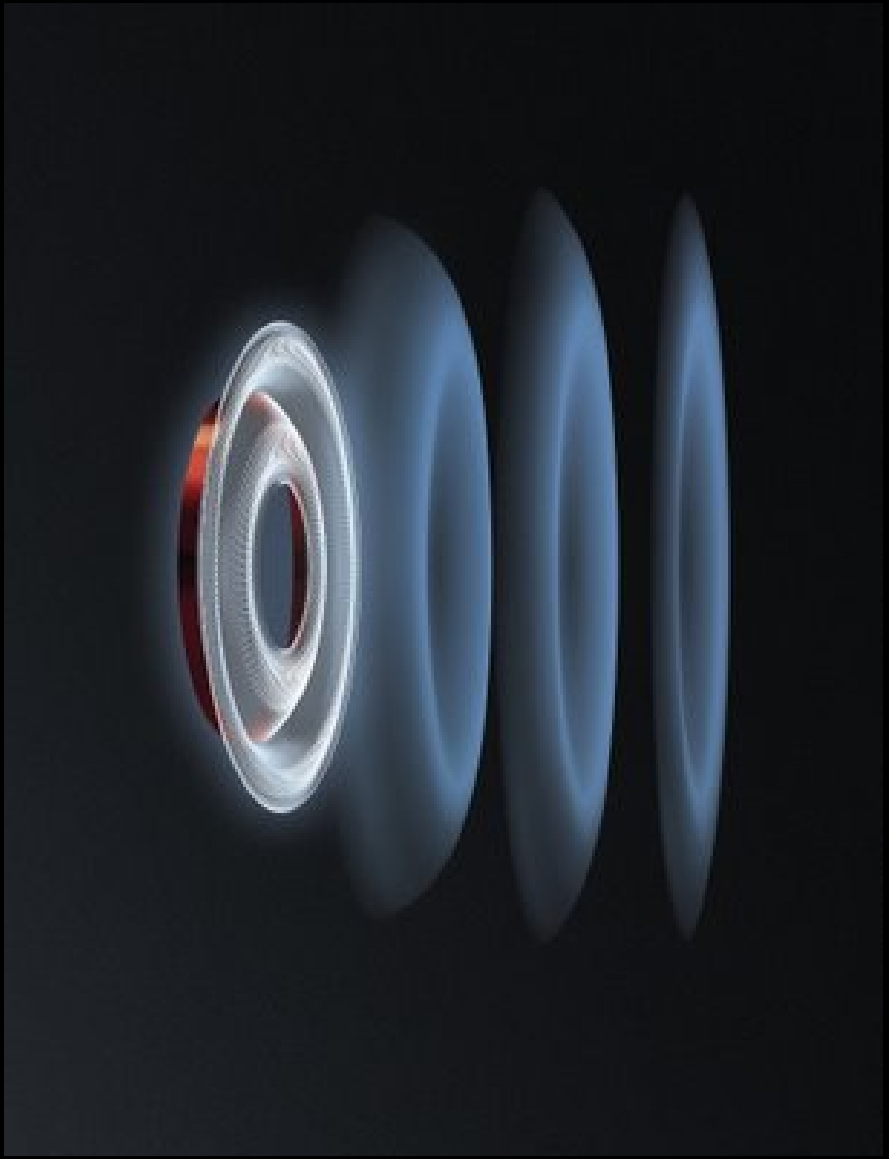
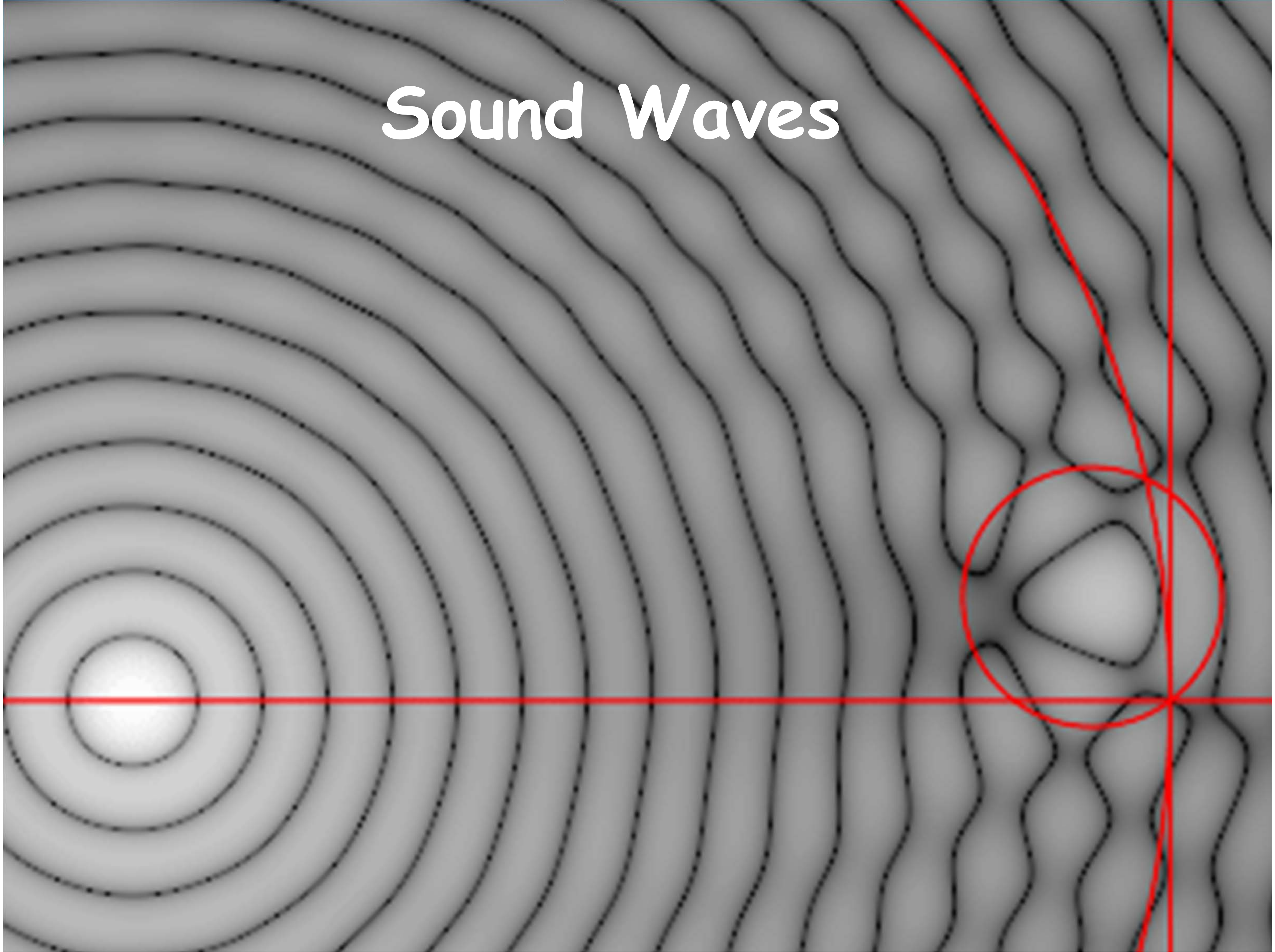
Sound Waves

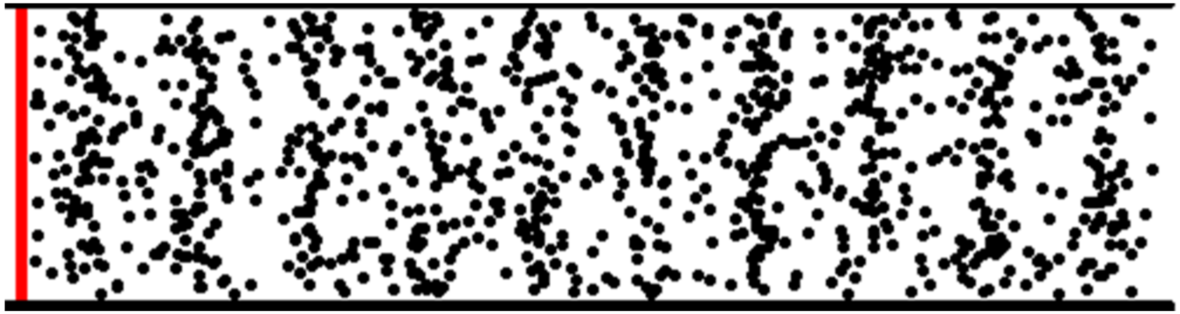
Sound Waves:

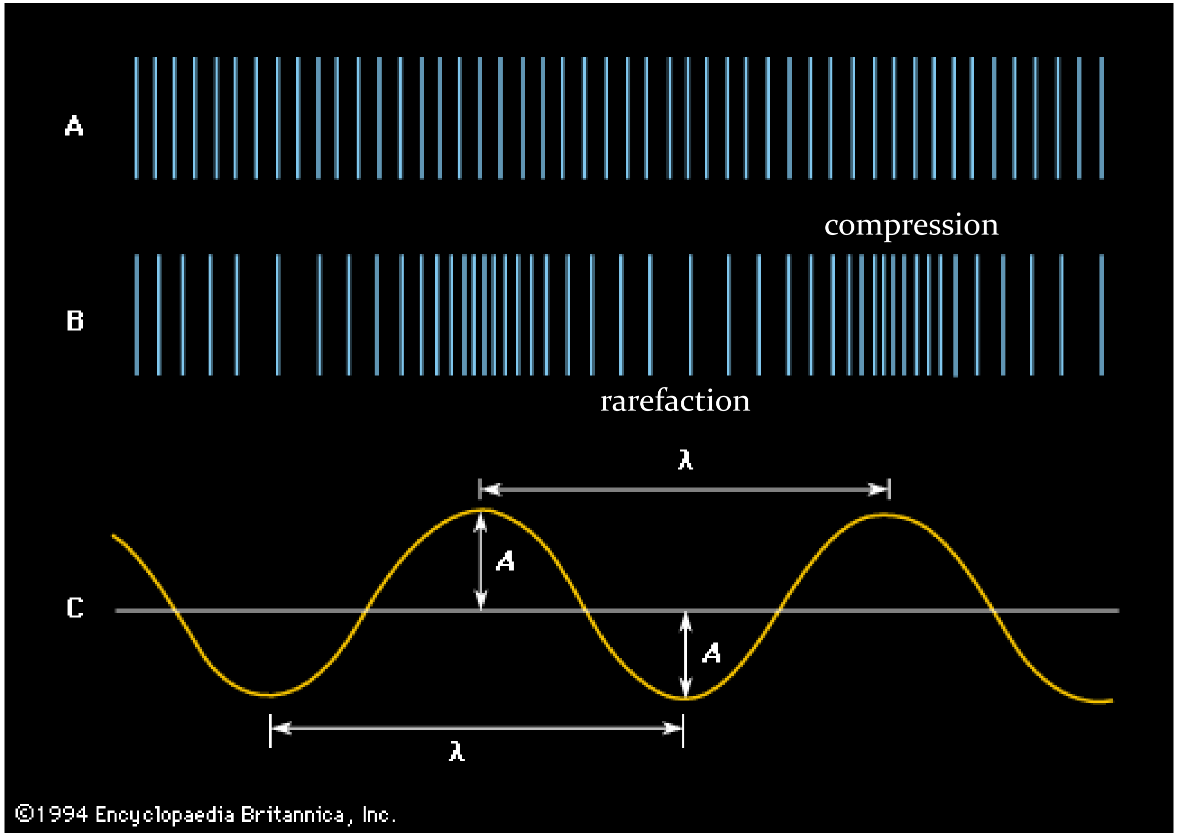


Sound Waves



Sound Waves




A
compression
B
rarefaction
λ
A
C
A
λ
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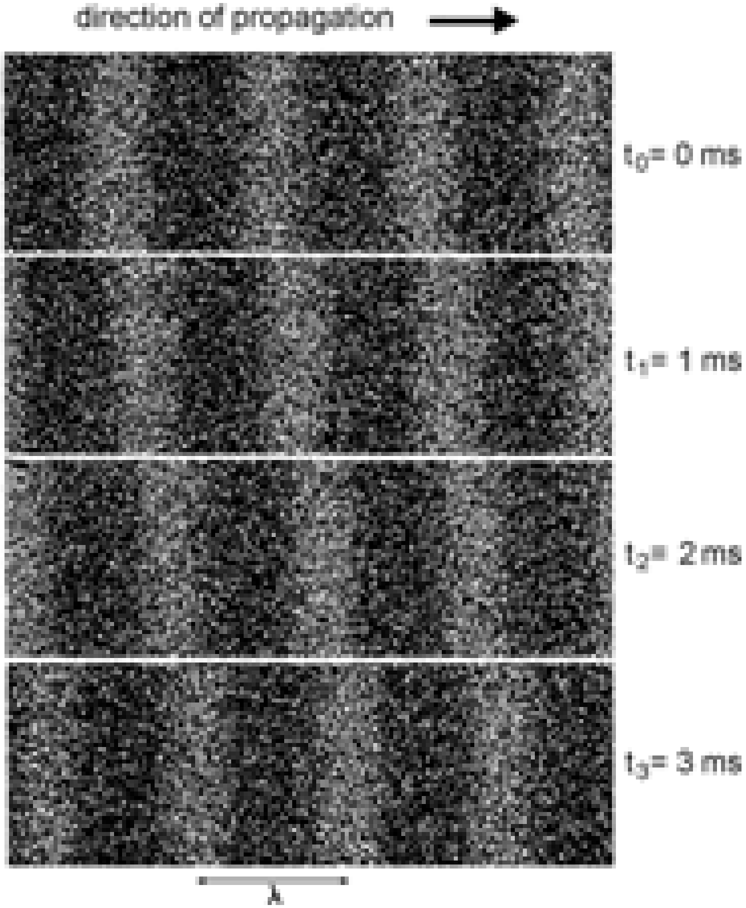
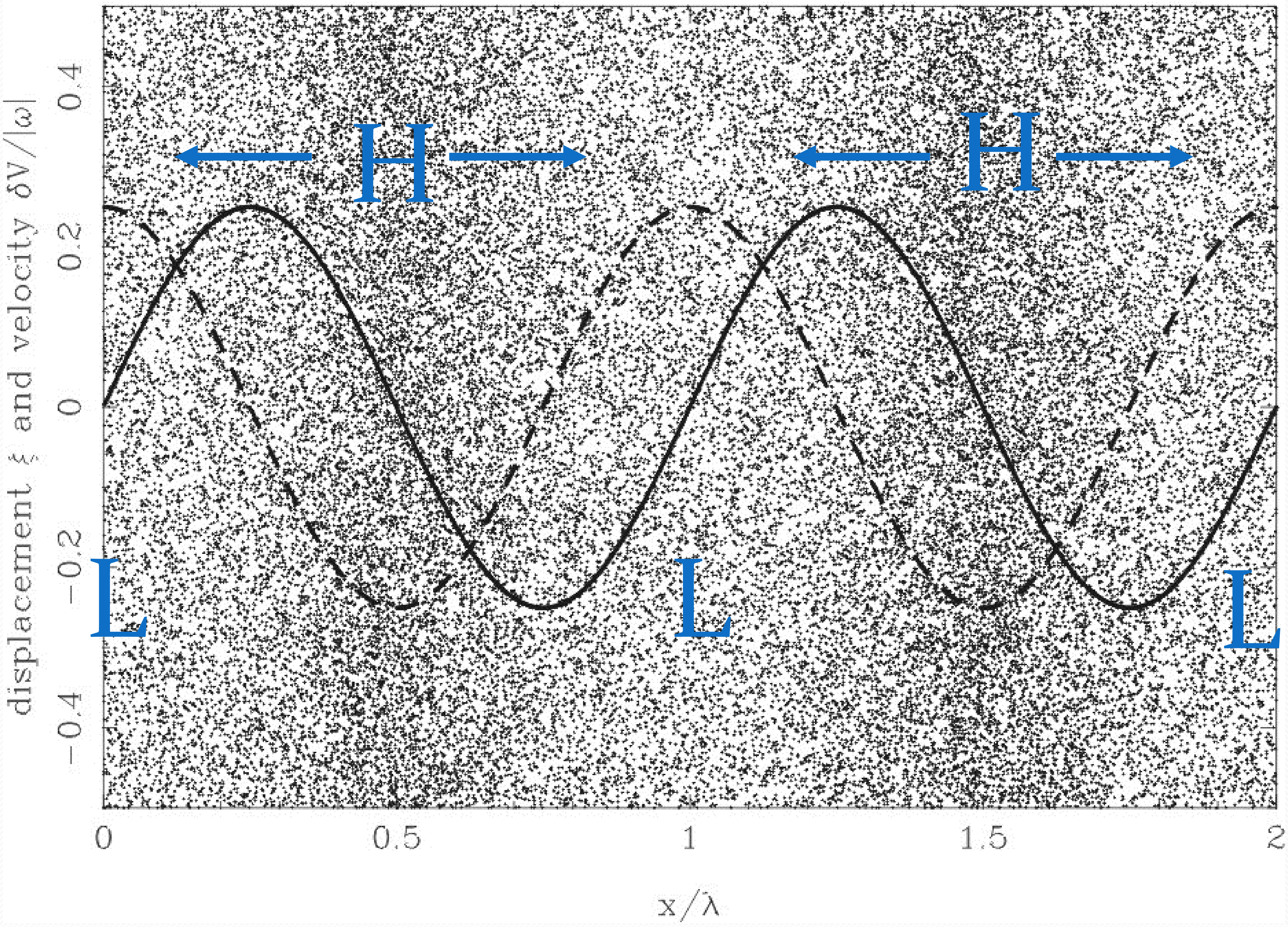

direction of propagation
t0= 0 ms
t1= 1 ms
t2= 2 ms
t3= 3 ms
λ

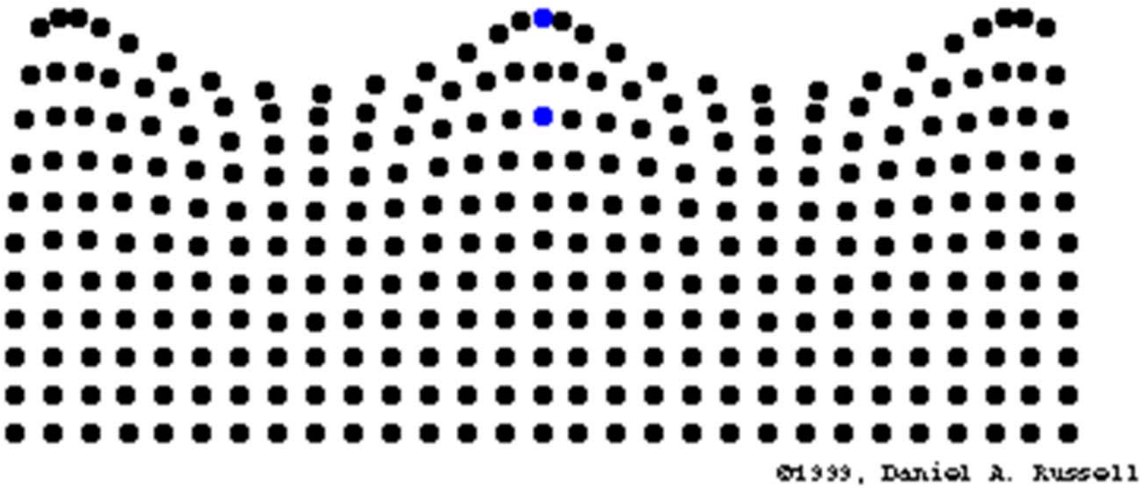
particle density, displacement and velocity in a sound wave

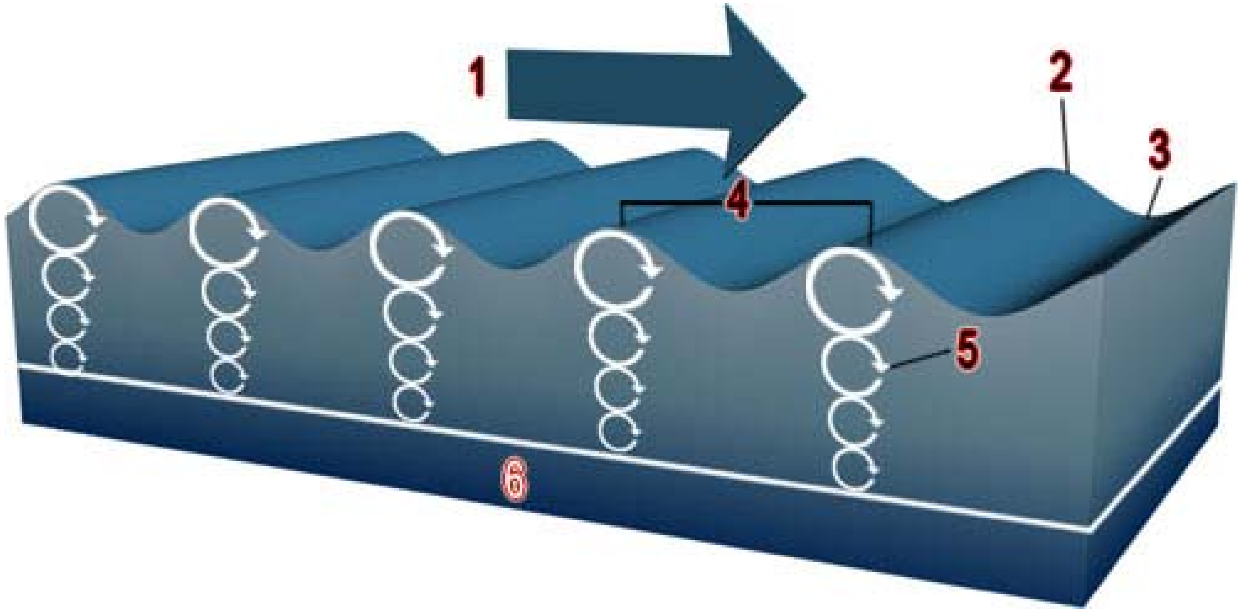


Gravity Waves

Gravity Waves

Gravity Waves





Gravity Waves

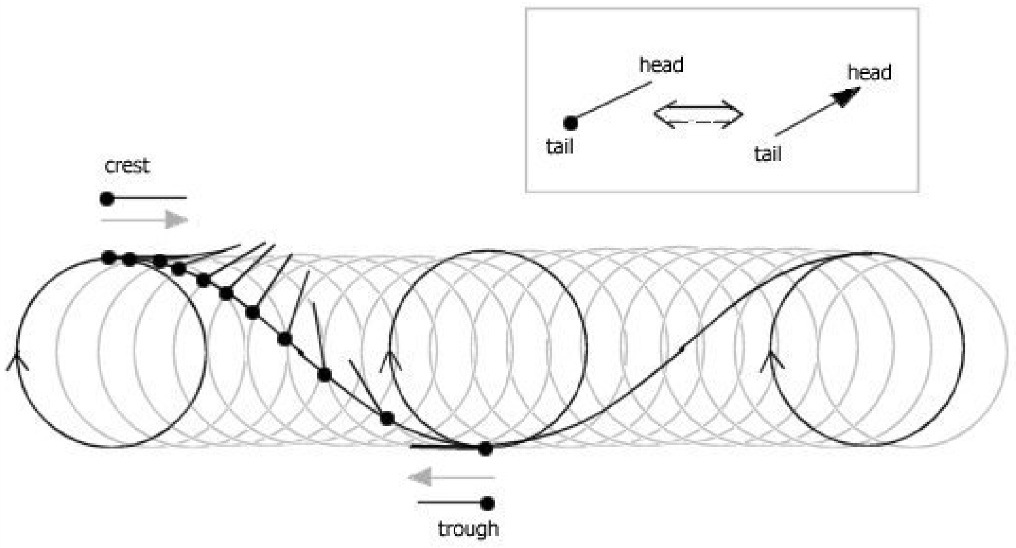
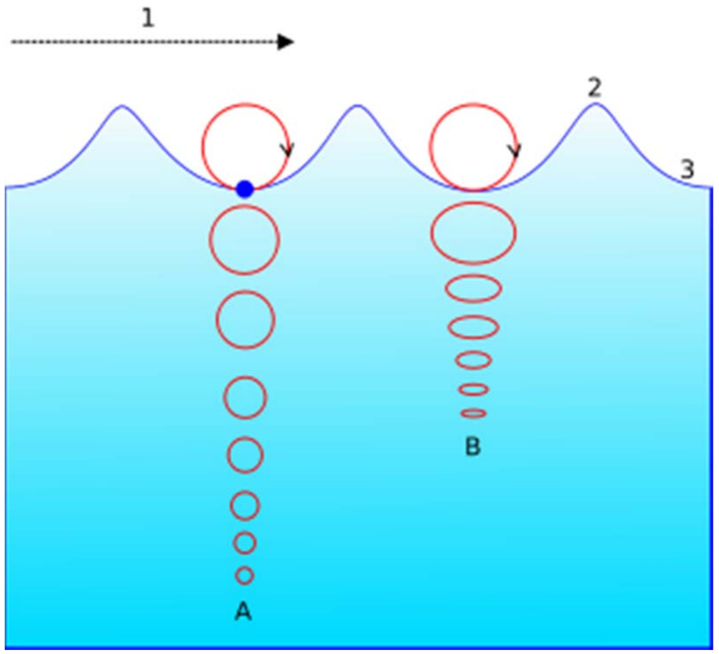
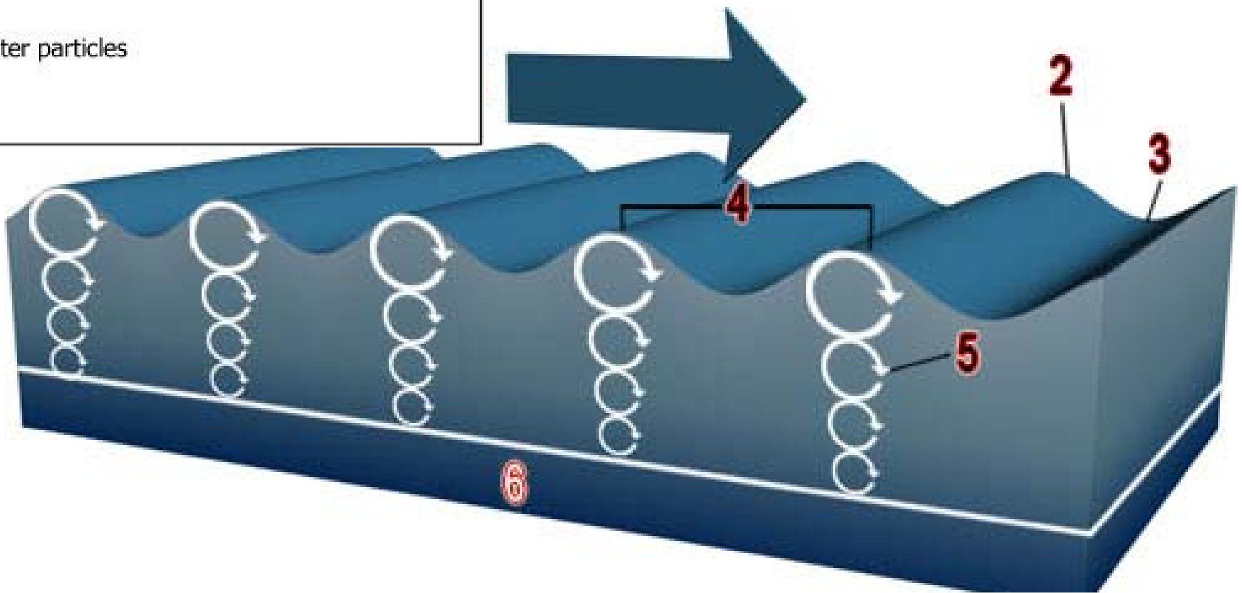


Fig 1

Selective velocity profiles for water particles in deep water wave.

