DYNAMICS OF GALAXIES

7. Dynamics of spiral galaxies

Piet van der Kruit
Kapteyn Astronomical Institute
University of Groningen
the Netherlands

Winter 2008/9

Tully-Fisher relation

For exponential disks:

$$M \propto \sigma_\circ h^2 \qquad V_{\max} \propto (\sigma_\circ h)^{1/2}$$

Then

$$M \propto V_{\max}^4 \sigma_\circ^{-1}$$

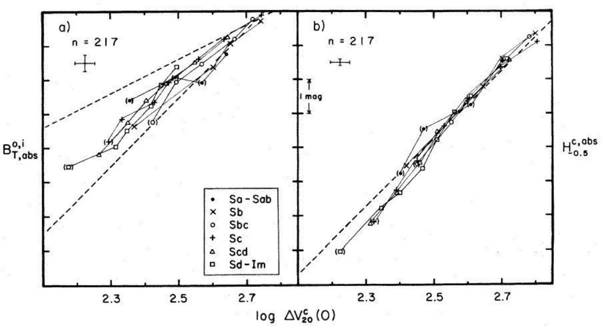
With Freeman's law and constant mass to light ratio M/L:

$$L \propto V_{\max}^4$$

This is the Tully-Fisher relation which has indeed been observed[1]. In practice $V_{\max}$ is measured from the total width of the HI-profile, corrected for inclination, at a level 20 or 50% of the peak.

[1] R.B. Tully & J.R. Fisher, A.&A. 54, 661 (1977)

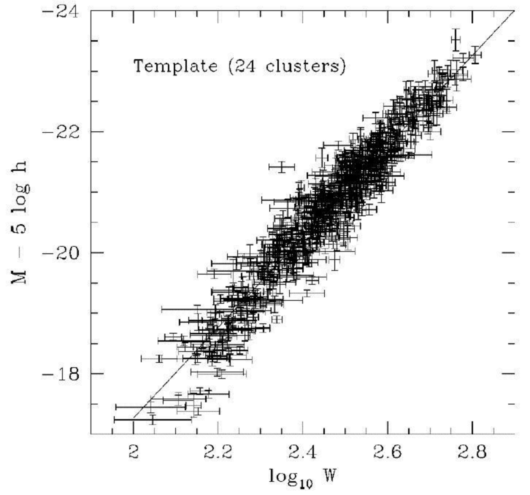
Aaronson & Mould[2] find exponents of 3.5 in B and 4.3 in H (1.6μ).



There is debate about the slope in observed relations.

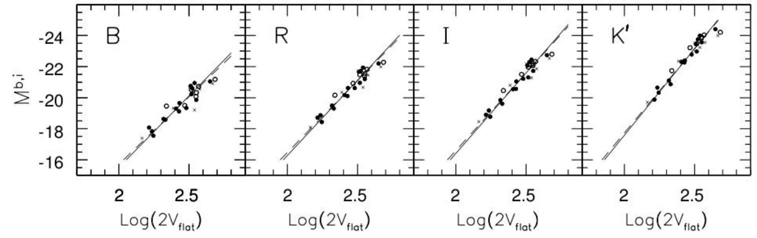
[2] M. Aaronson & J.R. Mould, Ap.J. 265, 1 (1983)

In the I-band Giovanelli *et al.*[a] find from 555 galaxies in 24 clusters a slope of 7.68 ± 0.13 (in magnitudes, which corresponds to 3.07 ± 0.05).



[a] R. Giovanelli & 6 other authors, Ap.J. 477, L1 (1997)

A recent study of the Ursa Major Cluster[3] shows that the relation is tightest in the K′-band and there the slope is 11.3 ± 0.5 (exponent 4.5 ± 0.2).



[3]M.A.M. Verheijen, Ph.D. thesis (1997) and Ap.J. 563, 694 (2001)

Rotation curves and mass distribution

Exponential disk

The exponential disk has a surface density distribution

$$\sigma(R) = \sigma_\circ e^{-(R/h)}$$

where $\sigma_\circ$ is the central surface density and h the scalelength. The total mass of the disk out to infinity is $M = 2\pi\sigma_\circ h^2$.

When it is self-gravitating and infinitessimally thin, the corresponding rotation curve has the analytic form[4]:

$$V_{\mathrm{rot}}^2(R) = \pi G h \sigma_\circ \left(\frac{R}{h}\right)^2 [I_0 K_0 - I_1 K_1]$$

I and K are modified Bessel functions *evaluated at $R/2h$*.

[4] K.C. Freeman, Ap.J. 160, 811 (1970)

This rotation curve has the properties

- that it rizes from the center to a maximum at $R = 2.2h$ with

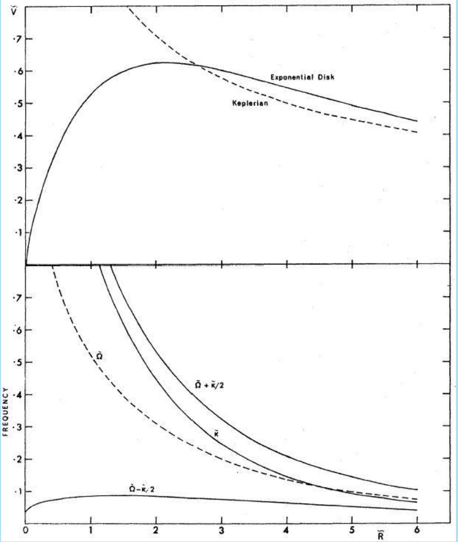
$$V_{\max} = 0.8796(\pi G h \sigma_0)^{1/2}$$

- and becomes Keplerian at large R.

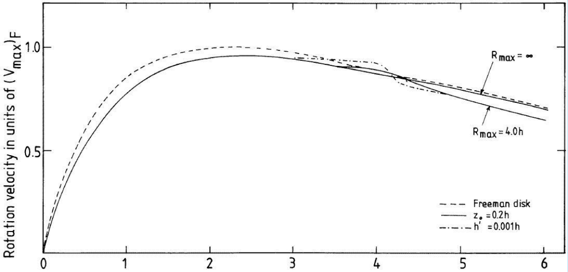
In the next figure the axes are dimensionless, such that $\tilde{R} = R/h$ and $\tilde{V} = V\sqrt{h/GM}$.

The lower half of the figure has the angular frequency Ω, the epicyclic frequency κ and the Lindblad resonance frequencies $\Omega \pm \kappa/2$.

These frequencies are in dimensionaless units of $\sqrt{GMh^3}$.



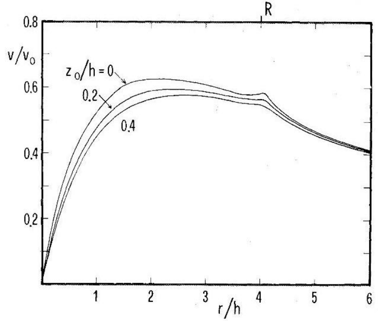
The rotation curve changes slightly when allowance is made for the finite thickness and the truncation[5].

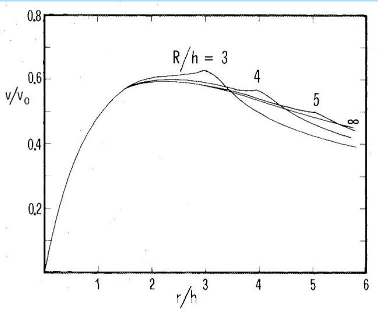


The dashed line has a infinitely thin disk, the full-drawn line has a finite thickness ($z_\circ = 0.2h$) without and with a shallow truncation (the scalelength changes by a factor 5 at $R_{\rm max}$). The dot-dashed curve has a very sharp edge.

[5]P.C. van der Kruit & L. Searle, A.&A. 110, 61 (1982)

Here are similar figures from another study[6] with a truncation as a linear drop in surface density over a radial range $\delta = 0.2h$.





On the left the thickness of the disk is varied and on the right the radius of the truncation.

[6] S. Casertano, Mon.Not.R.A.S. 203, 735 (1983)

Dark matter halo

Observations of spiral galaxies show flat rotation curves that do not show the Keplerian decline beyond the optical edge.

So add a dark halo with $\rho \propto R^{-2}$ at large R.

This can be an isothermal sphere[7] or some other analytical function[8].

In practice one may also directly infer a predicted rotation curve from the disk by calculated from the observed surface brightness profile.

[7] e.g. C. Carignan & K.C. Freeman, Ap.J. 294, 494 (1985)
[8] K. Begeman, Ph.D. thesis (1987)

In the general case that the disk density distribution is $\rho(R,z)$, the rotation curve from the corresponding self-gravitating disk is

$$V_{\rm c}^2(R) = -8GR \int_0^\infty r \int_0^\infty \frac{\partial \rho(r,z)}{\partial r} \, \frac{K(p)-E(p)}{(Rrp)^{1/2}} \, dz \, dr$$

with

$$p = x - (x^2-1)^{1/2} \ \text{and} \ x = \frac{R^2+r^2+z^2}{2Rr}$$

When the density distribution is separable in $\sigma(R)$ and $Z(z)$ this becomes

$$V_{\rm c}^2 = -8GR \int_0^\infty r\sigma(r) \int_0^\infty \frac{\partial Z(z)}{\partial z} \frac{K(p)-E(p)}{(Rrp)^{1/2}} \, dz \, dr$$

The vertical distribution can for example be assumed to be the isothermal sheet.

We may in addition have a bulge with observed surface density $\sigma(r)$; then for the self-gravitating case we have

$$V_c^2(R) = \frac{2\pi G}{R}\int_0^R r\sigma(r)\,dr +$$

$$\frac{4G}{R}\int_R^\infty \left[\arcsin\left(\frac{R}{r}\right) - \frac{R}{(r^2-R^2)^{1/2}}\right] r\sigma(r)\,dr$$

For the dark halo the assumed the density law

$$\rho(R) = \rho_\circ \left[1 + \left(\frac{R}{R_c}\right)^2\right]^{-1}$$

results in

$$V_c^2(R) = 4\pi G\rho_\circ R_c^2 \left[1 - \frac{R_c}{R}\arctan\left(\frac{R}{R_c}\right)\right]$$

To get the total rotation curve for a system consisting of three components add these circular velocities in quadrature:

$$V_{\rm circ}(R) = \left[V^2_{\rm disk}(R) + V^2_{\rm bulge}(R) + V^2_{\rm halo}(R)\right]^{1/2}$$

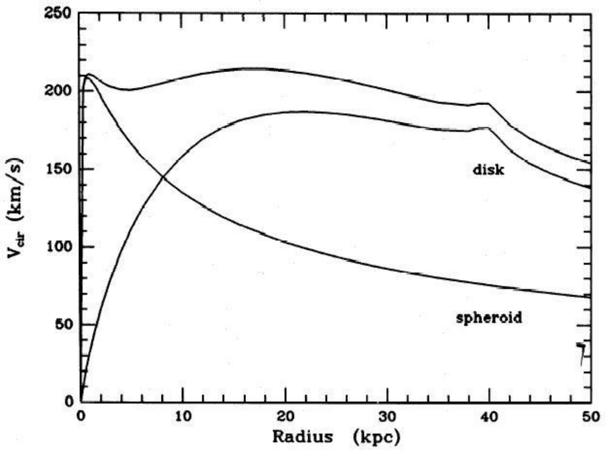
One can make things easier by fitting an exponential disk to the observations and use the analytic form of the corresponding rotation curve.

