Stellar structure and evolution

Sterrenstelsels & Kosmos
deel 1

Stellar energy sources

Or, Why does the Sun shine?

Gravitational energy

- Can the Sun power itself just by releasing energy from gravitational collapse?
- Let's examine the gravitational potential energy of a uniform sphere with constant density
    - Note that the Sun is ~spherical but doesn't have constant density; this isn't a problem if we use the *average* density

- The mass of a sphere with radius R and density ρ is $M = \frac{4\pi}{3}\rho R^3$
- The gravitational potential energy of a sphere is the *work* required to bring all of its material from infinity to the final configuration, *independent of the way that the sphere is assembled!*

- Let's consider the sphere as being broken up into *shells*, and let's focus on the shell at radius *r* in the sphere with width *dr*

- The volume of this shell is its surface area times its thickness:

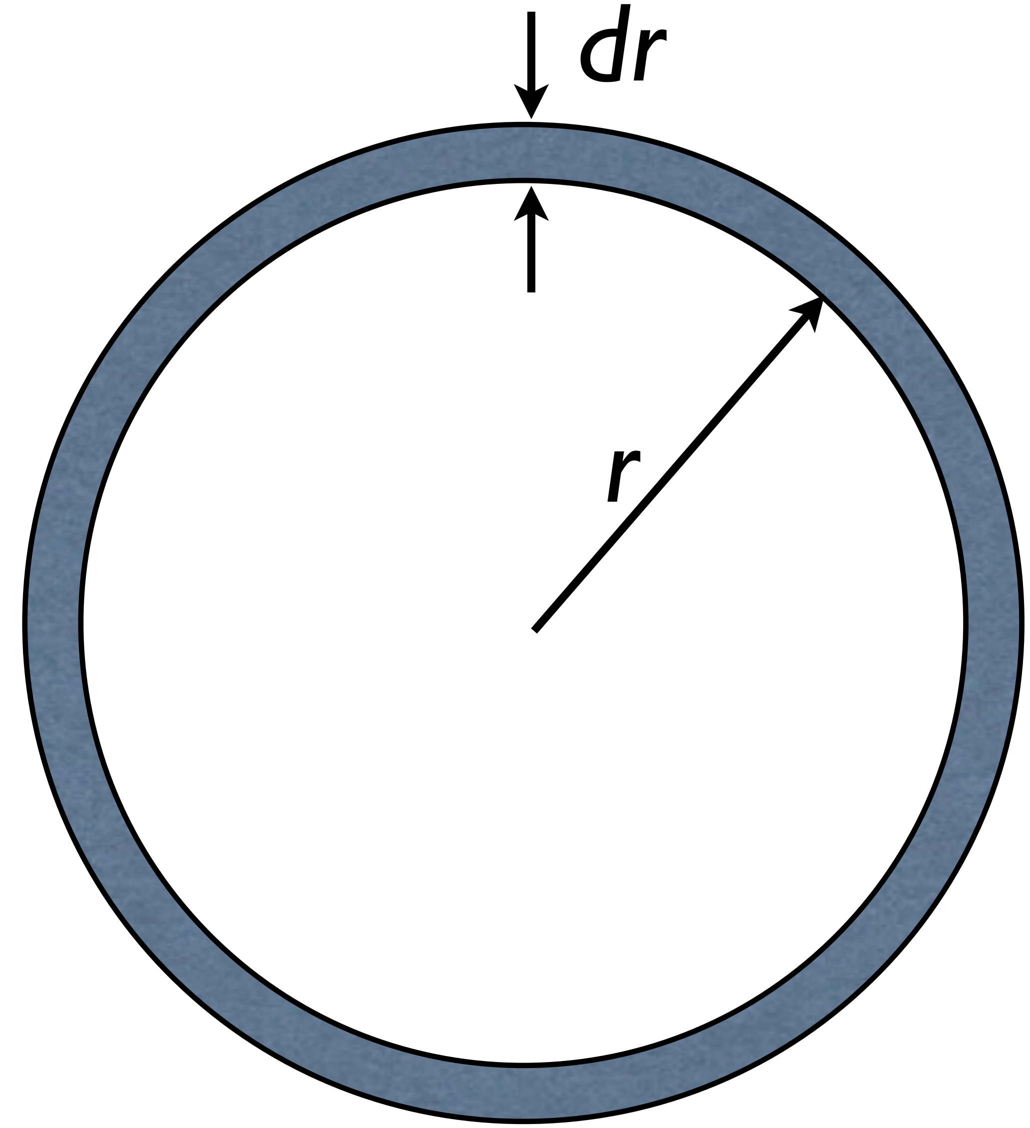
$$dV = 4\pi r^2 dr$$

- and its mass is its volume times its density:

$$dM = 4\pi r^2 \rho\, dr$$

- The mass assembled out to *r* is

$$M(r) = \frac{4\pi}{3} r^3 \rho$$



- Remember that material at *r* only feels force from the mass *M(r)* within it -- and then *as if all that mass were concentrated at a point in the center of the sphere!*
- Now, what is the gravitational potential energy?
  - For any two masses m_1, m_2 separated by a distance *r*, this is

$$U = -G\frac{m_1 m_2}{r}$$

- Let $m_1 = M(r)$ and $m_2 = dM$. Then the work need to bring this shell to r from infinity is

$$
\begin{aligned}
U &= -G\frac{M(r)dM}{r} \\
&= -\frac{16\pi^2}{3}G\rho^2 r^4 dr
\end{aligned}
$$

- Now we need to add up the work need to bring *all* the shells from infinity to $r=0$ out to $r=R$:

$$
\begin{aligned}
U &= \int_0^R dU(r) = -\frac{16\pi^2}{3}G\rho^2\int_0^R r^4 dr \\
&= -\frac{16\pi^2}{3}G\rho^2\frac{R^5}{5} = -\frac{3}{5}\left[\frac{4\pi}{3}\rho R^3\right]^2\frac{G}{R} \\
&= -\frac{3}{5}\frac{GM^2}{R}
\end{aligned}
$$

Note that the factor of 3/5 is only true for a sphere, but other configurations have similar constants, given some average length "R"

- So *how long* can this gravitational potential energy power the Sun?
- The Sun is losing energy from its surface at a rate of 4×10^{33} erg/s (its *luminosity*), so the lifetime of the Sun would be

$$
\begin{aligned}
t_g &= \frac{E}{dE/dt} = \frac{E}{L} \\
&= -\frac{U}{L} = \frac{2 \times 10^{48}\,\mathrm{erg}}{4 \times 10^{33}\,\mathrm{erg/s}} \\
&= 5 \times 10^{14}\,\mathrm{s} \approx 2 \times 10^{7}\,\mathrm{yr}
\end{aligned}
$$

assuming that E=-U ---
i.e., the only energy
source is gravitational
potential energy

- This is 20 million years...
- ...but we know the Sun has been around for 4.5 ***billion*** (4.5×10^9) years, so *gravitational potential energy is* ***insufficient*** (by a factor of >200) to power the Sun for so long!
- However, in the "protostar" phase, when a star is collapsing, gravitational potential energy is the *only* heat source.

Nuclear reactions

- Clearly we need *much* more energy than gravitational collapse can provide!
  - Chemical reactions are *less* energetic, so not useful (could power the Sun for ~20 *thousand* years!)
- How much energy do we *need*?

- We need enough energy to power the Sun for at least 4.5 billion years:

$$
\begin{aligned}
E &> 4.5 \times 10^9\,\mathrm{yr} \times 3 \times 10^7\,\mathrm{s/yr} \times 4 \times 10^{33}\,\mathrm{erg/s} \\
&\approx 5 \times 10^{50}\,\mathrm{erg}
\end{aligned}
$$

- Now let's consider how much energy this is *per nucleon* (proton or neutron) in the Sun:
    - The Sun has a mass of 2×10^{33} g, and it is made up of almost entirely hydrogen and helium, so the Sun has about 10^{57} nucleons
    - Converting ergs to MeV, we find that the Sun needs to produce *at least* 0.3 MeV/nucleon
- What can produce these sorts of energies?

- An atomic nucleus has some number *Z* of protons (with charge +e, where -e is the charge of an electron) and some number *N* of neutrons (without charge), *bound* by the "nuclear strong force"

- The *binding energy* of a nucleus is equal to the difference between the rest-mass energy of the protons and neutrons and the nucleus:

$$M_{\text{nucleus}}c^2 + BE = Zm_pc^2 + Nm_nc^2$$

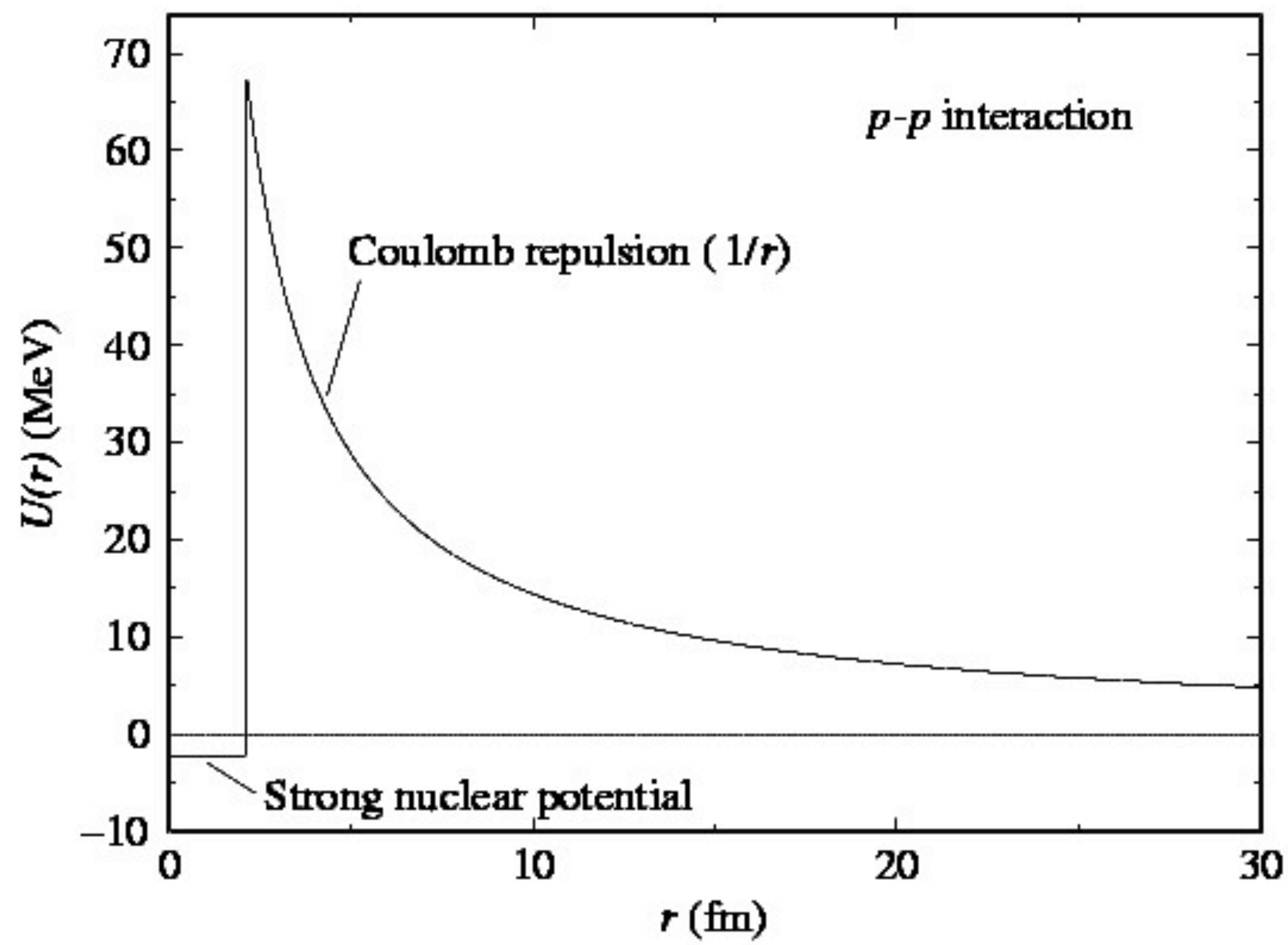
- Consider the binding energy of deuteron (D), an isotope of hydrogen with one proton and one neutron:

$$\begin{aligned} BE &= (m_p + m_n - m_D)c^2 \\ &= (1.6726 + 1.6749 - 3.3436) \times 10^{-24}\,\mathrm{g} \\ &\qquad \times (3 \times 10^{10}\,\mathrm{cm/s})^2 \\ &= 3.6 \times 10^{-6}\,\mathrm{erg} = 2.2\,\mathrm{MeV} \end{aligned}$$

