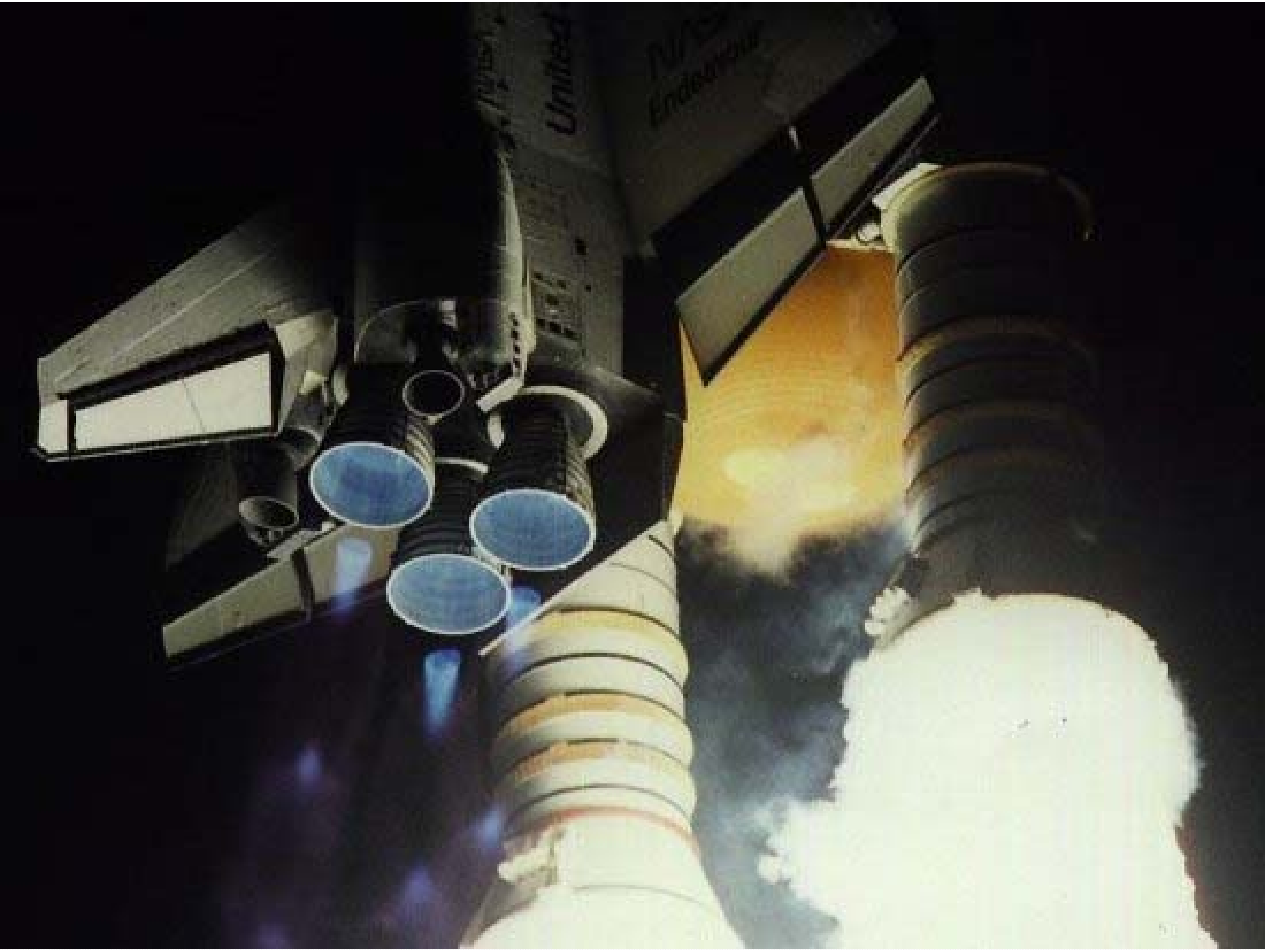
Tangential velocity along shock surface is <u>unchanged</u>

$$V_{t1} = V_1 \sin\theta_1 = V_{t2} = V_2 \sin\theta_2$$

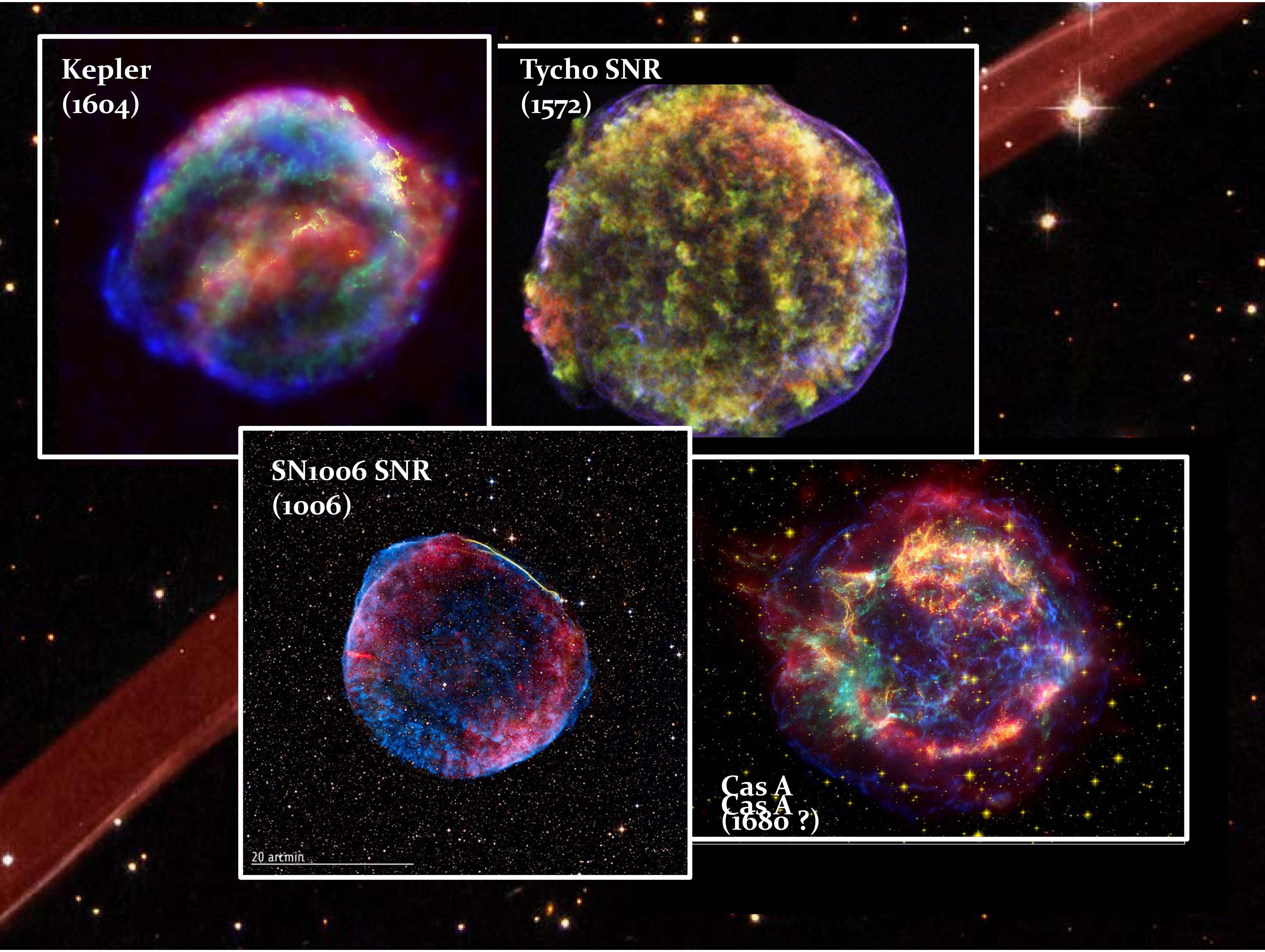
Example from Jet/Rocket engines




6062



Supernova Remnants


Kepler
(1604)
Tycho SNR
(1572)
SN1006 SNR
(1006)
20 arcmin
Cas A
Cas A
(1680 ?)

JOANNIS KEPPLERI
Sac. Cæs. Majest. Mathematici
DE
STELLA NOVA
IN PEDE SERPENTARII, ET QUI SUB EJUS EXORTUM DE NOVO INIIT,
TRIGONO IGNEO.
LIBELLUS ASTRONOMICIS, PHYSICIS, METAPHYSICIS, Meteorologicis & Astrologicis Disputationibus, ἐνδόξοις & παραδόξοις plenus.

ACCESSERUNT

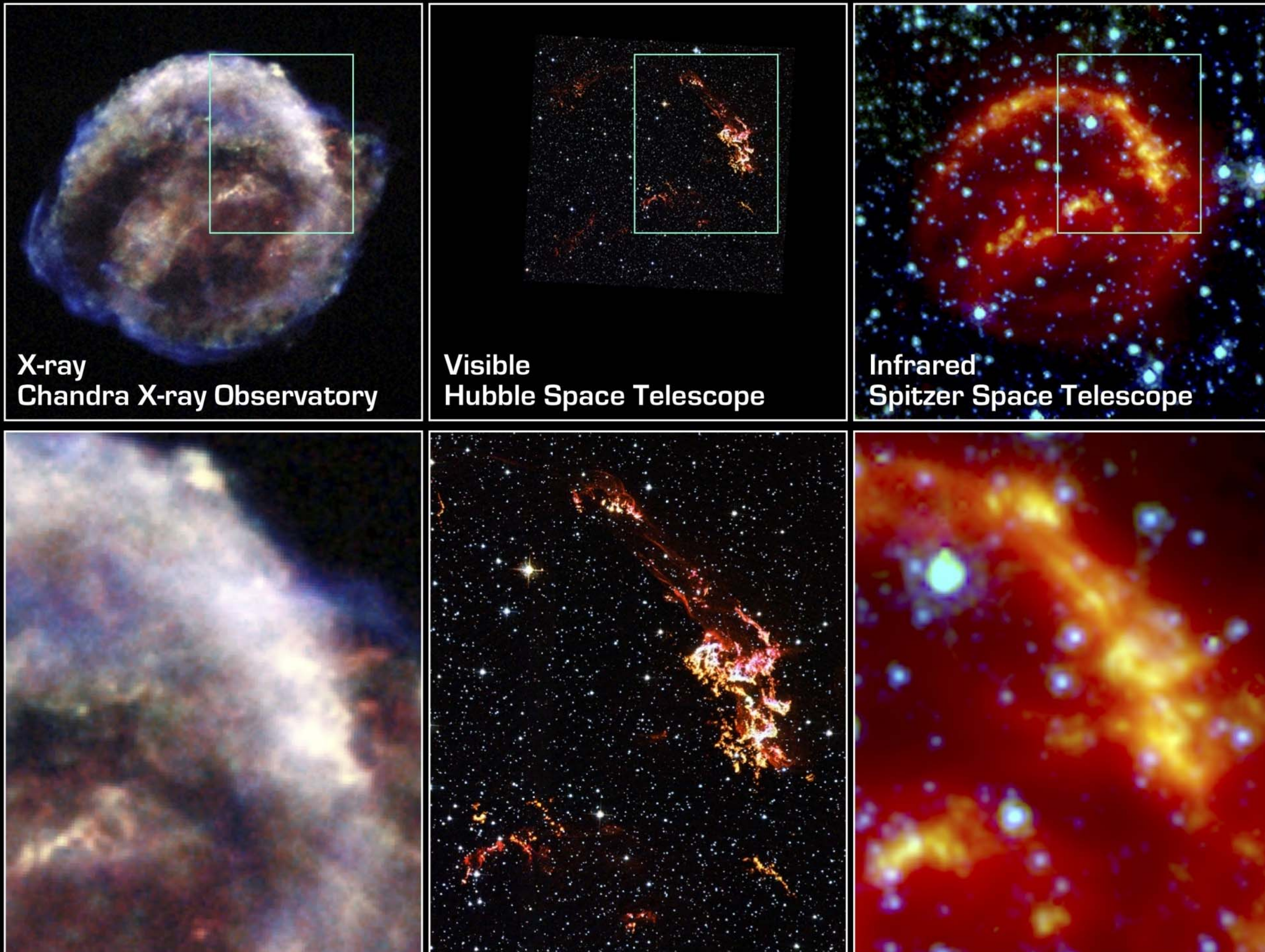
I. *DE STELLA INCOGNITA CYGNI: Narratio Astronomica.*

II. *DE JESU CHRISTI SERVATORIS VERO Anno Natalitio, consideratio novissimæ sententiæ LAURENTII SUSLYGÆ Poloni, quatuor annos in usitata Epocha desiderantis.*

Cum Privilegio S. C. Majest. ad annos XV.

GRANA DAT E FIMO SCRUTANS

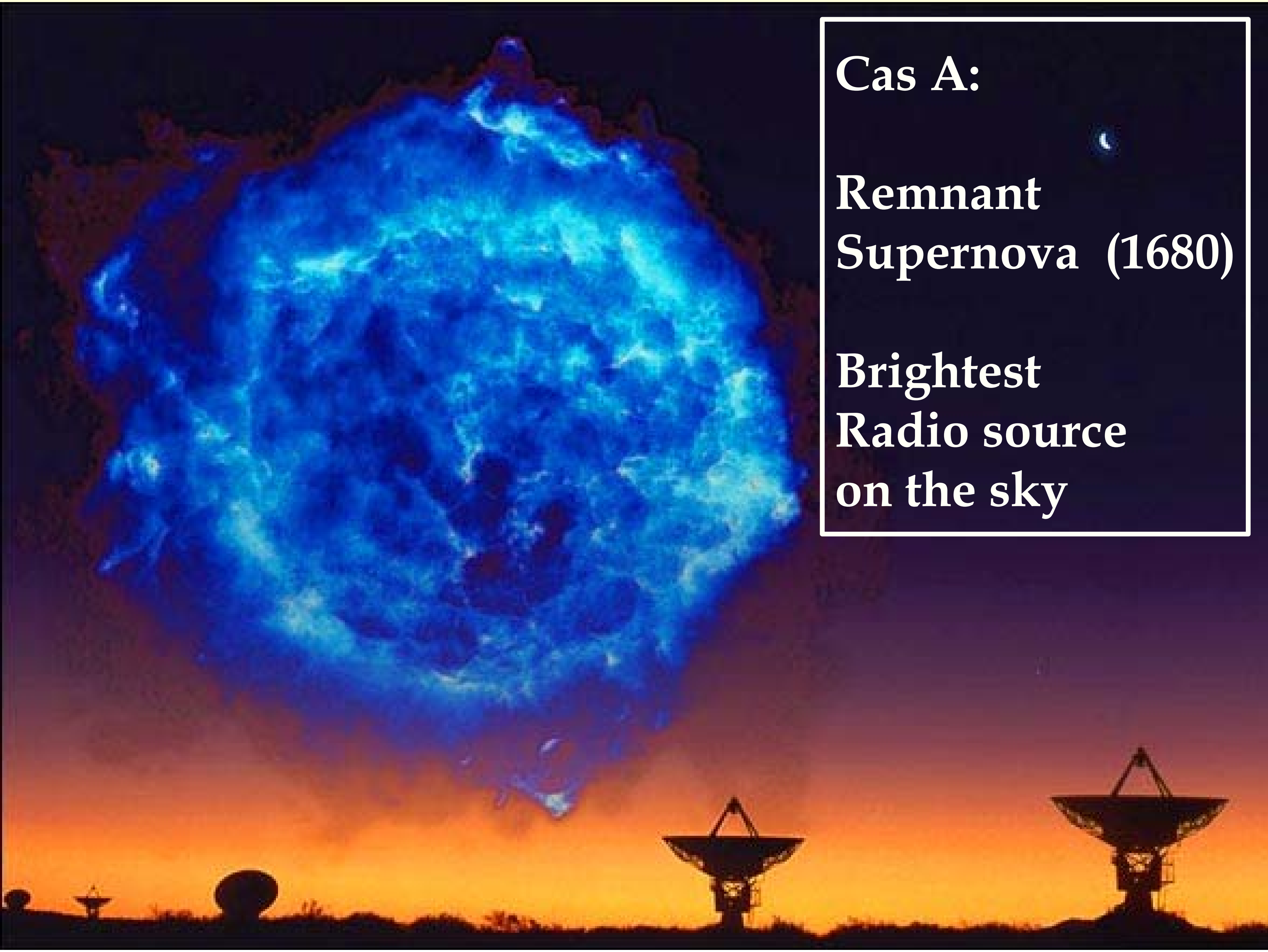
PRAGAE
Ex Officina calcographica PAULI SESSII.
ANNO M. DCVI.