If in addition there is gas, this should be treated in the same way.

In practice we have for the stars only surface *brightness* distributions, so we need an undetermined mass-to-light ratio M/L in order to turn this into a surface *density* distribution.

From the solar neighborhood we can only find that M/L is of order a few in solar units.

In principle one can make an approximately flat rotation curve by a careful tuning of the disk and bulge contributions, as here for the Galaxy.

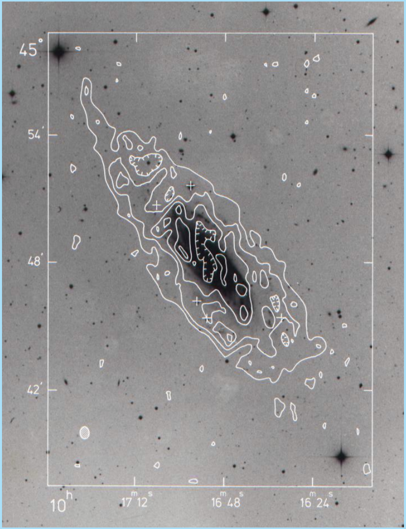


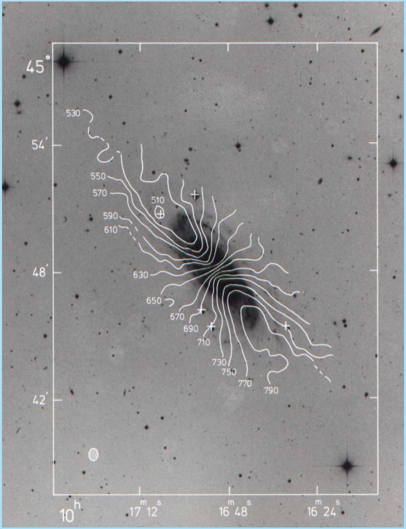
Maximum disk hypothesis

The following is from an analysis of the rotation curve of NGC 3198[a], which has essentially no bulge.

The HI extends out to 11 scalelengths.

[a]T.S. van Albada, J.N. Bahcall, K. Begeman & R. Sancisi, Ap.J. 295, 305 (1985)


45°
54′
48′
42′
10h
17m 12s
16m 48s
16m 24s

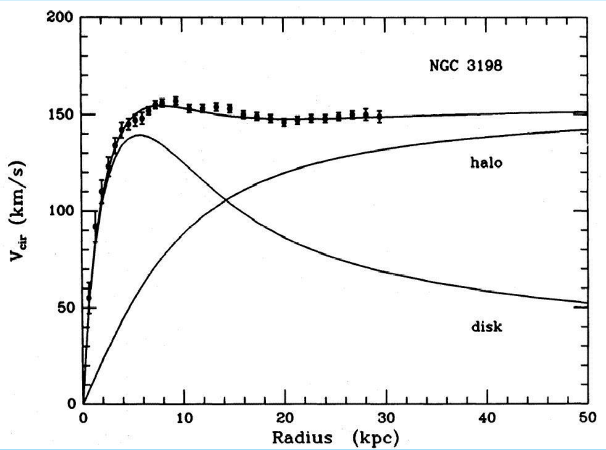

45°
54′
48′
42′
10h
17m 12s
16m 48s
16m 24s
530
550
570
510
590
610
630
650
670
690
710
730
750
770
790

The procedure then is to choose an M/L of the disk that gives the maximum amplitude of the disk rotation curve that is allowed by the observations.

The two free parameters of the dark halo, core radius $R_{\rm c}$ and central density $\rho_\circ$ are then used to fit the rotation curve.

This is called the "maximum disk hypothesis", since it is a fit to the rotation curve with the largest amount of mass possible in the disk (and the largest M/L).

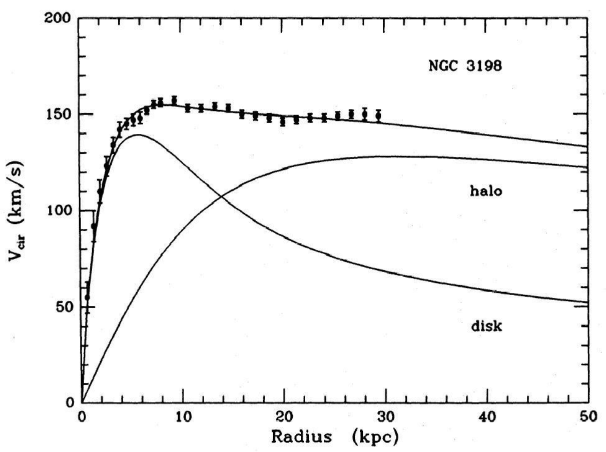
The maximum disk solution to the rotation curve of NGC 3198 looks as follows.



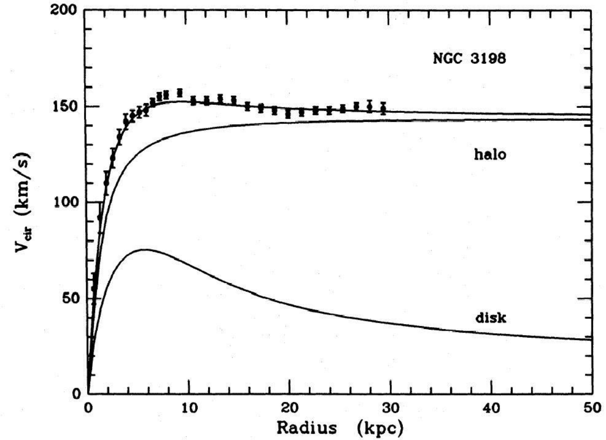
This particular model for NGC 3198 has a total mass of 15×10^{10} $M_\odot$ within 30 kpc.

Within this radius the ratio of dark to visible matter is 3.9. At the optical edge this ratio is 1.5.

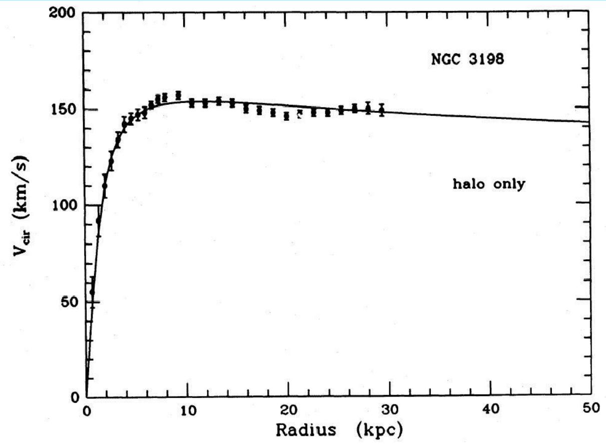
By adjusting the halo parameters one can minimize the dark halo mass by assuming that the rotation curve falls beyond the last measured point.


NGC 3198
halo
disk
V_{cir} (km/s)
Radius (kpc)
200
150
100
50
0
0
10
20
30
40
50

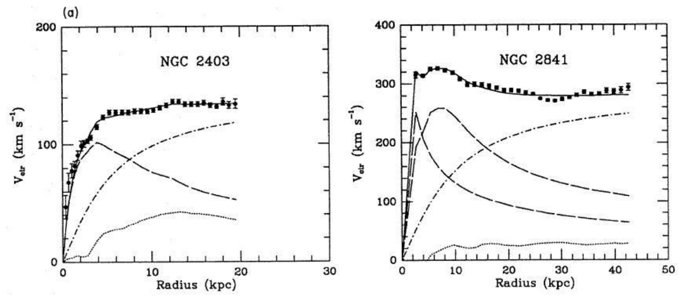
The difficulty with the maximum disk hypothesis is that it is possible to make similar good fits with lower disk masses ...



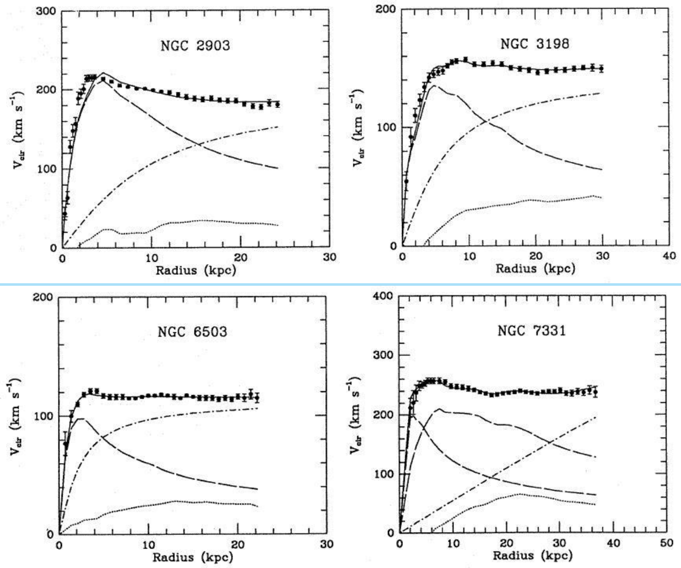
... and even no disk mass at all!



Begeman[9] observed 8 spirals, of which HI in NGC 2841 goes out to 17.8 h (43 kpc).



[9] K. Begeman, Ph.D. thesis (1987); K. Begeman, A.H. Broeils & R.H. Sanders, Mon.Not.R.A.S. 249, 523 (1991)


NGC 2903
NGC 3198
NGC 6503
NGC 7331
V_{cir} (km s^{-1})
Radius (kpc)

Begeman's maximum disk fits have

- $(M/L)_{\rm disk} = 3.1 \pm 1.2$ (9.4 for NGC 2841)
- $(M_{\rm halo})_{\rm R_{opt}} = 44 \pm 9\%$ (34 % for NGC 2841)

Broeils[10] made maximum disk fits to a sample of 23 galaxies with extended HI, accurate rotation curves and photometry. He studied the distribution of the parameters from the fits.

The global mass-to-light ratio M/L out to the maximum radius observed is in the range 10 to 20 (in B).

The ratio of the dark to luminous matter at some fiducial radius (either R_{25} or $R = 7h$) correlates well with the maximum rotation velocity and reasonably well with integrated magnitude and morphological type.

