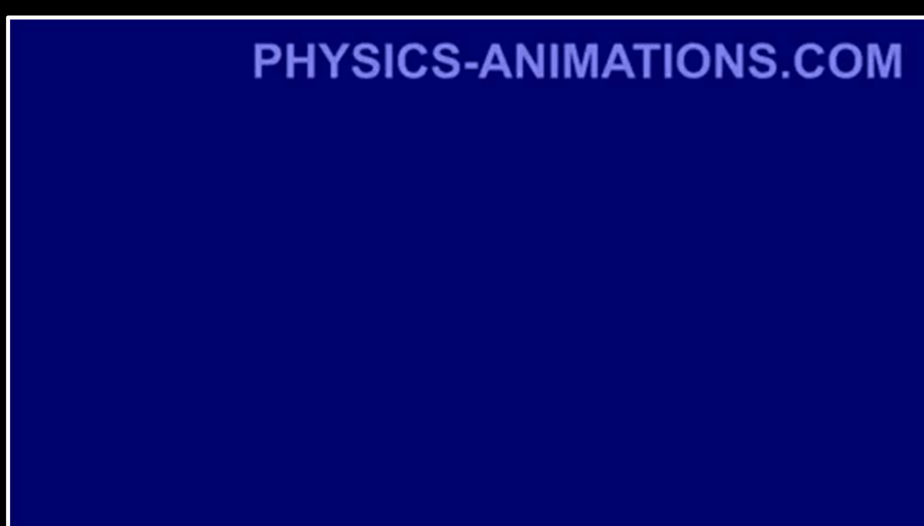
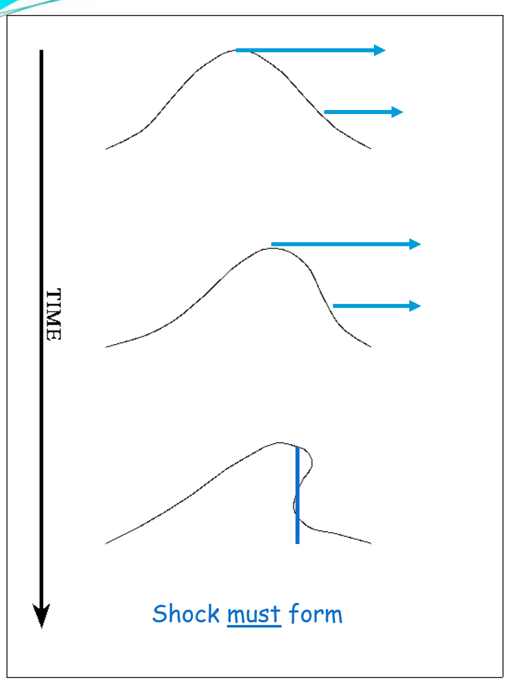
Shock Waves



Shocks

1. Shocks are sudden transitions in flow properties such as density, velocity and pressure;
2. In shocks the kinetic energy of the flow is converted into heat, (pressure);
3. Shocks are inevitable if sound waves propagate over long distances;
4. Shocks always occur when a flow hits an obstacle supersonically
5. In shocks, the flow speed along the shock normal changes from supersonic to subsonic



Wave Breaking

High-pressure/density regions move faster

$$u = \frac{2c_{s0}}{\gamma - 1}\left[\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right]$$

$$\approx c_{s0}\left(\frac{\Delta\rho}{\rho}\right)$$

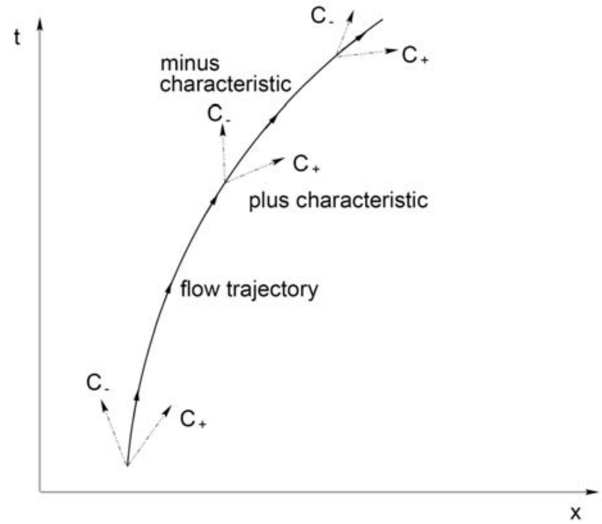
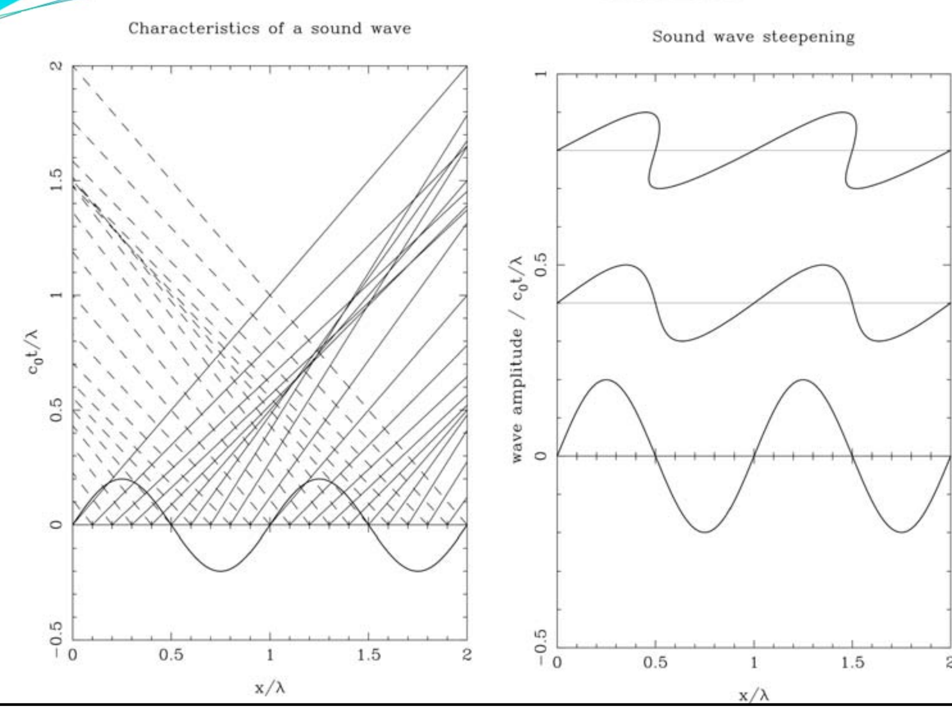
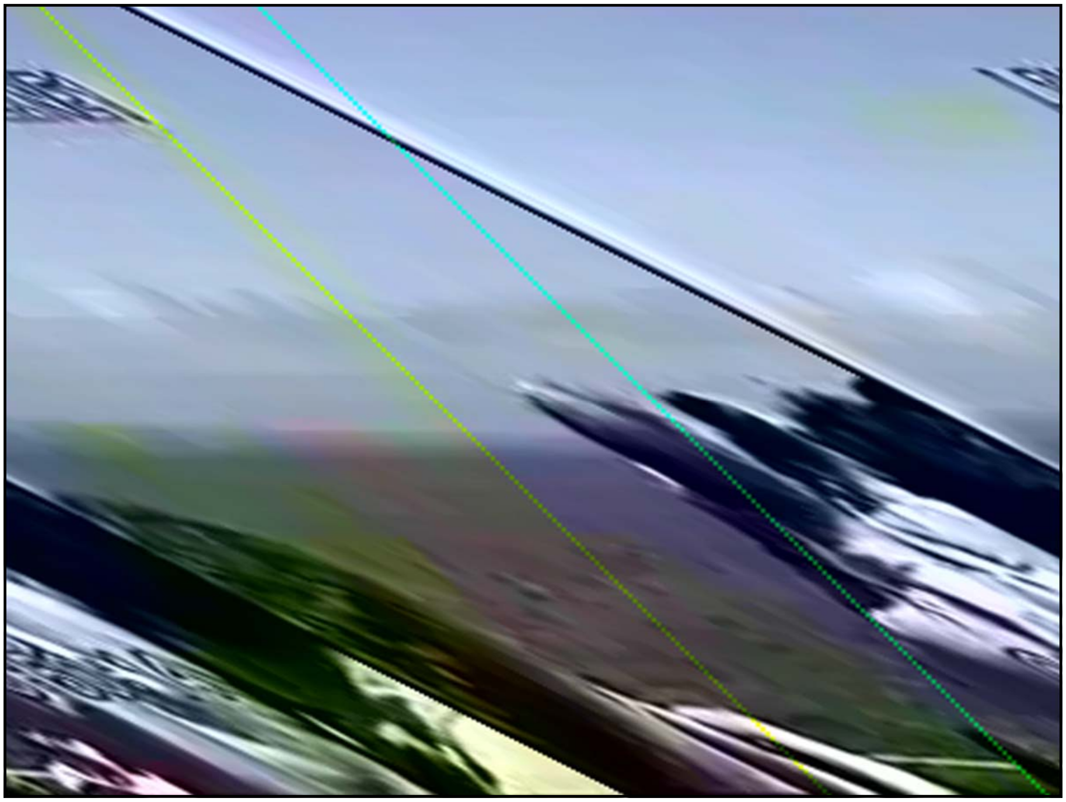


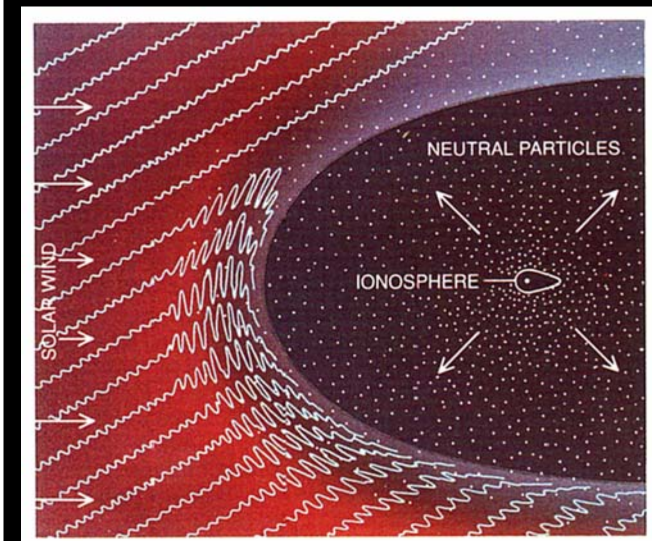
Figure 7.1: *Diagram showing the space-time flow line, defined by* $\mathrm{d}x = u\,\mathrm{d}t$, *and the two characteristics* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *defined by* $\mathrm{d}x = (u + c_\mathrm{s})\,\mathrm{d}t$ *and* $\mathrm{d}x = (u - c_\mathrm{s})\,\mathrm{d}t$. *From each point in the flow two characteristics originate along which* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are constant respectively. Note that the value of* $\mathcal{C}_\pm$ *can be different on the different characteristics so that the characteristic variables* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are* **not** *global constants!*



