

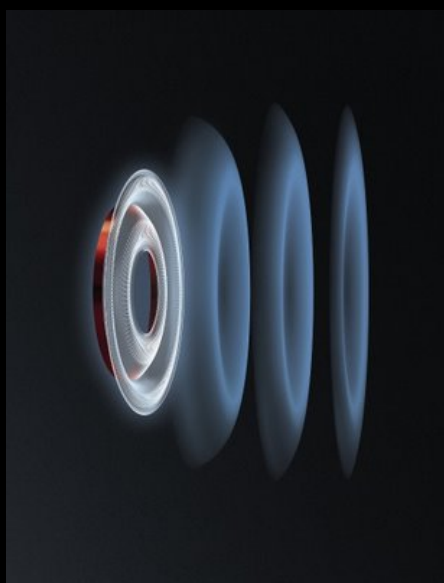


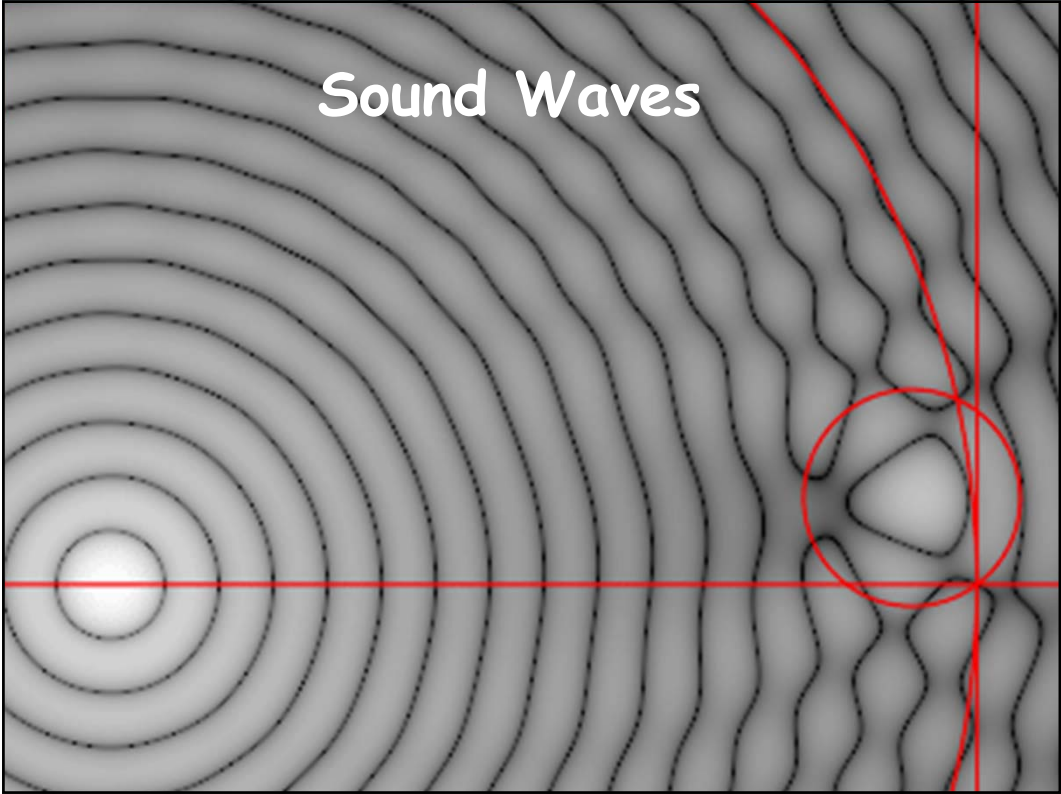
**Waves:
sea & ocean waves**

This section features three distinct images of waves. The top-left image shows close-up ripples on a blue sea surface. The middle image shows a line of white-capped waves breaking on a beach. The bottom-right image shows a large, curling wave with a white foam crest, set against a dark blue sky.

Sound Waves

Sound Waves:





Sound Waves

direction of propagation →

$t_0 = 0 \text{ ms}$

$t_1 = 1 \text{ ms}$

$t_2 = 2 \text{ ms}$

$t_3 = 3 \text{ ms}$

compression

rarefaction

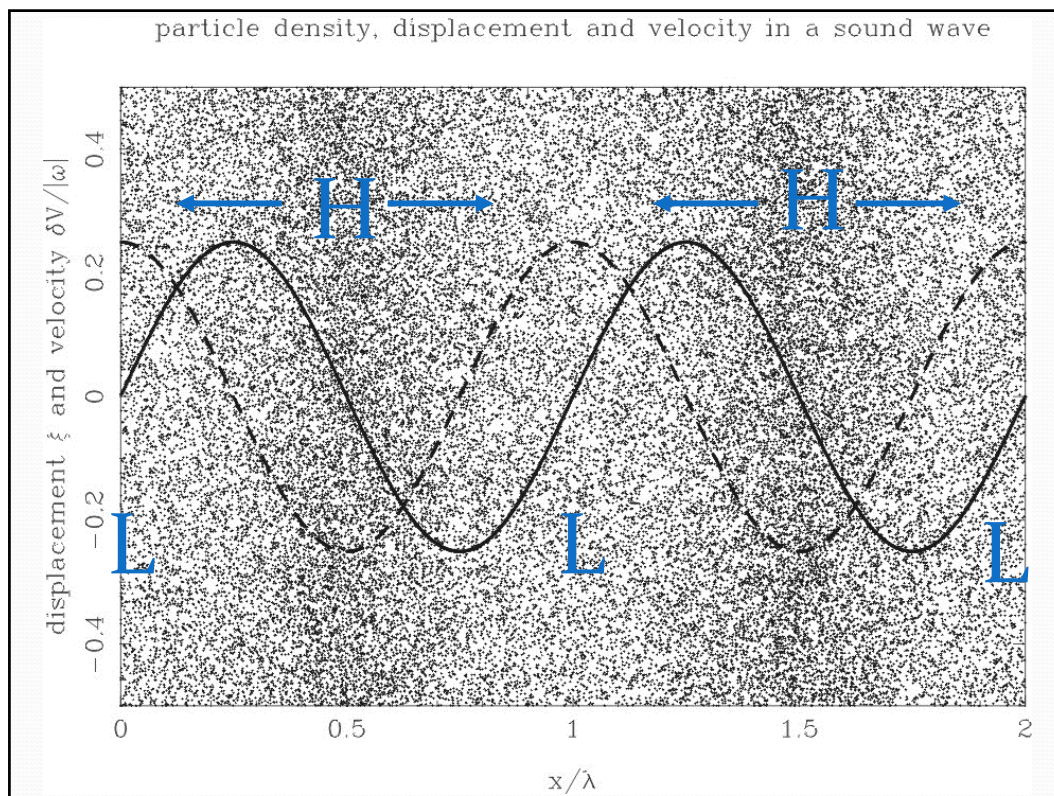
λ

A

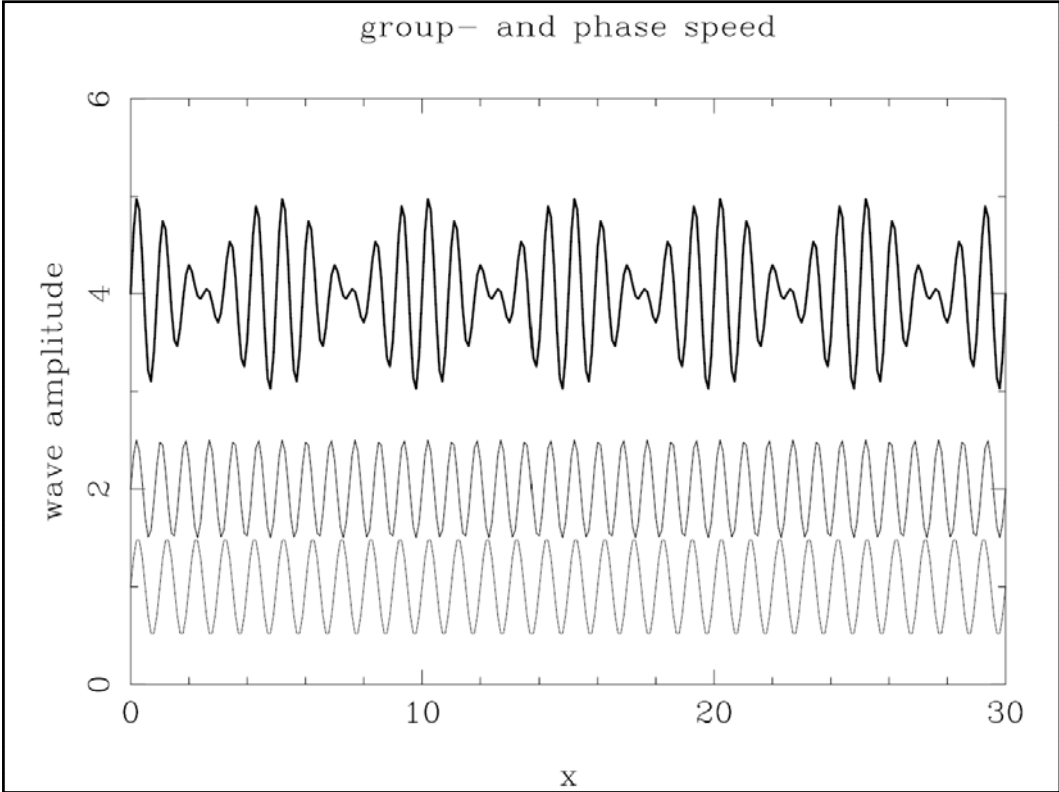
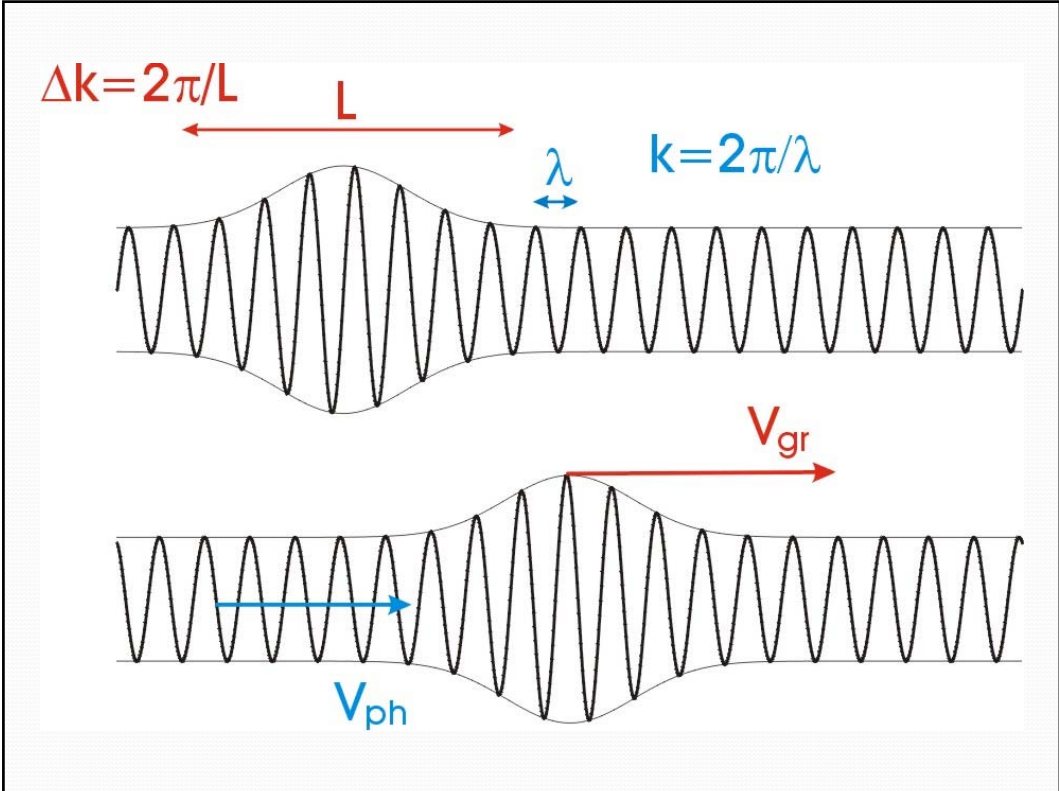
λ

λ

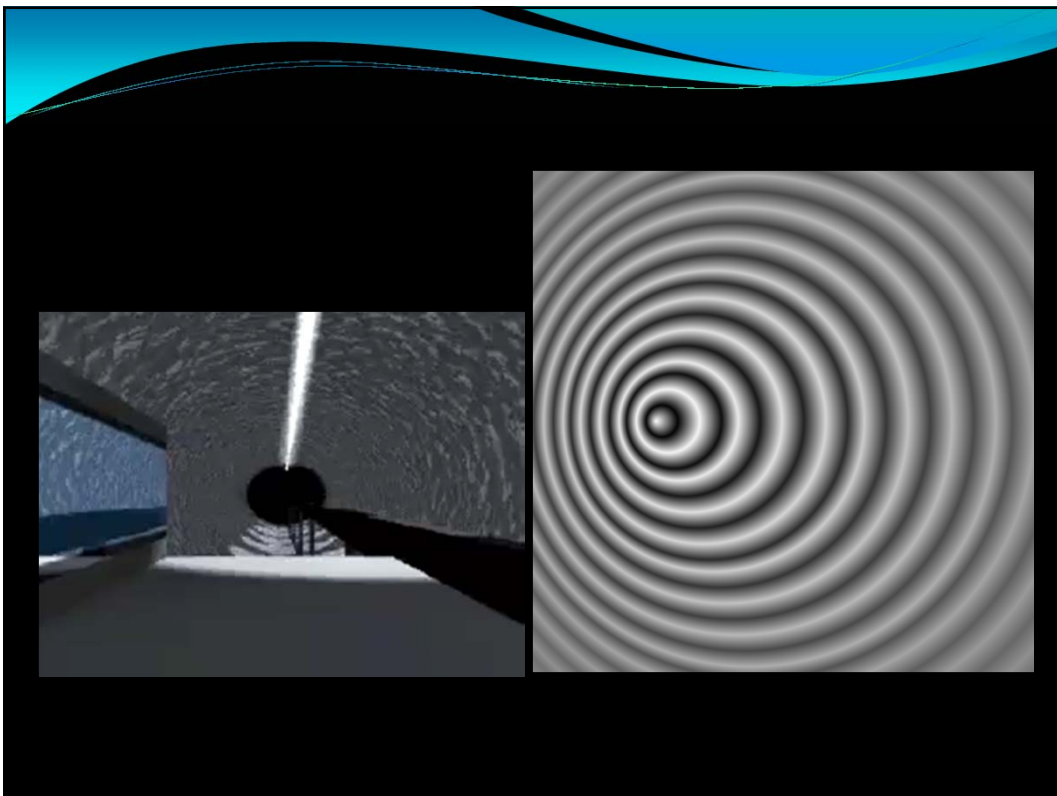
©1994 Encyclopaedia Britannica, Inc.