[10] A.H. Broeils, Ph.D. thesis (1992)

It is possible that these conclusions are influenced by the assumption of the maximum disk hypothesis.

The maximum disk hypothesis could lead to the following spurious results:

- Large $V_{\rm max}$ results in disk surface density and therefore large $(M/L)_{\rm disk}$.
- Large $(M/L)_{\rm disk}$ results in less dark matter.

Indeed:

$$\left(\frac{M}{L}\right)_{\rm disk} = (0.014 \pm 0.003)\,V_{\rm max} + (0.72 \pm 0.60)$$

with $r = 0.67$.

$$\left(\frac{M_{\rm dark}}{M_{\rm lum}}\right)_{\rm R_{25}} = (2.37 \pm 0.39) - (0.42 \pm 0.12)\left(\frac{M}{L}\right)_{\rm disk}$$

with $r = 0.62$.

Independent checks on the maximum disk hypothesis

There are independent ways in which the maximum disk hypothesis can be checked by independent measurement of M/L.

a. The truncation feature in the rotation curve:

The truncation feature in the rotation curve can in principle be used to estimate the mass of the disk. It has been done in two cases where the mass of the halo within the truncation radisu has been estimated:

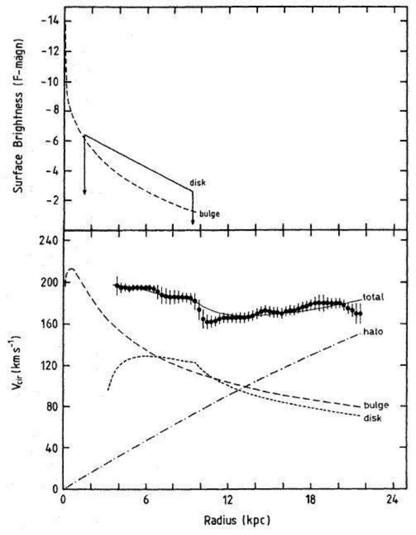
- NGC 5907[11]: $(M_{\rm halo})_{\rm R_{opt}} \approx 60\%$ (so not maximum disk)
- NGC 4013[12]: $(M_{\rm halo})_{\rm R_{opt}} \approx 25\%$

[11] S. Casertano, Mon.Not.R.A.S. 203, 735 (1983)
[12] R. Bottema, A.&A. 306, 345 (1996)

In NGC 4013 the disk and bulge must dominate dynamically in the inner regions.

The truncation feature is clearly visible.

However, the fit to the rotation curve is not maximum disk.

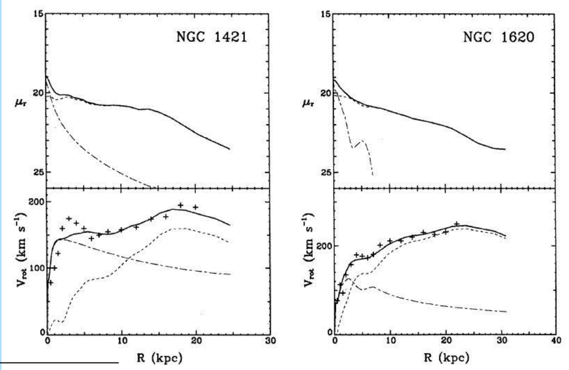


b. "Wiggles" in rotation curves

The inner parts op rotation curves can often be fit without a dark halo and features in luminosity profiles seem to correspond to features in rotation curves[13].

Top shows the light distributions of disk and bulge.

Bottom shows the rotation curve with constant M/L in both components.

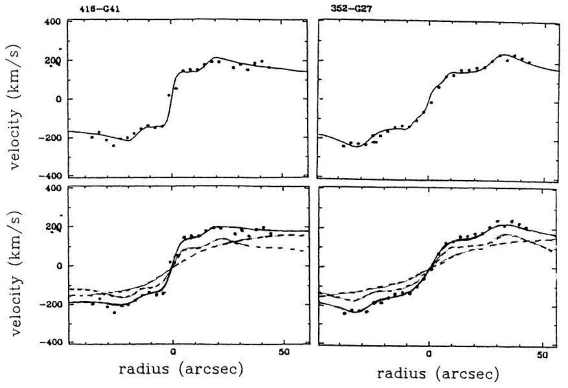


[13] E.g. S. Kent, A.J. 91, 1301 (1986)

This suggests maximum disks, but even if disks are not dynamically dominant in the inner parts the wiggles can still be reproduced.[14]

Top has the rotation curve from the photometry without a dark halo.

Bottom has reduced the disk mass by half and a dark halo added.



[14] P.C. van der Kruit, IAU Symp. 164, 227 (1995)

c. Maximum rotation versus scalelength

Another interesting argument is the following[15].

For a pure exponential disk the maximum in the rotation curve occurs at $R = 2.2h$ with an amplitude of

$$V_{\rm max} \propto \sqrt{h\sigma_\circ} \propto \sqrt{\frac{M_{\rm disk}}{h}}$$

For fixed disk-mass $M_{\rm disk}$ this gives

$$\frac{\partial \log V_{\rm max}}{\partial \log h} = -0.5$$

[15] S. Courteau & H.-W. Rix, Ap.J. 513, 561 (1999)

Remember that the Tully-Fisher relation is a tight correlation between maximum rotation and total luminosity of disk galaxies.

The total luminosity of an exponential disk is $L = 2\pi\mu_\circ h^2$.

Then at a given absolute magnitude (or mass) lower scalelength disks should have higher rotation.

So, if disk-dominated galaxies are maximum disk (in practice $V_{\rm disk} \sim 0.85 V_{\rm total}$) this should be seen in *scatter* in the Tully-Fisher relation

This is not observed and the estimate is that on average $V_{\rm disk} \sim 0.6 V_{\rm total}$.

d. Thickness of the HI-layer.

The thickness of the gas layer can be used to measure the surface density of the disk independent of the rotation curve.

The density distribution of the exponential, locally isothermal disk was:

$$\rho_*(R,z) = \rho_*(0,0)\,\exp\left(-R/h\right)\,\mathrm{sech}^{\,2}(z/z_\circ)$$

If the HI has a velocity dispersion $\langle V_{\rm z}^2\rangle_{\rm HI}^{1/2}$, and if the stars dominate the gravitational field

$$\rho_{\rm HI}(R,z) = \rho_{\rm HI}(R,0)\,\mathrm{sech}^{\,2p}(z/z_\circ)$$

$$p = \frac{\langle V_{\rm z}^2\rangle_*}{\langle V_{\rm z}^2\rangle_{\rm HI}}$$

The full width at half maximum of this distribution is:

$$W_{\rm HI} = 1.663 p^{-1/2} z_\circ \ \ \text{for } p \gg 1$$

$$W_{\rm HI} = 1.763 p^{-1/2} z_\circ \ \ \text{for } p = 1$$

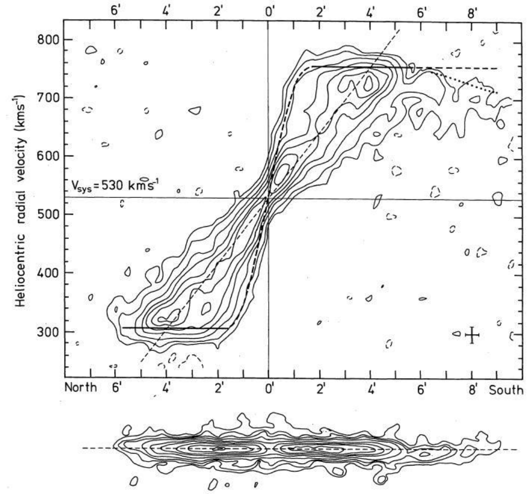
Then to within 3%

$$W_{\rm HI} = 1.7 \langle V_z^2 \rangle_{\rm HI}^{1/2} \left[\frac{\pi G (M/L) \mu_\circ}{z_\circ} \right]^{-1/2} \exp\left(R/2h\right)$$

So the gas layer increases exponentially in thickness with an e-folding of $2h$.

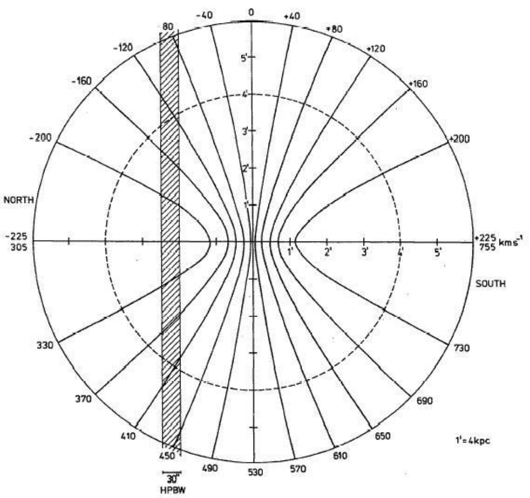
We now look at an analysis of the HI-layer in NGC 891[16] from measurements by Sancisi & Allen[17].

[16] P.C. van der Kruit, A.&A. 99, 298 (1981)
[17] R. Sancisi & R.J. Allen, A.&A. 74, 73 (1979)

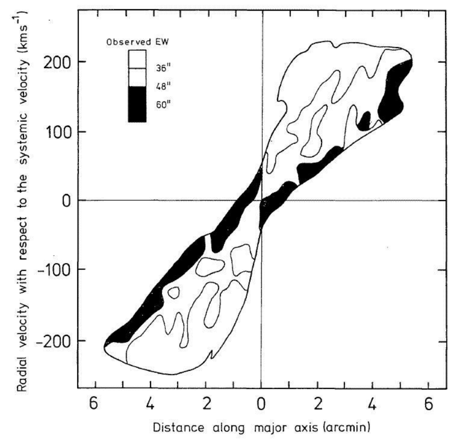

6'
4'
2'
0'
2'
4'
6'
8'
800
700
600
500
400
300
Heliocentric radial velocity (kms^{-1})
$V_{sys} = 530$ kms^{-1}
North
South

The position-velocity diagram (l, V-diagram) is a projection of the plane of the galaxy with only a ambiguity around the "line of nodes".

This can be seen when we draw lines of equal line of sight velocity on the plane of the galaxy.



Here is a measure of the thickness.