- So if we turned all the H into D in the Sun, we'd have more than enough energy to power it for 4.5 billion years!

- There is a problem, however!

- Electrostatic repulsion between two positively-charged protons makes forcing them together *difficult:* have to get over the "Coulomb barrier" to get to them to bind together.

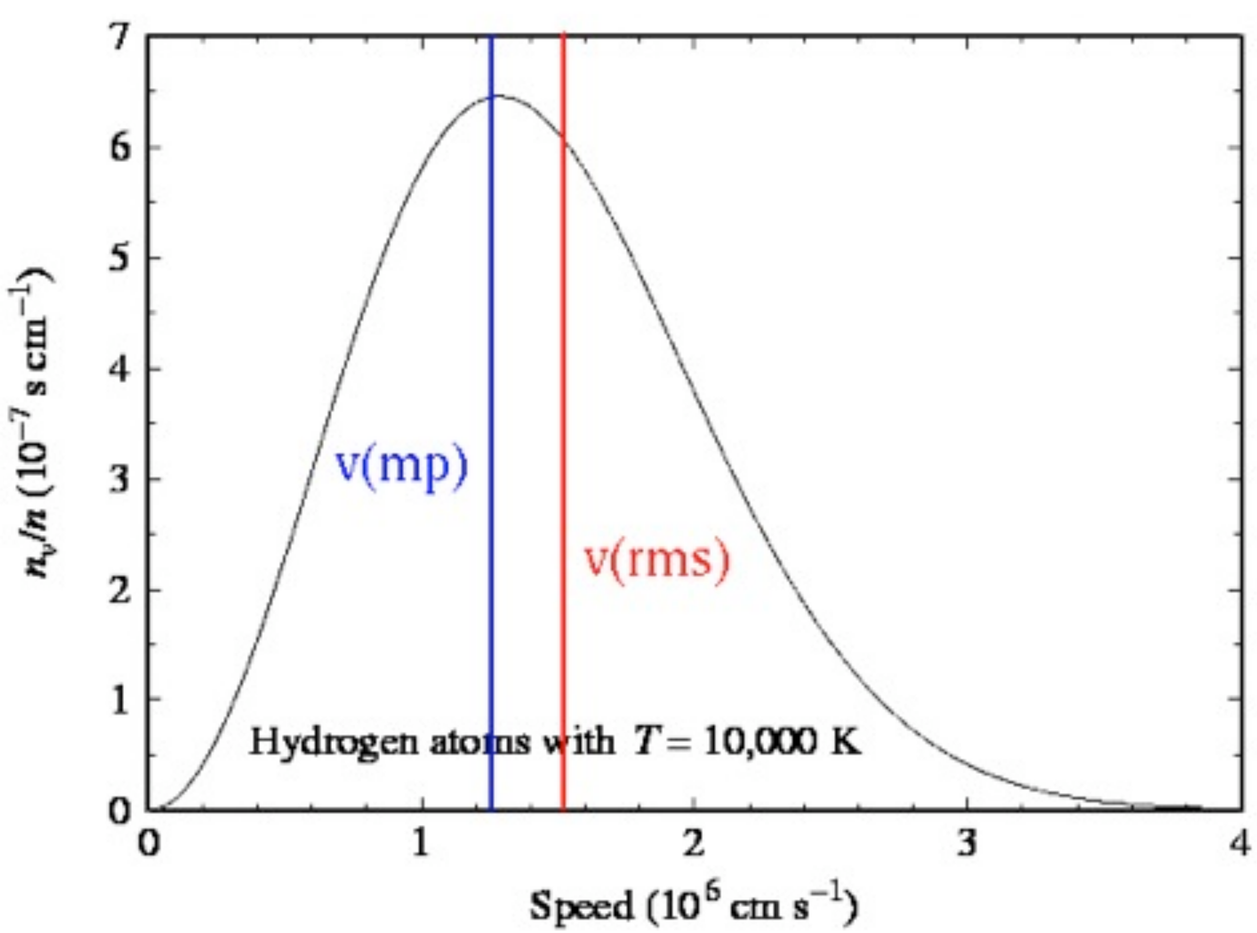


- For an *average* particle to get over the Coulomb barrier, it would need to have a temperature of

$$\begin{aligned} \frac{3}{2}kT &> \frac{e^2}{r_{\rm nuc}} \\ T &> \frac{2e^2}{3kr_{\rm nuc}} \\ &\approx 10^{10}\,{\rm K} \end{aligned}$$

- Thankfully, things aren't *that* bad...

- Particles at some temperature *T* will have a *distribution of velocities* called the “Maxwell-Boltzmann” distribution. The *probability* of finding a particle with energies between *E* and *E+dE* is $P(E) \propto \sqrt{E} e^{-E/kT} dE$
- Note that this distribution has a long tail to high energies...



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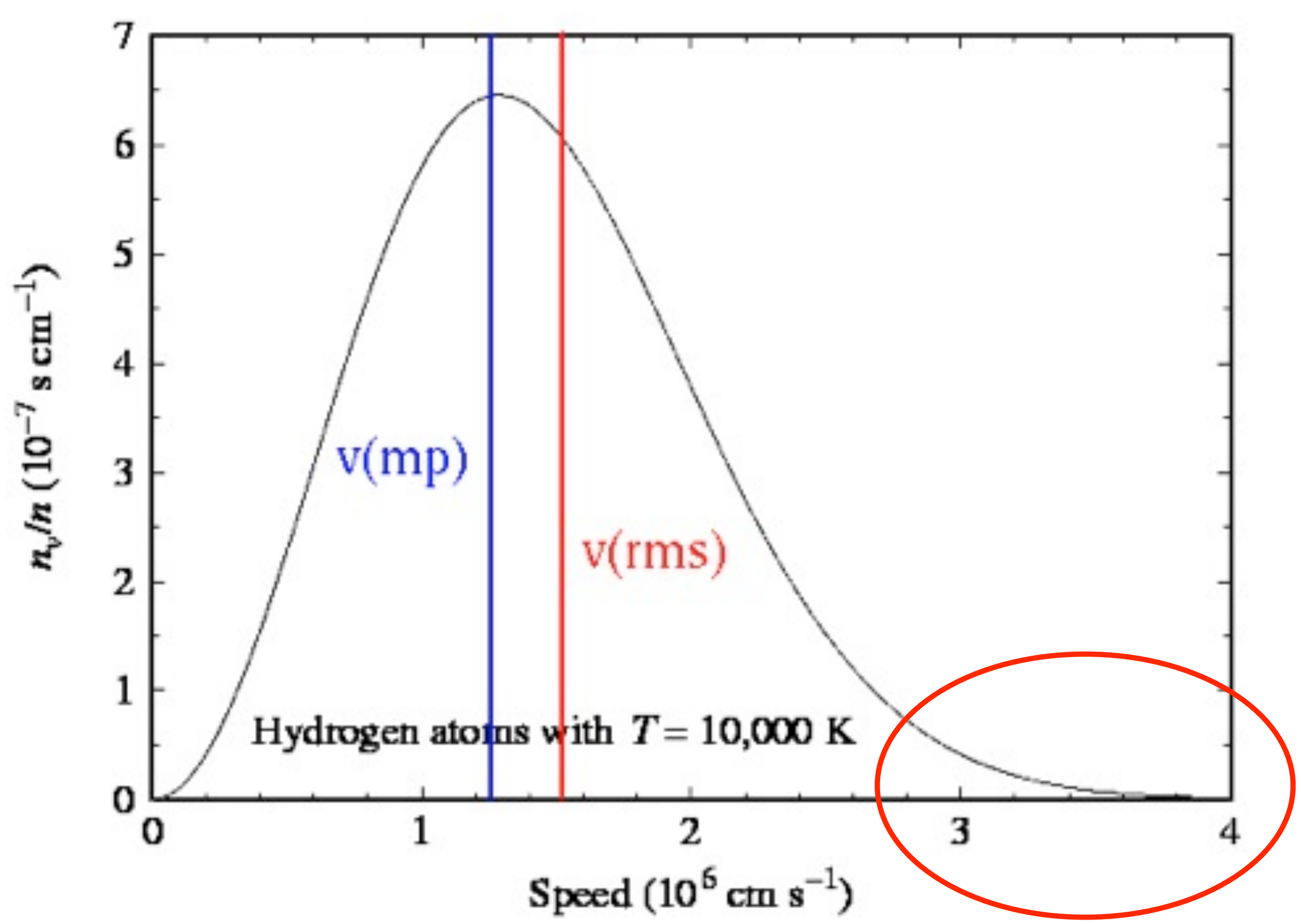
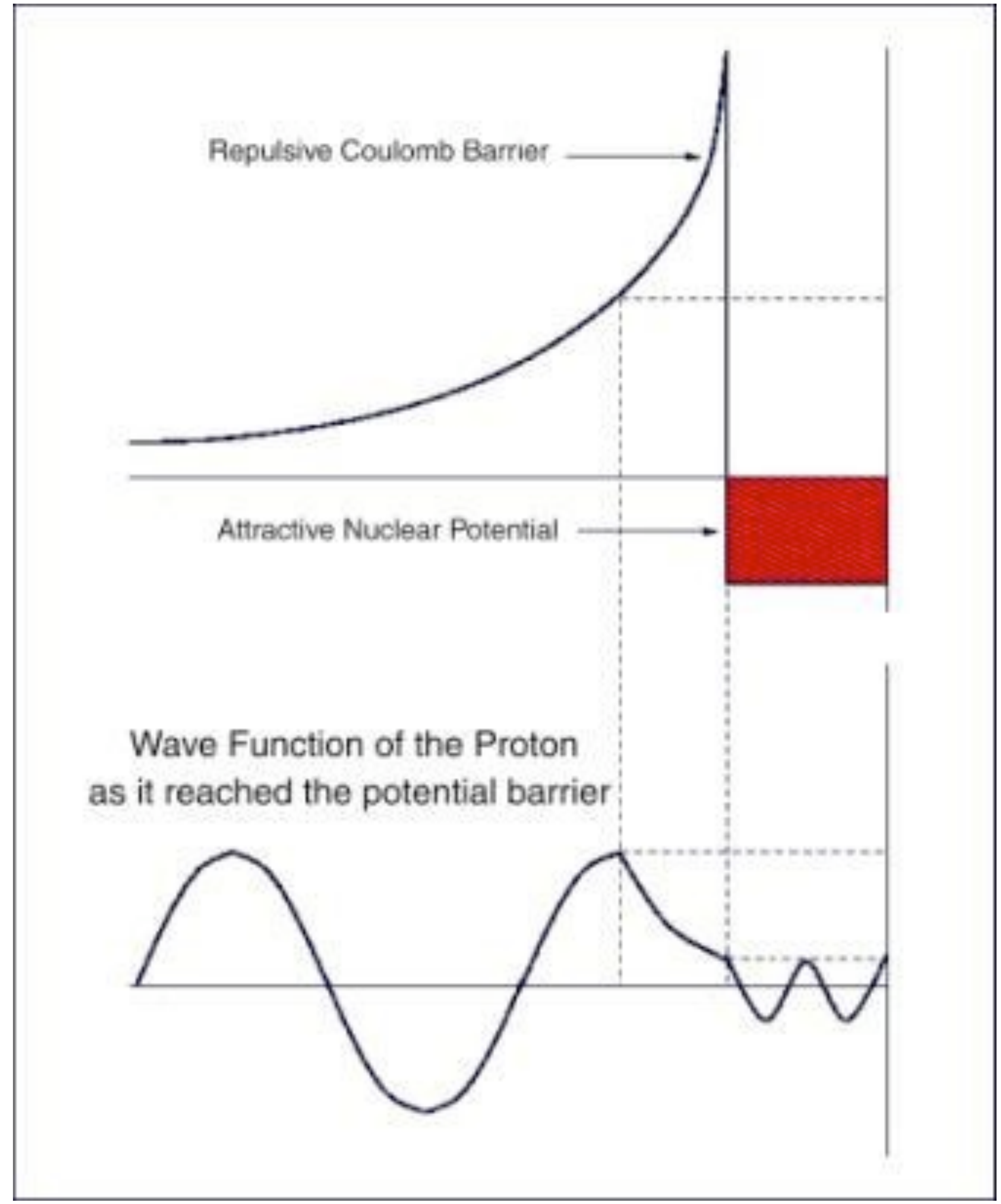