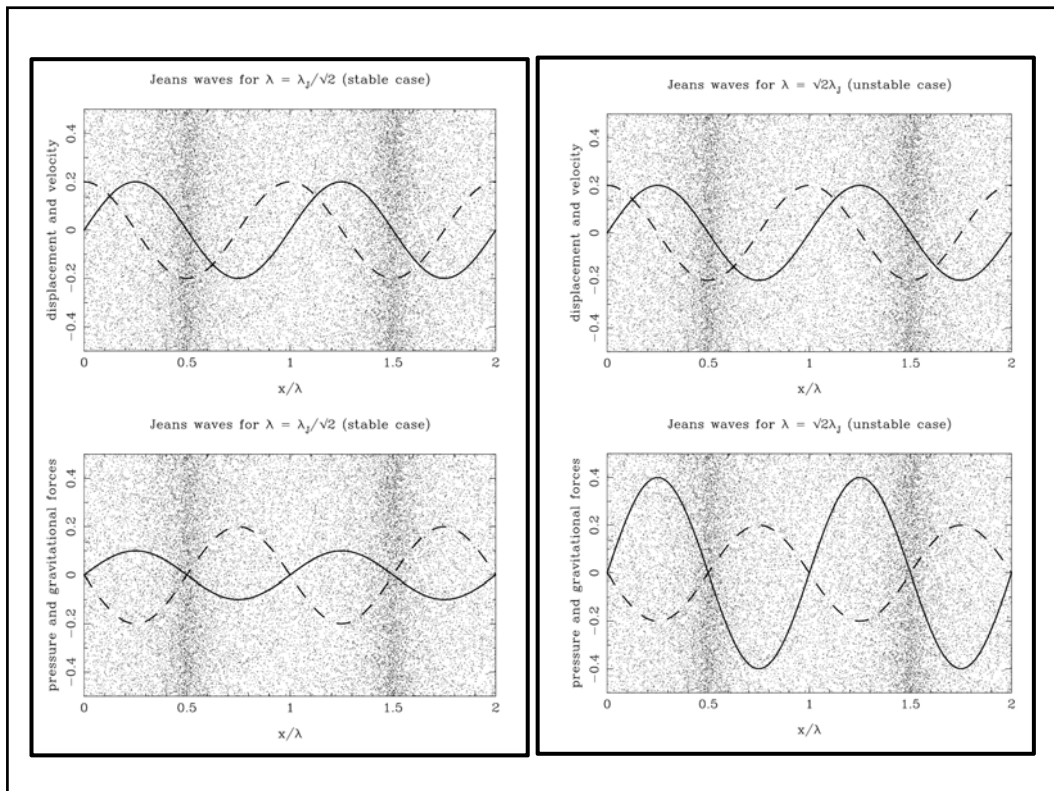
Phase & Group Velocity

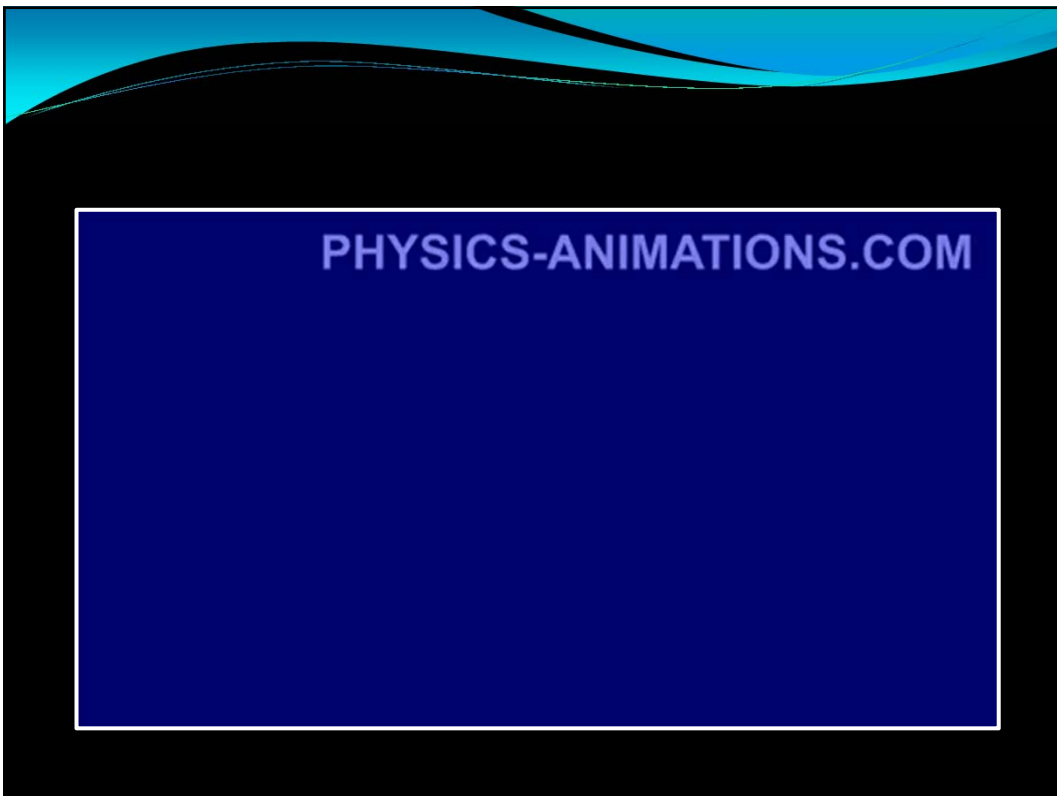
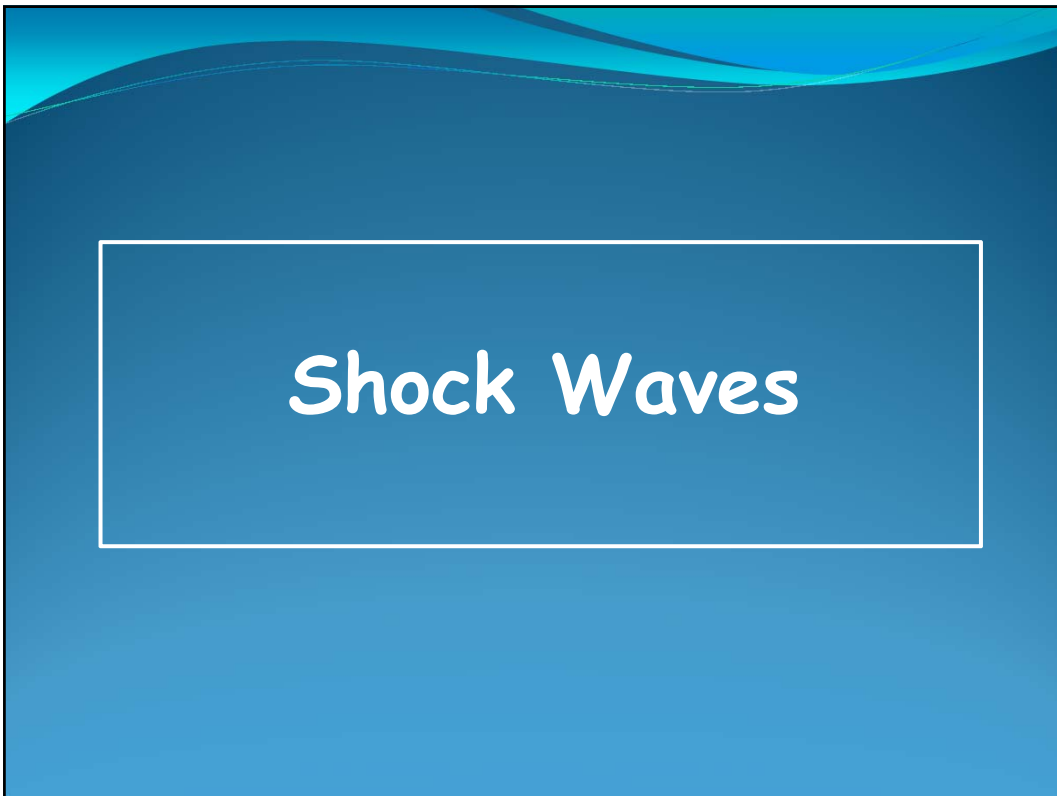


Doppler Effect



Jeans Instability





Shocks

1. Shocks are **sudden** transitions in flow properties such as density, velocity and pressure;
2. In shocks the kinetic energy of the flow is converted into **heat**, (pressure);
3. Shocks are **inevitable** if sound waves propagate over long distances;
4. Shocks always occur when a flow hits an obstacle **supersonically**
5. In shocks, the flow speed along the shock normal changes from **supersonic** to **subsonic**

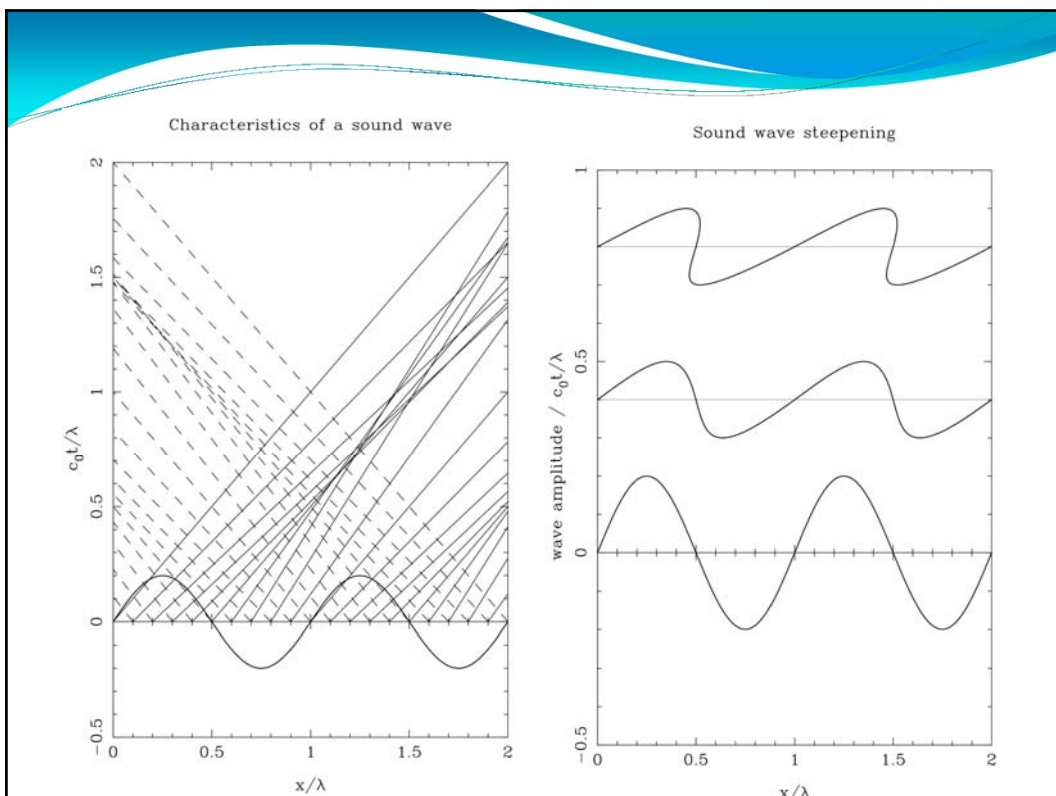
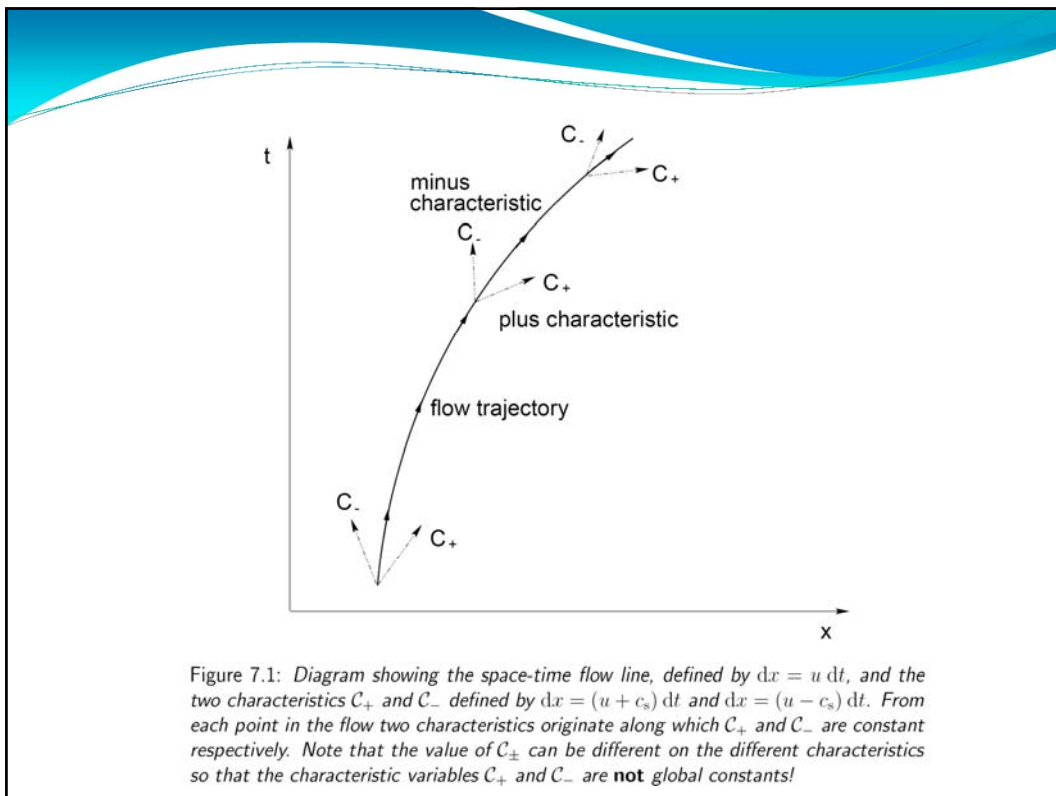
Shock must form