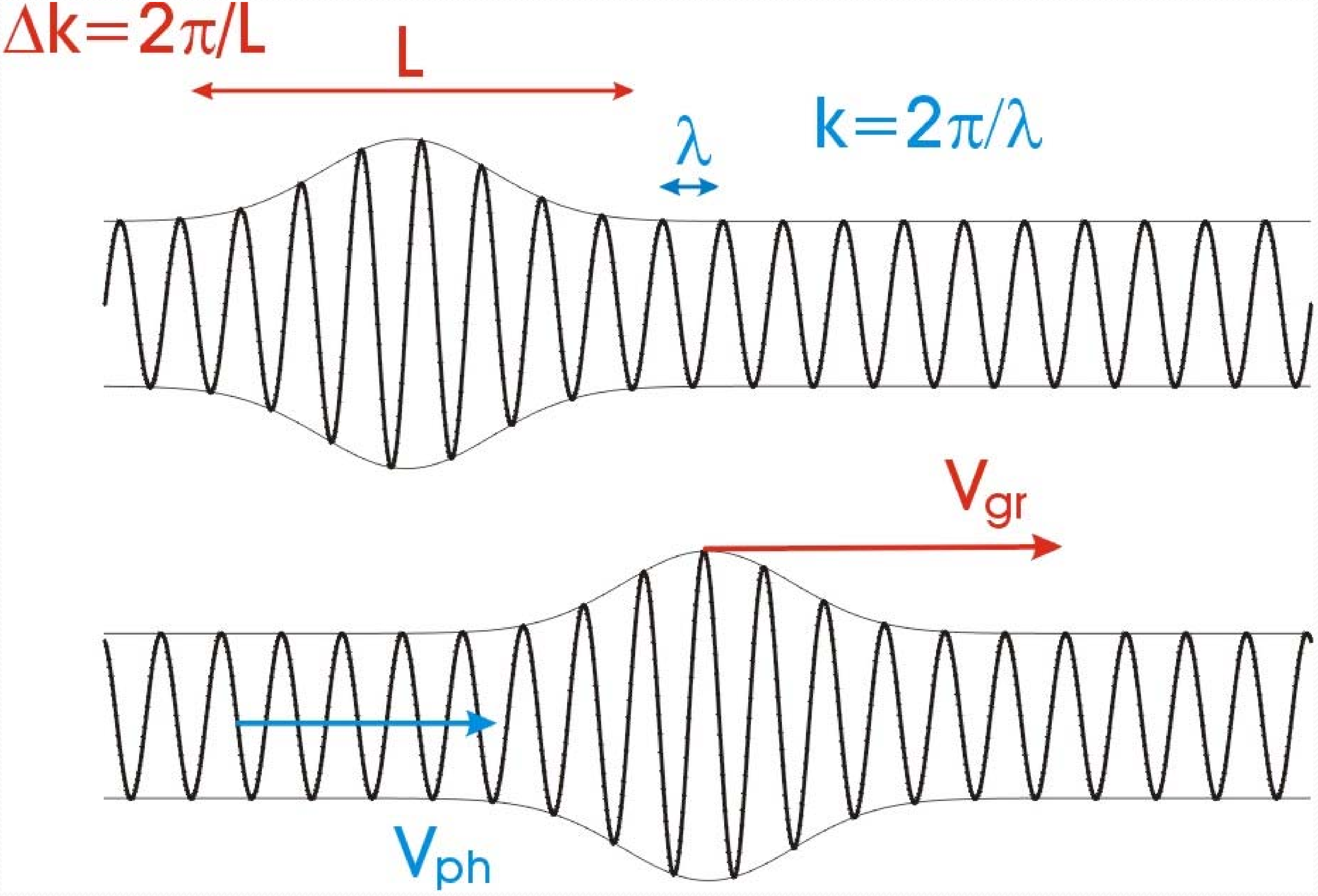


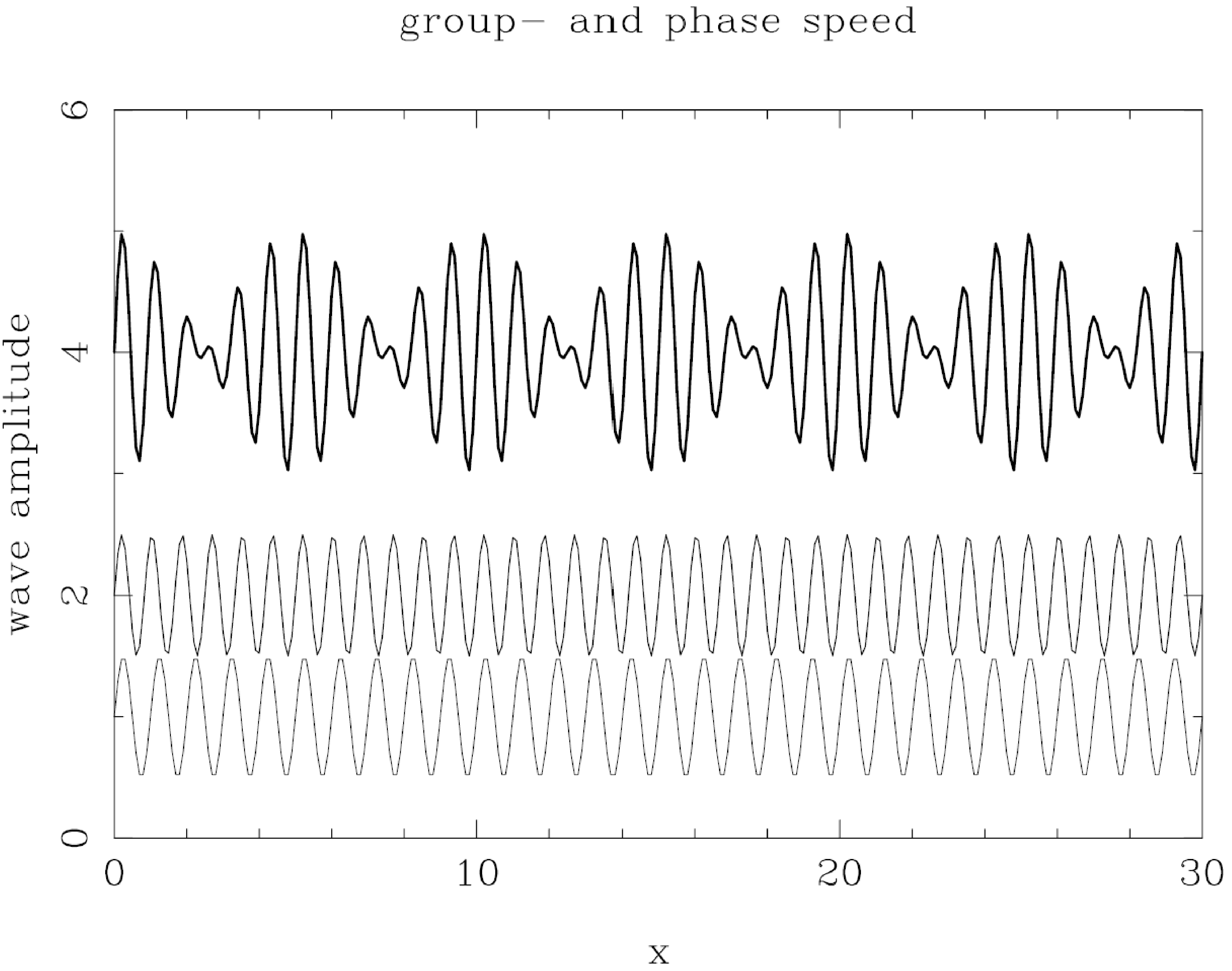


Kayak Surfing
on ocean gravity waves
Oregon Coast

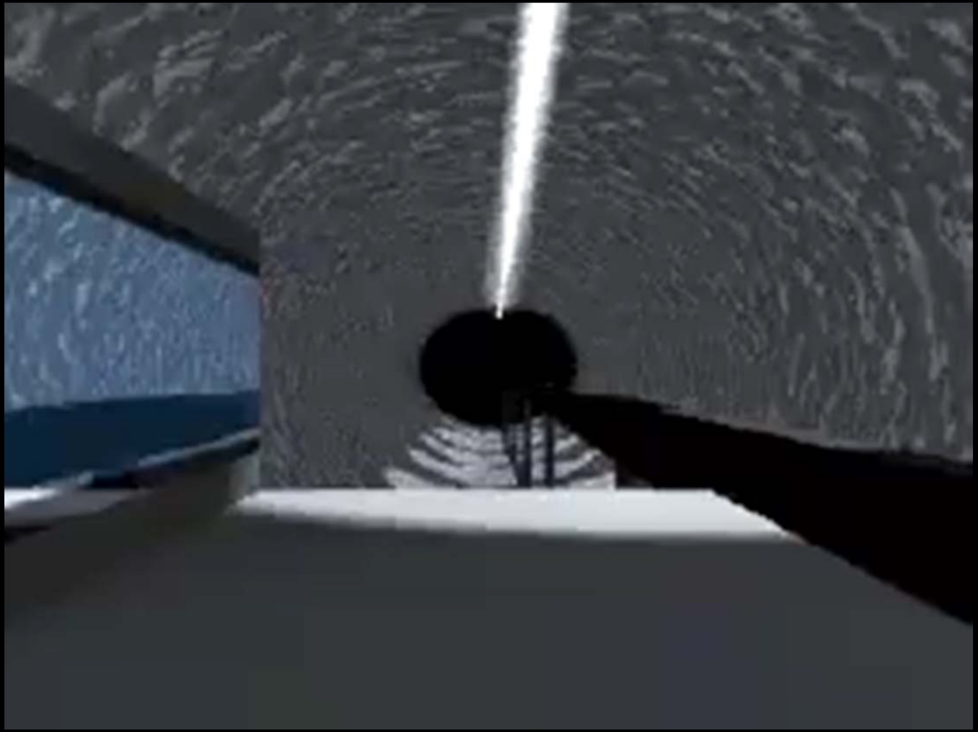
Waves:
sea & ocean waves

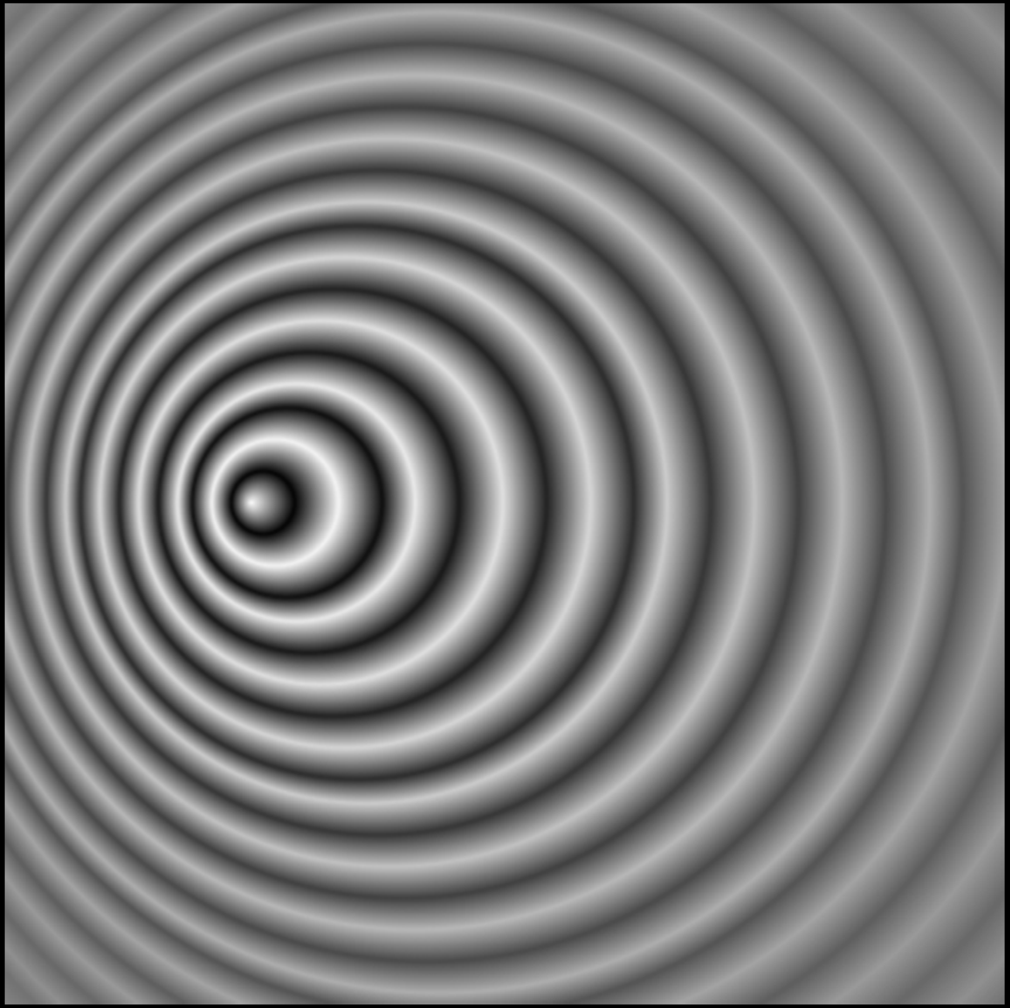
Phase & Group Velocity


$\Delta k = 2\pi/L$
L
λ
$k = 2\pi/\lambda$
V_{gr}
V_{ph}

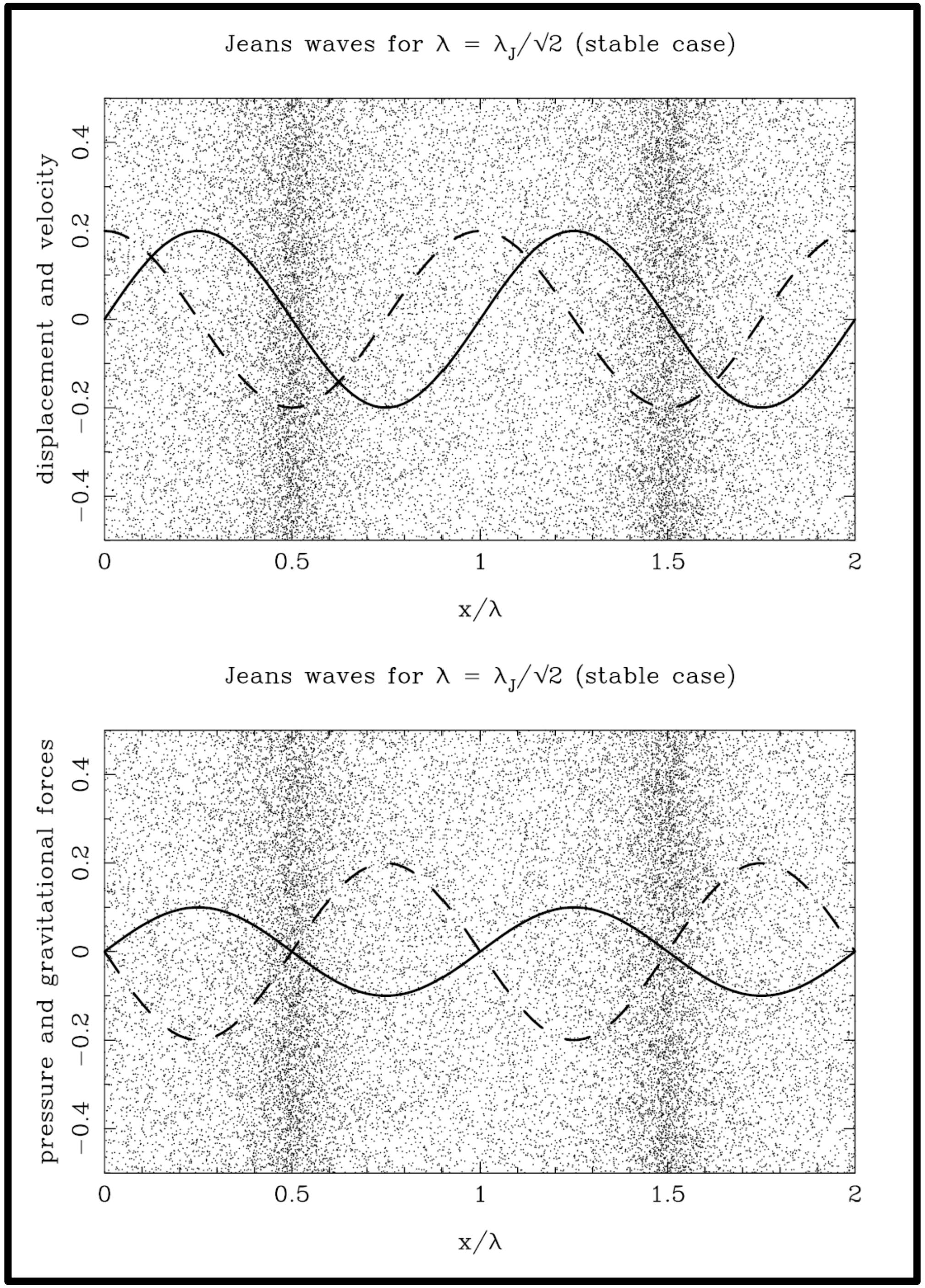

group- and phase speed
wave amplitude
0
2
4
6
0
10
20
30
x

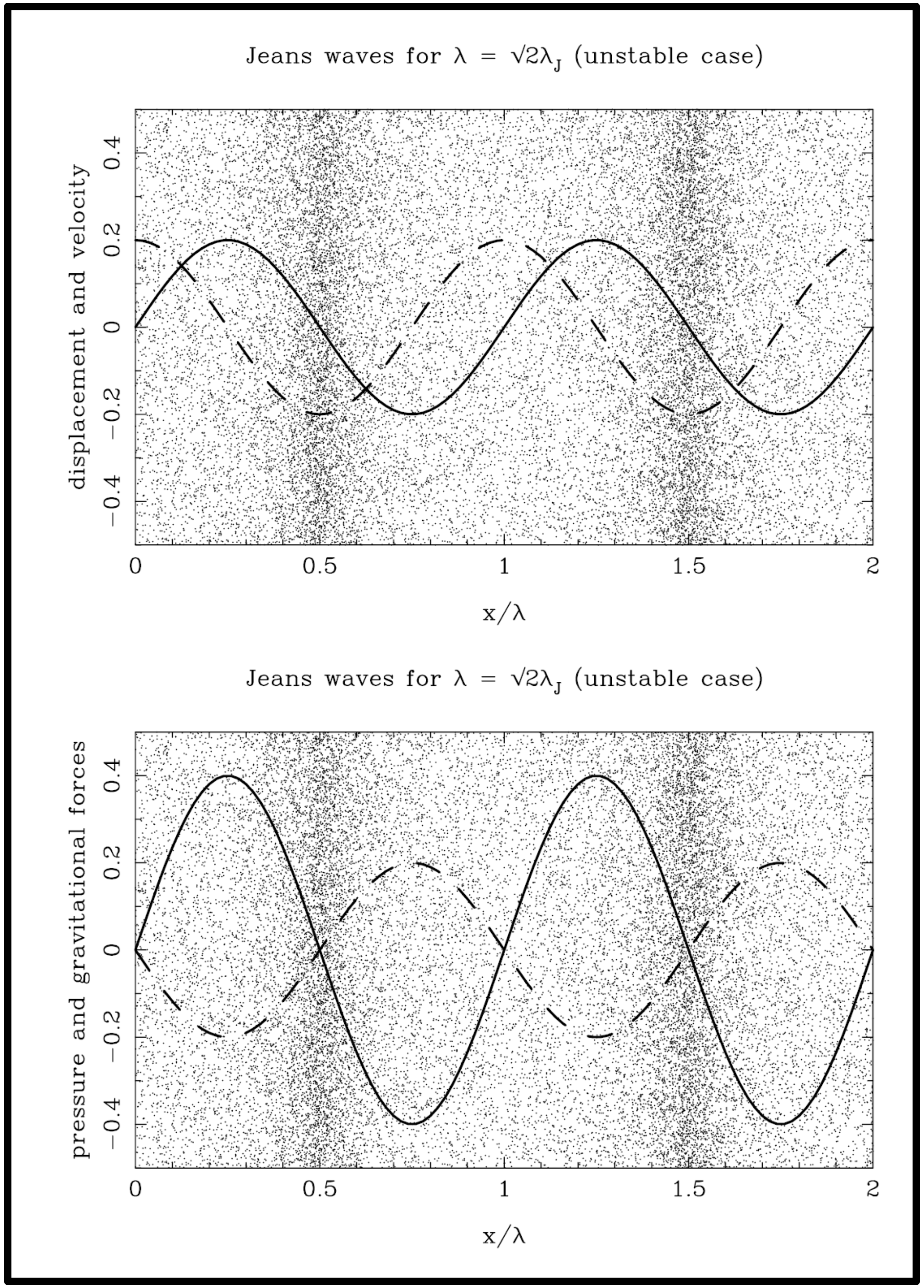
Doppler Effect



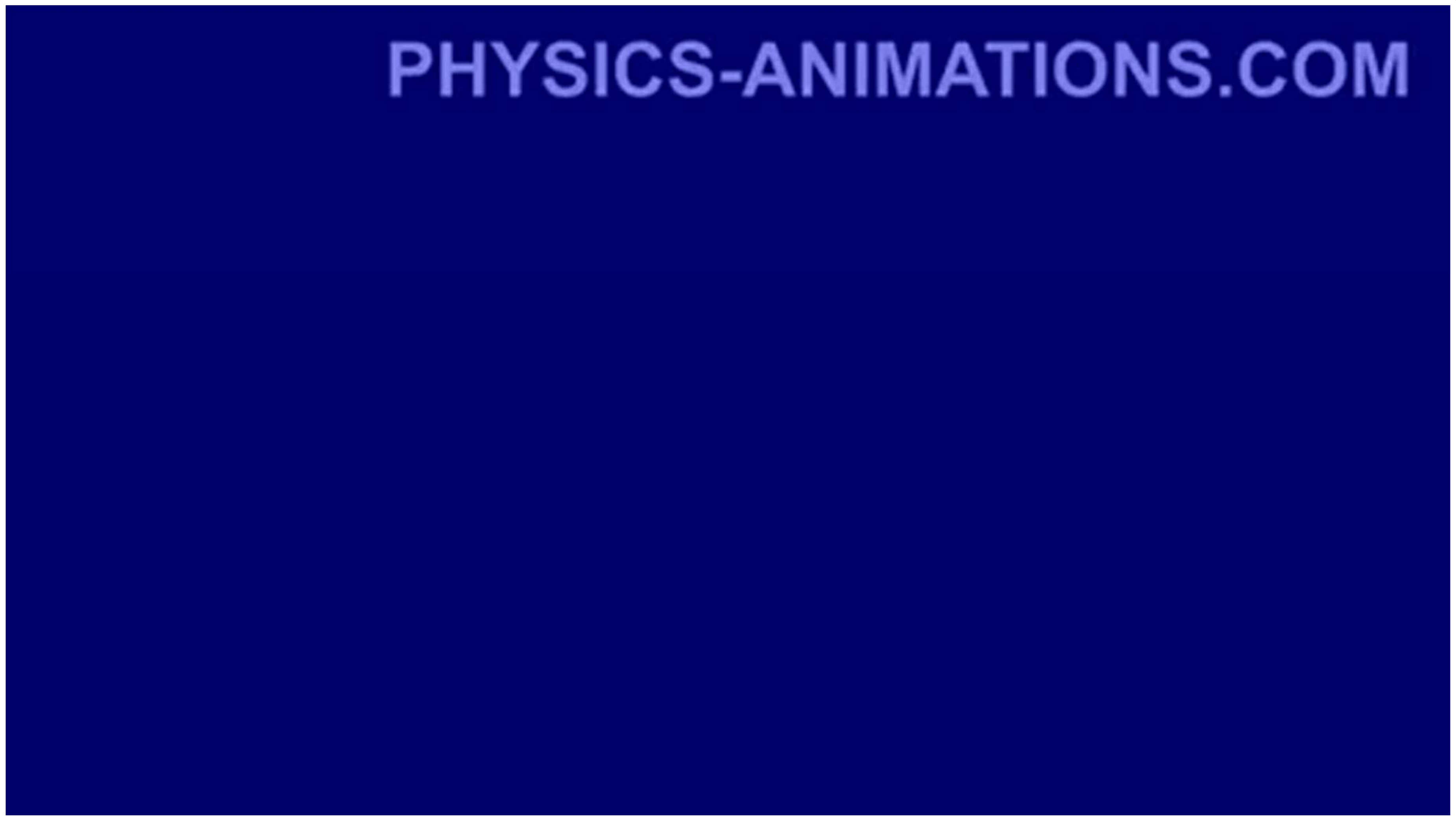


Jeans Instability


Jeans waves for $\lambda = \lambda_J/\sqrt{2}$ (stable case)
displacement and velocity
−0.4
−0.2
0
0.2
0.4
0
0.5
1
1.5
2
x/λ
Jeans waves for $\lambda = \lambda_J/\sqrt{2}$ (stable case)
pressure and gravitational forces
−0.4
−0.2
0
0.2
0.4
0
0.5
1
1.5
2
x/λ


Jeans waves for $\lambda = \sqrt{2}\lambda_J$ (unstable case)
displacement and velocity
−0.4
−0.2
0
0.2
0.4
0
0.5
1
1.5
2
x/λ
Jeans waves for $\lambda = \sqrt{2}\lambda_J$ (unstable case)
pressure and gravitational forces
−0.4
−0.2
0
0.2
0.4
0
0.5
1
1.5
2
x/λ

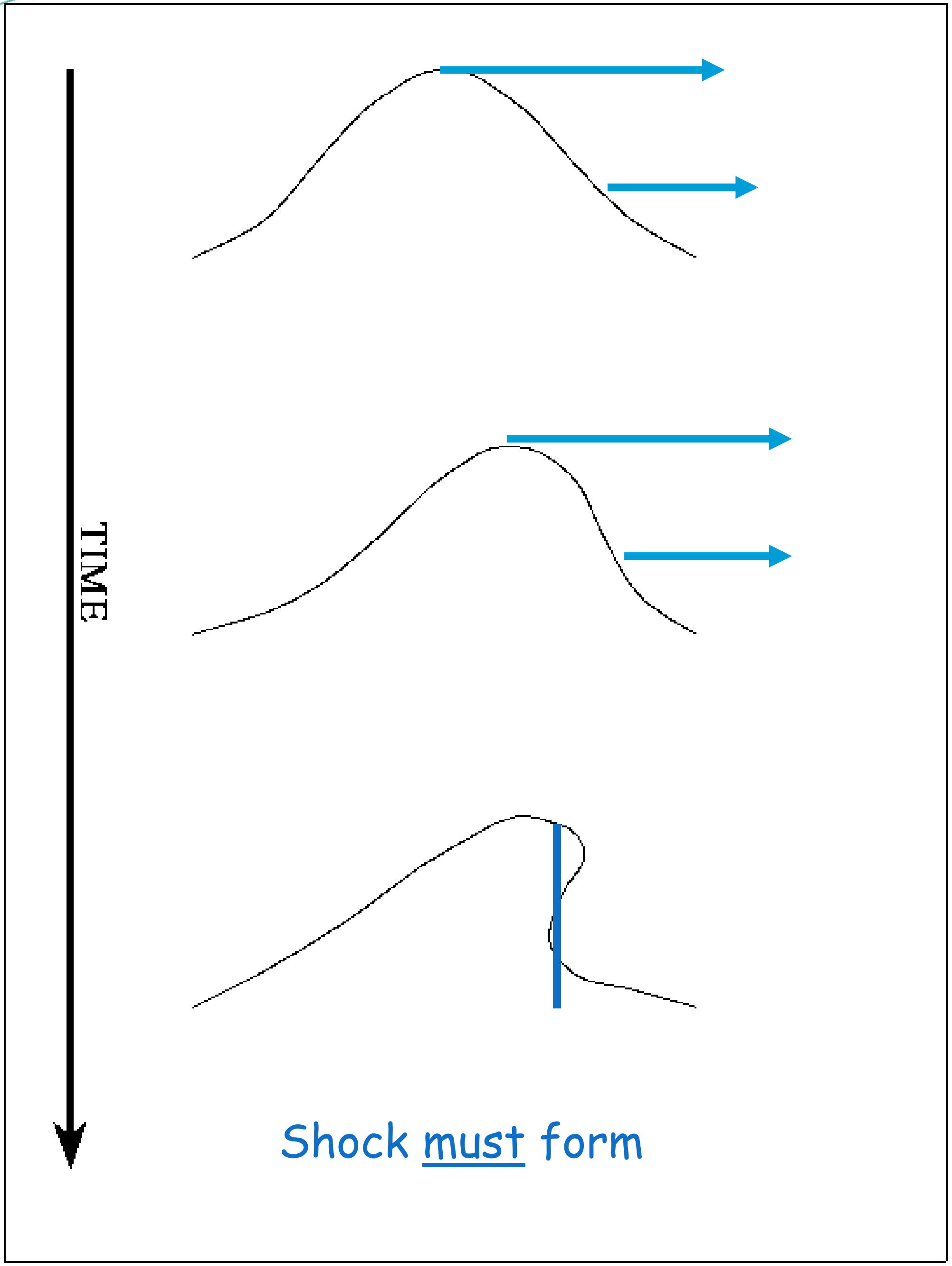
Shock Waves


PHYSICS-ANIMATIONS.COM

Shocks

1. Shocks are sudden transitions in flow properties such as density, velocity and pressure;
2. In shocks the kinetic energy of the flow is converted into heat, (pressure);
3. Shocks are inevitable if sound waves propagate over long distances;
4. Shocks always occur when a flow hits an obstacle supersonically
5. In shocks, the flow speed along the shock normal changes from supersonic to subsonic