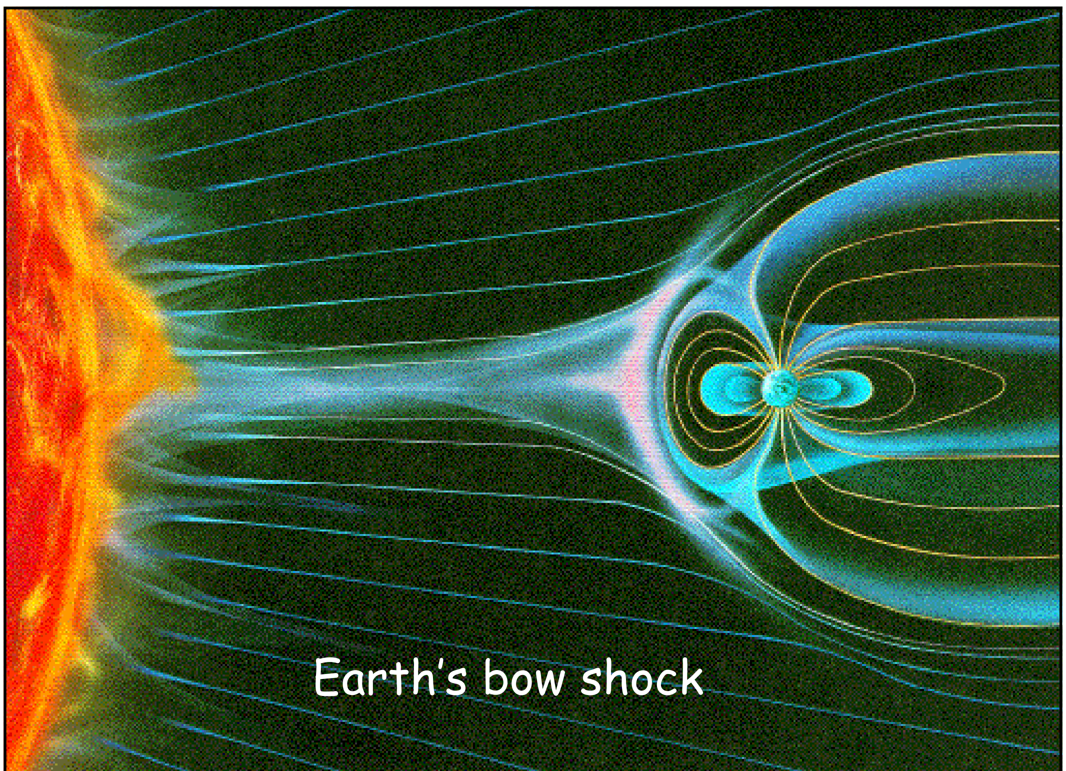


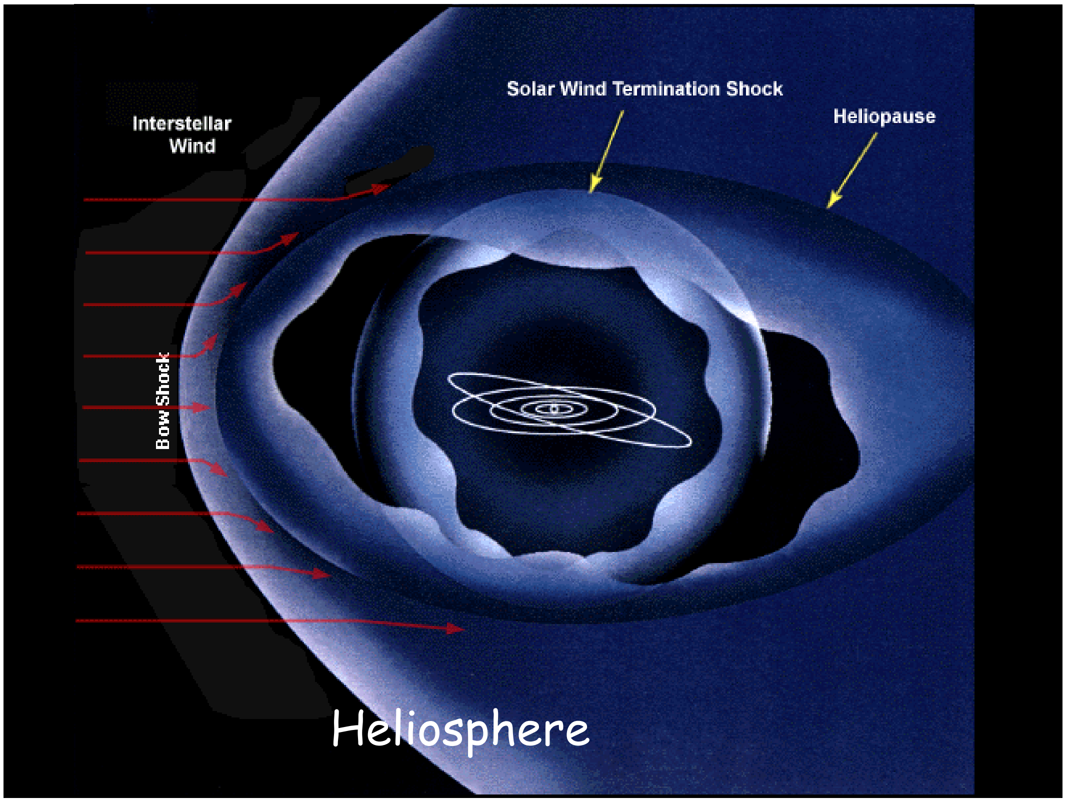
Chelyabinsk Meteorite (Feb. 2013): Sonic Boom