- Quantum mechanics comes to our rescue!
- A particle can "tunnel" through a potential barrier a distance *x* with a probability

$$P(x) \propto e^{-ax/\lambda}dx = e^{-axmv/h}dx$$

- where the particle has a wavelength $\lambda = h/mv$

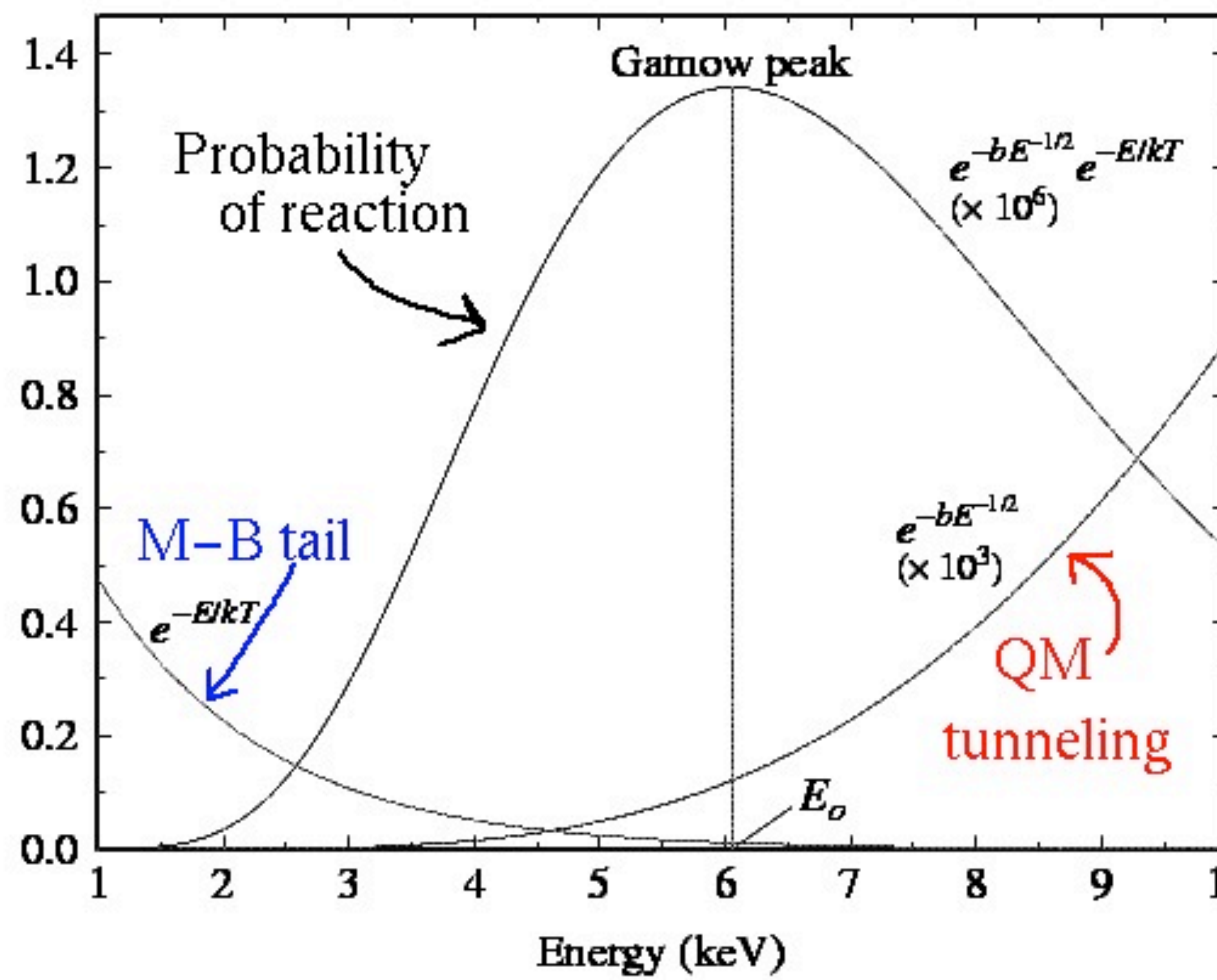


- If the particle has a velocity *v* and mass *m* has to penetrate an electrostatic barrier with energy $E = \frac{mv^2}{2} = \frac{Z_1 Z_2 e^2}{r_{\rm nuc}}$
- then the probability of tunneling that barrier is $P(r_{\rm nuc}) \propto e^{-aZ_1 Z_2 e^2/2hv} \propto e^{-b/E^{1/2}}$

- By combining the Maxwell-Boltzmann "tail" and the QM tunneling, we find that the probability for the particles to bind is

$$P(E) \propto e^{(-E/kt - b/E^{1/2})} dE$$

- This *enhanced probability* at some energy is called the "Gamow peak", after George Gamow



Note that the Gamow peak has been enhanced by a factor of 10^6 here!

- This means that two protons with energies of 3-10 keV can bind --- and there are *lots* of protons with that energy in the center of the Sun, where the temperature is ~16 million K (1.6×10^7 K, *not* 10^{10} K!)

Note that this is actually the first pp chain, contributing ~85% of the time in the Sun; there are two other chains that work at higher temperatures

- In fact, combining two protons together is just the first step in making helium out of hydrogen in the Sun:

$$\left.\begin{aligned} p + p &\rightarrow D + e^+ + \nu \\ D + p &\rightarrow {}^3\mathrm{He} + \gamma \end{aligned}\right\} \times 2$$

$$ {}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p + p $$

- The amount of energy released is

$$(4m_p - m_{^4\mathrm{He}})c^2 = 0.007(4m_pc^2)$$

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This comes from β-decay: $p \rightarrow n + e^{+} + \nu$

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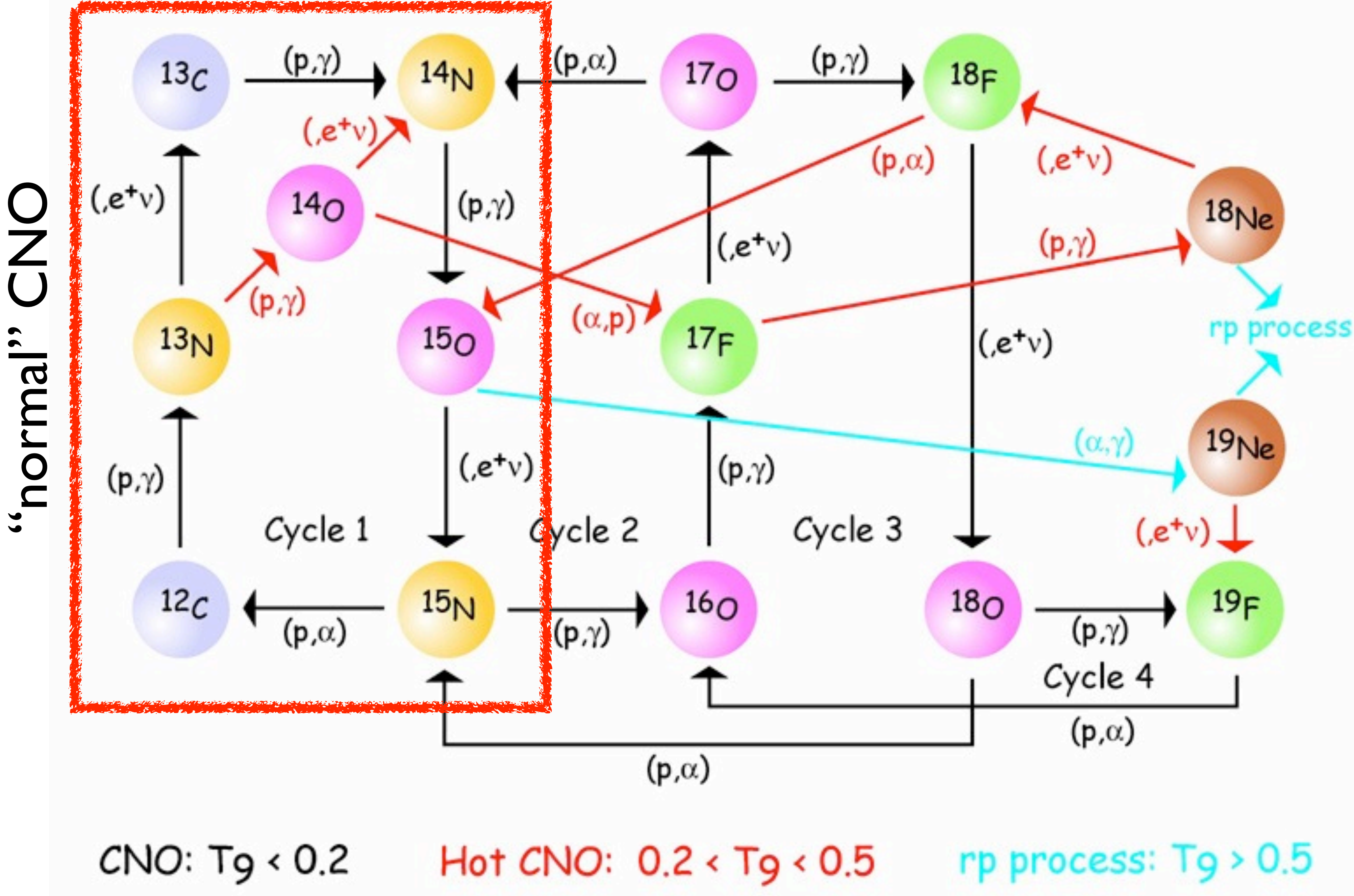
- If all of the hydrogen in the Sun were converted into helium, the available energy would be

$$E = 0.007 M_{\odot} c^2 = 1.3 \times 10^{52}\,\mathrm{erg}$$

- and so the Sun could live for

$$t_{\mathrm{nuc}} = \frac{E}{L} = 1 \times 10^{11}\,\mathrm{yr}$$

- ...but only about 10% of the hydrogen in the Sun becomes helium before the Sun becomes a red giant, so the Sun has lived about half of its lifetime so far!



