Constraints on dark matter due to primordial black hole evaporation

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July 6, 2005

Report of Klein Onderzoek done under the supervision of Prof. Dr. R.H. Sanders at the Kapteyn Astronomical Institute of the University of Groningen

Abstract

Four models of primordial black hole (PBH) formation that deal with primordial density fluctuations are compared. It is investigated if the evaporation of black holes can explain a critical dark matter density decrease from recombination redshift ($z = 1000$) of $\Omega_{dm} = 0.3$ to $\Omega_{dm} \sim 0.0$ now. In order to form a considerable amount of primordial black holes the spectral index of the power spectrum of the density fluctuations should be $n > 1$, which contradicts the cosmic microwave background radiation (CMBR) measurements by WMAP. It is suggested that PBH constraints provide information that CMBR measurements do not, in particular in the case that the power spectrum P(k) is not a power law but a spike, a step or broken scale-invariant. Only in the latter case a significant part of the cold dark matter is in the form of PBHs with mass M in the range $5 \times 10^{15}g \leq M \leq 10^{21}g$. An accurate determination of the mass spectrum of MACHOs, which if black holes have to be of a primordial origin, would yield information on the underlying primordial perturbation spectrum. Femtolensing observations of gamma-ray bursts by PBHs could preclude those in the range $10^{17}g \leq M \leq 10^{20}g$. From the fact that the total flux of evaporated black holes should not be larger than the observed diffuse gamma-ray background, a critical density of at least $\Omega_{PBH} < 10^{-8}$ for the mass range $2 \times 10^{13}g \leq M \leq 5 \times 10^{14}g$ is guaranteed. Only if all the cold dark matter (CDM), $\Omega_{CDM} = 0.3$, would consist of PBHs with masses of 6×10^{14}, $13.4 Gyrs$ of evaporation would yield a new critical dark matter density of $\Omega_{CDM} = 0.25$ now.

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1 Introduction

Distinct from black holes which form by the process of stellar collapse, black holes formed by mechanisms in the early universe can exist. These primordial black holes (PBHs) can be a justifiable alternative to the most popular candidate for dark matter: massive nonbaryonic elementary particles. The evaporation of these PBHs can explain why dark matter disappears from the recombination epoch until now. But with what amount do evaporating black holes contribute to the dark matter loss? Can evaporating PBHs explain a critical dark matter density decrease of $\Omega_{dm} = 0.3$ at the recombination redshift ($z = 1000$), $3.8 \times 10^5 years$ after the Big Bang, to $\Omega_{dm} \sim 0.0$ now? To answer this question it is investigated to what extent statements about how many PBHs form, that is what are the PBH number, mass and critical energy density in present and past, and the speed with which they evaporate, can be correct.

Only the mechanism for PBH formation which deals with primordial density perturbations, the density fluctuations in the early universe, is discussed here. Over-dense regions which strongly deviate from the background universe can evolve into black holes when the over-dense regions enter the cosmological horizon (Page and Hawking, 1976; Niemeyer and Jedamzik, 1998; Kim, Lee and MacGibbon, 1999; Green, 2001). However, there exists a variety of other formation scenarios, for example it is also considered that PBHs form due to the collision of bubbles formed at phase transitions (Hawking, Moss and Stewart, 1982), the collapse of cosmic string loops (Polnarev and Zemboricz, 1991), softening of the equation of state (Canuto, 1978) and in a braneworld cosmology (Clancy, Guedens and Liddle, 2003). An overview of recent developments is given by Carr (2005). These PBH formation mechanisms as candidates for dark matter evaporation won't be discussed in this paper.

Over the past thirty years, four density perturbation models for PBH formation have arisen. Early studies by Carr (1975), who assumed that all PBHs form at a given epoch have the same mass, found that an extended PBH mass function is only possible if the primordial power spectrum is scale-invariant: $P(k) \approx k$. Then the PBH initial mass spectrum is found to be $\frac{dn}{dM} \approx M^{-\frac{5}{2}}$. Using the Press-Schechter formalism, developed to describe galaxy formation via primordial density perturbations as Gaussian fluctuations, Kim, Lee and MacGibbon (1999) found the initial PBH mass function produced for other power law spectra: $P(k) \approx k^n$. They also found from the normalization of the fluctuation amplitude on the scales for PBH formation to the cosmic microwave background radiation (CMBR) anisotropy, significant PBH abundance is possible only if the density fluctuations have an $n > 1$, blue spectrum. For a $n = 1$ scale-invariant power spectrum normalized to that on large scales deduced from the Cosmic Background Explorer (COBE) observations of the CMBR anisotropy not a large number of PBHs is formed. Green (2001) calculates that the fraction of the energy density of the universe in PBHs at the time they form is $\sim 10^{-6\times10^6}$. The classical $M^{-\frac{5}{2}}$ initial mass function has therefore shown to be cosmologically irrelevant. In contrast with the preceding models Niemeyer and Jedamzik (1999) have shown using numerical simulations that, as a consequence of near critical gravitational collapse, at a fixed epoch PBHs with a range of masses form, leading to a broader mass function. Recently, Shibata and Sasaki (1999) devised a new approach to the formation of PBHs seeking to find criteria on the metric distribution rather than the density field, and in a form which can be applied to super-horizon initial perturbations. They found that the PBH mass spectrum is best computed using a theory of peaks, rather than the Press-Schechter-like calculations in the Kim-Lee-MacGibbon and the Niemeyer-Jedamzik model. Their model doesn't result in a drastic revision of the PBH formation rate but do put calculations on a sounder theoretical footing (Green, Liddle, Malik and Sasaki, 2004).

Upper limits on Ω_{PBH}, the fraction of the critical energy density of the universe which is in PBHs, are found from the effects of PBHs on their environment. The emission of Hawking radiation from the PBHs has to have contributed to the observed extragalactic diffuse gamma-ray background (DGB) (Kim, Lee and MacGibbon, 1999). And PBHs with initial mass $\sim 5 \times 10^{14} g$ are presently at the final stage of their evaporation and may emit enormous amounts of energy in the form of short-period gamma-ray bursts (GRBs) (Green, 2001). PBHs with initial masses $< 5 \times 10^{14} g$ should even have contributed more energy to the DGB. From these two energy density limits constraints on the spectral index of the density fluctuations have been derived. It has been shown that the evaporation of PBHs in general give the strongest upper bounds on the spectral index (Green, Liddle and Riotto, 1997). Possibly massive compact halo objects (MACHOs) discovered by microlensing of stars in the Large Magellanic Cloud are black holes with masses of $\sim 0.5 M_{\odot}$. Black holes of this mass can only be of primordial origin. Allowing a critical density of $\Omega_{MACHOs} = 0.1$ at around a solar mass and searches for microlensing of stars in the Large Magellanic Cloud excludes PBHs in the mass range $10^{26}g \leq M_{PBH} \leq 10^{34}g$ (Alcock et al., 2002). There is a possibility that a large fraction of the dark matter consists of PBHs in the mass range $5 \times 10^{15}g \leq M_{PBH} \leq 10^{21}g$ (Blais et al., 2002). Femtolensing observations of gamma-ray bursts by PBHs could preclude those in the range $10^{17}g \leq M_{PBH} \leq 10^{20}g$. (Gould, 1992)

In section 2 the PBH formation constraints by primordial density fluctuations are discussed by elaborating on the four proposed PBH mass spectra. In section 3 the physics of PBHs, the evaporation rate and lifetime of PBHs, the contribution to the DGB and short-period gamma-ray bursts and the constraint that the existence of PBHs gives on the spectral index of the power spectrum is discussed. In section 4 it is shown to what amount PBHs are an explanation for disappearing dark matter. In section 5 the conclusions are summarized.

2 PBH formation constraints of primordial density perturbation models

In the case of PBHs created due to density fluctuations in the early universe what constraints do the four different models give? What are the free parameters for each model? How large will the initial mass of a PBH be, and how will the mass be distributed over time and density? What are the observational constraints on these models?