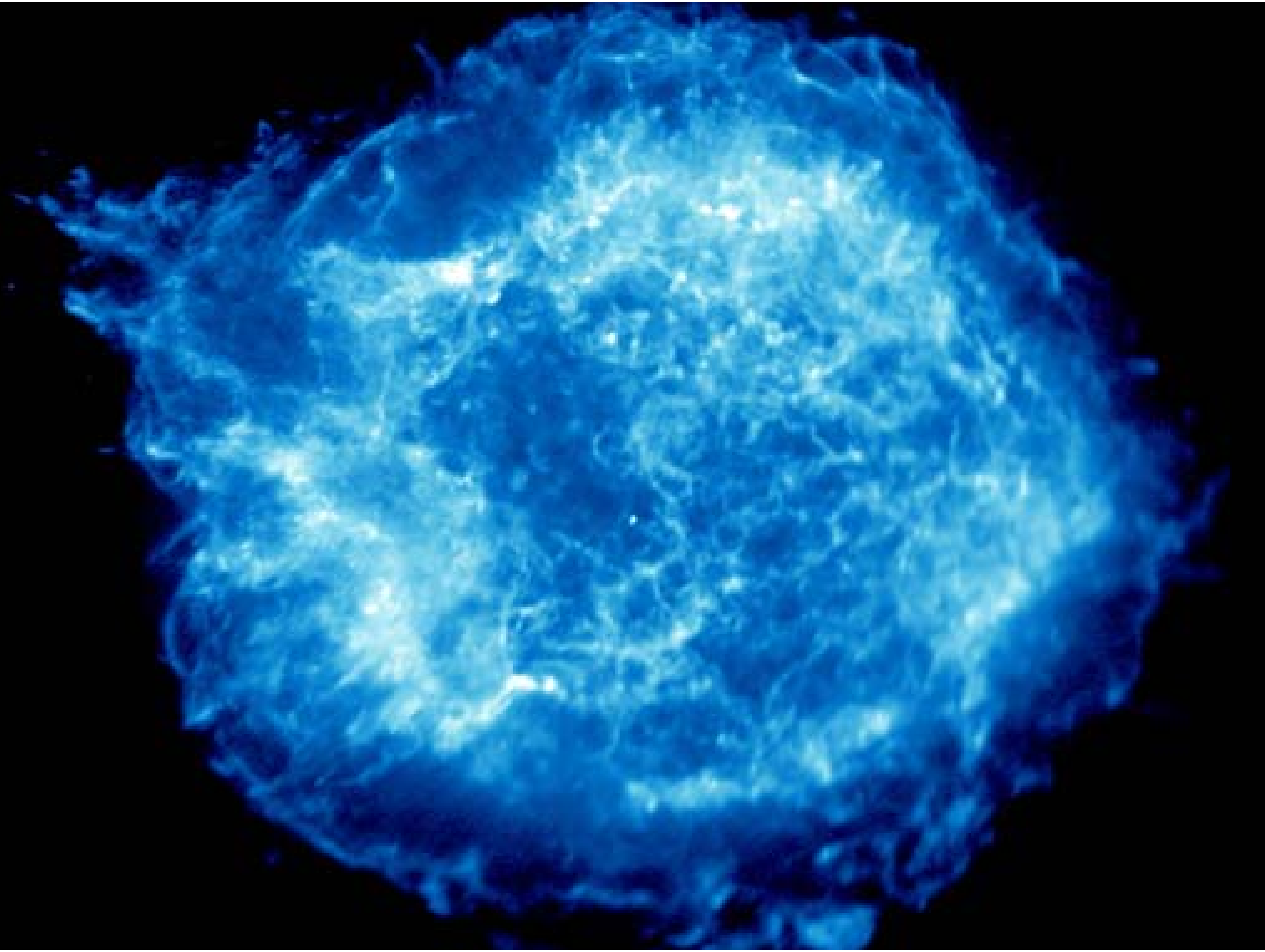


Kepler's Supernova Remnant • SN 1604

ssc2004-15b

NASA, ESA / JPL-Caltech / R. Sankrit & W. Blair (Johns Hopkins University)


Cas A:
Remnant
Supernova (1680)
Brightest
Radio source
on the sky

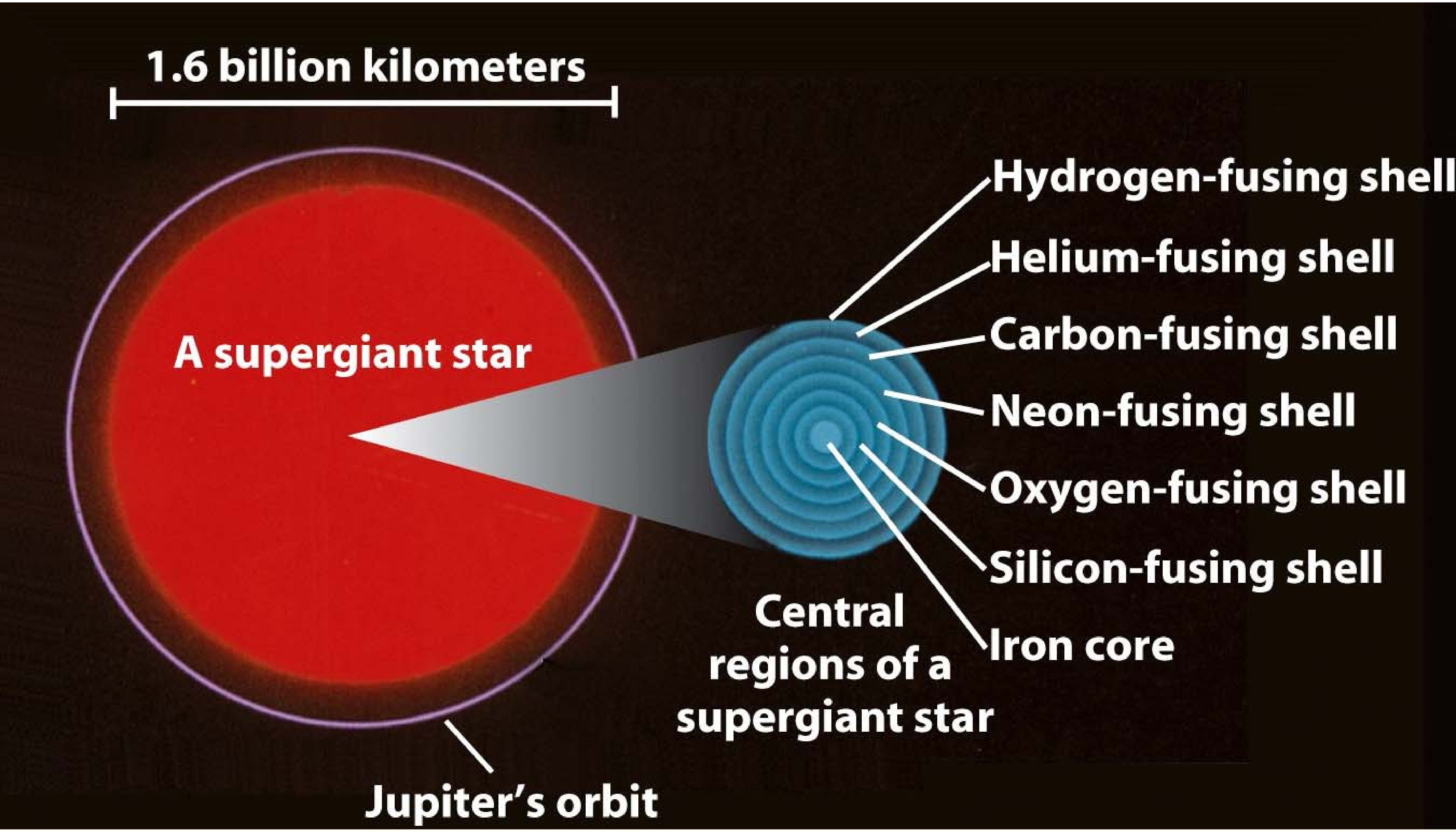


Cas A SNR flythrough

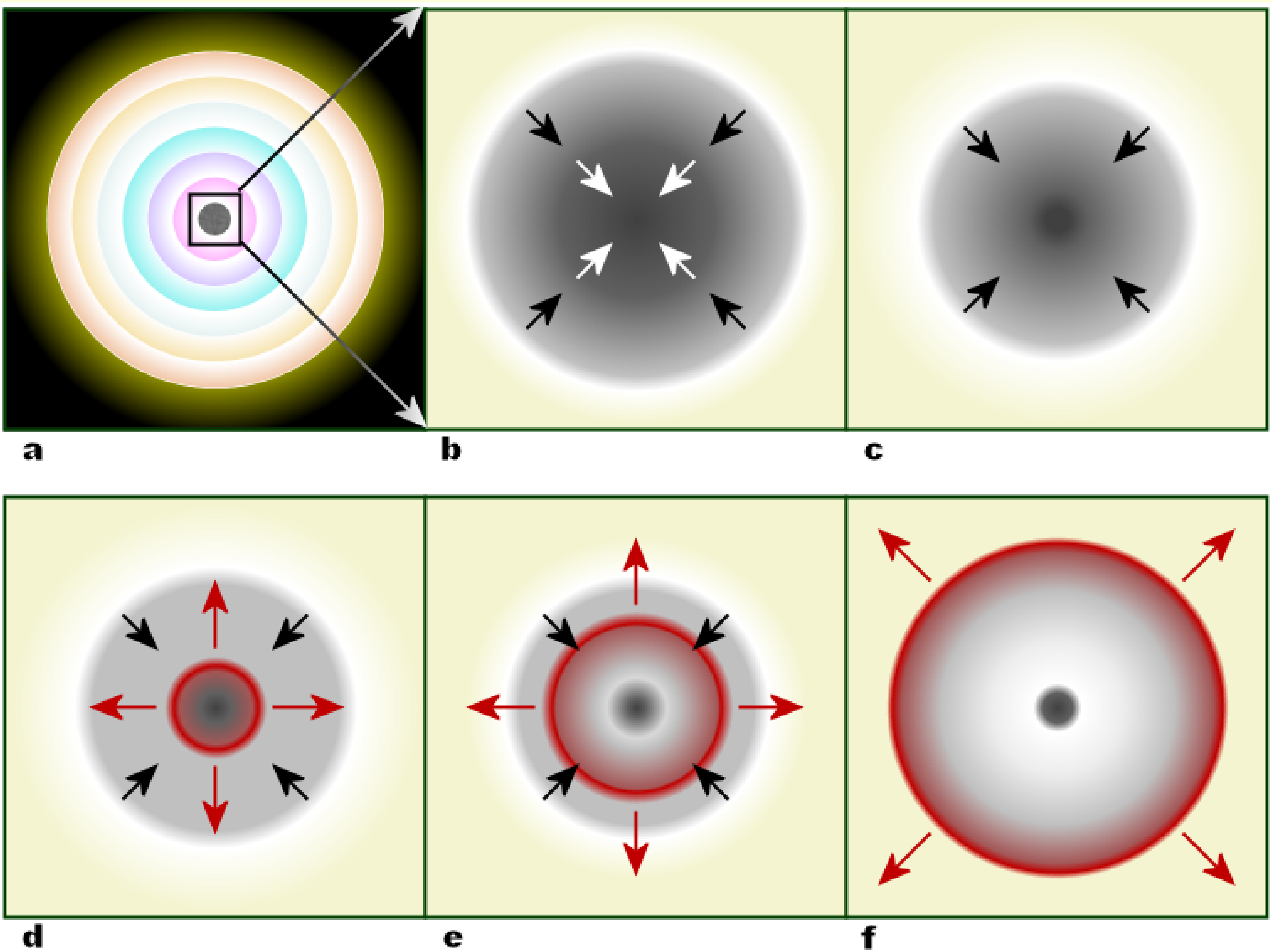
Theory of Supernova Blast Waves

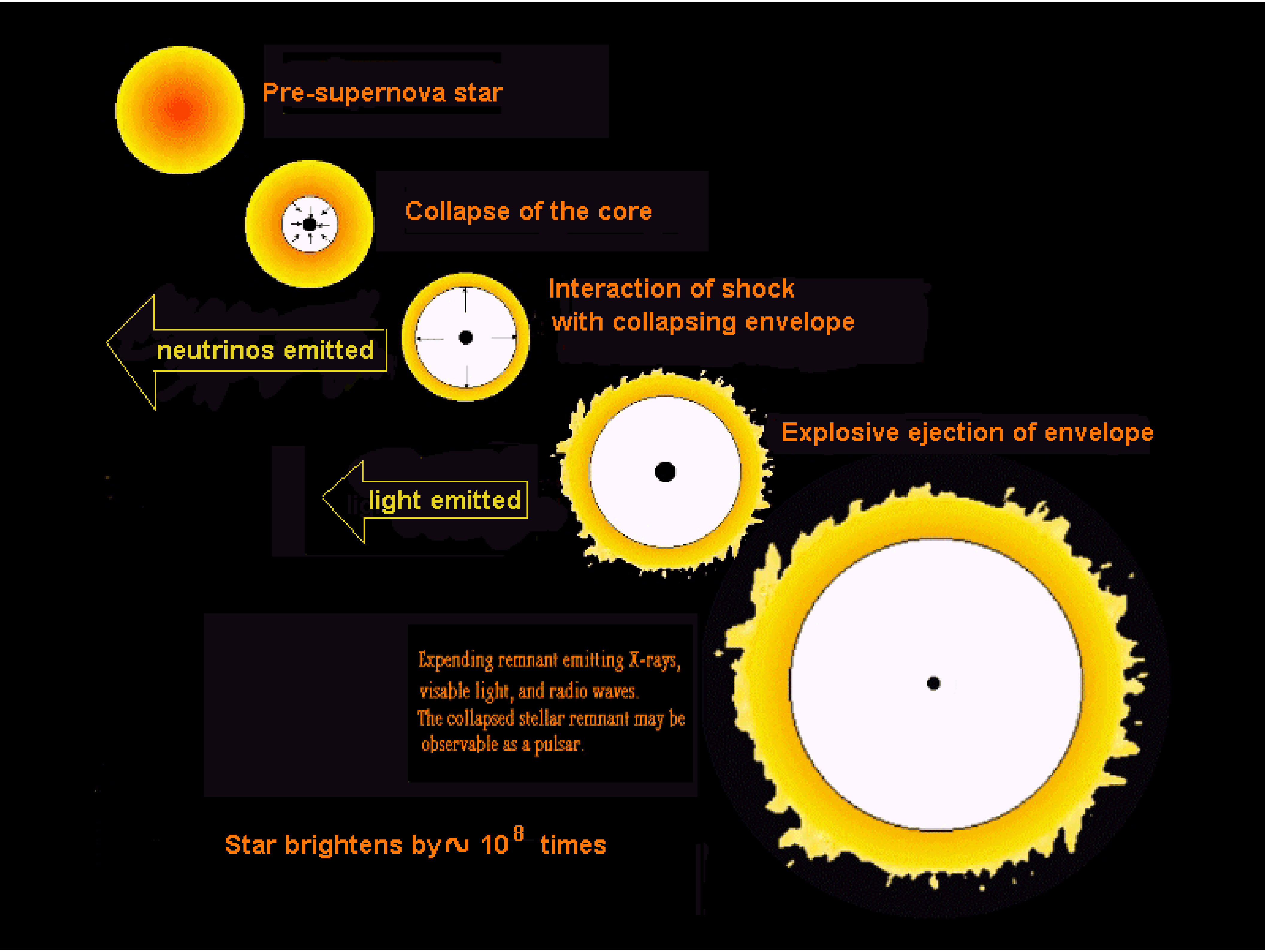
Supernovae:	
Type Ia	Subsonic deflagration wave turning into a supersonic detonation wave in outer layers. Mechanism: explosive carbon burning in a mass-accreting white dwarf
Type Ib-Ic & Type II	Core collapse of massive star

