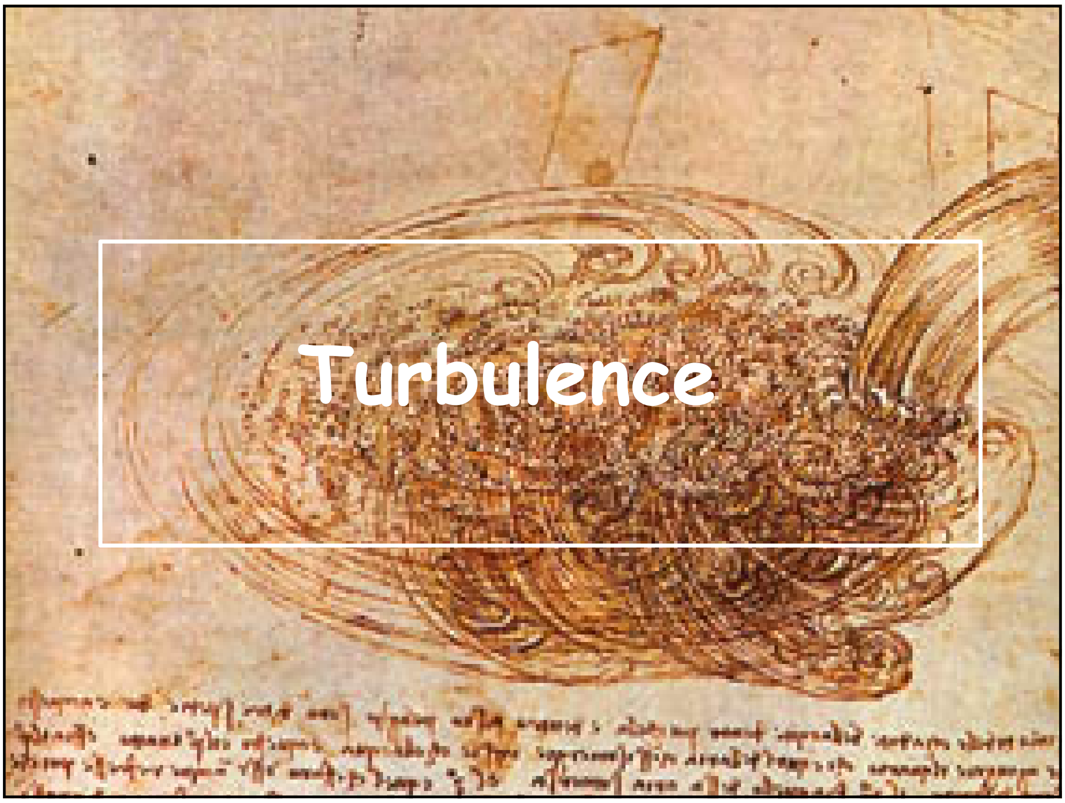
Turbulence

Instability

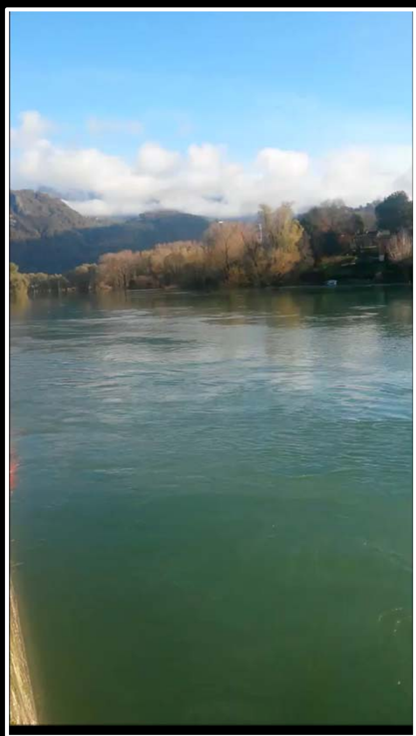
1) All flows become unstable above a certain Reynolds number.
2) At low Reynolds numbers flows are laminar.
3) For high Reynolds numbers flows are turbulent.
4) The transition occurs anywhere between R ~ 2000 and 10^6 , depending on the flow.
5) For laminar flow problems, flows can be solved using the energy equations developed previously.
6) For turbulent flows, the computational effort involved in solving those for all time and length scales is prohibitive.
7) An engineering approach to calculate time-average flow fields for turbulent flows has been developed.



Turbulence

the
Da Vinci
swirls

Merate, Italy (Nov. 2014)



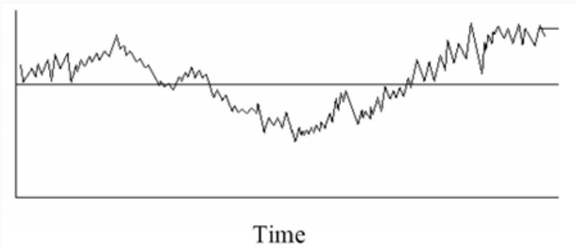
the Turbulence "Experience"

What is turbulence ?

1) Unsteady, aperiodic motion in which all three velocity components fluctuate, mixing matter, momentum and energy.

2) Decompose velocity into mean and fluctuating parts:

$$\vec{v}(t) = \vec{v}_0 + \vec{v}_1(t) + \ldots$$

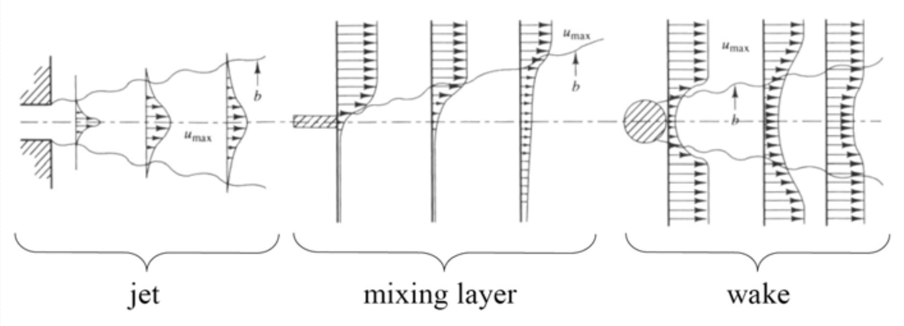


3) Similar fluctuations for pressure, temperature, and species concentration values

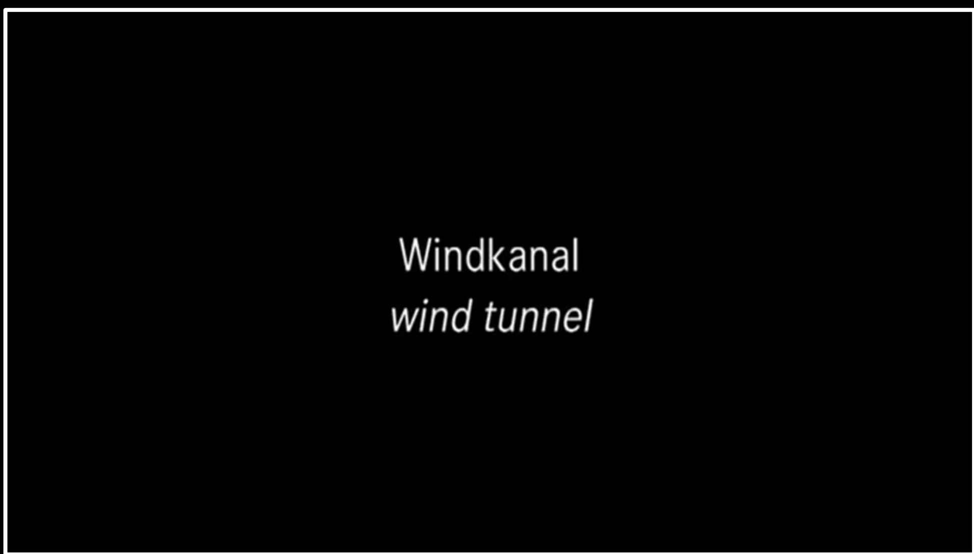
Examples of simple turbulent flows

1) Some examples of simple turbulent flows are
   - a jet entering a domain with stagnant fluid
   - a mixing layer
   - the wake behind objects such as cilinders