2.1 Carr-Hawking model

Carr (1975), Page and Hawking (1976) are the pioneers in investigating the formation of PBHs by the gravitational collapse of primordial density perturbations. According to them at a spatial region x a black hole will form if the density contrast δ, i.e. the deviation in energy density relative to the background energy density ρ_b,

$$\delta(x) = \frac{\rho(x) - \rho_b}{\rho_b} \tag{1}$$

fulfills $\frac{1}{3} \leq \delta \leq 1$. The lower bound is derived from the fact that the size of the region should be greater than the Jeans length at the time of collapse before gravity can overcome pressure. The upper bound is needed for the requirement that the region doesn't form a disconnected topology, i.e. a separate closed universe. In this Hawking-Carr model the power spectrum of the density fluctuations, described by the power law

$$P(k) \propto |k|^n, \tag{2}$$

where the spectral index n is scale-invariant, that is $n = 1$. Inflation produces a power spectrum that is close to this scale-invariant Harrison-Zel'dovich spectrum. In this case PBHs have an initial mass of

$$M_{BHi} = \gamma^{\frac{3}{2}} M_H = \gamma^{\frac{3}{2}} \frac{4\pi\rho}{3H^3}. \tag{3}$$

M_H is the horizon mass when the specific region crossed the horizon and γ determines the equation of state $p = \gamma\rho$. In general it is considered that during the formation of PBHs the universe is radiation dominated, for which $\gamma = 1/3$. The classical Carr-Hawking PBH initial mass function is given by

$$\frac{dn_{BH,CH}(M_{BHi})}{dM_{BHi}} \propto M_{BHi}^{-\frac{5}{2}}. \tag{4}$$

2.2 Kim-Lee-and-MacGibbon model

Using the Press-Schechter formalism for galaxy formation Kim, Lee and MacGibbon (1999) modernized the Carr-Hawking model to a model dependent on power laws other than the Harrisson-Zel'dovich power spectrum. COBE data suggest that it is impossible to form a significant number of PBHs in the case of a $n = 1$ power law spectrum. With the normalization of the fluctuation amplitude on the scales for PBH formation to that detected by COBE on much larger scales from the cosmic microwave background radiation (CMBR) anisotropy amplitude $\delta \sim 1.9 \times 10^{-5}$, significant PBH abundance is possible only if the density fluctuations have a $n > 1$, 'blue' spectrum. Green (2001) calculated a PBH density of $\sim 10^{-6\times10^6}$ for a $n = 1$ scale-invariant Harrison-Zel'dovich spectrum.

Kim, Lee and MacGibbon consider the same condition for the initial density contrast of a region, δ_i, as Carr (1975):

$$\beta^2 \left(\frac{M_i}{M_{Hi}}\right)^{-\frac{2}{3}} \leq \delta_i \leq \alpha^2 \left(\frac{M_i}{M_{Hi}}\right)^{-\frac{2}{3}}. \tag{5}$$

in a universe with a hard equation of state, that is $p = \gamma\rho$ with $0 \leq \gamma \leq 1$. Here α and β are constants of the order of $\sqrt{\gamma}$, M_i is the mass contained in the region of initial radius R, which is greater than the particle horizon, at the time t_i when the fluctuation develops, and M_{Hi} is the horizon mass at t_i. The PBHs form when the over-dense regions enter into the horizon. The resulting initial PBH mass M_{BHi} is approximately the horizon mass at that time t_H and is given by

$$M_{BHi} \simeq \gamma^{\frac{3}{2}} M_{Hi} \frac{t_H}{t_i} \tag{6}$$

and in terms of M_i, via

$$M_{BHi} \simeq \gamma^{\frac{3\gamma}{1+3\gamma}} M_i^{\frac{1+\gamma}{1+3\gamma}} M_{Hi}^{\frac{2\gamma}{1+3\gamma}}. \tag{7}$$

As the universe expands, larger PBHs are formed, so that PBHs with masses less than M_{BHi} coexist in the universe at time t_H.

For a $n > 1$ power-law spectrum, i.e. $P(k) \propto k^n$ the number density of black holes produced by the collapse of regions with mass between M_i and $M_i + dM_i$ is given by:

$$\frac{dn_{BH,KLG}(M_{BHi})}{dM_{BHi}} = \frac{n+3}{4}\sqrt{\frac{2}{\pi}}\gamma^{\frac{7}{4}}\rho_i M_{Hi}^{\frac{1}{2}} M_{BHi}^{-\frac{5}{2}} \sigma_H^{-1} \exp\left(-\frac{\gamma^2}{2\sigma_H^2}\right) \tag{8}$$

where ρ_i is the energy density when the PBHs form, immediately after reheating finishes at the end of inflation and the universe becomes radiation dominated. The horizon provides a sharp lower cut-off in the mass function at $M_{BH} = \gamma^{\frac{3}{2}} M_{Hi}$. For $n \neq 1$ the exponential term cuts the mass function off sharply so that it is peaked around $M_{BH} \sim M_{Hi}$. Only if $n = 1$ the mass function has the traditional form of $M_{BH}^{-\frac{5}{2}}$. The mass variance $\sigma_H = \sigma(M_H)$, roughly the mean square of the density perturbation, is related to the power spectrum as

$$\sigma_H^2 = \frac{1}{2\pi^2}\int_0^\infty W(kR)P(k)k^2 dk, \tag{9}$$

with $R = \frac{1}{aH}$ the comoving Hubble radius and $W(kR)$ a window function picking out scales around $k \sim \frac{1}{R}$. Using the time evolution of cosmological quantities during radiation and matter domination, one finds for power law power spectra

$$\sigma_H = \sigma(M_H, 0)\left(\frac{M_{H,eq}}{M_{H,0}}\right)^{\frac{1-n}{6}}\left(\frac{M_H}{M_{H,eq}}\right)^{\frac{1-n}{4}}, \tag{10}$$

where the subscript *0* and *eq* refer to values at the present epoch and matter-radiation equality and $\sigma(M_H, 0) = 9.5 \times 10^{-5}$ using the horizon crossing amplitude resolved by the COBE experiment.

PBH formation is limited to the epoch when the fluctuation arises. Since PBHs formed before the period of inflation are diluted away, this time is related to the reheating temperature by

$$t_{iRH} = 0.301 g_*^{-\frac{1}{2}} \frac{M_{Pl}}{T_{RH}^2} \prec \frac{MeV^2}{T_{RH}^2} seconds \tag{11}$$

Here g_* counts the degrees of freedom of the constituents in the early universe, in the standard model $g_* \sim 100$, and M_{Pl} is the Planck mass. And in terms of redshift this is

$$1 + z_{iRH} \approx \left(\frac{3}{2}\Omega_M^{\frac{1}{2}} t_{iRH} H_0\right)^{-\frac{2}{3}} \tag{12}$$

The minimum initial PBH mass corresponding to T_{RH} is

$$M_{RH} \preceq \frac{1}{8}\gamma^{\frac{3}{2}} M_{Pl} \frac{T_{Pl}^2}{T_{RH}^2}, \tag{13}$$

where T_{Pl} is the Planck Temperature.

2.3 Niemeyer-Jedamzik model

The effect of gravitational collapse as a critical phenomenon on PBH formation was studied by Niemeyer and Jedamzik (1998). They realized that one expects most regions collapsing to black holes to have an over-density close to a critical density of $\delta_c \approx 0.7$, twice the predicted value of the Hawking-Carr model. Numerical solutions showed that the following scaling relation, based on a scaling relation discovered in gravitational collapse of various near-critical space-times, generalized to collapsing density perturbations in an Einstein-de-Sitter universe, holds for PBH formation in the regime $0.7 < \delta_c < 1$:

$$M_i(\delta) = kM_H(\delta - \delta_c)^{\eta} \tag{14}$$

where k is a constant that depends on the shape of the fluctuation, $\eta \approx 0.37$ the universal density-perturbation-shape-independent scaling exponent and M_H the horizon mass at PBH formation.

Under the assumption that PBHs of different masses form at a single horizon mass the initial PBH number density per unit mass is calculated analytically by Niemeyer and Jedamzik to be

$$\frac{dn_{BH,NJ}(M_{BH})}{dM_{BH}} = \frac{\rho_i}{\sqrt{2\pi}\gamma\sigma(M_H)M_{BH}M_H}\left(\frac{M_{BH}}{kM_H}\right)^{\frac{1}{\gamma}} \exp\left(-\frac{[\delta_c + (\frac{M_{BH}}{kM_H})^{\frac{1}{\gamma}}]^2}{2\sigma^2(M_H)}\right) \tag{15}$$

where $\sigma(M_H)$ is the mass variance, the root-mean-square fluctuation amplitude, evaluated at horizon crossing (see equation 10). The physical number density of PBHs dilutes $\propto a^{-3}$ so that at any later time t

$$\frac{dn_{BH,NJ}}{dM_{BH}}(t) = \frac{dn_{BH,NJ}}{dM_{BH}}\left(\frac{T(t)}{T_i}\right)^3, \tag{16}$$

where T_i can be related to the initial horizon mass M_{Hi} by

$$M_{Hi} = M_{H0}\left(\frac{T_{eq}}{T_i}\right)^2\left(\frac{T_0}{T_{eq}}\right) \tag{17}$$

where the subscripts 0 and eq denote quantities evaluated at the present day and matter-radiation equality.