Core-Collapse SN



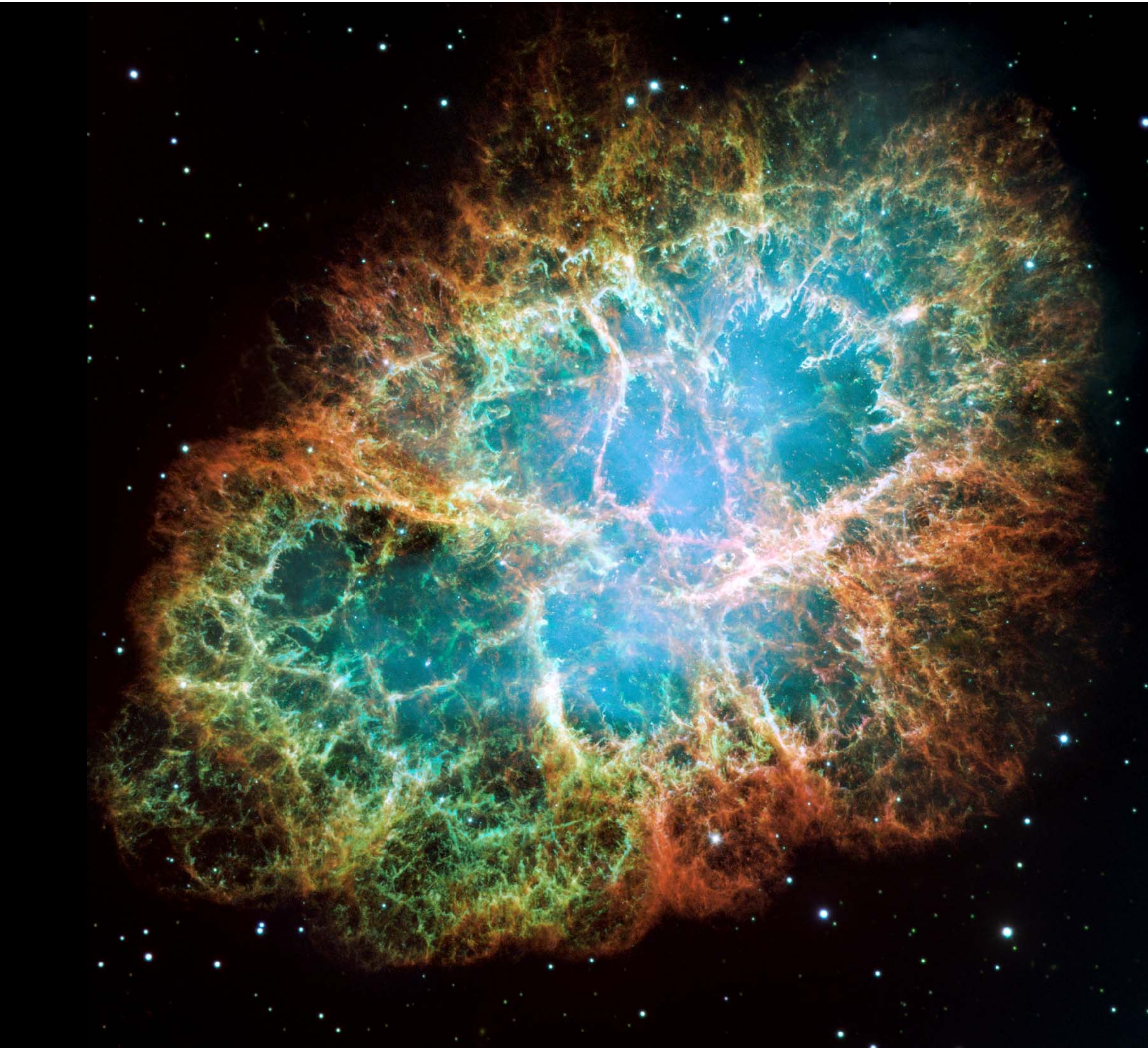
- In the last stages of its life, high-mass star:
  - iron-rich core
  - surrounded by concentric shells, hosting the various thermonuclear reactions
- The sequence of thermonuclear reactions stops here:
  - formation of elements heavier than iron requires
  - input of energy rather than causing energy to be released


a
b
c
d
e
f

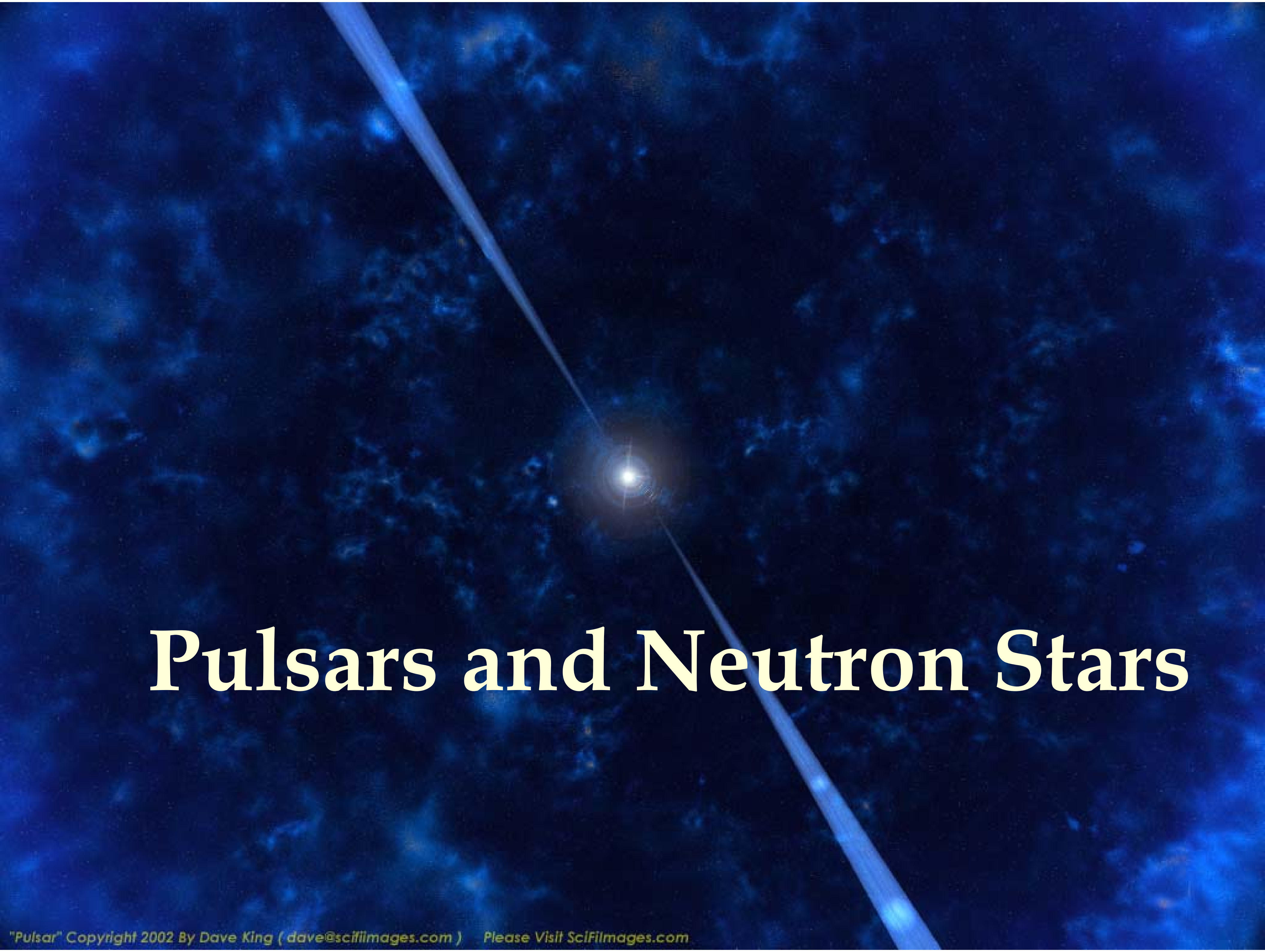


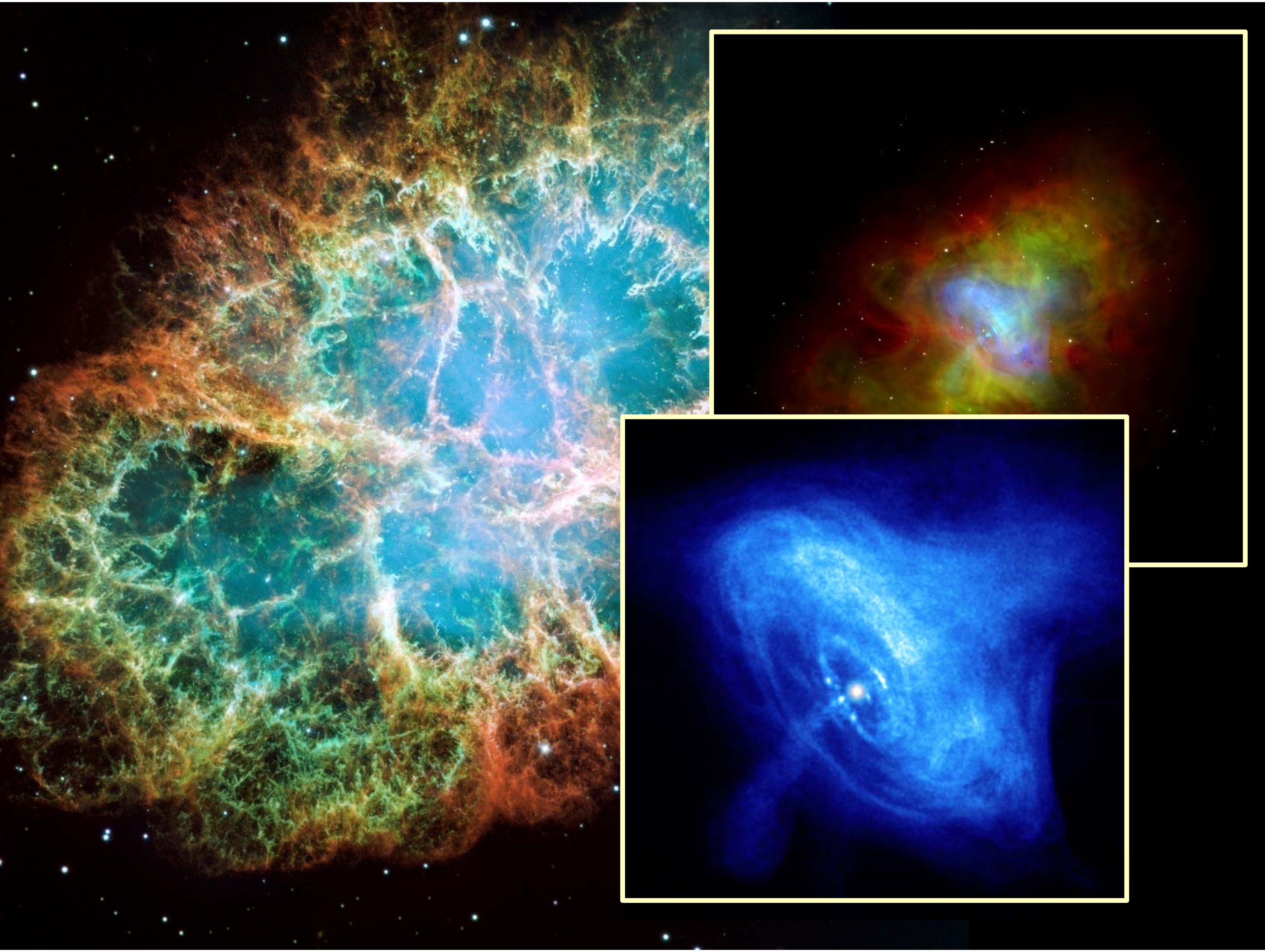
Pre-supernova star
Collapse of the core
Interaction of shock
with collapsing envelope
neutrinos emitted
Explosive ejection of envelope
light emitted
Expending remnant emitting X-rays,
visable light, and radio waves.
The collapsed stellar remnant may be
observable as a pulsar.
Star brightens by ~ 10^8 times