- There are other nuclear reactions in stars, like the CNO (bi)cycle: it also makes a helium nucleus out of four hydrogen nuclei, but because the CNO nuclei are strongly charged, it requires *higher temperatures* to get over the Coulomb barriers
    - ~15% of the Sun's luminosity comes from CNO

- At even higher temperatures, the "triple-alpha" (3α) process occurs: 3 helium nuclei become one ^{12}C nucleus:

$$
\begin{aligned}
{}^4\mathrm{He} + {}^4\mathrm{He} &\rightarrow {}^8\mathrm{Be} + \gamma \\
{}^8\mathrm{Be} + {}^4\mathrm{He} &\rightarrow {}^{12}\mathrm{C} + \gamma
\end{aligned}
$$

- But there's a problem: there's no stable ^{8}Be nucleus -- it wants to break up into 2 α's!
  - That's ok, because it takes a short time (3×10^{-16} s) for this to happen, and *there are so many helium nuclei* that there is *some* chance for another collision before breakup

- The 3α process is much less efficient per unit weight than hydrogen burning: only 7.3 MeV/3α
  - and it happens at high temperatures: $T \sim 10^8$ K
  - at even higher temperatures, other helium-burning reactions can happen
    - we'll return to this in a while!

reaction	typical log (T/K)	temperature sensitivity
pp	6.7-7.3	T^{6-3}
CNO	7-7.5	T^{18}
3α	8	T^{40}

Stellar structure

Or, how four equations (+four relations) tell us about the structure and evolution of stars

Conservation of mass

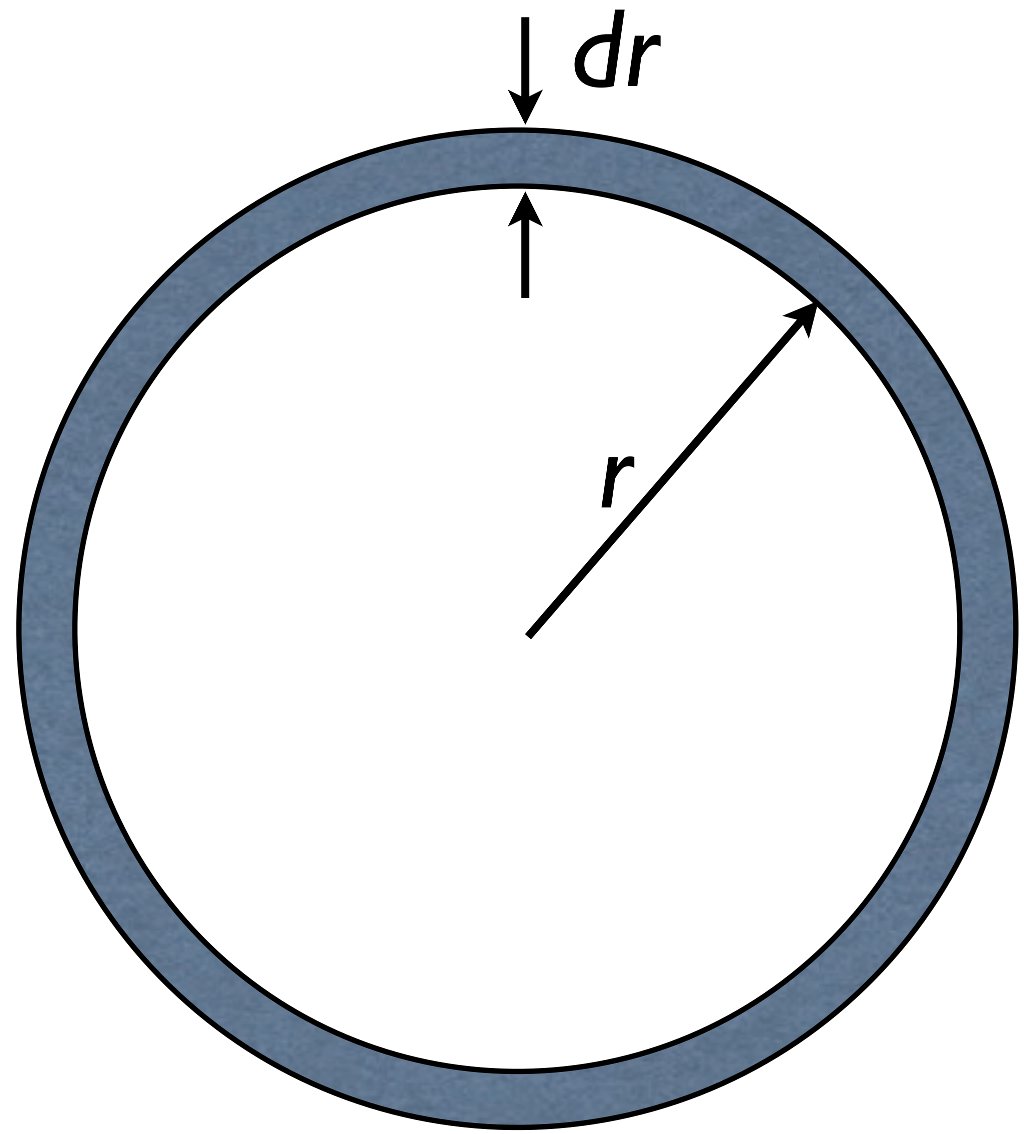
- Remember our spherical star, with a density that depends on radius, $\rho(r)$
- Then the mass contained in a shell of thickness *dr* at *r* is

$$dM = \rho(r)dV = 4\pi r^2 \rho(r)dr$$

- We can then determine the rate at which we add mass as we go out from the center:

$$\boxed{\frac{dM}{dr} = 4\pi r^2 \rho(r)}$$

- This is the *equation of mass conservation*: how the mass changes with radius and density



remember that the force at r depends ONLY on the mass within r!

Hydrostatic equilibrium

- The star should not collapse under its own mass! (under normal circumstances)
- This support is provided by *hydrostatic equilibrium*, the pressure difference between the top and bottom of each shell in our star

note that dP is negative, because there's more pressure from the bottom than from the top!

- How much pressure difference is needed?

- Consider a small cylinder with height *dr* and area *dA*, with mass

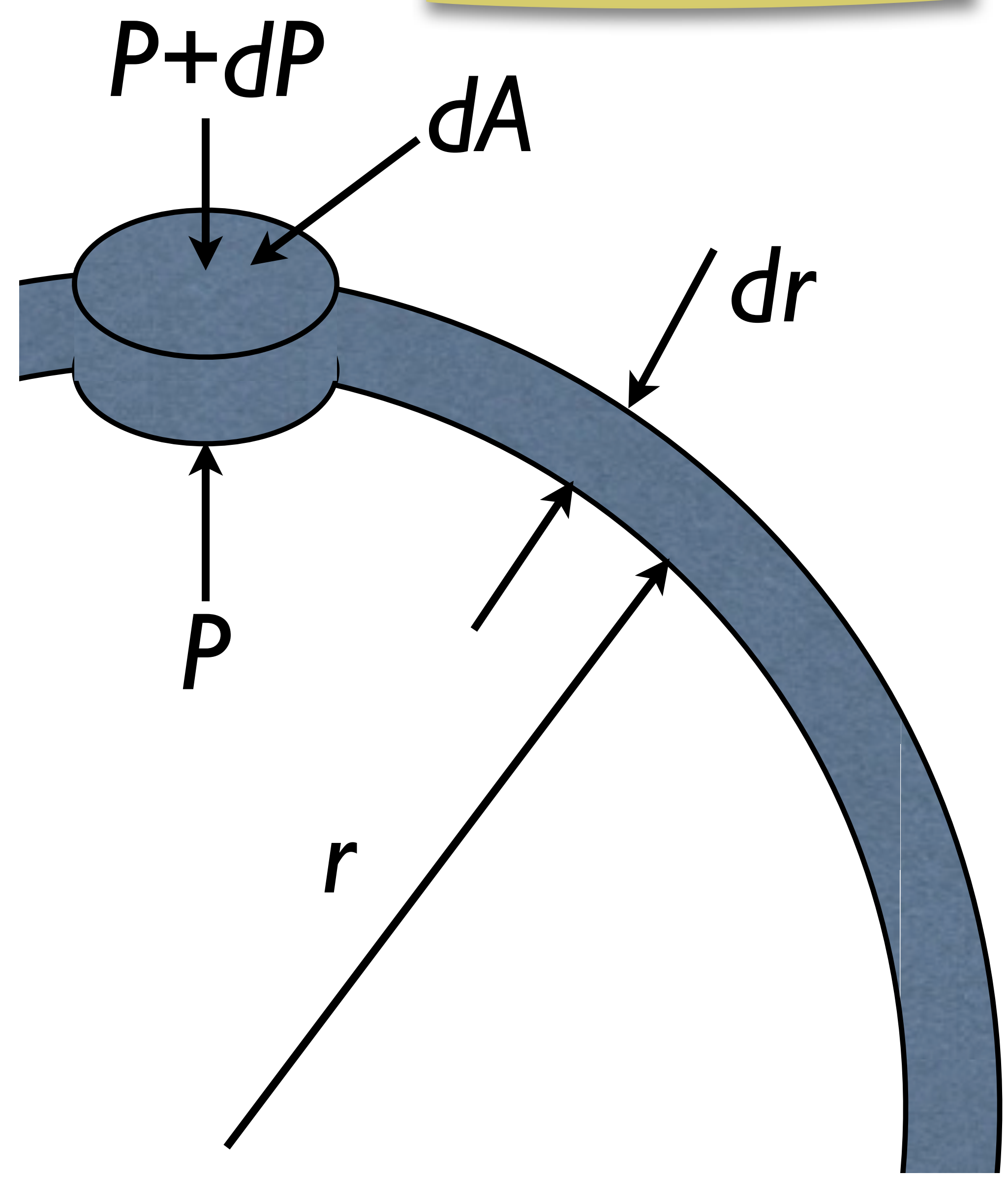
$$dm = \rho(r) dr\, dA$$

- The gravitational force acting on this cylinder is

$$F_g = -\frac{GM(r)dm}{r^2} = -\frac{GM(r)}{r^2}\rho(r)dr\, dA$$

- Remember that pressure is the *force per unit area*, so the *buoyant force* on the cylinder is

$$F_B = PdA - (P + dP)dA = -dP\, dA$$



- Hydrostatic equilibrium requires the buoyant force to balance the gravitational force, so that $F_B+F_g=0$:

$$-dP\,dA - \frac{GM(r)}{r^2}\rho(r)dr\,dA = 0$$

- Dividing by *dA* and *dr* and rearranging, we have the *equation of hydrostatic equilibrium:*