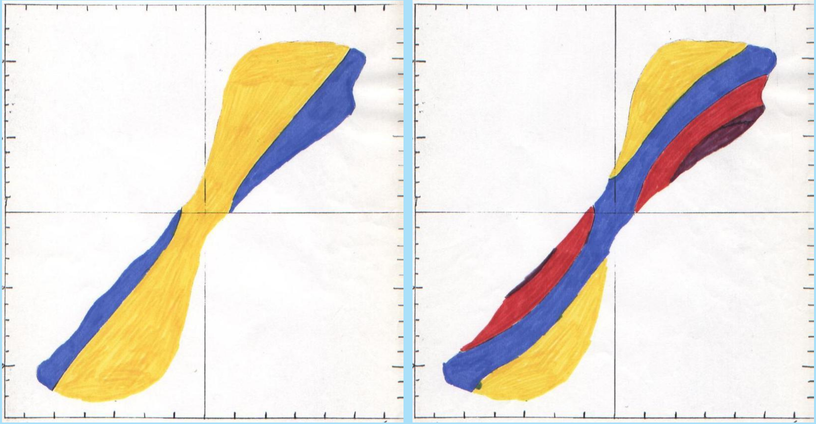


Three particular models were then calculated:

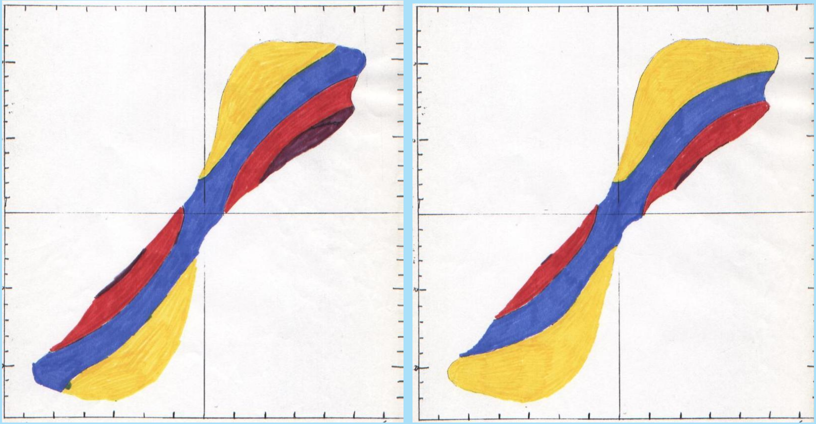
- Model I, which has 40% of the mass within the optical radius in the disk,
- Model II with all the mass (including the dark mass) in the disk,
- Model III with a constant thickness of the HI-layer.

The W_{HI} in the observations were then calculated for disks with inclinations of 87.5 and 90°.

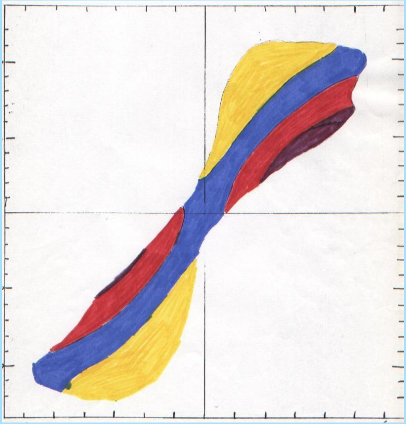
Here is the equivalent width in the (x, V)-diagram for Model I with inclinations of 90° (left) and $87\overset{\circ}{.}5$ (right).

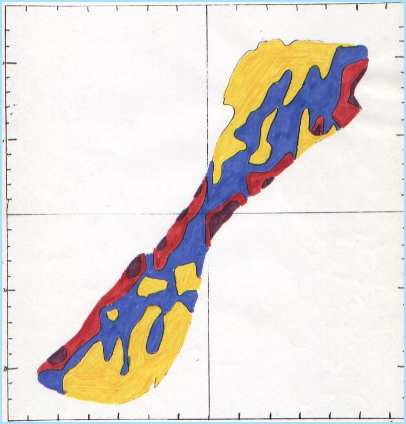


Here is the equivalent width in the (x, V)-diagram for Model I (left) and Model II (right) both at an inclination of 87°5.

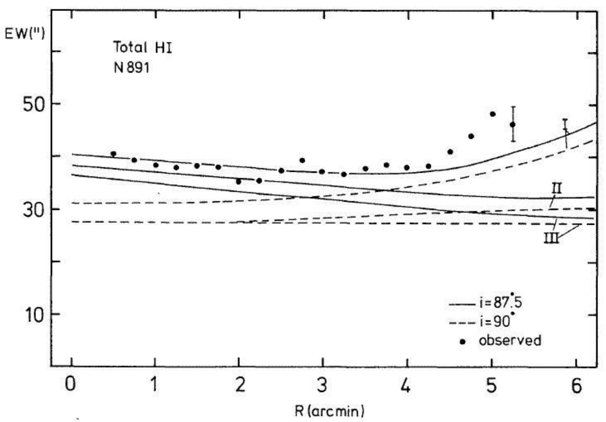


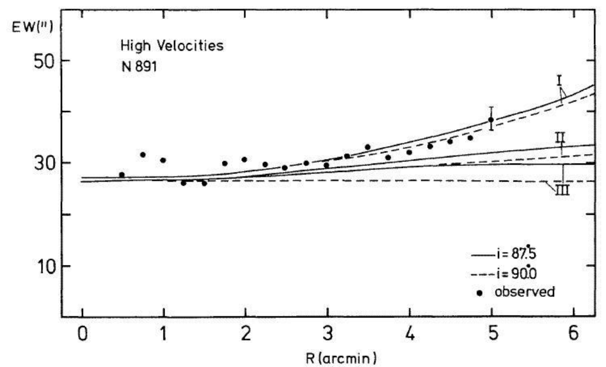
Here is the equivalent width in the observed (x, V)-diagram (left) and that for Model I with an inclination of 87°.5 (right).





Also the thickness over all velocities and the “high” velocities (190 to 230 km/s) can be compared to observations.





NGC 891 is not maximum disk. Also this analysis shows that the dark matter cannot be in the disk.

e. Thickness of the stellar disk

The vertical motions of the stars can be combined with the thickness of stellar disks to estimate of the disk surface densities σ.

For the isothermal sheet with space density

$$\rho(z) = \rho(0)\ \mathrm{sech}^{\,2}(z/z_\circ)$$

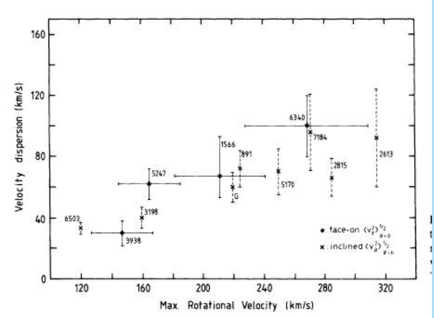
we had for the stellar velocity dispersion

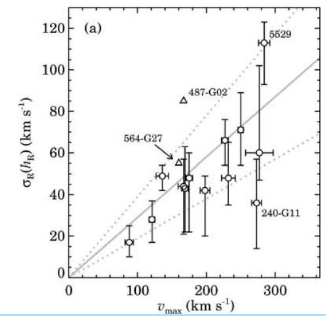
$$\langle V_{\rm z}^2\rangle^{1/2} = \sqrt{2\pi G\rho(0)}z_\circ = \sqrt{\pi G\sigma}z_\circ$$

Roelof Bottema[18] found that the stellar velocity dispersion at a fiducial radius correlates maximum in the rotation curve.

[18] R. Bottema, A.&A. 275, 16 (1993)

On the left Bottema's original correlation and on the right the same from a more recent study[19].





[19] M. Kregel, P.C. van der Kruit & K.C. Freeman, Mon.Not.R.A.S. 358, 503 (2005)

Using this relation we can estimate the disk surface density if we know $z_\circ$ and the rotation curve.

Statistical analysis of samples of galaxies gives[20] then is

$$\frac{V_{\rm rot,disk}}{V_{\rm rot,obs}} = 0.56 \pm 0.06.$$

A working definition[21] of this ratio for a maximum disk is

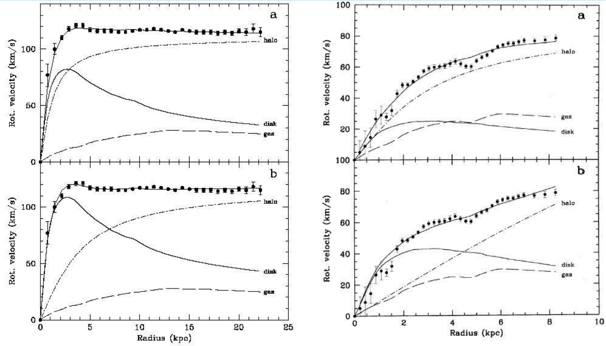
$$\frac{V_{\rm rot,disk}}{V_{\rm rot,obs}} = 0.85 \pm 0.10.$$

So, in general galaxy disk appear to be NOT maximum disk.

[20] R. Bottema, A.&A. 275, 16 (1993); M. Kregel, P.C. van der Kruit & K.C. Freeman, Mon.Not.R.A.S. 358, 5003 (2004)

[21] P.D. Sackett, Ap.J. 483, 103 (1997)

Bottema's analysis[22] on a high surface brightness and a low-surface brightness galaxy gives a model according to the stellar velocity dispersion as at the top and the maximum disk hypothesis as at the bottom.



[22] R. Bottema, A.&A. 328, 517 (1997)

f. Our Galaxy

The measured surface density[23] of the stellar disk in the solar neighbourhood is 50 to 80 $M_\odot$ pc^{-2} and the scalelength[24] of the disk 4 to 5 kpc.
With this it can be estimated that the luminous matter provides a maximum rotation velocity of 155 ± 30 km/s, while the observed value is 225 ± 10 km/s.

The Galaxy is then not maximum disk.

However, one can change the parameters within uncertainties to get different answers[25].

---

[23] K.H. Kuijken & G. Gilmore, M.N.R.A.S. 239, 605 (1989) find $46 \pm 9\ \mathrm{M_\odot pc^{-2}}$ and J.N. Bahcall, Ap.J. 287, 926 (1984) $80 \pm 20 \mathrm{M_\odot pc^{-2}}$

[24] P.C. van der Kruit, A&A. 157,230 (1986)

[25] e.g. J.A. Sellwood & R.H. Sanders, Mon.Not.R.A.S. 233, 611 (1988); P.D. Sackett, Ap.J. 483, 103 (1997)

Modified dynamics

Flat rotation curves may show that classic Newtonian gravity does not work at large distances[26]. For this purpose Modified Newtonian Dynamics (MOND)[27] was developed.

This has an accelaration $\vec{g}$, which is related to Newtonian acceleration $\vec{g}_{\rm N}$ as

$$\vec{g}\left(\frac{g}{a_\circ}\right)\left[1+\left(\frac{g}{a_\circ}\right)^2\right]^{-1/2} = \vec{g}_{\rm N}$$

with $a_\circ \sim 1.2 \times 10^{-8}$ cm sec^{-2}.