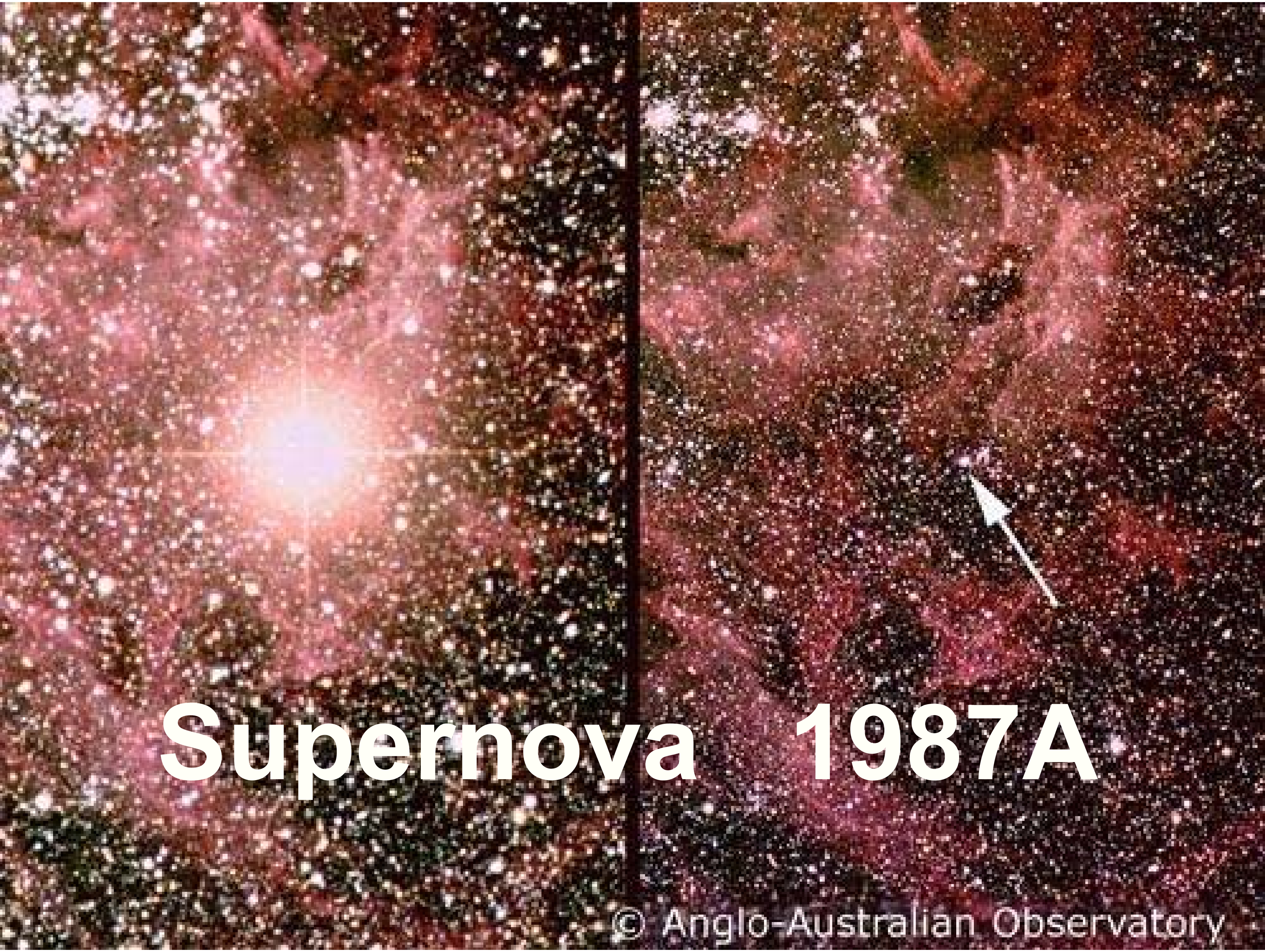
Supernova II Explosion: SN1054



Pulsars and Neutron Stars

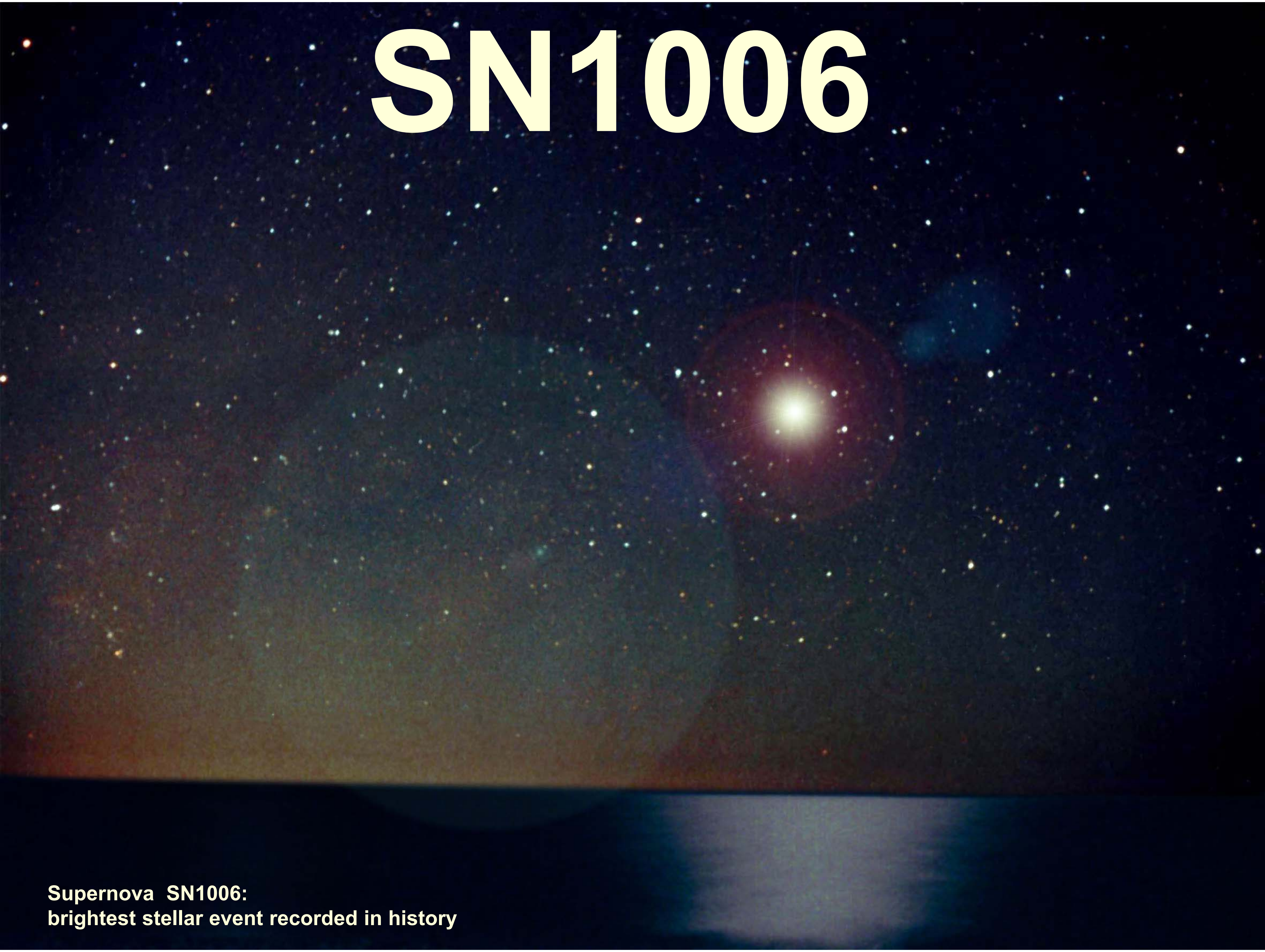





Supernova 1987A
© Anglo-Australian Observatory

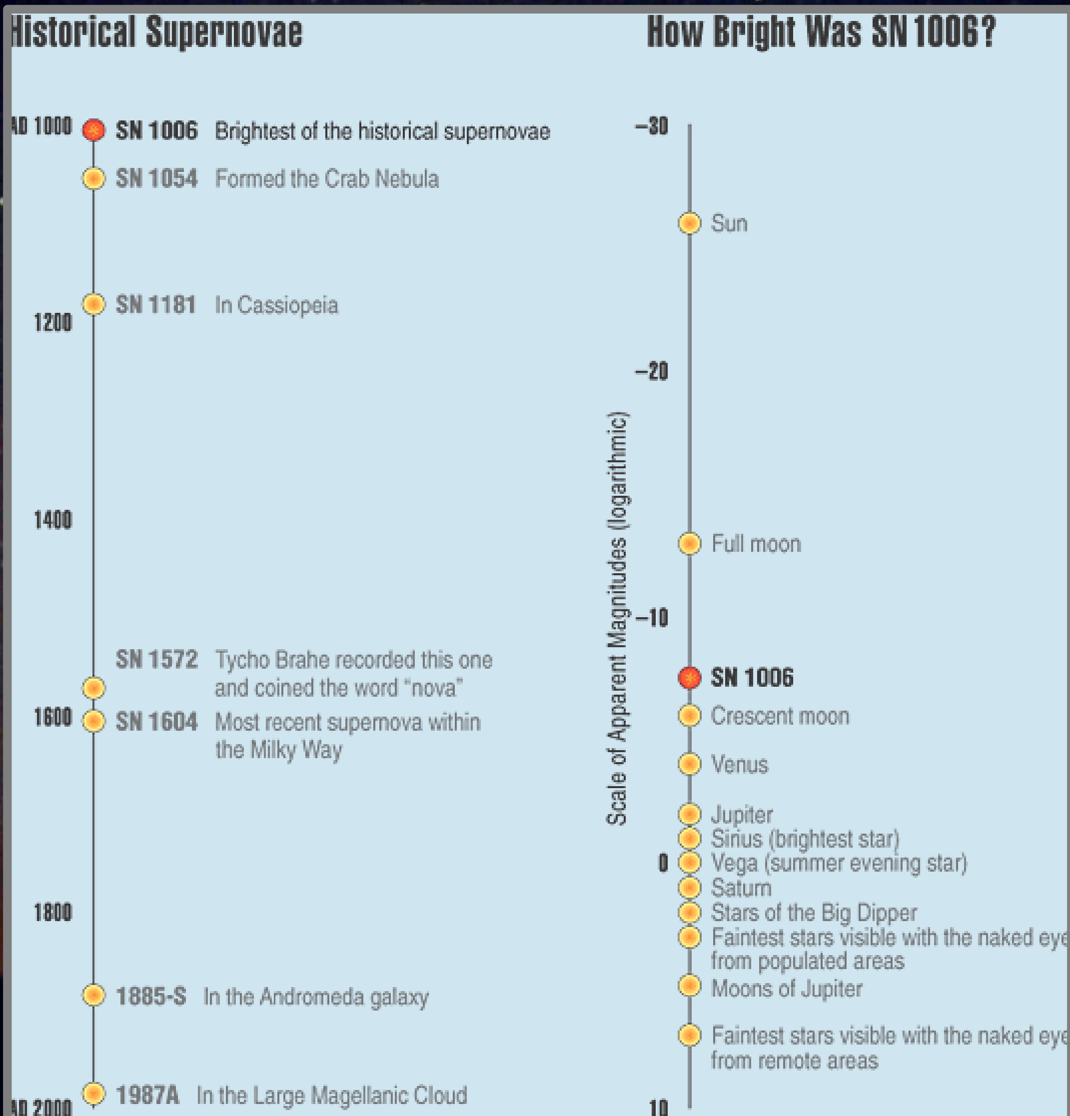
Thermonuclear SN (Supernova Ia)

SN1006



Supernova SN1006:
brightest stellar event recorded in history

SN1006

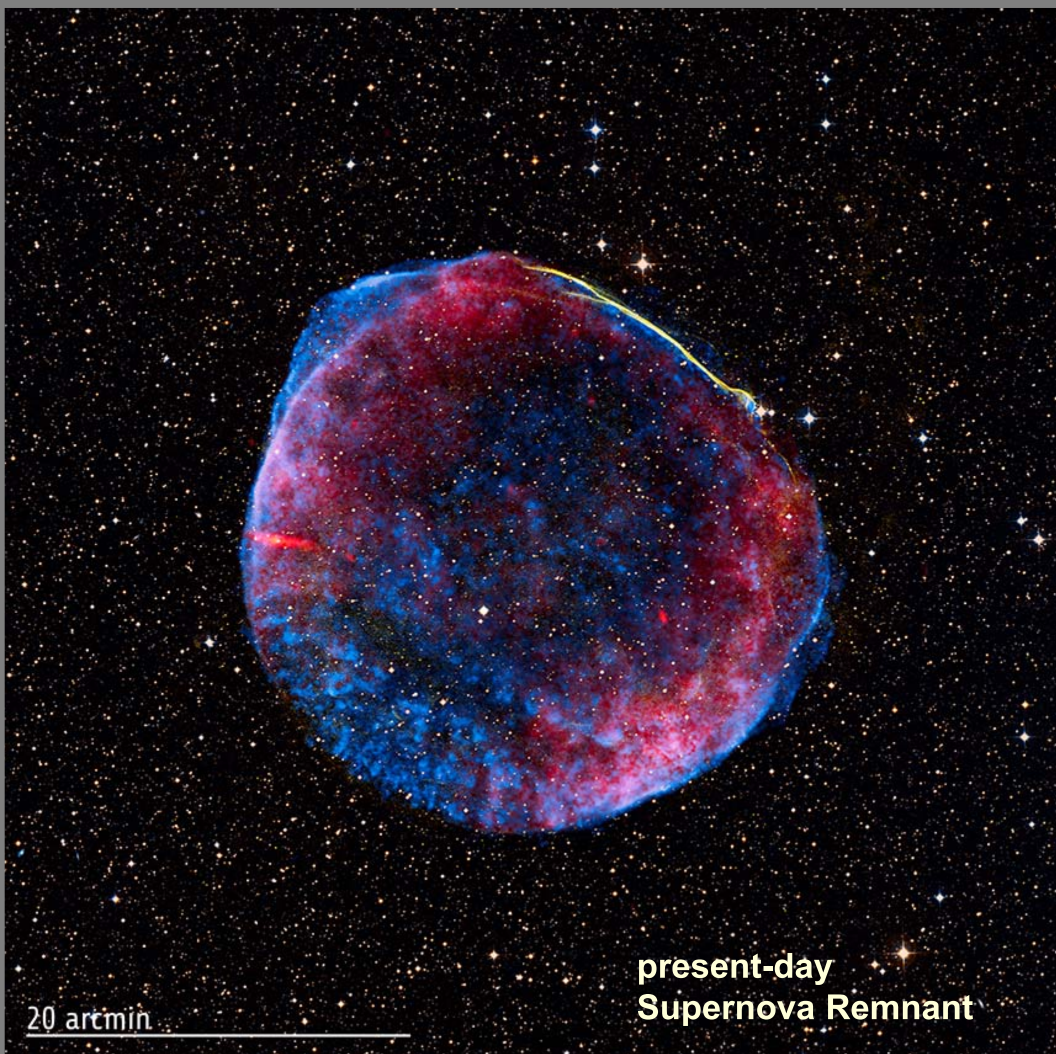


Supernova SN1006:

- **brightness:** m = -7.5
- **distance:** d=2.2 kpc
- **recorded:** China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