Wave Breaking



Shock must form

High-pressure/density
regions move faster

$$u = \frac{2c_{s0}}{\gamma - 1}\left[\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right]$$

$$\approx c_{s0}\left(\frac{\Delta\rho}{\rho}\right)$$

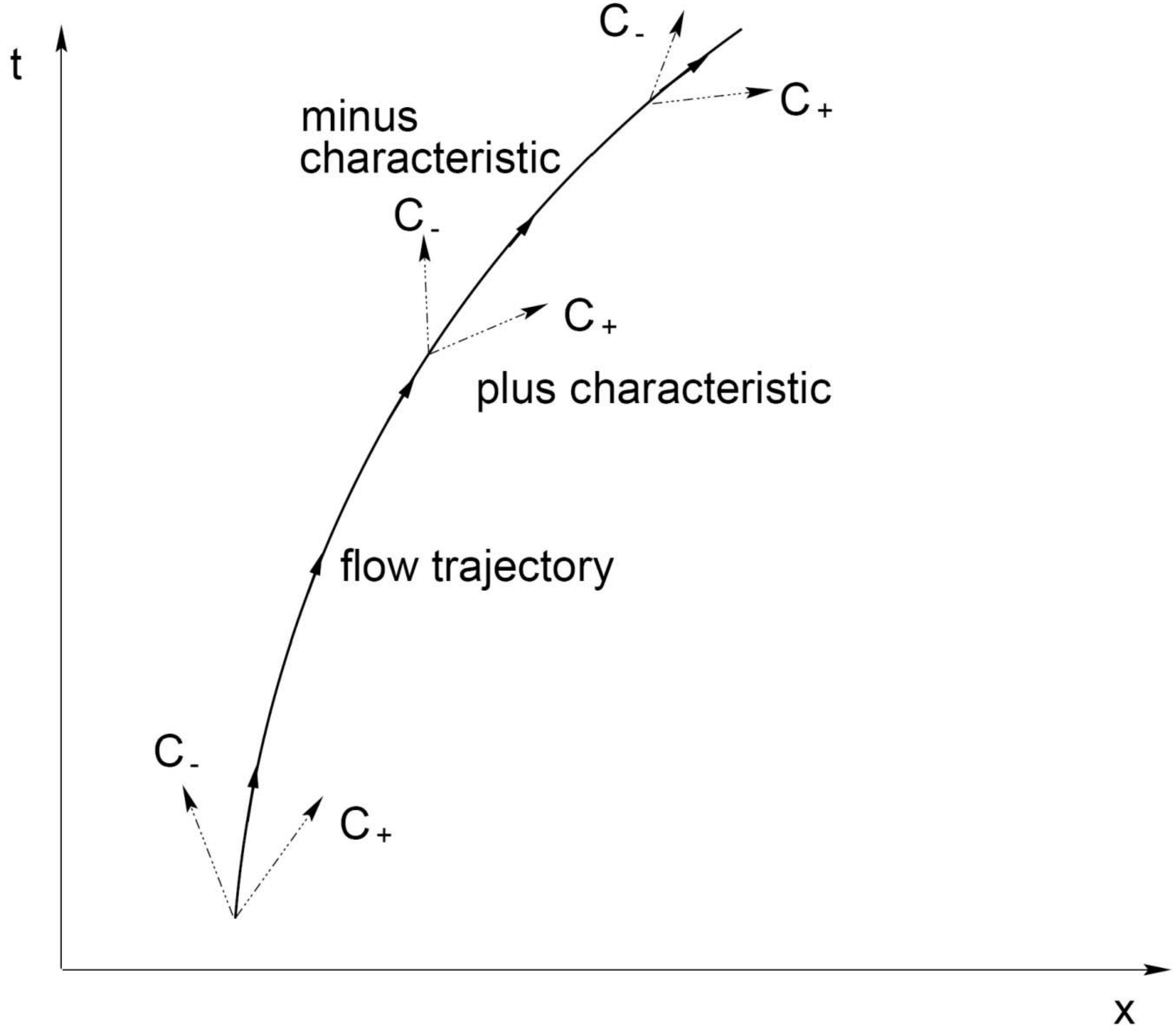
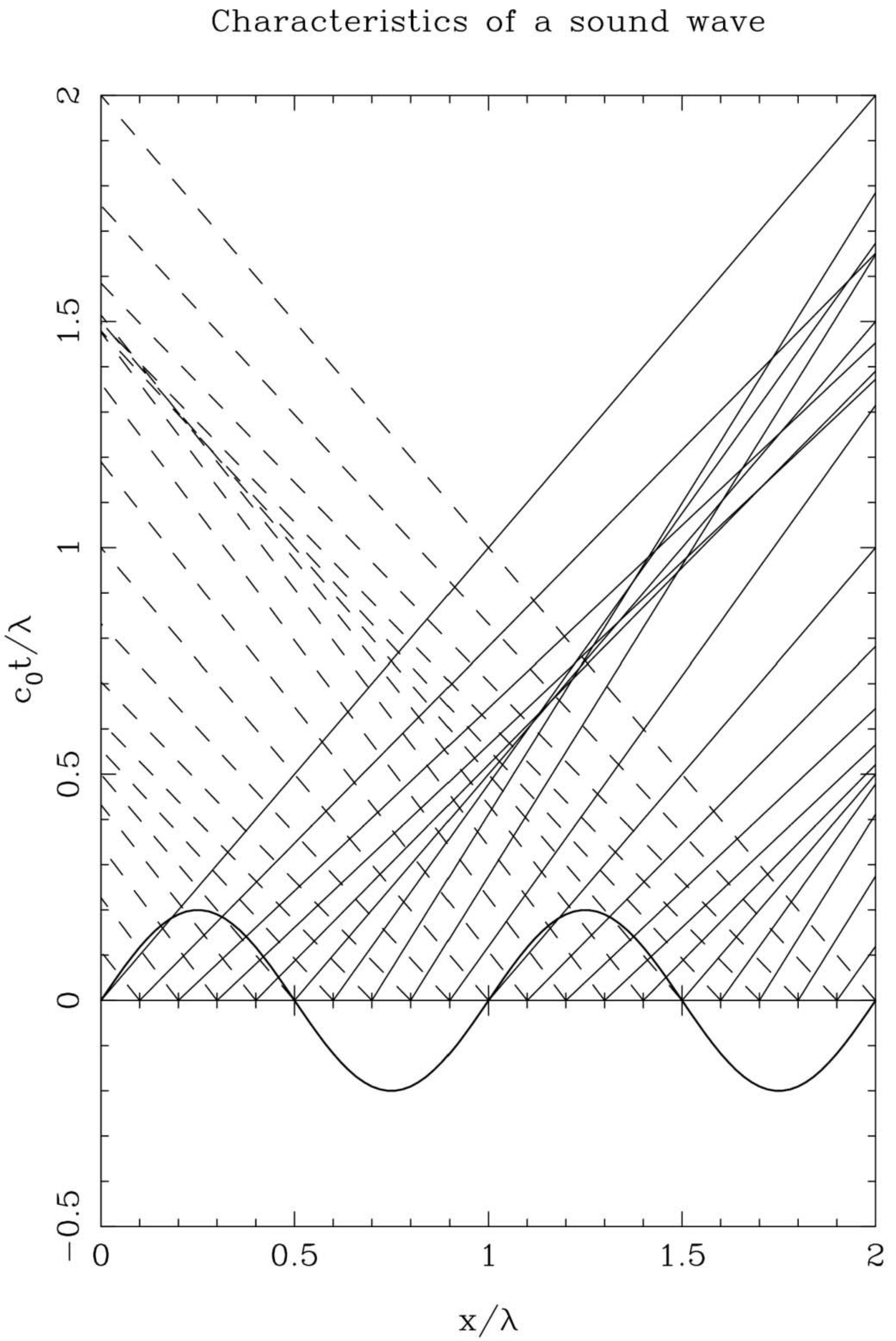
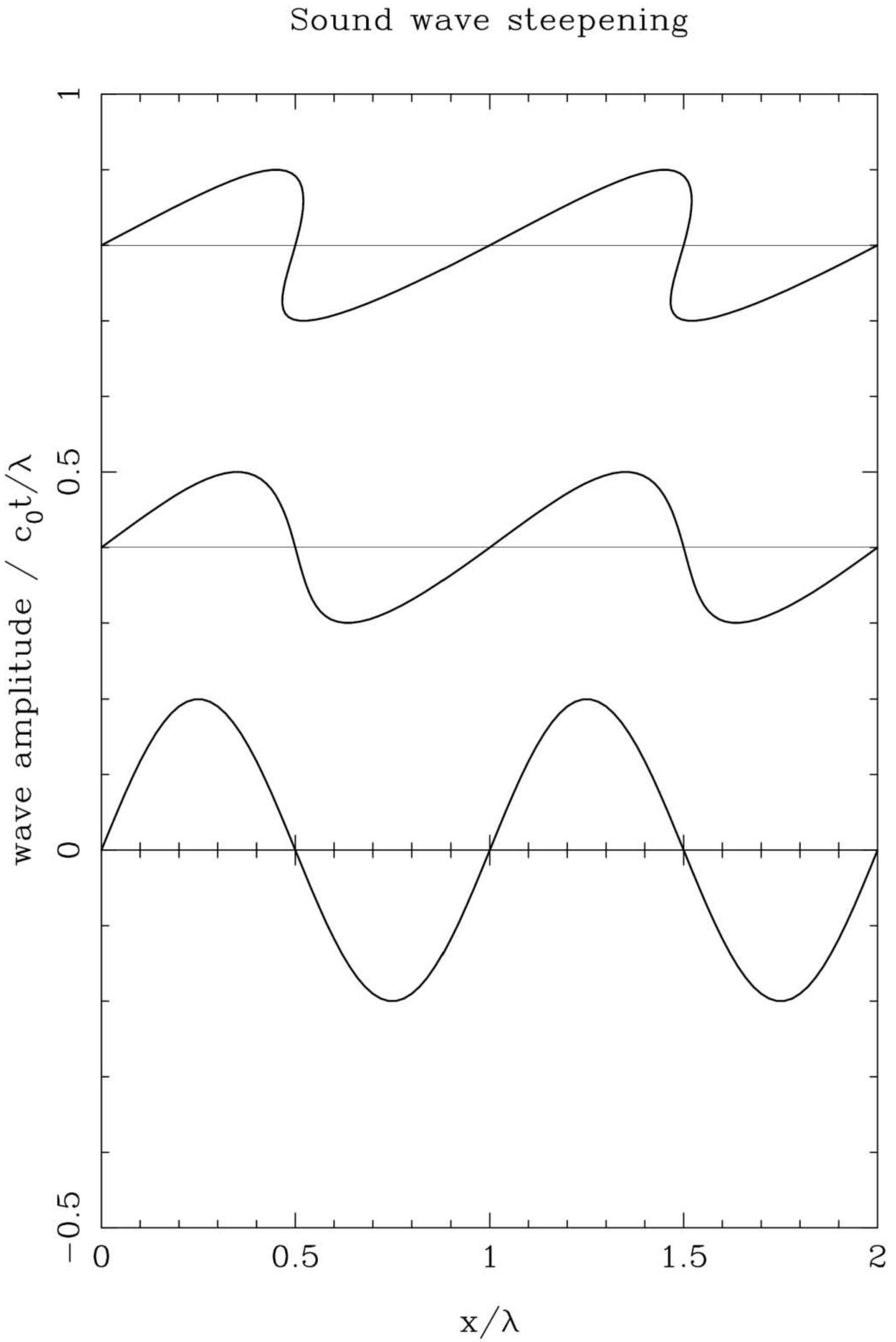
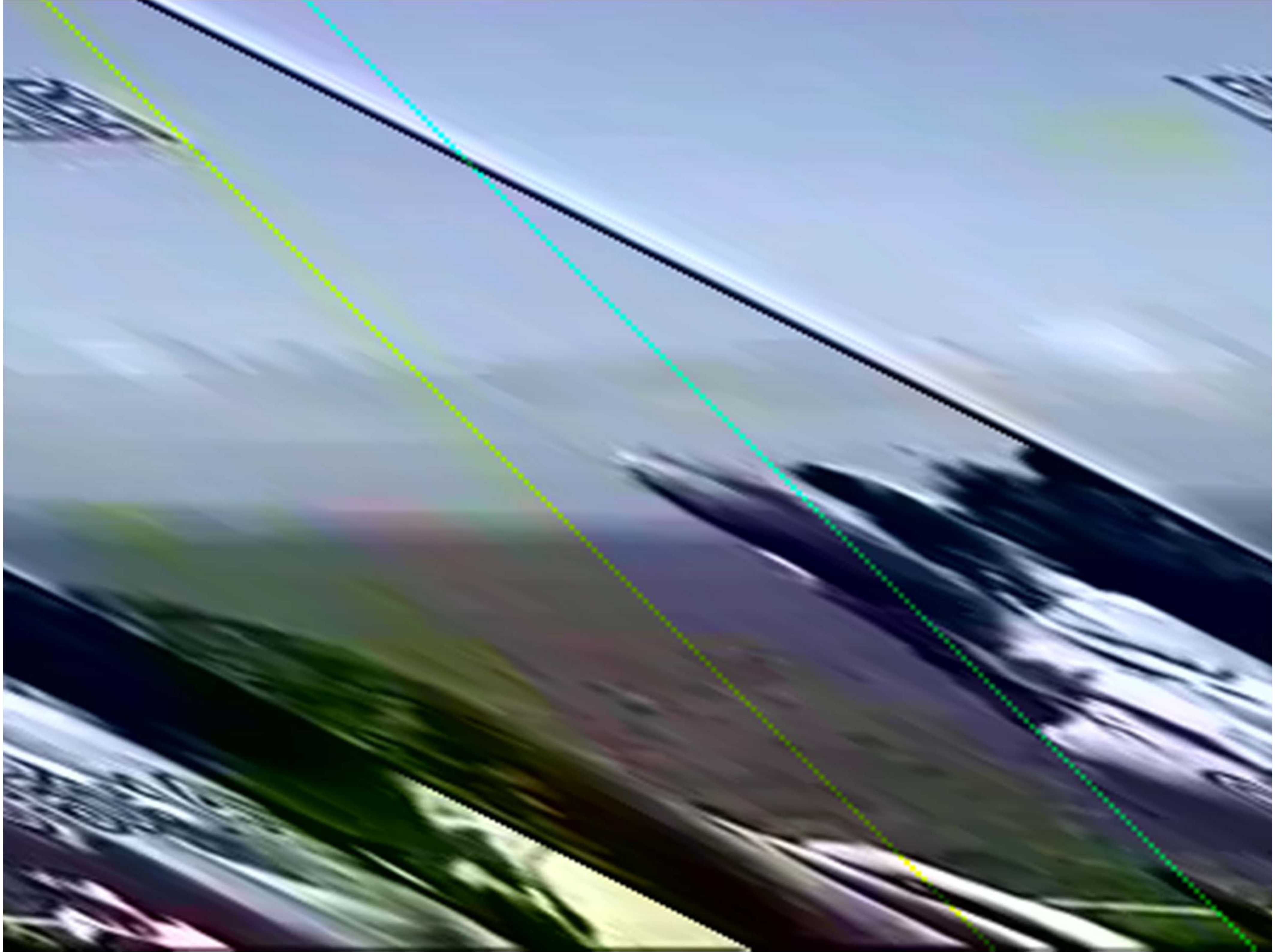


Figure 7.1: *Diagram showing the space-time flow line, defined by* $\mathrm{d}x = u\,\mathrm{d}t$*, and the two characteristics* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *defined by* $\mathrm{d}x = (u + c_\mathrm{s})\,\mathrm{d}t$ *and* $\mathrm{d}x = (u - c_\mathrm{s})\,\mathrm{d}t$*. From each point in the flow two characteristics originate along which* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are constant respectively. Note that the value of* $\mathcal{C}_\pm$ *can be different on the different characteristics so that the characteristic variables* $\mathcal{C}_+$ *and* $\mathcal{C}_-$ *are* **not** *global constants!*