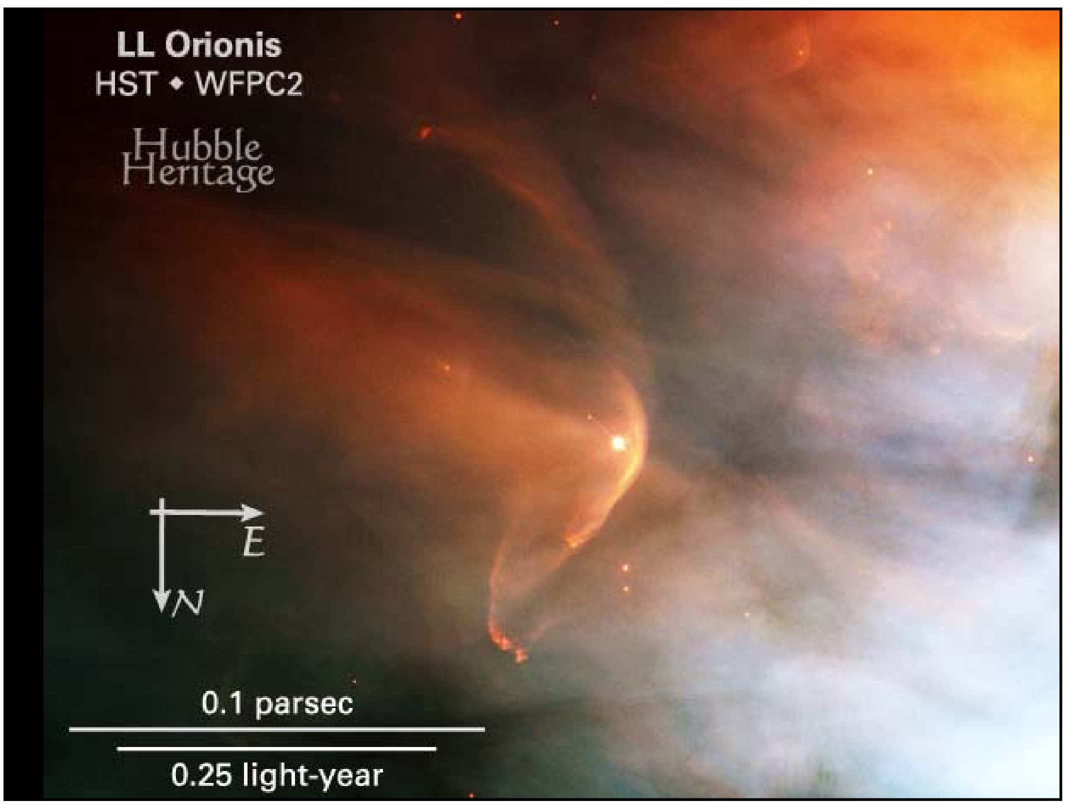
Examples of Astrophysical shocks



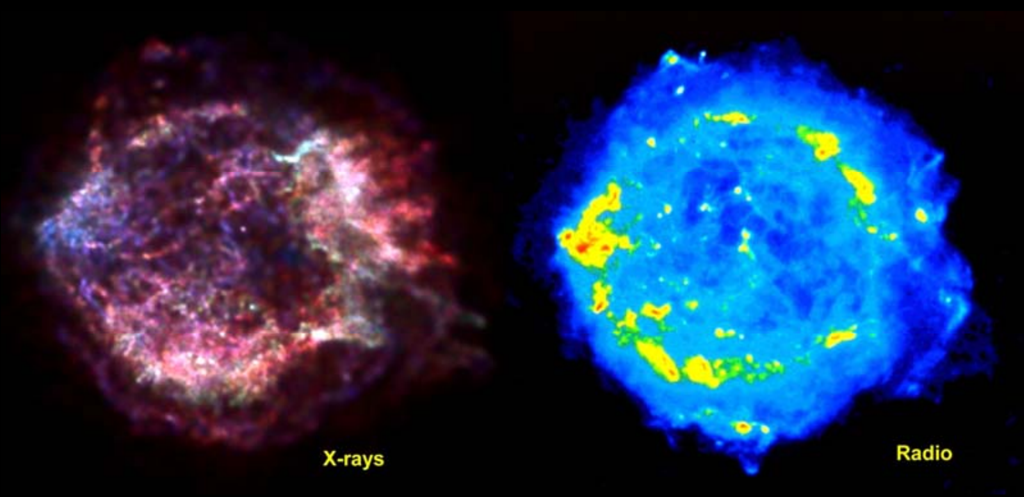
Cometary bow-shocks




Solar Wind Termination Shock
Heliopause
Interstellar
Wind
Bow Shock
Heliosphere

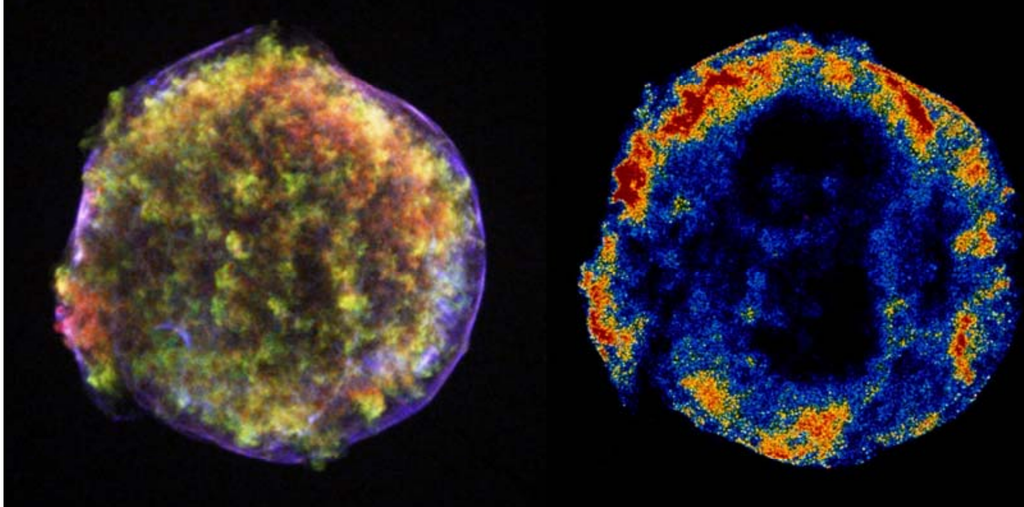

LL Orionis
HST • WFPC2
Hubble
Heritage
E
N
0.1 parsec
0.25 light-year

Supernova Remnant Cassiopeia A

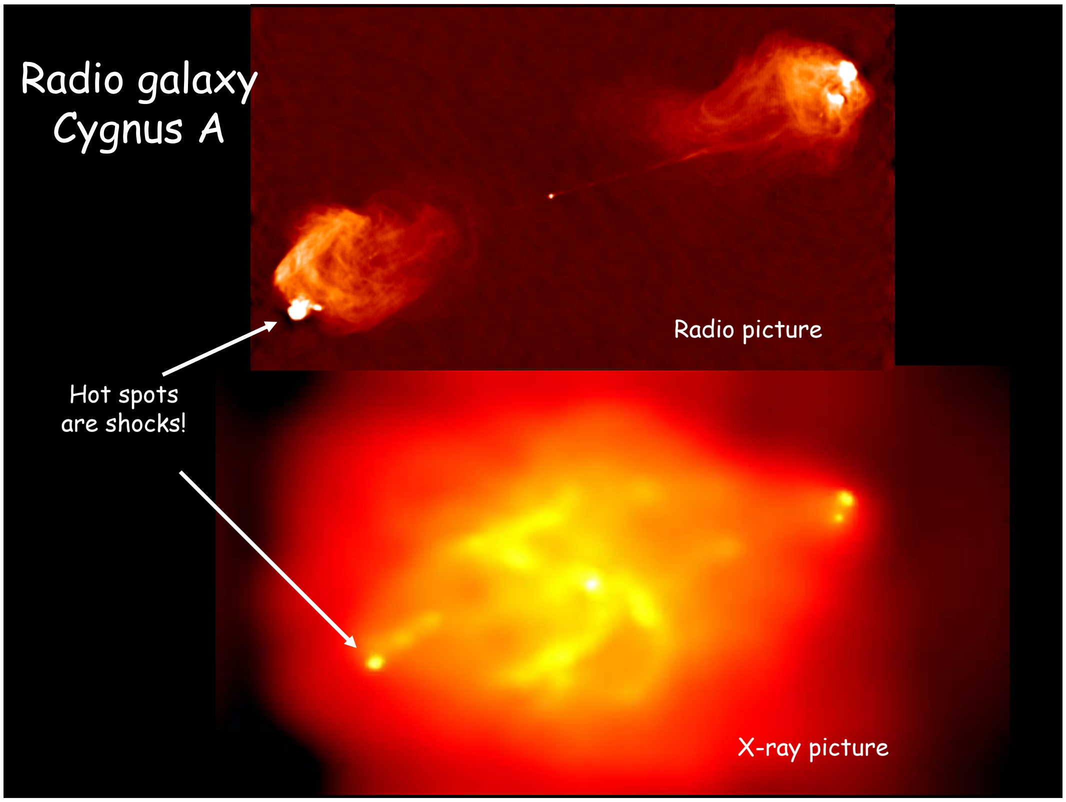


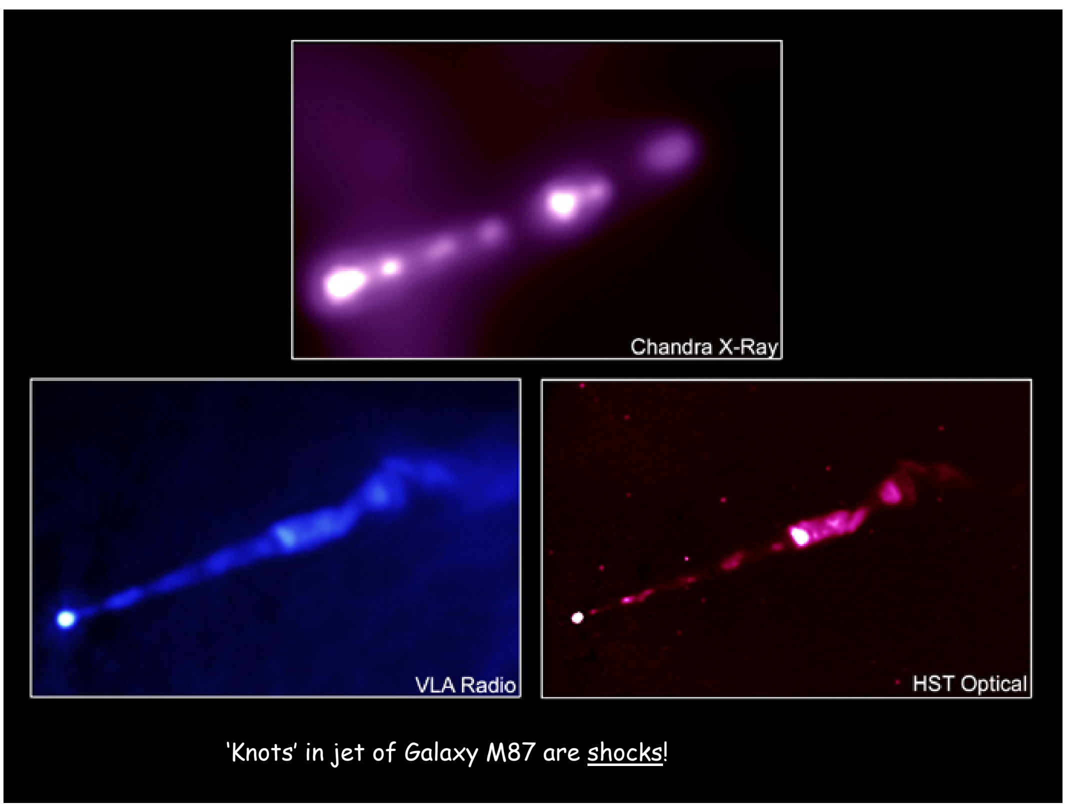
Supernova blast waves

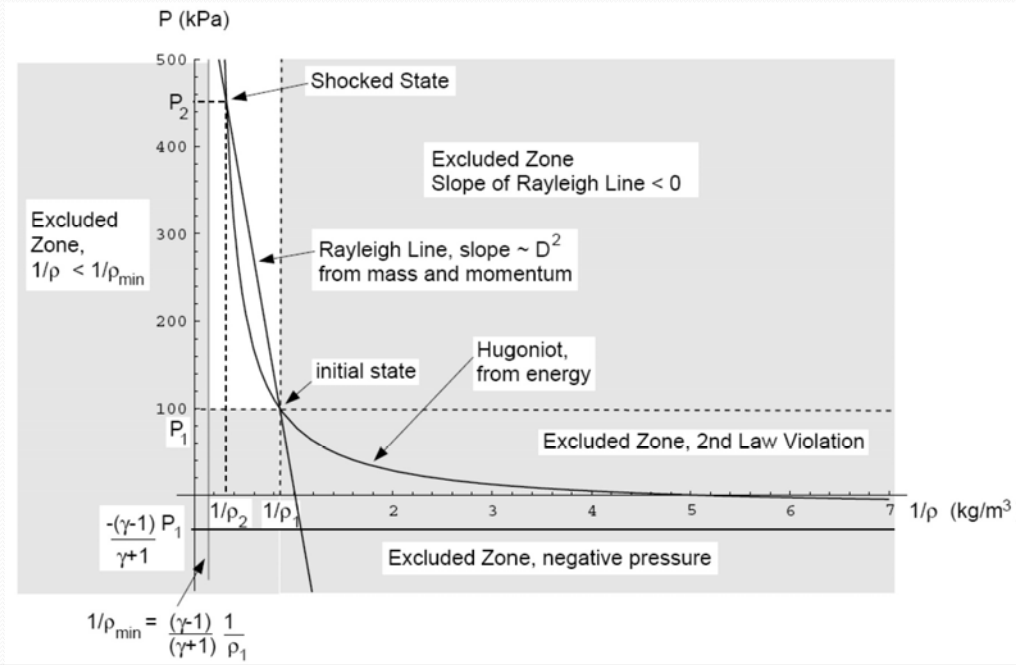
Tycho's Remnant (SN 1572AD)

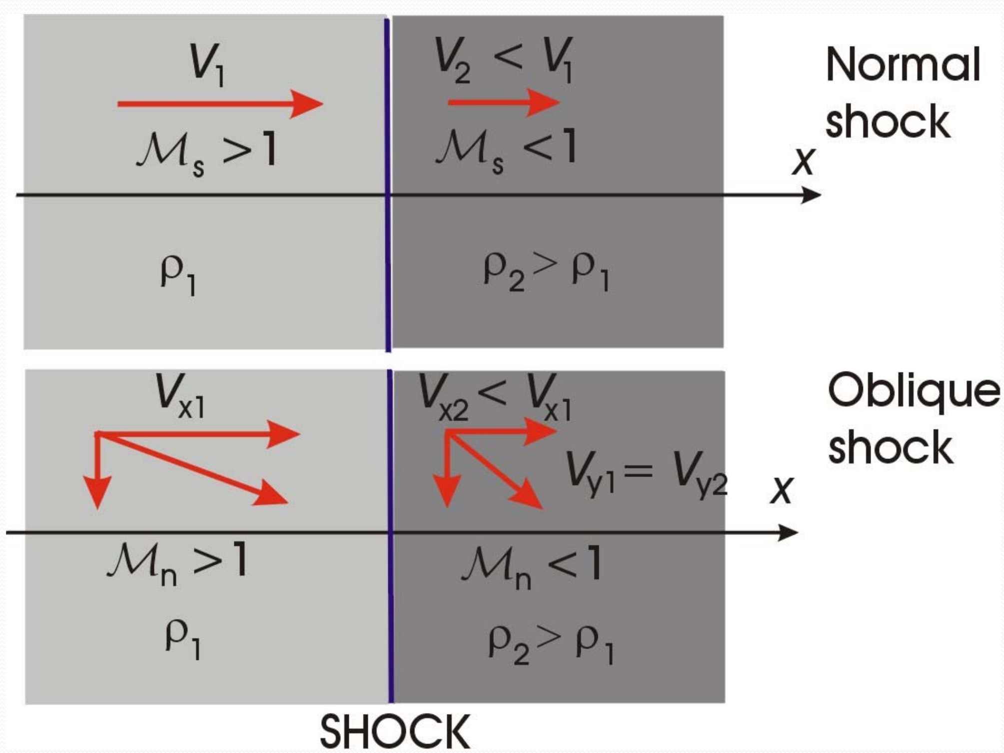


X-Rays (CHANDRA Observatory)   Radio (21cm)


Radio galaxy Cygnus A
Radio picture
Hot spots are shocks!
X-ray picture


Chandra X-Ray
VLA Radio
HST Optical
'Knots' in jet of Galaxy M87 are shocks!


P (kPa)
500
400
300
200
100
P_2
P_1
Shocked State
Excluded Zone
Slope of Rayleigh Line < 0
Excluded Zone,
$1/\rho < 1/\rho_{min}$
Rayleigh Line, slope ~ D^2
from mass and momentum
initial state
Hugoniot,
from energy
Excluded Zone, 2nd Law Violation
$1/\rho_2$ $1/\rho_1$ 2 3 4 5 6 7 $1/\rho$ (kg/m^3)
$\frac{-(\gamma-1)\, P_1}{\gamma+1}$
Excluded Zone, negative pressure
$1/\rho_{min} = \frac{(\gamma-1)}{(\gamma+1)} \frac{1}{\rho_1}$


V_1
$V_2 < V_1$
$\mathcal{M}_s > 1$
$\mathcal{M}_s < 1$
Normal shock
x
ρ_1
$\rho_2 > \rho_1$
V_{x1}
$V_{x2} < V_{x1}$
$V_{y1} = V_{y2}$
Oblique shock
x
$\mathcal{M}_n > 1$
$\mathcal{M}_n < 1$
ρ_1
$\rho_2 > \rho_1$
SHOCK

Summary : Shock Physics

Across an infinitely thin steady shock you have, in the shock frame where the shock is at rest, the following Rankine-Hugoniot Jump conditions:

Mass-flux conservation

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

Momentum-flux conservation

$$\rho_1 \left(V_{n1}\right)^2 + P_1 = \rho_2 \left(V_{n2}\right)^2 + P_2$$

$$V_{t1} = V_{t2}$$

Energy-flux conservation

$$\tfrac{1}{2}\left(V_{n1}\right)^2 + \frac{\gamma P_1}{(\gamma - 1)\rho_1} = \tfrac{1}{2}\left(V_{n2}\right)^2 + \frac{\gamma P_2}{(\gamma - 1)\rho_2}$$

Summary: Rankine-Hugoniot relations (for normal shock)

Fundamental parameter:
Mach Number

$$\mathcal{M}_s \equiv \frac{\text{shock speed}}{\text{sound speed}} = \frac{V_1}{c_{s1}}$$

R-H Jump Conditions relate the up- and downstream quantities at the shock:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2 + 2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_s^2 - (\gamma-1)}{\gamma+1}$$

From normal shock to oblique shocks:

All relations remain the same if one makes the replacement:

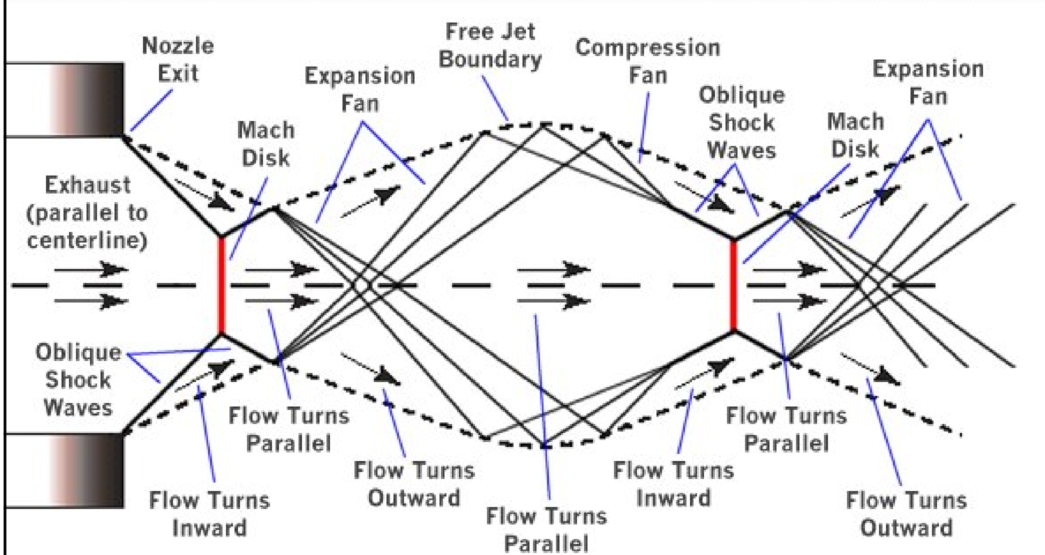
$$V_1 \Rightarrow V_{n1} = V_1 \cos\theta_1 \ ,$$
$$\mathcal{M}_S \Rightarrow \mathcal{M}_n = V_{n1} / c_{s1} = \mathcal{M}_S \cos\theta_1$$

θ is the angle between upstream velocity and normal on shock surface

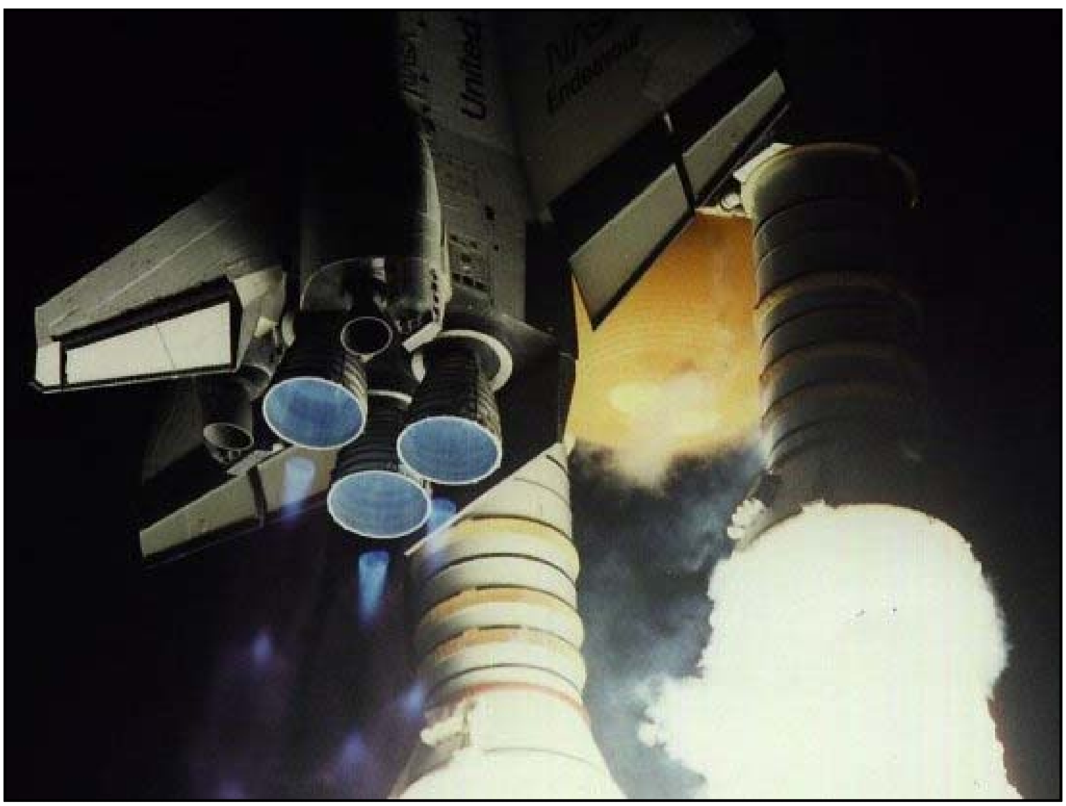
Tangential velocity along shock surface is unchanged