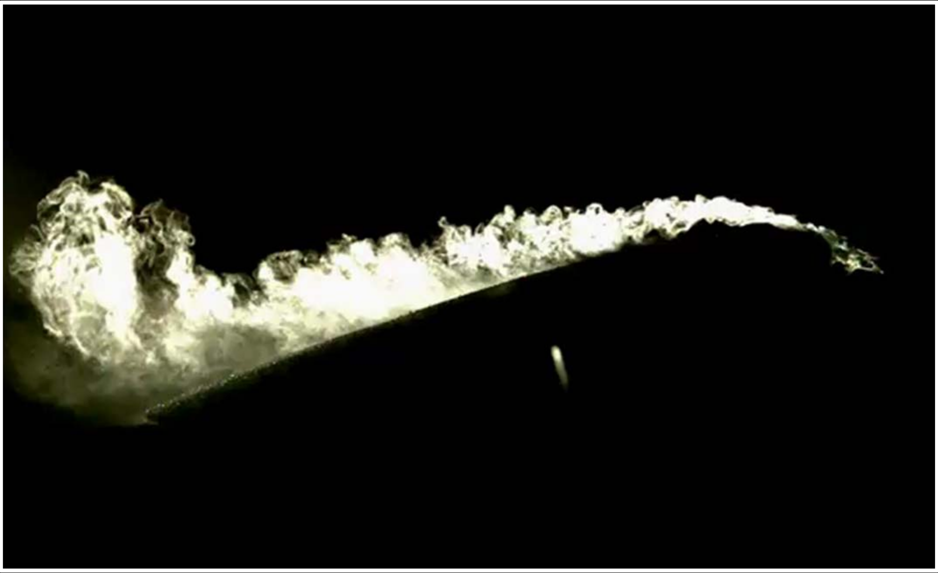
2) Such flows are often used as test cases to validate the ability of computational fluid dynamics software to accurately predict fluid flows.



Turbulent Wakes Mercedes



Turbulent Whirls over Edge



What is turbulence ?

Turbulent flows have the following characteristics:

1) Turbulent flows have **irregularity** or randomness. A full deterministic approach is very difficult.Turbulent flows are usually described statistically. Turbulent flows are always chaotic. But not all chaotic flows are turbulent. Waves in the ocean, for example, can be chaotic but are not necessarily turbulent.

2) The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent. If a flow is chaotic, but not diffusive, it is not turbulent. The trail left behind a jet plane, that seems chaotic, but does not diffuse for miles is not turbulent.

3) Turbulent flows always occur at **high Reynold numbers**. They are caused by the complex interaction between the viscous terms and the inertia terms in the momentum equations.

4) Turbulent flows are **rotational;** ie., they have non-zero vorticity. Mechanisms such as the stretching of 3-D vortices play a key role in turbulence.

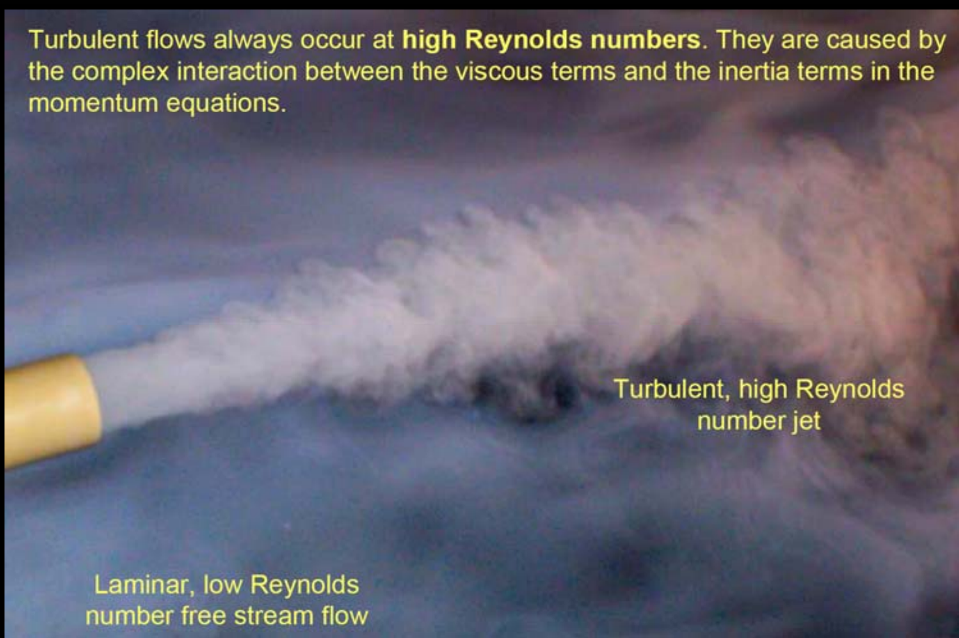
What is turbulence ?

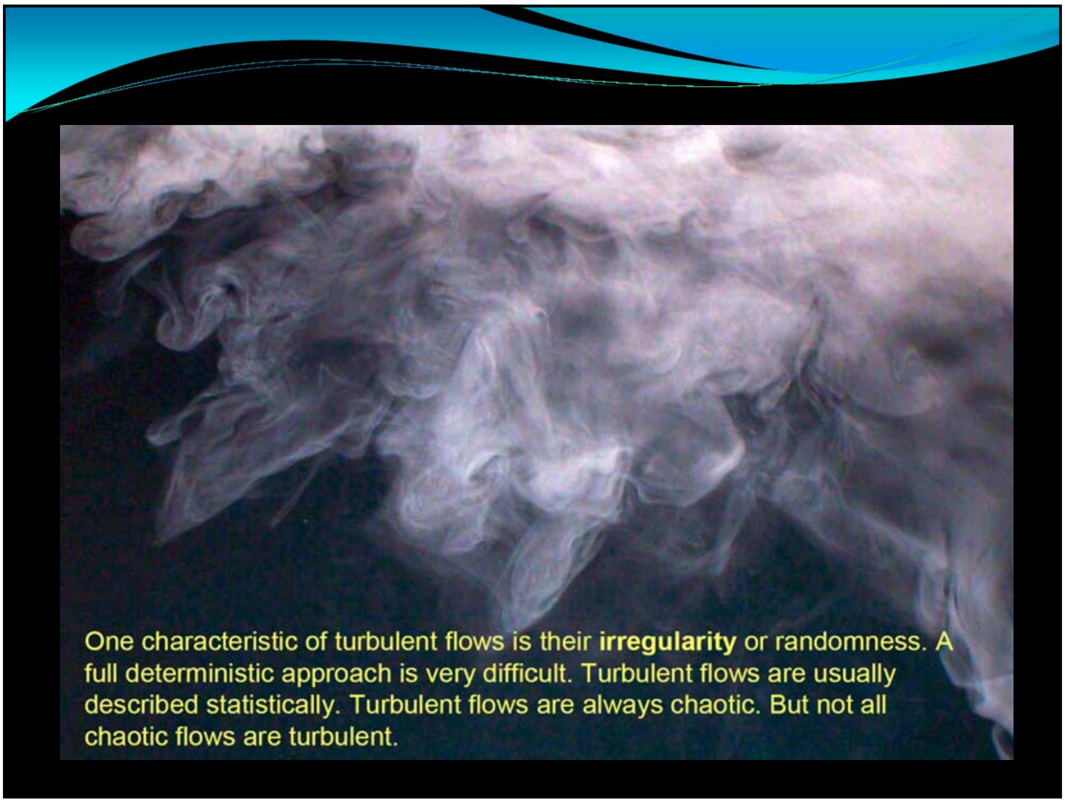
Turbulent flows have the following characteristics:

5) Turbulent flows are **dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied. Random motions that have insignificant viscous losses, such as random sound waves, are not turbulent.

6) Turbulence is a **continuum** phenomenon. Even the smallest eddies are significantly larger than the molecular scales. Turbulence is therefore governed by the equations of fluid mechanics.

7) Turbulent flows are flows. Turbulence is a **feature of fluid flow**, not of the fluid. When the Reynolds number is high enough, most of the dynamics of turbulence are the same whether the fluid is an actual fluid or a gas. Most of the dynamics are then independent of the properties of the fluid.