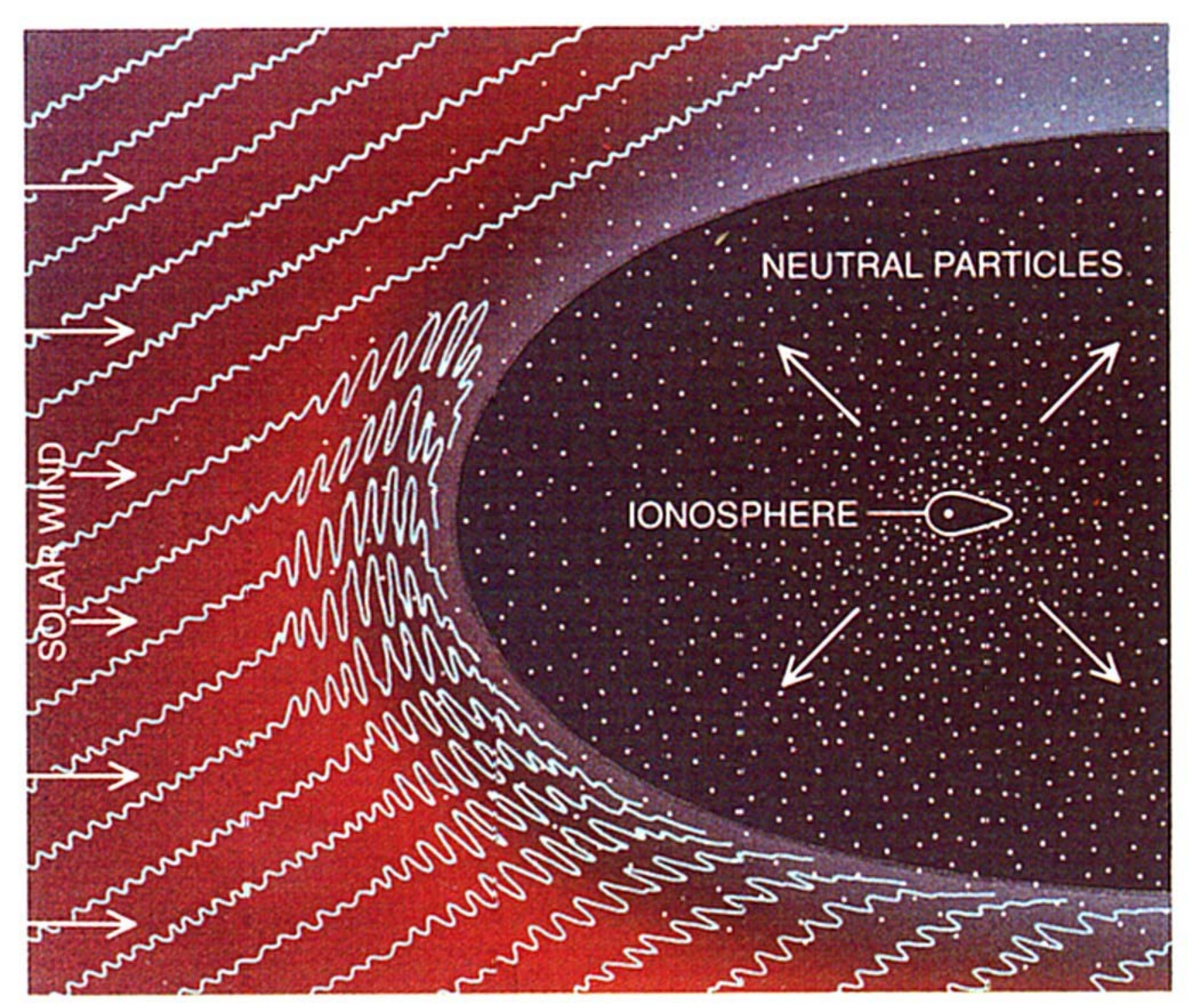

Characteristics of a sound wave


Sound wave steepening

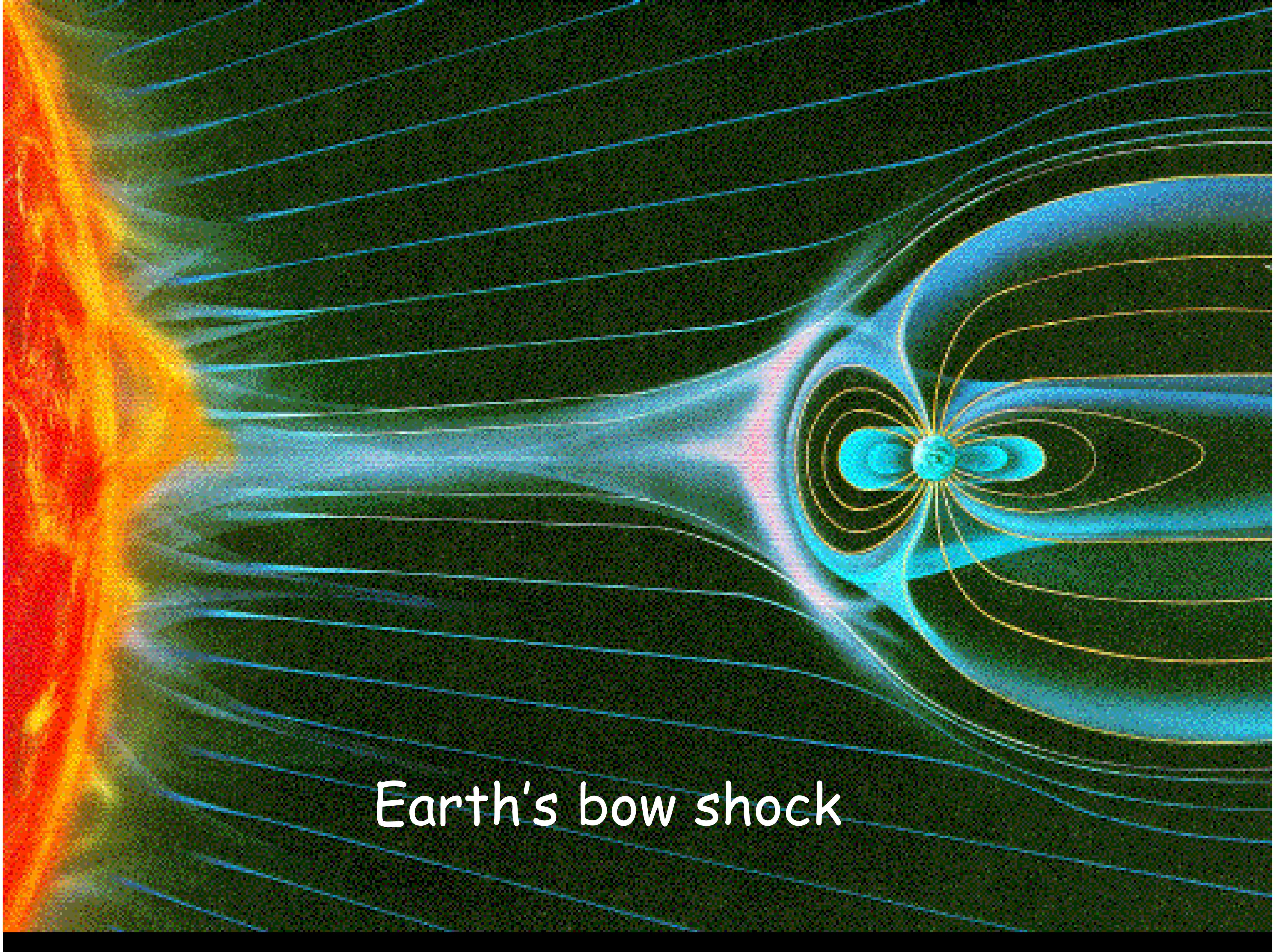


Chelyabinsk Meteorite (Feb. 2013): Sonic Boom

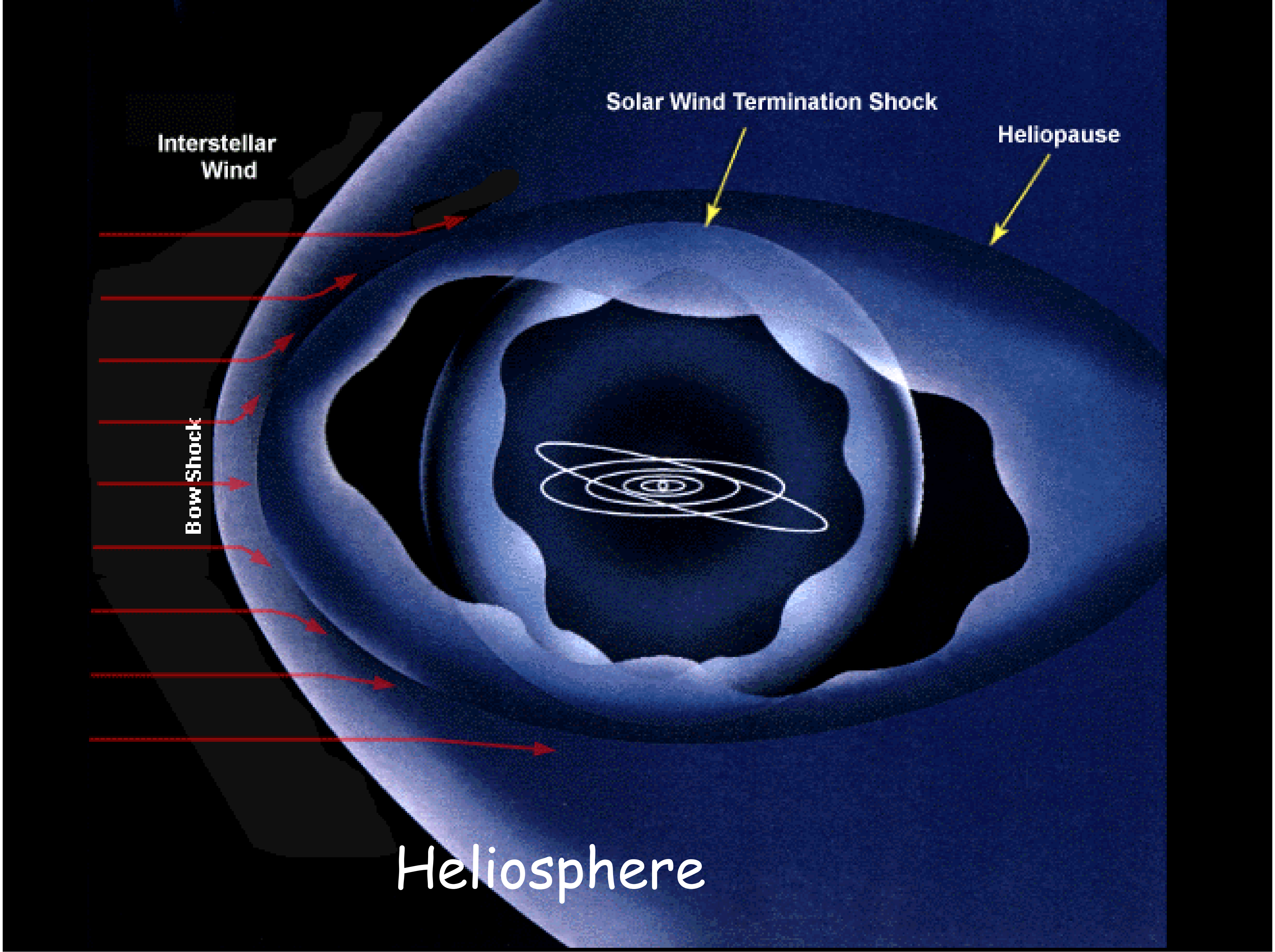
Examples of Astrophysical shocks



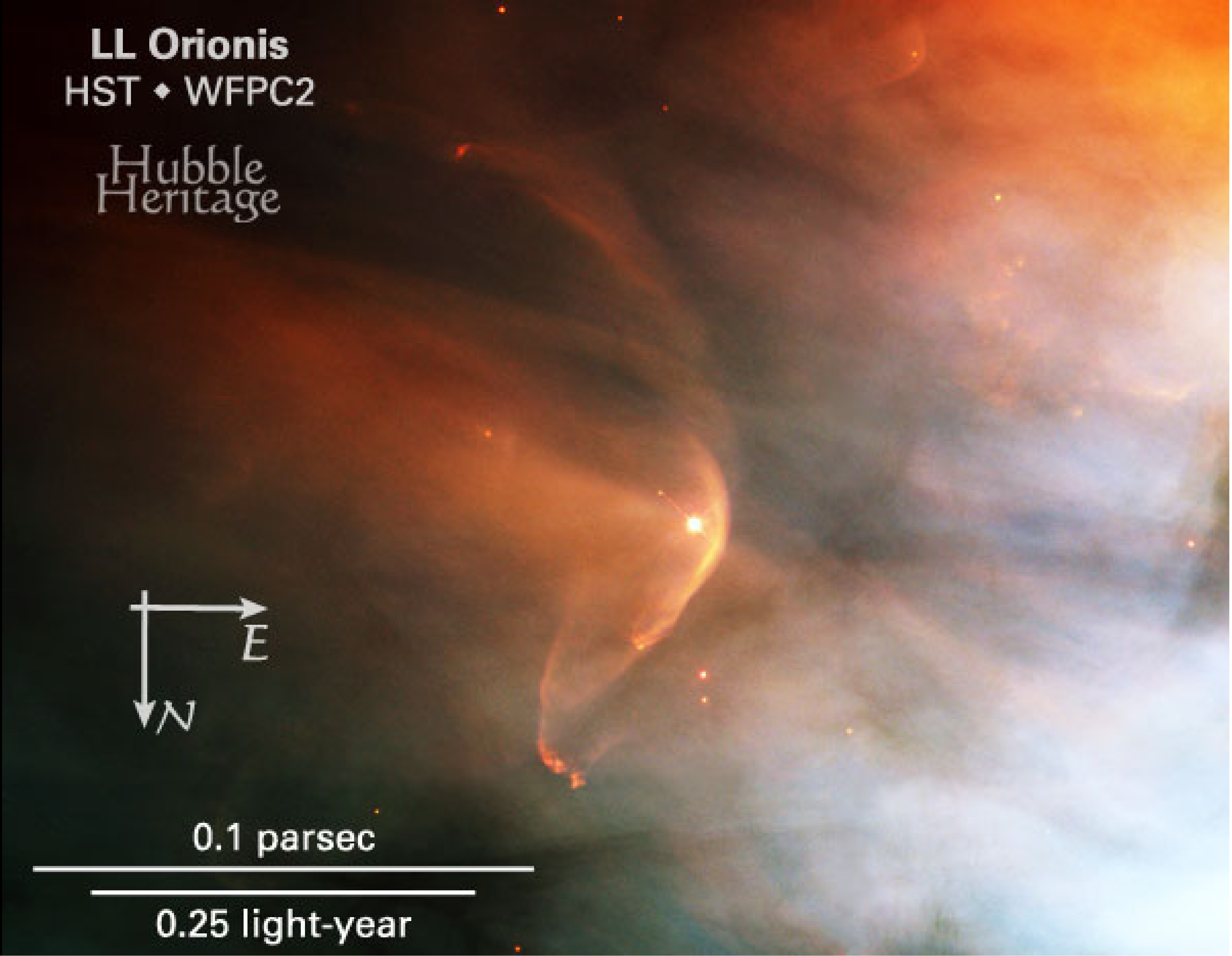
Cometary bow-shocks



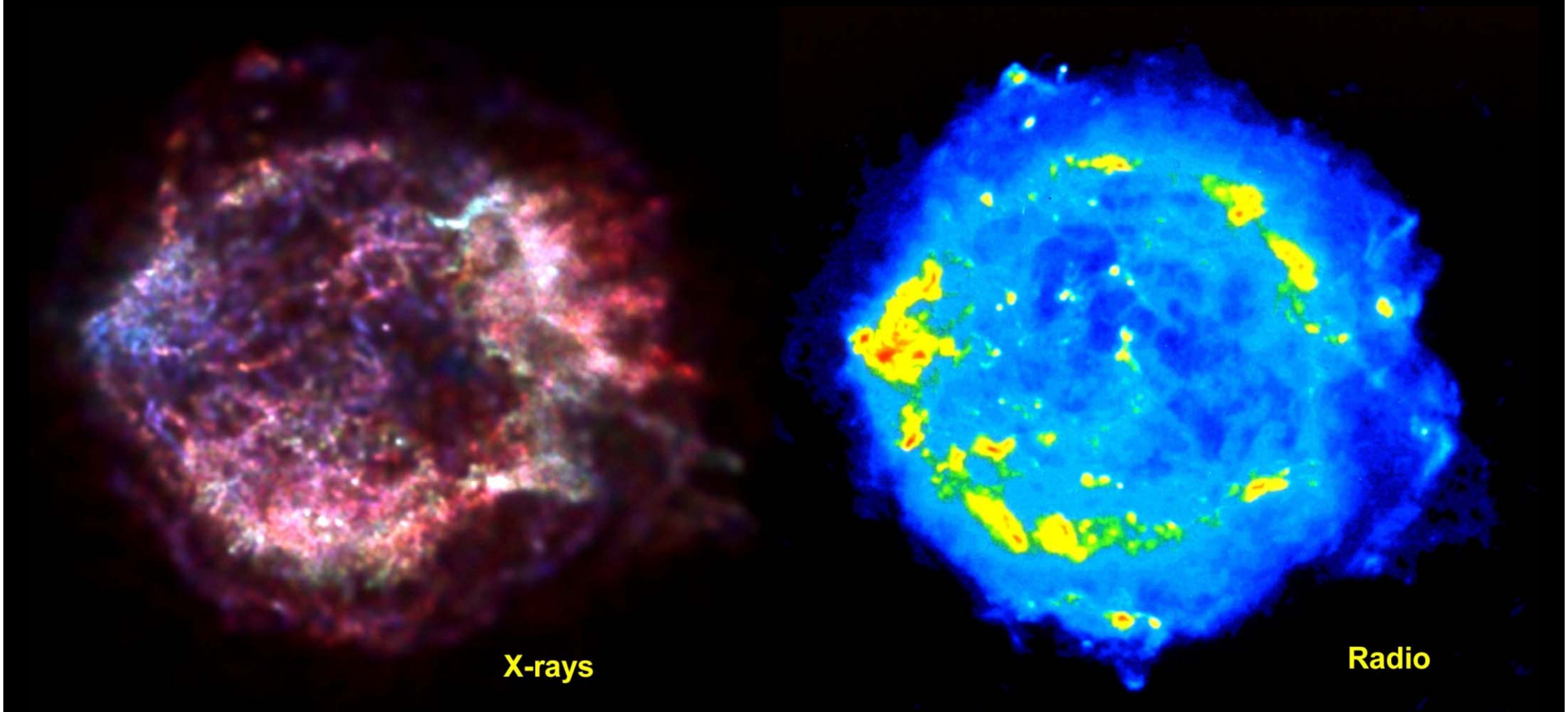
Earth’s bow shock



Heliosphere

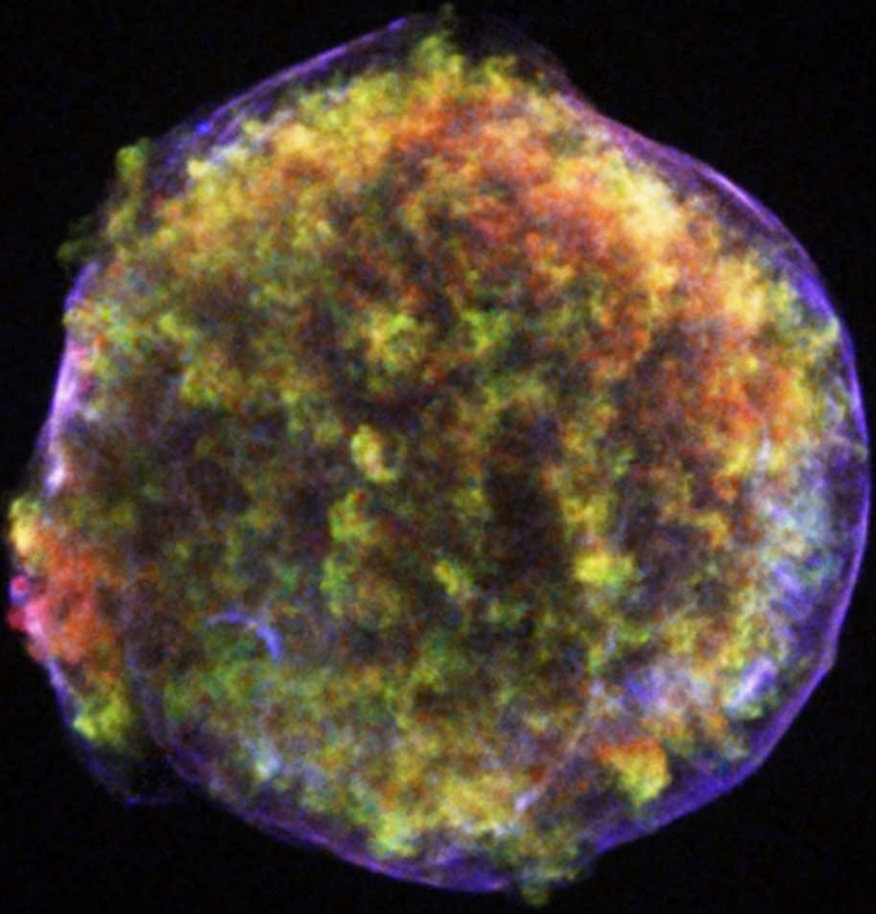

LL Orionis
HST ◆ WFPC2
Hubble Heritage
E
N
0.1 parsec
0.25 light-year

Supernova Remnant Cassiopeia A

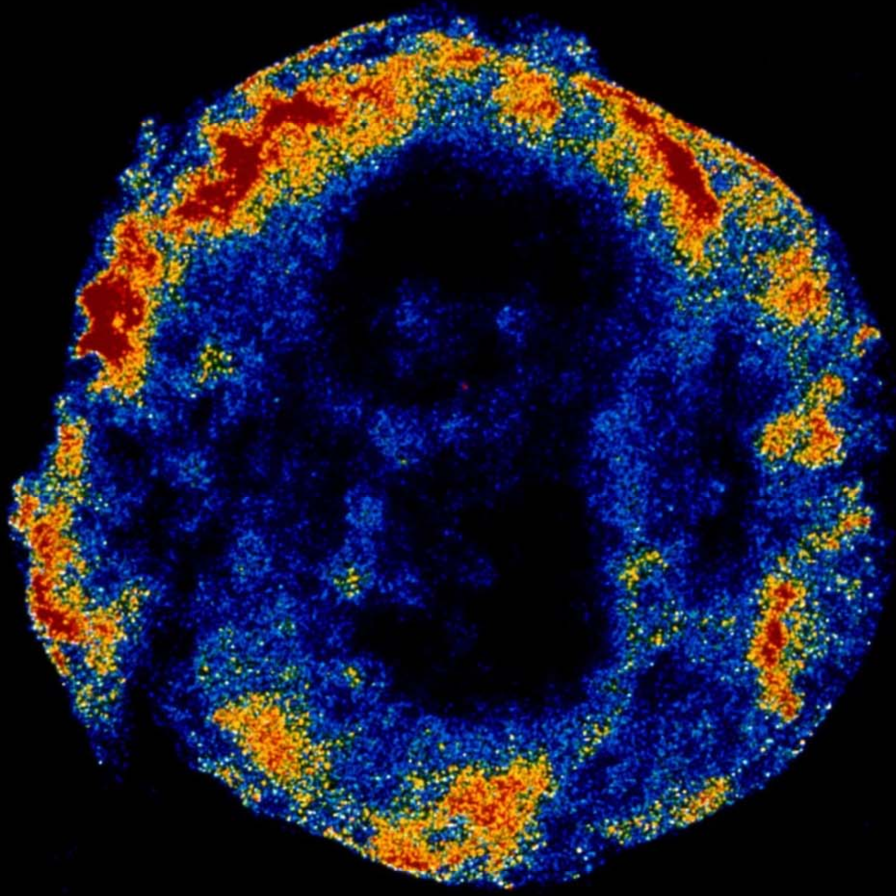


Supernova blast waves

Tycho's Remnant (SN 1572AD)

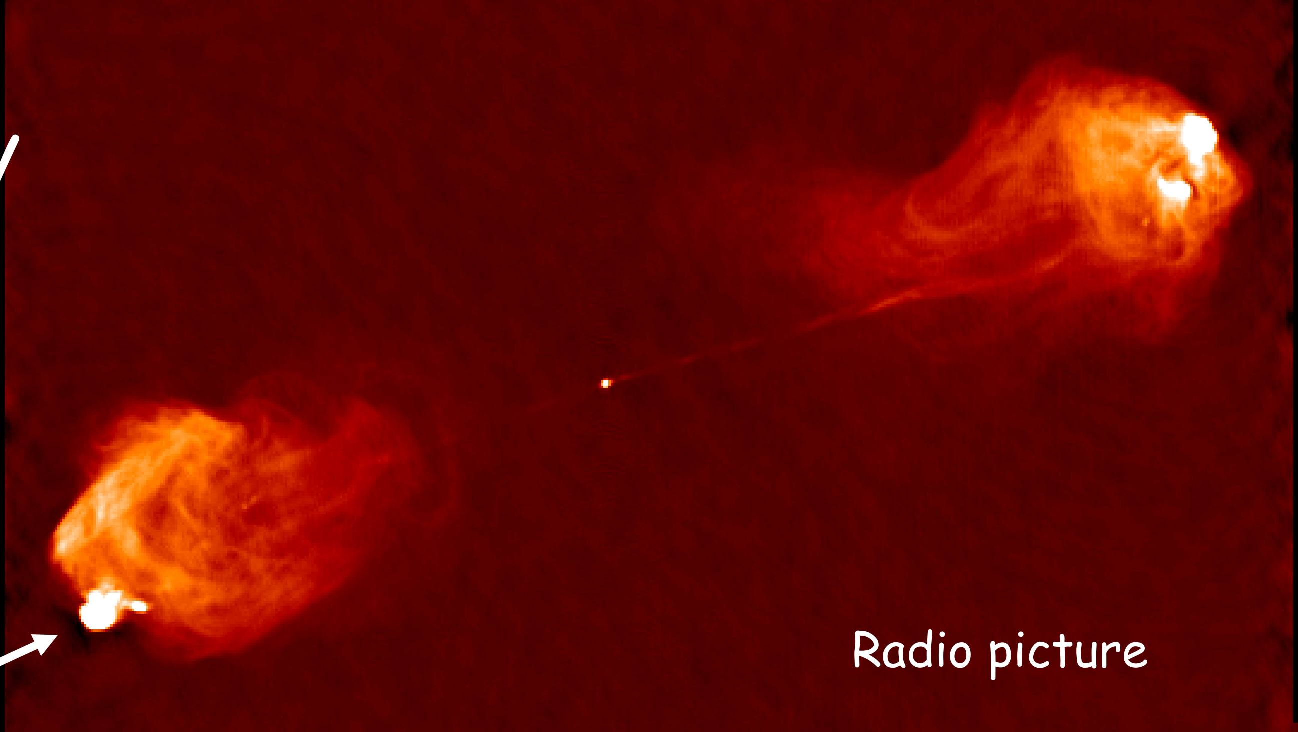


X-Rays (CHANDRA Observatory)

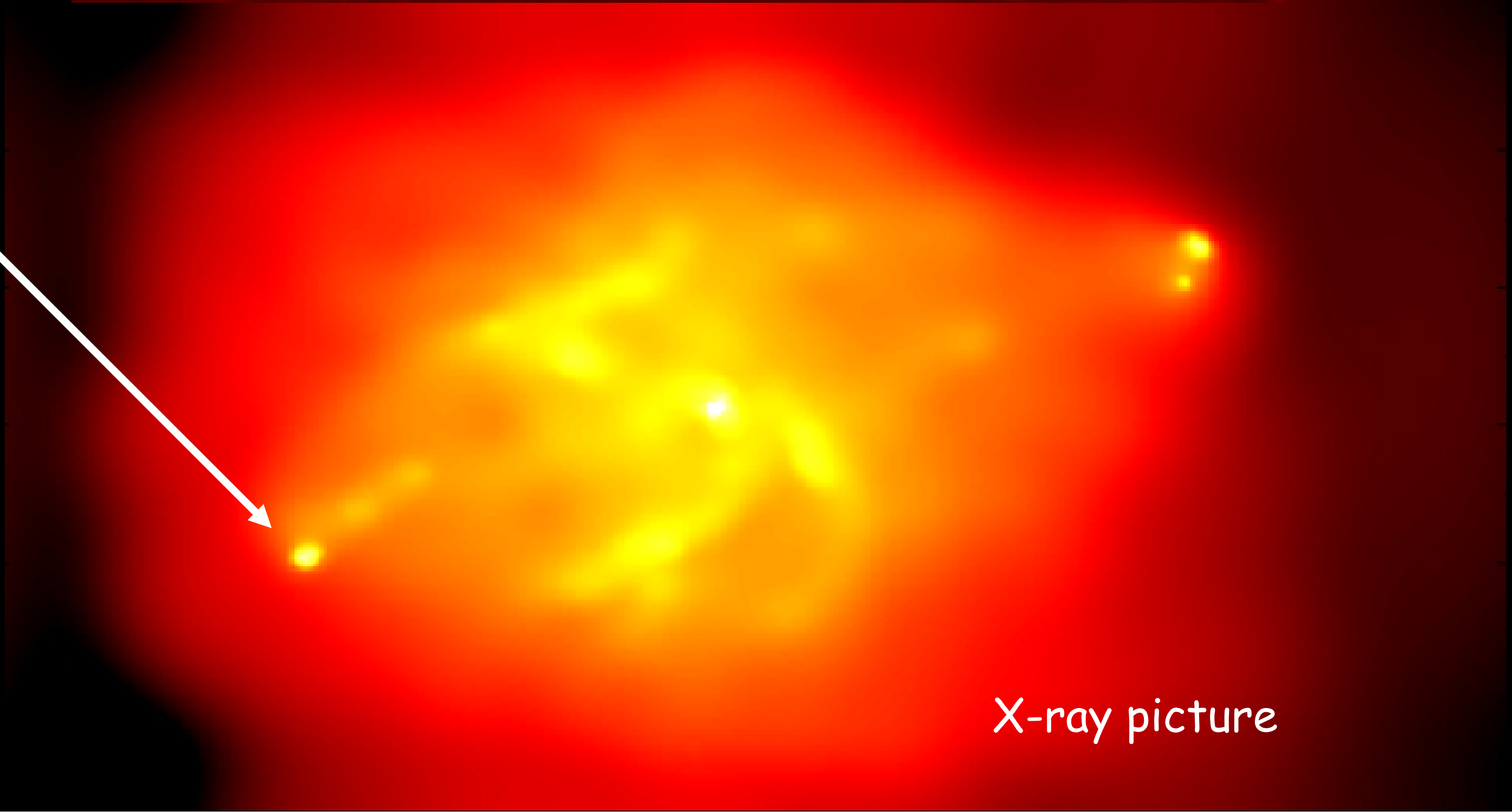


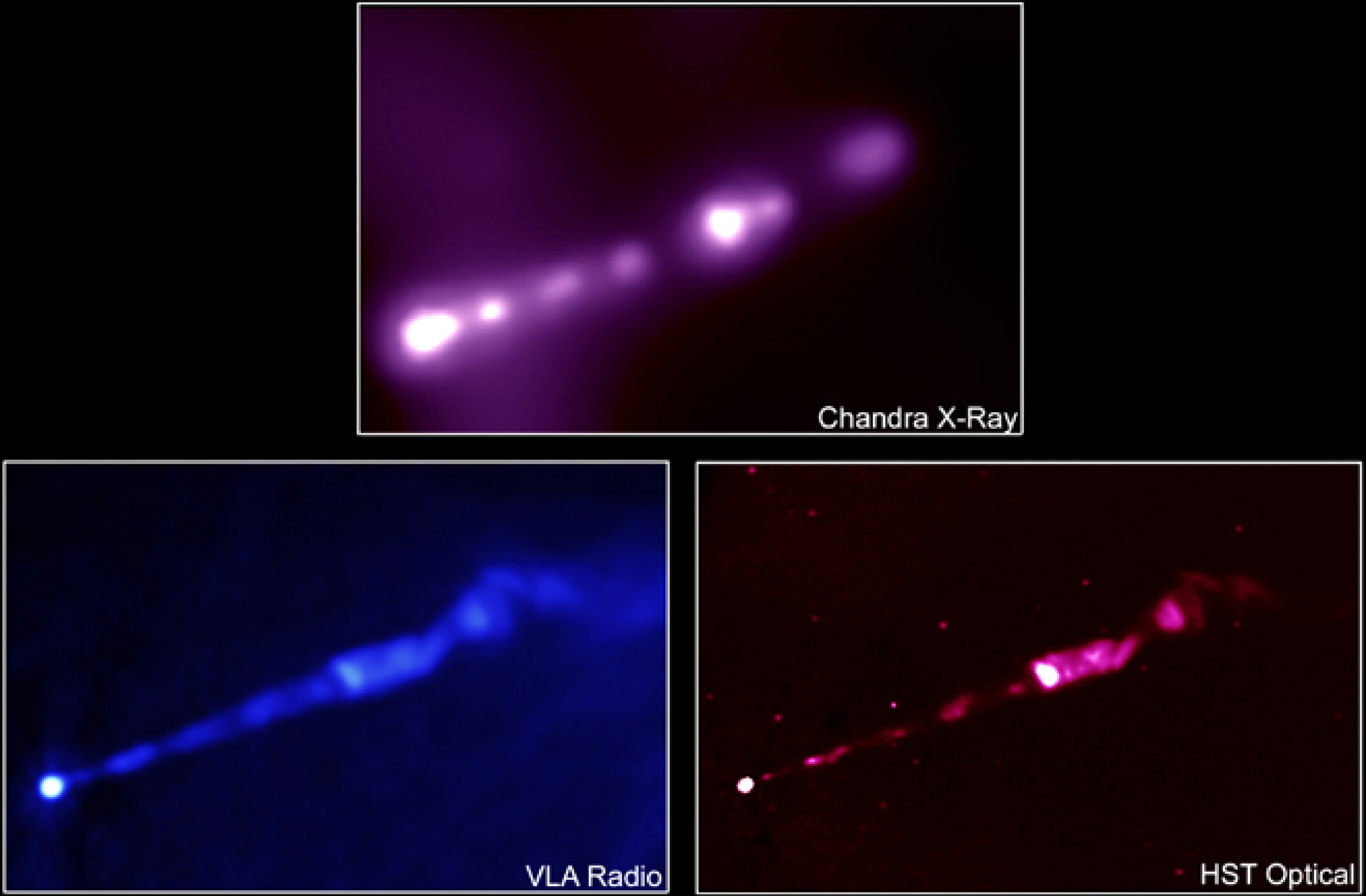
Radio (21cm)

Radio galaxy Cygnus A

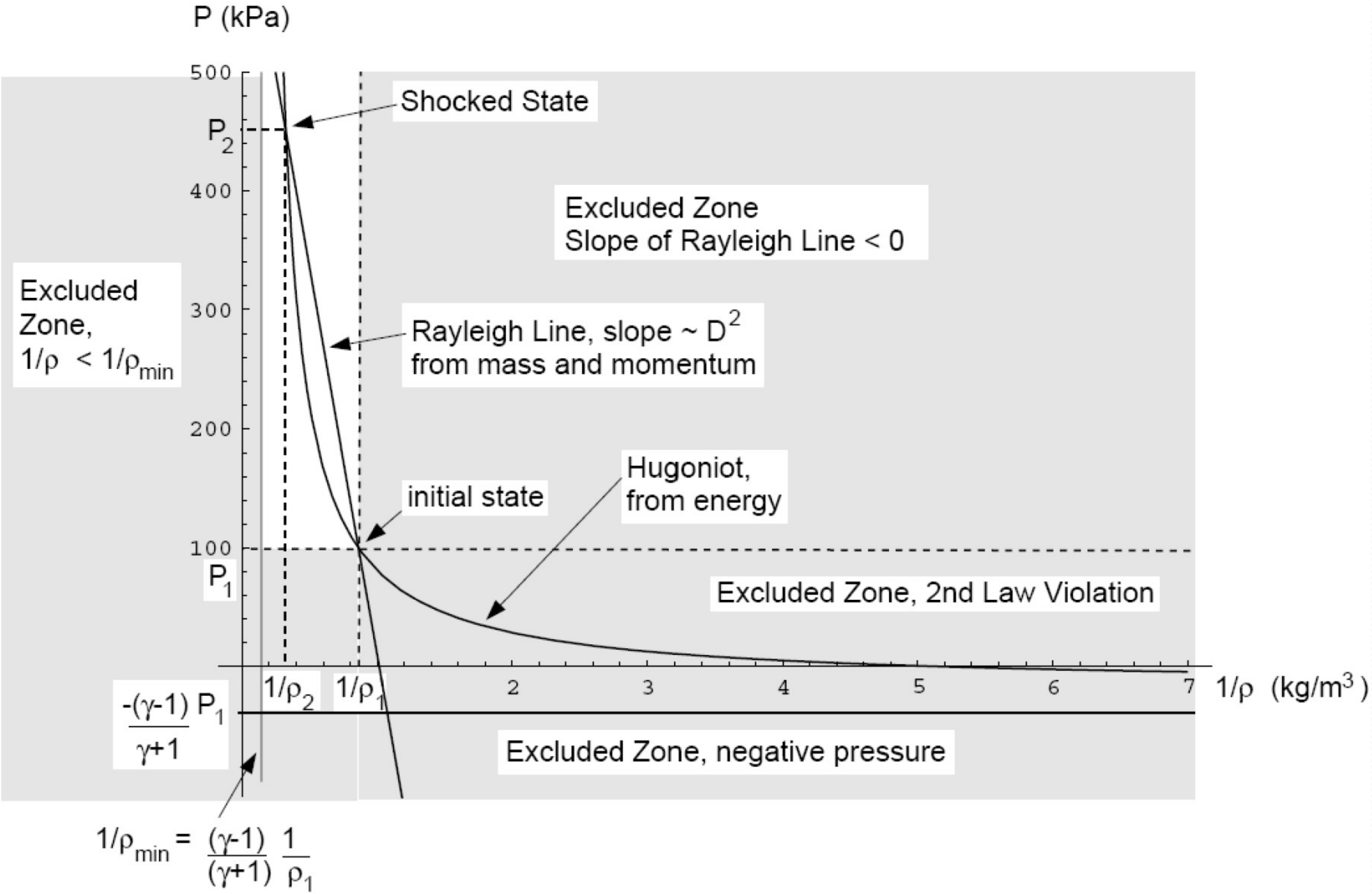

Radio picture

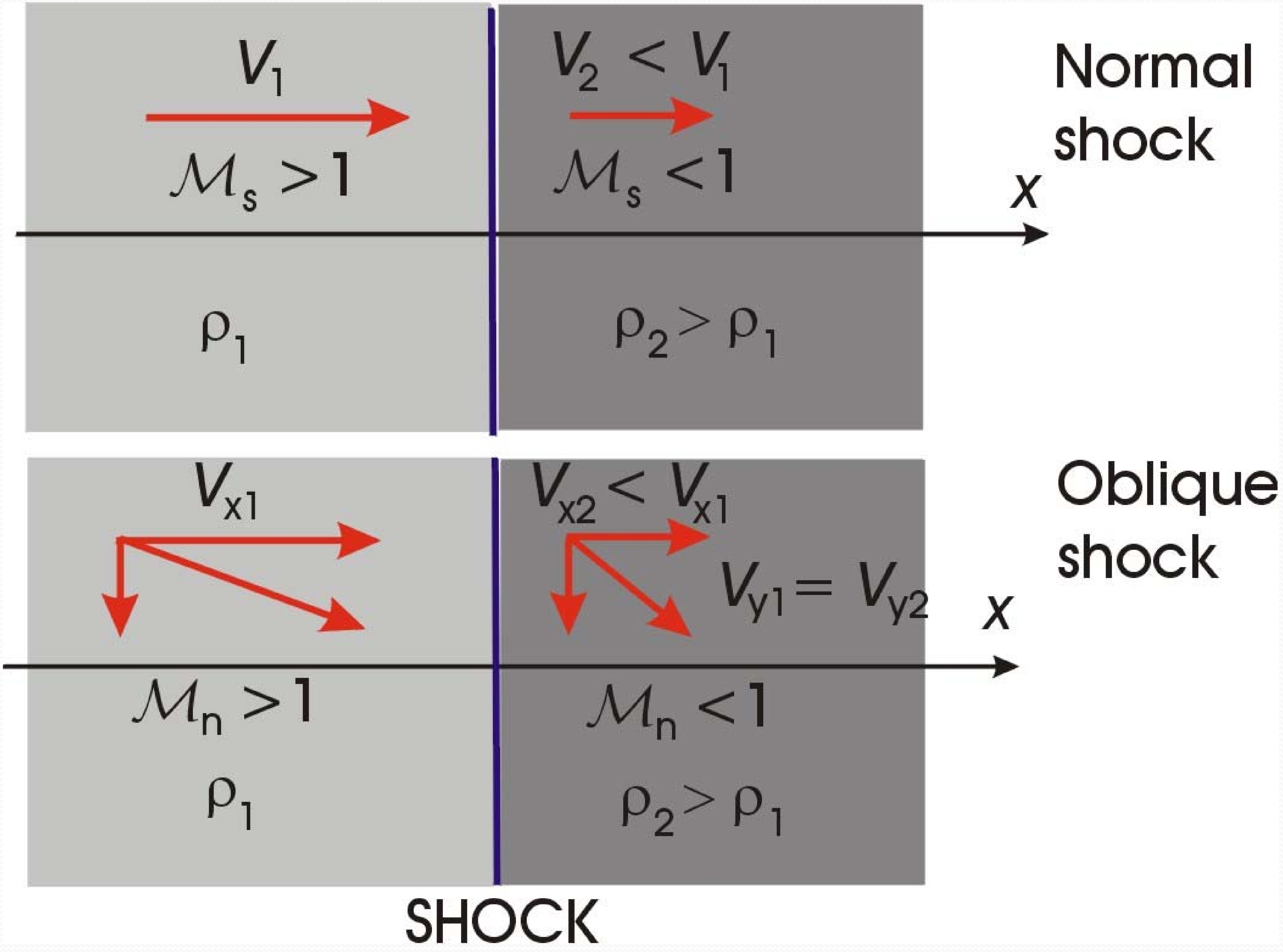
Hot spots are shocks!


X-ray picture



'Knots' in jet of Galaxy M87 are shocks!