$$V_{t1} = V_1 \sin\theta_1 = V_{t2} = V_2 \sin\theta_2$$

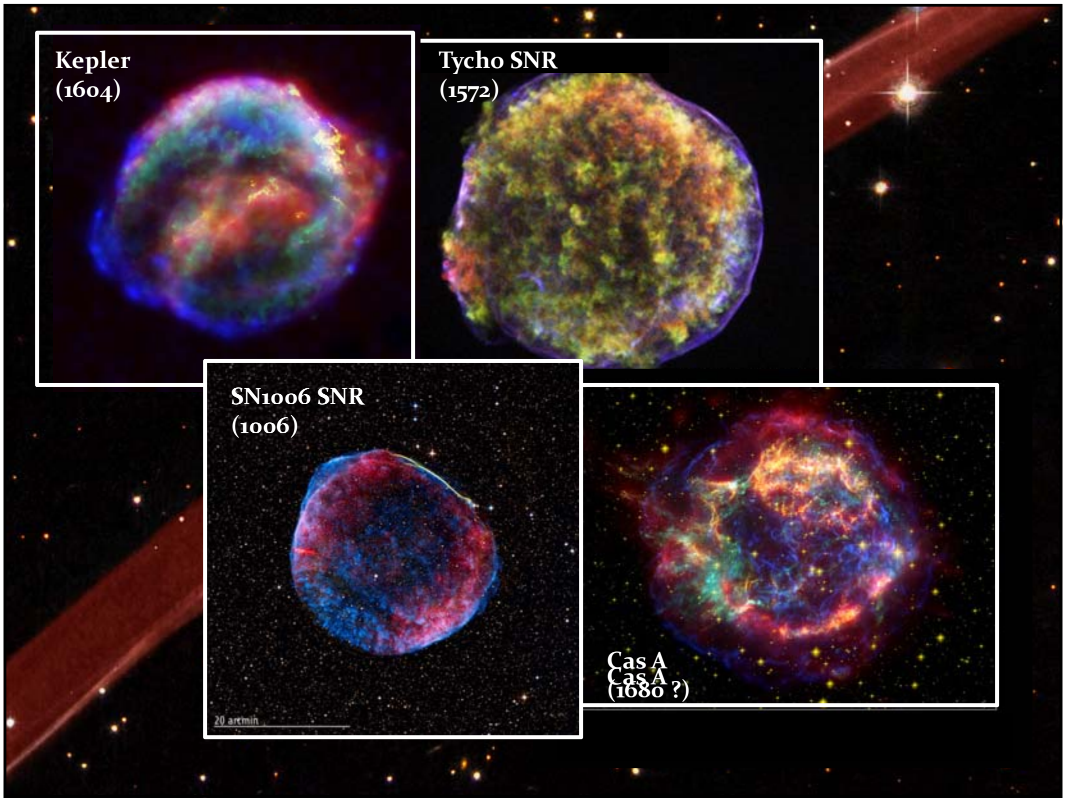
Example from Jet/Rocket engines



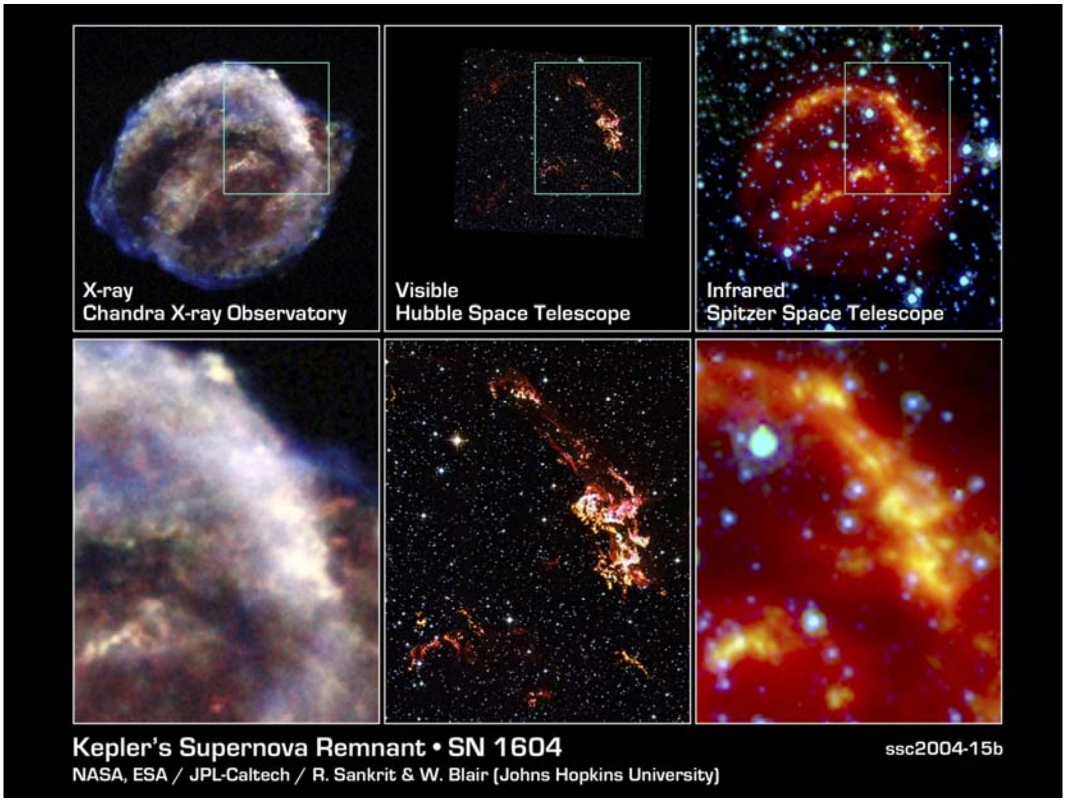

6062

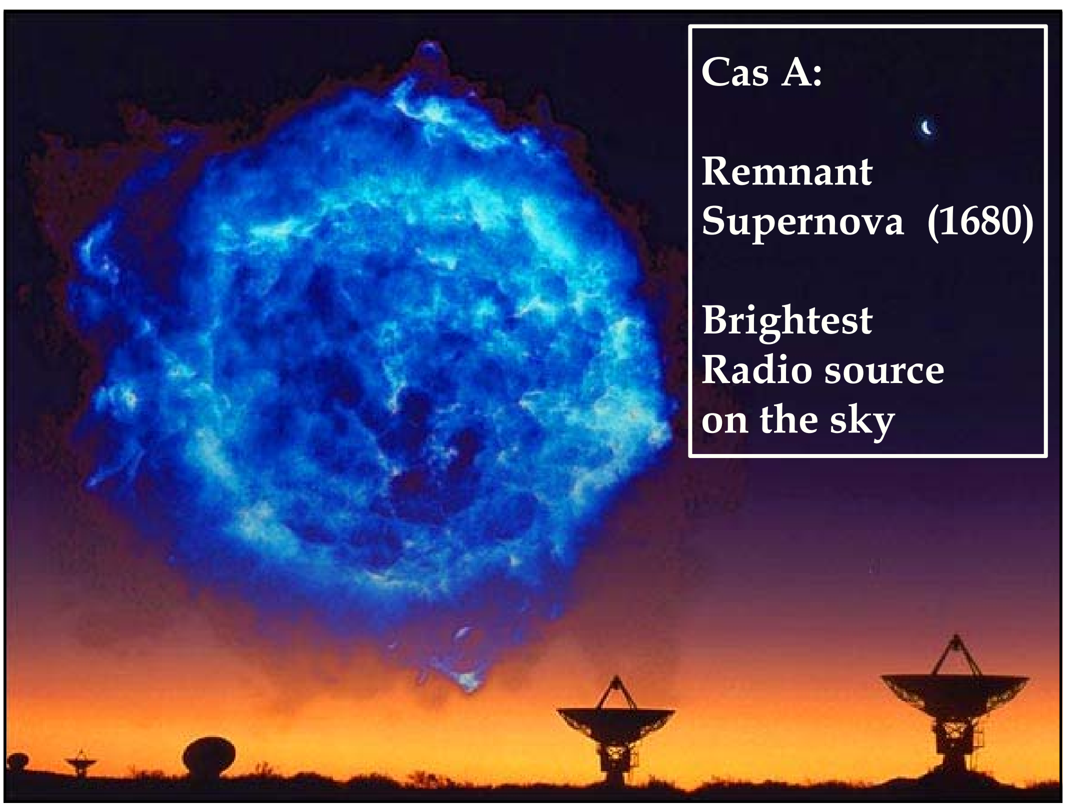


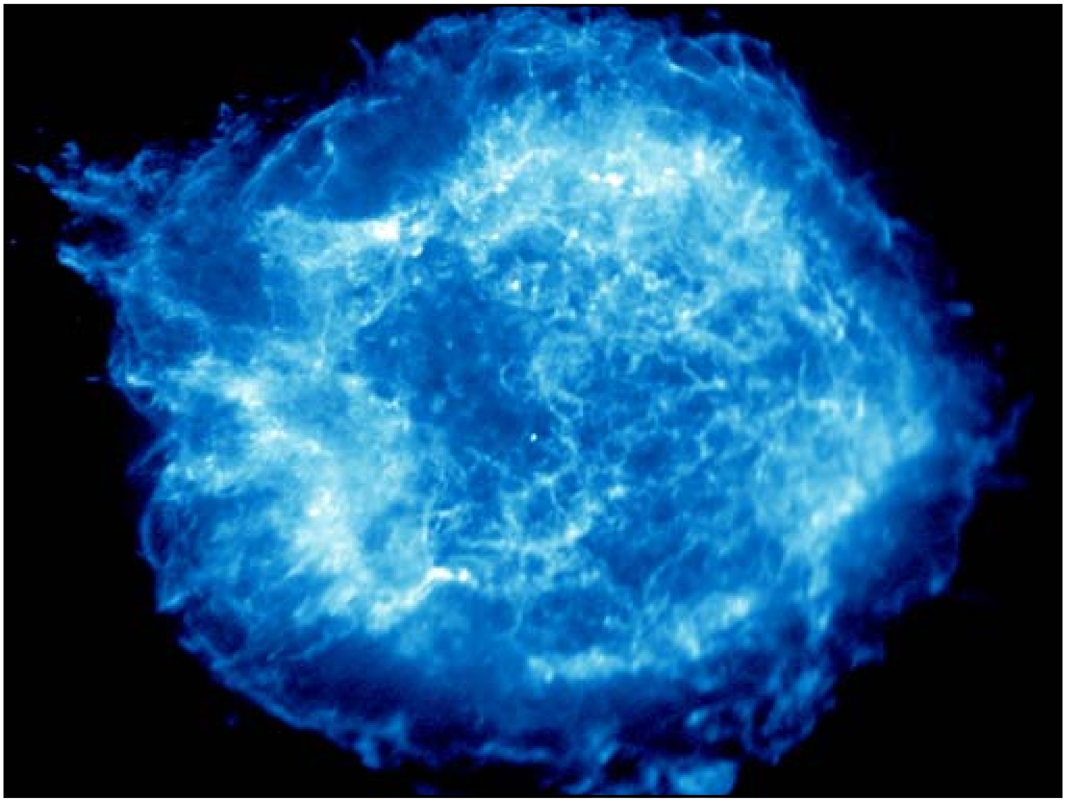
Supernova Remnants

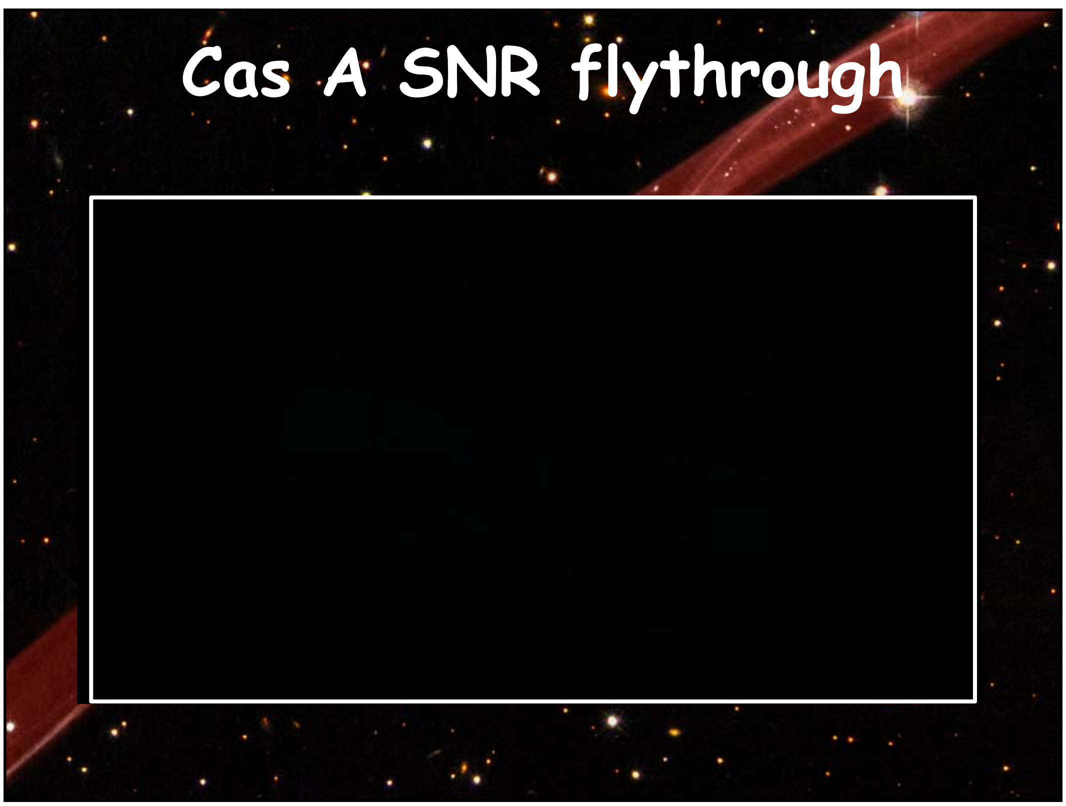

Kepler
(1604)
Tycho SNR
(1572)
SN1006 SNR
(1006)
20 arcmin
Cas A
(1680 ?)