PBHs in cosmological interesting numbers are formed during the evolution of the early universe if the Niemeyer-Jedamzik PBH initial mass function has a maximum at

$$M_{bh}^{max} = k\left(\frac{\sigma^2}{\delta_c}\right)^{\gamma} M_H \approx 0.6 M_H, \tag{18}$$

with $k \approx 3.3$, $\frac{\sigma}{\delta_c} \approx 0.15$ and $\delta_c \approx 0.33$. Depending on the value of σ, a fraction of all PBHs formed at each epoch will have masses significantly smaller than this maximum mass. Using the Niemeyer-Jedamzik scaling relation it is no longer valid to assume a one-to-one-correspondence between M_{BH} and redshift z. The formation of black holes with a continuous initial mass function allows the formation of microscopic PBHs at all epochs.

2.4 Shibata-Sasaki model

More recently, Shibata and Sasaki devised a new approach to the formation of PBHs seeking to find criteria on the metric distribution rather than the density field, and in a form which can be applied to super-horizon initial perturbations. In this approach the initial data are formulated after horizon crossing, and hence the criterion cannot be related to the initial perturbations produced by, for instance, a period of inflation. Shibata and Sasaki numerically explored a range of initial configurations, all spherically symmetric, for the metric variable ψ in a radiation-dominated universe, defined from the spatial part of the metric on uniform-expansion hyper-surfaces as

$$g_{ij} = a^2 \psi^4 \gamma_{ij}, \tag{19}$$

where γ_{ij} is the metric of the spatial 3-sections. They were able to show that the central value of ψ, denoted ψ_0, was a good indicator of PBH formation, that takes place provided that ψ_0 exceeds a thresh-hold value $\psi_{0,th}$. The precise value of this threshold depends on the environment of the initial configuration and lies in the range from 1.4 for a density peak surrounded by a low-density region to 1.8 for a peak surrounded by a flat Friedman-Robertson-Walker (FRW) region (Shibata and Sasaki, 1999; Green, Liddle, Malik and Sasaki, 2004).

Within this new formalism the PBH mass spectrum is best computed using the theory of peaks, rather than the standard Press-Schechter-like calculation. The PBH formation criterion is expressed in terms of the peak value of the fluctuation, ψ_0 at $t = 0$, that is at some early time when the perturbation is on super-horizon scales, since ψ is constant on super-horizon scales. For a power-law power spectrum, applied to an initial value of ζ, $\mathcal{P}_\zeta(k) = A_\zeta(k/k_0)^{n-1}$, where $A_\zeta = A_{\mathcal{R}_c} = (0.8 \pm 0.1) \times 2.95 \times 10^{-9}$ for $k_0 = 0.05Mpc^{-1}$, the fraction of the Universe in peaks above the threshold is given by

$$\Omega_{PBH,peaks} = \frac{(n-1)^{\frac{3}{2}}}{(2\pi)^{\frac{1}{2}}6^{\frac{3}{2}}} \left(\frac{\zeta_{th}}{\sigma_\zeta^2(M)}\right)^2 \exp\left(-\frac{\zeta_{th}^2}{2\sigma_\zeta^2(M)}\right) \tag{20}$$

where for a Gaussian window function the mass associated with the filter $M = \rho(2\pi)^{\frac{3}{2}}R^3$, and where

$$\sigma_\zeta(M) = \frac{5+3w}{2(1+w)}\sigma_\Delta(M) = \left(\frac{A_\zeta\Gamma((n-1)/2))}{2(k_0R)^{n-1}}\right)^{\frac{1}{2}} \tag{21}$$

In figure 1 various calculations of the abundance Ω_{PBH} for power-law primordial power spectra with spectral indexes $n = 1.25$ and 1.5 are shown. The traditional calculation with $\Delta_{th} = 1/3$ is compared with the peaks theory calculation for the thresholds $\zeta_{th} = 0.7$ and 1.2. The figure shows that the Press-Schechter and peak theory correspond closely. If the expressions were exactly the same, the thresholds would be related by

$$\mathcal{P}_\Delta(k) = \frac{4(1+w)^2}{(5+3w)^2}\mathcal{P}_{\mathcal{R}_c}, \tag{22}$$

that in radiation domination would give $\Delta_{th} = \frac{4}{9}\zeta_{th}$. It turns out that this correspondence holds quite accurately even at the low abundance $\Omega_{PBH} \sim 10^{-20}$ which are close to observational bounds. Only at much lower abundances the relation doesn't hold; there peaks theory gives higher values than the Press-Schechter formalism. So, in general there's a good correspondence: $\zeta_{th} = 1.2$ is equivalent to $\Delta_{th} \simeq 0.5$ and $\zeta_{th} = 0.7$ to $\Delta_{th} \simeq 0.3$.

Given the uncertainties in the observational constraints involved Shabati and Sasaki's formulation doesn't result in a drastic revision of the PBH formation rate, but do put the calculation on a sounder theoretical footing. The mass function can be approximated by that of the standard calculation in the region of interest, if the threshold density Δ_{th} is taken in the range 0.3 to 0.5. This range of threshold values is however significantly lower than the value $\Delta_{th} \simeq 0.7$ suggested by the simulations of Niemeyer and Jedamzik, but equals the $\Delta_{th} \simeq 0.3$ of the classical Hawking-Carr model.

2.5 A comparison

The three initial mass functions, Hawking-Carr, Kim-Lee-and-MacGibbon and Niemeyer-Jedamzik are plotted in figure 2 with the parameters of each mass function chosen such that the present energy density in PBHs is the same $\Omega_{PBH} = 1 \times 10^{-8}$, which is derived from the constraint that the emission of PBHs should not exceed the observed diffuse gamma-ray background (DGB). The Kim-Lee mass function, which arises from assuming that all PBHs which form at a given epoch have the same mass, is very sharply peaked. The Niemeyer-Jedamzik mass function, which assumes a constant fabrication of PBHs with all kinds of masses at different epochs, is far broader with a long tail of low mass PBHs which would have evaporated since $z \sim 700$ and would contribute to the DGB (Green, 2001). Figure 1 shows that if the spectral index of the density perturbations increases more massive PBHs would form.

3 Basic PBH physics

Now the four most important PBH mass functions have been investigated, the questions that need to be answered are how fast do the PBHs evaporate and how many of them evaporate today. From that and the mass spectra it can be learned how much dark matter disappears from recombination epoch until now via the method of black hole evaporation. At what rate does a PBH evaporate? When is a PBH completely evaporated? How much energy is released? Can the total PBH emission equal the diffuse gamma-ray background (DGB)? Do PBHs give a strict constraint on the power spectrum of primordial density fluctuations? Are the today evaporating black holes short-period gamma-ray bursts? If so, do they constrain the PBH initial mass functions?

3.1 PBH emission rate and lifetime

PBHs evaporate via the emission of Hawking radiation (Page and Hawking, 1976). A rotating charged black hole emits particles at a rate

$$\frac{dN_s}{d\omega dt} = \frac{\Gamma_s}{2\pi}\left[\exp\frac{\omega - l\Omega - q\Phi}{\frac{\kappa}{2\pi}} + (-1)^{2s}\right]^{-1} \tag{23}$$

per degree of particle freedom. Here κ, Ω and Φ are the surface gravity, angular velocity and electric potential respectively, s is the particle spin, l is the angular momentum and q is the particle charge. The absorption probability for the emitted species Γ_s is in general a function of $\omega, \Omega, \Phi, \kappa$ together with the internal degrees of freedom and rest mass of the emitted particle (Kim, Lee and MacGibbon, 1999).