P (kPa)
500
400
300
200
100
P_2
P_1
Shocked State
Excluded Zone
Slope of Rayleigh Line < 0
Excluded Zone, 1/ρ < 1/ρ_min
Rayleigh Line, slope ~ D^2 from mass and momentum
Hugoniot, from energy
initial state
Excluded Zone, 2nd Law Violation
1/ρ_2
1/ρ_1
2
3
4
5
6
7
1/ρ (kg/m^3)
-(γ-1) P_1 / γ+1
Excluded Zone, negative pressure
1/ρ_min = (γ-1)/(γ+1) 1/ρ_1


V_1
$V_2 < V_1$
Normal shock
$\mathcal{M}_s > 1$
$\mathcal{M}_s < 1$
x
ρ_1
$\rho_2 > \rho_1$
V_{x1}
$V_{x2} < V_{x1}$
Oblique shock
$V_{y1} = V_{y2}$
x
$\mathcal{M}_n > 1$
$\mathcal{M}_n < 1$
ρ_1
$\rho_2 > \rho_1$
SHOCK

Summary :
Shock Physics

Across an infinitely thin steady shock you have, in the shock frame where the shock is at rest, the following Rankine-Hugoniot Jump conditions:

Mass-flux conservation

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

Momentum-flux conservation

$$\rho_1 \left(V_{n1}\right)^2 + P_1 = \rho_2 \left(V_{n2}\right)^2 + P_2$$

$$V_{t1} = V_{t2}$$

Energy-flux conservation

$$\tfrac{1}{2}\left(V_{n1}\right)^2 + \frac{\gamma P_1}{(\gamma - 1)\rho_1} = \tfrac{1}{2}\left(V_{n2}\right)^2 + \frac{\gamma P_2}{(\gamma - 1)\rho_2}$$

Summary:
Rankine-Hugoniot relations
(for normal shock)

Fundamental parameter:
Mach Number

$$\mathcal{M}_s \equiv \frac{\text{shock speed}}{\text{sound speed}} = \frac{V_1}{c_{s1}}$$

R-H Jump Conditions
relate the up- and downstream
quantities at the shock:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1}$$

From normal shock to oblique shocks:

All relations remain the same if one makes the replacement:

$$V_1 \Rightarrow V_{\mathrm{n1}} = V_1 \cos\theta_1 \ ,$$

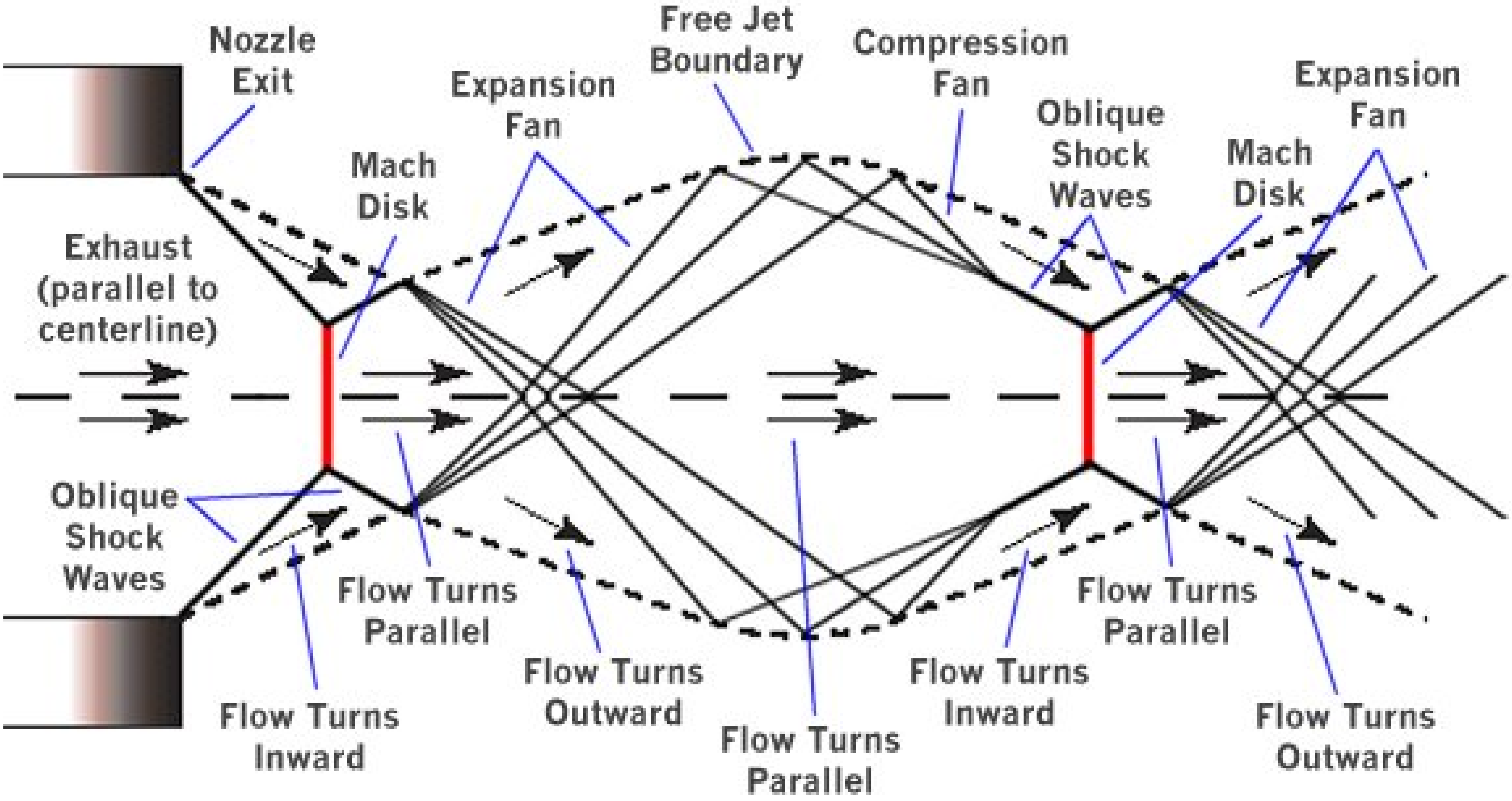
$$\mathcal{M}_S \Rightarrow \mathcal{M}_{\mathrm{n}} = V_{\mathrm{n1}} / c_{s1} = \mathcal{M}_S \cos\theta_1$$

θ is the angle between upstream velocity and normal on shock surface

Tangential velocity along shock surface is <u>unchanged</u>

$$V_{\mathrm{t1}} = V_1 \sin\theta_1 = V_{\mathrm{t2}} = V_2 \sin\theta_2$$

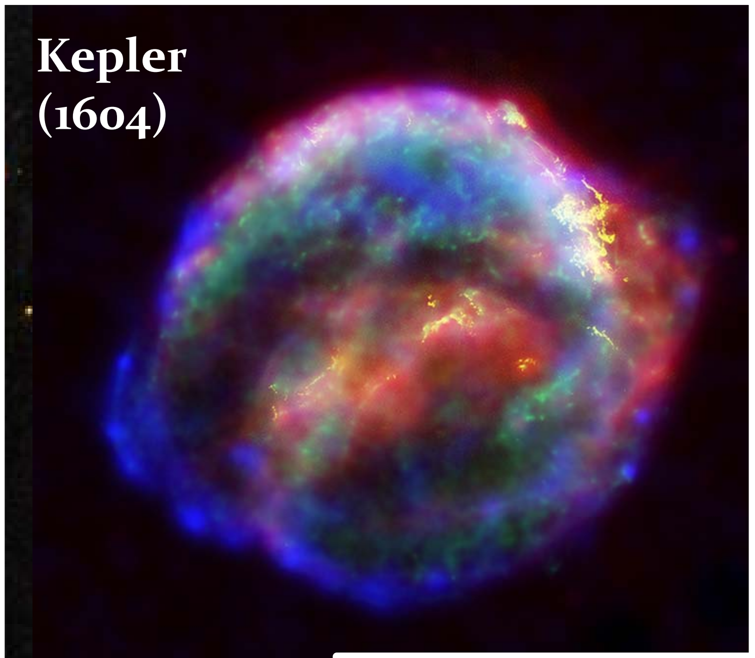
Example from Jet/Rocket engines



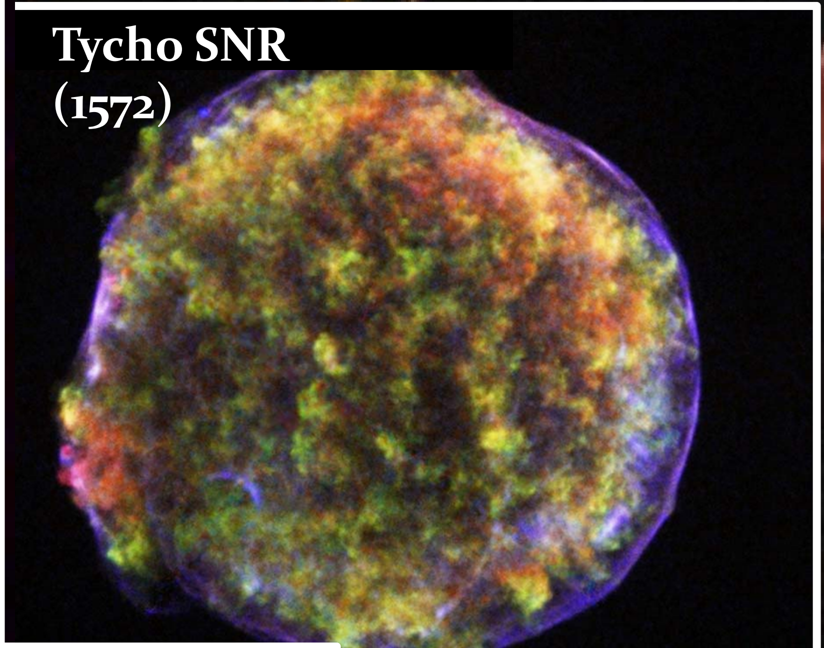

6062

[redacted]

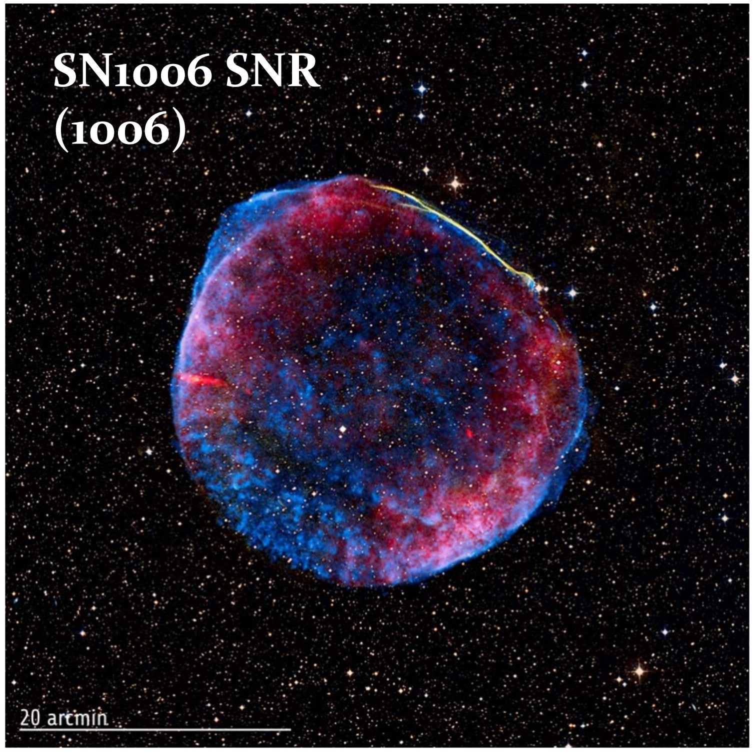
Supernova Remnants



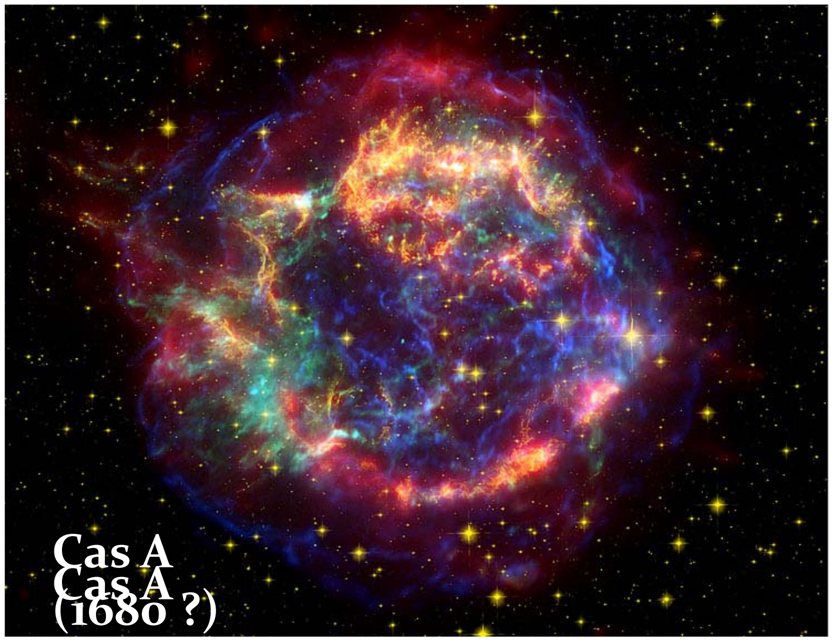
Kepler
(1604)



Tycho SNR
(1572)



SN1006 SNR
(1006)



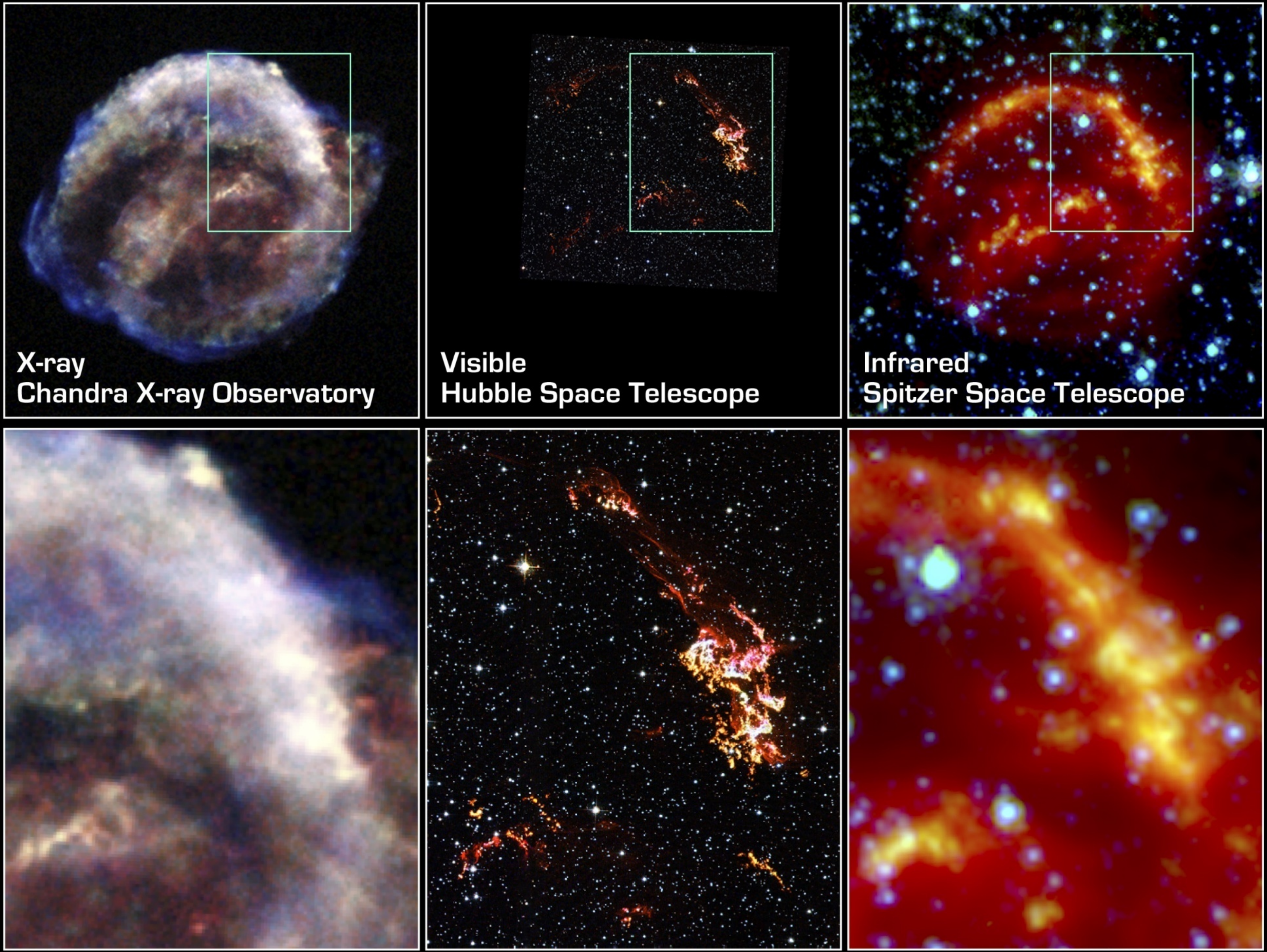
Cas A
Cas A
(1680 ?)

De Stella Nova

JOANNIS KEPPLERI
Sac. Cæſ. Majeſt. Mathematici
DE
STELLA NOVA
IN PEDE SERPENTARII, ET
QUI SUB EJUS EXORTUM DE
NOVO INIIT,
TRIGONO IGNEO.
LIBELLUS ASTRONOMICIS, PHYSICIS, METAphyſicis, Meteorologicis & Aſtrologicis Diſputationibus,
ἐνδόξοις & παραδόξοις plenus.
ACCESSERUNT
I. DE STELLA INCOGNITA CYGNI: Narratio Aſtronomica.
II. DE JESU CHRISTI SERVATORIS VERO Anno Natalitio, conſideratio noviſſimæ ſententiæ LAURENTII SUSLYGÆ Poloni, quatuor annos in uſitata Epocha deſiderantis.
Cum Privilegio S. C. Majeſt. ad annos XV.

GRANA DAT E FIMO SCRUTANS

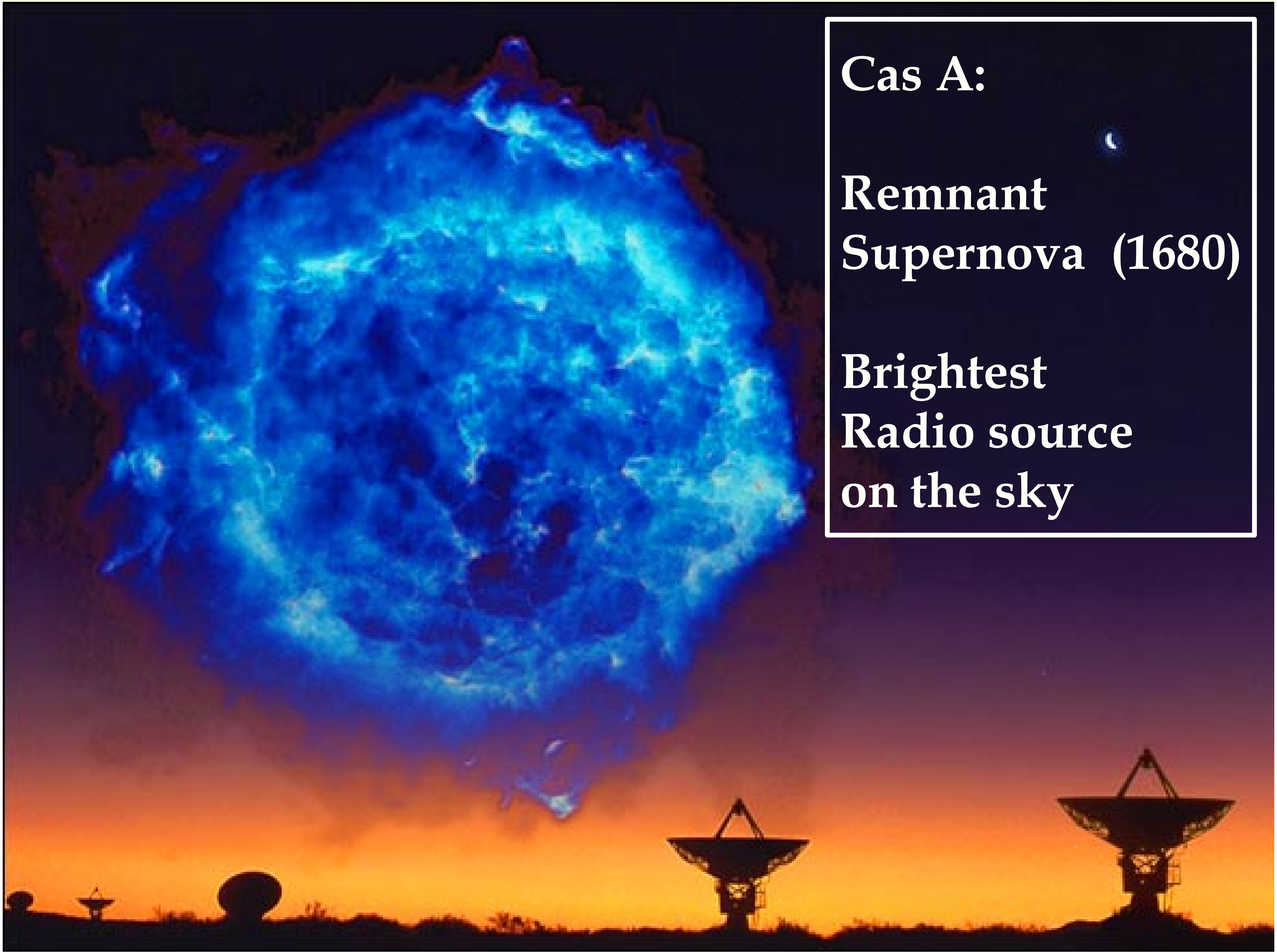
PRAGAE
Ex Officina calcographica PAULI SESSII.
ANNO M. DCVI.



Kepler's Supernova Remnant • SN 1604

ssc2004-15b

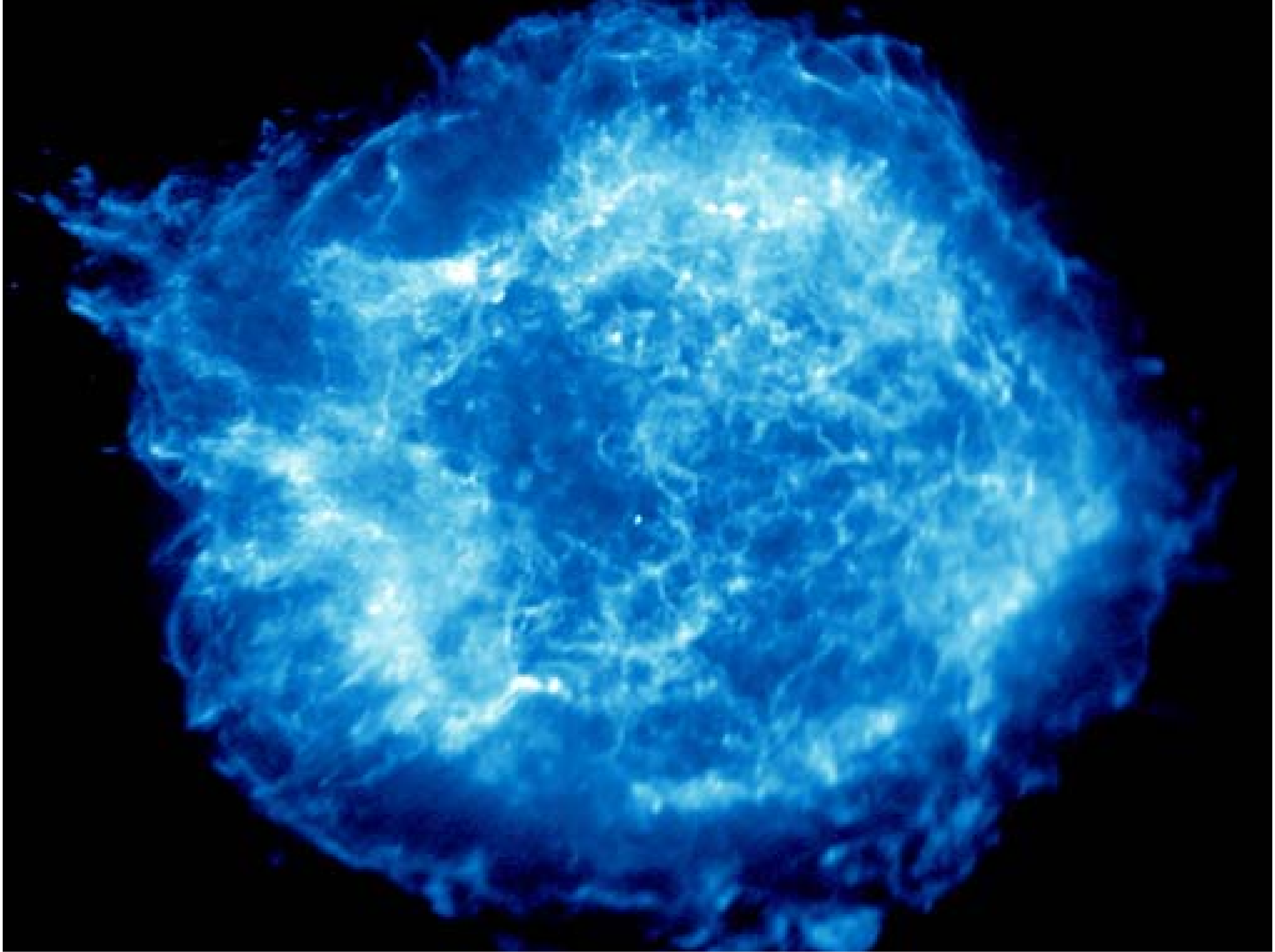
NASA, ESA / JPL-Caltech / R. Sankrit & W. Blair (Johns Hopkins University)



Cas A:

Remnant
Supernova (1680)