JOANNIS KEPPLERI
Sac. Cæs. Majest. Mathematici
DE
STELLA NOVA
IN PEDE SERPENTARII, ET
QUI SUB EJUS EXORTUM DE
NOVO INIIT,
TRIGONO IGNEO.
LIBELLUS ASTRONOMICIS, PHYSICIS, META-
physicis, Meteorologicis & Astrologicis Disputationibus,
plenus.
ACCESSERUNT
I. DE STELLA INCOGNITA CYGNI:
Narratio Astronomica.
II. DE JESU CHRISTI SERVATORIS VERO
Anno Natalitio, consideratio novissimæ sententiæ LAU-
RENTII SUSLYGÆ Poloni, quatuor annos in usitata
Epocha desiderantis.
Cum Privilegio S. C. Majest. ad annos XV.
GRANA DAT E FIMO SCRUTANS
PRAGAE
Ex Officina calcographica PAULI SESSII.
ANNO M. DCVI.
Ad fol. 76.
De
Stella
Nova


X-ray
Chandra X-ray Observatory
Visible
Hubble Space Telescope
Infrared
Spitzer Space Telescope
Kepler's Supernova Remnant • SN 1604
NASA, ESA / JPL-Caltech / R. Sankrit & W. Blair (Johns Hopkins University)
ssc2004-15b


Cas A:
Remnant
Supernova (1680)
Brightest
Radio source
on the sky

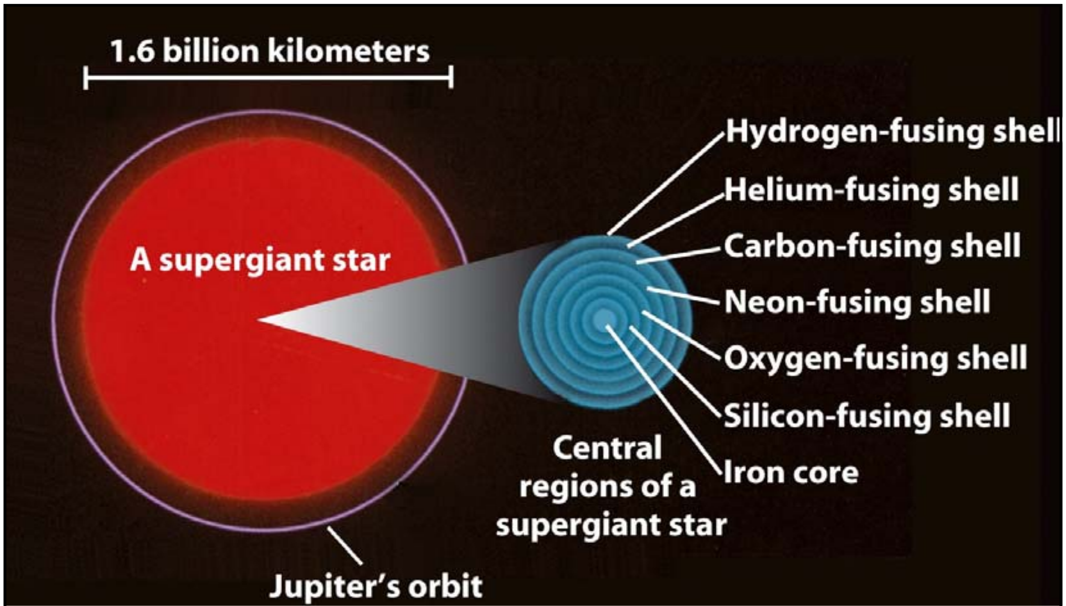



Cas A SNR flythrough

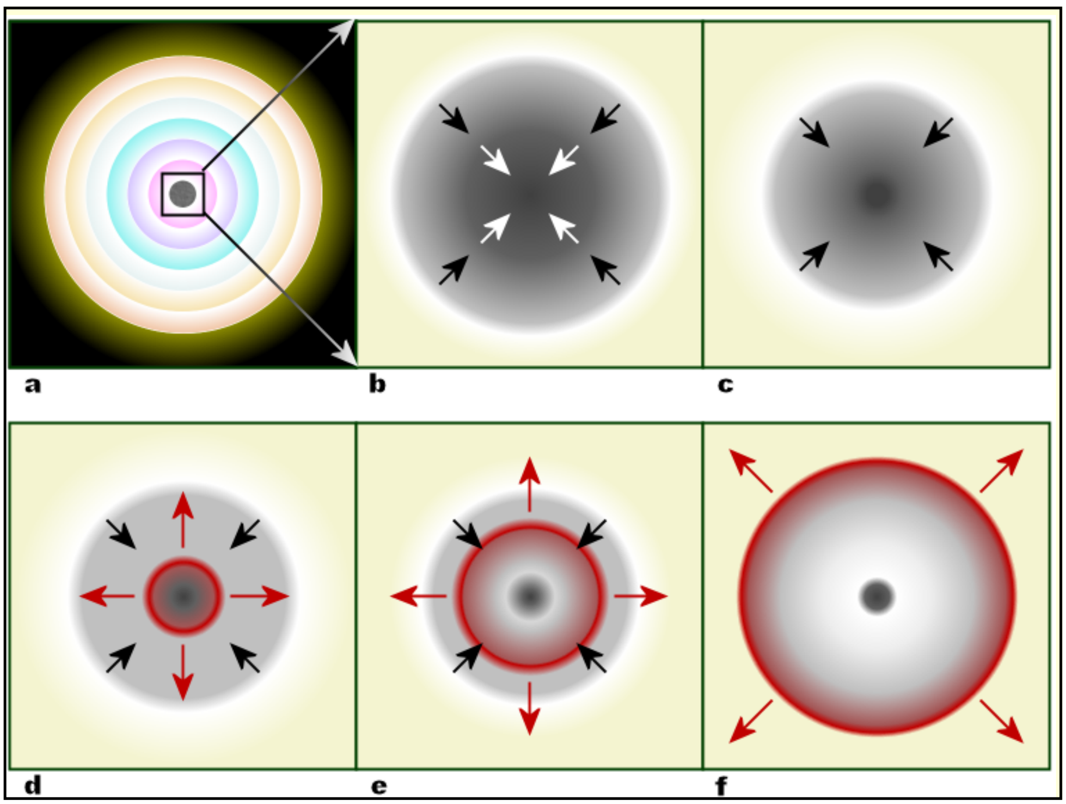
Theory of Supernova Blast Waves

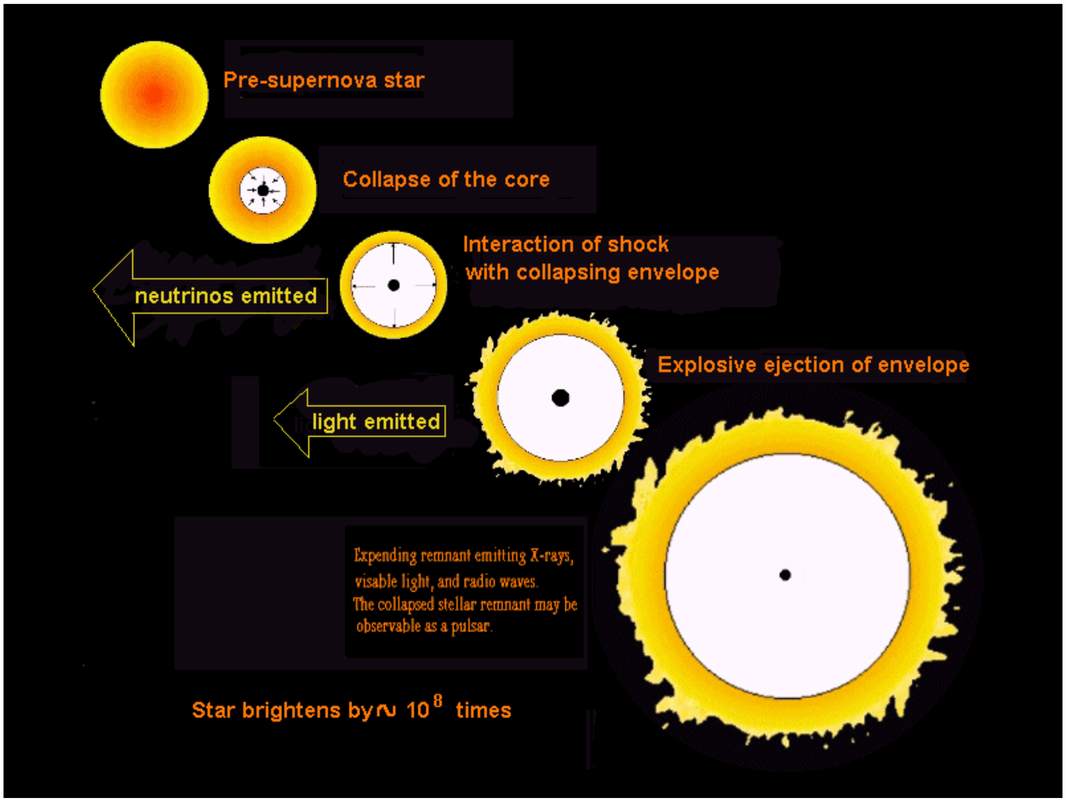
Supernovae: Type Ia	Subsonic deflagration wave turning into a supersonic detonation wave in outer layers. Mechanism: explosive carbon burning in a mass-accreting white dwarf
Type Ib-Ic & Type II	Core collapse of massive star

Core-Collapse SN

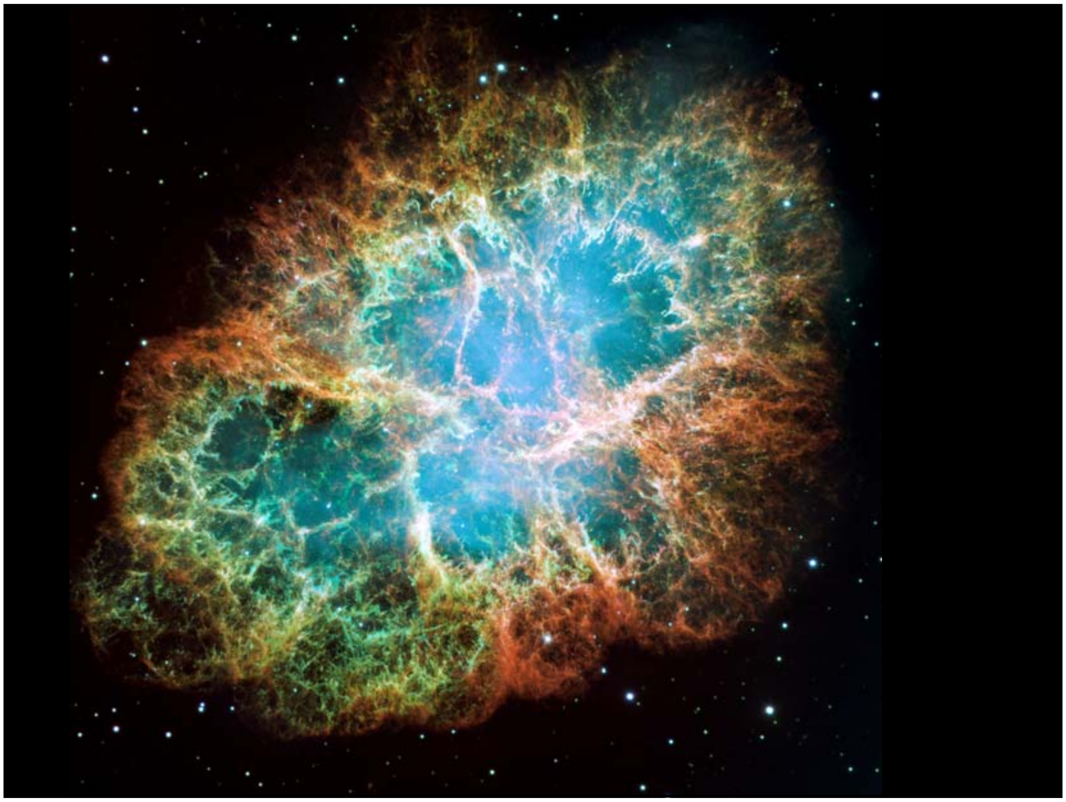


- In the last stages of its life, high-mass star:
  - iron-rich core
  - surrounded by concentric shells, hosting the various thermonuclear reactions
- The sequence of thermonuclear reactions stops here:
  - formation of elements heavier than iron requires
  - input of energy rather than causing energy to be released