[26] e.g. R.H. Sanders, Mon.Not.R.A.S. 223, 539 (1986); K. Begeman, A.H. Broeils & R.H. Sanders, Mon.Not.R.A.S. 249,523 (1991)

[27] e.g. M. Milgrom, Ap.J. 270, 365 (1983)

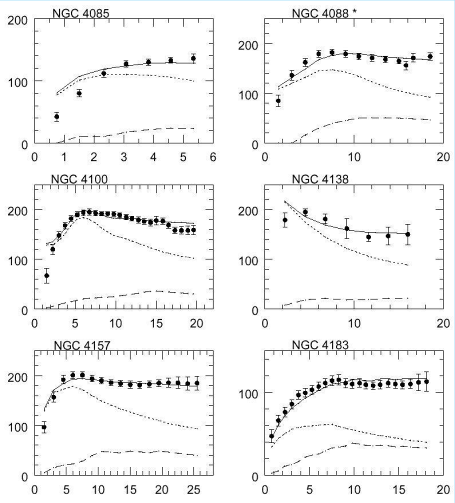
- For large accelerations $g/a_\circ$ this reduces to Newtonian gravity. So on small scales (in the solar system or the inner parts of galaxies) we have $g = g_{\rm N} \propto R^{-2}$ and Keplerian rotation with $V_{\rm rot}^2 \propto R^{-1}$.
- But at low accelerations is becomes $g = (g_{\rm N} a_\circ)^{1/2}$. Since now $g \propto R^{-1}$ this gives rise to $V_{\rm rot}^2 \propto R^0 =$ constant.

The result is that flat rotation curves can be produced without introducing a dark halo .

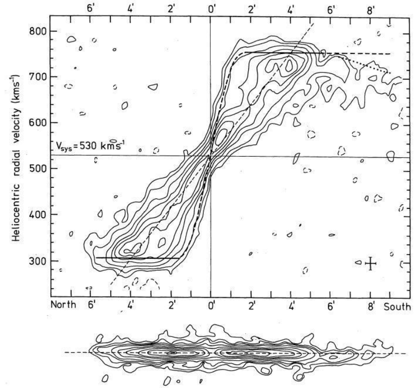
Here are some fits to actual rotation curves[a].

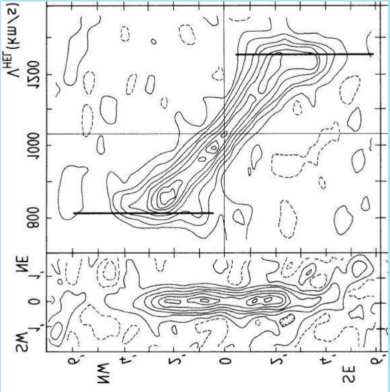
The full lines are the MOND-fits and the other lines show Newtonian curves for the stars and gas.

[a] R.H. Sanders & M.A.M. Verheijen, Ap.J. 503, 97 (1998)

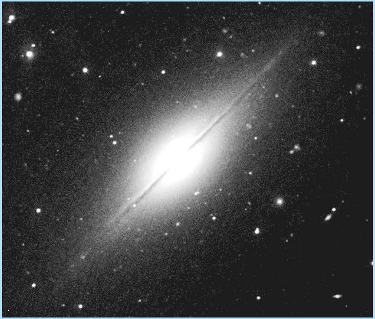


NGC 891 and NGC 7814 have the same rotation curves...





but completely different light distributions.



This is inconsistent with MOND.

Vertical dynamics

Observations of stellar velocity dispersions

1. Z-velocity dispersion

If disks have constant mass-to-light ratios M/L, the density can be described by

$$\rho(R,z) = \rho(0,0)\,\exp\left(-R/h\right)\,\mathrm{sech}^{\,2}(z/z_\circ)$$

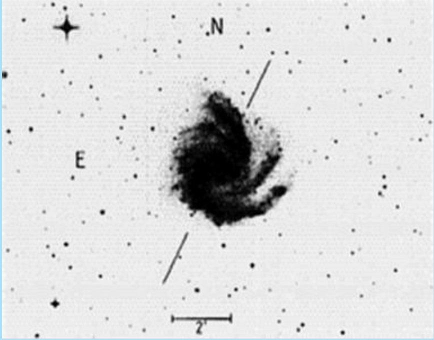
The vertical velocity dispersion then is

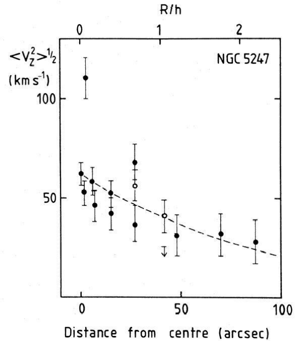
$$\langle V_z^2 \rangle^{1/2} = \sqrt{2\pi G \rho(R,0)}\, z_\circ$$

and it is expected that

$$\langle V_z^2 \rangle^{1/2} \propto \exp\left(-R/2h\right)$$

This can be tested by observations in face-on systems, e.g. NGC 5247[28].





[28] P.C. van der Kruit & K.C. Freeman 1986, Ap.J. 303, 556 (1968)

The fit is

$$\langle V_{\rm z}^2 \rangle^{1/2} = (62 \pm 7) \exp\left[-(0.42 \pm 0.10)\ R/h\right] {\rm km\ s}^{-1}$$

This is consistent with M/L about constant.

R- and θ -velocity dispersions

From fundamental kinematics we have

$$\frac{\langle (V_\theta - V_{\rm t})^2 \rangle}{\langle V_{\rm R}^2 \rangle} = \frac{B}{B-A}$$

So, if we know the rotation curve we know the ratio of the radial and tangential velocity dispersion.

The other property to consider is the asymmetric drift.

The hydronamic equation can be written as

$$\begin{aligned} -K_{\rm R} \quad = \quad & V_{\rm t}^2 - \langle V_{\rm R}^2 \rangle \frac{\partial}{\partial R} \ln(\nu \langle V_{\rm R}^2 \rangle) + \\ & \frac{1}{R} \left\{ \langle V_{\rm R}^2 \rangle - \langle (V_\theta - V_{\rm t})^2 \rangle + \right. \\ & \left. \langle V_{\rm z} V_{\rm R} \rangle \frac{\partial}{\partial z} (\ln \nu \langle V_{\rm z} V_{\rm R} \rangle) \right\} \end{aligned}$$

Poisson's equation is

$$\frac{\partial K_{\rm R}}{\partial R} + \frac{K_{\rm R}}{R} + \frac{\partial K_{\rm z}}{\partial z} = -4\pi G \rho$$

For small z it can be shown that

$$\frac{\partial K_{\rm R}}{\partial R} + \frac{K_{\rm R}}{R} = 2(A-B)(A+B)$$

and for a flat rotation curve $A=-B$, so that

$$\frac{\partial K_{\rm z}}{\partial z} = -4\pi G\rho$$

Then

$$\langle V_{\rm z} V_{\rm R} \rangle = 0$$

Obviously we have

$$K_{\rm R} = V_{\rm rot}^2 / R$$

For an exponential disk with constant M/L

$$\frac{\partial}{\partial R} \ln \nu = -\frac{1}{h}$$

The asymmetric drift equation then becomes

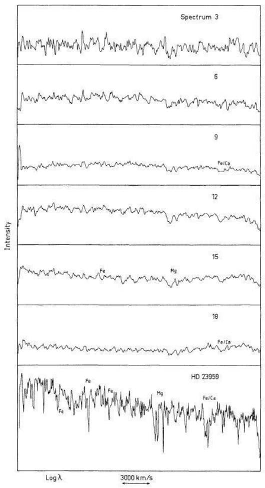
$$V_{\rm rot}^2 - V_{\rm t}^2 = \langle V_{\rm R}^2 \rangle \left[\frac{R}{h} - R \frac{\partial}{\partial R} \ln \langle V_{\rm R}^2 \rangle - \left\{ 1 - \frac{B}{B-A} \right\} \right]$$

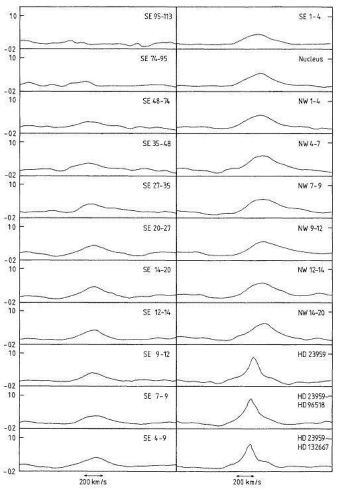
There are now two possibilities for observing. The first is to measure $\langle V_{\rm R}^2 \rangle^{1/2}$ directly from spectra.

The difficulty is the line-of-sight integration. This has to be treated by modeling as was done in the edge-on galaxy NGC 5170[29].

The profiles now have become asymmetric. In the next figure we see here the spectra and the cross-correlation peaks between galaxy and template spectra.

[29] R. Bottema, P.C. van der Kruit & K.C. Freeman, Ap.J. 178. 77 (1987)

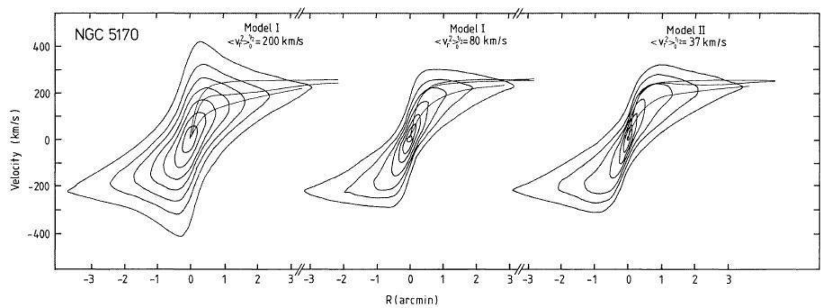

Spectrum 3
6
9
Fe/Ca
12
15
Fe
Mg
18
Fe/Ca
HD 23959
Intensity
Log λ
3000 km/s


SE 95-113
SE 1-4
SE 74-95
Nucleus
SE 48-74
NW 1-4
SE 35-48
NW 4-7
SE 27-35
NW 7-9
SE 20-27
NW 9-12
SE 14-20
NW 12-14
SE 12-14
NW 14-20
SE 9-12
HD 23959
SE 7-9
HD 23959–HD 96518
SE 4-9
HD 23959–HD 132667
200 km/s
200 km/s

Using an estimate of the circular motion from the HI-rotation curve one can calculate the profiles in a stellar "I,V-diagram".

To do this one needs an assumed radial variation of the velocity dispersion, the rotation curve (and from that the Oort constants) and the density distribution of the stars.

In the figure here we see a few such simulations. The three lines in each panel are form top to bottom: the circular motion from HI-observations, the stellar rotation velocity and peaks of Gaussians fitted to the resulting profiles.


NGC 5170
Model I
$<v_r^2>_0^{1/2} = 200$ km/s
Model I
$<v_r^2>_0^{1/2} = 80$ km/s
Model II
$<v_r^2>_0^{1/2} = 37$ km/s
Velocity (km/s)
R (arcmin)

The second option is to measure the asymmetric drift.