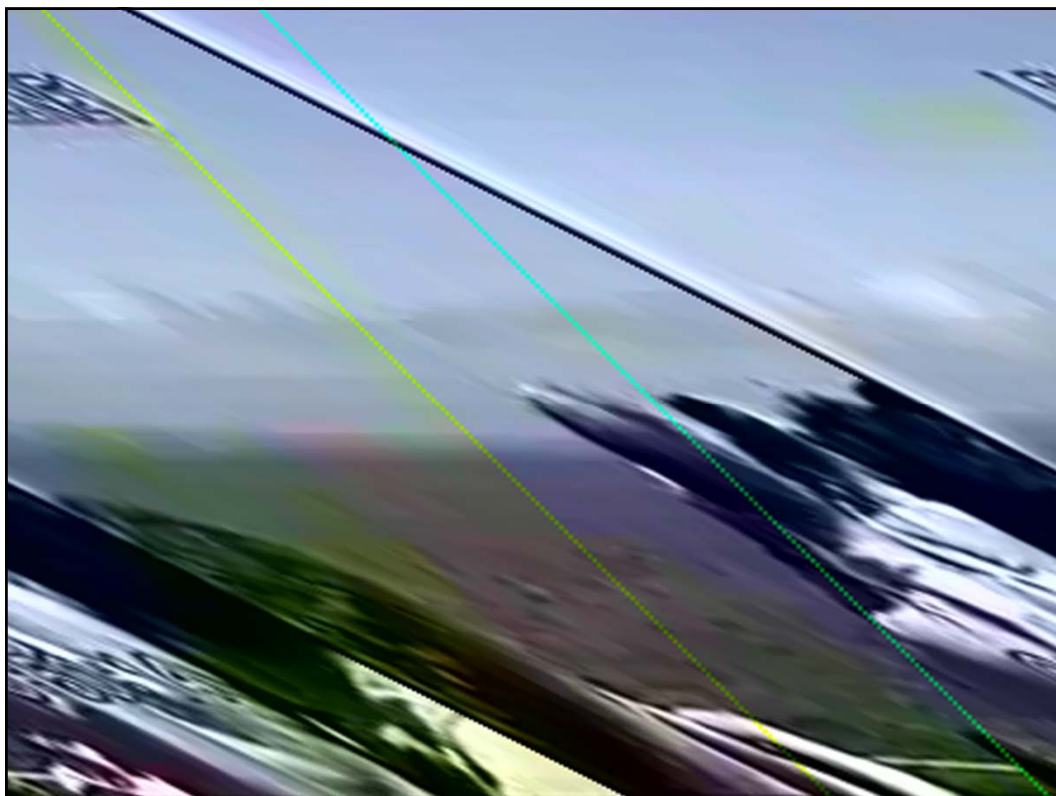
Wave Breaking

High-pressure/density regions move faster

$$u = \frac{2c_{s0}}{\gamma - 1} \left[\left(\frac{\rho}{\rho_0} \right)^{(\gamma-1)/2} - 1 \right]$$

$$\approx c_{s0} \left(\frac{\Delta\rho}{\rho} \right)$$

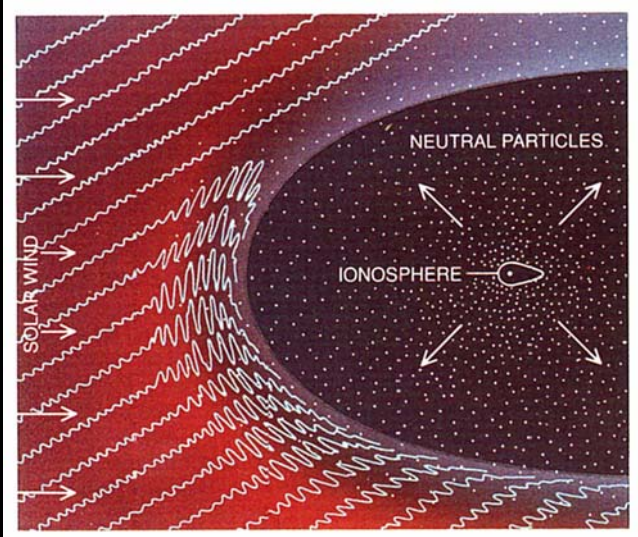




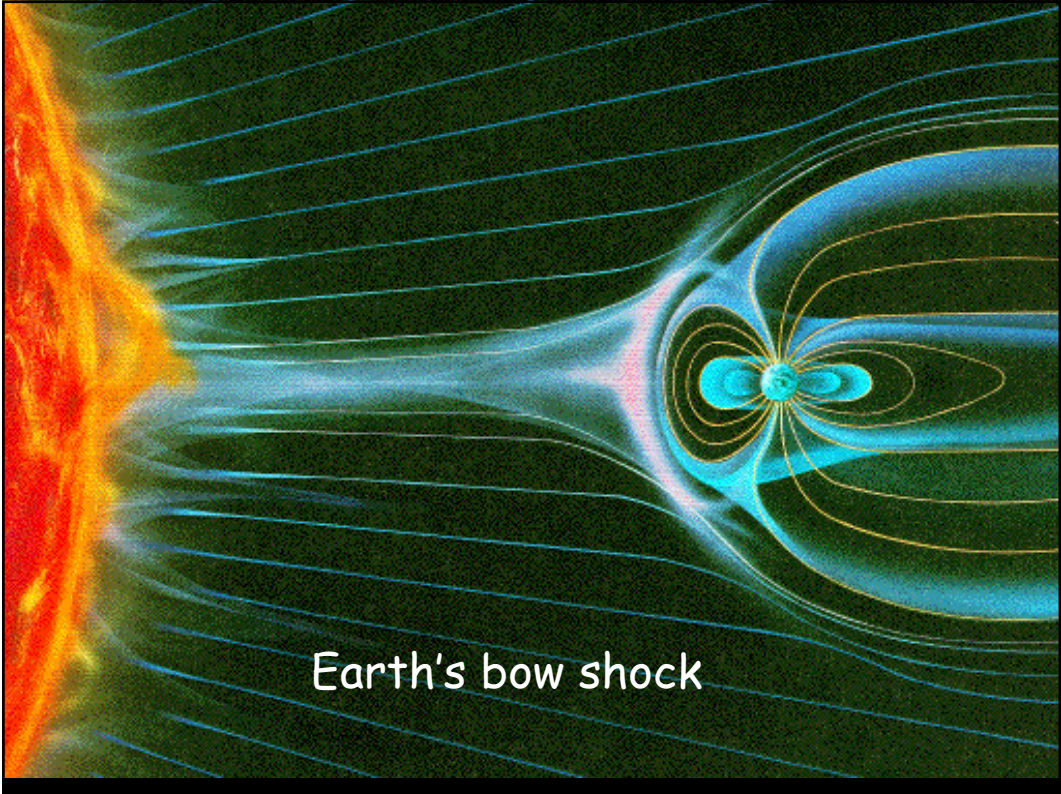
Chelyabinsk Meteorite (Feb. 2013): Sonic Boom



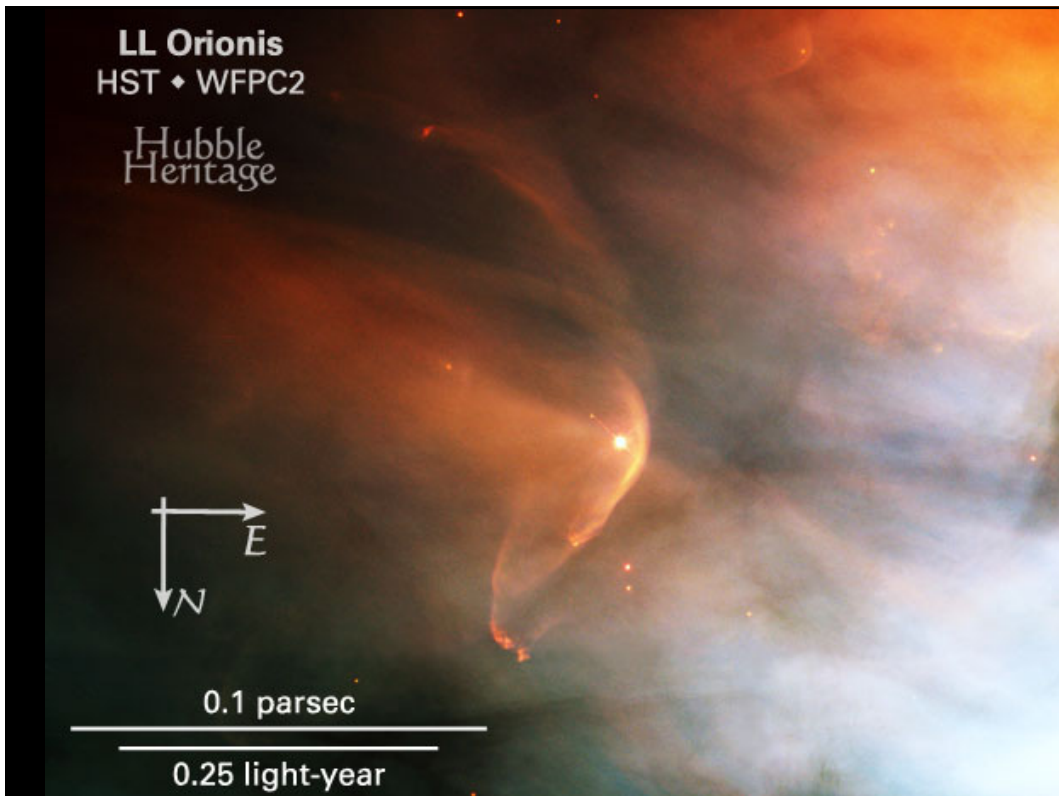
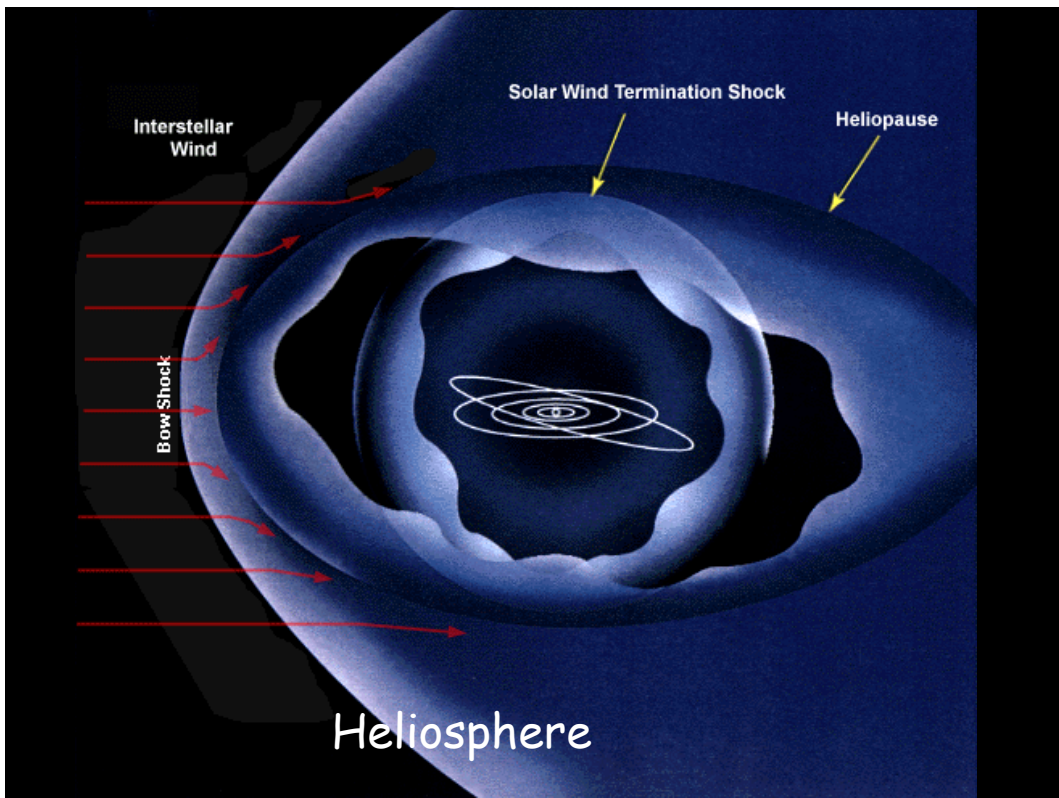
Examples of Astrophysical shocks