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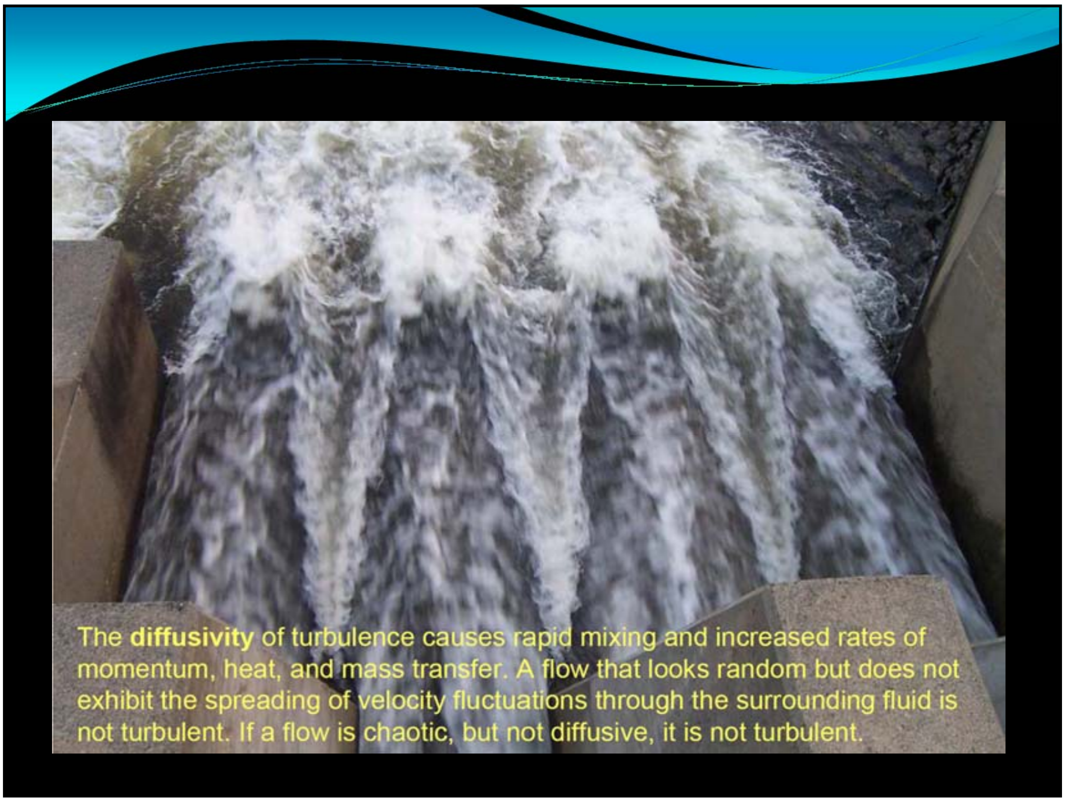
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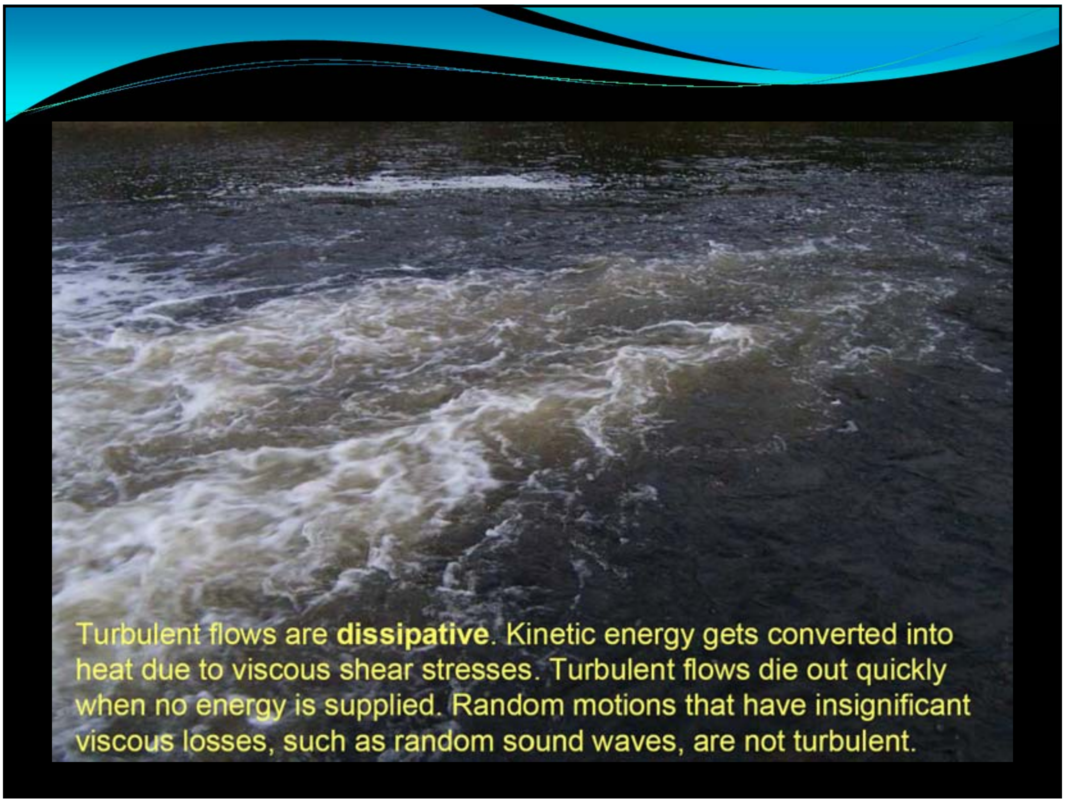
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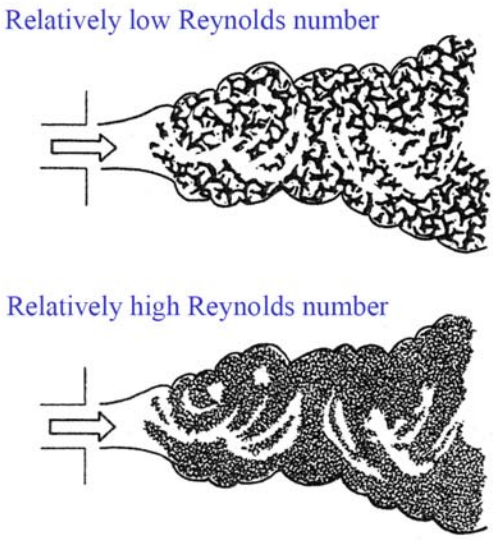
Turbulent flows are **rotational**; that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.
Vortices

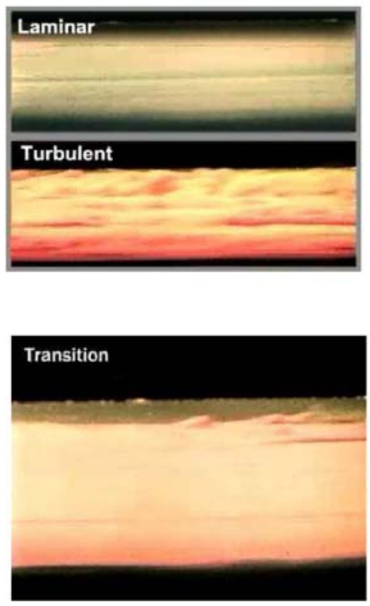
Different Reynold numbers

- Change in appearance and nature of turbulence as a function of Reynolds number:
- Turbulence in jet



Transition to Turbulence

- Photographs show flow in a boundary layer
- Below Re_{crit} the flow is laminar and adjacent fluid layers side past each other in an orderly fashion.
- The flow is stable. Viscous effects lead to small disturbances being dissipated.
- Above the transition point Re_{crit} small disturbances in the flow start to grow.
- A complicated series of events takes place that eventually leads to the flow becoming fully turbulent.