Brightest
Radio source
on the sky

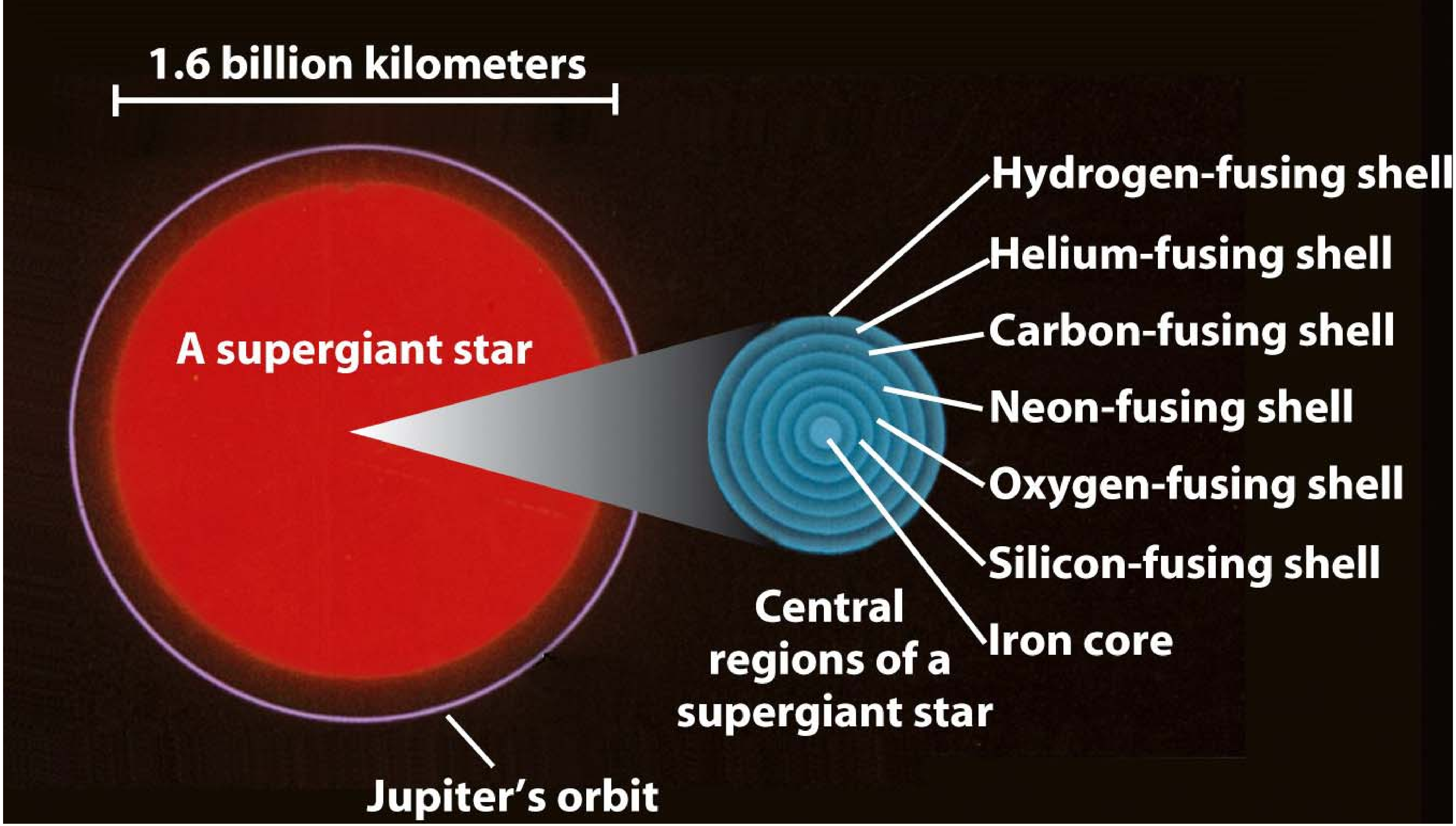


Cas A SNR flythrough

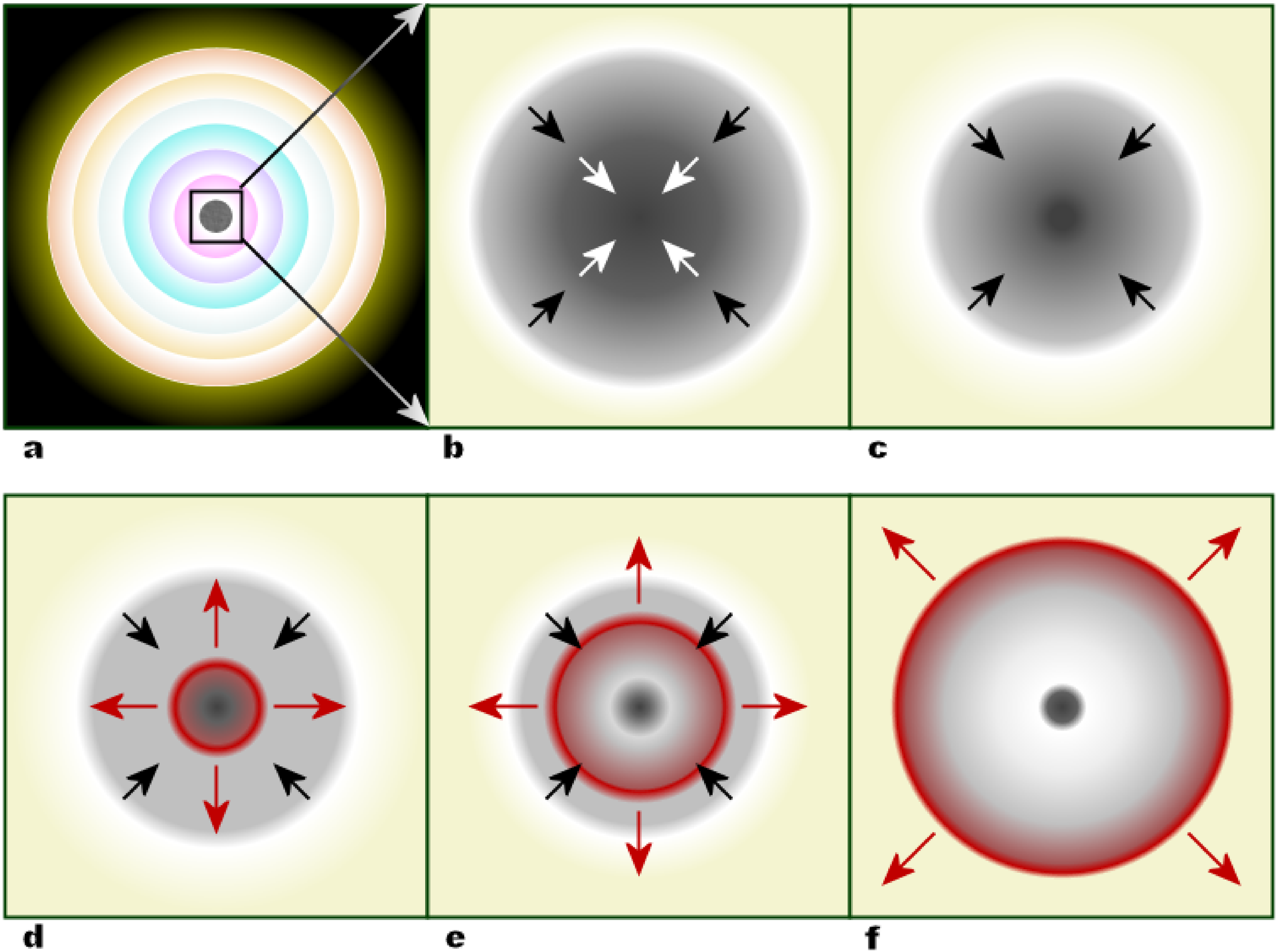
Theory of Supernova Blast Waves

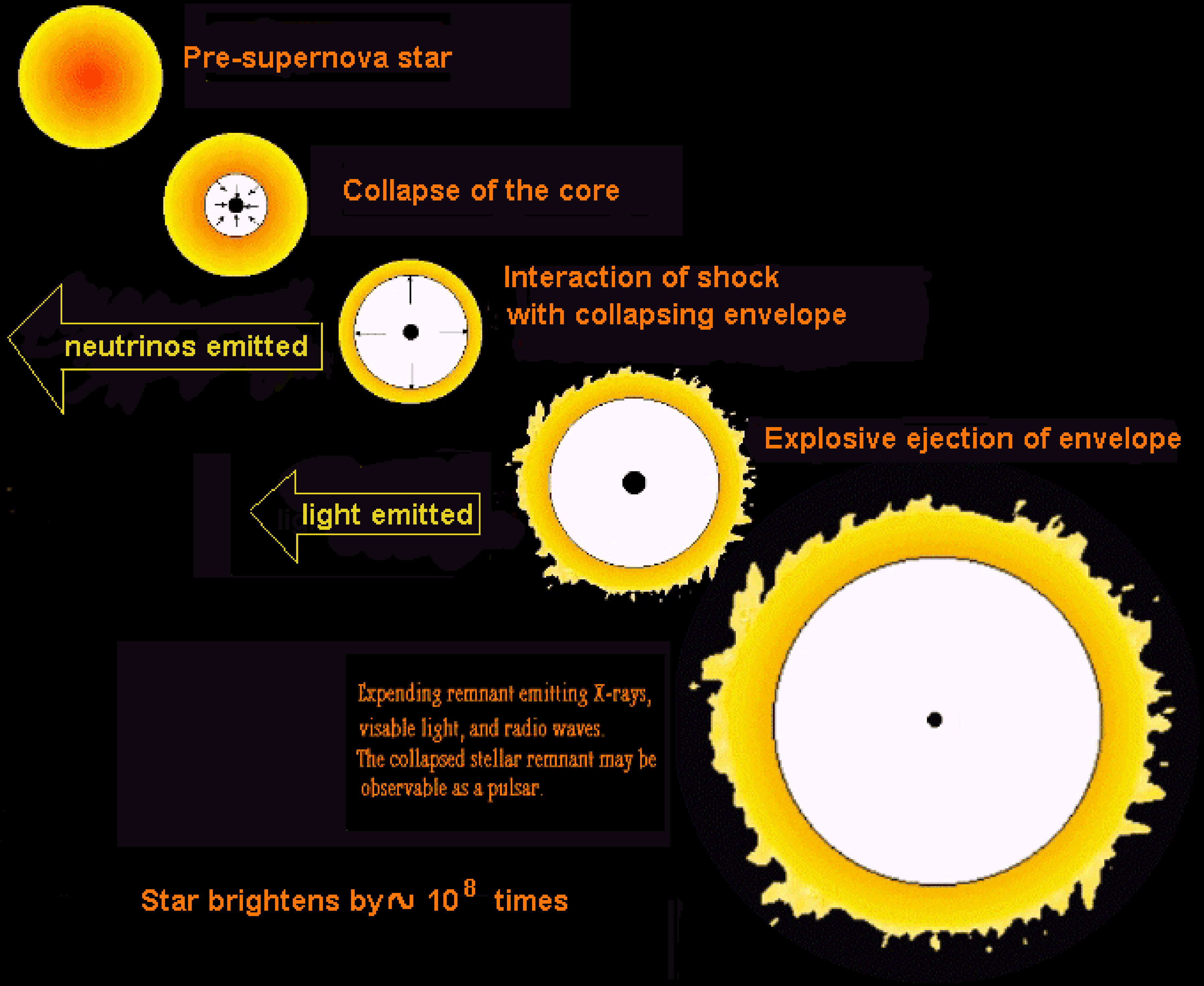
Supernovae:	
Type Ia	Subsonic deflagration wave turning into a supersonic detonation wave in outer layers. Mechanism: explosive carbon burning in a mass-accreting white dwarf
Type Ib-Ic & Type II	Core collapse of massive star

Core-Collapse SN



- In the last stages of its life, high-mass star:
  - iron-rich core
  - surrounded by concentric shells, hosting the various thermonuclear reactions
- The sequence of thermonuclear reactions stops here:
  - formation of elements heavier than iron requires
  - input of energy rather than causing energy to be released


a
b
c
d
e
f



Pre-supernova star

Collapse of the core

Interaction of shock
with collapsing envelope

neutrinos emitted

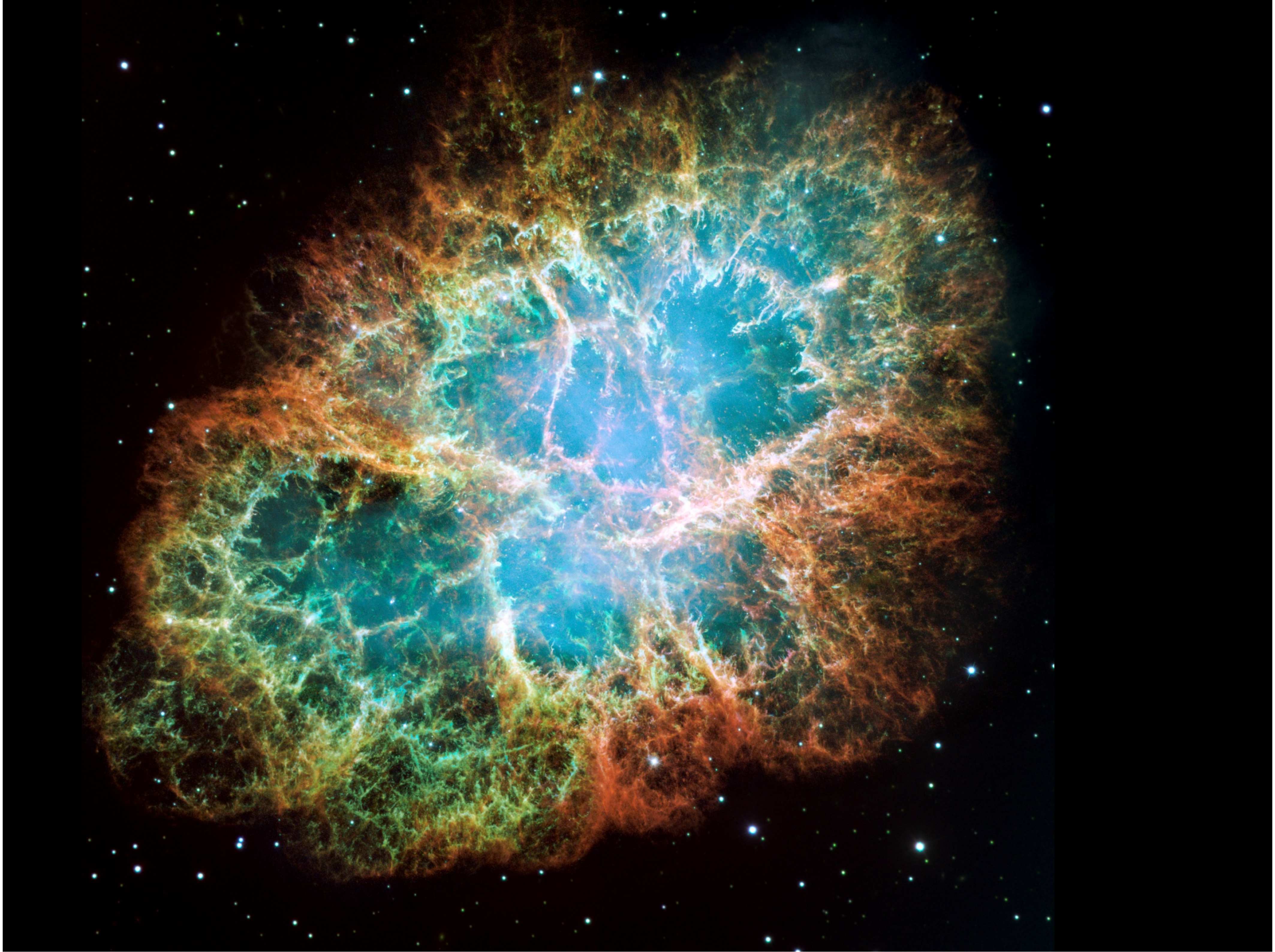
Explosive ejection of envelope

light emitted

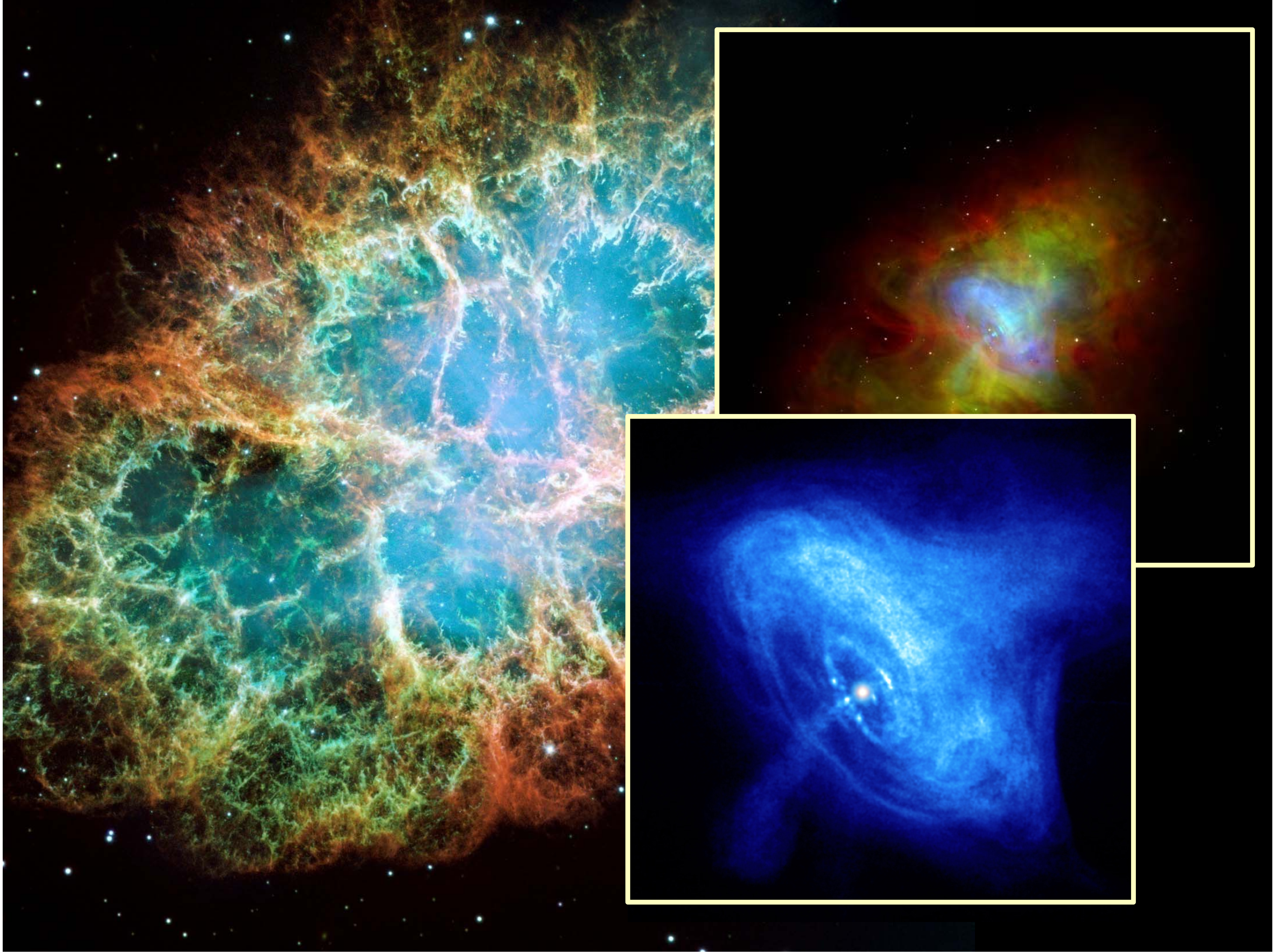
Expending remnant emitting X-rays,
visable light, and radio waves.
The collapsed stellar remnant may be
observable as a pulsar.

Star brightens by $\sim 10^{8}$ times

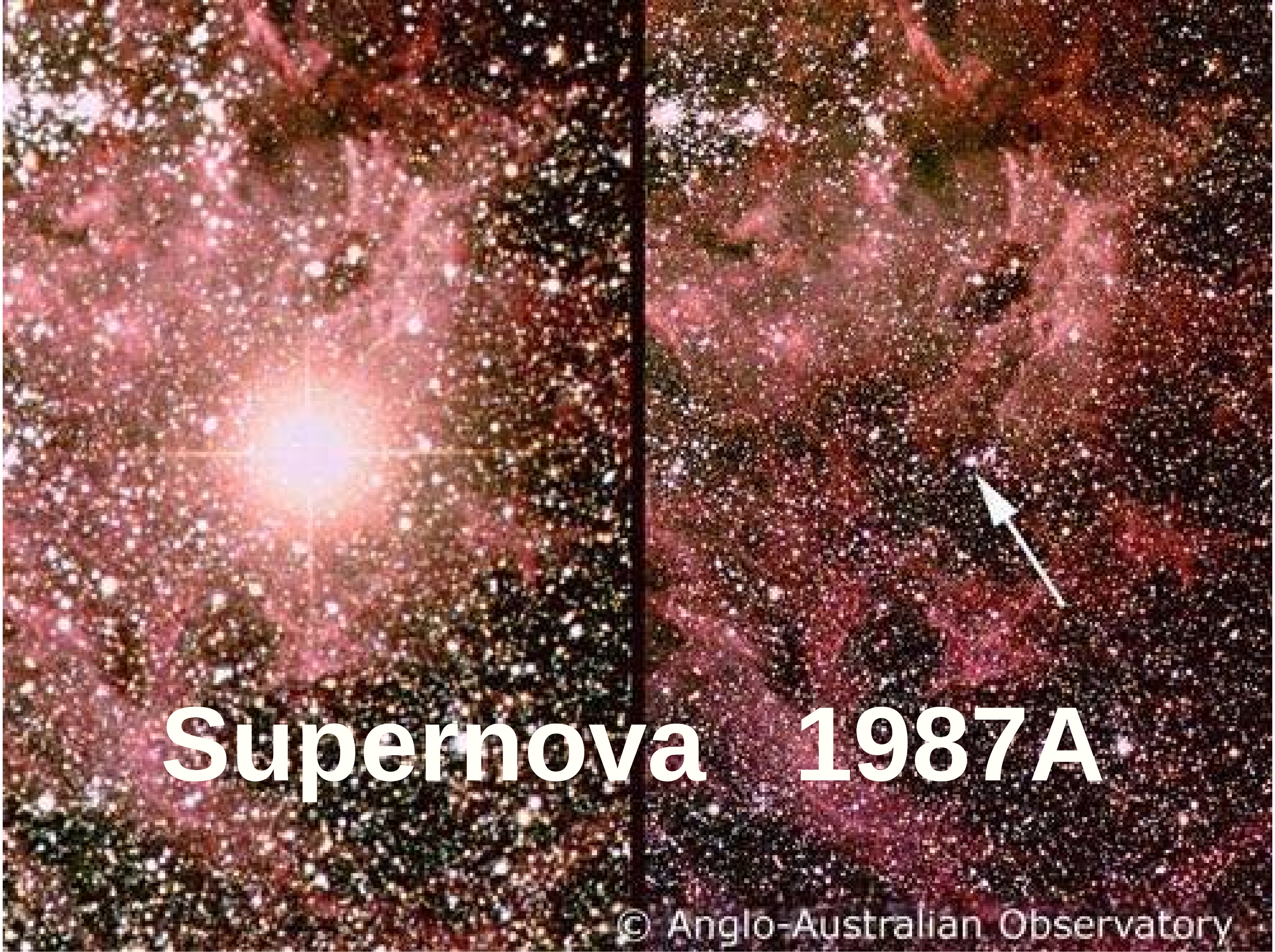
Supernova II Explosion: SN1054



Pulsars and Neutron Stars



Supernova 1987A



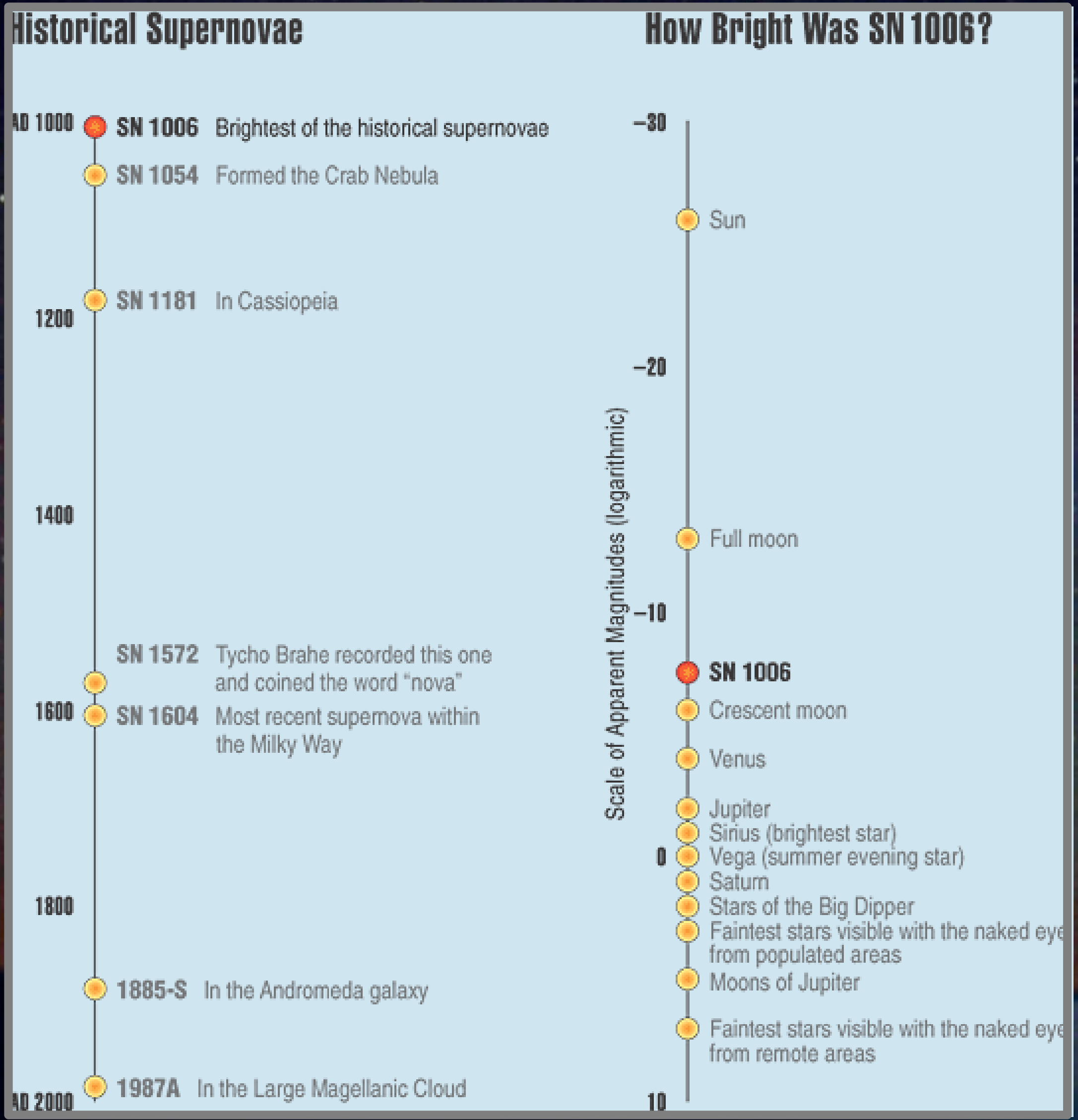
© Anglo-Australian Observatory

Thermonuclear SN
(Supernova Ia)

SN1006

Supernova SN1006:
brightest stellar event recorded in history

SN1006

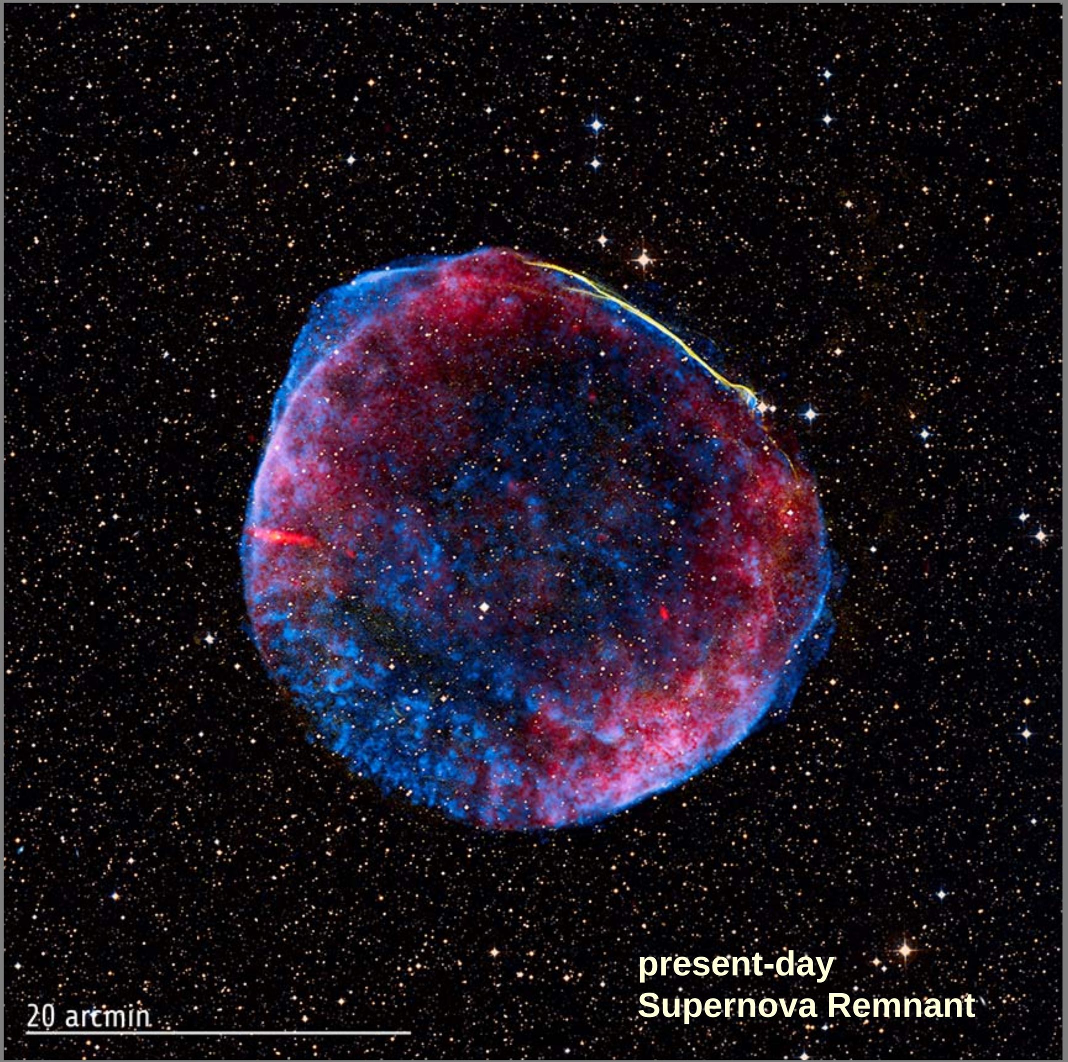


Supernova SN1006:

- brightness: m = -7.5
- distance: d=2.2 kpc
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

SN1006



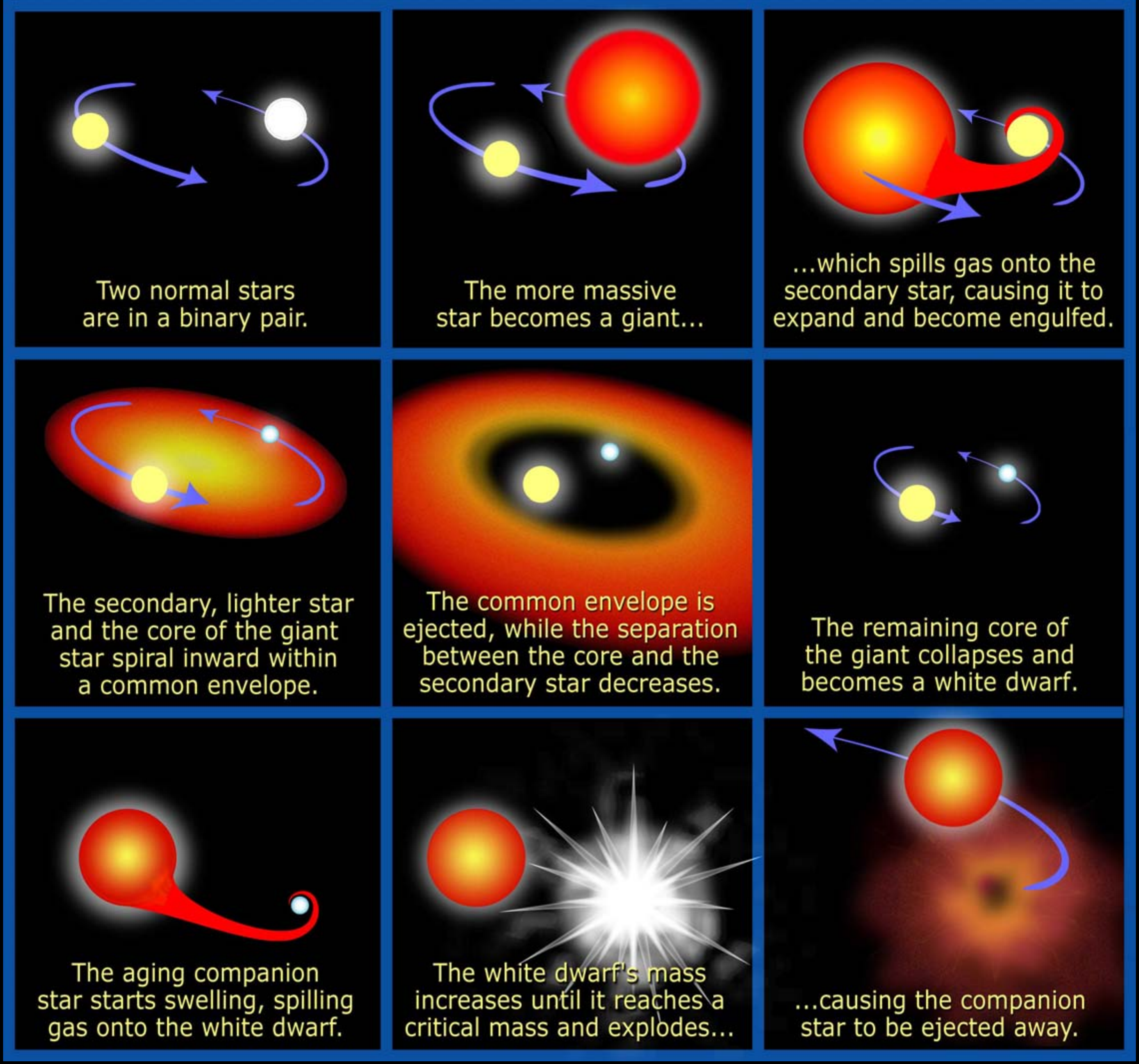
present-day
Supernova Remnant

Supernova SN1006:

-	brightness:	m = -7.5
-	distance:	d=2.2 kpc
-	recorded:	China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

The progenitor of a Type Ia supernova



Two normal stars are in a binary pair.