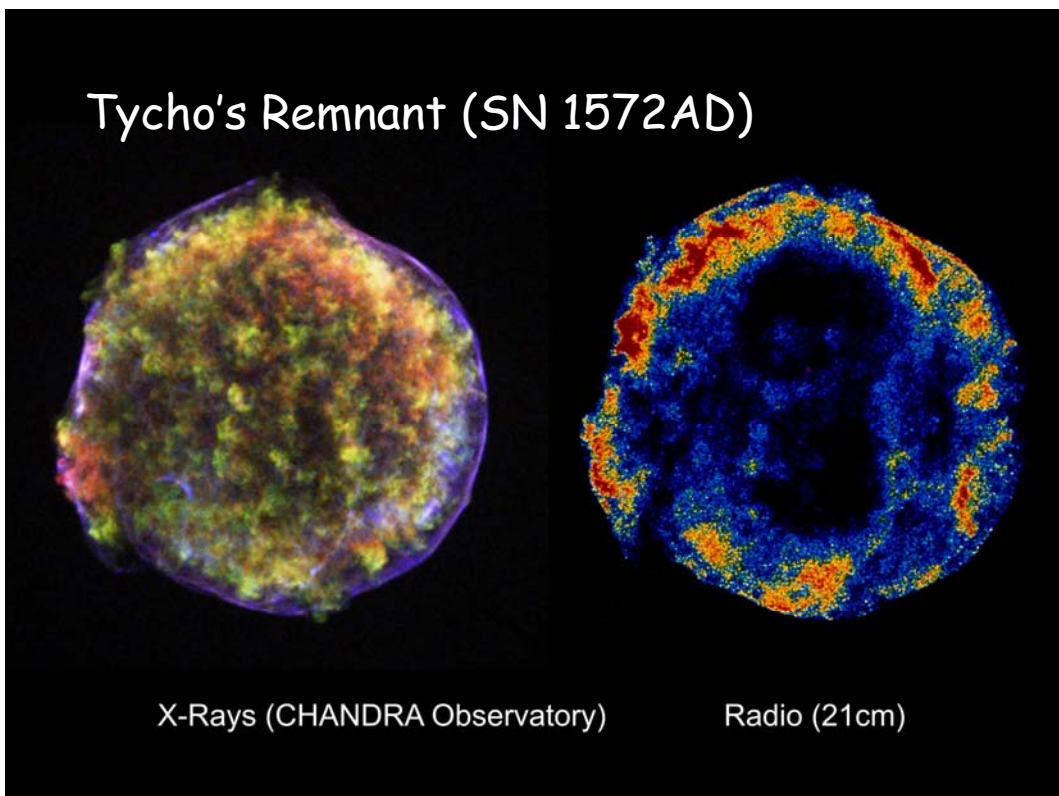
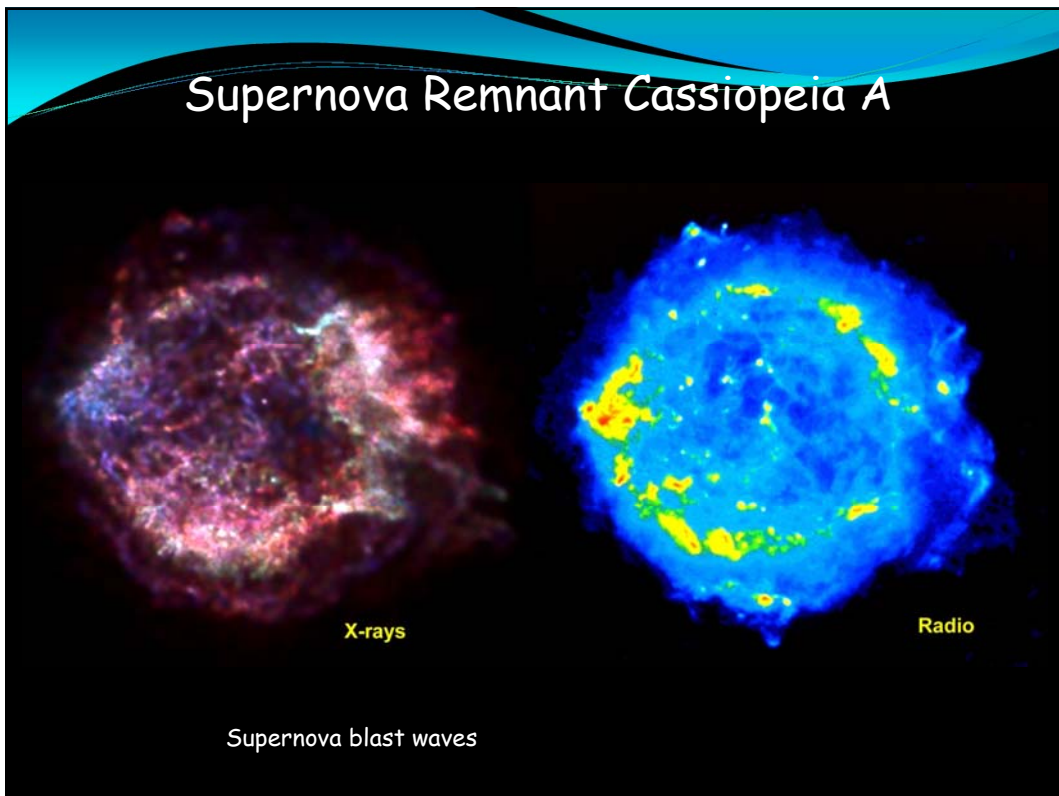


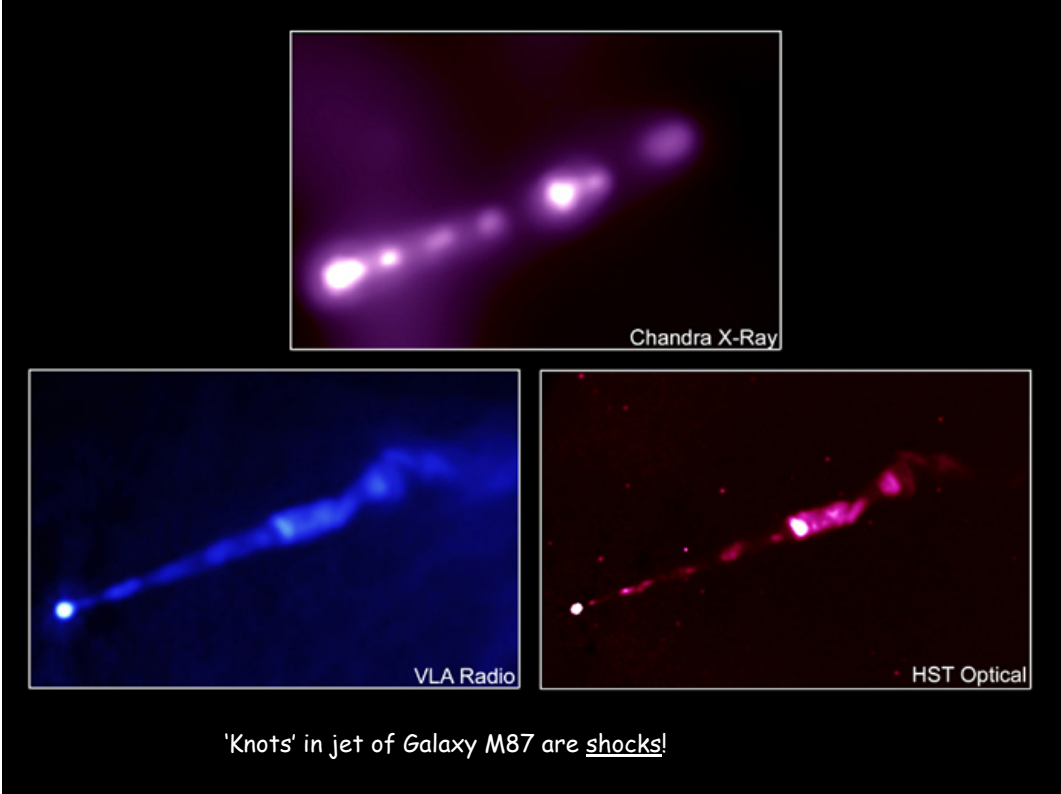
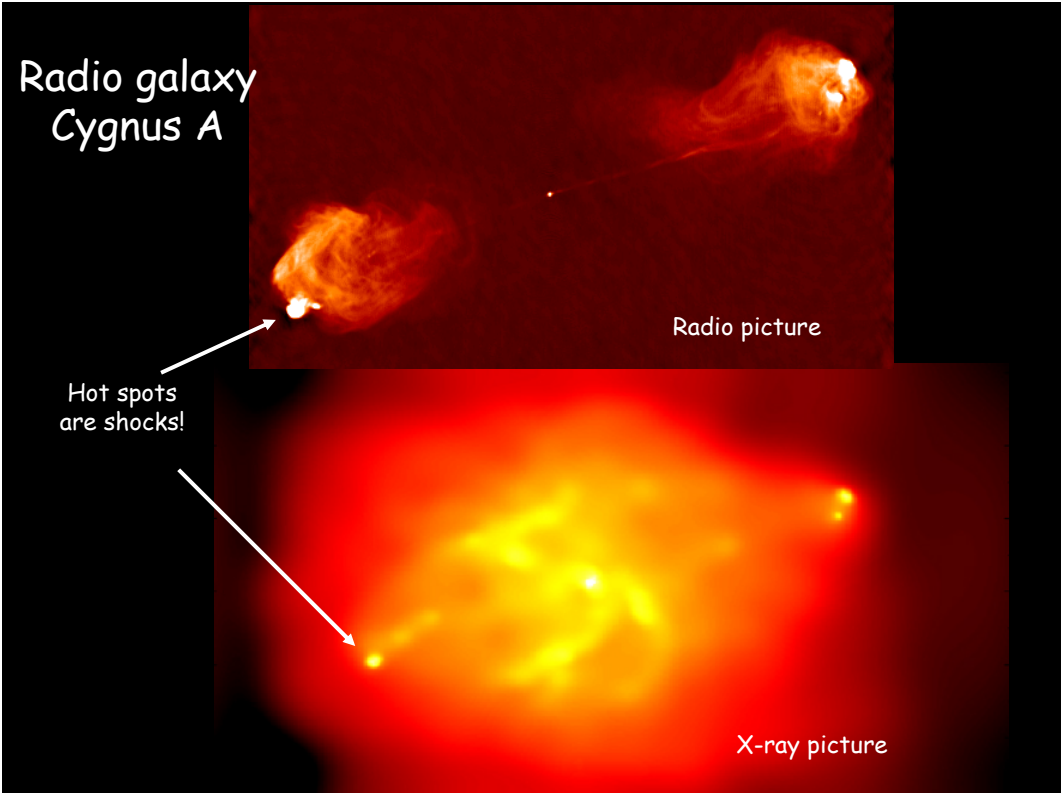
Cometary bow-shocks



Earth's bow shock







Summary : Shock Physics

Across an infinitely thin steady shock you have, in the shock frame where the shock is at rest, the following Rankine-Hugoniot Jump conditions:

Mass-flux conservation

$$\rho_1 V_{n1} = \rho_2 V_{n2}$$

Momentum-flux conservation

$$\rho_1 (V_{n1})^2 + P_1 = \rho_2 (V_{n2})^2 + P_2$$

$$V_{t1} = V_{t2}$$

Energy-flux conservation

$$\frac{1}{2}(V_{n1})^2 + \frac{\gamma P_1}{(\gamma-1)\rho_1} = \frac{1}{2}(V_{n2})^2 + \frac{\gamma P_2}{(\gamma-1)\rho_2}$$

Summary: Rankine-Hugoniot relations (for normal shock)

Fundamental parameter:
Mach Number

$$\mathcal{M}_s \equiv \frac{\text{shock speed}}{\text{sound speed}} = \frac{V_1}{c_{s1}}$$

R-H Jump Conditions
relate the up- and downstream
quantities at the shock:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2 + 2} \Rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_s^2 - (\gamma-1)}{\gamma+1}$$

From normal shock to oblique shocks:

All relations remain the same if one makes the replacement:

$$V_1 \Rightarrow V_{n1} = V_1 \cos \theta_1 ,$$

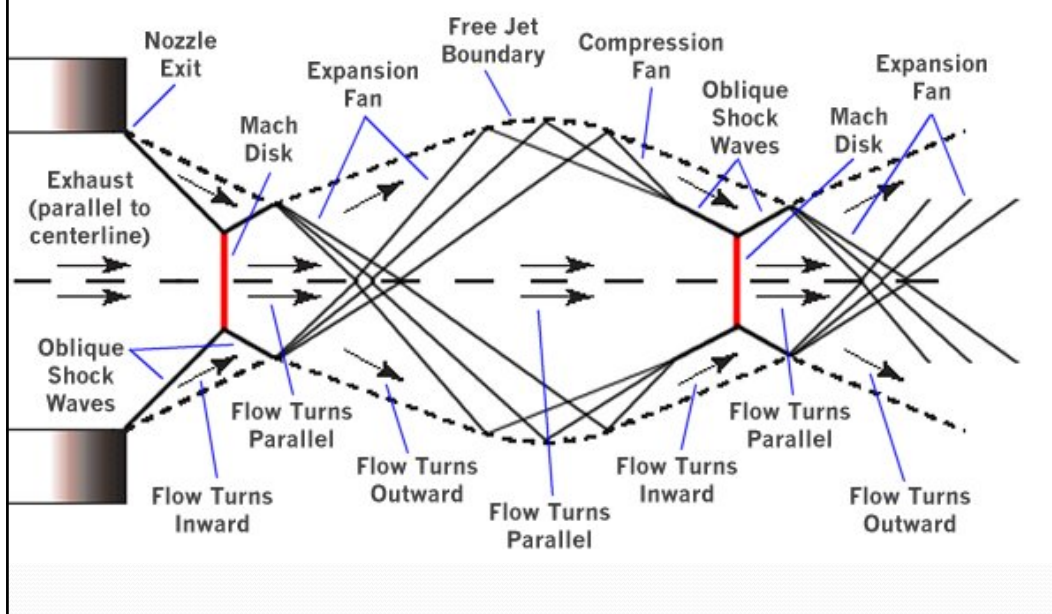
$$\mathcal{M}_s \Rightarrow \mathcal{M}_n = V_{n1} / c_{s1} = \mathcal{M}_s \cos \theta_1$$

θ is the angle between upstream velocity and normal on shock surface

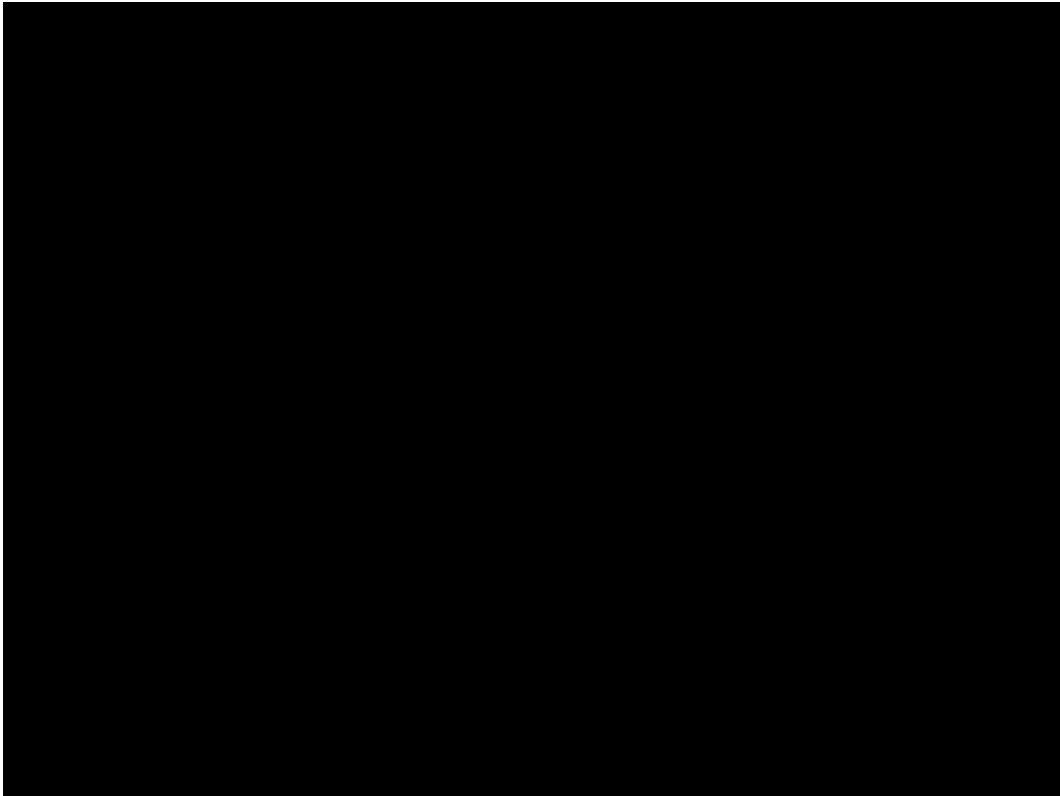
Tangential velocity along shock surface is unchanged