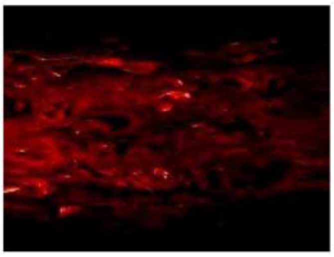


Transition

Boundary Layer Flow over Flat Plate

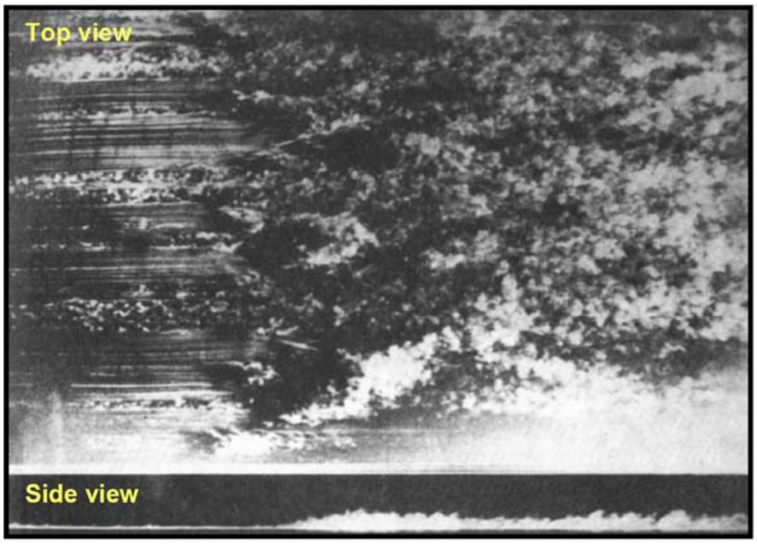
T-S waves

Turbulent spots



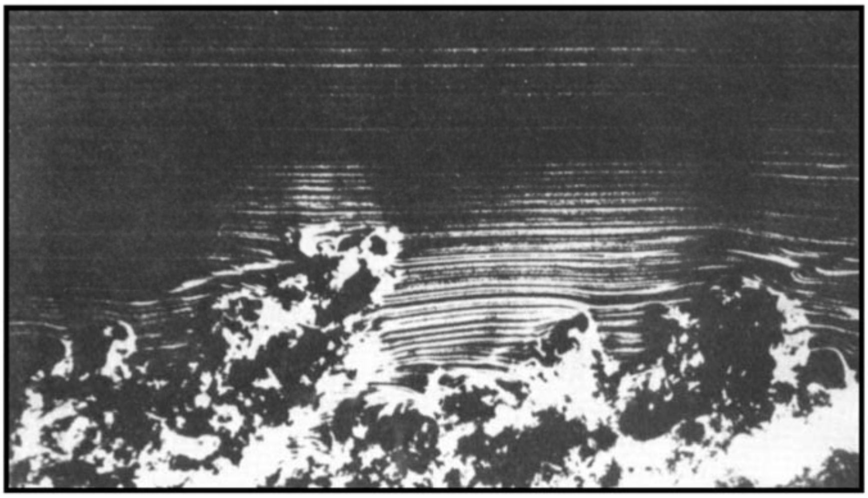
Fully turbulent flow

Turbulent Boundary Layer



Merging of turbulent spots and transition to turbulence in a natural flat plate boundary layer.

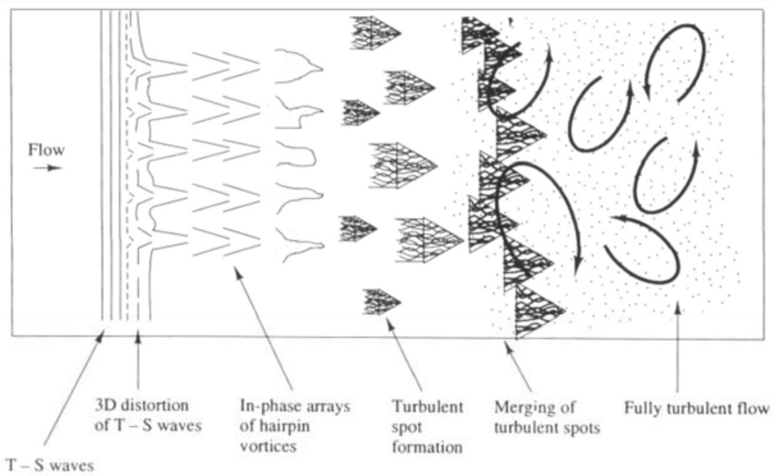
Turbulent Boundary Layer

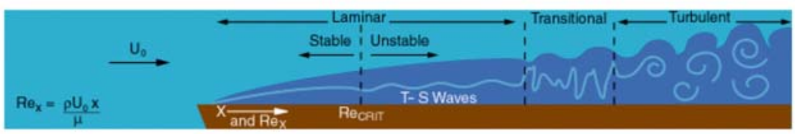


Close-up view of the turbulent boundary layer.

Transition

Boundary Layer Flow over Flat Plate

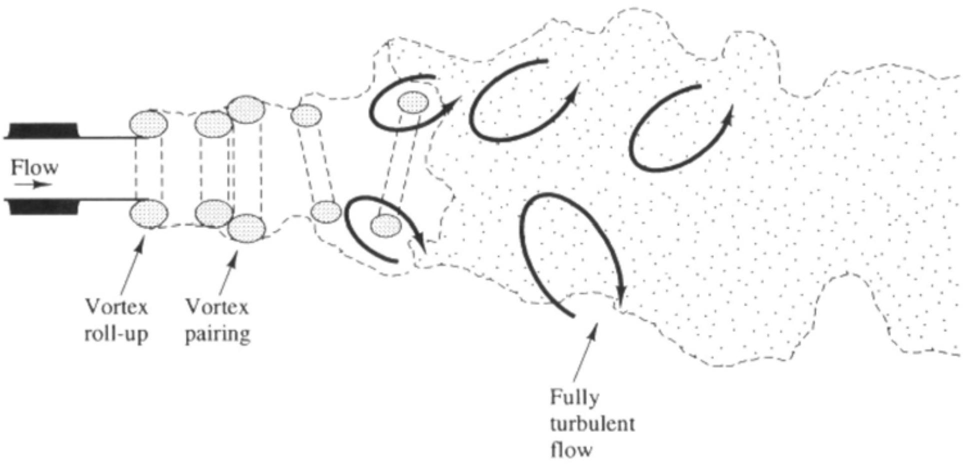




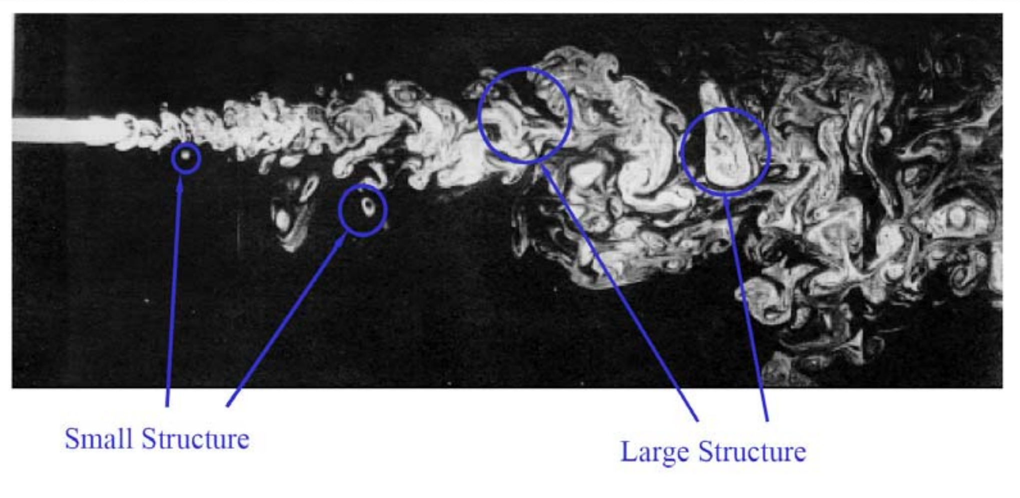
Vorticity and vortex stretching

1) Existence of eddies implies rotation or vorticity
2) Vorticity concentrated along contorted vortex lines or bundles
3) As end points of a vortex line move randomly further apart, the vortex line increases in length but decreases in diameter. Vorticity increases because angular momentum is nearly conserved. Kinetic energy increases at rate equivalent to the work done by large-scale motion that stretches the bundle.
4) Viscous dissipation in the smallest eddies converts kinetic energy into thermal energy.
5) Vortex-stretching cascade process maintains the turbulence and dissipation is approximately equal to the rate of production of turbulent kinetic energy.
6) Typically energy gets transferred from the large eddies to the smaller eddies. However, sometimes smaller eddies can interact with each other and transfer energy to the (i.e. form) larger eddies, a process known as backscatter.