The relevant equation was

$$V_{\rm rot}^2 - V_{\rm t}^2 = \langle V_{\rm R}^2 \rangle \left[\frac{R}{h} - R \frac{\partial}{\partial R} \ln \langle V_{\rm R}^2 \rangle - \left\{ 1 - \frac{B}{B-A} \right\} \right]$$

So we see that we need to measure:

- $V_{\rm rot}$, A and B from HI-synthesis or emission line spectroscopy.
- $V_{\rm t}$ from absorption line spectroscopy.
- h from surface photometry.

For a flat rotation curve:

$$\frac{B}{B-A} = 0.5 \text{ and } \kappa^2 = \frac{2V_{\rm rot}^2}{R^2}$$

For small asymmetric drift:

$$V_{\rm rot}^2 - V_{\rm t}^2 \approx 2V_{\rm rot}(V_{\rm rot} - V_{\rm t})$$

Now consider two possibilities:

- Model I with $\langle V_{\rm R}^2 \rangle / \langle V_{\rm z}^2 \rangle$ constant. Then

$$\langle V_{\rm R}^2 \rangle^{1/2} \propto \exp\left(-R/2h\right)$$

$$V_{\rm rot} - V_{\rm t} = \frac{\langle V_{\rm R}^2 \rangle}{2 V_{\rm rot}} \left(\frac{2R}{h} - 0.5 \right)$$

- Model II with Q constant. Then

$$\langle V_{\rm R}^2 \rangle^{1/2} \propto R \exp\left(-R/h\right)$$

$$V_{\rm rot} - V_{\rm t} = \frac{\langle V_{\rm R}^2 \rangle}{2 V_{\rm rot}} \left(\frac{3R}{h} - 2.5 \right)$$

How different are these models? For comparison calculate a Q (arbitrarily set to unity at one scalelength) for the first model:

R/h	Q
1.0	1.17
1.5	**1.00**
2.0	0.96
3.0	1.06
4.0	1.31
5.0	1.73

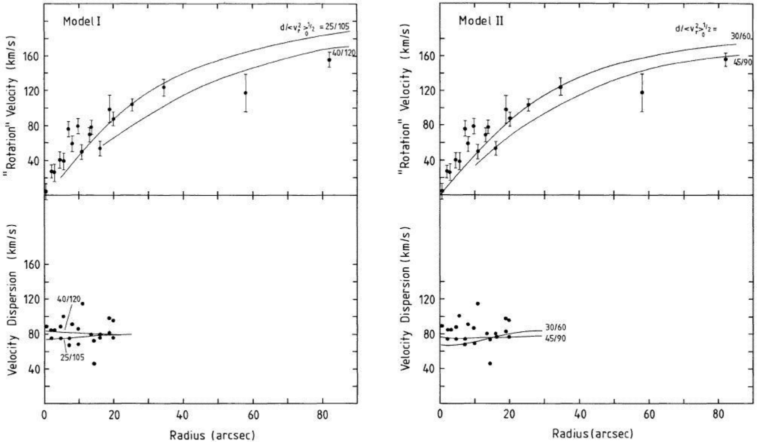
We see that the models are really not different up to four h.

At large R there is a contribution from the gas, which lowers the total velocity dispersion and increases the surface density and therefore lowers Q.

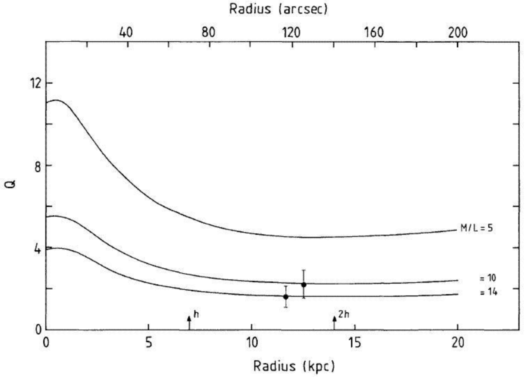
Numerical experiments on dynamics of stellar disks give $Q \sim 1.5 - 2.0$ at all radii.

Back to the data on NGC 5170.

The fits to the data of NGC 5170 are as follows.

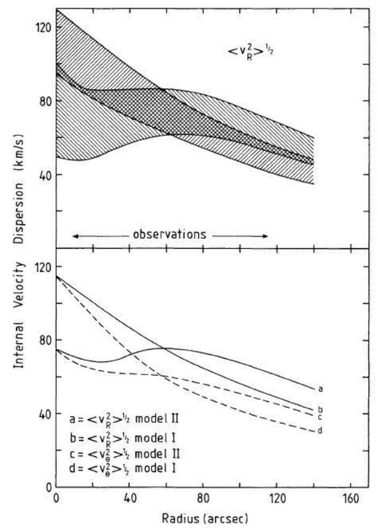


The resulting Q is as the lines in the figure below for Model I and the dots plus error bars in Model II for various assumed values for the disk M/L.



The velocity dispersions have the following radial distribution.

There is little difference between the two models.

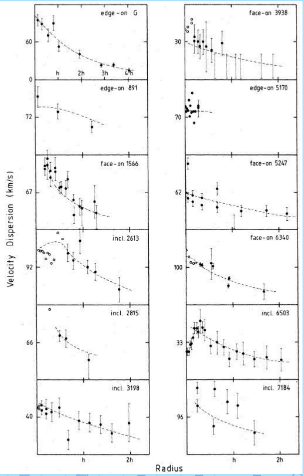


The Bottema relations

R. Bottema[a] observed stellar velocity dispersions in a set of 12 galaxies.

He then defined as fiducial values the radial velocity dispersion at one scalelength for inclined systems and the vertical velocity dispersion in the center for face-on systems.

This difference should roughly correct for the ratio between these dispersions.

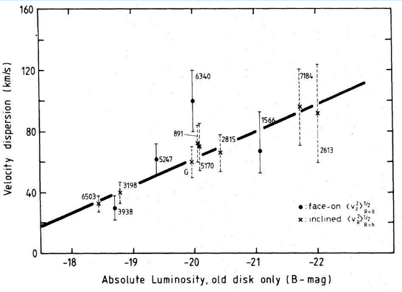


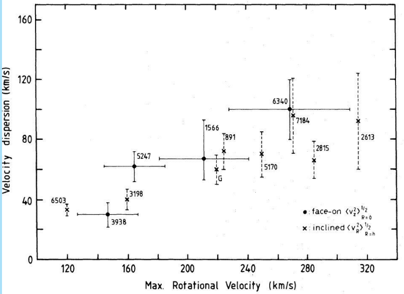
[a] Ph.D. thesis (1995); Bottema, A.&A. 275, 16 (1993)

He then found the following relations

$$\langle V_{\rm R}^2\rangle_{\rm R=h}^{1/2} = \langle V_{\rm z}^2\rangle_{\rm R=0}^{1/2} = -17 \times M_{\rm B} - 279 \ \ {\rm km/s}$$

$$\langle V_{\rm R}^2\rangle_{\rm R=h}^{1/2} = \langle V_{\rm z}^2\rangle_{\rm R=0}^{1/2} = 0.29 V_{\rm rot} \ \ {\rm km/s}$$





Can we understand these relations?

From the definition of Q we have

$$Q \propto \langle V_{\rm R}^2 \rangle^{1/2} \kappa \sigma^{-1}$$

For a flat rotation curve

$$\kappa \propto V_{\rm rot} R^{-1}$$

An exponential disk has

$$\sigma \propto \mu_\circ (M/L) \exp{(-R/h)}$$

Combining these equations gives

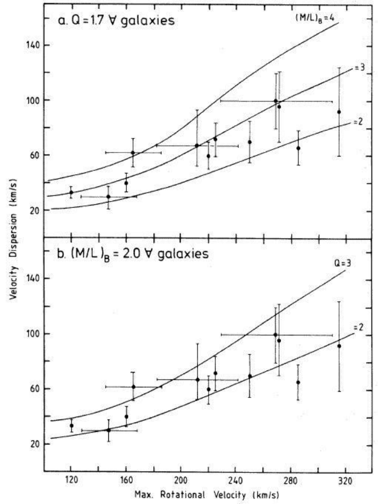
$$\langle V_{\rm R}^2 \rangle_{\rm h}^{1/2} \propto \mu_\circ (M/L) Q h V_{\rm rot}^{-1}$$

Now $L \propto \mu_\circ h^2$ and the Tully-Fisher relation gives $L \propto V_{\rm rot}^n$ with $n \approx 4$, so

$$\langle V_{\rm R}^2 \rangle_{\rm h}^{1/2} \propto \mu_\circ (M/L) Q V_{\rm rot} \propto \mu_\circ (M/L) Q L^{1/4}$$

So we expect that $\mu_\circ$, M/L and Q or at least their product are constant between disks.

With the actual observed central surface brightness the following curves result for either constant Toomre Q or constant M/L.



We had for hydrostatic equilibrium at the center

$$\langle V_{\rm z}^2 \rangle_{\rm R=0}^{1/2} = (2.3 \pm 0.1)\sqrt{G \sigma_\circ z_{\rm e}}$$

$\sigma_\circ$ is the central surface density and the range in the constant results from the choice of n.

The maximum rotation velocity of the exponential disk then is

$${}_{\rm disk} = 0.88\sqrt{\pi G \sigma_\circ h} = (0.69 \pm 0.03) \langle V_{\rm z}^2 \rangle_{\rm R=0}^{1/2} \sqrt{\frac{h}{z_{\rm e}}}$$

With the Bottema relation between this central velocity dispersion and the maximum observed rotation velocity we get

$$\frac{V_{\rm disk}}{V_{\rm rot}} = (0.21 \pm 0.08)\sqrt{\frac{h}{z_{\rm e}}}$$

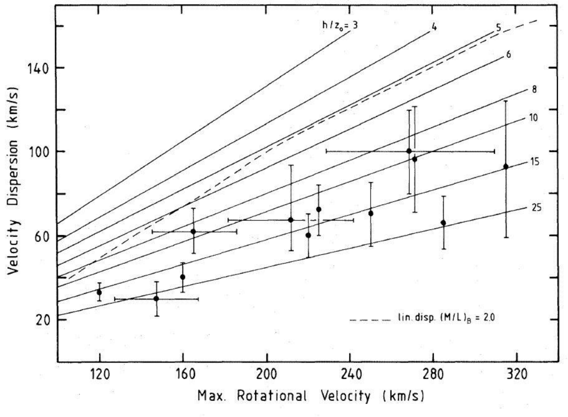
Analysis of a sample of edge-on galaxies gives for the ratio of scaleparameters 7.3 ± 2.2[30], so that

$$\frac{V_{\rm disk}}{V_{\rm rot}} = (0.57 \pm 0.22)$$

So disks in general are not maximum disk.