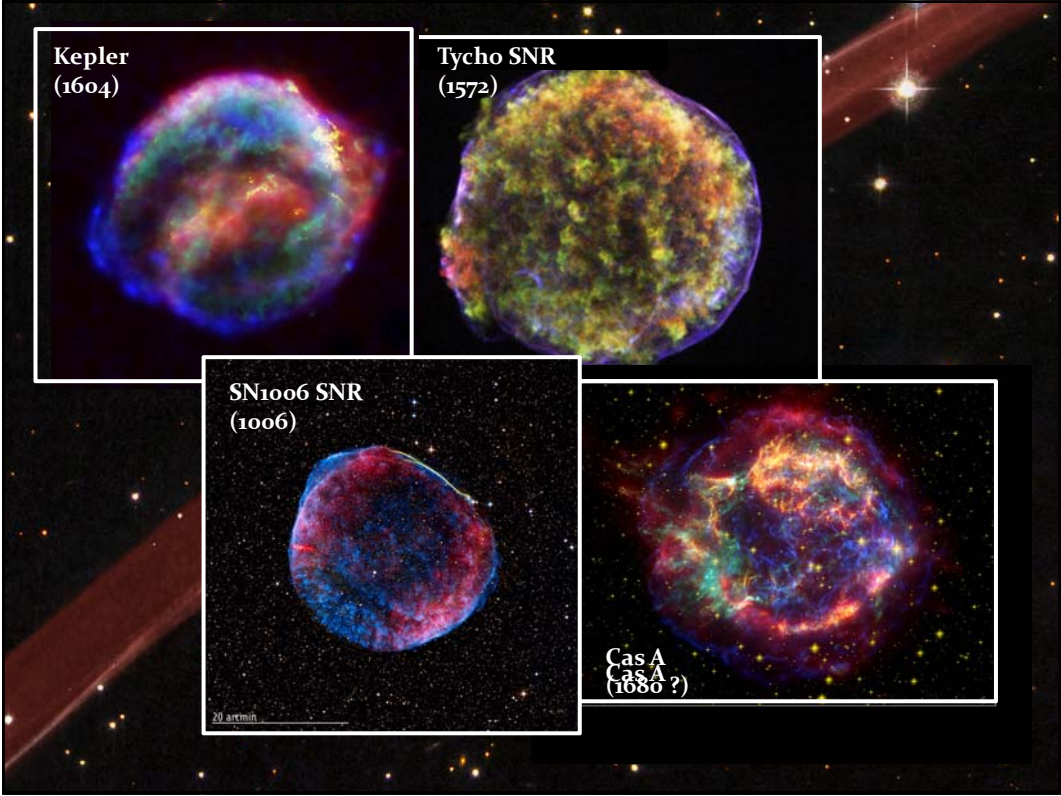
$$V_{t1} = V_1 \sin \theta_1 = V_{t2} = V_2 \sin \theta_2$$

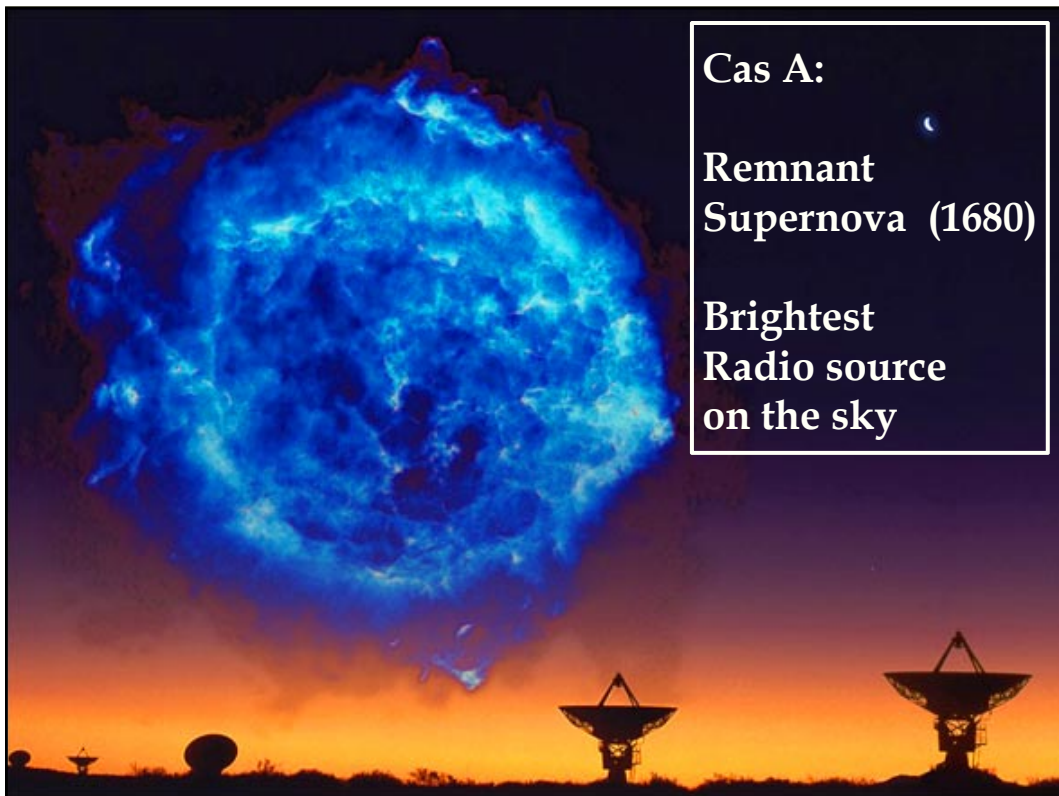
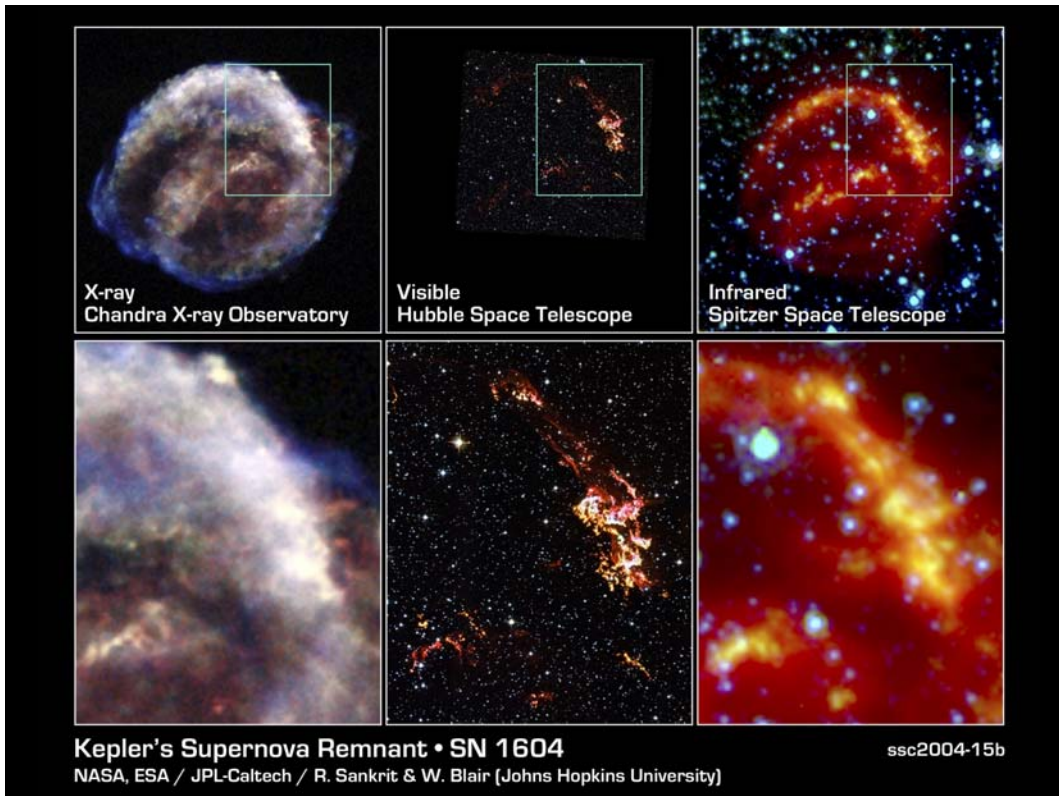
Example from Jet/Rocket engines

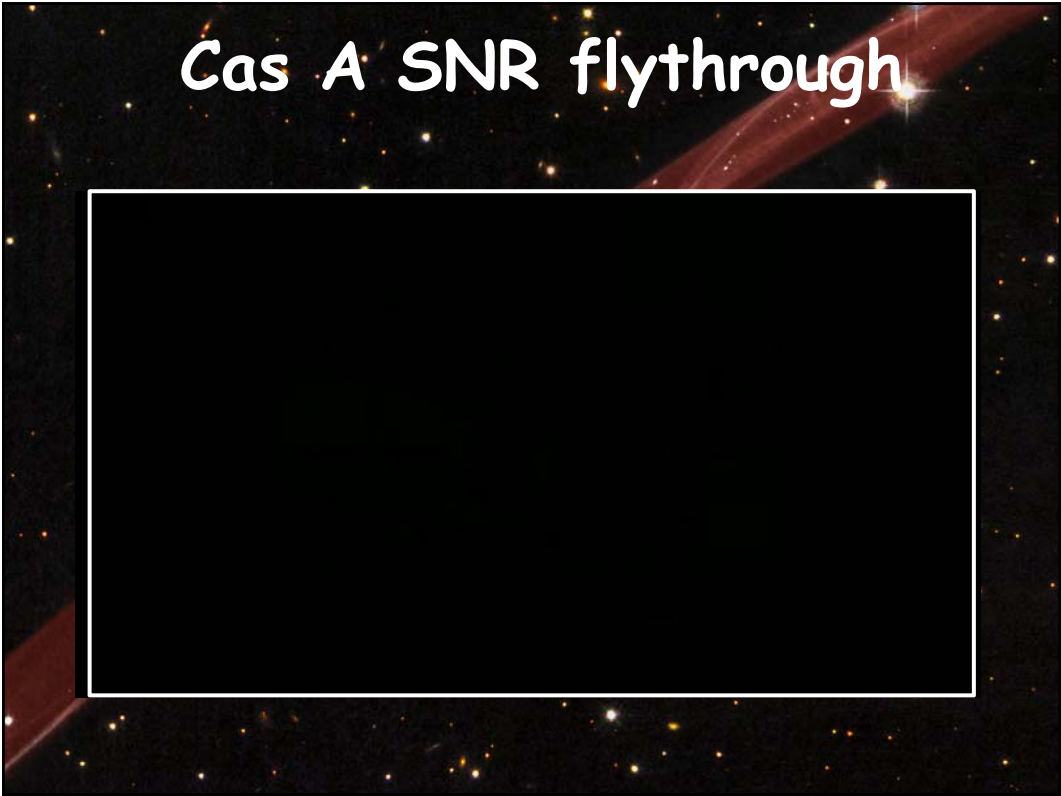
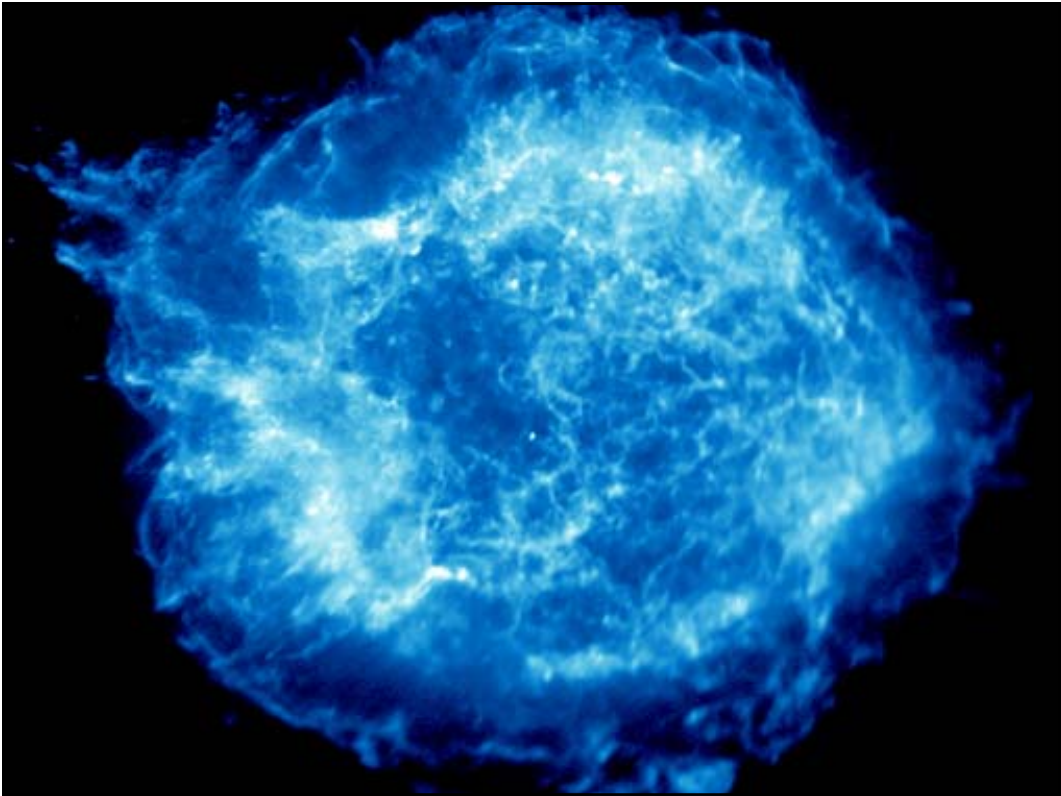