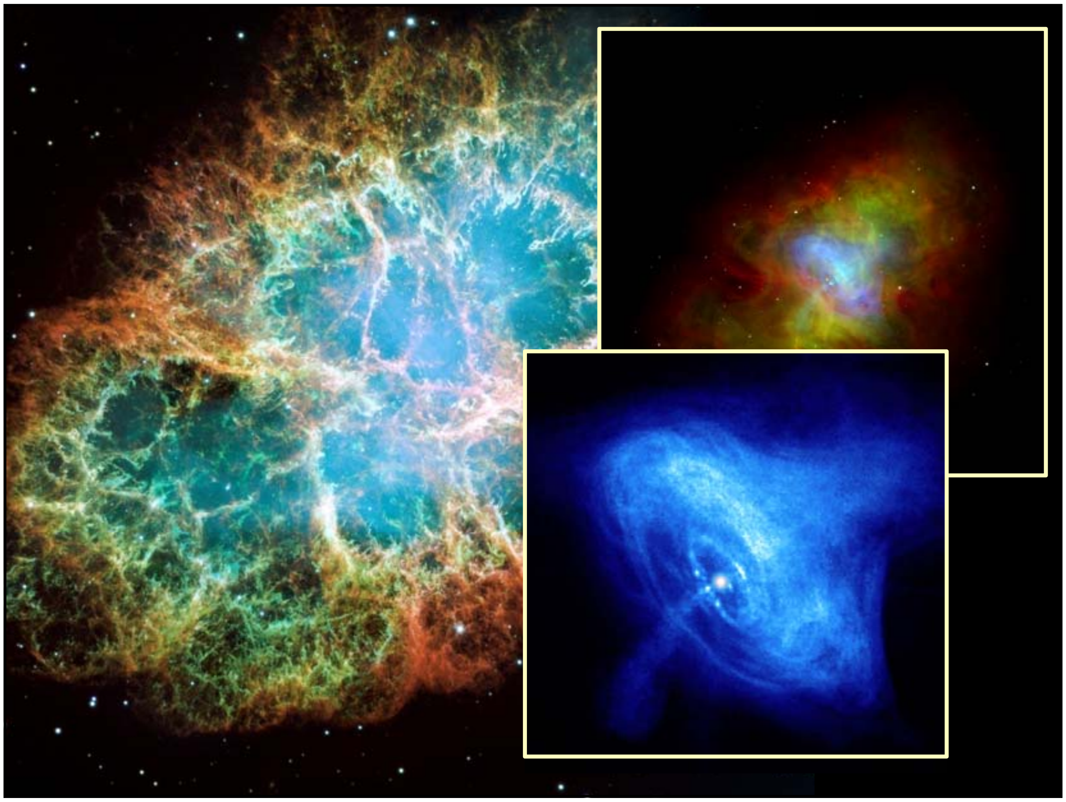


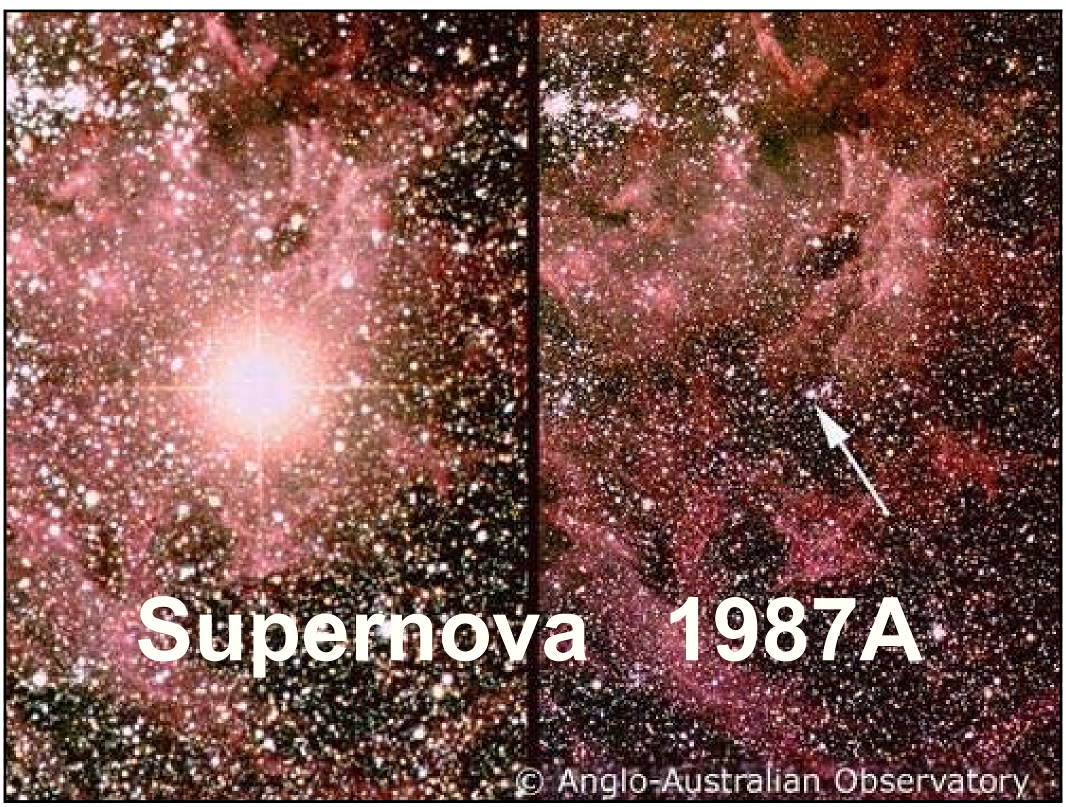


Supernova II Explosion: SN1054

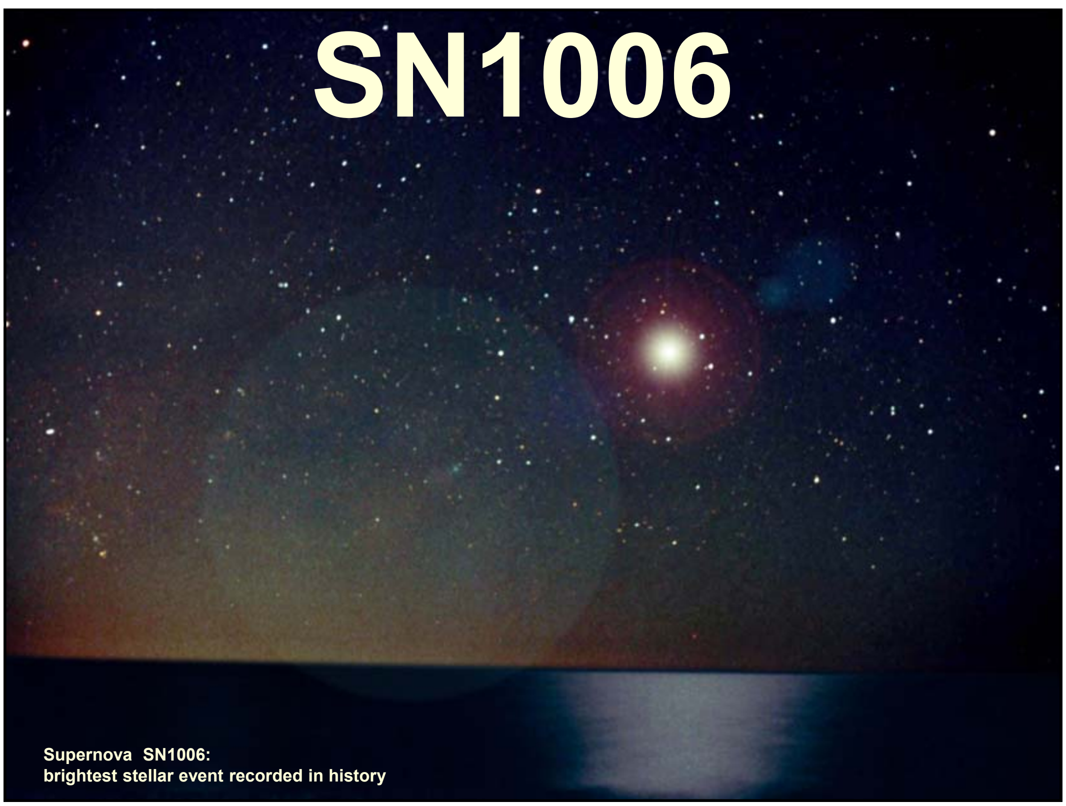


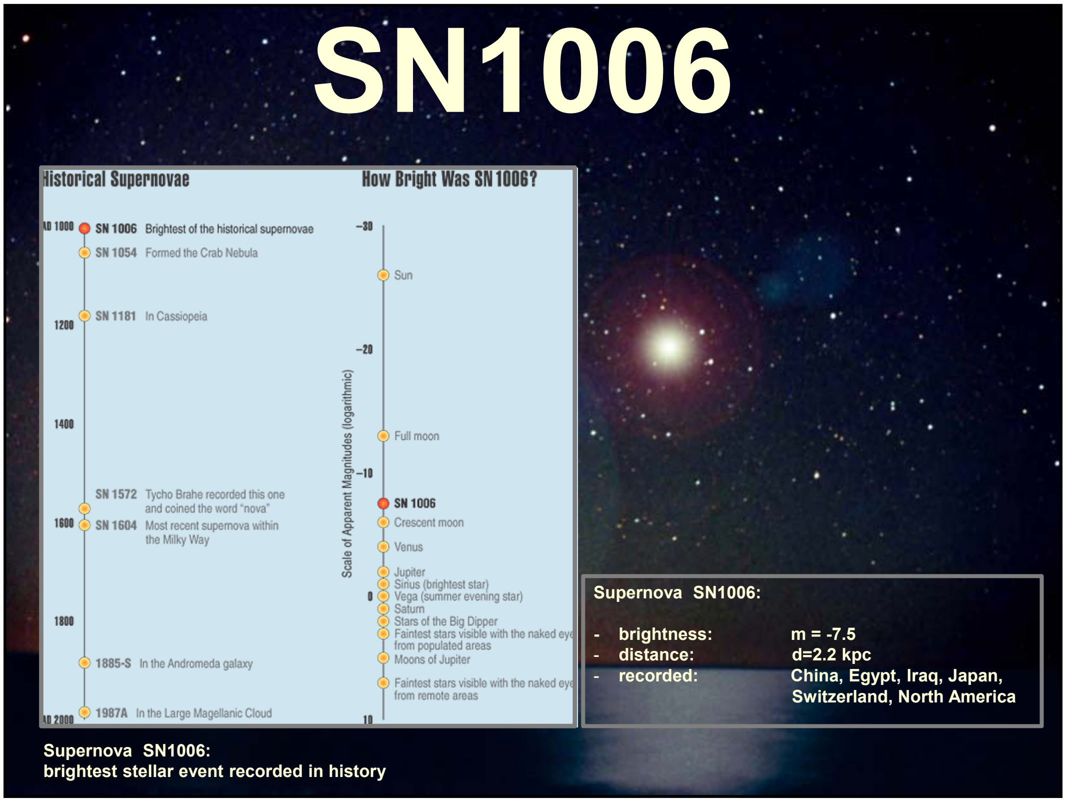

Pulsars and Neutron Stars
"Pulsar" Copyright 2002 By Dave King (dave@scifiimages.com)   Please Visit SciFiImages.com

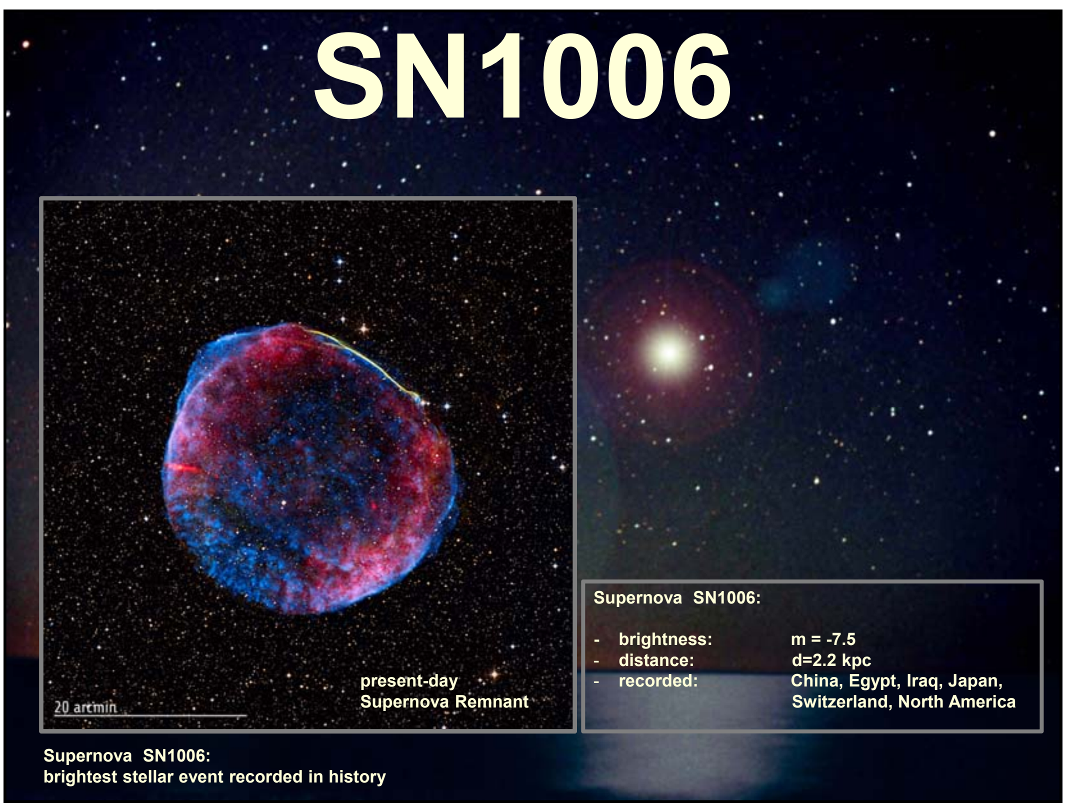


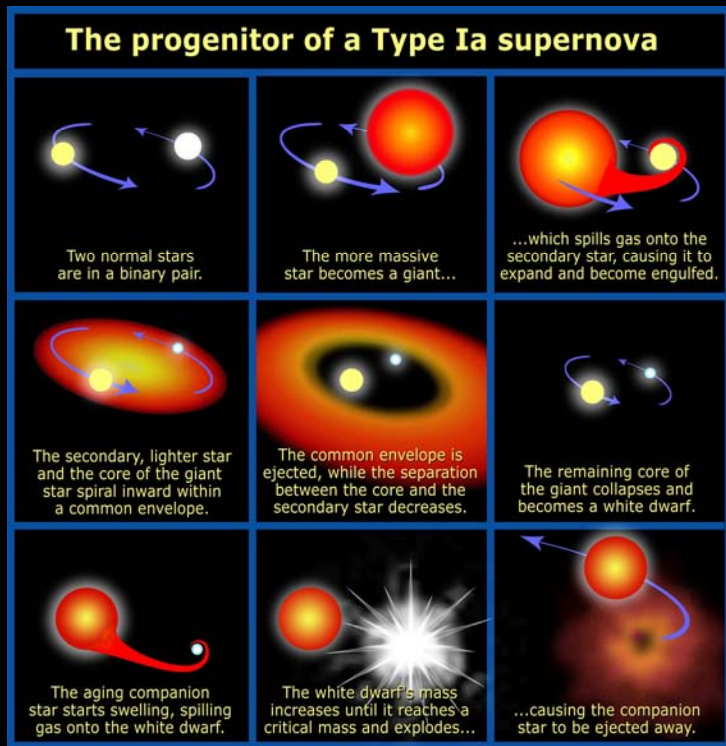

Supernova 1987A
© Anglo-Australian Observatory

Thermonuclear SN
(Supernova Ia)