[30] M. Kregel, P.C. van der Kruit & R. de Grijs, Mon.Not.R.A.S. 334, 646 (2002)

Bottema[31] first showed with this argument that his relations implied that for maximum disk situations the stellar disks should be much flatter than observed.



[31] R. Bottema, A.&A. 275, 16 (1993)

For a flat rotation curve we have

$$\kappa = 2\sqrt{B(B-A)} = \sqrt{2}\frac{V_{\rm rot}}{R}$$

From the definition of Q and applying at $R = h$ we get

$$\langle V_{\rm R}^2 \rangle_{\rm R=h}^{1/2} = \frac{3.36G}{\sqrt{2}} Q \frac{\sigma(R=h)h}{V_{\rm rot}}$$

Using hydrostatic equilibrium (also at $R = h$) gives[32]

$$\frac{\langle V_{\rm z}^2 \rangle^{1/2}}{\langle V_{\rm R}^2 \rangle^{1/2}} = \sqrt{\frac{(7.2 \pm 2.5)}{Q} \frac{z_{\rm e}}{h}}$$

[32] P.C. van der Kruit & R. de Grijs, A.&A. 352, 129 (1999)

In the solar neighborhoud this axis ratio of the velocity ellisoid is ~ 0.5[33] and for the Galaxy we have $z_{\rm e} \sim 0.35$ kpc and $h \sim 4$ kpc, so that

$$Q \sim 2.5.$$

Taking all data and methods together it is found that this applies in all galaxies; disks are locally stable according to the Toomre criterion.
Numerical studies give such values for Q when disks are marginaly stable.

[33] W. Dehnen & J. Binney, Mon.Not.R.A.S. 298, 387 (1998)

Swing amplification and global stability

Swing amplification[34] of disturbances occurs as a result of the shear in rotating disks and turns these disturbances into growing trailing spiral waves.

It can be formulated in a criterion for prevention of this instability[35]

$$X = \frac{R\kappa^2}{2\pi G m \sigma(R)} \gtrsim 3$$

Here m is the number of spiral arms.

[34] A. Toomre, in a Cambridge conference on Structure and Evolution of Galaxies (1981)

[35] J.R. Sellwood, IAU Symp. 100, 197 (1983)

For a flat rotation curve this can be rewritten as

$$\frac{QV_{\rm rot}}{\langle V_{\rm R}^2\rangle^{1/2}} \gtrsim 3.97m$$

and with Bottema's relation it translates into

$$Q \gtrsim 1.1m$$

To prevent strong asymmetric $m = 1$ or bar-like $m = 2$ instabilities we require $Q \gtrsim 2$.

Numerical studies have indicated that disks with velocity dispersions as observed show global instabilities when evolving by themselves.

Disks can be stabilised by massive halos and therefore global stability requires that the disk mass has to be less than a certain fraction of the total mass, according to the criterion[36]

$$Y = V_{\rm rot} \left(\frac{h}{G M_{\rm disk}} \right)^{1/2} \gtrsim 1.1$$

This implies that within $R_{\rm max}$ the mass in the halo $M_{\rm halo} > 75\%$. This is also not true for maximum disk.

[36] G. Efstathiou, G. Lake & J. Negroponte, Mon.Not.R.A.S. 199, 1069 (1982)

The criterion can be rewritten as

$$Y = 0.615 \left[\frac{Q R V_{\rm rot}}{h \langle V_{\rm R}^2 \rangle^{1/2}} \right]^{1/2} \exp \left(-\frac{R}{2h} \right) \gtrsim 1.1$$

Evaluating this at $R = h$ and using the Bottema relation gives

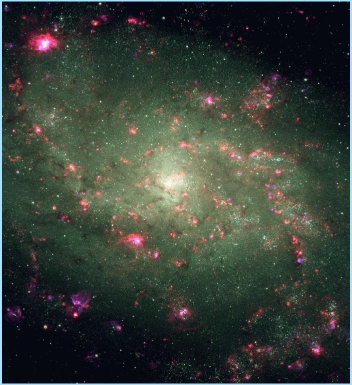
$$Q \gtrsim 2$$

Spiral structure

Density wave theory

We distinguish two types of spiral structure, grand design ...

and flocculent.



A comparative study of these two classes[37] suggests that in grand-design spiral strcuture there seems to be a strong underlying spiral wave in the stellar disk, while not in flocculent ones.

The density wave theory[38] was a response to the "winding dilemma", where material arms would wind up in a matter of 10^8 years or less.

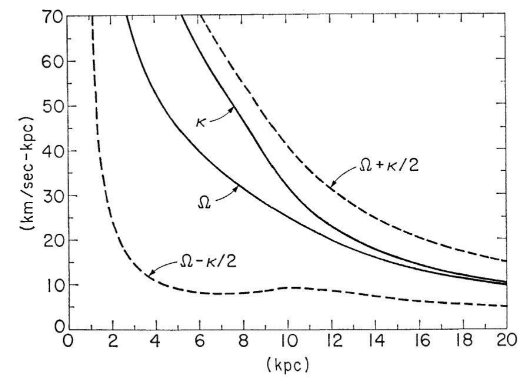
The density wave is a spiral pattern, whose shape does not change with time, and which moves through the stellar and insterstellar disk.

[37] B.G. Elmegreen & D.B. Elmegreen, Ap.J.Suppl. 54, 127 (1984)

[38] C.C. Lin & F.H. Shu, Ap.J. 140,646 (1964), C.C. Lin, C. Yuan & F.H. Shu, Ap.J. 155, 721 (1969)

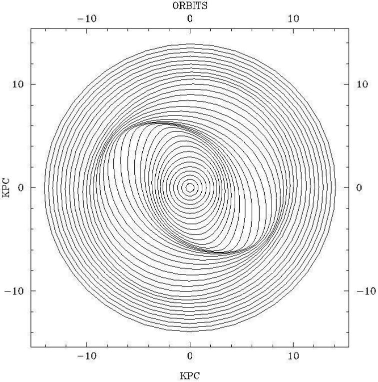
At the basis of a good description we can take the deduction that in the disk of our Galaxy (and in many others) the inner Lindblad resonance $\Omega - \kappa/2$ is fairly constant.

In this resonance a star goes through two epicycles during one revolution around the center. That means it describes a closed oval orbit in a rotating coordinate system with $\Omega - \kappa/2$.



In a disk where this property is constant over most radii we can get the following situation, where the stars are forced in orbits that line up as a spiral pattern.

In a coordinate frame, rotating with the pattern speed $\Omega_\mathrm{p} = \Omega - \kappa/2$, the spiral pattern remains unchanged.



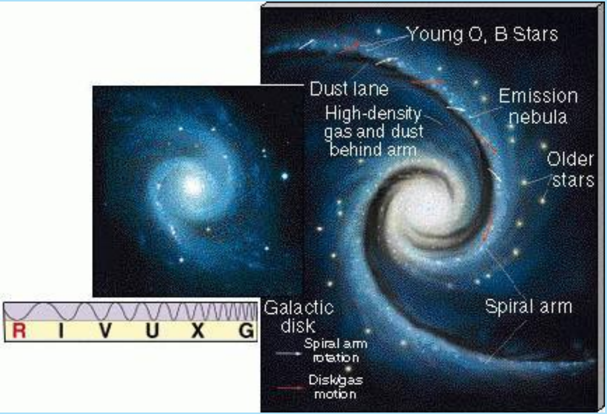
In the original density wave theory the density perturbations maintain themselves. The response of the stars to the perturbed gravitational field by the density concentrations in the arms results in a self-sustaining pattern of density perturbations.

It was realized later by Toomre and others that the dissipation of energy in the waves is quick enough ($\sim 10^8$ years) that rejuvenation is required regularly.

It took until the first part of the seventies, before the underlying wave in the stellar disk was discovered in surface photometry[39].

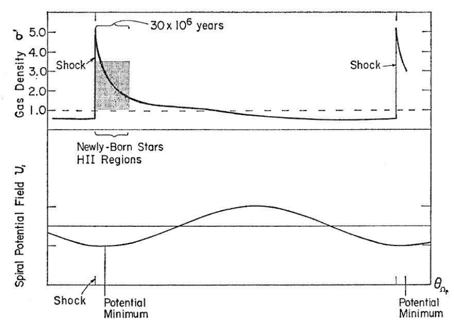
[39] F. Schweizer, Ap.J.Suppl. 31, 313 (1976)

The strongest confirmation came from studies of the interstellar medium.



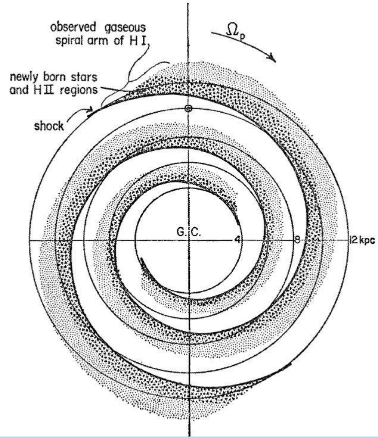
The reponse of the gas and dust is a-linear, since the relative velocities involved are supersonic[40].

This gives shocks at the inner sides of the spiral arms and associated dustlanes and star formation.



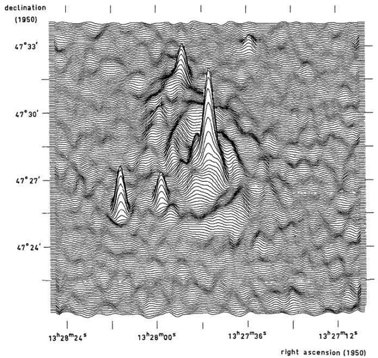
[40]W.W. Roberts, Ap.J. 158, 123 (1969)

The “delay” between dustlanes and HII-regions concerns the time between onset of gravitational instability and birth of MS-stars.

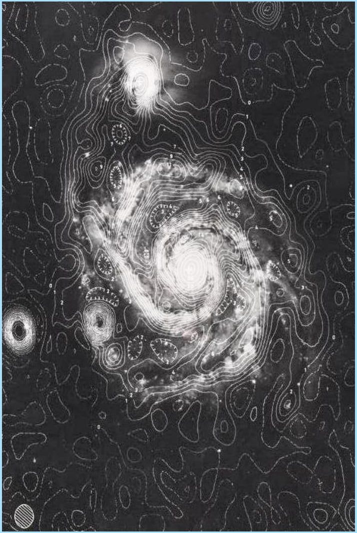


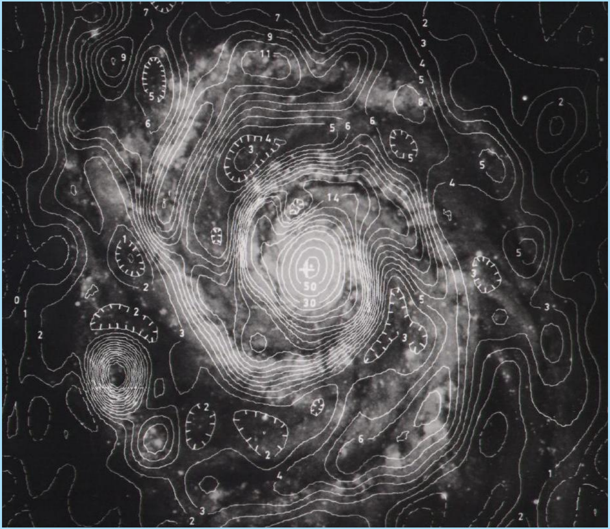
It was also confirmed by radio continuum studies with the new WSRT[41] in M51.