In the standard picture of PBH evaporation, non-rotating, uncharged black holes are considered. It has been shown that the angular velocity $\Omega \to 0$ before most of the black hole evaporates and that a black hole with mass $\leq 10^6 M_\odot$ discharges faster than it evaporates (Page, 1976). All particles inside the black hole, mostly neutrinos, photons and gravitons, with rest mass less than the black hole temperature T_{BH}, where

$$T_{BH} = \frac{\hbar c^3}{8\pi G M_{BH}} = 1.06\left(\frac{10^{13} g}{M_{BH}}\right) GeV \tag{24}$$

and M_{BH} is the PBH mass in grams, are emitted. The rate of mass loss therefore depends on the number of particle degrees of freedom $\alpha(M)$:

$$\frac{dM_{BH}}{dt} = -\frac{\alpha(M_{BH})}{M_{BH}^2}. \quad (25)$$

In the standard model of particle physics $\alpha(M_{BH}) \leq 7.8\times10^{26}g^3s^{-1}$ for $M_{BH} \sim 5\times 10^{14}g$. PBHs of this mass, which are evaporating today, would have a temperature $T_{BH} > 20MeV$. Modern models indicate $\alpha(M_{BH}) = 5.34 \times 10^{25}\phi(M_{BH})gsec^{-1}$ where $\phi(M_{BH})$ is a function of the number directly emitted species normalized to unity for $M_{BH} \gg 10^{17}g$. Black holes with masses $5 \times 10^{14}g \ll M_{BH} \ll 10^{17}$ emit $e^{\pm}$, neutrinos and photons and have initially $\phi(M_{BH}) = 1.569$. If black holes can emit three lepton families, six quark flavors, the photon and direct pions, then $\phi(M_{BH}) \leq 13.9$. Including the emission of weak gauge and higgs bosons, $\phi(M_{BH}) \leq 15.4$ for $T_{BH} < 100GeV$ and $M_{BH} \geq 10^{11}g$. At higher energies or in non-standard models like super-symmetry or super-strings, $\phi(M_{BH})$ may be greater but in general remains less than 100 (Kim, Lee and MacGibbon, 1999). The primordial black hole lifetime is found, by integrating the mass loss rate over time, to be:

$$\tau_{evap} \cong 1.2 \times 10^3 \frac{G^2 M_{BH}^3}{\phi(M_{BH})} = 6.2 \times 10^{-27} M_{BH}^3 \phi(M_{BH})^{-1} sec. \quad (26)$$

3.2 Total PBH evaporation flux

The total flux of gamma-rays from PBHs can be determined from the PBH initial mass spectrum. This in turn depends on the fluctuation amplitude and spectral index, and t_i, the time when the fluctuations develop in the case of the Kim-Lee-MacGibbon mass function. Since the number density of PBHs decreases as R^{-3}, the number density of PBHs at the lifetime $t_l \geq t_H$ is

$$n'_{BH}(t_l) \simeq \left(\frac{R_1}{R_i}\right)^{-3} \int_{M_{*(t_l)}}^{M_{BHl}} n_{BH}(M_{BHi}) dM_{BHi}, \quad (27)$$

where $M_*(t_1)$, the initial mass of a PBH whose lifetime is t_1, is given by

$$M_*(t_1) \simeq \left[\frac{\phi(M_*(t_1))}{6.24 \times 10^{-27}} \left(\frac{t_1}{1sec}\right)\right]^{\frac{1}{3}} g. \quad (28)$$

At t_l PBHs with initial mass M_{BHi} have evaporated a mass of

$$M_{evap} \simeq (M_{BHi}^3 - 1.6 \times 10^{26} \phi(M_{BHi}) t_l)^{\frac{1}{3}} \tag{29}$$

and are emitting photons with flux $f_\gamma(M_{evap}, \omega)$, where ω is the angular frequency at emission and which is red-shifted by the expansion of the universe to a present angular frequency ω_0 of

$$\omega = \frac{R_0}{R_1}\omega_0. \tag{30}$$

If a photon effectively interacts τ times during flight the total photon flux per unit solid angle reaching Earth at the present time due to the evaporation of black holes would thus be

$$\frac{dJ}{d\omega_0} = \frac{1}{4\pi}\int_{t_{min}}^{t_0} \left(\frac{R_0}{R_1}\right)\left(\frac{R_l}{R_i}\right)^{-3} dt_l \int_{M_*(t_l)}^{M_{BHl}} f_\gamma(M_{evap}, \omega) n_{BH}(M_{BHi}) \exp -\tau dM_{BHi}. \tag{31}$$

The optical depth τ depends on the energy and the redshift z at emission (Kim, Lee and MacGibbon, 1999).

3.3 Short-period gamma-ray bursts as PBHs

PBHs with initial mass $M_{BH} = M_* \sim 5 \times 10^{14} g$ would be evaporating today. Short period gamma-ray bursts (GRBs), with a duration $< 200ms$, occurring at a rate of $\sim 10 yr^{-1}$ may be due to the evaporation of PBHs located in the galactic halo (Cline, Matthey and Otwinowski, 2003). These short events, which make up roughly 2 percent of the total GRB population, have simple time histories and hard spectra, relative to the longer duration GRBs and are consistent with a Euclidean source distribution, suggesting a local origin.

Comparing the short period GRB rate with the local PBH evaporation rate it can be indicated which mass spectrum is the most relevant. To calculate the local PBH evaporation rate one needs the PBH mass function and local density enhancement (Green, 2001). In the classical, cosmologically irrelevant, Hawking-Carr case the bound on the global number density of PBHs per logarithmic mass interval, $\mathcal{N}_g$ at $M = M_*$ is taken to be $\leq 10^5 pc^{-3}$ from the diffuse gamma-ray background. Combined with a local density enhancement factor $\eta = \frac{\rho_{local}}{\rho_{global}} = 5 \times 10^5$ the local number density of PBHs per logarithmic mass interval at $M = M_*$

is obtained to be $\mathcal{N}_{local} = \eta \mathcal{N}_{global} \sim 10^{10} pc^{-3}$. The local evaporation rate is then given by

$$\frac{dn_{BH}}{dt} = \frac{dn_{BH}}{dM_{BH}} \frac{dM_{BH}}{dt} = \frac{\alpha(M_*)}{M_*^3} \mathcal{N}_{local} \approx 10 yr^{-1}, \tag{32}$$

using the fact that evaporating PBHs can only be detected within $\sim 1pc$. Now the initial PBH number density is given by

$$\mathcal{N}_{global} = \frac{\Omega_{PBH,0} \rho_c}{2M_*}, \tag{33}$$

where $\Omega_{PBH,0}$ is the present day fraction of the critical energy density ρ_c in PBHs. MacGibbon and Carr's (1991) evaluation of the diffuse gamma-ray bound gives $\Omega_{PBH} \leq 7.6(\pm 2.6) \times 10^{-9} h^{-1.95 \pm 0.15}$, assuming the $M^{-\frac{5}{2}}$ mass function, where h is the Hubble parameter in units of $100 kms^{-1} Mpc^{-1}$. This gives $\mathcal{N}_{global} = 4.2 \times 10^3 pc^{-3}$.

A plot of the present day PBH number density as a function of mass for the classical Carr $M^{-\frac{5}{2}}$, Kim-Lee and Niemeyer-Jedamzik mass functions with parameters chosen such that the present day number density of evaporating PBHs is $\mathcal{N}_{global} = 4 \times 10^3 pc^{-3}$ (figure 3) shows that the Kim-Lee and classical $M^{-\frac{5}{2}}$ mass function both have a sharp lower cut-off. Thus the diffuse gamma-ray constraint on the present day PBH evaporation rate is the same for both mass functions.

For the Niemeyer-Jedamzik mass function the relevant constraint on PBHs with mass $M = M_*$ on the mass variance is $\sigma(M_H = 1.54 \times 10^{15} g) < 0.023$. This leads to $\mathcal{N}_g < 4.5 \times 10^{-157} h^2 pc^{-3}$ so that the present day rate of PBH evaporation is completely negligible (Green, 2001).

Using the bounds which arise from the diffuse gamma-ray background, which constrain the abundance of all PBHs which have evaporated since a redshift of $z = 700$, it is calculated that for a PBH mass function with significant width it is not possible to produce a present day evaporation rate comparable with the observed short-period gamma-ray burst rate. But for a sharply peaked mass function, which arises if all PBHs which form at a given epoch have the same mass, it is.