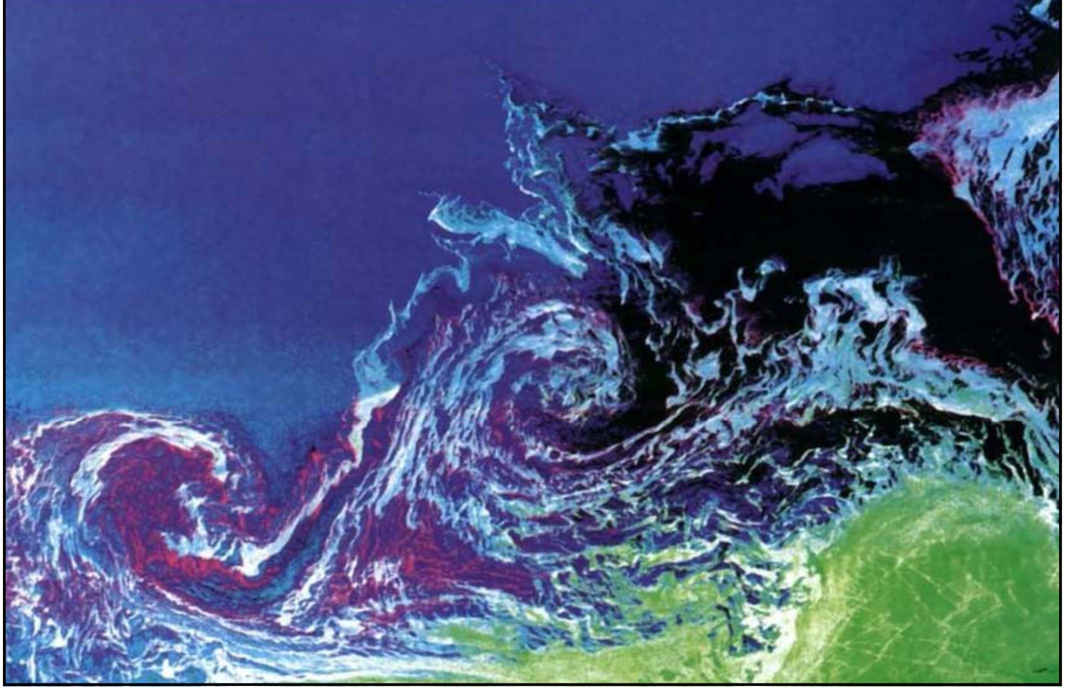
Transition in a jet flow

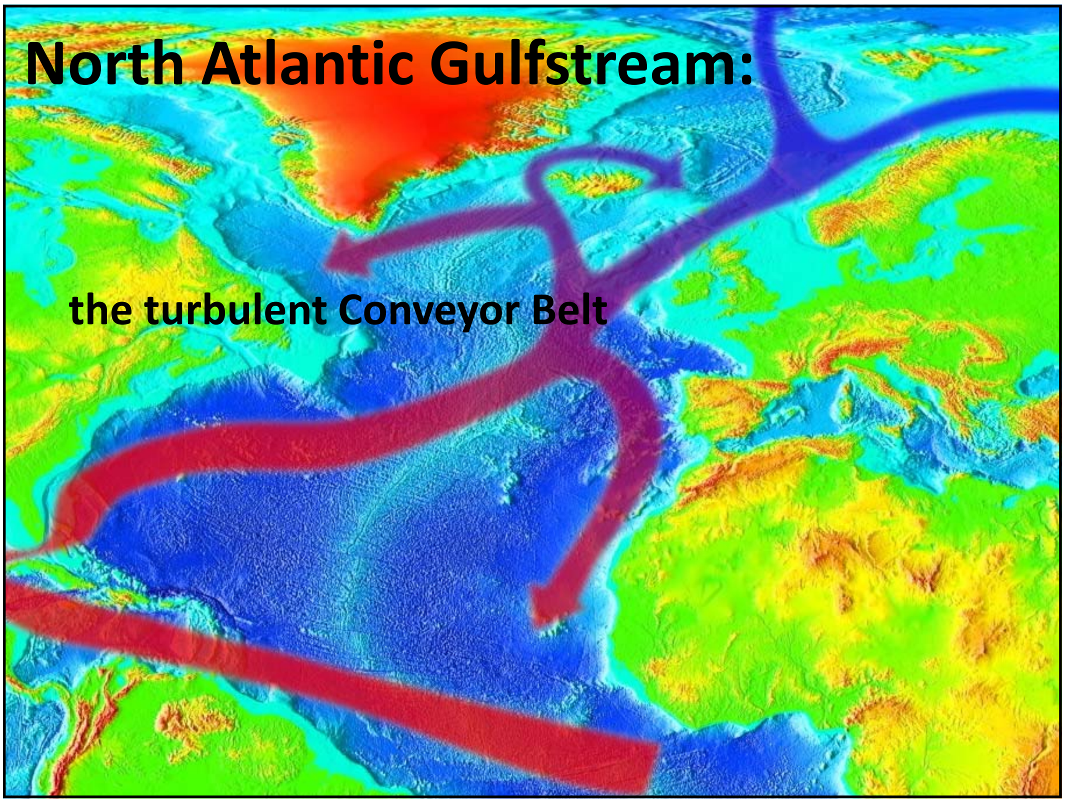


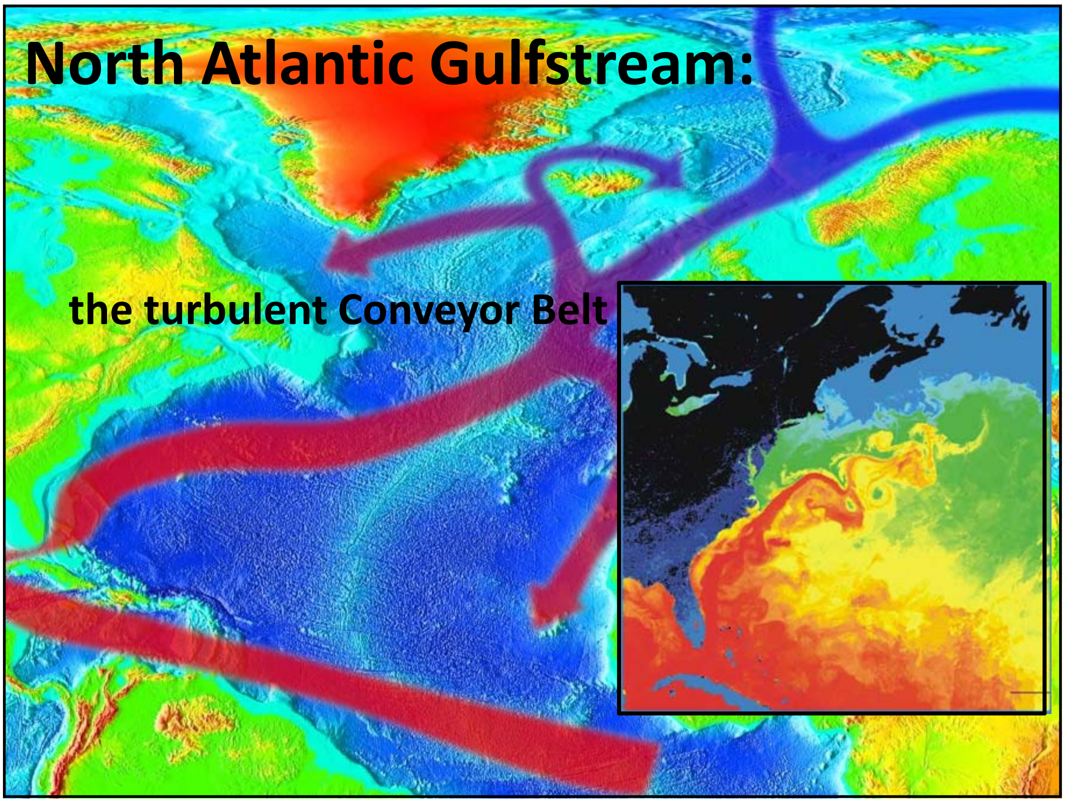
Large-scale vs. Small-scale



Weddell Sea off Antarctica




North Atlantic Gulfstream:
the turbulent Conveyor Belt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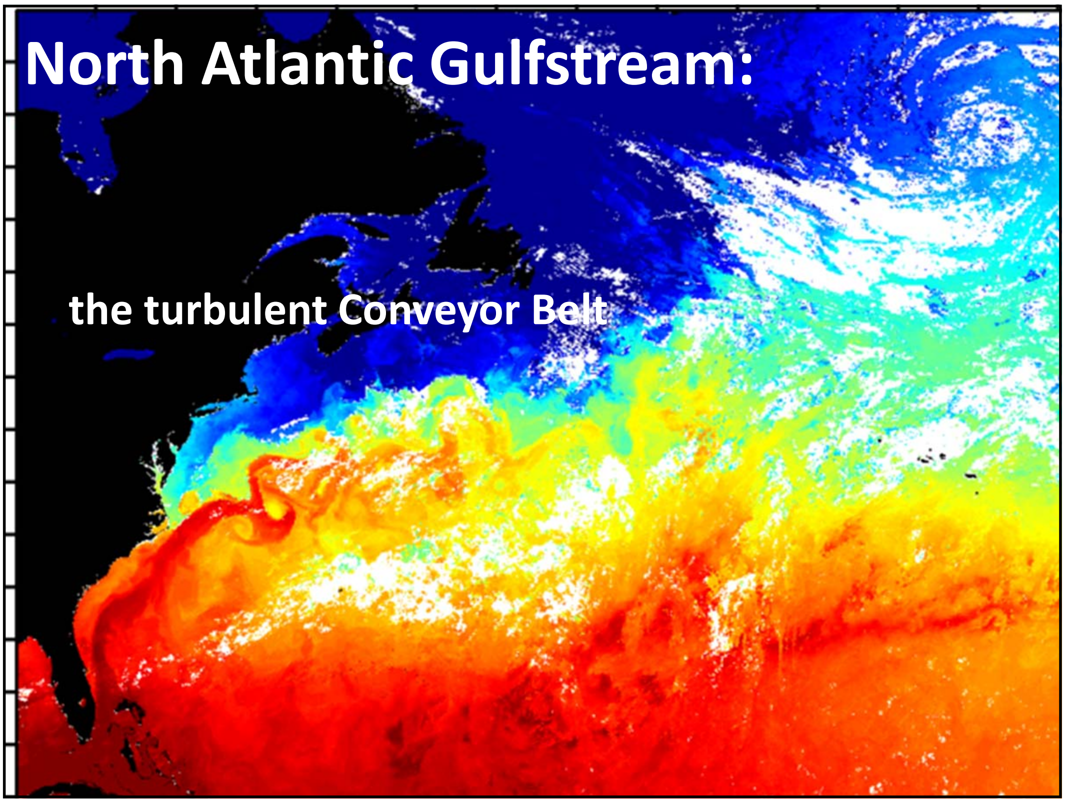
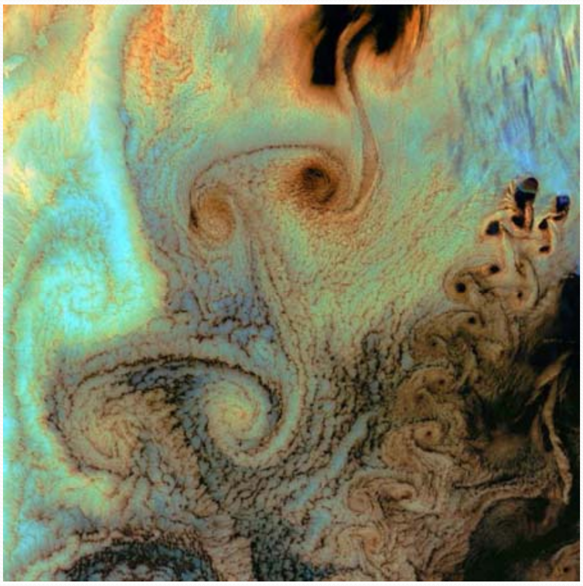
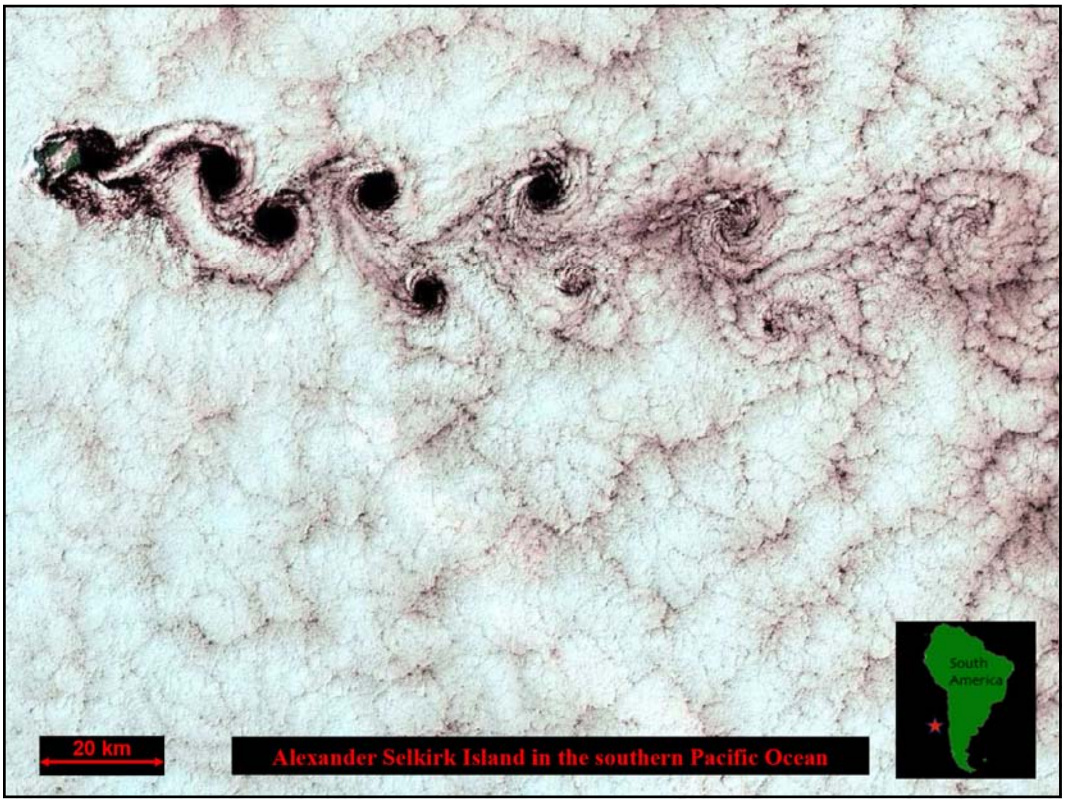
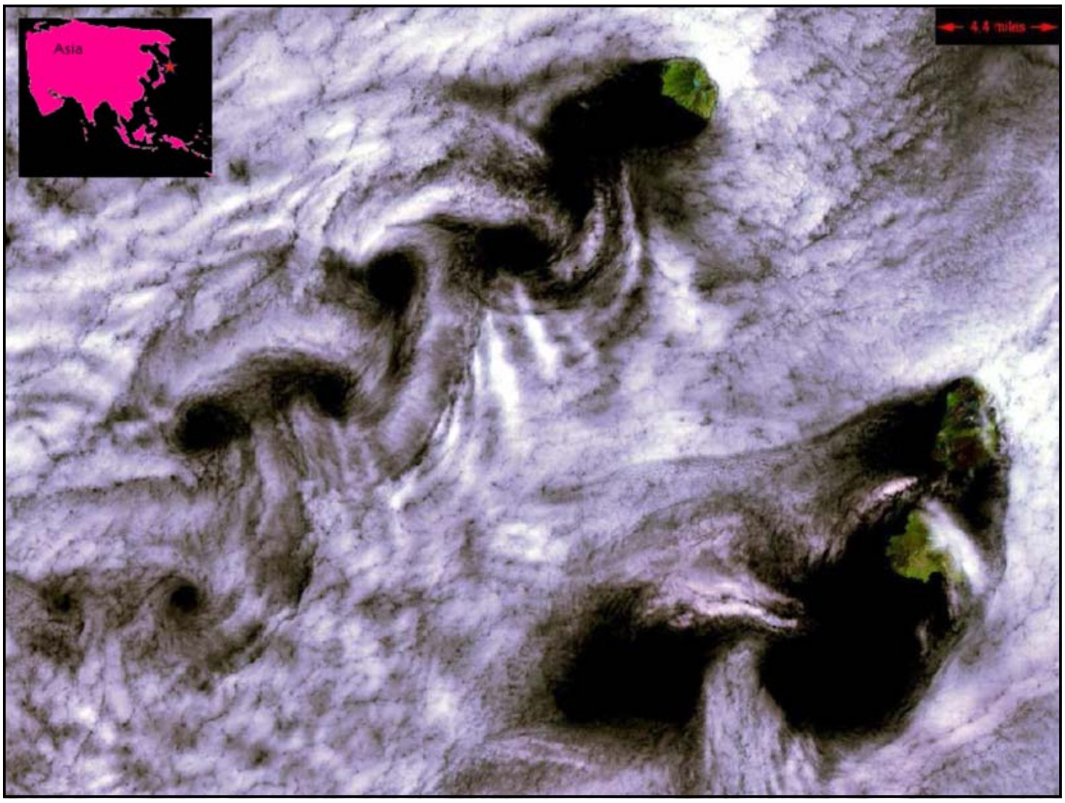