SN1006



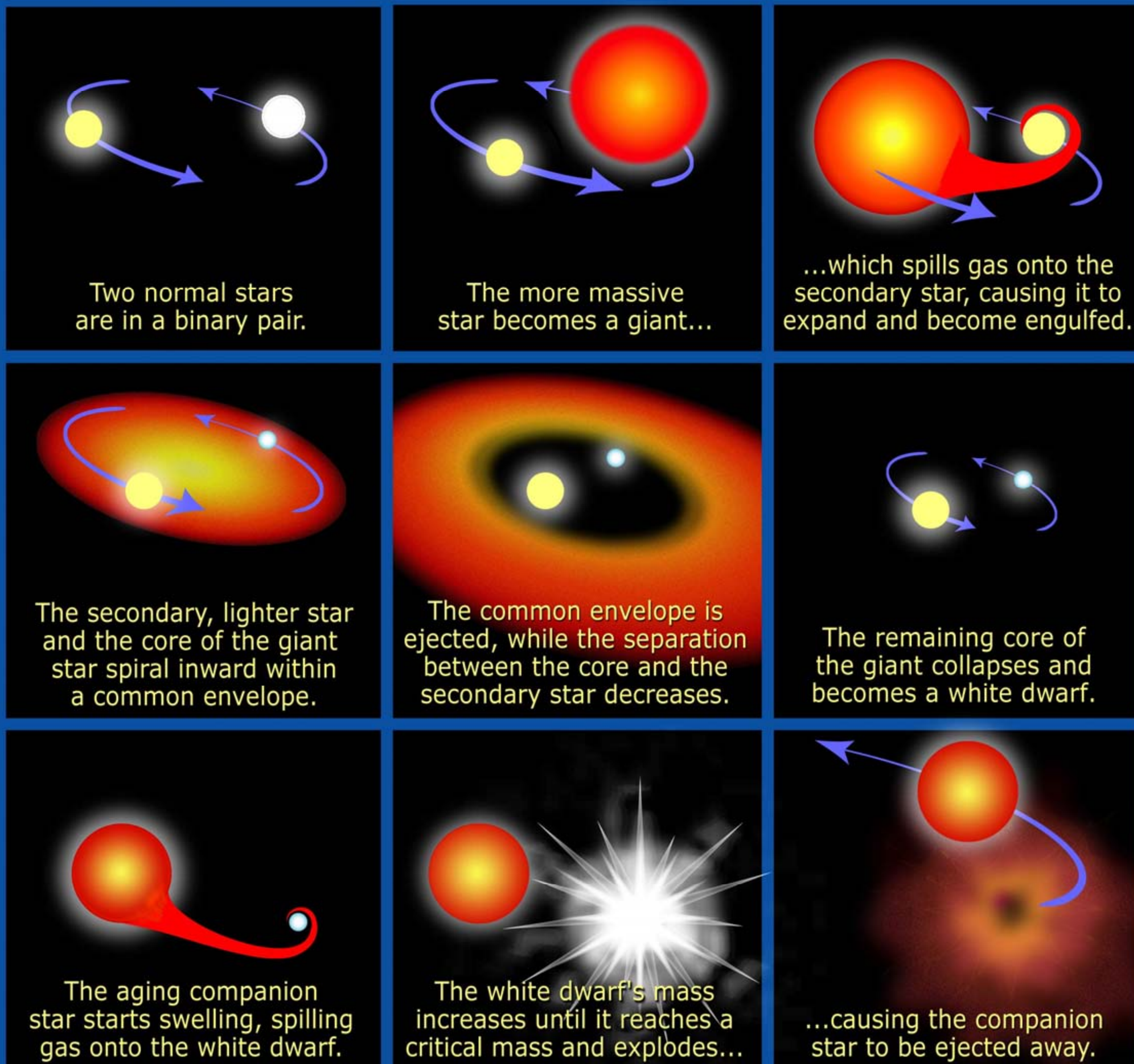
present-day
Supernova Remnant

Supernova SN1006:

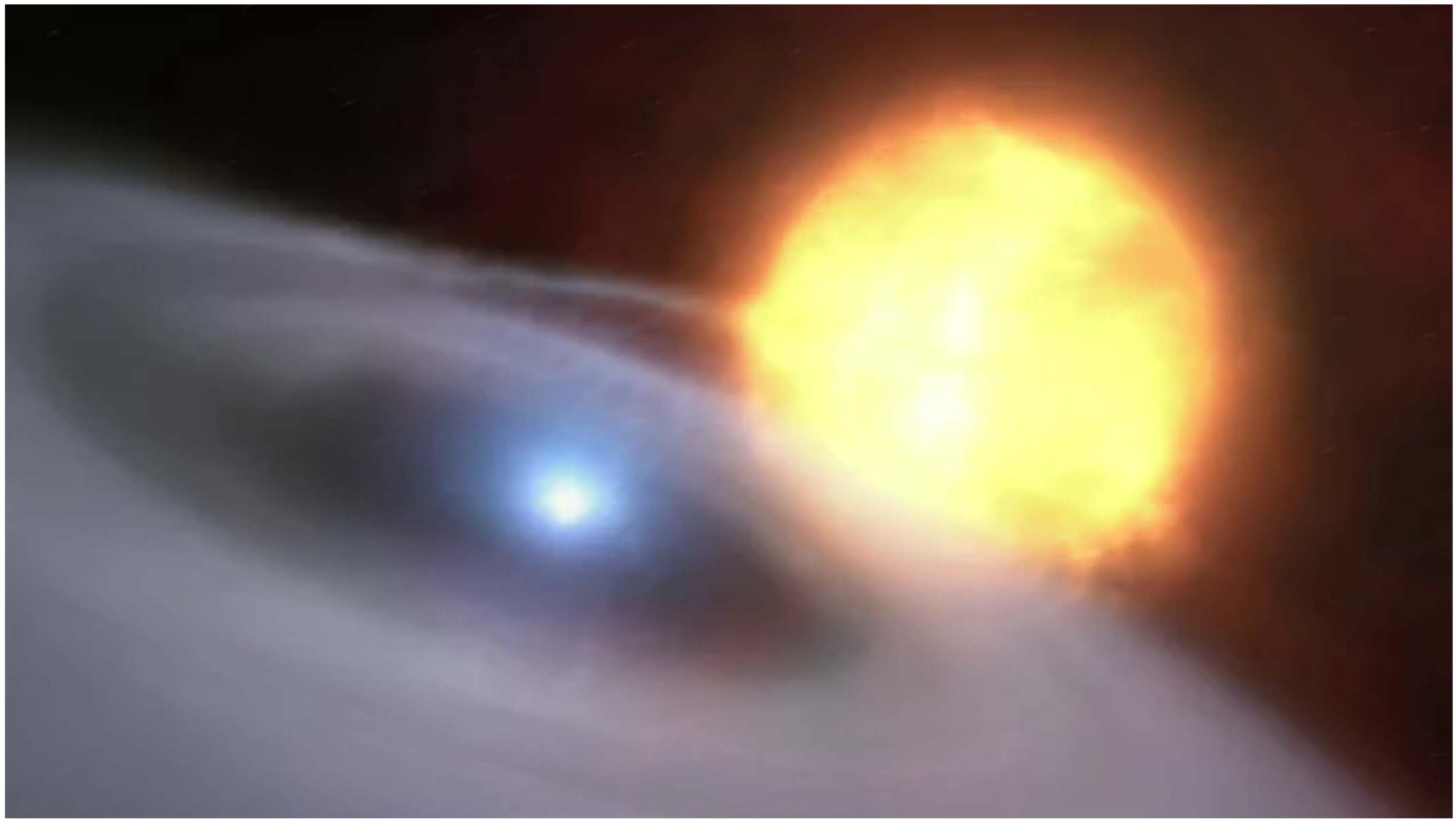
-	brightness:	m = -7.5
-	distance:	d=2.2 kpc
-	recorded:	China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

The progenitor of a Type Ia supernova

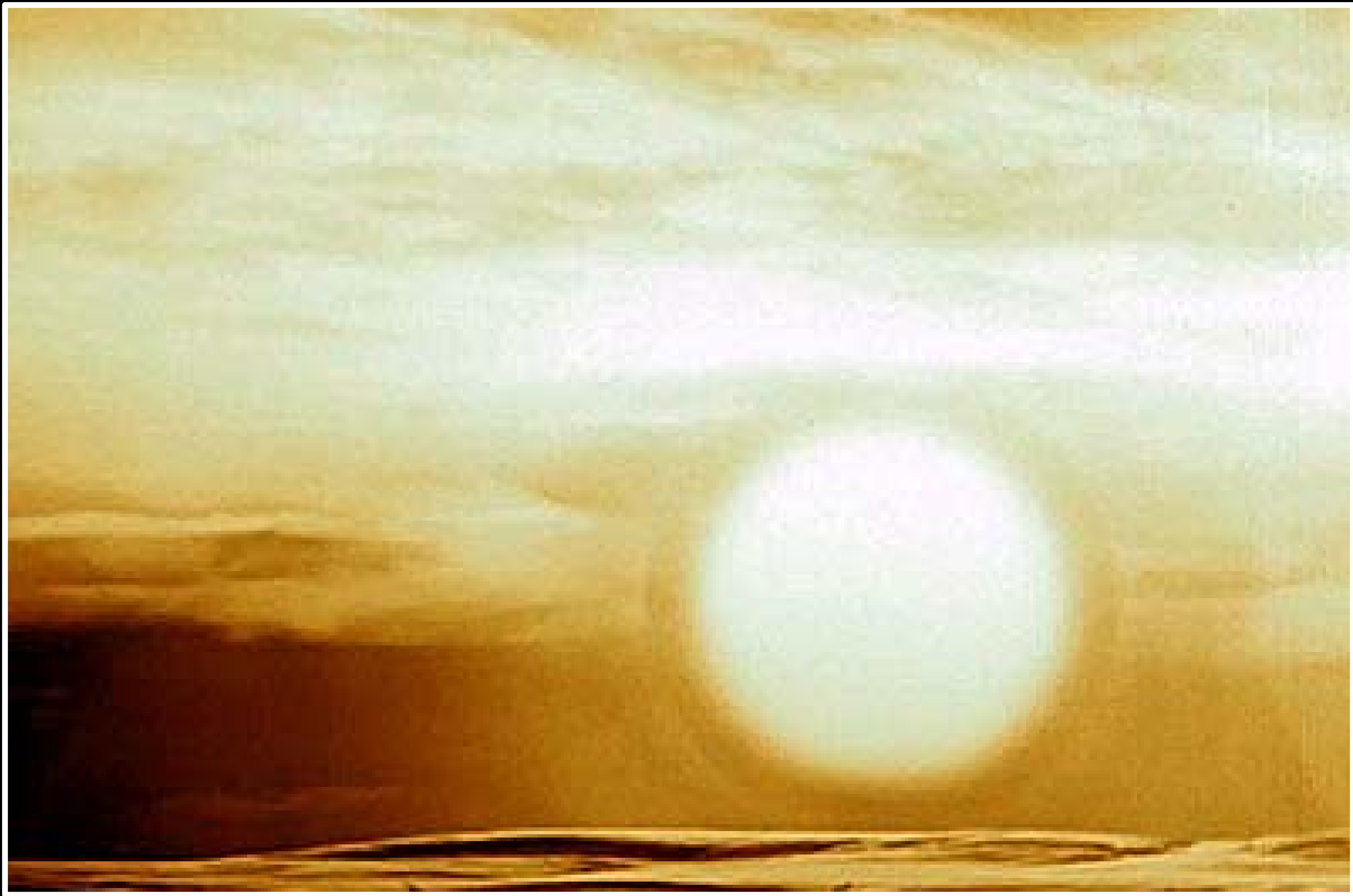


Supernova Ia Explosion



Blast Waves

Tsar Bomba Nuclear Explosion



Tsar Bomba Nuclear Explosion

Tsar Bomba Nuclear Explosion

Hiroshima, the Shockwave

Sedov-Taylor Expansion Law

Blast waves

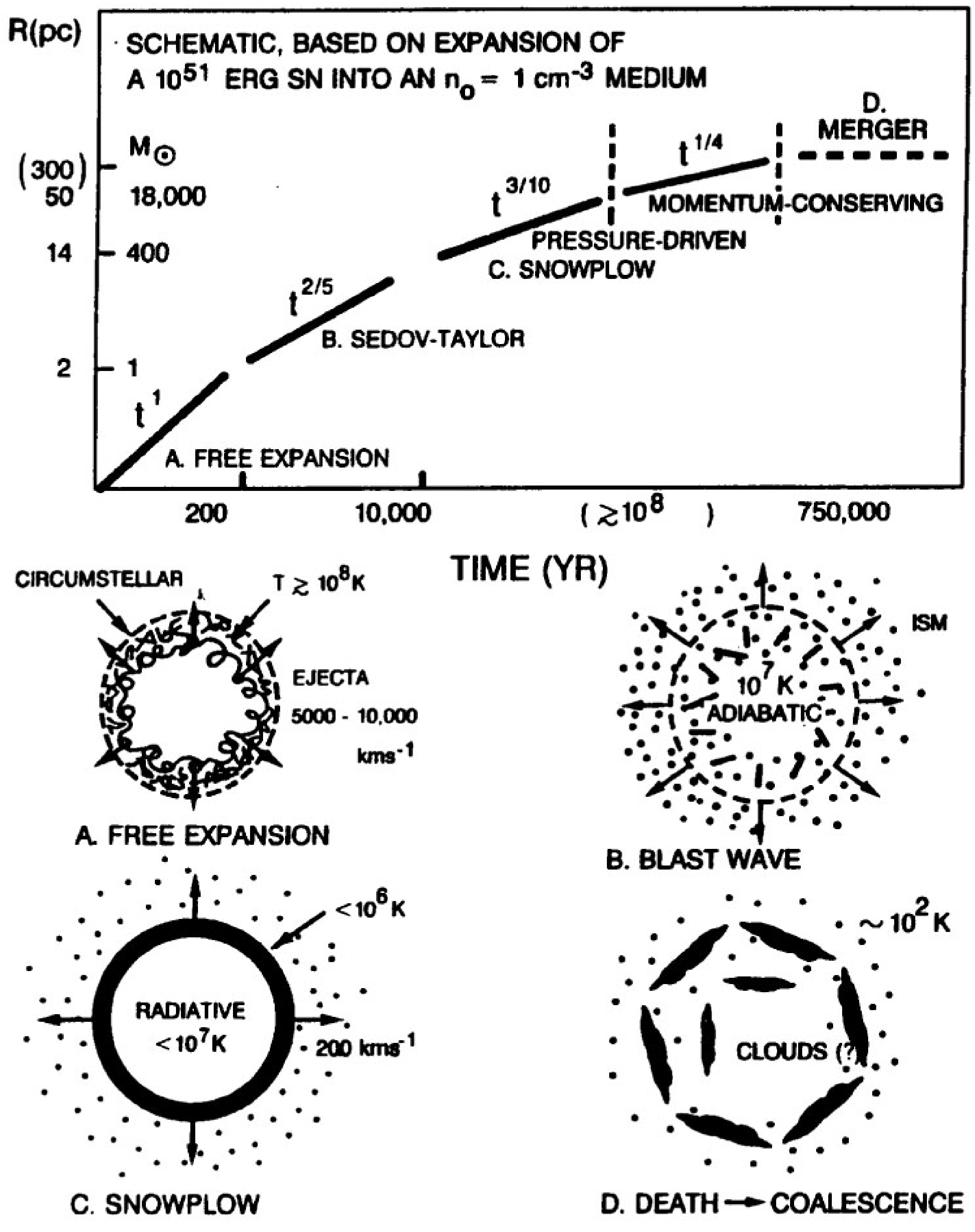
Main properties:

1. Strong shock propagating through the Interstellar Medium, or through the wind of the progenitor star;

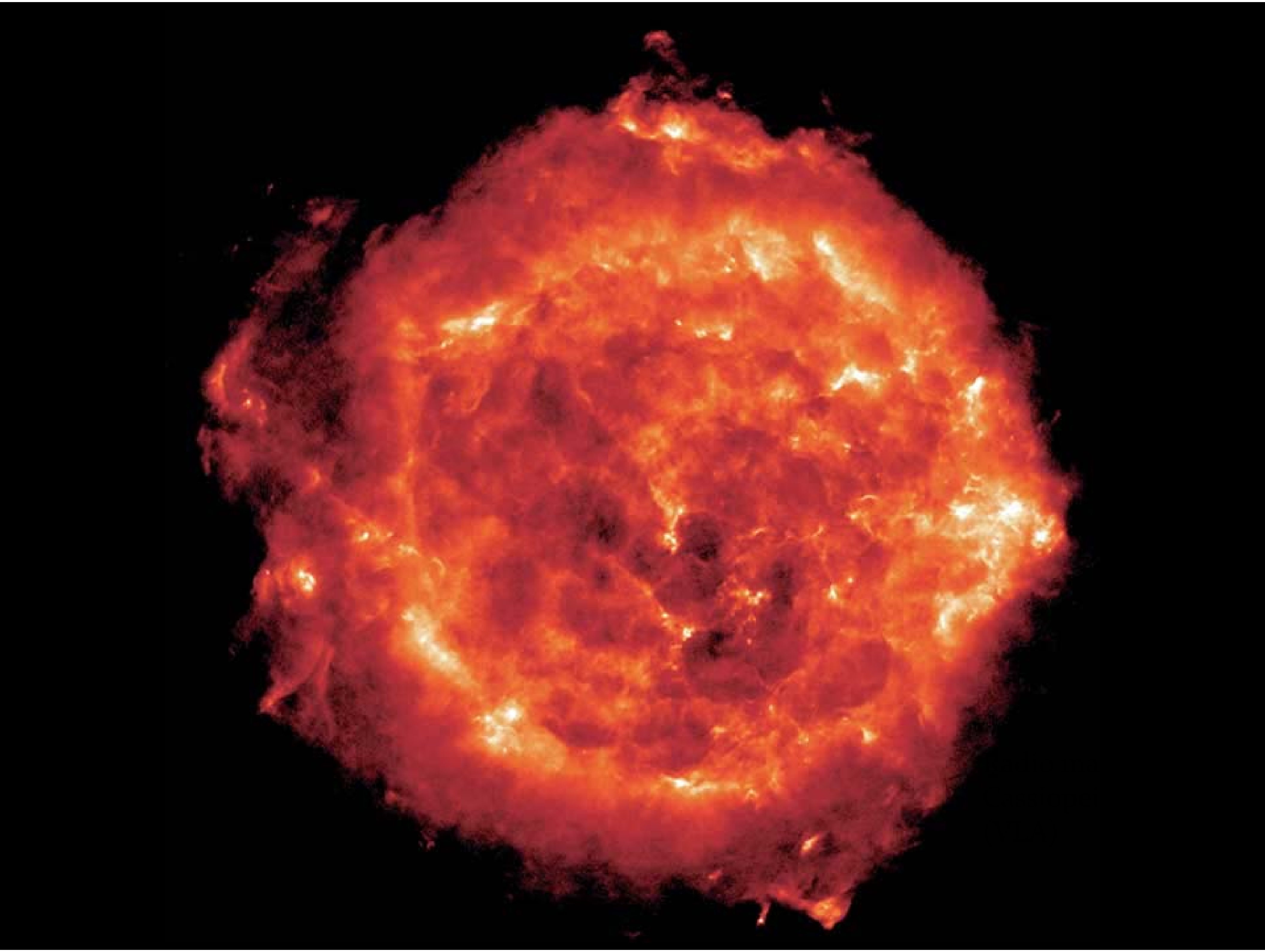
2. Different expansion stages:

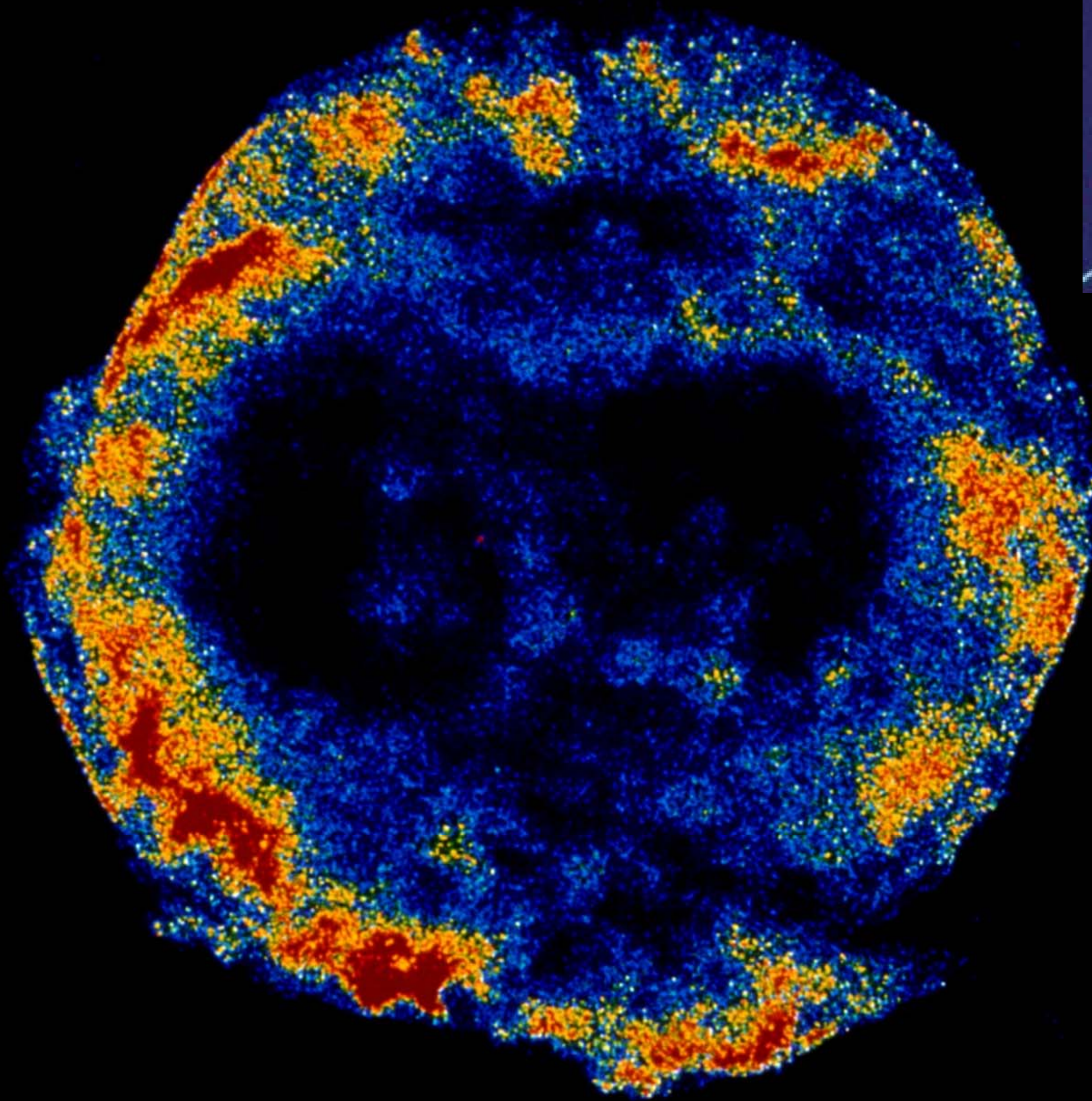
   - Free expansion stage ($t < 1000$ yr) $R \propto t$
   - Sedov-Taylor stage (1000 yr $< t <$ 10,000 yr) $R \propto t^{2/5}$
   - Pressure-driven snowplow (10,000 yr $< t <$ 250,000 yr) $R \propto t^{3/10}$

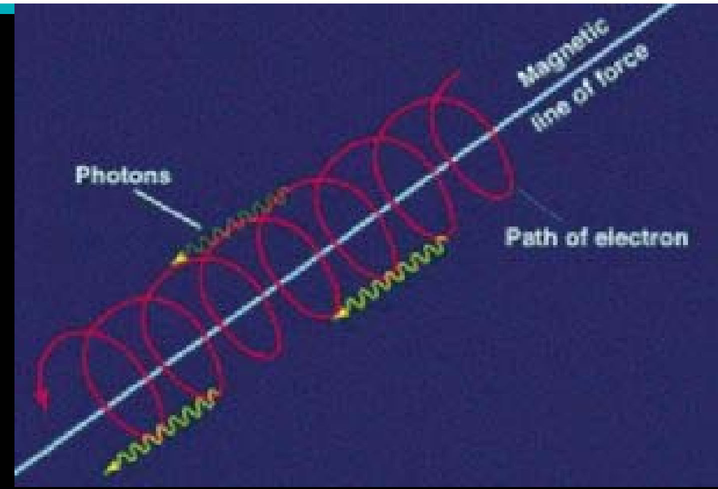
STANDARD SNR EVOLUTION

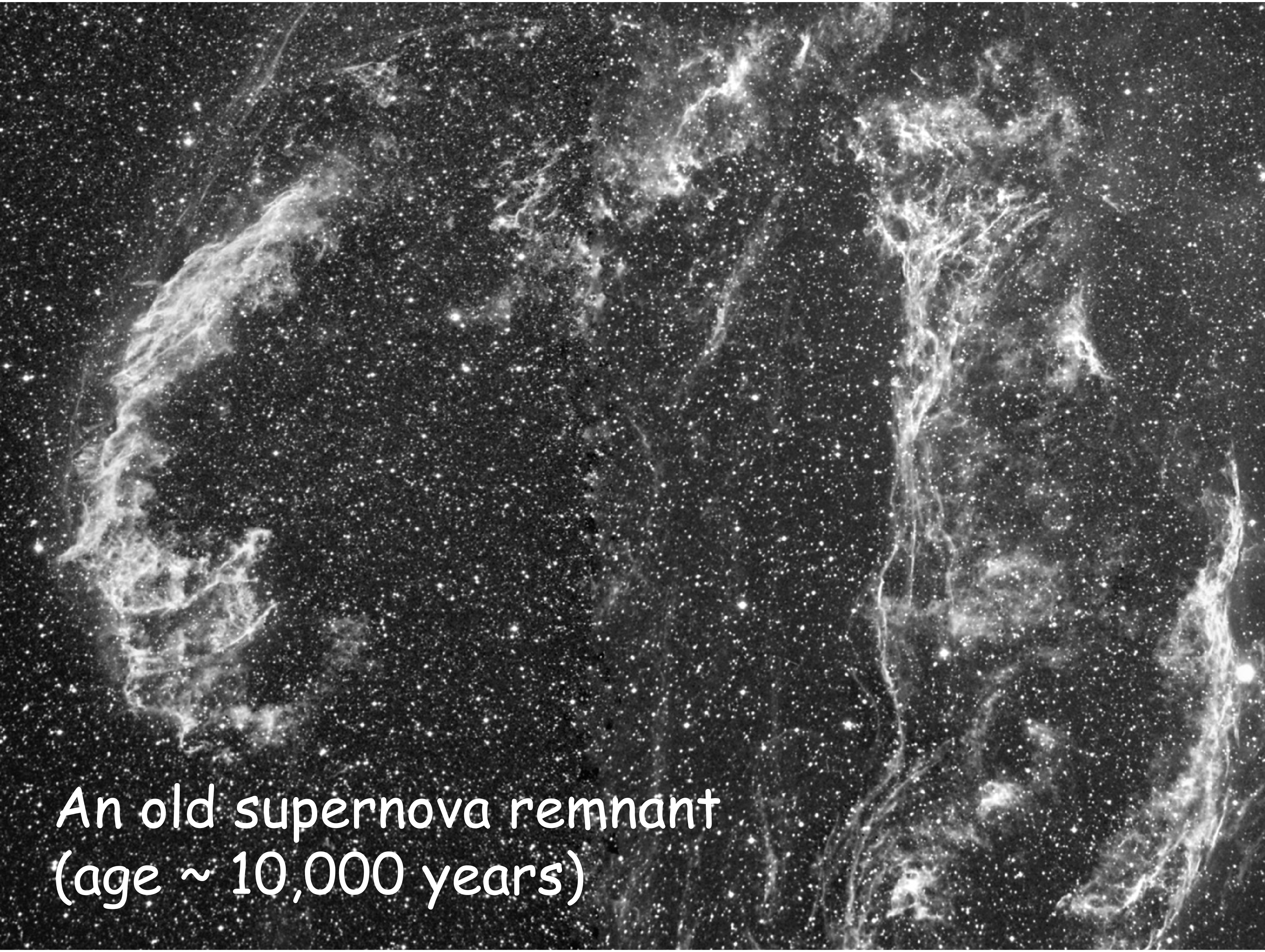


Tsar Bomba Nuclear Explosion






Magnetic
line of force
Photons
Path of electron


An old supernova remnant
(age ~ 10,000 years)

Free-expansion phase

Energy budget:

$$\left|E_{\rm grav}\right| = \tfrac{3}{5}\frac{GM_c^2}{R_c} \simeq 10^{53}\ \text{erg} \Rightarrow \begin{cases} \text{99\% into neutrino's} \\ \text{1\% into mechanical energy} \end{cases}$$

Expansion speed:

$$V_{\rm exp} \simeq \sqrt{\frac{2E_{\rm mech}}{M_{\rm ej}}} = 3000\left(\frac{E_{\rm mech}}{10^{51}\ \text{erg}}\right)^{1/2}\left(\frac{M_{\rm ej}}{10\ M_\odot}\right)^{-1/2}\ \text{km/s}$$