3.4 PBH constraints on the spectral index

In the inflationary universe, the abundance of PBHs is extremely sensitive to the spectral index n, which specifies the dependence of the power spectrum of primordial density fluctuations on the co-moving wavenumber k. Thus a constraint on the abundance of PBHs will also yield a constraint on the index n. The simplest constraint is to require that the present mass density of PBHs of mass $M > 10^{14}g$ must not over-close the universe $\Omega_{PBH} < 1$, which leads to a limit $n \leq 1.25$ or a weaker limit $n \leq 1.3$ in case of a cosmology with a second period of thermal inflation which also leads to a gap $10^{18} < M < 10^{26}g$ in the PBH initial mass fraction (Green and Liddle, 1997). More effective constraints on the spectral index and PBH abundances can be derived from the condition that the PBH gamma-ray flux should not be larger than the COMPTEL- and EGRET-observations of the diffuse gamma-ray background (DGB) flux constraints on the spectral index and PBH abundances can be derived. From the exponential dependence of the Kim-Lee-and-MacGibbon PBH mass spectrum it is deduced that the number density of PBHs increases with T_{RH} for a given spectral index. The upper limit on n decreases as the reheating temperature grows. The constraint on n becomes stronger as the reheating temperature increases. The upper limits of the spectral index are in the range $7 \times 10^7 GeV \preceq T_{RH} \preceq 4 \times 10^8 GeV$, corresponding to an initial mass range $2 \times 10^{13}g \leq M_{BH} \leq 5 \times 10^{14}$ that can contribute to the observed DGB flux. Due to the interactions of gamma-rays with the background matter in the universe only PBHs surviving later than $z < 700$ can contribute to the DGB flux observed today. From the conditions that $\Omega_{BH} < 1$ throughout the history of the universe and $\Omega_{relic} < 1$, the limit on n is obtained to be $n \preceq 1.23 - 1.25$ (see figure 4). The Niemeyer-Jedamzik PBH mass distribution, in which PBHs may form in considerable numbers at any formation epoch, has an effect on the spectral index limit similar to raising T_{RH} (Kim, Lee and MacGibbon, 2002).

Fang and He (2002) constrain the abundance of PBHs and n with observations of $Ly\alpha$ emission of the ionizing photon background at the epoch of reionization. This constraint is independent of the reheating temperature T_{RH} if $T_{RH} > 10^{10} GeV$. The abundance of evaporated and evaporating black holes centered around a mass of $M_{PBH} = 2 \times 10^{14}g$ given by this constraint is $\Omega_{BH} < 10^{-4}$.

In their approach Fang and He make use of the co-moving Kim-Lee mass function, which is independent of the time at which the PBH forms, and calculate the PBH-contributed ionizing photon background at $z = 6$ as a function of n. Compared to the observed photon background flux at the redshift of reionization this leads to a limit of $n < 1.27$, assuming that the flux is emitted by massive stars.

Currently, the best constraints on the spectral index come from the CMBR measurements by WMAP (Spergel et al., 2003), resulting in $n = 0.99 \pm 0.04$. However the spectral index may vary with the wavenumber k and PBH constraints probe much smaller scales than CMBR measurements do. Moreover if the optical depth of the CMB photons to the last scattering surface $\tau_c > 0$, the original index could well be $n > 1$. The constraint on n given by the PBHs is independent of the optical depth τ_c (Fang and He, 2002). Consequently the PBH constraints provide information that CMBR measurements do not, in particular in the case that the power spectrum $P(k)$ is not a power law but a spike or step (Sahlén, 2003).

4 PBHs as evaporating dark matter

Now the PBH mass functions, their different PBH number densities and the evaporation rates are discussed it can be investigated if the different models can explain a critical dark matter density decrease of $\Omega_{dm} = 0.3$ at recombination redshift to $\Omega_{dm} \sim 0$ now.

4.1 PBHs in the mass range $2 \times 10^{13}g \leq M \leq 5 \times 10^{14}g$

With their relative short lifetimes, $10^5 - 10^{10}yrs$ (see table 1), PBHs in the mass range $2 \times 10^{13}g \leq M \leq 5 \times 10^{14}g$ will be completely evaporated by now and would have contributed to the DGB. From the fact that this contribution shouldn't exceed the observed DGB emission the critical density of this mass range is pinned down to $\Omega_{PBH} < 10^{-8}$ throughout the universe. Observations of $Ly\alpha$ emission of the ionizing photon background at the epoch of reionization constrain the abundance of black holes centered around a mass of $M_{PBH} = 2 \times 10^{14}g$ to $\Omega_{BH} < 10^{-4}$. Therefore no considerable dark matter evaporation can be expected from this PBH mass range.

4.2 PBHs in the mass range $5 \times 10^{14}g \leq M \leq 10^{34}g$

Massive compact halo objects (MACHOs) discovered by micro-lensing of stars in the Large Magellanic Cloud are black holes with masses of $\sim 0.5M_{\odot}$. Black holes of this mass can only be of primordial origin. Allowing a critical density of $\Omega_{MACHOs} = 0.1$ at around a solar mass and searches for microlensing of stars in the Large Magellanic Cloud excludes PBHs in the mass range $10^{26}g \leq M_{PBH} \leq 10^{34}g$ (Alcock et al., 2002). However there are no constraints in the mass range $1 \times 10^{15}g \leq M_{PBH} \leq 10^{26}g$. Blais et al. (2002) propose a PBH spectrum with a peak at $M_{PBH} = 10^{18}g$ (see figure 5), produced by a jump in the derivative of the inflaton potential, a broken scale invariance (BSI) model. The BSI model contains two extra parameters: the strength of the jump, p and its location, k_s. A primordial spectrum with large oscillations near k_s gives a pronounced bump in the mass variance and could yield a significant fraction of the cold dark matter (CDM). Supposing all CDM is in PBHs $\Omega_{PBH,0}(M_{peak}) = 0.3$ and using

Mass (g)	Lifetime (yrs)	Mass loss after 13.4$Gyrs$ (g)	Percentage of mass loss (%)
2.0×10^{13}	1.0×10^{5}	2.0×10^{13}	100
5.0×10^{13}	1.6×10^{6}	5.0×10^{13}	100
1.0×10^{14}	1.3×10^{7}	1.0×10^{14}	100
5.0×10^{14}	1.6×10^{10}	1.4×10^{14}	28
6.0×10^{14}	2.7×10^{10}	9.9×10^{13}	16
7.0×10^{14}	4.3×10^{10}	7.2×10^{13}	10
1.0×10^{15}	1.3×10^{11}	3.5×10^{13}	3.5
5.0×10^{15}	1.6×10^{13}	1.4×10^{12}	2.8×10^{-2}
1.0×10^{16}	1.3×10^{14}	3.5×10^{11}	3.5×10^{-3}
1×10^{17}	2.0×10^{17}	2.3×10^{9}	2.3×10^{-6}
1×10^{18}	2.0×10^{20}	2.3×10^{7}	2.3×10^{-9}

Table 1: *The lifetime, mass loss after* 13.4$Gyrs$ *and percentage mass loss for different PBH masses. The lifetime is calculated with equation 26, the mass loss by integration over time of equation 25, using* $\alpha(M_{BH}) = 5.34 \times 10^{25} \phi(M_{BH}) g sec^{-1}$ *with* $\phi(M_{BH}) = 1$ *for* $M_{BH} \gg 10^{17} g$, $\phi(M_{BH}) = 1.569$ *for* $5 \times 10^{14} g \leq M_{BH} \leq 10^{17} g$ *and* $\phi(M_{BH}) = 15.4$ *for* $10^{11} g \leq M_{BH} < 5 \times 10^{14} g$

the gamma-ray constraint $\Omega_{PBH,0}(M \ll M_{peak}) = 10^{-8}$ the range of PBH masses is fixed to a mass window $5 \times 10^{15} g \leq M_{PBH} \leq 10^{21} g$. Relaxing the requirement on the present density of PBHs allows a slightly weaker constraint for the upper bound of the mass window. Setting $\Omega_{PBH,0} = 0.1$ gives $M_s \simeq 4 \times 10^{21} g$ and $\Omega_{PBH,0} = 0.01$ gives $M_s \simeq 4 \times 10^{23} g$ (see figure 6). A observational constraint on the allowed range of MACHO masses could therefore give a severe constraint on this BSI-model. Femtolensing observations of gamma-ray bursts by PBHs could preclude those in the range $10^{17} g \leq M \leq 10^{20} g$ (Gould, 1992) and would be a good test of this model.

But suppose if all CDM would be PBHs of mass $5 \times 10^{15} g$ of which $1.4 \times 10^{12} g$ would evaporate in 13.4$Gyrs$ no considerable amount of dark matter density would have been evaporated. And that would be the same if CDM consists only of PBHs with a higher mass. In the case that the CDM consists only of PBHs with mass $6 \times 10^{14} g$ the critical dark matter density would decrease from $\Omega_{CDM} = 0.3$ at $z = 1000$ to $\Omega_{CDM} = 0.25$ now (see table 2) due to the evaporation.