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r) = -g(r)\rho(r)}$$

  - where *g(r)* is the *local gravitational acceleration*

- This equation says that the denser the fluid (here, the star) is, the faster the pressure changes with radius
    - denser fluid ➔ higher mass shell ➔ stronger gravitational force ➔ bigger pressure difference required
    - also, bigger $g(r)$ ➔ stronger gravitational force ➔ bigger pressure difference required

Energy generation

- The *luminosity* produced in our shell of mass *dm* is equal to the energy released by the nuclear reactions per unit mass per second, ε, minus the energy that goes into heat per second, *dQ/dt*:

$$dL = \epsilon dm - dQ/dt$$

- From the first law of thermodynamics, *dQ=dE +PdV*, where *E* is the internal energy and *V* is the volume, so we divide by *dm* and convert to *dr*:

$$\boxed{\frac{dL}{dr} = 4\pi r^2 \rho\epsilon - \frac{dE}{dt} - P\frac{dV}{dt}}$$

last two terms represent gravitational heating

Energy transport

- Now we need to get that energy out of the star!
- There are three processes by which this can happen:
  - Radiation: photons escape through diffusion
  - Convection: blobs of gas rise and fall, carrying heat energy
  - Conduction: electrons carry energy

- Let's consider *radiative transport* first.
- For moving particles, special relativity tells us that the energy of a particle in motion is

$$E^2 = p^2c^2 + m^2c^4$$

- A photon has no mass, but it *does* have a momentum p=E/c ➔ must exert force on particles it encounters ➔ exerts a *pressure* on its surroundings

- Consider a flux F_{rad} of photons leaving our little cylinder (so with units erg/s/cm^2): the momentum transferred from the photons to the volume element is

$$dp = \frac{dF_{\rm rad}}{c} = \frac{F_{\rm rad}}{c}\frac{dr}{l}$$

- where *l* is the *mean free path* of photons --- the average distance a photon will travel before being absorbed or scattered

  - In the Sun *l*~1 cm! Because this distance is so short, it takes 5×10^5 years for a photon to get from the center of the Sun to its surface!

- Now, the momentum transferred is actually the *radiation pressure*, so $dp = -dP_{\mathrm{rad}}$
- and therefore $dP_{\mathrm{rad}} = -\frac{F_{\mathrm{rad}}}{c}\frac{dr}{l}$
- We define the *opacity* as the probability per unit mass that the photons experience absorption or scattering over one mean free path: $\bar{\kappa}\rho \equiv 1/l$
- and we can write $\boxed{\frac{dP_{\mathrm{rad}}}{dr} = -\frac{\bar{\kappa}\rho}{c}F_{\mathrm{rad}}}$

- Now if the radiation obeys a *blackbody distribution* of energies, we can compute that the radiation pressure depends on the temperature through $P_{\rm rad} = aT^4/3$
  - where a=4σ/c is the radiation constant (here σ is the Stefan-Boltzmann constant)
- Then we can take the derivative with respect to *r*: $\boxed{\dfrac{dP_{\rm rad}}{dr} = \dfrac{4}{3}aT^3\dfrac{dT}{dr}}$

- Then we can combine these equations for $dP_{\rm rad}/dr$ together and solve for dT/dr to find the radiative temperature gradient

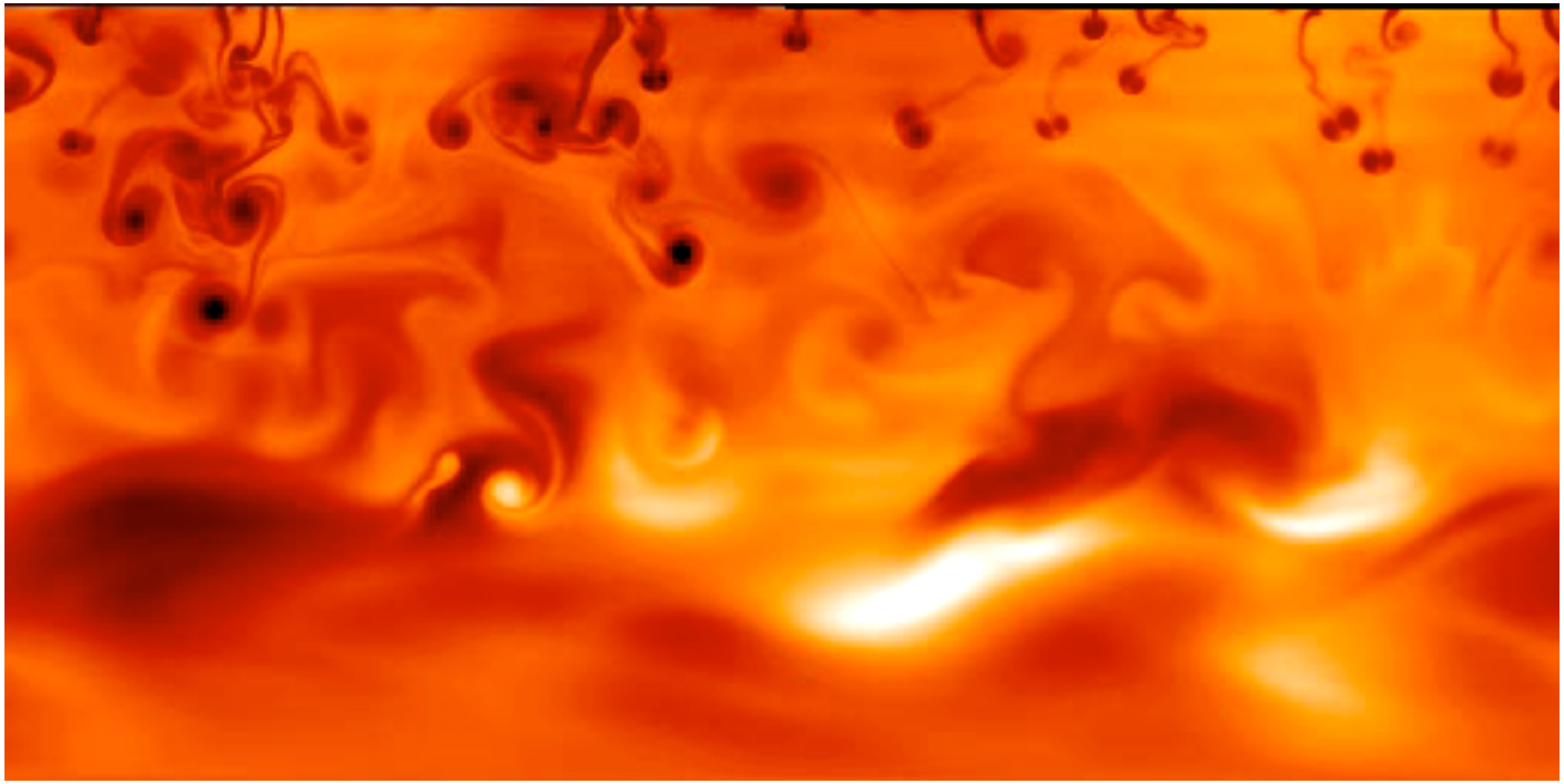
$$\frac{dT}{dr} = -\frac{3\bar{\kappa}\rho}{4acT^3}F_{\rm rad}$$

- Now, the flux at r is just the luminosity at r spread out over the entire shell:

$$L = 4\pi r^2 F_{\rm rad}$$

- So if radiation is carrying *all* the energy flux, the temperature must change across the shell as $\boxed{\dfrac{dT}{dr} = -\dfrac{3}{4ac}\dfrac{\bar{\kappa}\rho}{T^3}\dfrac{L}{4\pi r^2}}$

- If the temperature difference is too steep across a shell --- for example, when the CNO cycle (with its strong temperature dependence) is dominating the energy production at the core of a high-mass star, or near the surface of the Sun, where the gas is cool enough to absorb energy through ionization of the elements --- then radiation *does not transport* all of the energy



- Then we need convection!
- In convection, hot blobs of fluid (or gas) carry energy to low-pressure regions and then cool and fall back to high-pressure regions

- Convection is important to the structure and evolution of stars because convective regions
    - make stars bigger: it takes a lot of "room" to carry the energy in this way --- the star will expand and cool to allow this to happen
    - mix material through stars: any convective region is completely mixed with the stuff in that region, so any new elements made in some region will be mixed throughout the region

- Finally, in white dwarfs (which we'll come to shortly), energy is actually carried by conduction
    - conduction is the transfer of energy by electrons: the heat you feel when you touch hot metal (like the hot surface of a clothes iron) is being transferred through conduction

The equation of state & other relations

- Now we have four equations but eight(!) unknowns: $m(r)$, $T(r)$, $L(r)$, $P(r)$, $\rho(r)$, $\kappa(r)$, $\varepsilon(r)$, $E(r)$
- So we need four other relations!
- $\kappa=\kappa(\rho, T, \text{composition})$: from pre-computed opacity tables
- $\varepsilon=\varepsilon(\rho, T, \text{composition})$: from nuclear physics

- *P*=*P*(ρ,T,composition) and *E*=*E*(ρ,T,composition): these come from the *equation of state* of the matter
  - For an *ideal gas*, *PV*=*NkT* implies $\boxed{P = \frac{\rho}{m} kT}$
    - where *m* is the average mass per particle in the gas and *k* is the Boltzmann constant
  - and the internal energy *E* is $E = \frac{3}{2m} kT$
  - For, say, *degenerate* matter, there is a different equation of state, which we'll see later

Summary of stellar structure equations

Mass conservation: $$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium: $$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$$

Energy generation: $$\frac{dL}{dr} = 4\pi r^2 \rho\epsilon - \frac{dE}{dt} - P\frac{dV}{dt}$$

(Radiative) energy transport: $$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L}{4\pi r^2}$$