Alaska's Aleutian Islands

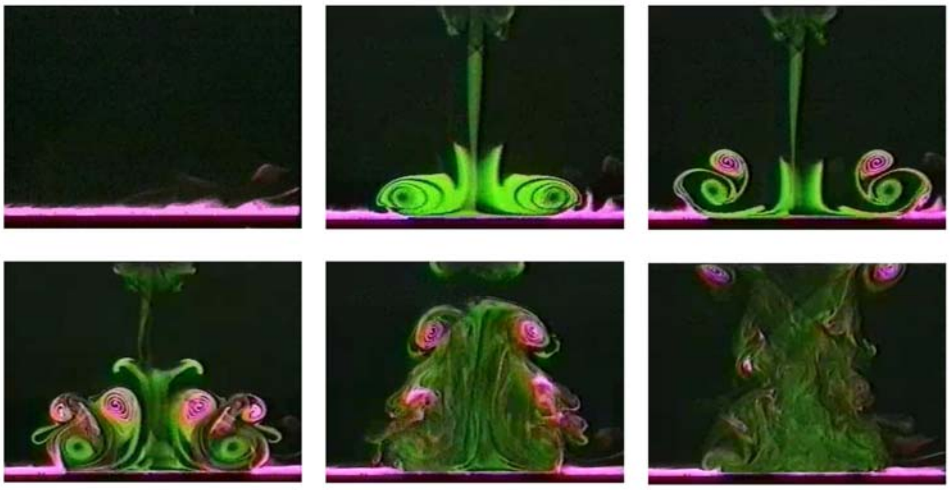
- As airflows over and around objects, spiralling eddies, known Von Karman vortices, may form.
- The vortices in this image were created when prevailing winds sweeping east across the northern Pacific Ocean encountered Alaska Aleutian Islands




20 km
Alexander Selkirk Island in the southern Pacific Ocean
South
America


Asia
4.4 miles

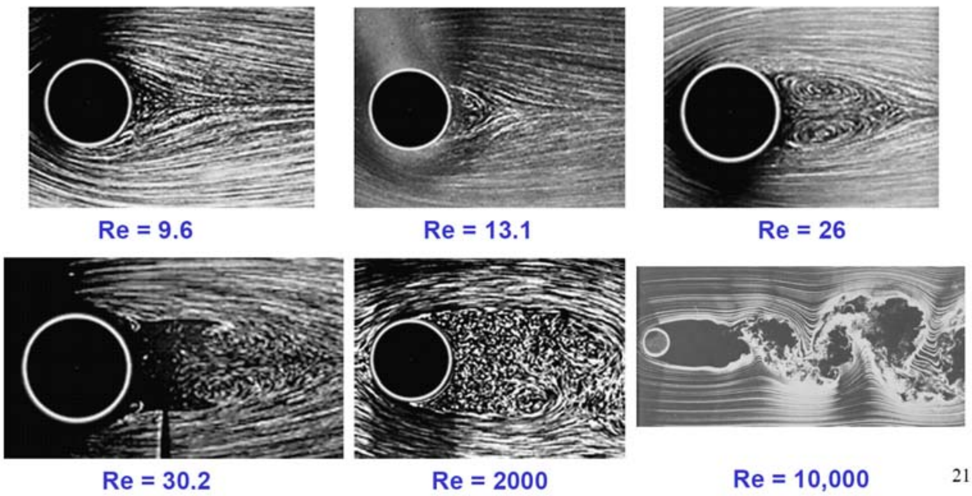
Smoke Ring



A smoke ring (green) impinges on a plate where it interacts with the slow moving smoke in the boundary layer (pink). The vortex ring stretches and new rings form. The size of the vortex structures decreases over time.

Flow transitions around a cylinder

- For flow around a cylinder, the flow starts separating at Re=5.
  For Re < 30, the flow is stable
  Oscillations appear for higher Re.
- The separation point moves upstream, increasing drag up to Re=2000.



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Turbulence Modeling

1) The objective of turbulence modeling is to develop equations that will predict the *time averaged* velocity, pressure, and temperature fields without calculating the complete turbulent flow pattern as a function of time.

   - saves a lot of work
   - most of the time it is all we need to know
   - we may also calculate other statistical properties, such as RMS values.

2) Important to understand: the time averaged flow pattern is a statistical property of the flow

   - it is not an existing flow pattern !
   - it does not usually satisfy the steady Navier-Stokes equations !
   - the flow never actually looks that way !!

L.F. Richardson

Big whirls have little whirls
Which feed on their velocity;
And little whirls have lesser whirls
And so on to viscosity
in the molecular sense

Turbulent eddies

- Consider fully turbulent flow at high Reynolds number: Re=UL/ν
- Turbulence can be considered to consist of eddies of different sizes.
- An *eddy* precludes precise definition, but it is conceived to be a turbulent motion, localized over a region of size l. That is, the flow is at least coherent over this region.
- The region occupied by a larger eddy can also contain smaller eddies.
- Eddies of size *l* have a
  - characteristic velocity $u(l)$
  - timescale $\tau(l)=l/u(l)$
- Eddies in the largest size range are characteristic by the lengthscale l_o which is comparable to the flow length scale L.
- Their characteristic velocity $u_o=u(l_o)$ is on the order of the r.m.s. turbulence intensity $u'=(2U_k/3)^{1/2}$ which is comparable to U.
- Here the turbulent kinetic energy is defined as $U_k = \frac{1}{2}\langle u_i u_i \rangle = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$
- The Reynolds number of these eddies $\mathrm{Re}_0 = u_0 l_0 / \nu$ is therefore large (comparable to Re) and the direct effects of viscosity on these eddies are negligibly small.

Energy Transfer

1) The large eddies are unstable and break up, transferring their energy to somewhat smaller eddies.

2) These smaller eddies undergo a similar breakup process and transfer their energy to yet smaller eddies.

3) This *energy cascade* – in which energy is transferred to successively smaller and smaller eddies - continues until the Reynolds number is

$$\mathrm{Re}(l) = \frac{u(l)\,l}{\nu}$$

sufficiently small that the eddy motion is stable, and molecular viscosity is effective in dissipating the kinetic energy.

4) At these small scales, the kinetic energy of turbulence is converted into heat.

Dissipation

1) Note that dissipation takes place at the end of the sequence of processes.

2) The rate of dissipation ϵ is determined by the first process in the sequence, which is the transfer of energy from the largest eddies.

3) These eddies have energy of order u_o^2 and timescale $\tau_o = l_o/u_o$ so the rate of transfer of energy can be supposed to scale as

$$\varepsilon = \frac{u_0^{\ 2}}{\tau_0} = \frac{u_0^{\ 3}}{l_0}$$

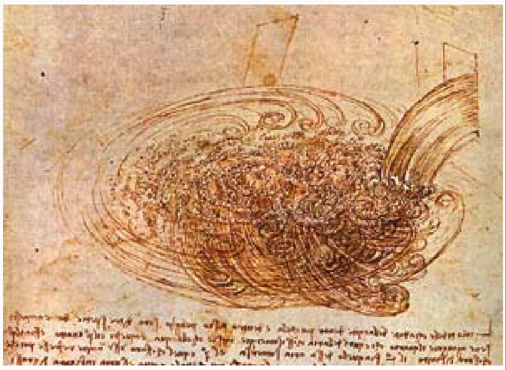
4) Consequently, consistent with experimental observations in free shear flows, this picture of the energy cascade indicates that ϵ is proportional to u_o^3/l_o independent of ν (at high Reynolds numbers).

Kolmogorov Theory

1) Many questions on turbulence remain unanswered:
   - what is the size of the smallest eddies that are responsible for dissipating the energy ?
   - As l decreases, do the characteristic velocity and timescales u(l) and τ(l) increase, decrease or remain the same ? The assumed decrease of the Reynolds number $u_o l_o/\nu$ by itself is not sufficient to determine these trends.

2) Kolmogorov's theory describes how energy is transferred from larger to smaller eddies
   How much energy is contained by the eddies of a given size ?
   How much energy is dissipated by eddies of each size ?

3) These and others are answered by Kolmogorov's theory of turbulence (1941)



Kolmogorov Hypotheses

Kolmogorov theory is based on two hypotheses:

1) Kolmogorov hypothesis of local isotropy

2) Kolmogorov first similarity hypothesis

Kolmogorov Hypothesis: Local Isotropy

1) For homogeneous turbulence, the turbulent kinetic energy U_k is the same everywhere. For isotropic turbulence the eddies also behave the same in all directions.