The more massive star becomes a giant...

...which spills gas onto the secondary star, causing it to expand and become engulfed.

The secondary, lighter star and the core of the giant star spiral inward within a common envelope.

The common envelope is ejected, while the separation between the core and the secondary star decreases.

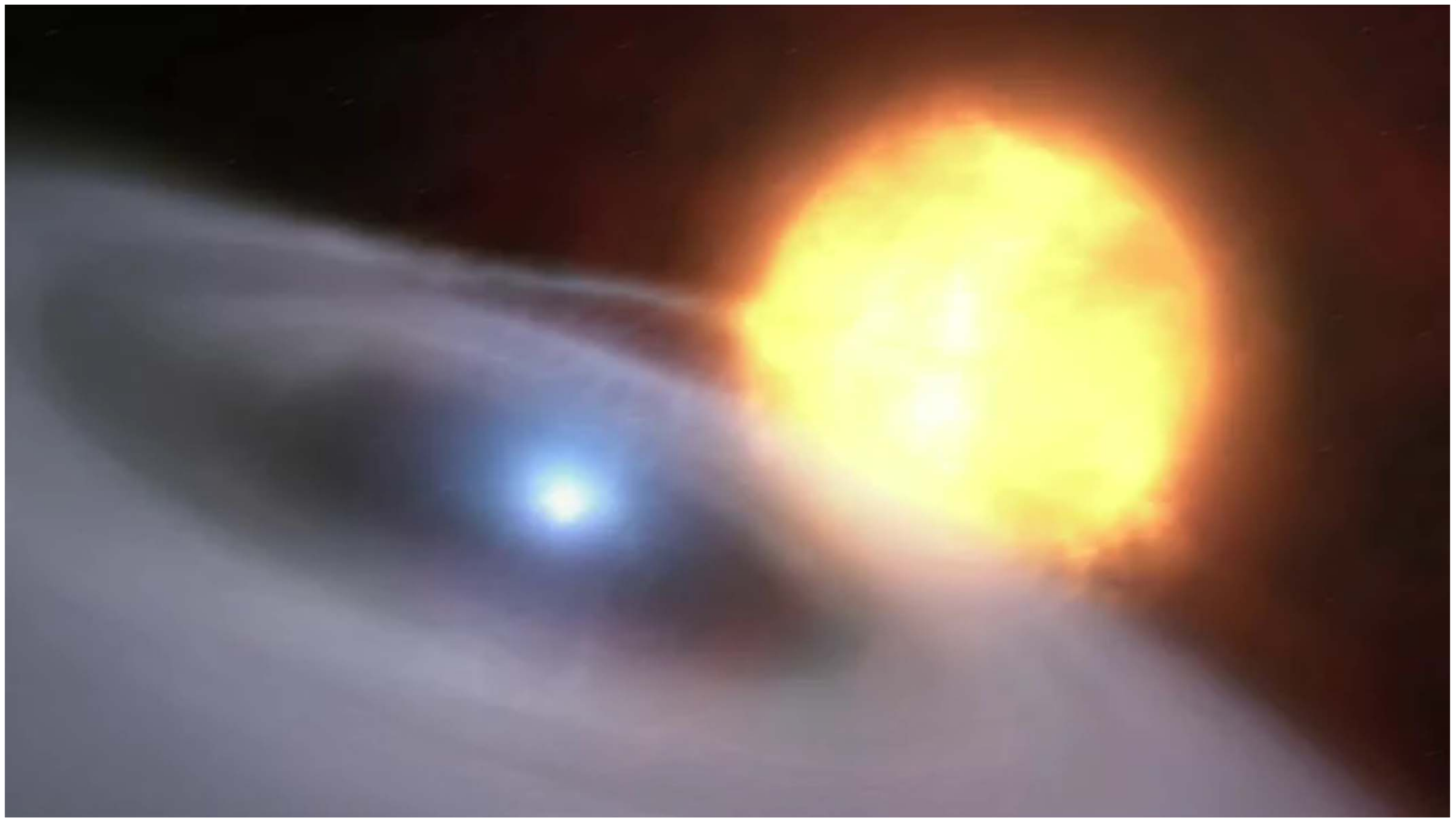
The remaining core of the giant collapses and becomes a white dwarf.

The aging companion star starts swelling, spilling gas onto the white dwarf.

The white dwarf's mass increases until it reaches a critical mass and explodes...

...causing the companion star to be ejected away.

Supernova Ia Explosion



Blast Waves

Tsar Bomba Nuclear Explosion

Tsar Bomba Nuclear Explosion

Tsar Bomba Nuclear Explosion

Hiroshima, the Shockwave

Sedov-Taylor Expansion Law

Blast waves

Main properties:

1. Strong shock propagating through the Interstellar Medium, or through the wind of the progenitor star;

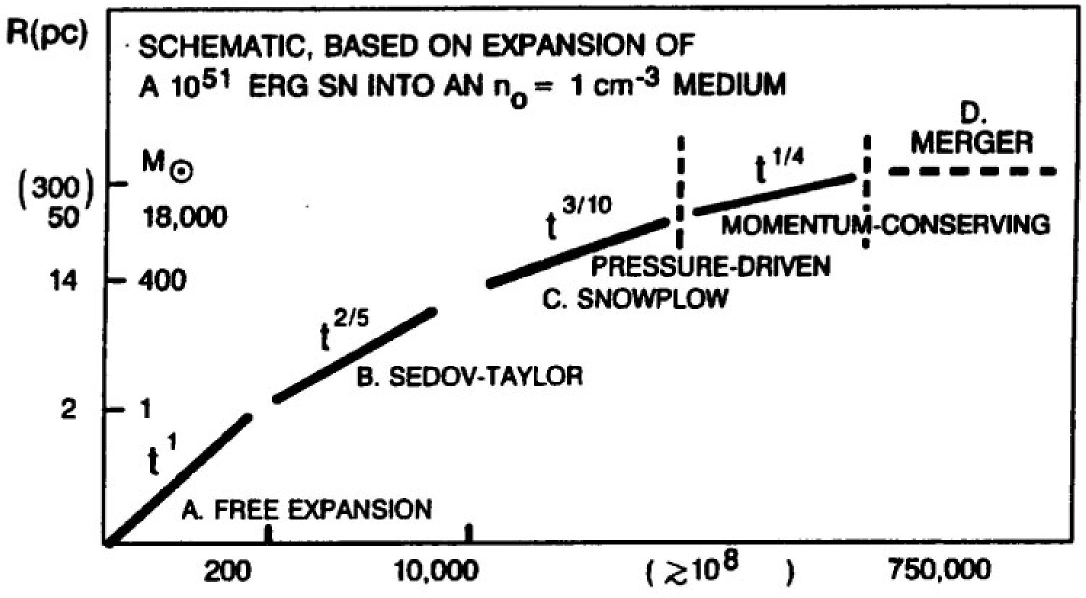
2. Different expansion stages:

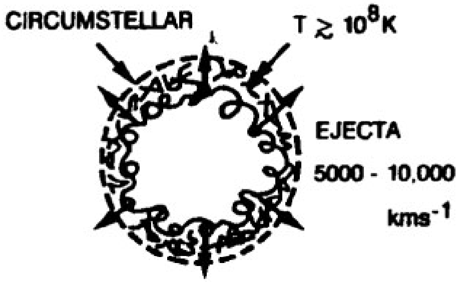
   - Free expansion stage ($t < 1000$ yr) $R \propto t$

   - Sedov-Taylor stage ($1000 \text{ yr} < t < 10{,}000$ yr) $R \propto t^{2/5}$

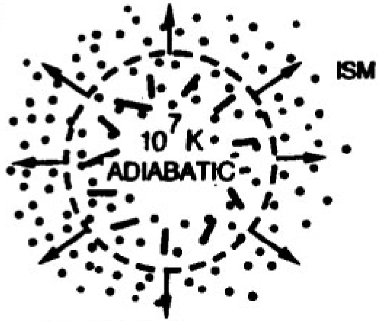
   - Pressure-driven snowplow ($10{,}000 \text{ yr} < t < 250{,}000$ yr) $R \propto t^{3/10}$

STANDARD SNR EVOLUTION

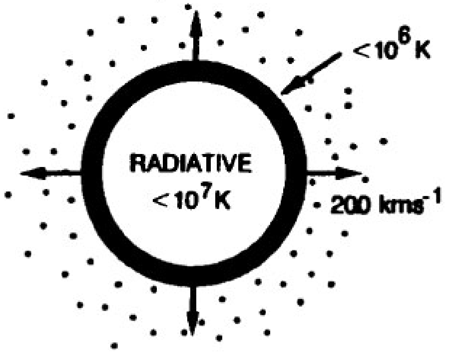




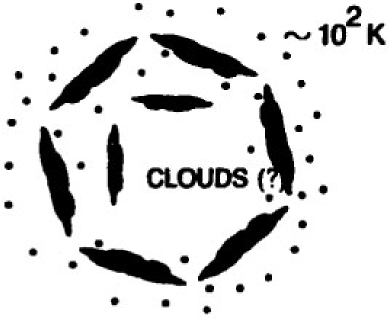
A. FREE EXPANSION



B. BLAST WAVE

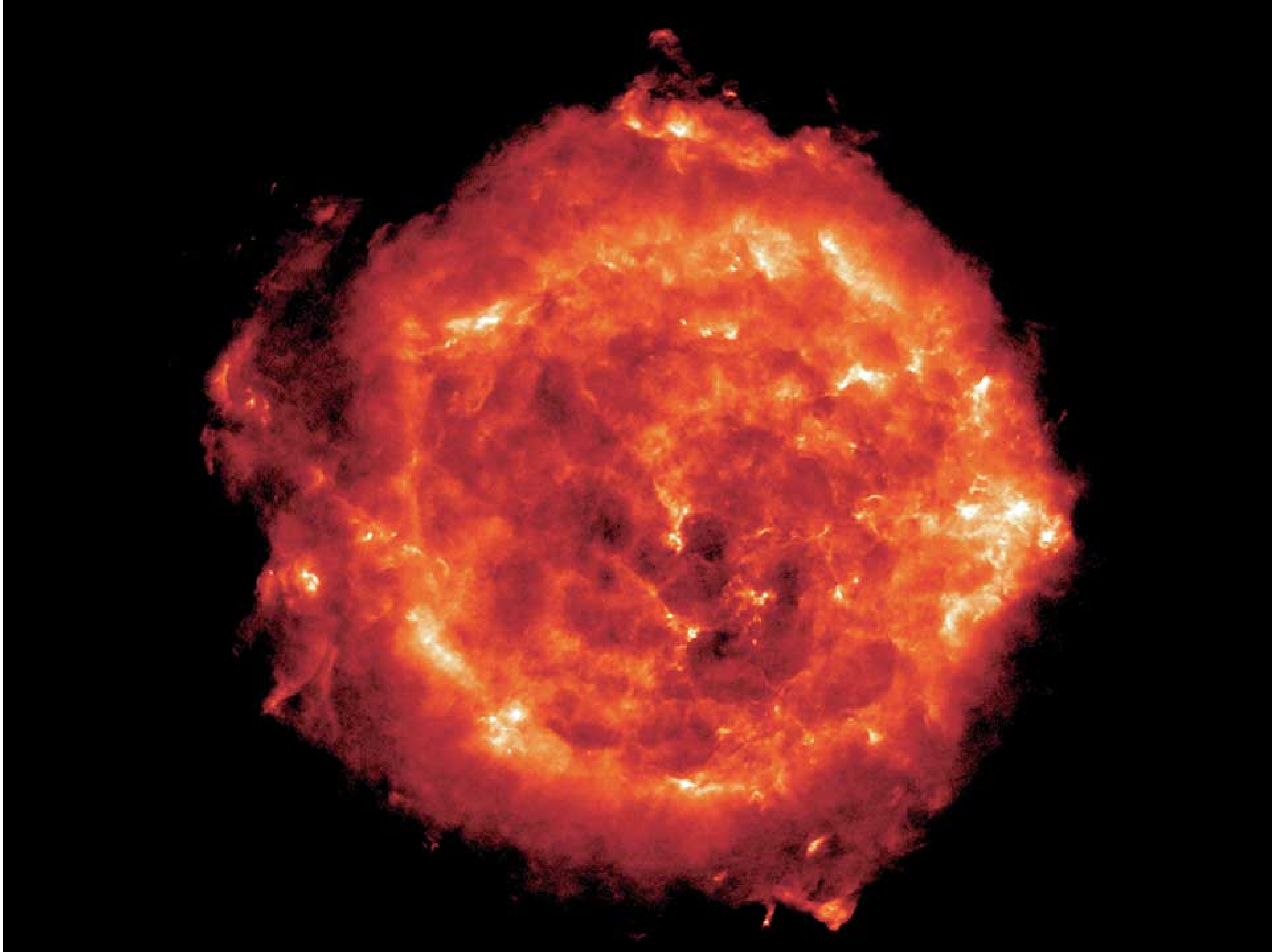


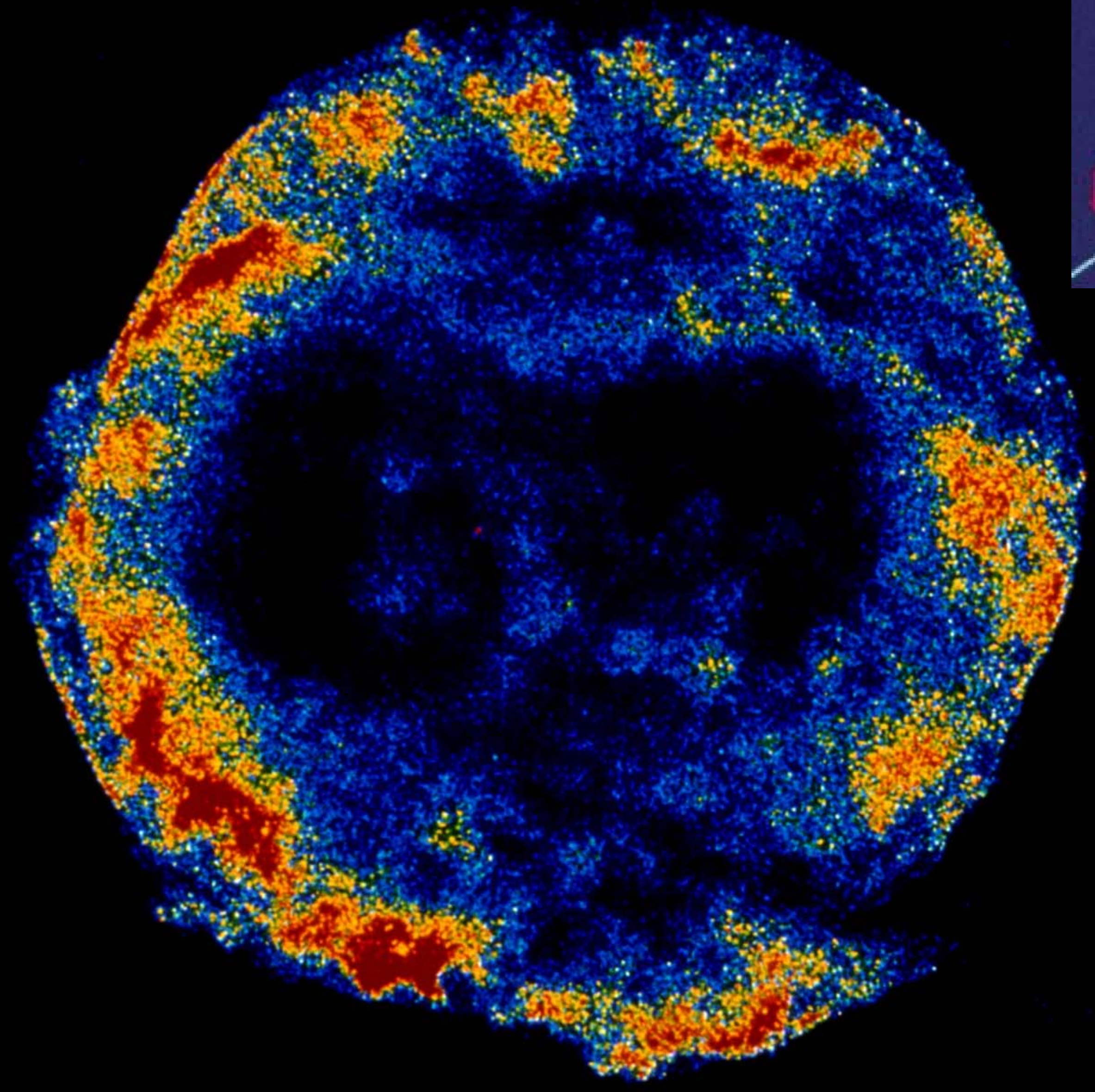
C. SNOWPLOW

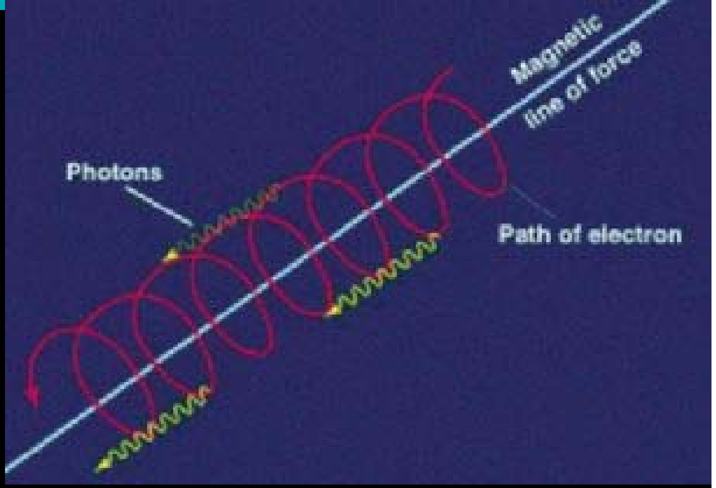


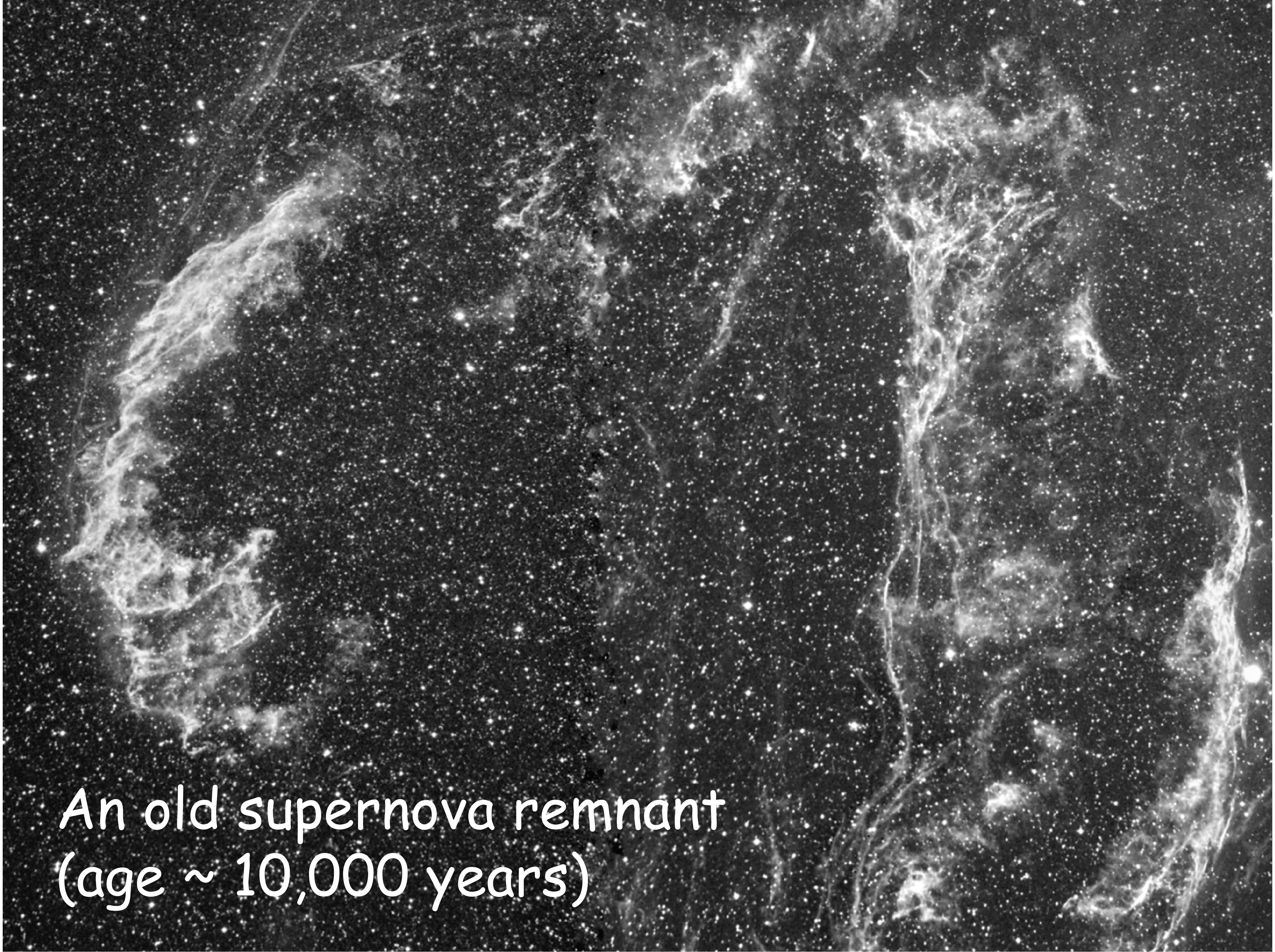
D. DEATH → COALESCENCE

Tsar Bomba Nuclear Explosion






Magnetic line of force
Photons
Path of electron



An old supernova remnant
(age ~ 10,000 years)

Free-expansion phase

Energy budget:

$$\left|E_{\text{grav}}\right| = \tfrac{3}{5}\frac{GM_c^2}{R_c} \simeq 10^{53}\ \text{erg} \Rightarrow \begin{cases} \text{99\% into neutrino's} \\ \text{1\% into mechanical energy} \end{cases}$$

Expansion speed:

$$V_{\text{exp}} \simeq \sqrt{\frac{2E_{\text{mech}}}{M_{\text{ej}}}} = 3000\left(\frac{E_{\text{mech}}}{10^{51}\ \text{erg}}\right)^{1/2}\left(\frac{M_{\text{ej}}}{10\ M_{\odot}}\right)^{-1/2}\ \text{km/s}$$

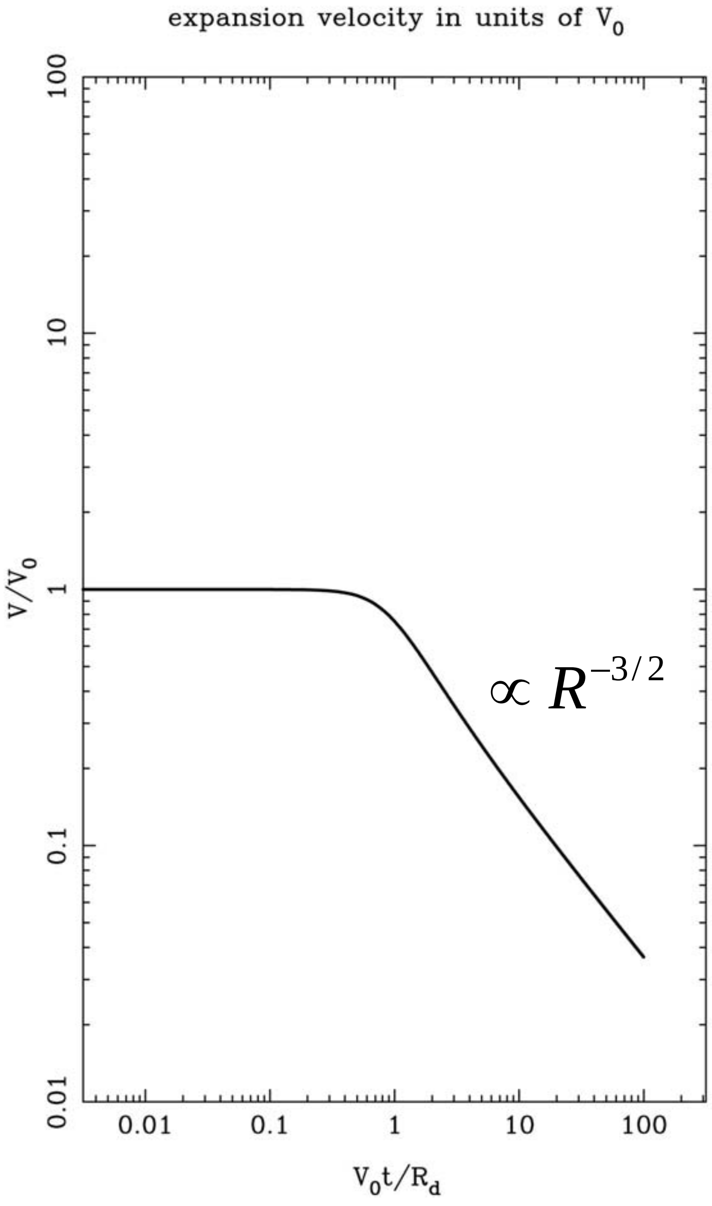
Sedov-Taylor stage

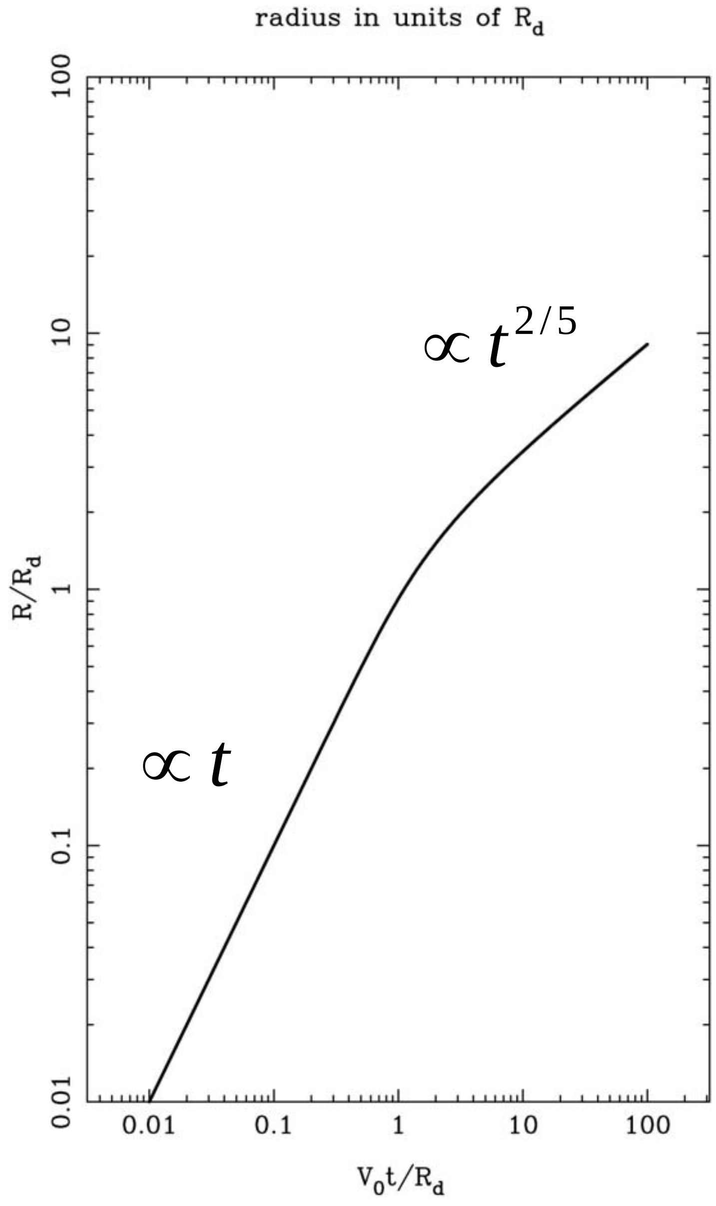
- Expansion starts to decelerate due to swept-up mass