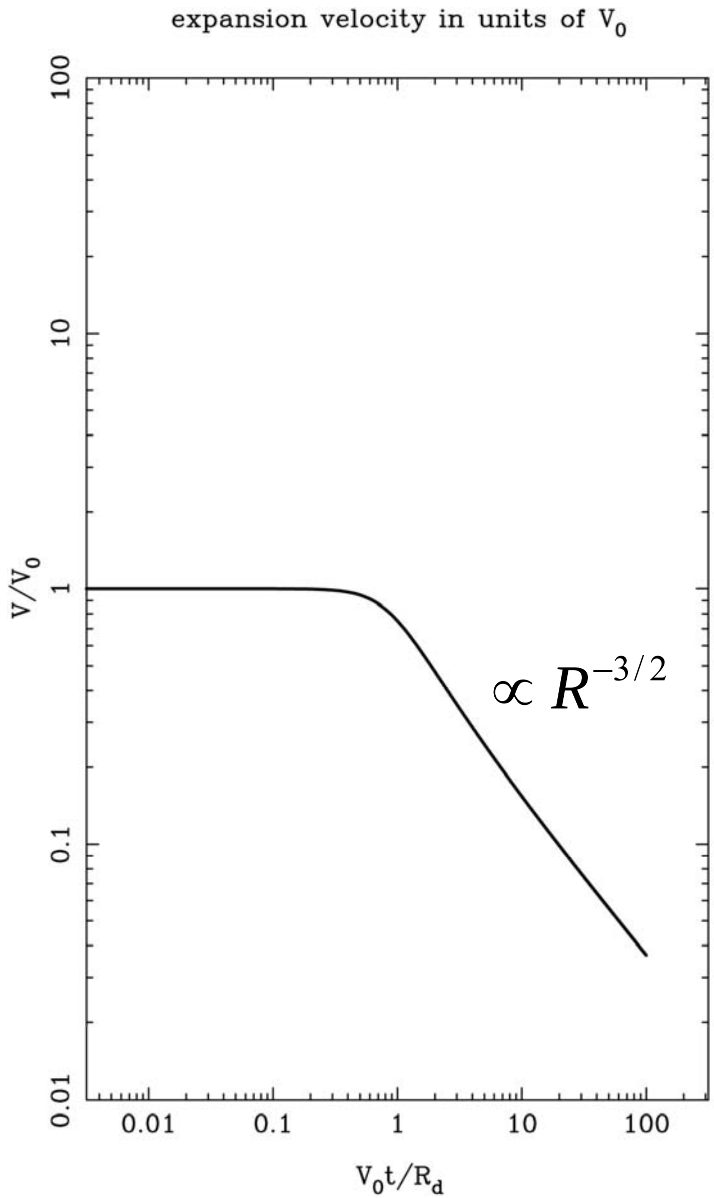
Sedov-Taylor stage

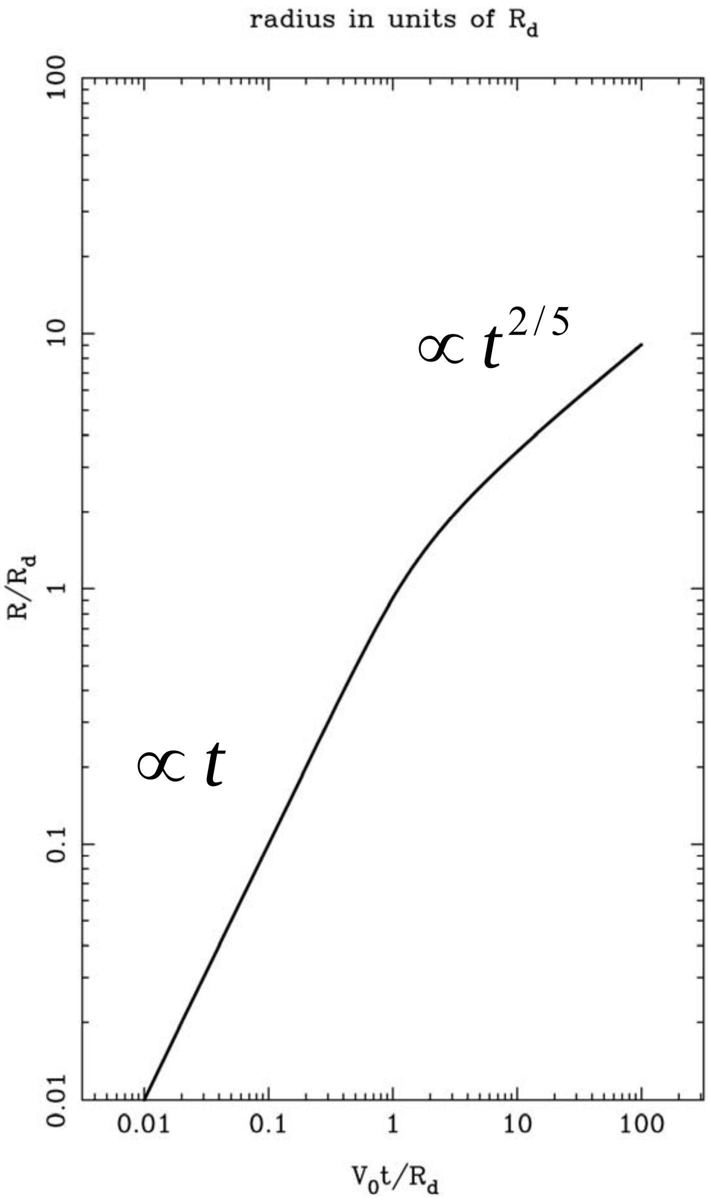
- Expansion starts to decelerate due to swept-up mass

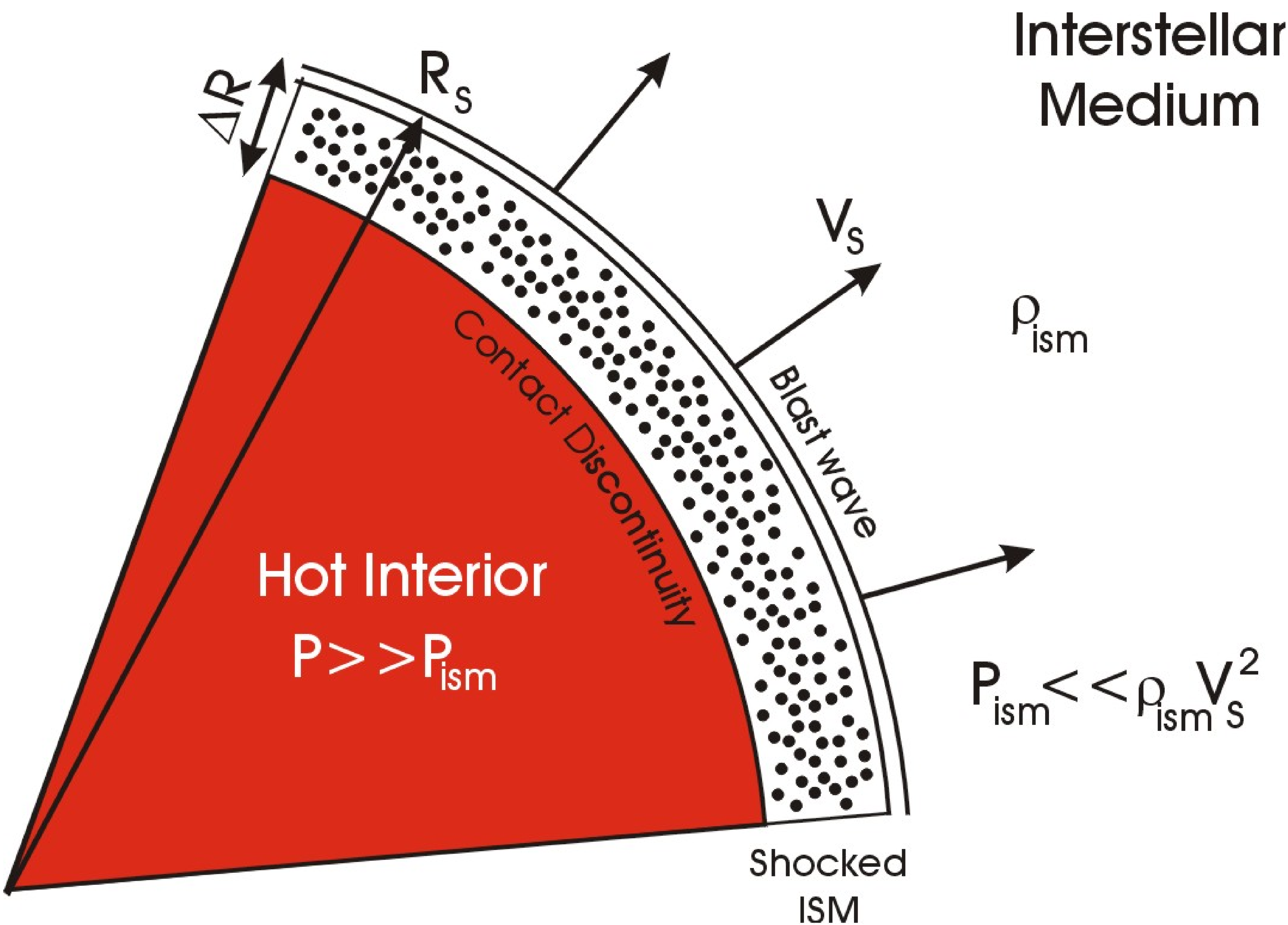
- Interior of the bubble is reheated due to reverse shock

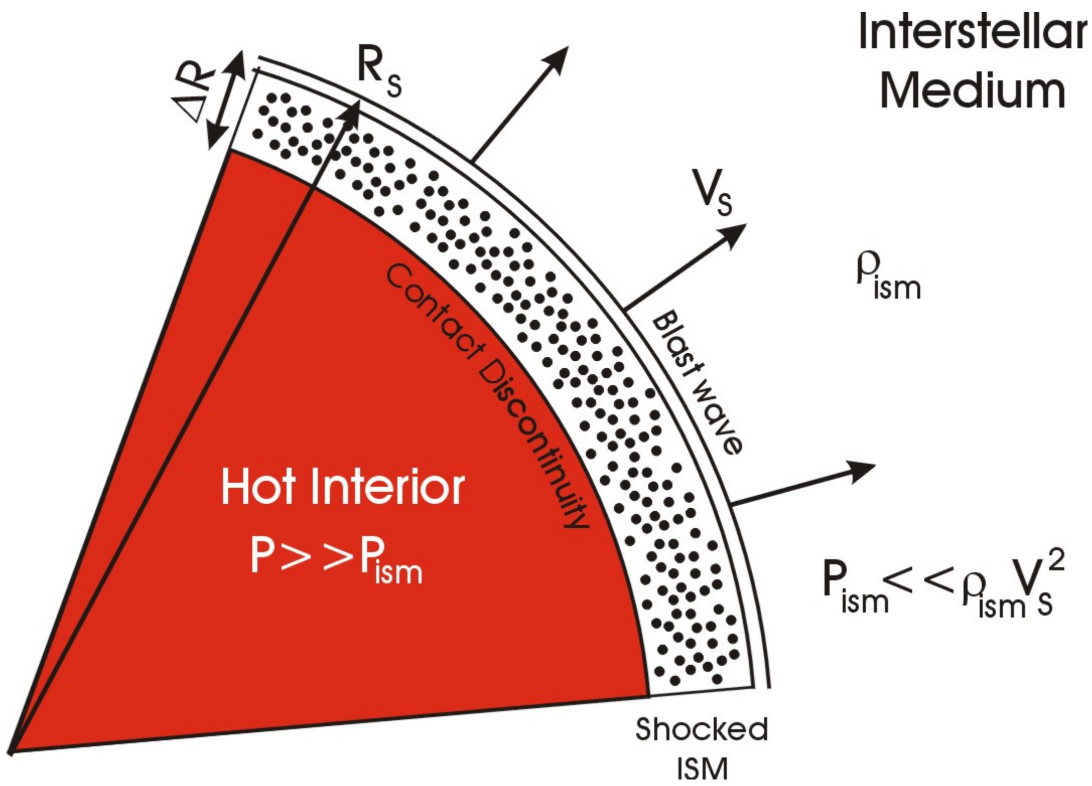
- Hot bubble is preceded in ISM by strong blast wave

$$V_s = \sqrt{\frac{2E_{\rm snr}}{M_{\rm ej}}} \times \left(\frac{1}{1 + (R/R_{\rm d})^3} \right)^{1/2} = V_0 \left(\frac{1}{1 + (R/R_{\rm d})^3} \right)^{1/2}$$






Interstellar
Medium
ΔR
R_S
V_S
ρ_ism
Contact Discontinuity
Blast wave
Hot Interior
P>>P_ism
P_ism<<ρ_ism V_S^2
Shocked
ISM



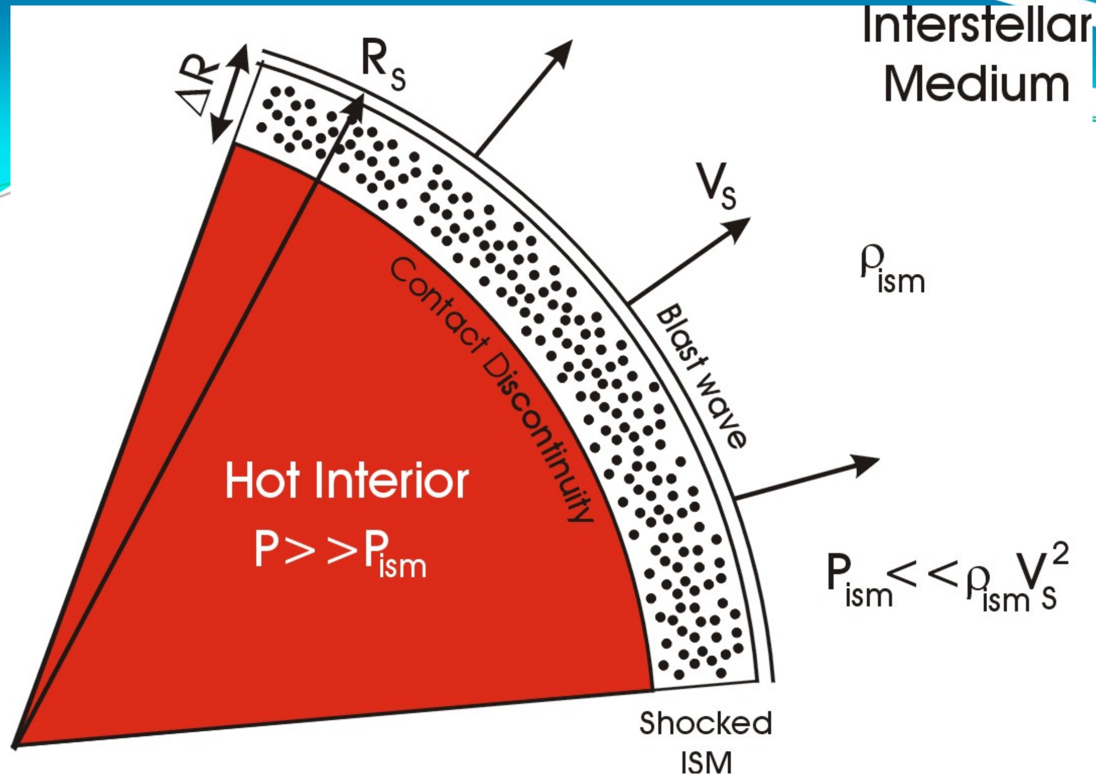
Shock relations
for strong
(high-Mach number)
shocks:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1} \Rightarrow \frac{2\gamma}{\gamma+1}\mathcal{M}_s^2$$

$$\Leftrightarrow \quad P_2 = \frac{2}{\gamma+1}\rho_1 V_1^2$$

as $\mathcal{M}_s^2 \equiv \left(\frac{V_1}{c_{s1}}\right)^2 = \frac{\rho_1 V_1^2}{\gamma P_1} \Rightarrow \infty$