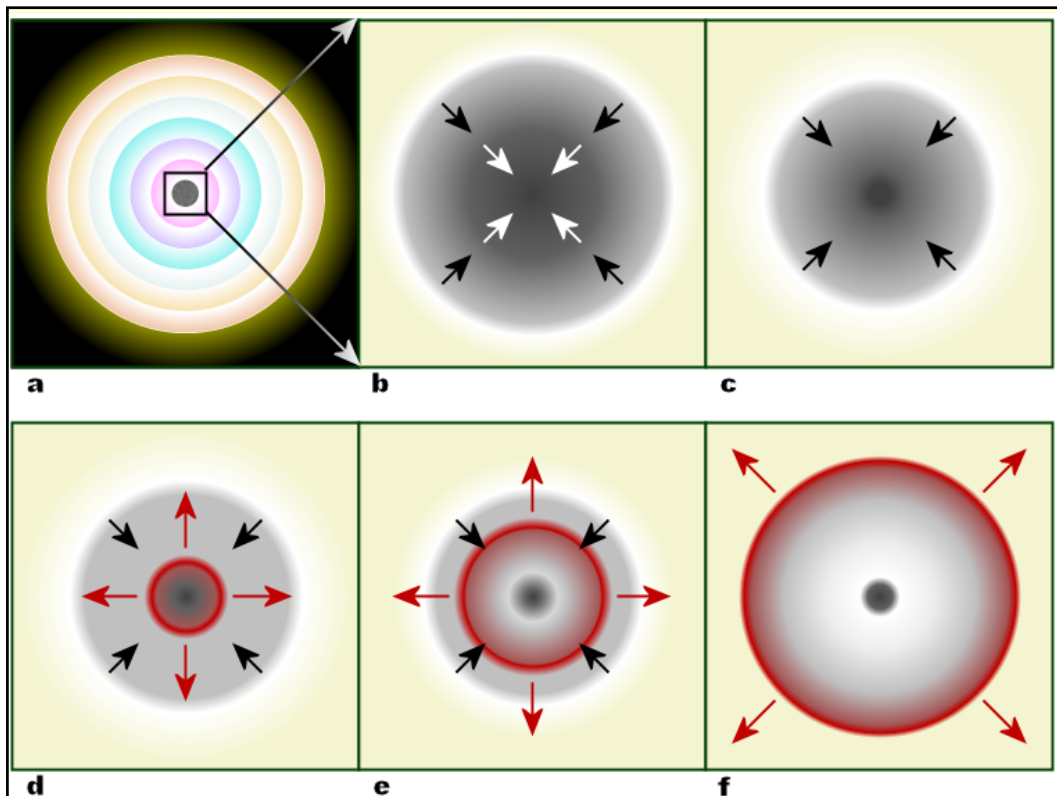
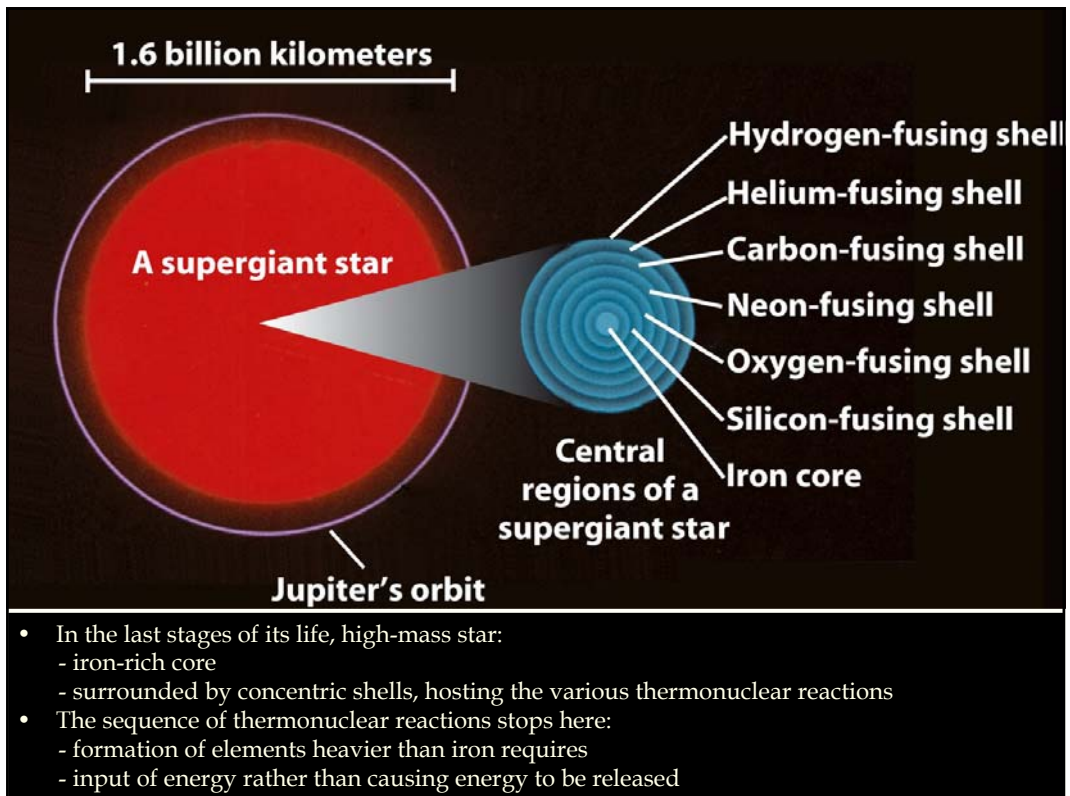


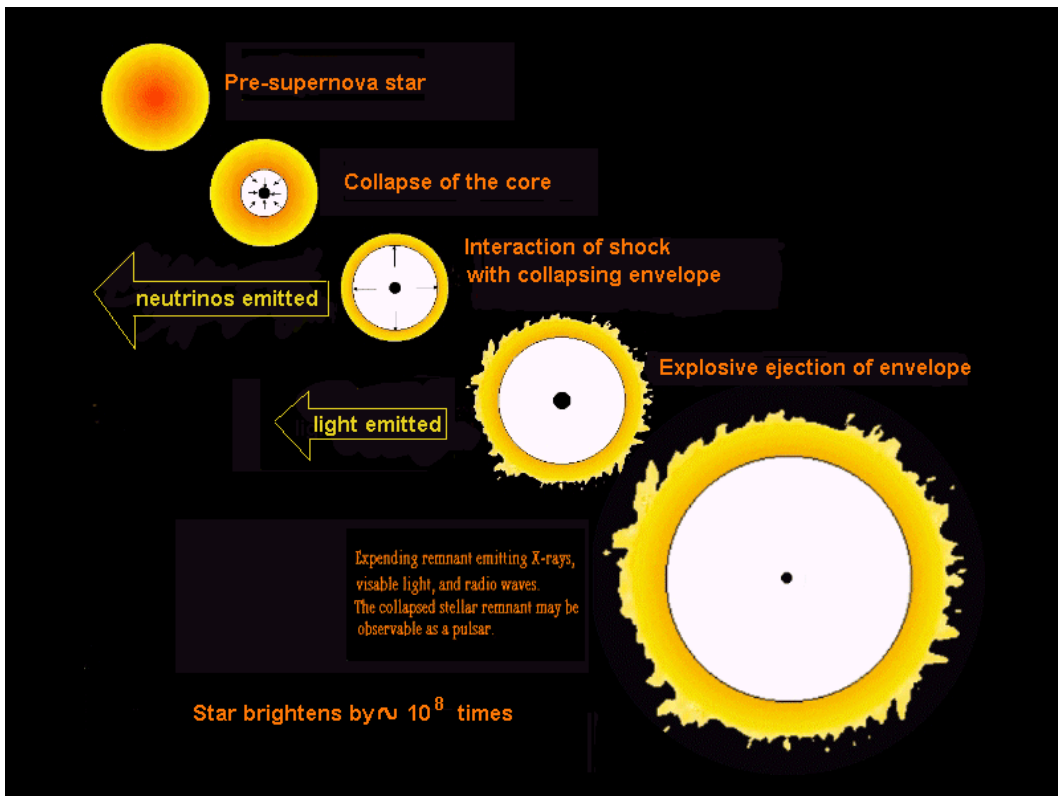


Theory of Supernova Blast Waves

<u>Supernovae:</u>	
Type Ia	Subsonic deflagration wave turning into a supersonic detonation wave in outer layers. <u>Mechanism:</u> explosive carbon burning in a mass-accreting white dwarf
Type Ib-Ic & Type II	<u>Core collapse</u> of massive star

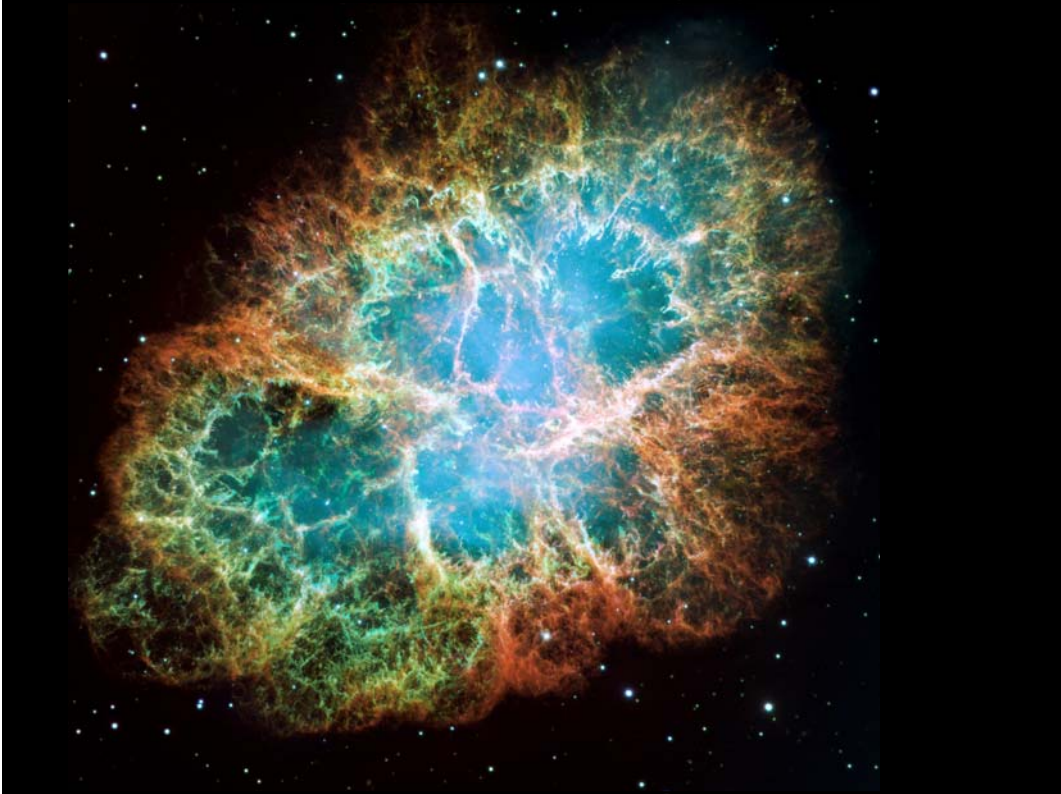
Core-Collapse SN

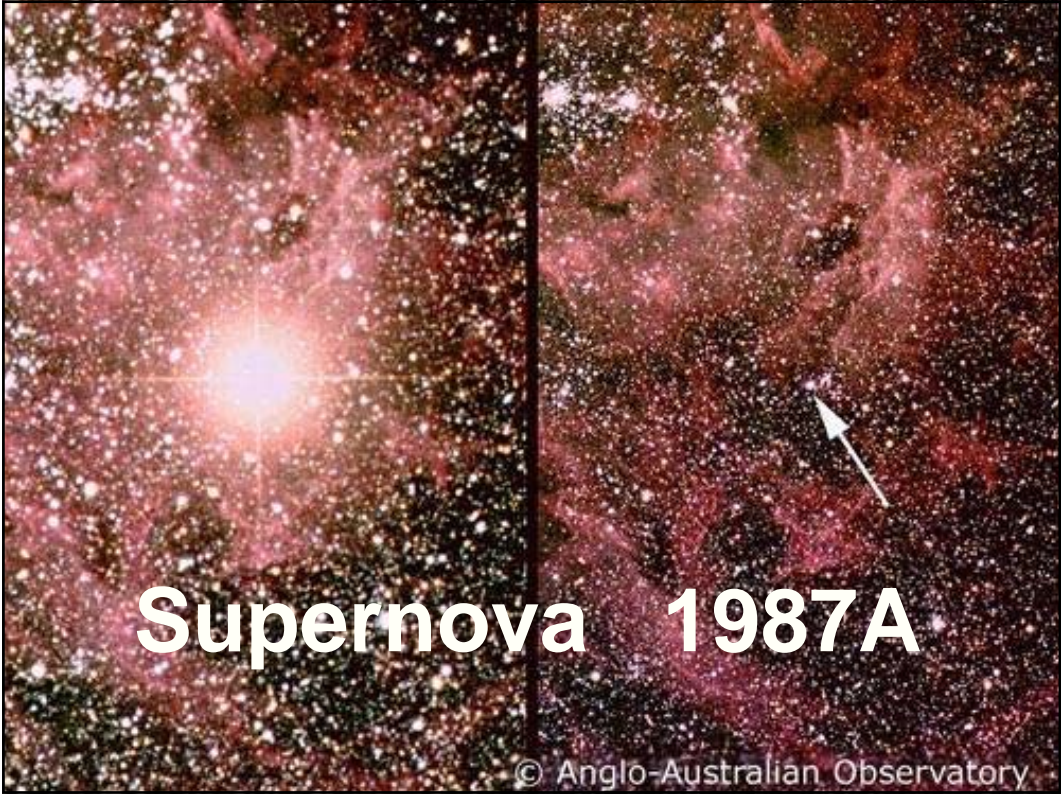
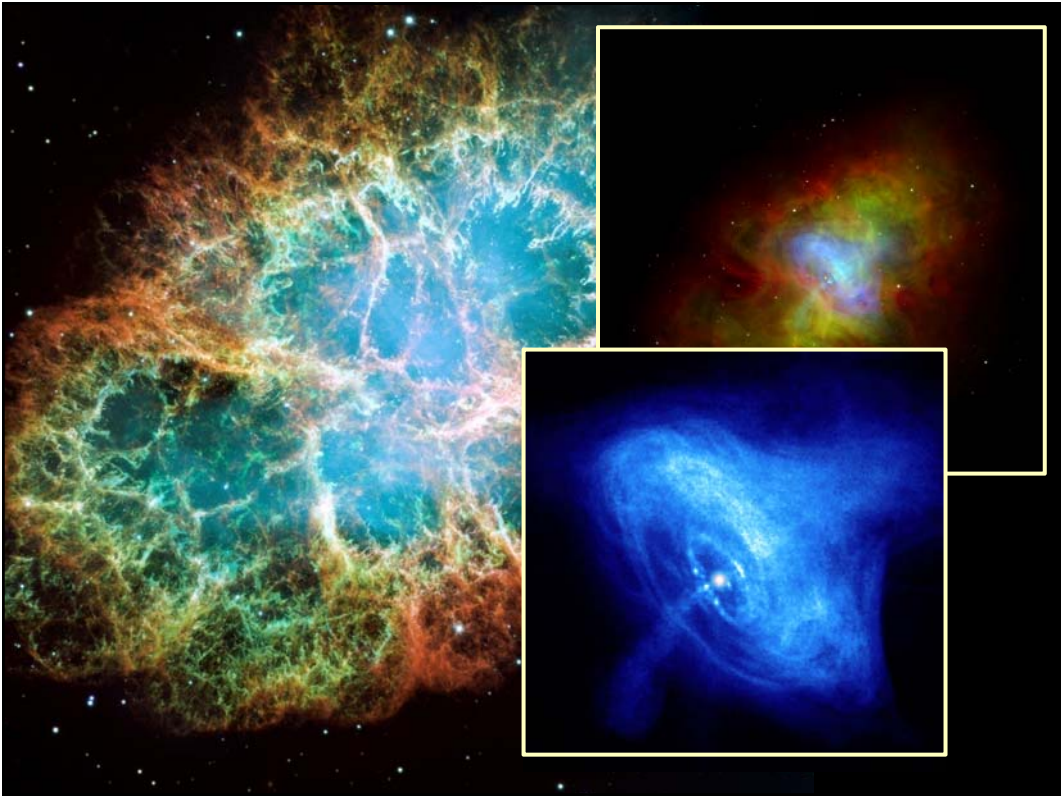