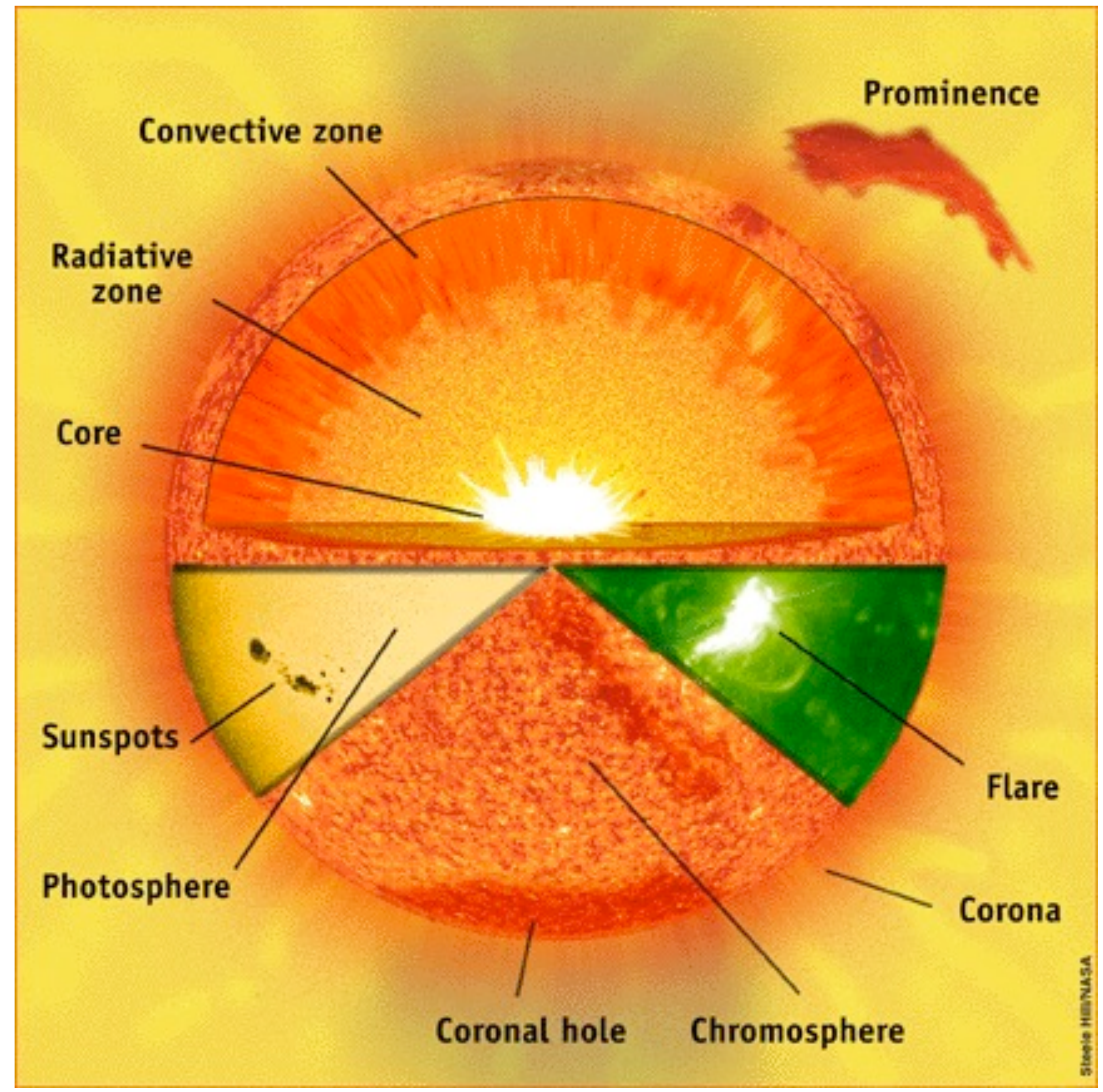
+equation of state, opacity tables and nuclear energy generation

Stellar Models

- To generate a model of a star, we write a computer program that uses these four equations + the four supporting relations, assumes an initial mass, a composition, and some boundary conditions
  - Then we get the structure of the star at some time *t*

Structure of the Sun

- These equations allow us to "peer inside" the Sun
- We can identify three major regions "inside" the Sun:
  - The core, where most of the luminosity is produced
  - The radiative zone, where the temperature gradient is small
  - The convective zone, where the temperature gradient is large



- To get the *present-day* Sun, we have to make the Sun at the time t=0, which is defined at the time at which the star first starts burning hydrogen in equilibrium: that is, the time when all the elements involved in hydrogen burning have reached their "equilibrium abundances"

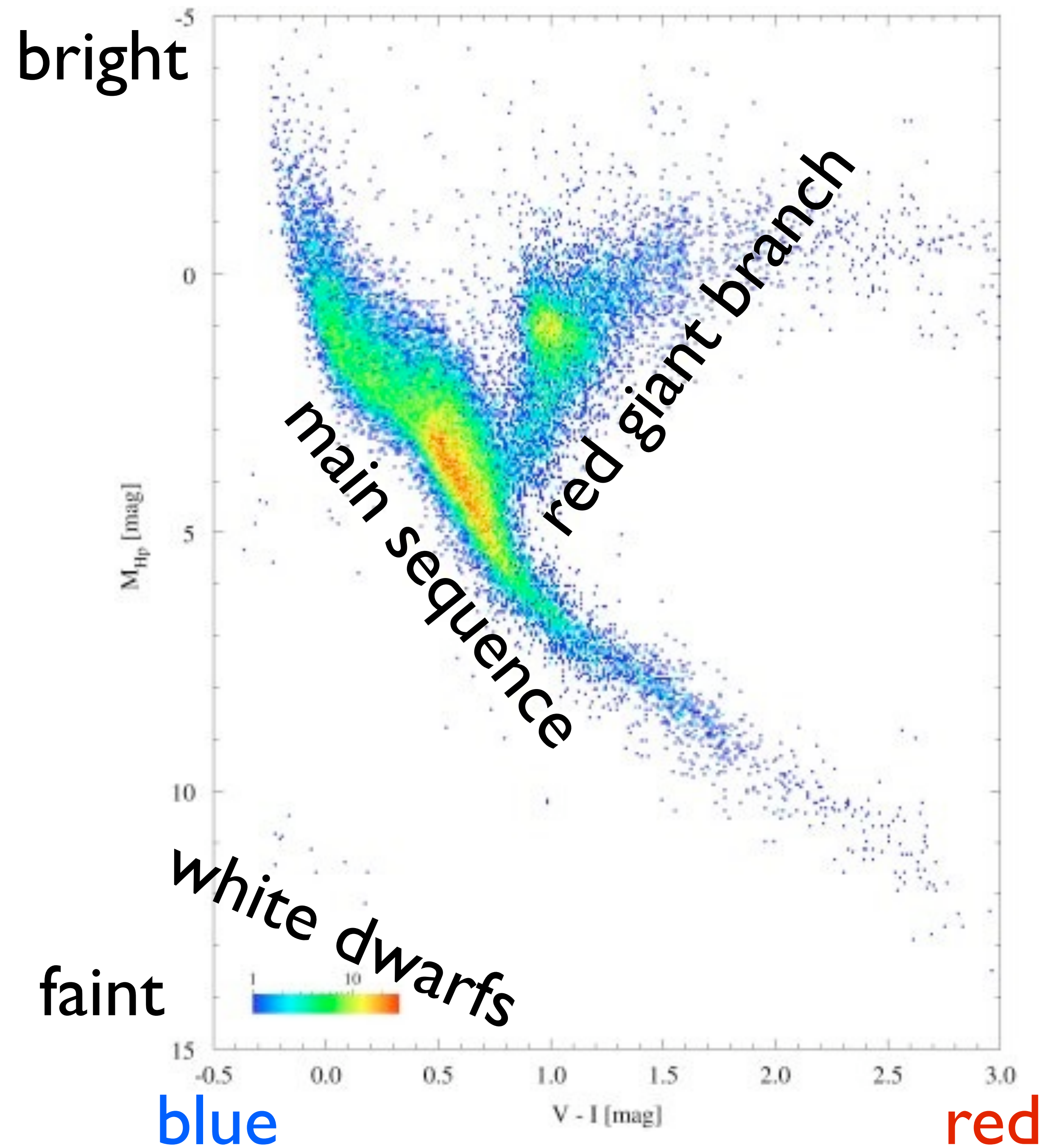
- We then let the Sun burn hydrogen for some amount of time, then we compute a new model with the new composition after the burning has gone on for some time
    - As burning proceeds, the amount of helium will increase and the amount of hydrogen will decrease
    - This will change the structure because energy generation in the pp-chain and CNO cycles depends on the amount of hydrogen available, and the mass of a typical particle will change, which changes the equation of state
    - We keep following this evolution until the Sun is 4.5 billion years old and then compare the properties of the model -- its radius, luminosity, and temperature, which all increase with time -- to our Sun

The colour-magnitude diagram of local stars

- Let's turn to the observations now!
- We can classify stars by their **colours** and their **luminosities**, which we rank using magnitudes: $M = -2.5\log(L) + \mathrm{constant}$
- Colours are given as the ratio of luminosity in two different bands, expressed as a magnitude:
$$\begin{aligned} C &= -2.5\log(L_1/L_2) \\ &= -2.5\log(L_1) + 2.5\log(L_2) \\ &= M_1 - M_2 \end{aligned}$$

- Hotter stars are bluer: their spectra peak farther into the blue (shorter wavelengths)
- Cooler stars are redder: their spectra peak farther into the red (longer wavelengths)

- When we look at the colours and magnitudes of local stars, we see a clear **main sequence** of stars along with a distinct **red giant branch**
  - we also see very hot, very faint stars: these the white dwarfs, which we'll come back to soon!