SN1006
Historical Supernovae
AD 1000
SN 1006 Brightest of the historical supernovae
SN 1054 Formed the Crab Nebula
1200
SN 1181 In Cassiopeia
1400
SN 1572 Tycho Brahe recorded this one and coined the word "nova"
1600
SN 1604 Most recent supernova within the Milky Way
1800
1885-S In the Andromeda galaxy
AD 2000
1987A In the Large Magellanic Cloud
How Bright Was SN 1006?
Scale of Apparent Magnitudes (logarithmic)
−30
Sun
−20
Full moon
−10
SN 1006
Crescent moon
Venus
Jupiter
Sirius (brightest star)
0
Vega (summer evening star)
Saturn
Stars of the Big Dipper
Faintest stars visible with the naked eye from populated areas
Moons of Jupiter
Faintest stars visible with the naked eye from remote areas
10
Supernova SN1006:
- brightness: m = -7.5
- distance: d=2.2 kpc
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America
Supernova SN1006:
brightest stellar event recorded in history


SN1006
20 arcmin
present-day
Supernova Remnant
Supernova SN1006:
- brightness: m = -7.5
- distance: d=2.2 kpc
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America
Supernova SN1006:
brightest stellar event recorded in history

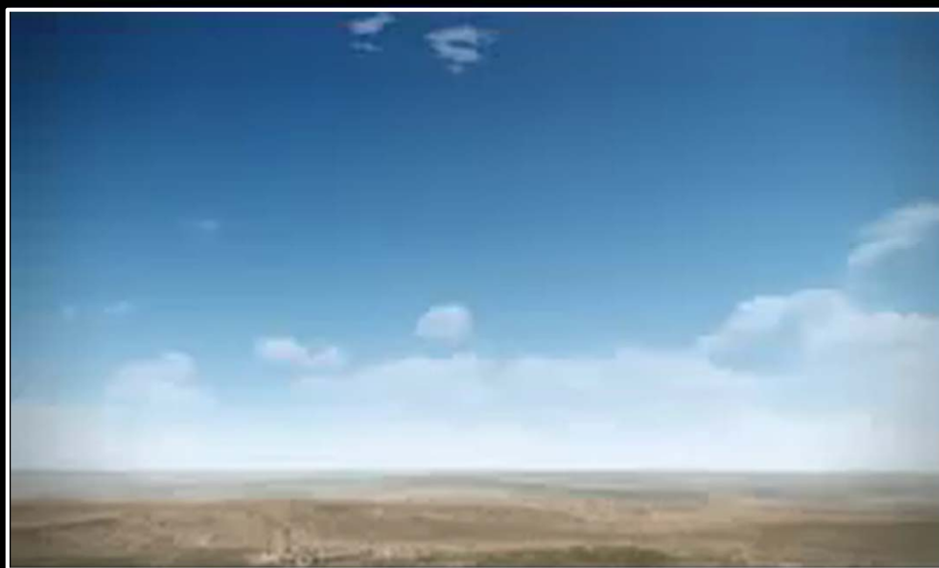


Supernova Ia Explosion

Blast Waves

Tsar Bomba Nuclear Explosion

Tsar Bomba Nuclear Explosion



Tsar Bomba Nuclear Explosion

Hiroshima, the Shockwave

Sedov-Taylor Expansion Law

Blast waves

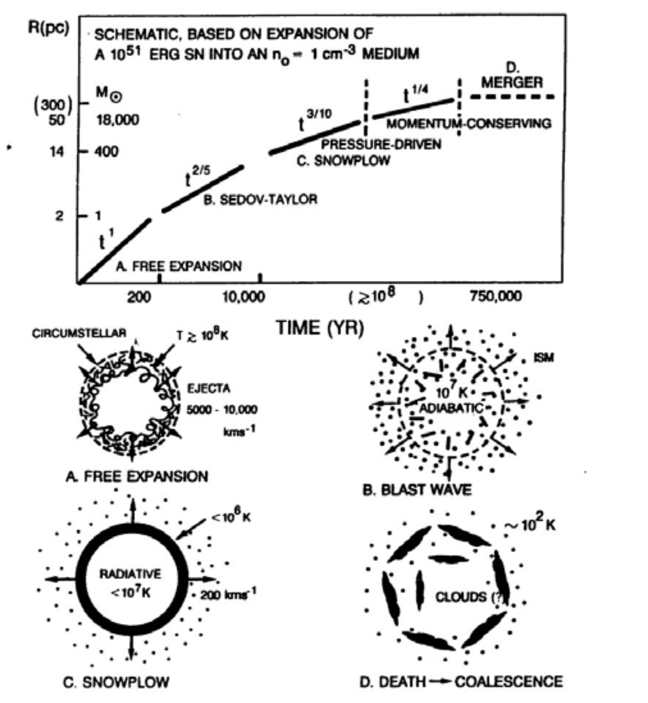
<u>Main properties:</u>

1. <u>Strong</u> shock propagating through the Interstellar Medium, or through the wind of the progenitor star;

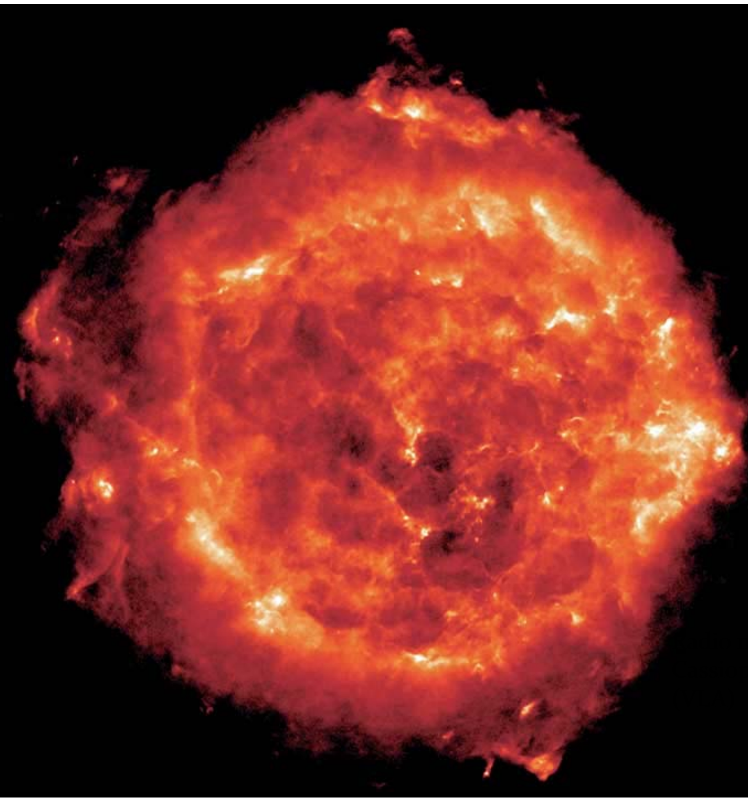
2. Different expansion stages:

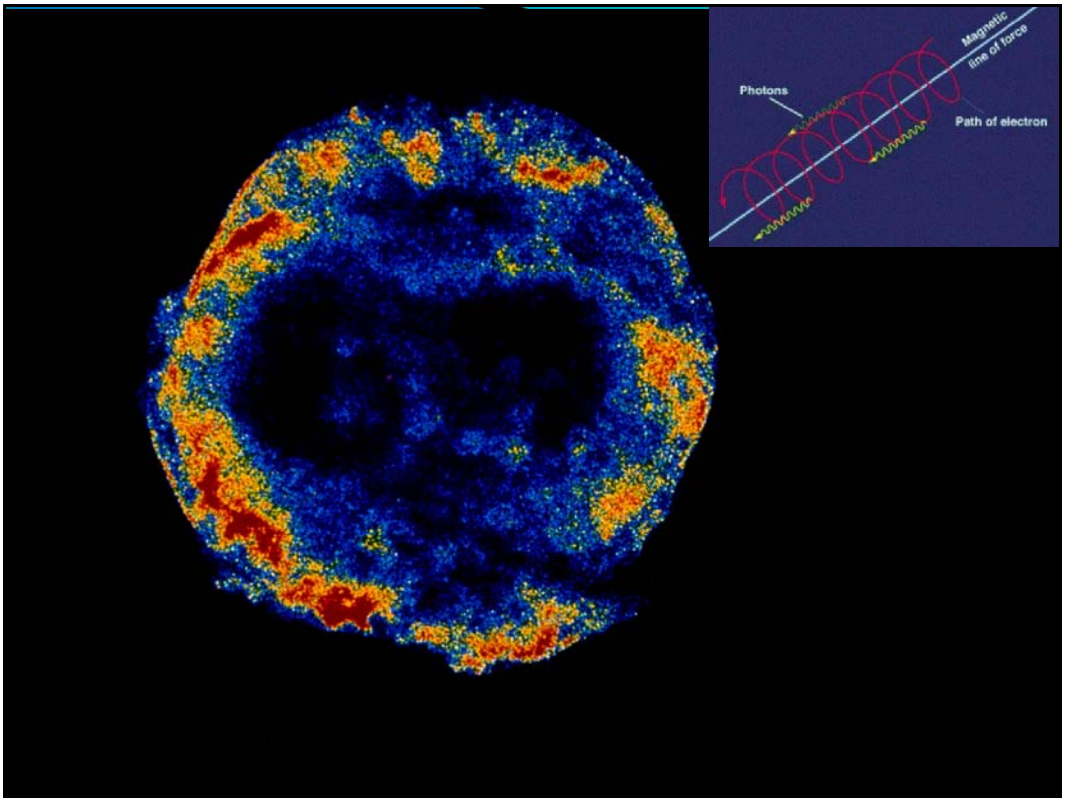
   - Free expansion stage (t < 1000 yr) $R \propto t$
   - Sedov-Taylor stage (1000 yr < t < 10,000 yr) $R \propto t^{2/5}$
   - Pressure-driven snowplow (10,000 yr < t < 250,000 yr) $R \propto t^{3/10}$

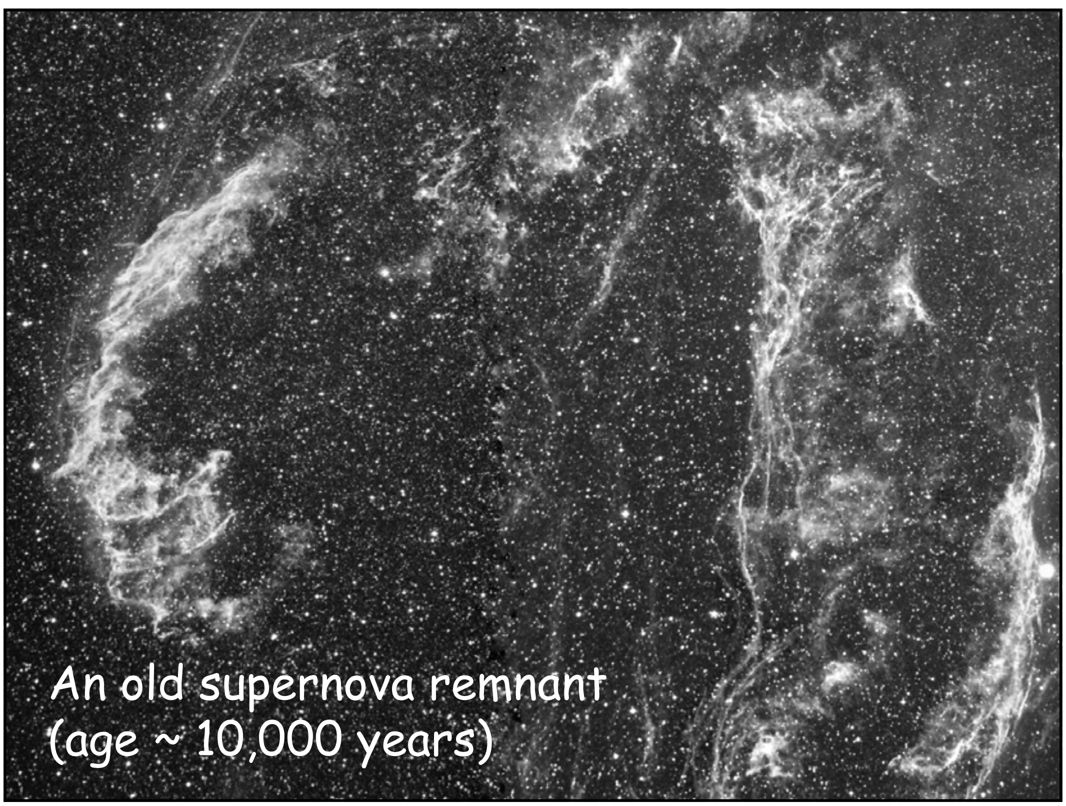
STANDARD SNR EVOLUTION



Tsar Bomba Nuclear Explosion




Magnetic line of force
Photons
Path of electron


An old supernova remnant
(age ~ 10,000 years)