Mass (g)	Ω_{CDM} after evaporation
6×10^{14}	0.25
7×10^{14}	0.27
1×10^{15}	0.29
5×10^{15}	0.3
1×10^{16}	0.3

Table 2: *The critical dark matter density after* $13.4 Gyrs$ *of PBH evaporation starting from* $\Omega_{CDM} = 0.3$*, consisting of PBHs with the indicated mass.*

4.3 PBHs in the mass range $30 M_\odot < M_{PBH} < 10^4 M_\odot$

The Poisson noise, enhanced by gravitational clustering in the matter-dominated era, leads to a plateau in the power spectrum at large wavenumbers. $Ly\alpha$ forest observations constrain this Poisson term, which constrains the black hole masses, as the constituents of cold dark matter (CDM), to be less than approximately a few times $10^4 M_\odot$ (Afshordi, McDonald and Spergel, 2003). Improved $Ly\alpha$ forest measurements should be able to narrow down this mass range and determine their abundance. However large this abundance these PBHs wouldn't contribute much to a decrease in the critical dark matter density due to evaporation.

5 Conclusions

This literature study on the formation of PBHs and its constraints on the loss of dark matter due to PBH evaporation has lead to the following conclusions and result:

C1 The classical Carr-Hawking initial PBH mass spectrum $M_{PBH} = M^{-\frac{5}{2}}$, assuming the scale-invariant Harrison-Zel'dovich spectrum with $n = 1$, a stage of radiation domination and a density contrast in the range $\frac{1}{3} \leq \delta \leq 1$, has shown to be cosmologically irrelevant. COBE data suggest that it is impossible to form a significant number of PBHs in the case of a $n = 1$ spectrum.

C2 The Kim-Lee-and-MacGibbon model describes PBH formation for $n > 1$ primordial power spectra in a universe with a hard equation of state. As the universe expands, more massive PBHs are formed. Since PBHs formed before the period of inflation are diluted away, the time of formation is related to the reheating temperature.

C3 The Niemeyer-Jedamzik model assumes a density contrast in the range $0.7 \leq \delta \leq 1$ and deduces a mass spectrum in which it is no longer valid to assume a one-to-one-correspondence between the mass of a PBH and its time of formation. At different epochs a range of masses is formed.

C4 Shibata and Sasaki put the calculation of the mass spectra on a sounder theoretical footing via a peak theory, which is independent of inflation. However the peak theory corresponds closely to the Kim-Lee-and-MacGibbon model. If the spectral index increases, more massive PBHs will be formed.

C5 Comparing the short period GRB rate with the local PBH evaporation rate a sharply peaked mass spectrum, which arises if all PBHs which form at a given epoch have the same mass, is the most relevant. This supports the Kim-Lee and Shibata-Sasaki mass spectra over the Niemeyer-Jedamzik mass spectrum.

C6 The range of initial PBH masses which can contribute to the diffuse gamma-ray background is $2 \times 10^{13} g - 5 \times 10^{14} g$ corresponding to a reheating temperature of $7 \times 10^{7} GeV = 4 \times 10^{8} Gev$. In this range the upper limit on the spectral index is $n \leq 1.23 - 1.25$. From the fact that the emitted PBH flux shouldn't exceed the DGB the PBH abundance in this mass range is constrained to $\Omega_{PBH} < 10^{-8}$.

C7 Observations of $Ly\alpha$ emission of the ionizing photon background constrain the PBH abundance to $\Omega_{PBH} < 10^{-4}$ for the mass range centered around $M_{PBH} = 2 \times 10^{14} g$ and limit the spectral index to $n < 1.27$.

C8 Currently, the best constraints on the spectral index come from the CMBR measurements by WMAP, resulting in $n = 0.99 \pm 0.04$. However the spectral index may vary with the wavenumber k and PBH constraints probe much smaller scales than CMBR measurements do. Moreover if the optical depth of the CMB photons to the last scattering surface $\tau_c > 0$, the original index could well be $n > 1$. The constraint on n given by the PBHs is independent of the optical depth τ_c. Consequently the PBH constraints provide information that CMBR measurements do not, in particular in the case that the power spectrum $P(k)$ is not a power law but a spike, step or broken scale invariant.

C9 Allowing a critical density of $\Omega_{MACHOs} = 0.1$ at around a solar mass and searches for microlensing of stars in the Large Magellanic Cloud excludes PBHs in the mass range $10^{26} g \leq M_{PBH} \leq 10^{34} g$.

R1 A critical dark matter decrease due to evaporating black holes from recombination redshift until now of $\Delta\Omega_{CDM} = 0.3$ can be ruled out. The observed DGB rules out a considerable amount of PBHs in the mass range $2 \times 10^{13} g \leq M_{PBH} \leq 5 \times 10^{14} g$. The BSI model proposes a large amount of CDM in the mass range $5 \times 10^{15} g \leq M_{PBH} \leq 10^{21} g$. However for PBH masses $\gg 5 \times 10^{15} g$, dark matter evaporation is of no importance, because of their long lifetimes and corresponding slow mass loss. At its best, if the CDM consists only of PBHs with a mass of $6 \times 10^{14} g$, a decrease of $\Delta\Omega_{CDM} = 0.05$ is established.

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Figures

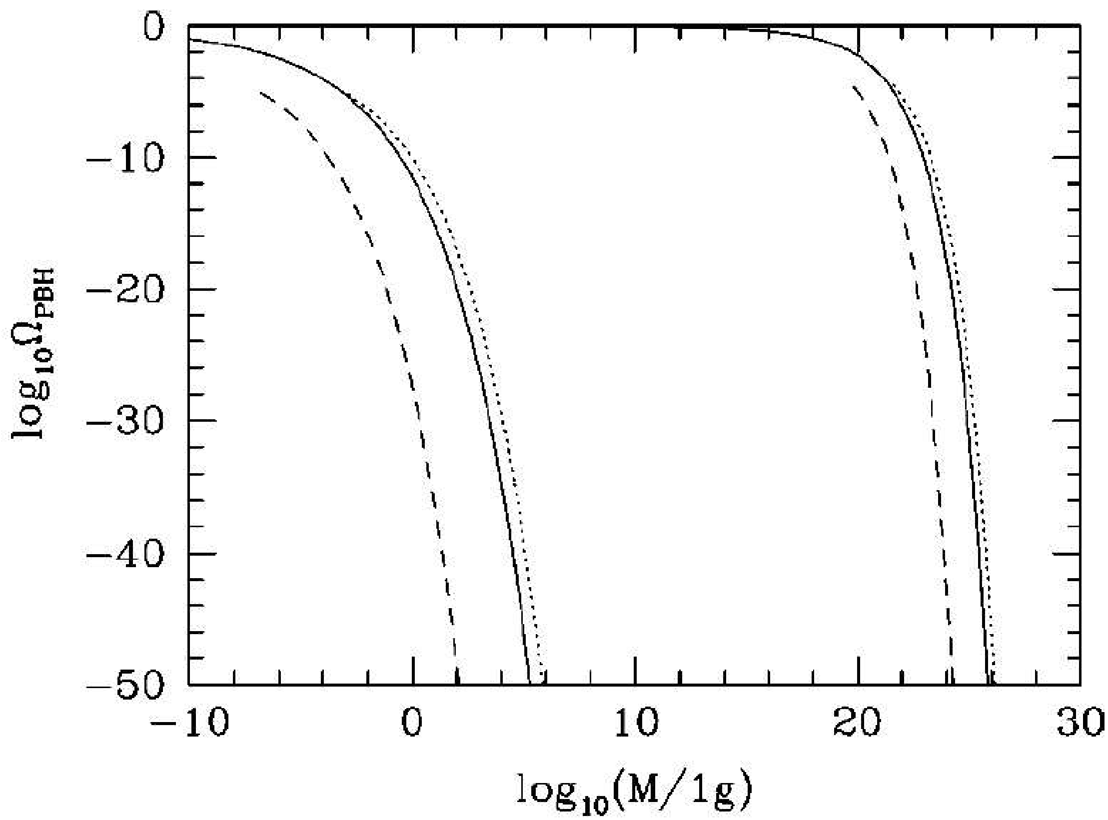

Figure 1: *PBH abundance as a function of horizon mass for power-law power spectra with* $n = 1.25$ *and* $n = 1.5$ *(left- and righthand sets of curves respectively) calculated using the Press-Schechter formalism with* $\Delta_{th} = 1/3$ *(solid line) and the peak formalism with* $\zeta = 0.7$ *and* 1.2 *(dotted and dashed line respectively).* Anne M. Green et al., 2004.