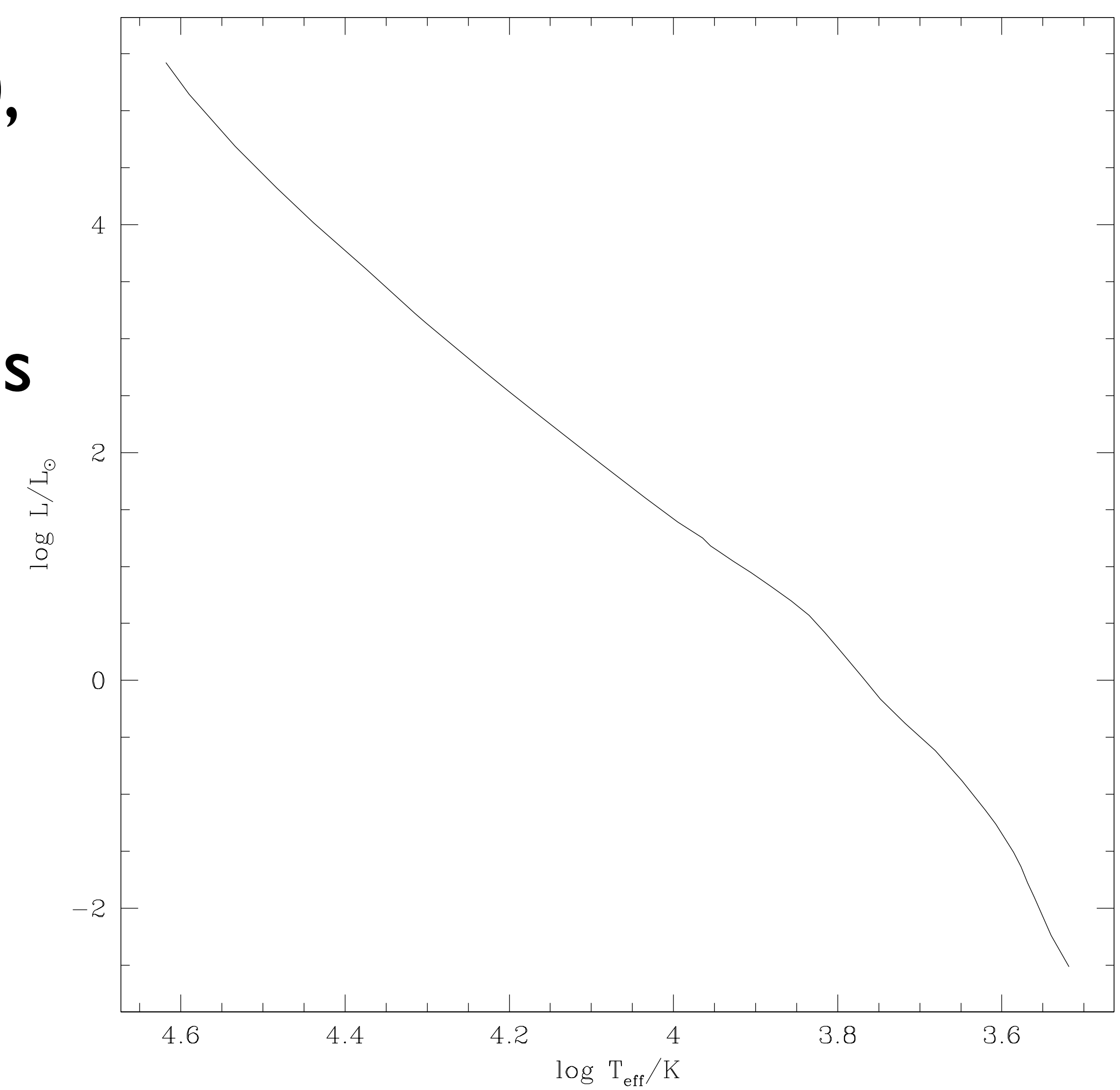


A note about astronomical terms

- There are two *almost* equivalent diagrams of the evolution or photometric properties of stars:
  - Colour-magnitude diagram (CMD): a plot of (absolute or apparent) magnitude as a function of colour
  - Hertzsprung-Russell (HR) diagram: a plot of stellar luminosity as a function of temperature

The "zero-age main sequence"

- If we make models of stars of many different masses (but the same composition) at *t*=0, when equilibrium hydrogen burning first begins, we find a distinct pattern of luminosities and temperatures that looks like the observed main sequence:
  - this is the "zero-age main sequence"



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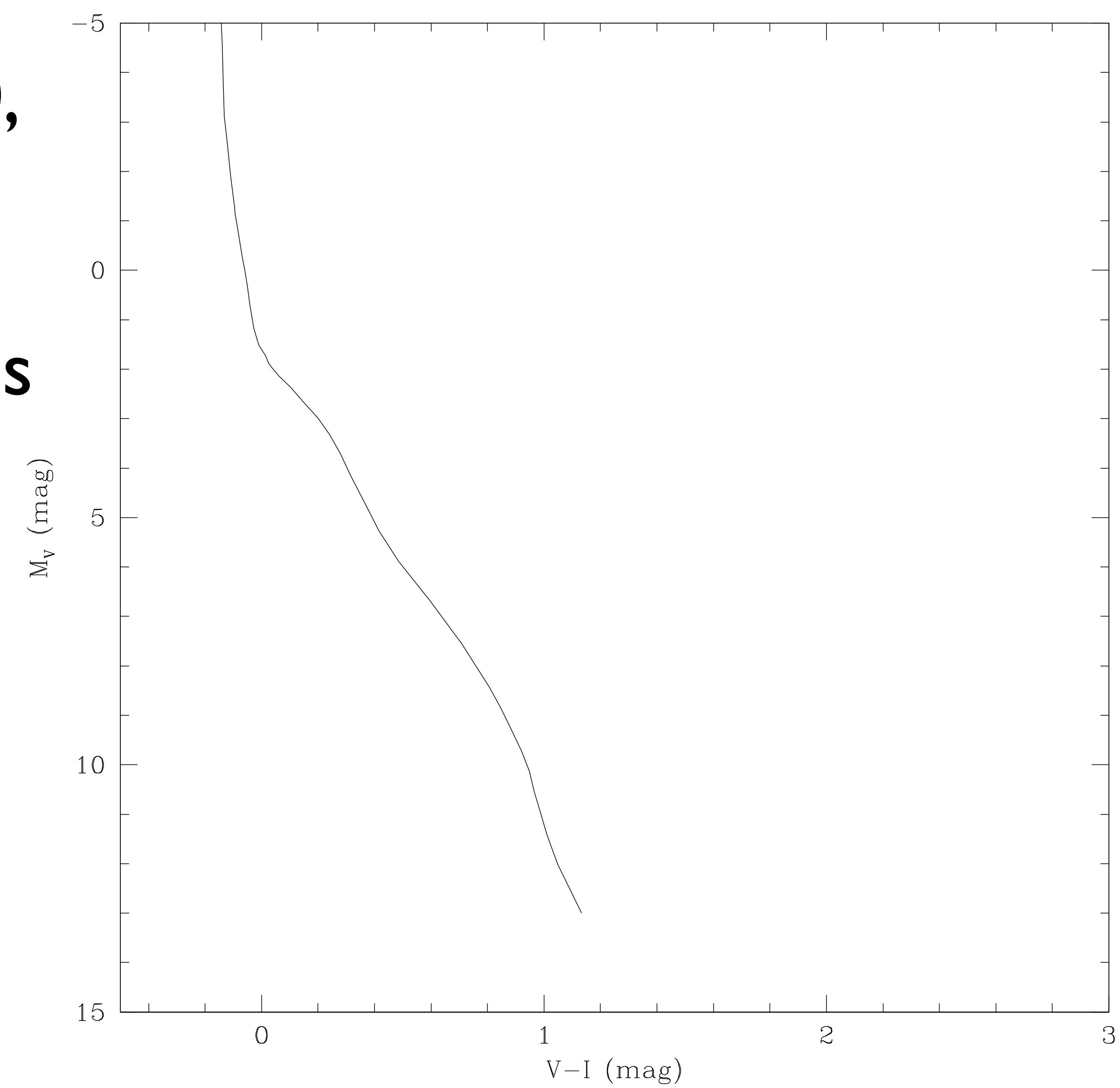


Stellar evolution

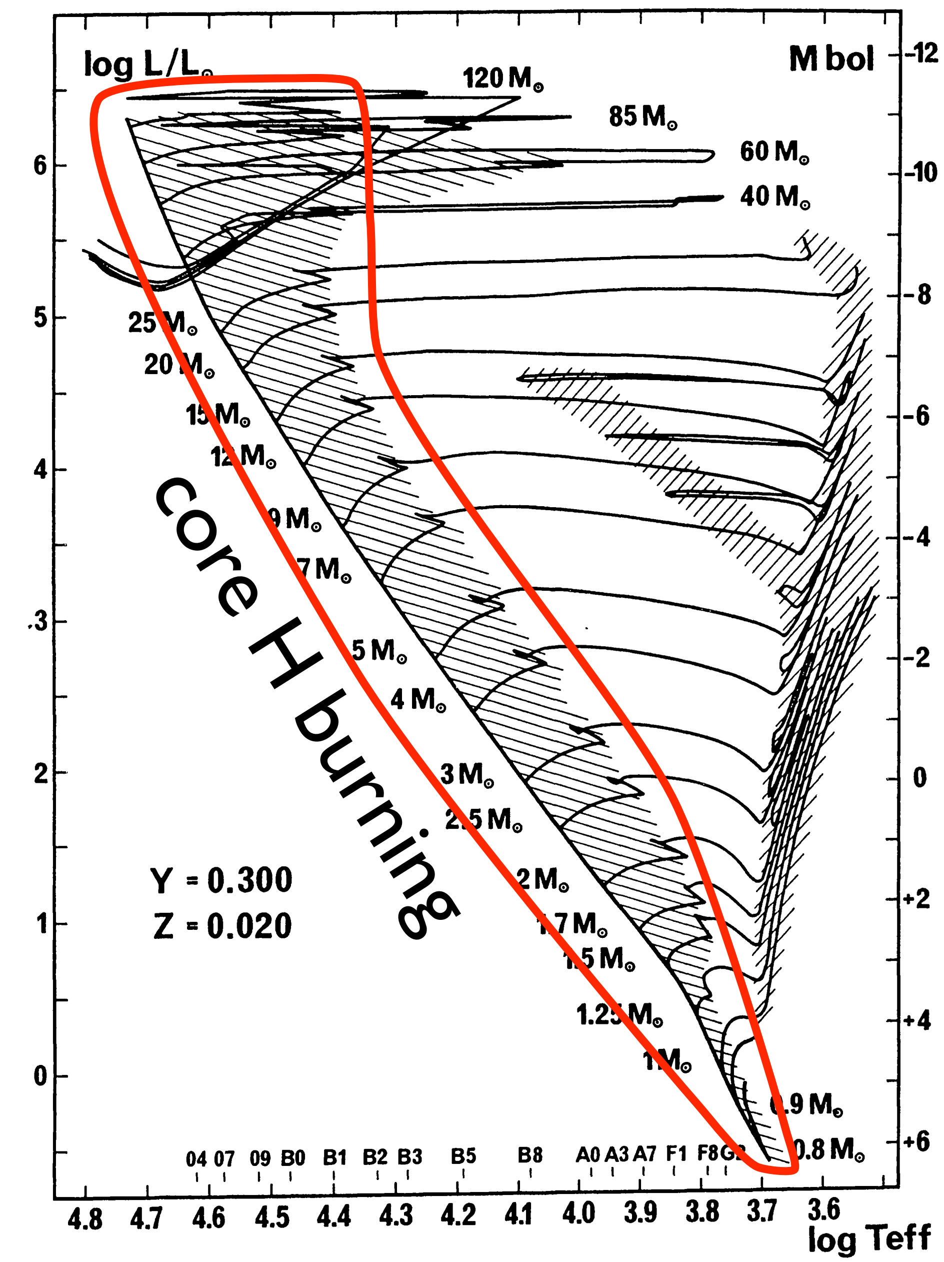
- All stars will go through (at least) **two** major burning phases:
  - Hydrogen burning
  - Helium burning
- ...but for stars with masses less than ~0.8 $M_\odot$, hydrogen burning takes longer than the current age of the Universe, so they never get to helium burning!