Free-expansion phase

Energy budget:

$$\left|E_{\rm grav}\right| = \tfrac{3}{5}\frac{GM_c^2}{R_c} \simeq 10^{53}\ \text{erg} \Rightarrow \begin{cases} \text{99\% into neutrino's} \\ \text{1\% into mechanical energy} \end{cases}$$

Expansion speed:

$$V_{\rm exp} \simeq \sqrt{\frac{2E_{\rm mech}}{M_{\rm ej}}} = 3000\left(\frac{E_{\rm mech}}{10^{51}\ \text{erg}}\right)^{1/2}\left(\frac{M_{\rm ej}}{10\ M_\odot}\right)^{-1/2}\ \text{km/s}$$

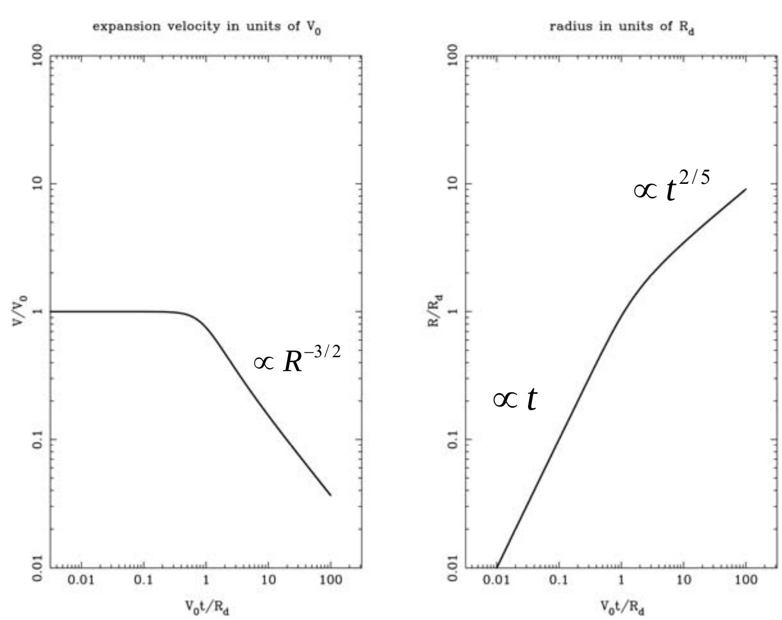
Sedov-Taylor stage

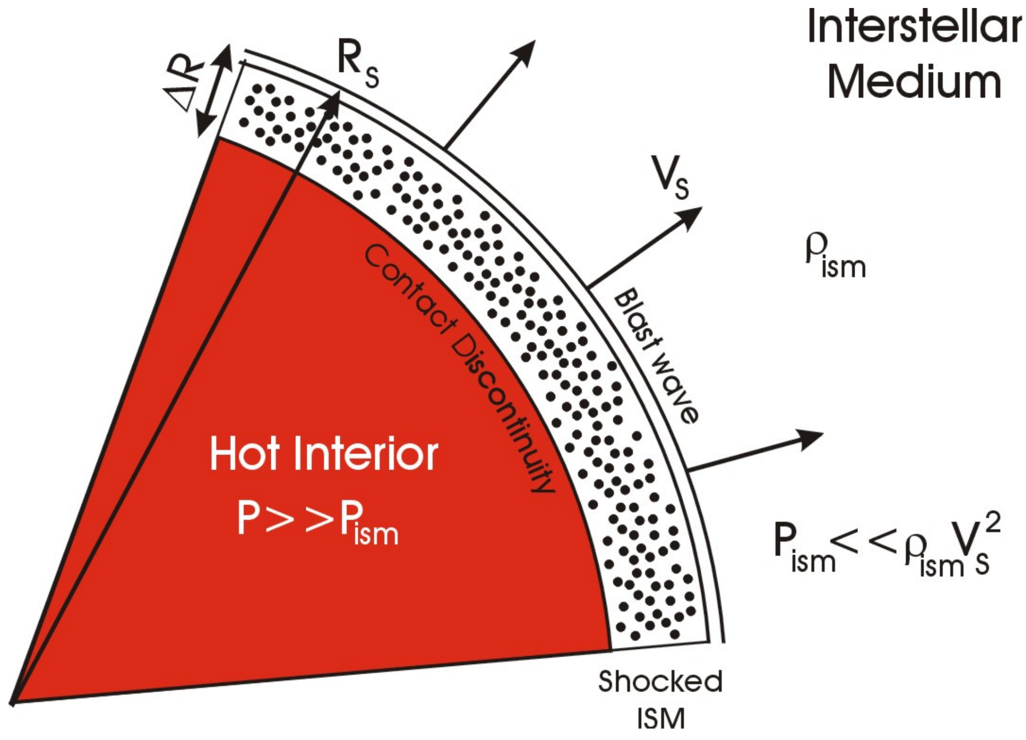
- Expansion starts to decelerate due to swept-up mass

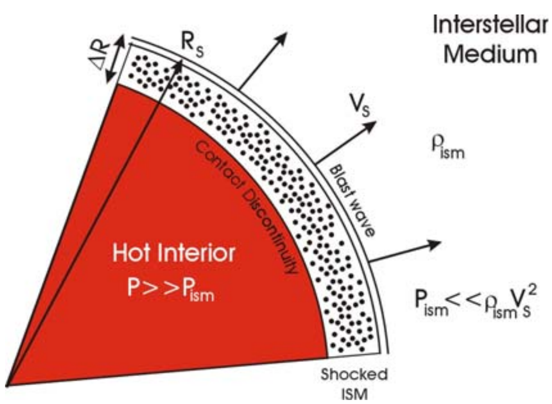
- Interior of the bubble is <u>reheated</u> due to <u>reverse shock</u>

- Hot bubble is preceded in ISM by strong <u>blast wave</u>

$$V_s = \sqrt{\frac{2E_{\rm snr}}{M_{\rm ej}}} \times \left(\frac{1}{1+\left(R/R_{\rm d}\right)^3}\right)^{1/2} = V_0\left(\frac{1}{1+\left(R/R_{\rm d}\right)^3}\right)^{1/2}$$

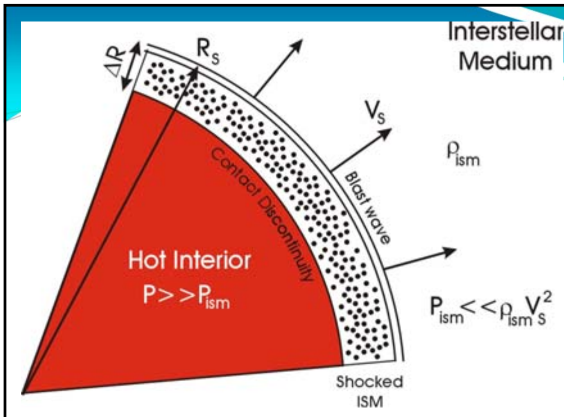






Shock relations for strong (high-Mach number) shocks:

$$\left.\begin{aligned} \frac{\rho_2}{\rho_1} &= \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1} \\ \frac{P_2}{P_1} &= \frac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1} \Rightarrow \frac{2\gamma}{\gamma+1}\mathcal{M}_s^2 \\ &\Leftrightarrow P_2 = \frac{2}{\gamma+1}\rho_1 V_1^2 \end{aligned}\right\} \text{as } \mathcal{M}_s^2 \equiv \left(\frac{V_1}{c_{s1}}\right)^2 = \frac{\rho_1 V_1^2}{\gamma P_1} \Rightarrow \infty$$

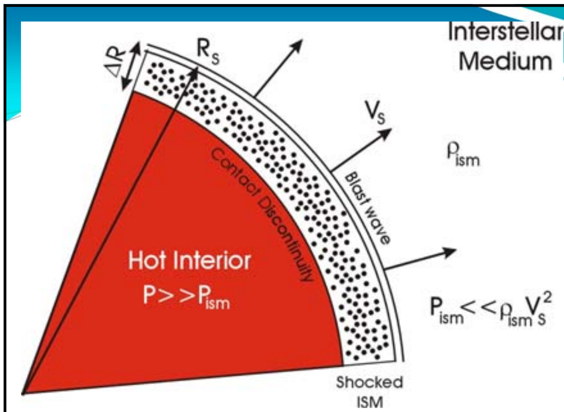


$$P_2 \approx \frac{2\gamma}{\gamma+1} \mathcal{M}_s^{\,2} P_1 = \frac{2}{\gamma+1} \rho_{\rm ism} V_s^{\,2}$$

Pressure behind strong shock (blast wave)

$$P_{\rm i} = (\gamma - 1) e_{\rm i} \approx (\gamma - 1) \frac{E_{\rm SNR}}{\frac{4\pi}{3} R_{\rm S}^3}$$

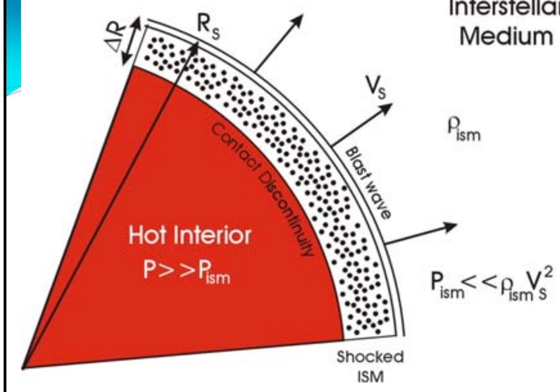
Pressure in hot SNR interior



At contact discontinuity: equal pressure on both sides!

$$\frac{2}{\gamma+1} \rho_{\rm ism} V_s^{\,2} \approx (\gamma - 1) \frac{E_{\rm SNR}}{\frac{4\pi}{3} R_{\rm s}^3}$$

This procedure is allowed because of high sound speeds in hot interior and in shell of hot, shocked ISM:
No large pressure differences are possible!

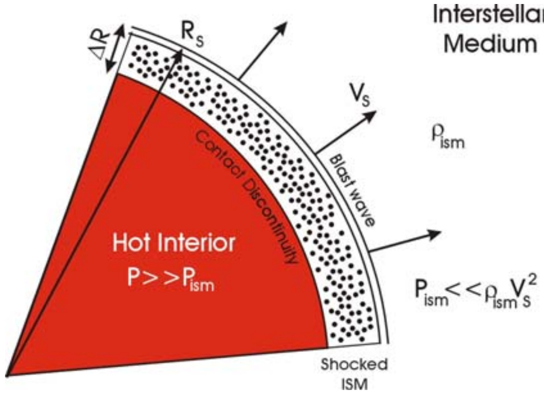


At contact discontinuity: <u>equal</u> pressure on both sides!

$$\frac{2}{\gamma+1}\rho_{\rm ism}V_s^{\,2} \approx (\gamma-1)\frac{E_{\rm SNR}}{\frac{4\pi}{3}R_s^3}$$

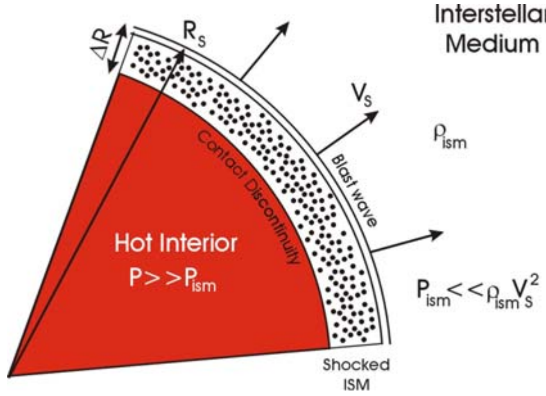
$$V_s = \frac{dR_s}{dt} \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} R_s^{-3/2}$$

Relation between velocity and radius gives expansion law!



$$R_s^{\,3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

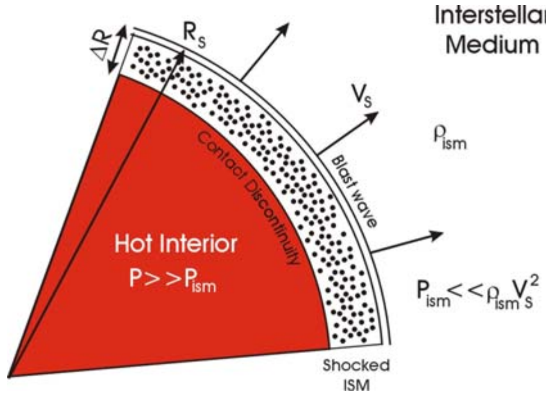
Step 1: write the relation as difference equation



$$R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

Step 2: write as total differentials and.........



$$R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

$$R_s(t) \simeq C_\gamma \left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/5} t^{2/5} \quad ,$$

$$C_\gamma = \left(\frac{5}{2}\right)^{2/5}\left(\frac{8\pi}{3\left(\gamma^2-1\right)}\right)^{1/5} \simeq 1.96$$

......integrate to find the Sedov-Taylor solution

Sedov & Taylor