The compression holds at least for the magnetic field and possibly the relativistic electrons, so the synchrotron radiation will be enhanced at the inside of the arms and at the dustlanes.

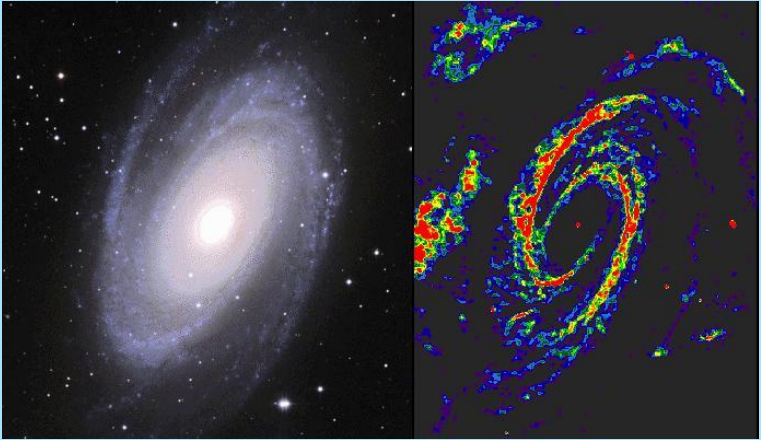


[41] D.S. Mathewson, P.C. van der Kruit & W.N. Brouw, A.&A. 17, 468 (1972)





The next thing was to try and measure the streaming motions due to the density wave. This was tried in M81 using HI.



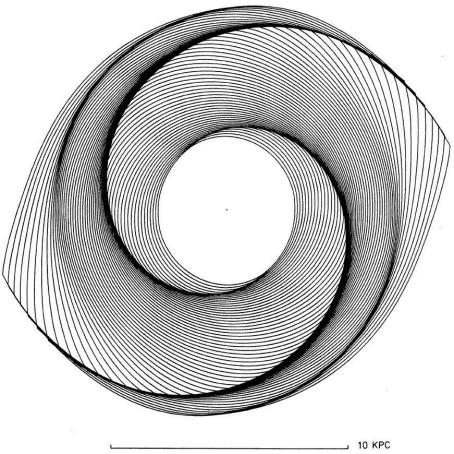
The Ph.D. thesis of H.C.D. Visser[42] analysed this in detail.

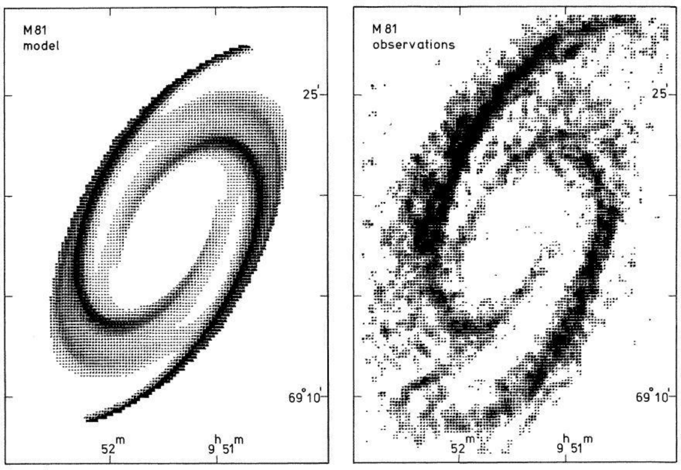
He used the surface photometry of Scheizer and HI-measurements at Westerbork.

With that he was able to find an internally consistent representation of the observations of at the same time both the HI surface density distribution and the HI velocity field.

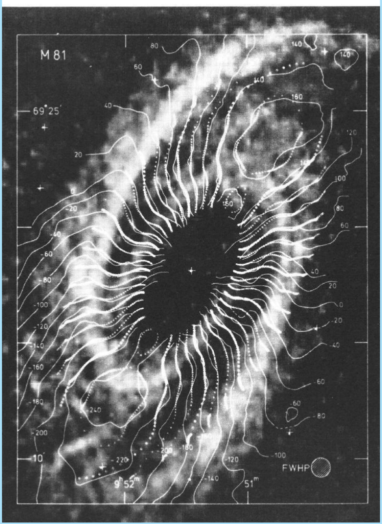
Here are the (non-linear) streamlines of the gas.

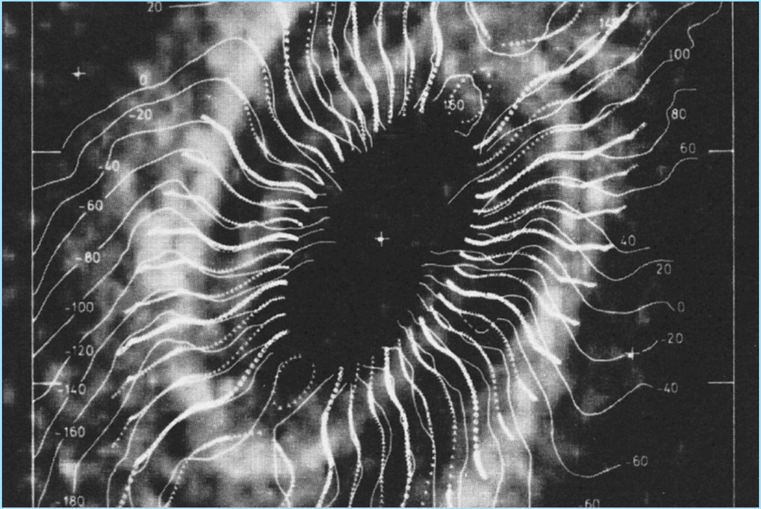
[42] 1978; see also A.&A. 88, 159 (1980)


10 KPC

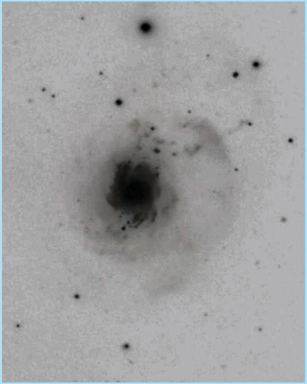

M 81
model
25'
69°10'
52m
9h51m
M 81
observations
25'
69°10'
52m
9h51m

The streaming motions are of the order of 10 km s^{-1}.



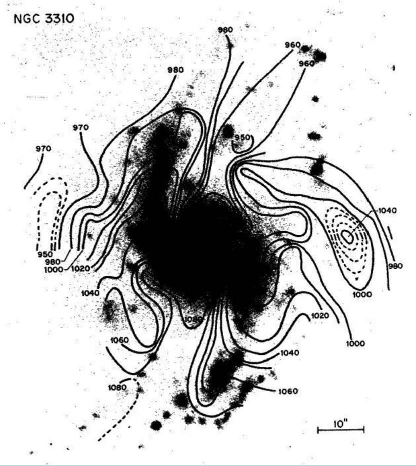

20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
160
100
80
60
40
20
0
-20
-40
-60
-60

A very exceptional case is the disturbed, star burst galaxy NGC 3310, which is probably an example of a recent merger[43].

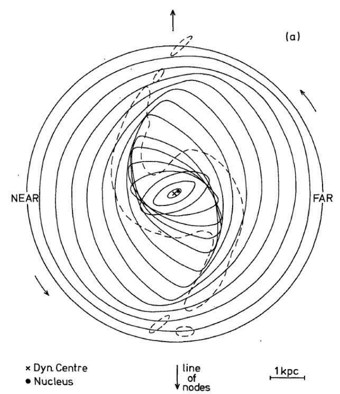


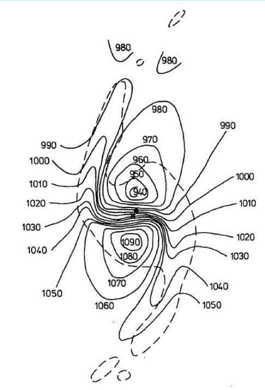
[43] P.C. van der Kruit & A.G. de Bruyn, A.&A. 48, 373 (1976); P.C. van der Kruit, A.&A. 49, 161 (1976)

The velocity field shows strong signs of streaming motions related to the spiral arms.



The streaming motions are here up to a third or so of the rotation velocity.





Stochastic star formation model

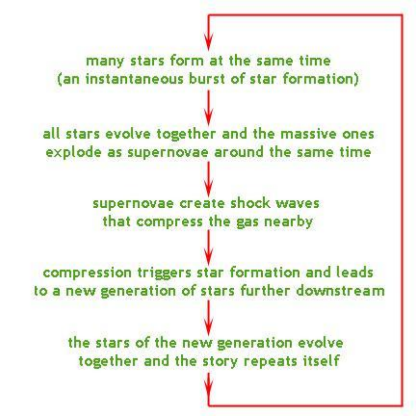
Density waves may be generated by tidal interactions, such as in M51 or in NGC 3310, or through Toomre's swing amplification.

The flocculent spiral stucture is probably the result of stochastic self-propagating star formation[44].

Since the propagation and induced star formation is never 100%, also this will die out unless there is also spontaneous star formation.

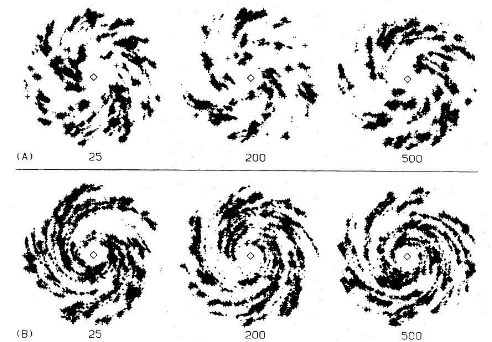
[44] H. Gerola & P.E. Seiden, Ap.J. 223, 129 (1978)

In this model star formation through supernova explosions is postulated to stimulate star formation in the neighborhood.



Such structures are then drawn out by differential rotation into arm-like features.

On the next page some simulations.



It has been suggested[45] that grand-design spiral structure is produced by bars or tidal encounters, while flocculent spiral structure results if the disk is left by itself.

[45] J. Kormendy & C.A. Norman, Ap.J. 233, 539 (1979)