Sterrenstelsels en Kosmos
Assignment 1
Due: 19 November 2009

1. To give you an idea of the conditions in the center of the Sun, use

   (a) the equation of hydrostatic equilibrium to estimate the pressure at the center of the Sun, P_c, and

   (b) the equation of state for an ideal gas to estimate the temperature at the center of the Sun, T_c.

   The average density of the Sun is $\bar{\rho} = 1.4\,\mathrm{g/cm^3}$ and the typical mass of a particle in the Sun is $0.62 m_H$, where m_H is the mass of a hydrogen atom. You can find other constants, like the Boltzmann constant, the mass of the hydrogen atom, and the mass and radius of the Sun on page 553 of Kutner.

   Compare the numbers you estimated above with the values inferred from accurate models: $P_c = 2.3 \times 10^{17}\,\mathrm{dyne/cm^2}$ and $T_c = 1.6 \times 10^7\,\mathrm{K}$. Do your estimates agree with these numbers? Examine your assumptions and try to figure out why they agree or disagree.

2. Derive the relation between stellar mass and luminosity for stars on the main sequence. Below you will use this relation to estimate how long stars live at a given mass.

   First, write down the "order of magnitude equivalent" equations for mass conservation, hydrostatic equilibrium, and radiative transfer equations of stellar structure, as well as the equation of state for an ideal gas. *Make sure you keep terms in the opacity $\bar{\kappa}$ as you write down these equations!*

   To get you started, the equation of mass conservation,

   $$\frac{dM}{dr} = 4\pi r^2 \rho(r), \tag{1}$$

   can be rewritten as

   $$\frac{M}{R} \propto R^2 \rho, \tag{2}$$

   where M and R are the radius of the star and ρ is some measure of its density. We then have

   $$\rho \propto M R^{-3}. \tag{3}$$

   Use this relation to eliminate ρ from the other equations.

   Next, use the approximation

   $$\frac{dT}{dr} \approx \frac{T}{R} \tag{4}$$

   at $L(r) = L(R)$ and your previous equations to write the radiative transfer equation in terms of luminosity L as a function of mass M and opacity $\bar{\kappa}$.

   (a) For hot—that is, high-mass—stars, the opacity is a constant. What is the relation between luminosity and mass for these stars?

   (b) For cooler stars (like the Sun), the opacity goes as $\bar{\kappa} \propto \rho T^{-3.5}$. What is the relation between luminosity, mass, and radius for these stars? For solar mass stars, $R \propto M$ (roughly). What is the relation between luminosity and mass in this case?

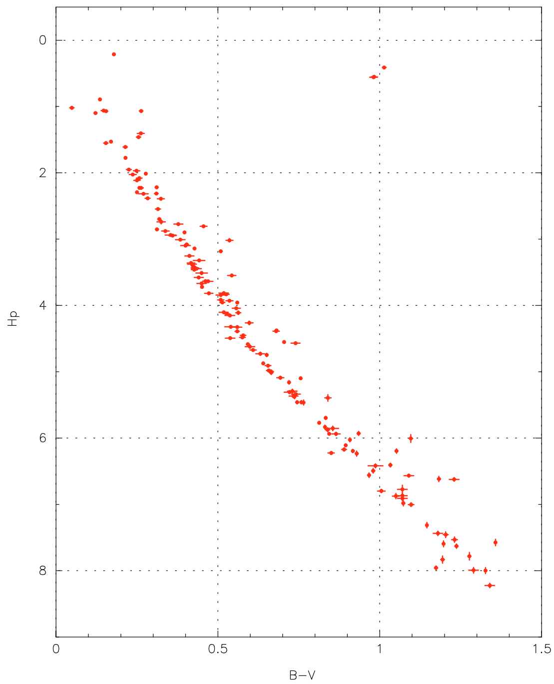

Figure 1: The colour–absolute magnitude diagram of the Hyades cluster, from van Leeuwen (2009)

Observationally, $L \propto M^{3.8}$, so $L \propto M^4$ is a reasonable approximation for the main sequence (except for the highest- and lowest-mass stars).

Now, the higher the luminosity, the more nuclear burning is required. The amount of fuel available is $\sim M$, and the timescale for nuclear burning is

$$\tau \propto \frac{M}{L}. \tag{5}$$

Then how does the timescale for nuclear burning vary with mass on the main sequence? If the Sun will live 10^{10} years on the main sequence, how long will a star with $M = 10\,M_\odot$ live on the main sequence?

3. The Hyades open cluster has a radial velocity of $v_r = 35\,\mathrm{km/s}$, a proper motion of $\mu = 0.078\,\mathrm{arcsec/yr}$, and its convergent point is located at an angle of $A = 26°$ from the line-of-sight to the Sun.

   (a) How far away is the cluster from the Sun in parsecs (pc)? What is its distance modulus $m - M$?

   (b) The cluster's colour–*absolute* magnitude diagram (sometimes known as the "Hertzsprung-Russell diagram") is shown in Figure 1. Use the distance modulus you found above to write down the *apparent* magnitude scale on the right-hand side of the diagram.

   (c) What is the cluster's space motion (velocity)?