Supernova II Explosion: SN1054





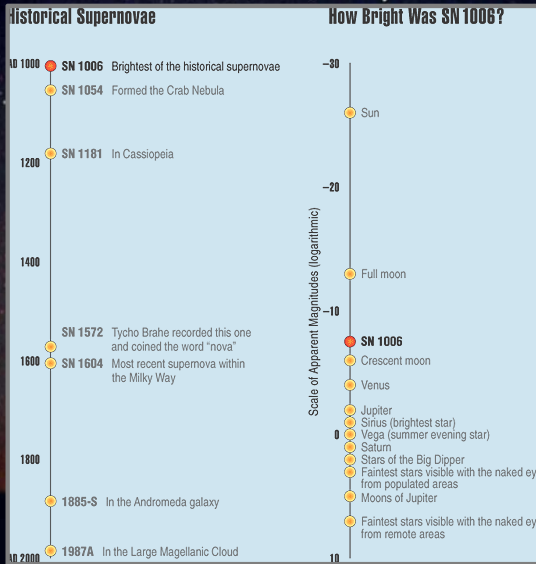


Thermonuclear SN (Supernova Ia)

SN1006

Supernova SN1006:
brightest stellar event recorded in history

SN1006



Supernova SN1006:

- brightness: $m = -7.5$
- distance: $d = 2.2 \text{ kpc}$
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history

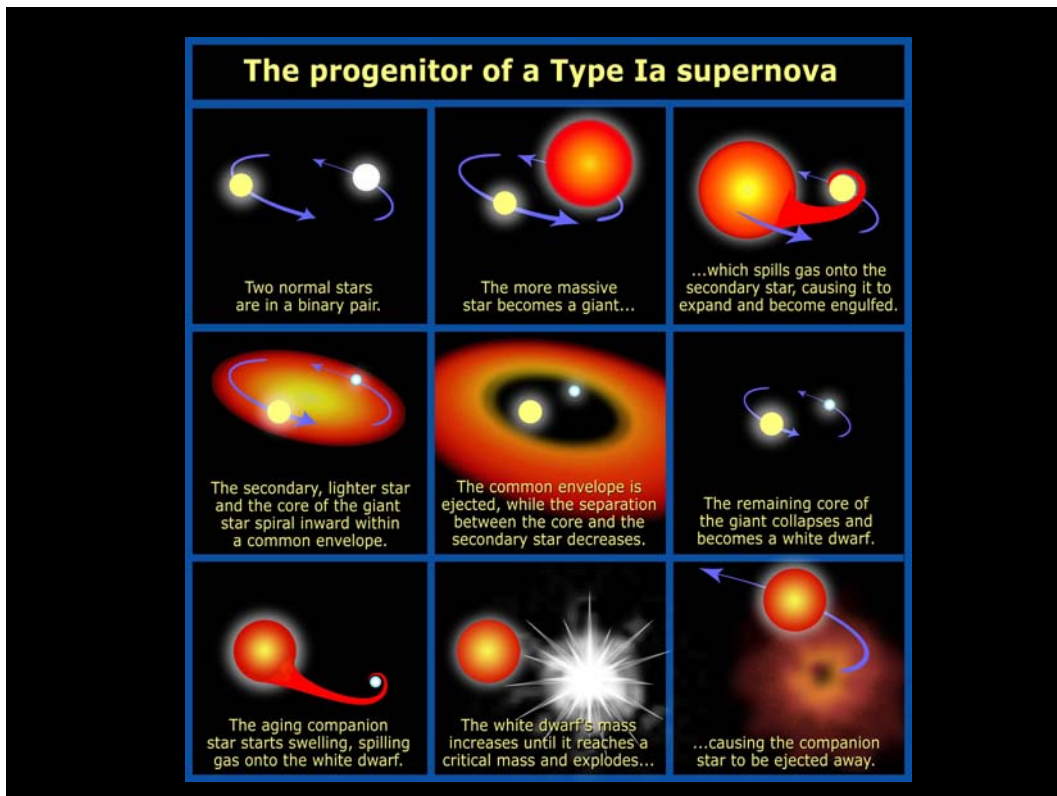
SN1006



Supernova SN1006:

- brightness: $m = -7.5$
- distance: $d = 2.2 \text{ kpc}$
- recorded: China, Egypt, Iraq, Japan, Switzerland, North America

Supernova SN1006:
brightest stellar event recorded in history



Blast Waves

Tsar Bomba Nuclear Explosion



Tsar Bomba Nuclear Explosion



Tsar Bomba Nuclear Explosion



Hiroshima, the Shockwave



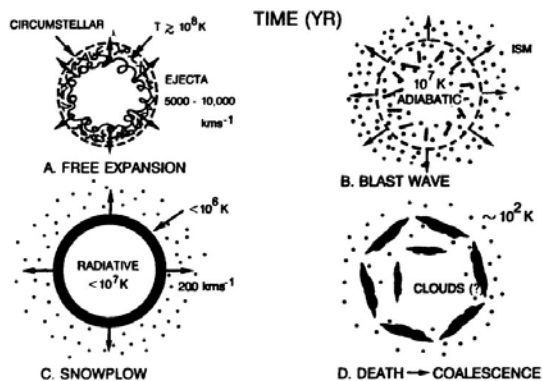
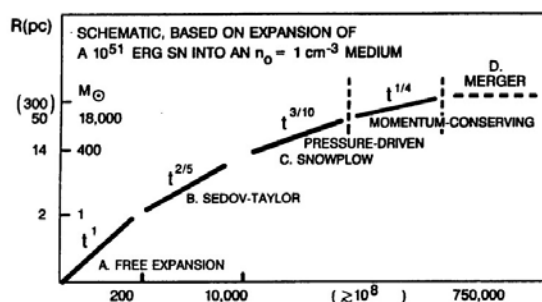
Sedov-Taylor
Expansion Law

Blast waves

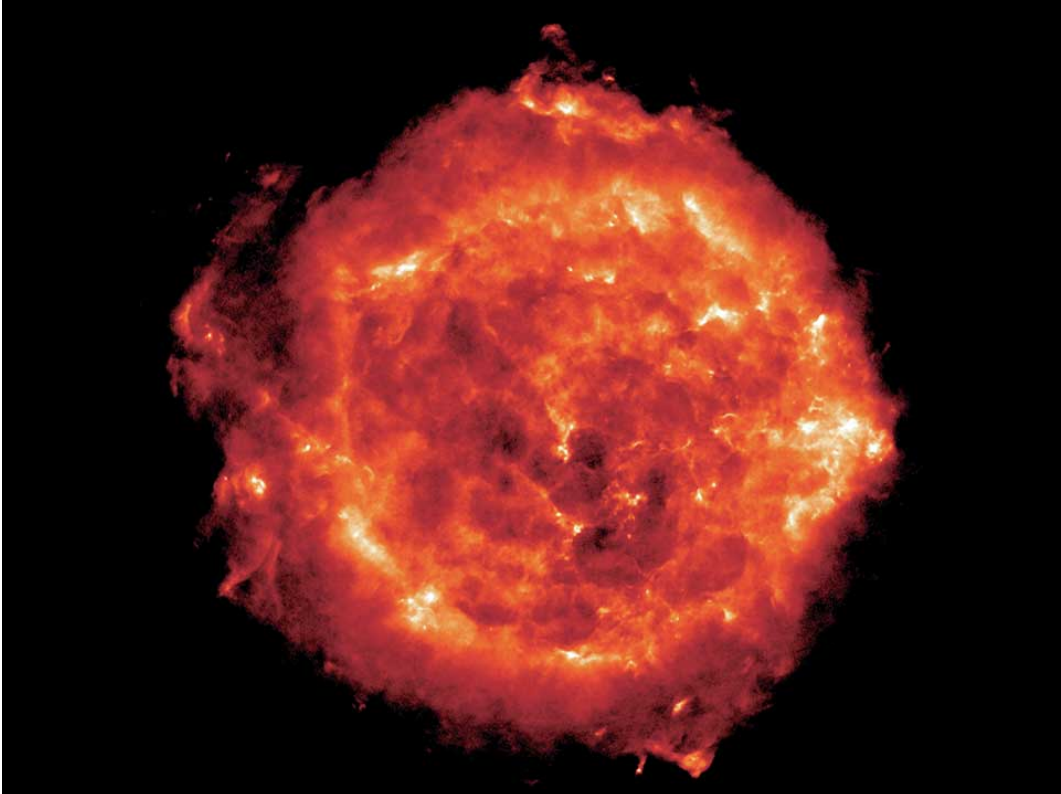
Main properties:

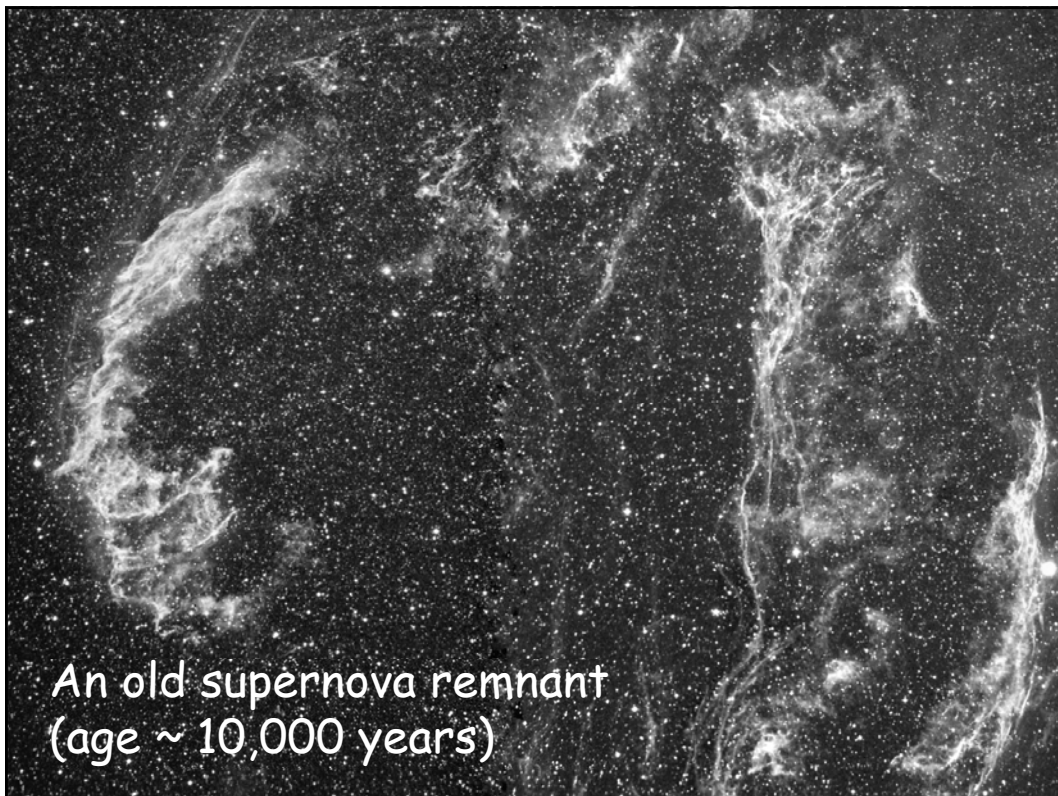
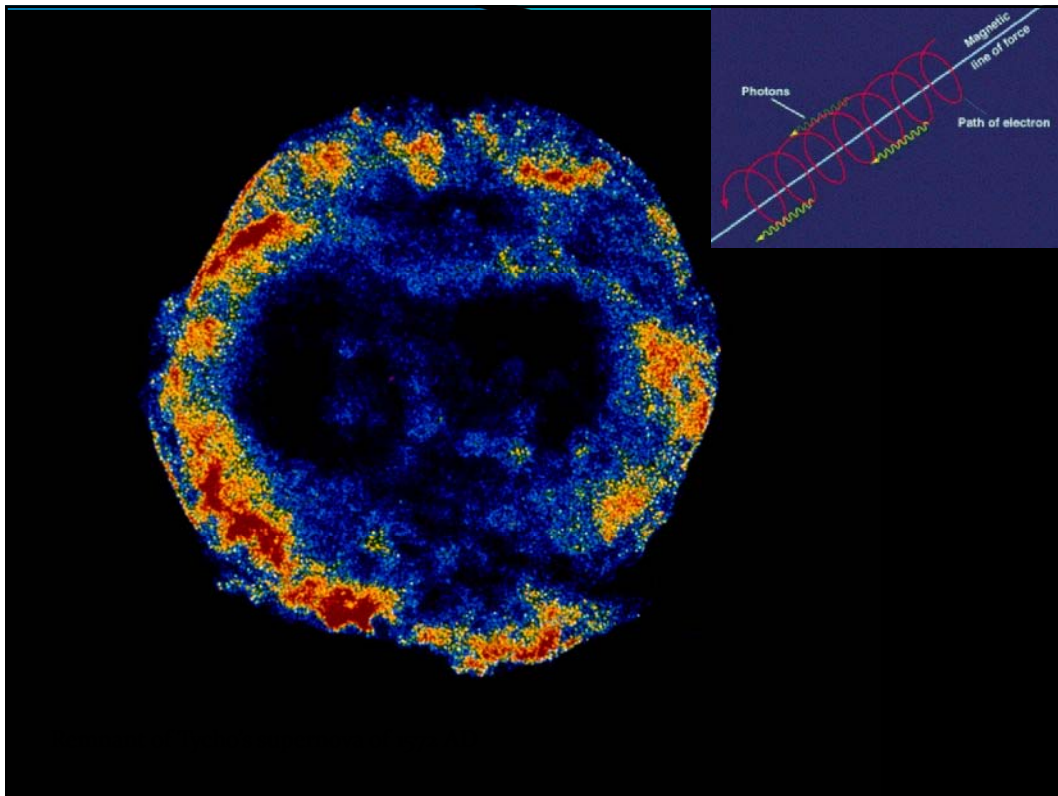
1. Strong shock propagating through the Interstellar Medium, or through the wind of the progenitor star;
2. Different expansion stages:
 - Free expansion stage ($t < 1000$ yr) $R \propto t$
 - Sedov-Taylor stage ($1000 \text{ yr} < t < 10,000$ yr) $R \propto t^{2/5}$
 - Pressure-driven snowplow ($10,000 \text{ yr} < t < 250,000$ yr) $R \propto t^{3/10}$