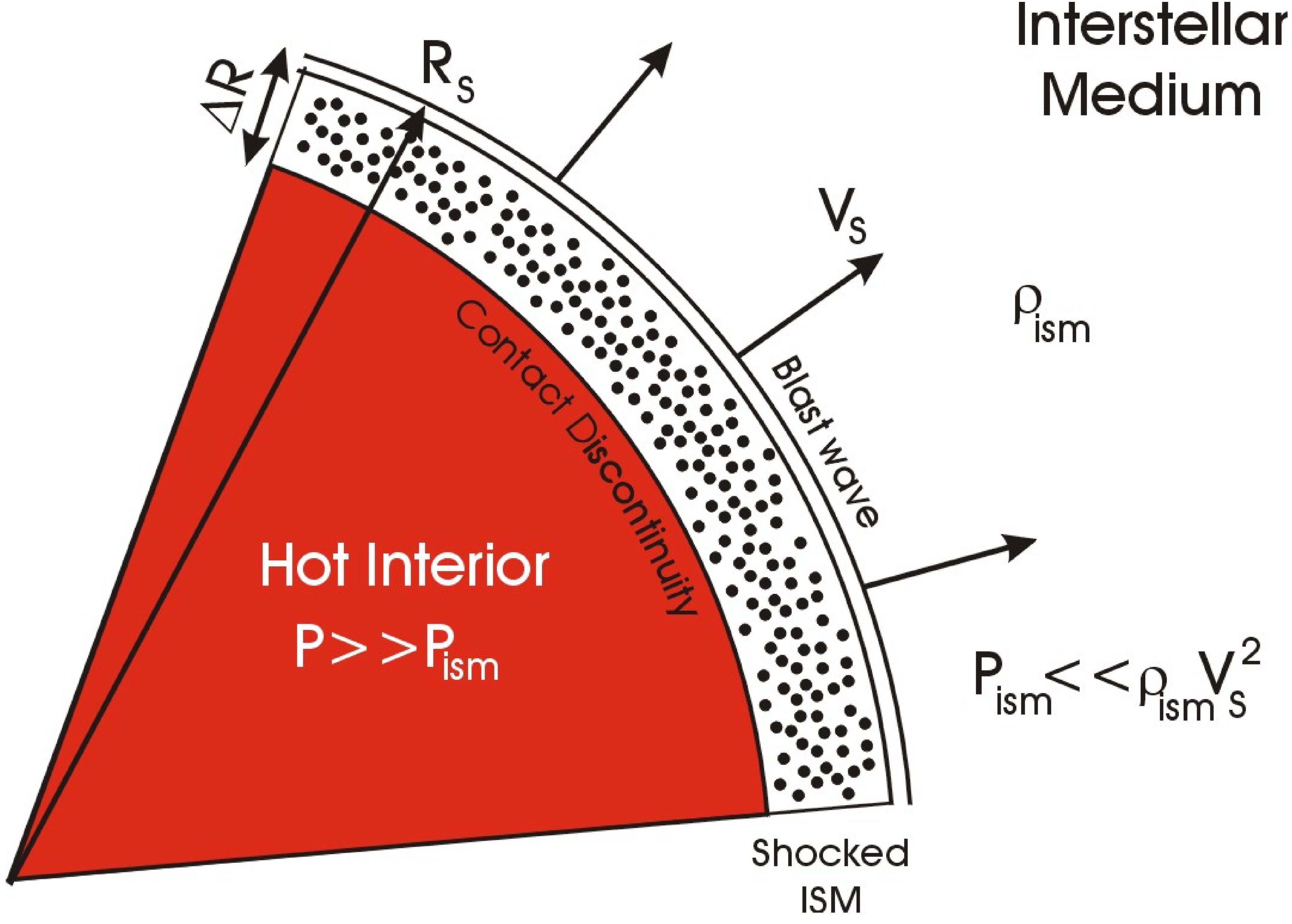
- Interior of the bubble is reheated due to reverse shock

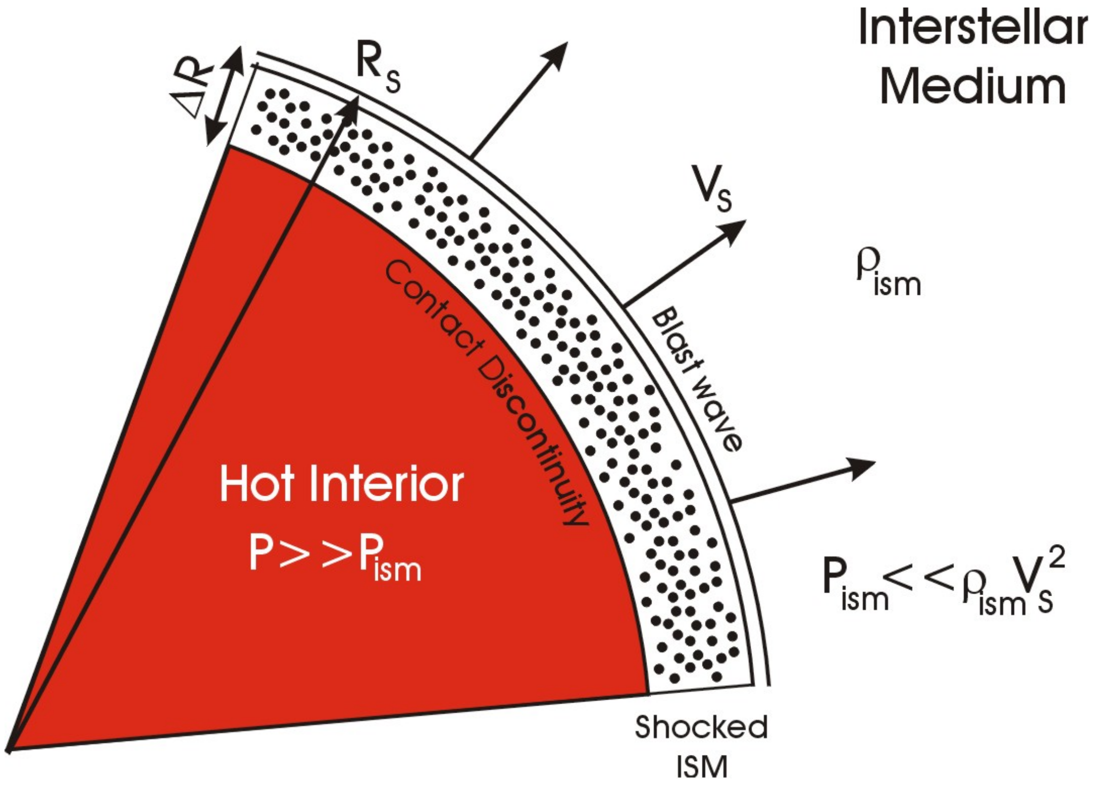
- Hot bubble is preceded in ISM by strong blast wave

$$V_s = \sqrt{\frac{2E_{\mathrm{snr}}}{M_{\mathrm{ej}}}} \times \left(\frac{1}{1+\left(R/R_{\mathrm{d}}\right)^3} \right)^{1/2} = V_0 \left(\frac{1}{1+\left(R/R_{\mathrm{d}}\right)^3} \right)^{1/2}$$



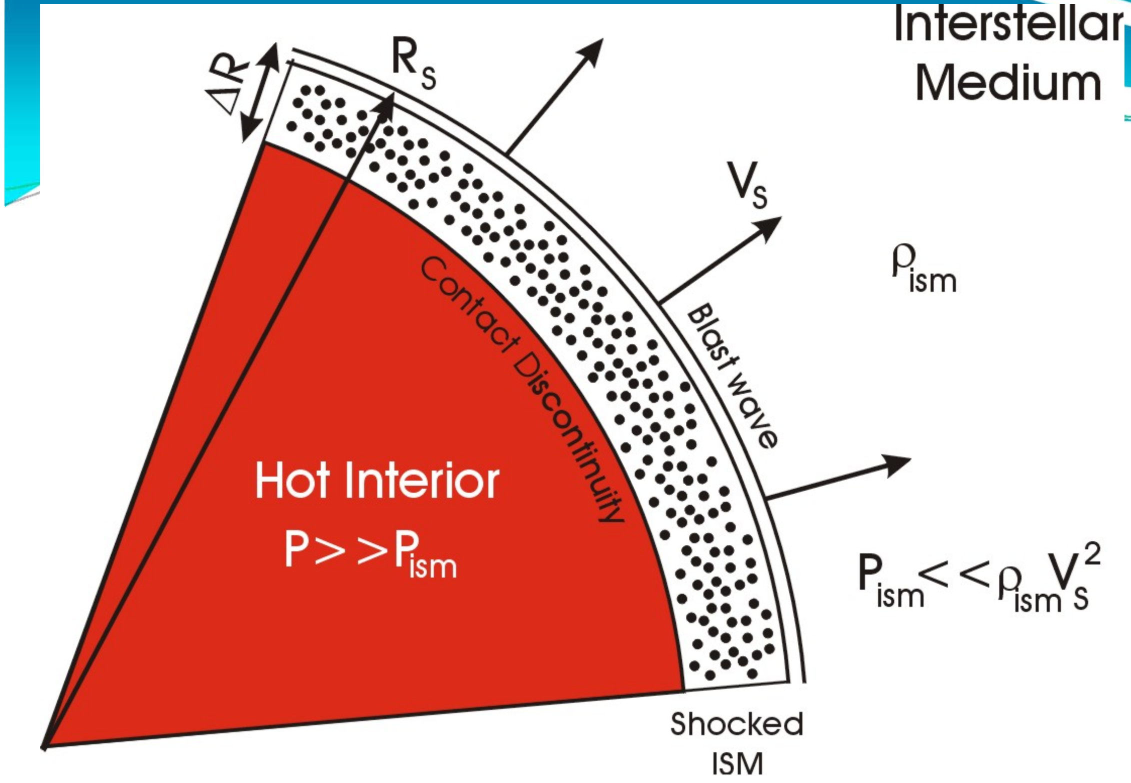



Interstellar Medium
ΔR
R_S
V_S
ρ_ism
Contact Discontinuity
Blast wave
Hot Interior
P>>P_ism
P_ism<<ρ_ism V_S^2
Shocked ISM



Shock relations for strong (high-Mach number) shocks:

$$\left.\begin{array}{l}\dfrac{\rho_2}{\rho_1} = \dfrac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \quad \Rightarrow \quad \dfrac{\gamma+1}{\gamma-1} \\ \\ \dfrac{P_2}{P_1} = \dfrac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1} \quad \Rightarrow \quad \dfrac{2\gamma}{\gamma+1}\mathcal{M}_s^2 \\ \\ \qquad\qquad\qquad \Leftrightarrow \quad P_2 = \dfrac{2}{\gamma+1}\rho_1 V_1^2 \end{array}\right\} \text{as } \mathcal{M}_s^2 \equiv \left(\frac{V_1}{c_{s1}}\right)^2 = \frac{\rho_1 V_1^2}{\gamma P_1} \Rightarrow \infty$$

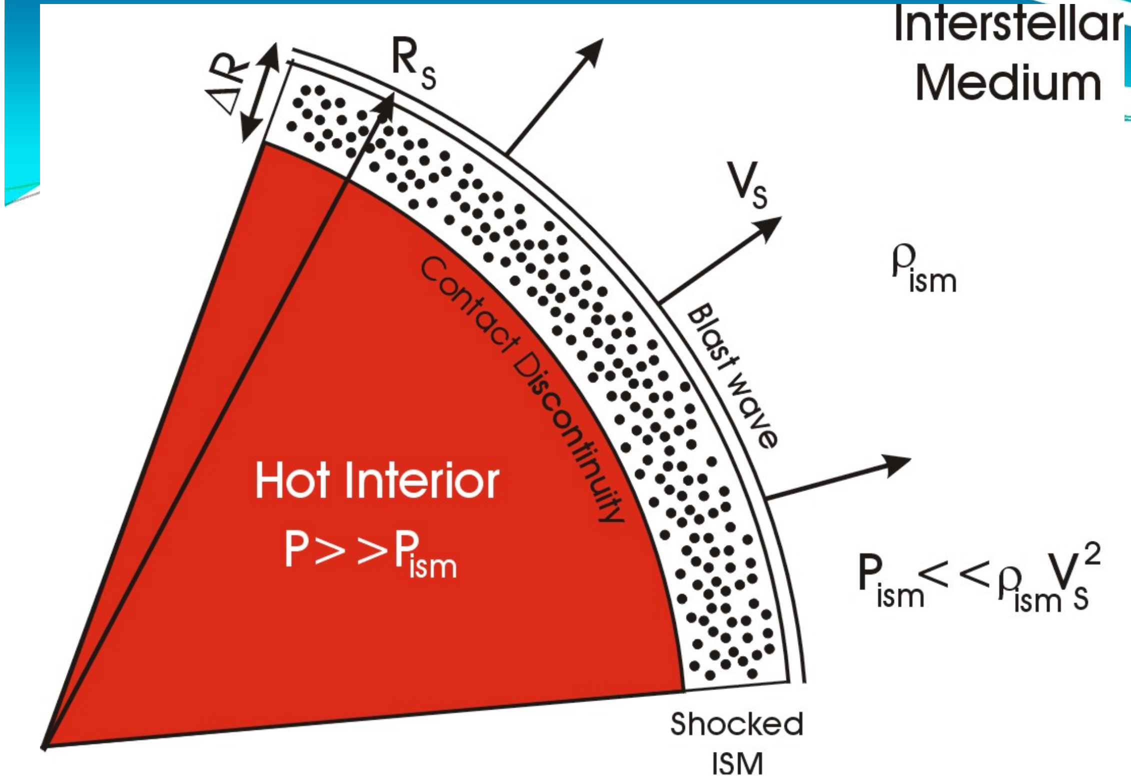


$$P_2 \approx \frac{2\gamma}{\gamma+1}\mathcal{M}_s^{\,2} P_1 = \frac{2}{\gamma+1}\rho_{\mathrm{ism}} V_s^{\,2}$$

Pressure behind strong shock (blast wave)

$$P_{\mathrm{i}} = (\gamma-1)e_{\mathrm{i}} \approx (\gamma-1)\frac{E_{\mathrm{SNR}}}{\frac{4\pi}{3}R_{\mathrm{S}}^3}$$

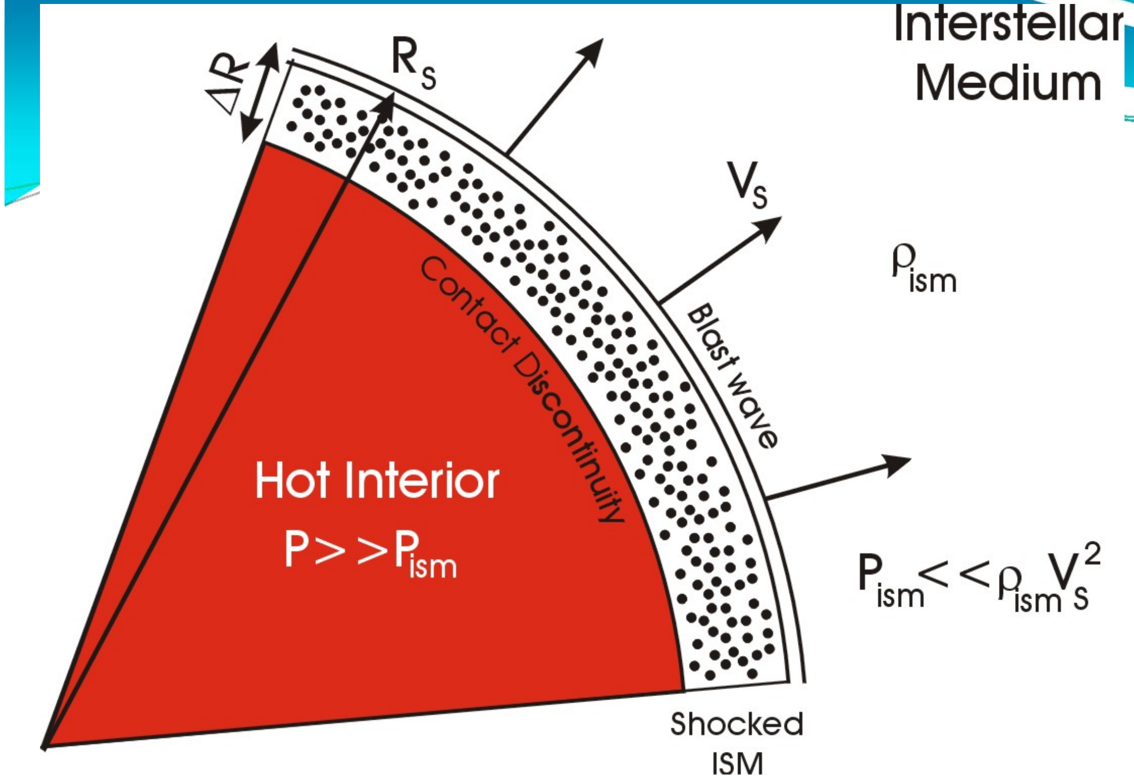
Pressure in hot SNR interior



At contact discontinuity: equal pressure on both sides!

$$\frac{2}{\gamma+1}\rho_{\mathrm{ism}}V_s^{\,2} \approx (\gamma-1)\frac{E_{\mathrm{SNR}}}{\frac{4\pi}{3}R_s^3}$$

This procedure is allowed because of high sound speeds in hot interior and in shell of hot, shocked ISM:
No large pressure differences are possible!

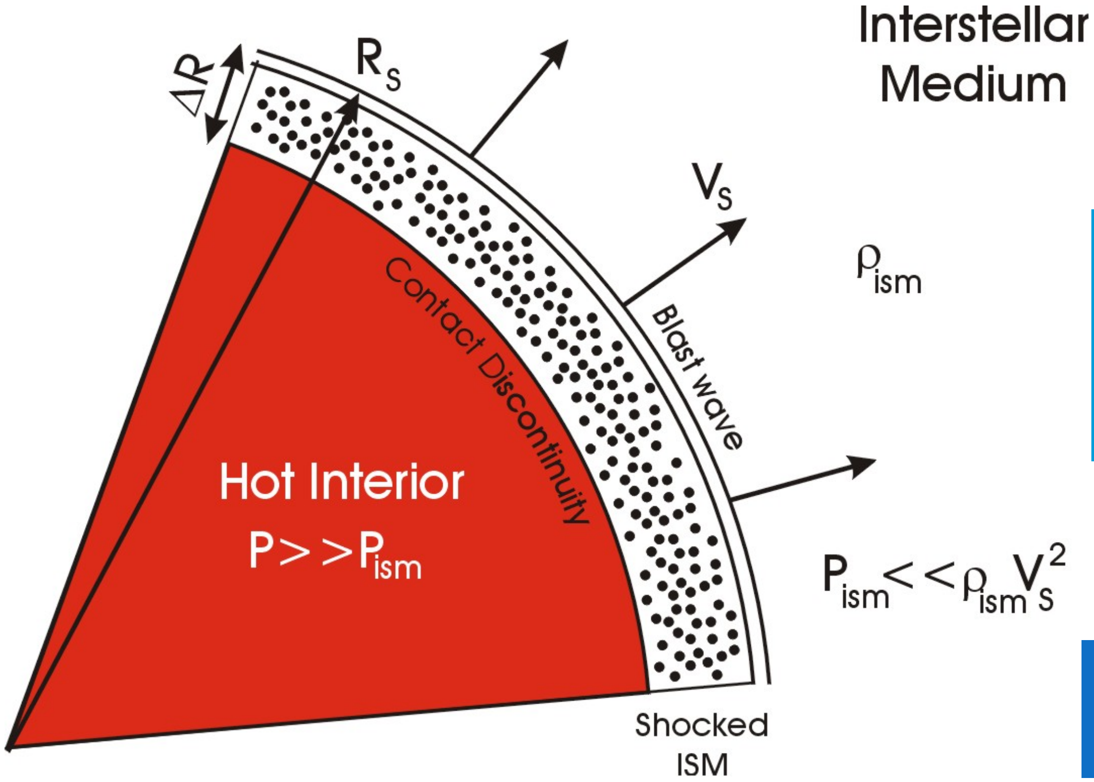


At contact discontinuity: equal pressure on both sides!

$$\frac{2}{\gamma+1}\rho_{\rm ism}V_s^{\,2} \approx (\gamma-1)\frac{E_{\rm SNR}}{\frac{4\pi}{3}R_s^3}$$

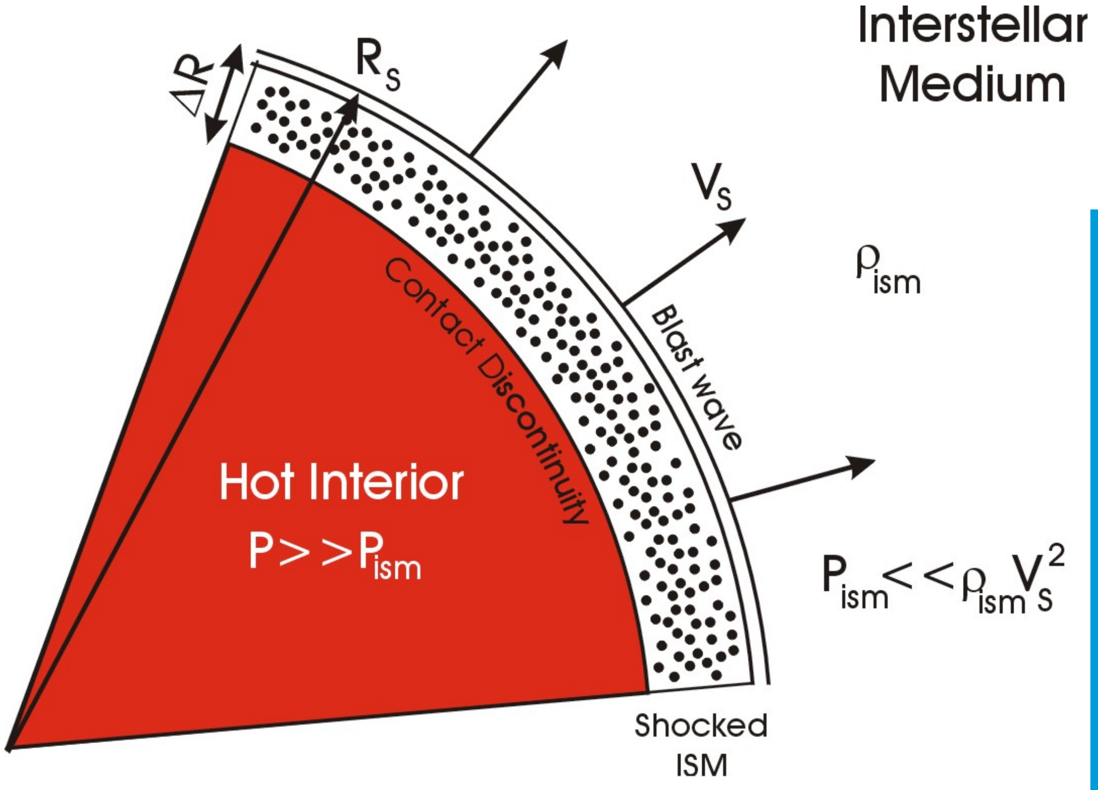
$$V_s=\frac{dR_s}{dt}\simeq\sqrt{\frac{8\pi}{3(\gamma^2-1)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2}R_s^{-3/2}$$

Relation between velocity and radius gives expansion law!



$$\boxed{R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2 - 1\right)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt}$$

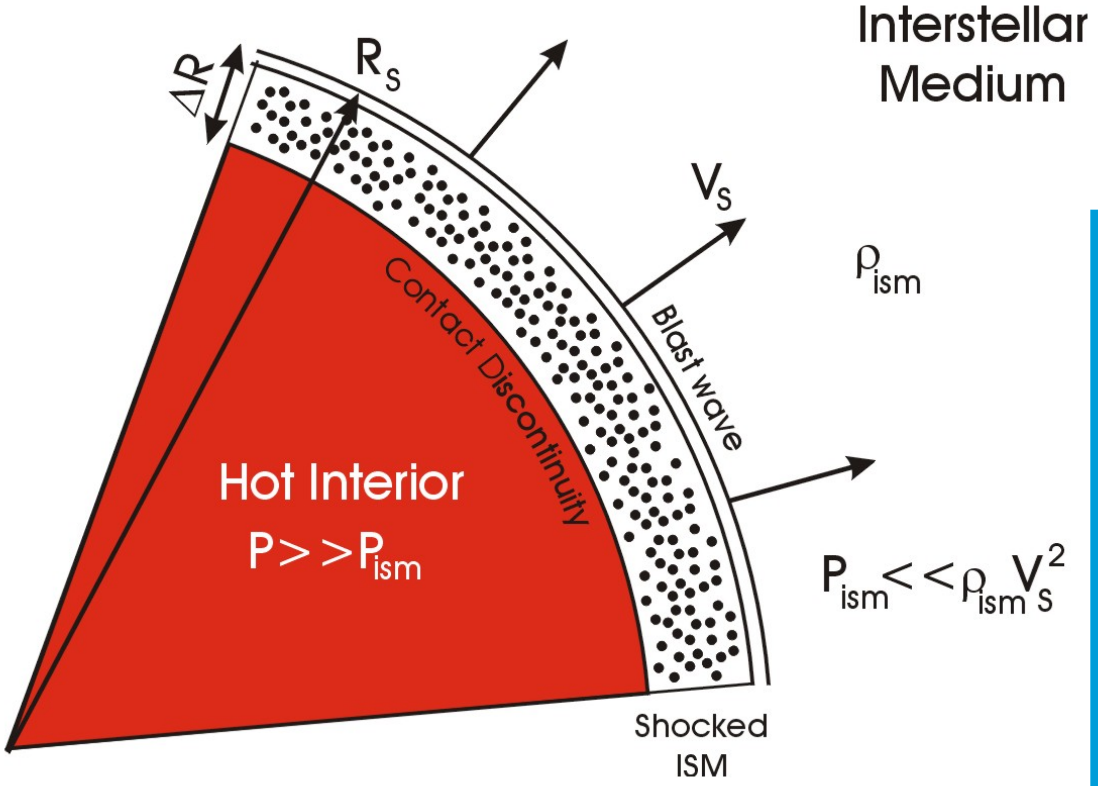
Step 1: write the relation as difference equation



$$R_s^{3/2} dR_s \simeq \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{snr}}{\rho_{ism}}\right)^{1/2} dt$$

Step 2: write as total differentials and.........



……integrate to find the Sedov-Taylor solution

$$R_s^{\,3/2} dR_s \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

$$\frac{2}{5} d\left(R_s^{5/2}\right) \simeq \sqrt{\frac{8\pi}{3\left(\gamma^2-1\right)}}\left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/2} dt$$

$$R_s(t) \simeq C_\gamma \left(\frac{E_{\rm snr}}{\rho_{\rm ism}}\right)^{1/5} t^{2/5} \quad ,$$

$$C_\gamma = \left(\frac{5}{2}\right)^{2/5}\left(\frac{8\pi}{3\left(\gamma^2-1\right)}\right)^{1/5} \simeq 1.96$$

Sedov & Taylor