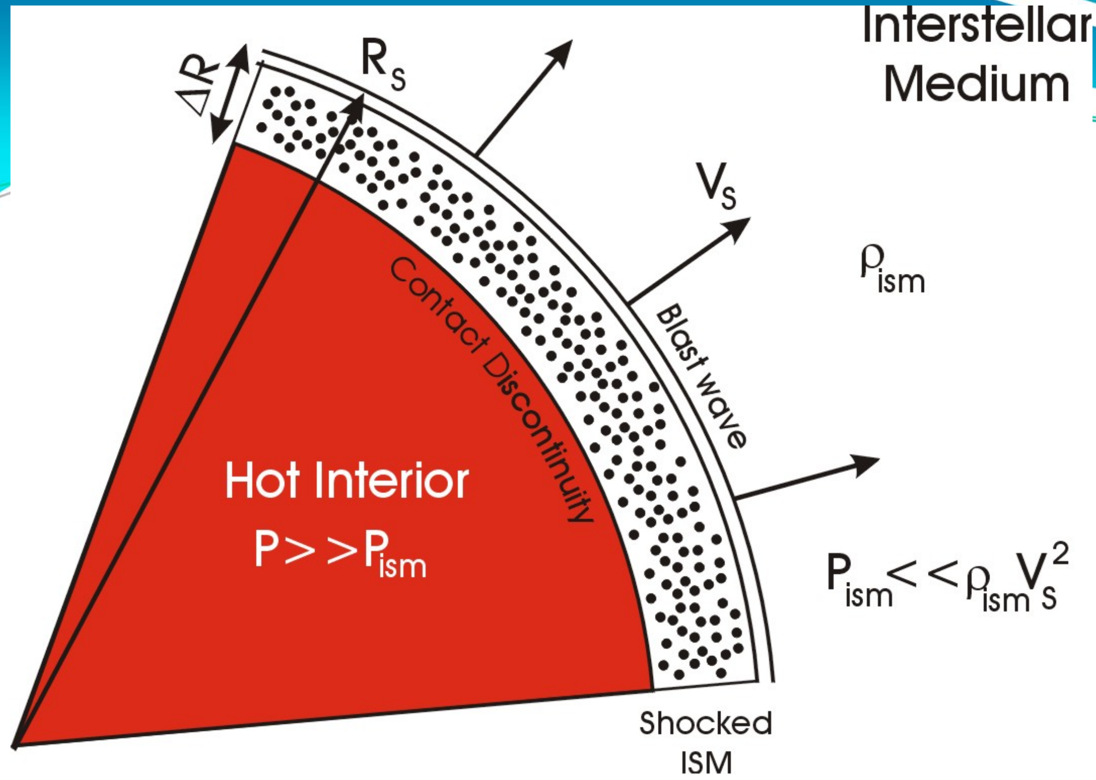


$$P_2 \approx \frac{2\gamma}{\gamma+1}\mathcal{M}_s^{\,2} P_1 = \frac{2}{\gamma+1}\rho_{\rm ism} V_s^{\,2}$$

Pressure behind strong shock (blast wave)

$$P_{\rm i} = (\gamma-1)e_{\rm i} \approx (\gamma-1)\frac{E_{\rm SNR}}{\frac{4\pi}{3}R_{\rm S}^3}$$

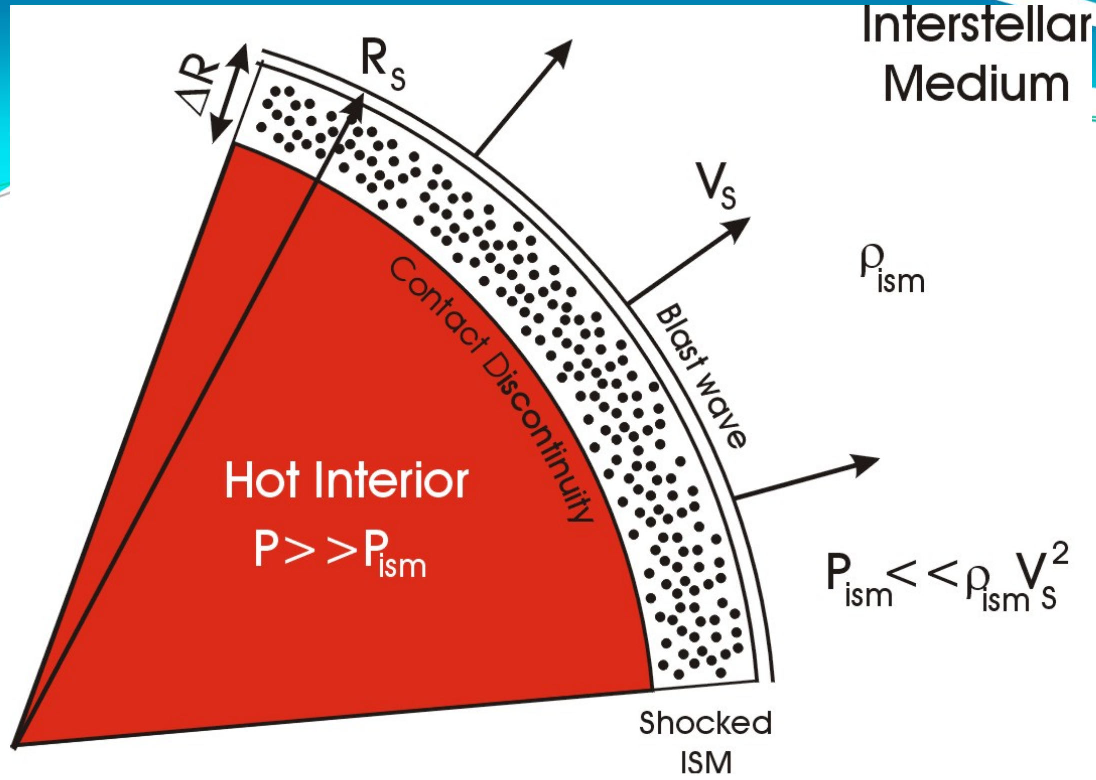
Pressure in hot SNR interior



At contact discontinuity: <u>equal</u> pressure on both sides!

$$\frac{2}{\gamma+1}\rho_{\rm ism}V_s^{\,2} \approx (\gamma-1)\frac{E_{\rm SNR}}{\frac{4\pi}{3}R_s^3}$$

This procedure is allowed because of high sound speeds in hot interior and in shell of hot, shocked ISM:
No large pressure differences are possible!

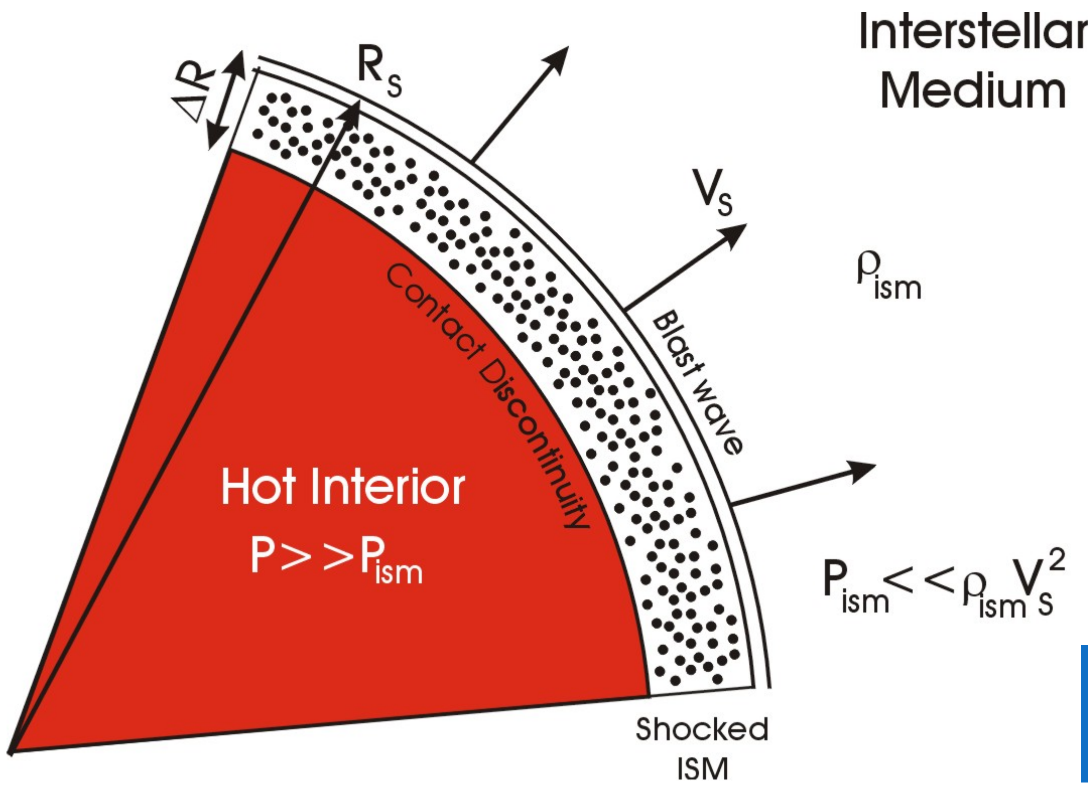


At contact discontinuity: equal pressure on both sides!

$$\frac{2}{\gamma+1}\rho_{\text{ism}} V_s^{\;2} \approx (\gamma-1)\frac{E_{\text{SNR}}}{\frac{4\pi}{3} R_s^3}$$

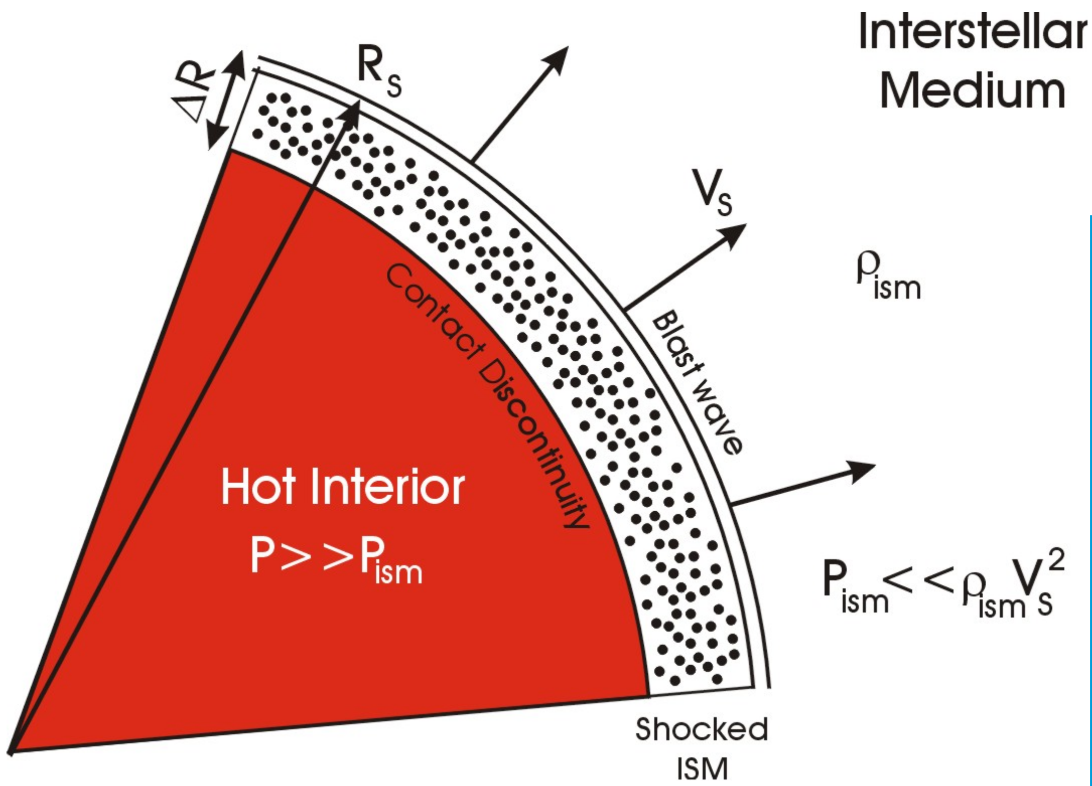
$$V_s = \frac{dR_s}{dt} \simeq \sqrt{\frac{8\pi}{3(\gamma^2-1)}}\left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}}\right)^{1/2} R_s^{-3/2}$$

Relation between velocity and radius gives expansion law!



$$R_s^{\,3/2}\,dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

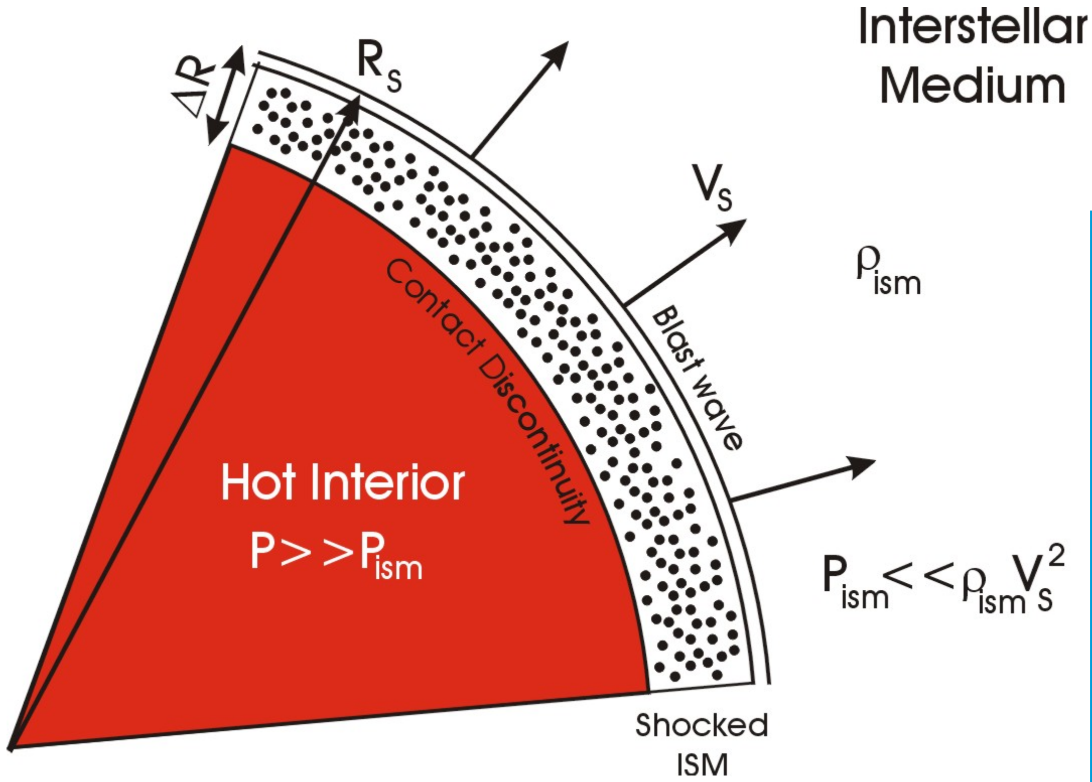
Step 1: write the relation as difference equation



$$R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2 - 1\right)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2 - 1\right)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

Step 2: write as total differentials and.........



......integrate to find the Sedov-Taylor solution

$$R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3(\gamma^2-1)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3(\gamma^2-1)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

$$R_s(t) \simeq C_\gamma \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/5} t^{2/5} \quad ,$$

$$C_\gamma = \left(\frac{5}{2}\right)^{2/5} \left(\frac{8\pi}{3(\gamma^2-1)}\right)^{1/5} \simeq 1.96$$

Sedov & Taylor