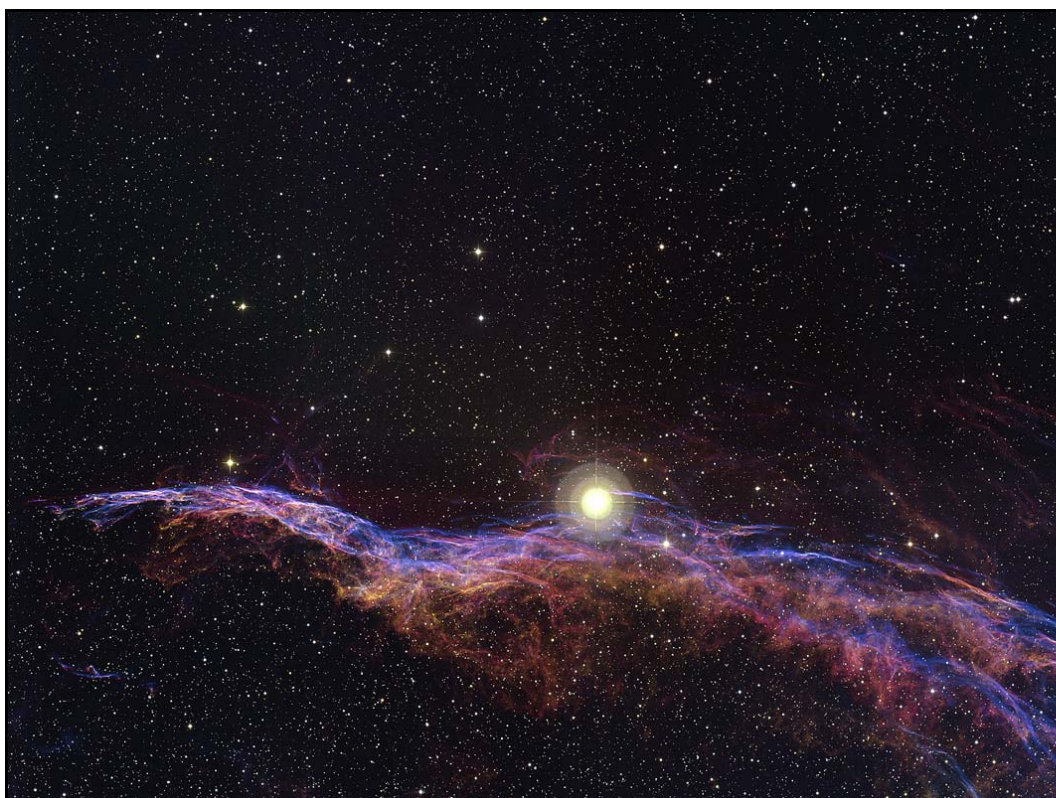
STANDARD SNR EVOLUTION



Tsar Bomba Nuclear Explosion







Free-expansion phase

Energy budget:

$$|E_{\text{grav}}| = \frac{3}{5} \frac{GM_c^2}{R_c} \approx 10^{53} \text{ erg} \Rightarrow \begin{cases} 99\% \text{ into neutrino's} \\ 1\% \text{ into mechanical energy} \end{cases}$$

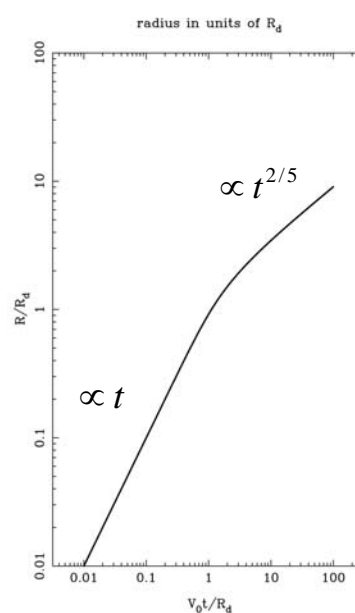
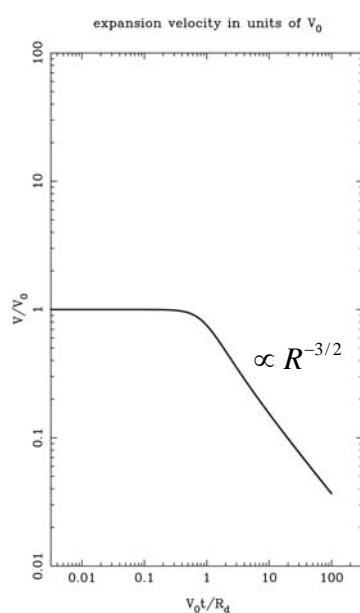
Expansion speed:

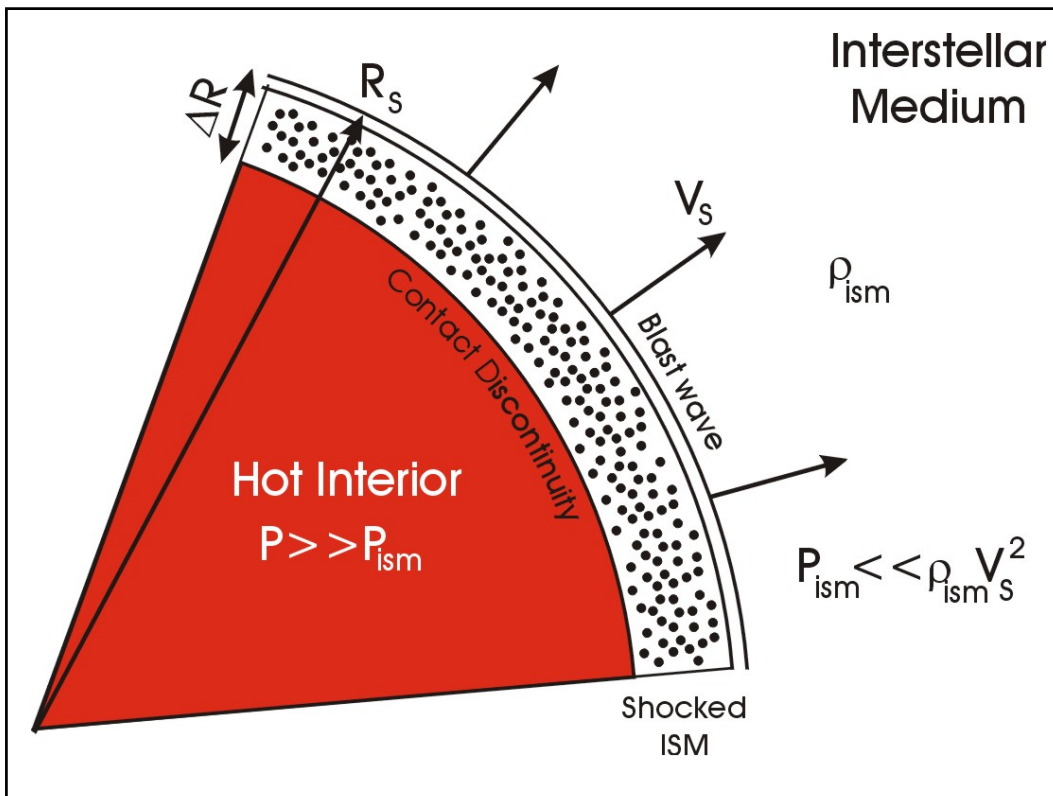
$$V_{\text{exp}} \approx \sqrt{\frac{2E_{\text{mech}}}{M_{\text{ej}}}} = 3000 \left(\frac{E_{\text{mech}}}{10^{51} \text{ erg}} \right)^{1/2} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{-1/2} \text{ km/s}$$

Sedov-Taylor stage

- Expansion starts to decelerate due to swept-up mass
- Interior of the bubble is reheated due to reverse shock
- Hot bubble is preceded in ISM by strong blast wave

$$V_s = \sqrt{\frac{2E_{\text{snr}}}{M_{\text{ej}}}} \times \left(\frac{1}{1 + (R/R_d)^3} \right)^{1/2} = V_0 \left(\frac{1}{1 + (R/R_d)^3} \right)^{1/2}$$





Interstellar Medium

Hot Interior
 $P \gg P_{\text{ism}}$

Contact Discontinuity

Shocked ISM

Blast wave

v_s

R_s

ΔR

ρ_{ism}

$P_{\text{ism}} \ll \rho_{\text{ism}} v_s^2$

Shock relations for strong (high-Mach number) shocks:

$$\left. \begin{aligned} \frac{\rho_2}{\rho_1} &= \frac{(\gamma+1)\mathcal{M}_s^2}{(\gamma-1)\mathcal{M}_s^2+2} \Rightarrow \frac{\gamma+1}{\gamma-1} \\ \frac{P_2}{P_1} &= \frac{2\gamma\mathcal{M}_s^2-(\gamma-1)}{\gamma+1} \Rightarrow \frac{2\gamma}{\gamma+1}\mathcal{M}_s^2 \end{aligned} \right\} \text{as } \mathcal{M}_s^2 \equiv \left(\frac{V_1}{c_{s1}}\right)^2 = \frac{\rho_1 V_1^2}{\gamma P_1} \Rightarrow \infty$$

$$\Leftrightarrow P_2 = \frac{2}{\gamma+1} \rho_1 V_1^2$$

Interstellar Medium

Hot Interior
 $P \gg P_{\text{ism}}$

Contact Discontinuity

Blast wave

Shocked ISM

ΔR

R_s

v_s

ρ_{ism}

$P_{\text{ism}} \ll \rho_{\text{ism}} v_s^2$

$$P_2 \approx \frac{2\gamma}{\gamma+1} \mathcal{M}_s^2 P_1 = \frac{2}{\gamma+1} \rho_{\text{ism}} v_s^2$$

Pressure behind strong shock (blast wave)

$$P_i = (\gamma - 1) e_i \approx (\gamma - 1) \frac{E_{\text{SNR}}}{\frac{4\pi}{3} R_s^3}$$

Pressure in hot SNR interior

Interstellar Medium

Hot Interior
 $P \gg P_{\text{ism}}$

Contact Discontinuity

Blast wave

Shocked ISM

ΔR

R_s

v_s

ρ_{ism}

$P_{\text{ism}} \ll \rho_{\text{ism}} v_s^2$

At contact discontinuity:
equal pressure on both sides!

$$\frac{2}{\gamma+1} \rho_{\text{ism}} v_s^2 \approx (\gamma - 1) \frac{E_{\text{SNR}}}{\frac{4\pi}{3} R_s^3}$$

This procedure is allowed because of high sound speeds in hot interior and in shell of hot, shocked ISM:
No large pressure differences are possible!

Interstellar Medium

At contact discontinuity: equal pressure on both sides!

$$\frac{2}{\gamma + 1} \rho_{\text{ism}} V_s^2 \approx (\gamma - 1) \frac{E_{\text{SNR}}}{\frac{4\pi}{3} R_s^3}$$

$$V_s = \frac{dR_s}{dt} \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} R_s^{-3/2}$$

Relation between velocity and radius gives expansion law!

Interstellar Medium

$$R_s^{3/2} dR_s \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} dt$$

Step 1: write the relation as difference equation

Diagram illustrating the structure of a supernova remnant. The interior is labeled "Hot Interior" with $P \gg P_{\text{ism}}$. The outer boundary is the "Blast wave" moving at velocity v_s . The region between the blast wave and the "Contact Discontinuity" is the "Shocked ISM". The contact discontinuity separates the hot interior from the "Interstellar Medium" (ISM) with density ρ_{ism} . The radius is R_s and the thickness of the shell is ΔR . The pressure in the shocked ISM is given by $P_{\text{ism}} \ll \rho_{\text{ism}} v_s^2$.

$$R_s^{3/2} dR_s \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} dt$$

$$\frac{2}{5} d(R_s^{5/2}) \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} dt$$

Step 2: write as total differentials and.....

Diagram illustrating the structure of a supernova remnant, identical to the one above.

$$R_s^{3/2} dR_s \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} dt$$

$$\frac{2}{5} d(R_s^{5/2}) \approx \sqrt{\frac{8\pi}{3(\gamma^2 - 1)}} \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/2} dt$$

.....integrate to find the Sedov-Taylor solution

$$R_s(t) \approx C_\gamma \left(\frac{E_{\text{snr}}}{\rho_{\text{ism}}} \right)^{1/5} t^{2/5},$$

$$C_\gamma = \left(\frac{5}{2} \right)^{2/5} \left(\frac{8\pi}{3(\gamma^2 - 1)} \right)^{1/5} \approx 1.96$$

Sedov & Taylor