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$$

2) Kolmogorov argued that the directional biases of the large scales are lost in the chaotic scale-reduction process as energy is transferred to successively smaller eddies.

3) Here, the term local isotropy means isotropy at small scales. Large scale turbulence may still be anisotropic.

4) l_{El} is the length scale that forms the demarcation between the large scale anisotropic eddies $(l<l_{El})$ and the small scale isotropic eddies $(l<l_{El})$.

For many high Reynolds number flows l_{EL} can be estimated as

$$l_{El} \approx l_o/6$$

At sufficiently high Reynolds numbers, the small-scale turbulent motions $(l<l_o)$ are statistically isotropic.

Kolmogorov Hypothesis: first similarity

1) Kolmogorov argued that not only does the directional information get lost as the energy passes down the cascade, but that all information about the geometry of the eddies gets lost also.

2) As a result, the statistics of the small-scale motions are universal:
they are similar in every high Reynolds number turbulent flow, independent of the mean flow field and the boundary conditions.

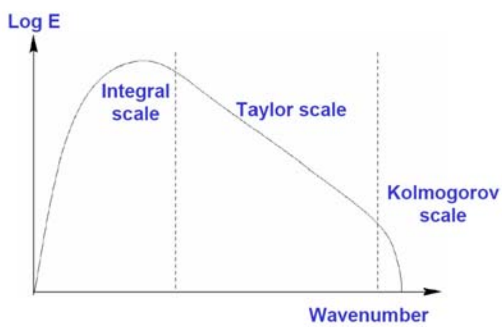
3) These small scales eddies depend on the rate at which they receive energy from the larger scales (which is approximately equal to the dissipation rate ϵ) and the viscous dissipation, which is related to the kinematic viscosity ν.

In every turbulent flow at sufficiently high Reynolds number, the statistics of the small scale motions have a universal form that is uniquely determined by ϵ and ν.

Kolmogorov Energy Spectrum

- Energy cascade, from large to small scale
- E is energy contained in eddies of wavelength λ
- There are three main turbulent length scales:
  - Integral scale
  - Taylor scale
  - Kolmogorov scale

  To each of these scales corresponds a Reynolds number.
- ***Length scales:***
  - Largest eddies. Integral length scale $\quad l_0 = U_k^{3/2} / \varepsilon$
  - Length scales at which turbulence is isotropic. Taylor microscale $\quad \lambda_T = \left(15 \nu u'^2 / \varepsilon\right)^{1/2}$
  - Smallest eddies. Kolmogorov length scale $\quad \eta = \left(\nu^3 / \varepsilon\right)^{1/4}$
    These eddies have a velocity scale $\quad u_\eta = \left(\nu \varepsilon\right)^{1/4}$
    time scale $\quad \tau_\eta = \left(\nu / \varepsilon\right)^{1/2}$



ϵ - energy dissipation rate (m²/s³)
U_k - turbulent kinetic energy (m²/s²)
ν - is kinematic viscosity (m²/s)

Kolmogorov Theory: Integral Scale

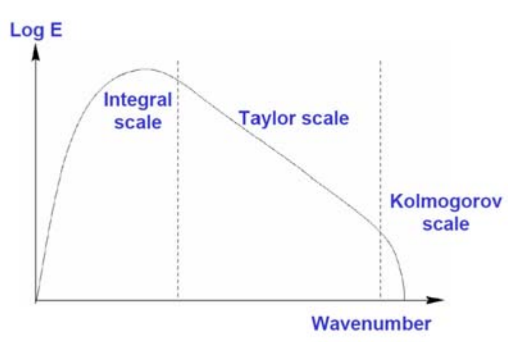
- The integral scale is the lengthscale l_o at which we find the largest eddies.
- Their size can be estimated on the basis of:
  - eddies of size lo have a characteristic velocity u_o, and timescale $\tau_o = l_o/u_o$
  - their characteristic velocity $u_o = u(l_o)$ is on the order of the rms turbulence intensity $u' = (2U_k/3)^{1/2}$
  - assume that energy of eddy with velocity scale u_o is dissipated in time τ_o
- From this, the length scale l_o can be derived:

$$l_0 \propto \frac{U_k^{3/2}}{\varepsilon}$$

where ϵ is the energy dissipation rate. The proportionality constant is of the order one. This length scale is usually referred to as the *integral scale* of turbulence.

- The Reynolds number associated with these large eddies is referred to as the turbulence Reynolds number Re_L, which is defined as

$$\mathrm{Re}_L = \frac{k^{1/2} l_0}{\nu} = \frac{k^2}{\varepsilon \nu}$$



ϵ - energy dissipation rate (m^2/s^3)
k - turbulent kinetic energy (m^2/s^2)
ν - is kinematic viscosity (m^2/s)

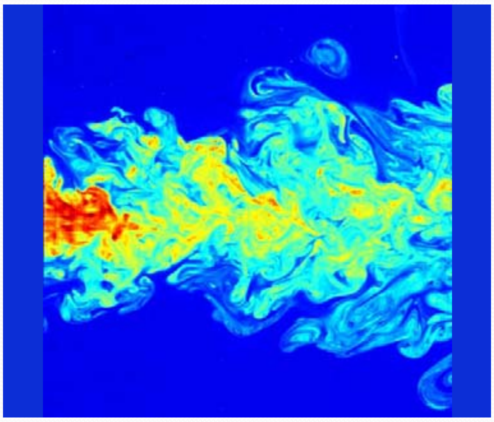
Kolmogorov Theory: Kolmogorov Energy Spectrum

- The Kolmogorov energy spectrum specifies how the turbulent kinetic energy is distributed among the eddies of different sizes.
- In steady state, the energy fed into the largest eddies can neither accumulate nor dissipate viscously.
- Only route is to get progressively transferred via nonlinear interactions – through the advective term in the equation of motion - to eddies of smaller and smaller scale.
- Eddies on scale λ, with associated velocity $v\lambda$, have also rate of energy dissipation rate (on dimensional grounds),

$$\varepsilon \sim v_\lambda^3 / \lambda$$

- Comparison with expression energy dissipation rate for largest eddies, we get

Kolmogorov Law



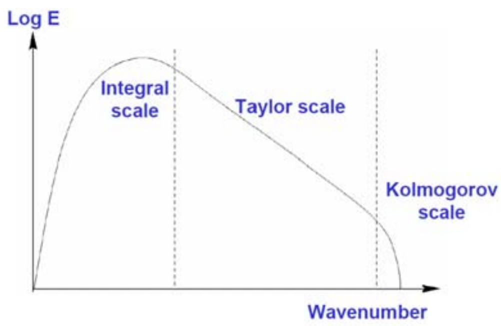
$$v_\lambda \sim U_k^{1/2} \left(\frac{\lambda}{l_o} \right)^{1/3}$$

Kolmogorov Theory: Kolmogorov Energy Spectrum

Kolmogorov Velocity Law

$$v_\lambda \sim U_k^{1/2} \left(\frac{\lambda}{l_o} \right)^{1/3}$$

- The eddy-cascade process leads to a velocity spectrum as a function of eddy size λ that depends on the 1/3 power of λ.
- This law, Kolmogorov's law, demonstrates that the largest eddies have the most velocity (turbulent energy),
- whereas the smallest eddies carry most of the vorticity, $\sim v_\lambda/\lambda$



Where does the cascade process end ?

Kolmogorov Theory: Kolmogorov Energy Spectrum

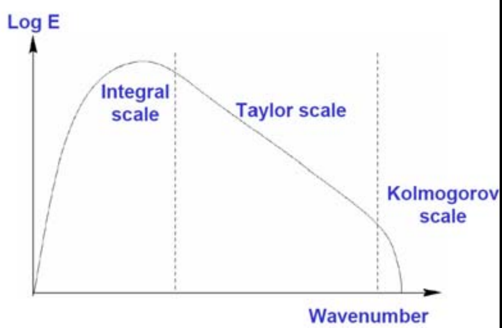
Where does the cascade process end ?

Eddies have so small a scale λ_{diss} that the viscous dissipation rate per units mass becomes comparable to the energy cascaded downward into this spectral/scale region.

$$\lambda_{diss} \sim \left(\frac{\text{Re}}{\text{Re}_{cr}} \right)^{-3/4} l_0$$

where Re is the Reynolds number of the flow associated with the largest eddies, and Re_{cr} the Reynolds number at which there is stability (against turbulence).

Typical numbers for $\text{Re}_{cr} \sim 10^2 - 10^3$ for viscous shear flows.



Kolmogorov Scale