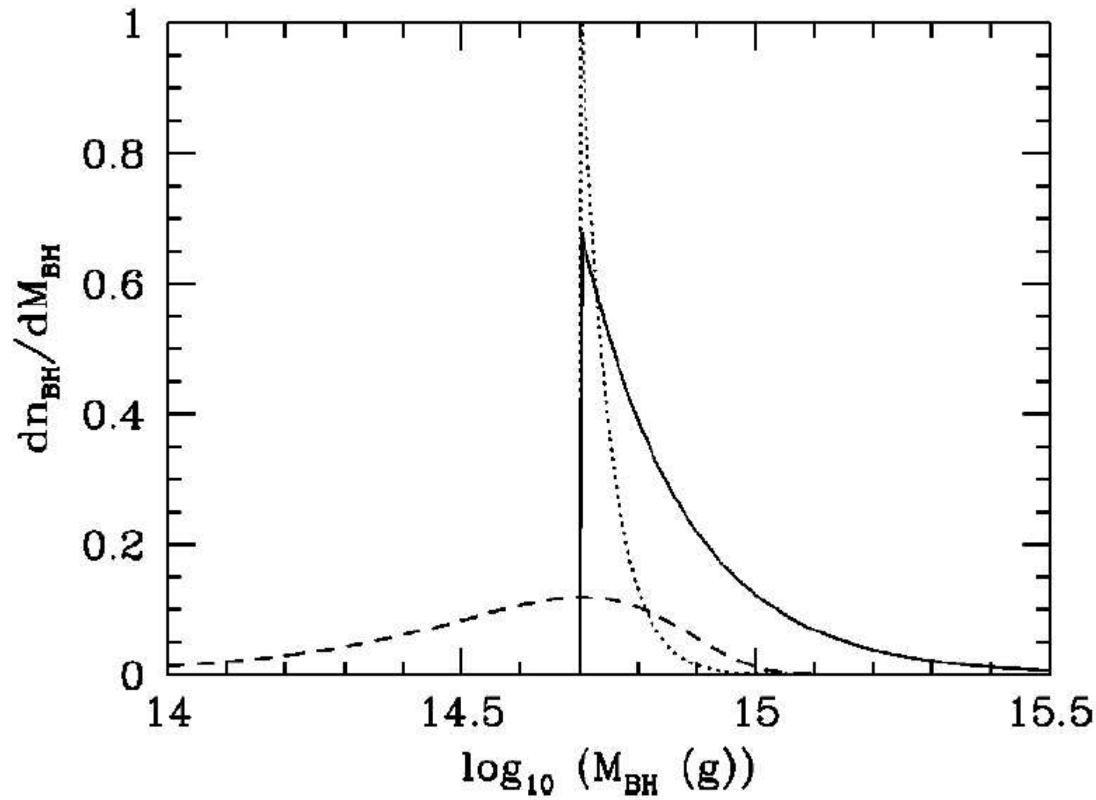

Figure 2: *The PBH initial number density, as a function of mass, for the Carr* $M_{BH}^{-5/2}$ *(solid line), Kim-Lee-MacGibbon (dotted line) and Niemeyer-Jedamzik (dashed) mass functions with parameters chosen such that the present day densities, ignoring evaporation, are the same* ($\Omega_{PBH} = 1 \times 10^{-8}$)*. The Carr and Niemeyer-Jedamzik mass functions are multiplied by a factor of ten.* Anne M. Green, 2001.

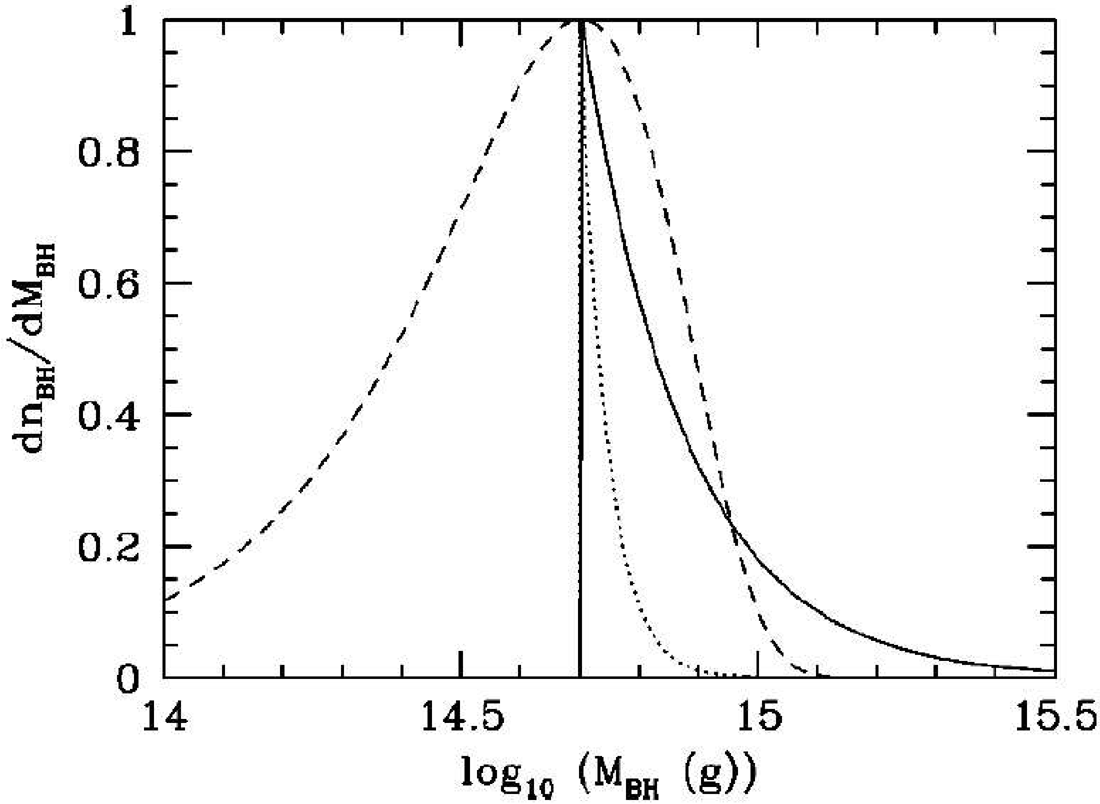

Figure 3: *The present day PBH number density, as a function of mass, for the Carr* $M_{BH}^{-5/2}$ *(solid line), Kim-Lee-MacGibbon (dotted line) and Niemeyer-Jedamzik (dashed) mass functions with parameters chosen such that the present day number density of evaporating PBHs is* $N_g = 4 \times 10^3 pc^{-3}$. Anne M. Green, 2001.

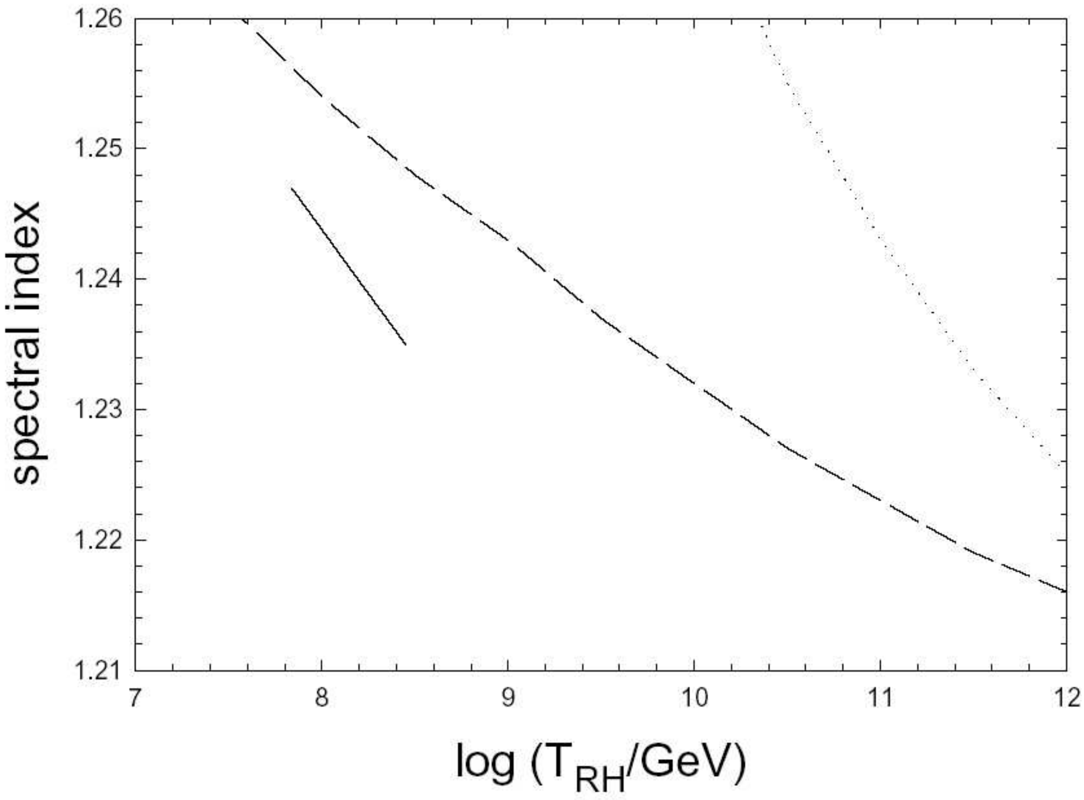

Figure 4: *The upper limits on the spectral index. The solid line between* 7×10^7 *GeV*- 4×10^8 *GeV is obtained from the condition that the PBH gamma-ray flux should not exceed the observed DGB flux. The dashed line is obtained from the condition that* $\Omega_{BH} < 1$ *throughout the history of the universe. The dotted line is obtained from the condition that* $\Omega_{relic} < 1$. Kim, Lee and MacGibbon, 1999.

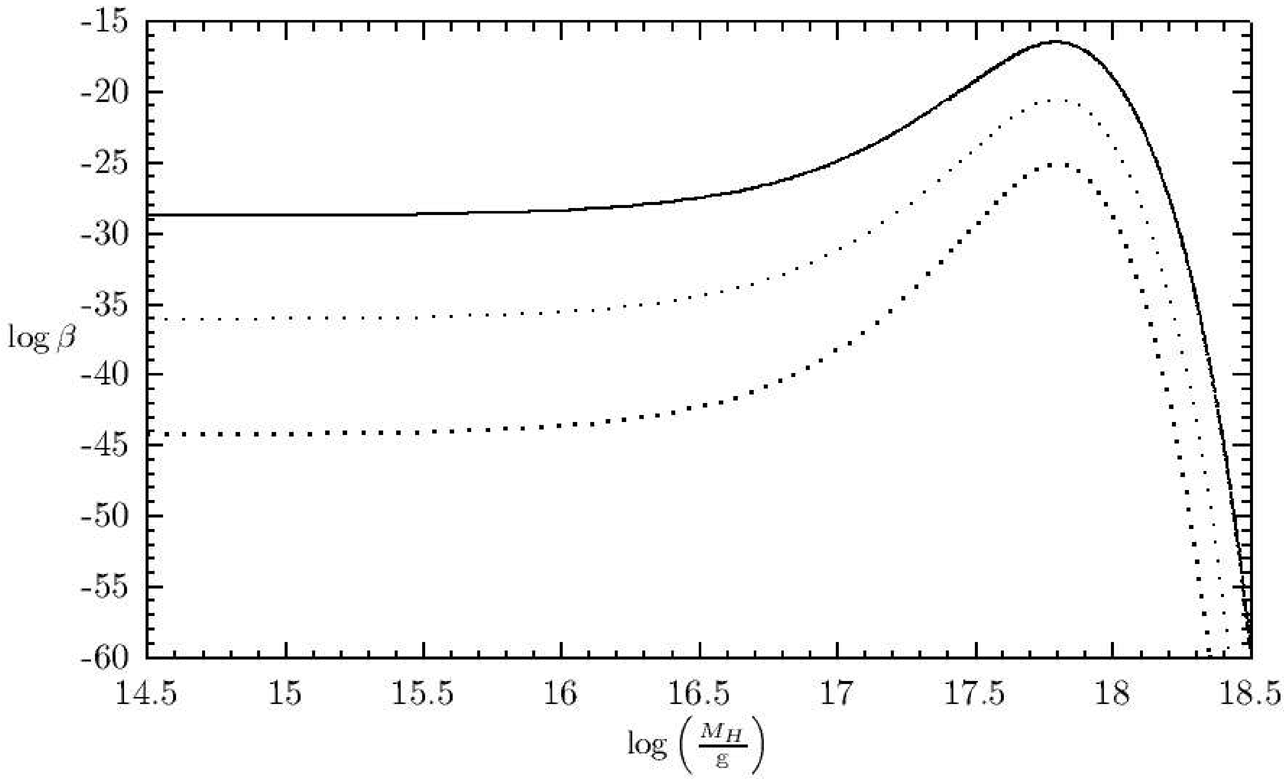

Figure 5: *The initial mass fraction $\beta(M_H)$ of PBHs of mass $M_{PBH} \geq M_H$ as a function of mass for the BSI-spectrum containing a jump in the inflaton potential derivative for $M_H(t_{k_s}) = 10^{18}g$. From bottom to top the strength of the jump is $p = 10^{-3}$, 9×10^{-4} and 8×10^{-4}. This function is no longer a decreasing function of mass, but has a peak at M_{peak} instead.* Blais et al., 2002.

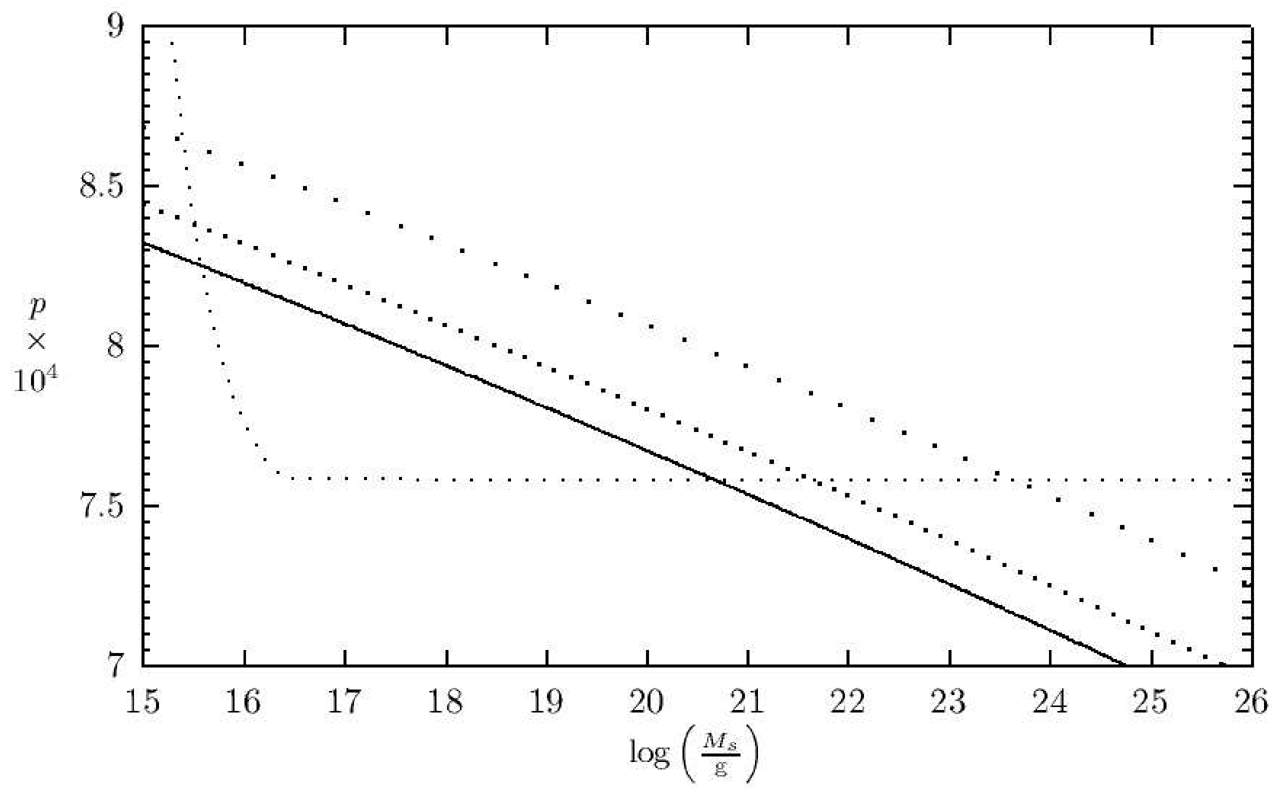

Figure 6: *The strength of the jump as a function of mass $M_s \equiv M_H(t_{k_s})$. The solid line represents those points for which $\Omega_{PBH,0}(M_{peak}) = 0.3$. Below the line the gravitational constraint is violated. The two lines parallel to the solid line represent, from bottom to top those points for which $\Omega_{PBH,0}(M_{peak}) = 0.1$ and 0.01, respectively. Below the dotted line, the gamma-ray background constraint is violated.* Blais et al., 2002.