Stellar evolution

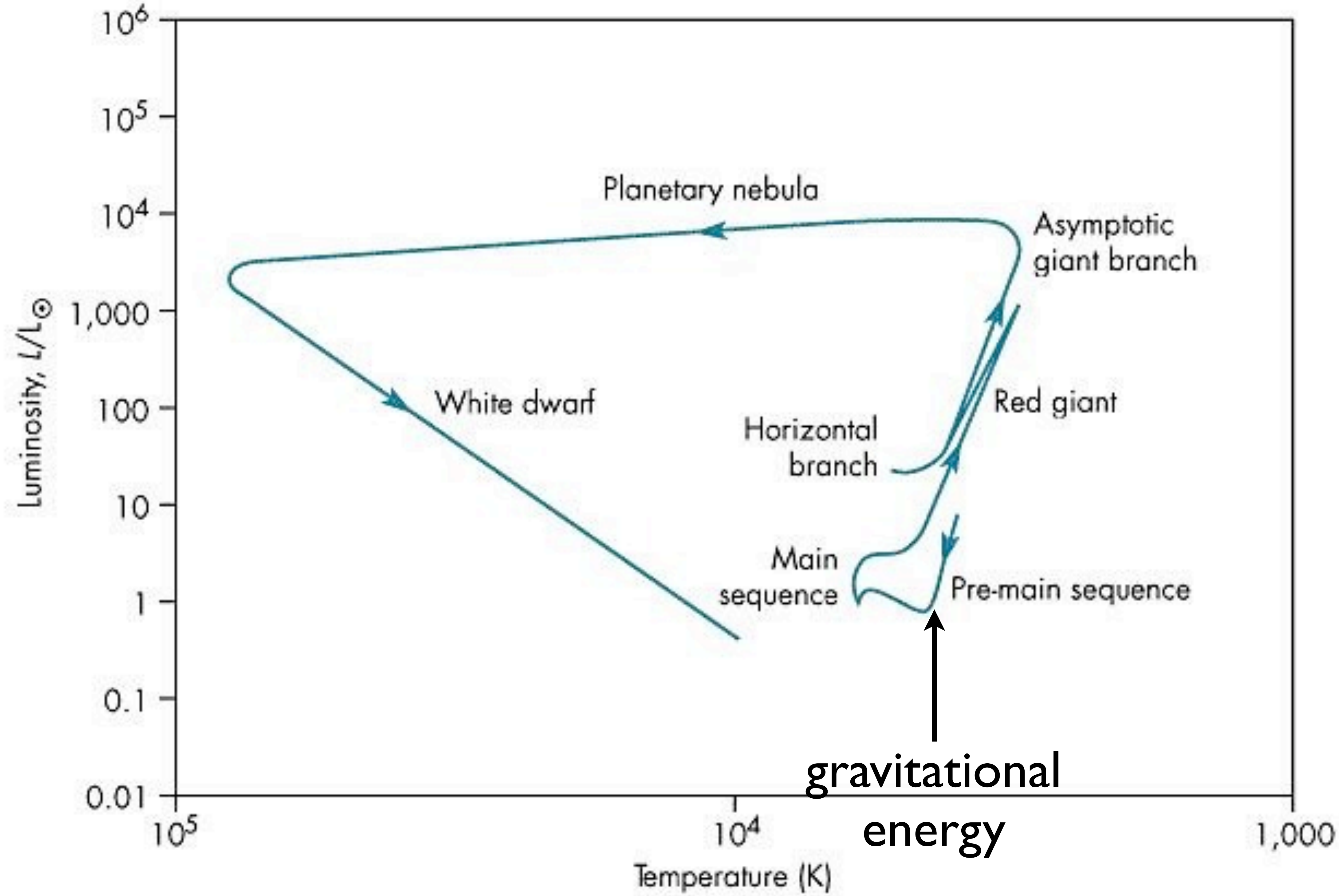
- As stars process their hydrogen into helium, they *evolve* in the Hertzsprung-Russell diagram, roughly from right (hotter) to left (cooler)

- Because the energy released from hydrogen burning is so efficient, stars spend ~90% of their lives on the main sequence

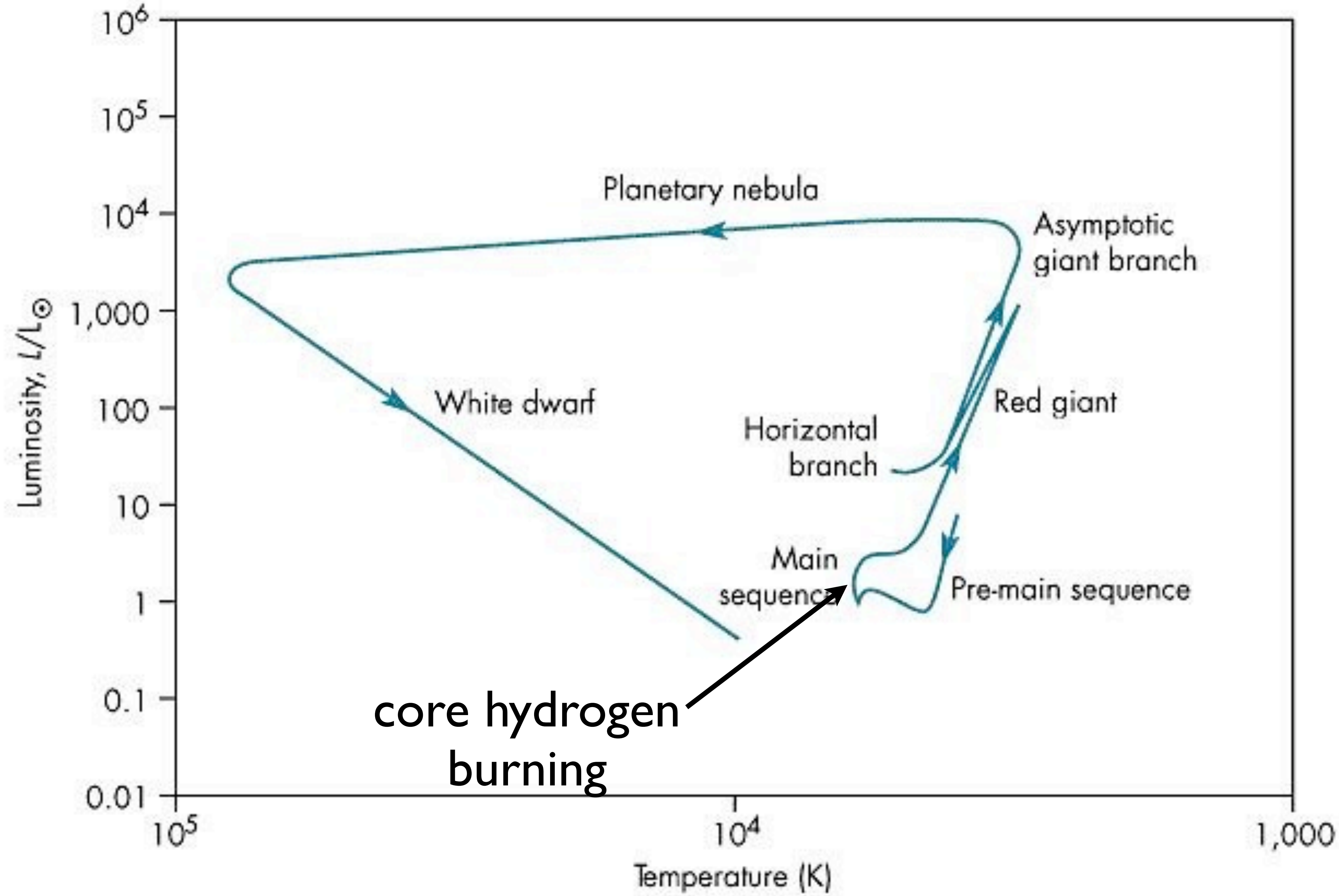
like the Sun, stars still evolve on the MS, which is why the observed MS has some thickness



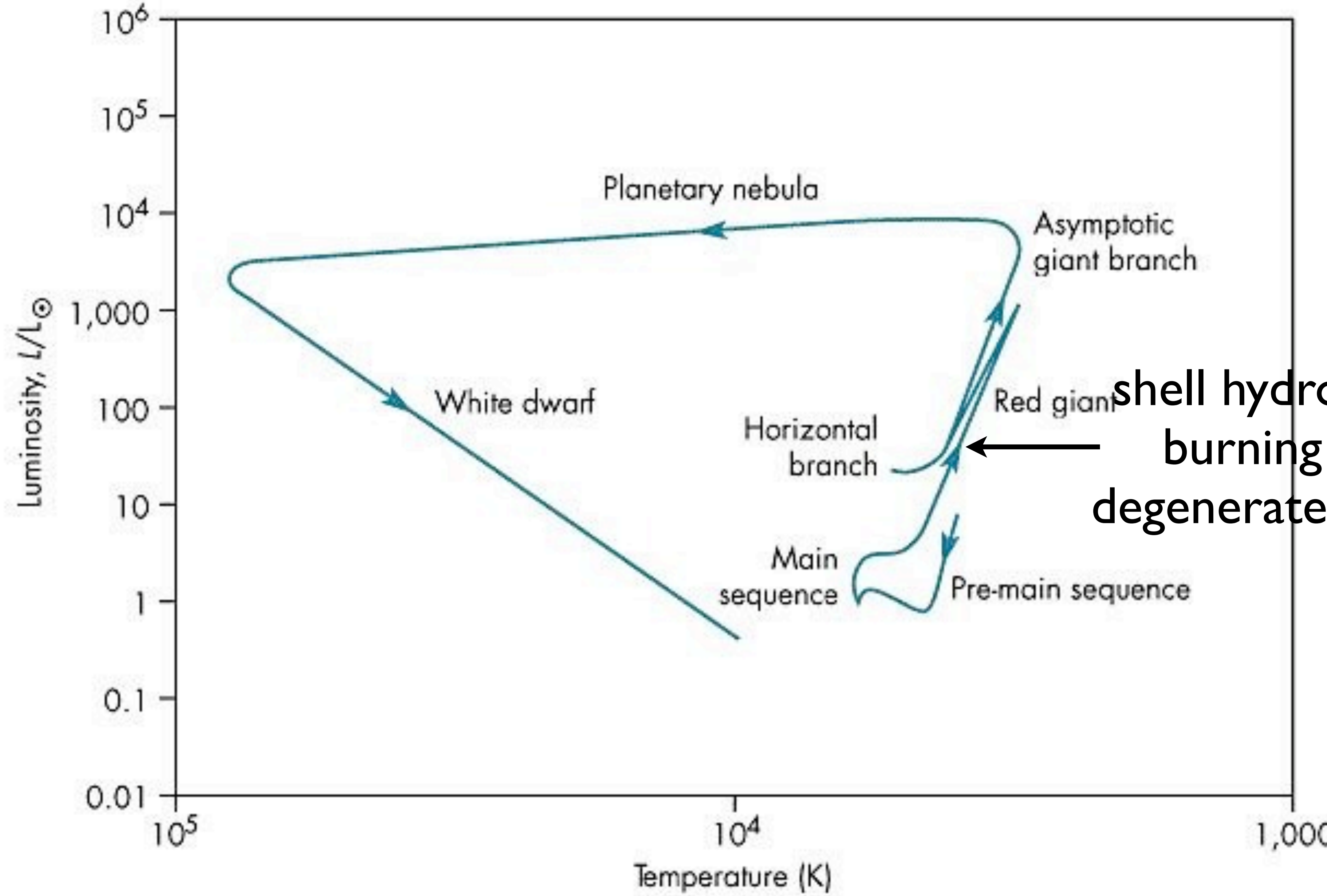
Stellar evolution: low mass stars



Stellar evolution: low mass stars



Stellar evolution: low mass stars



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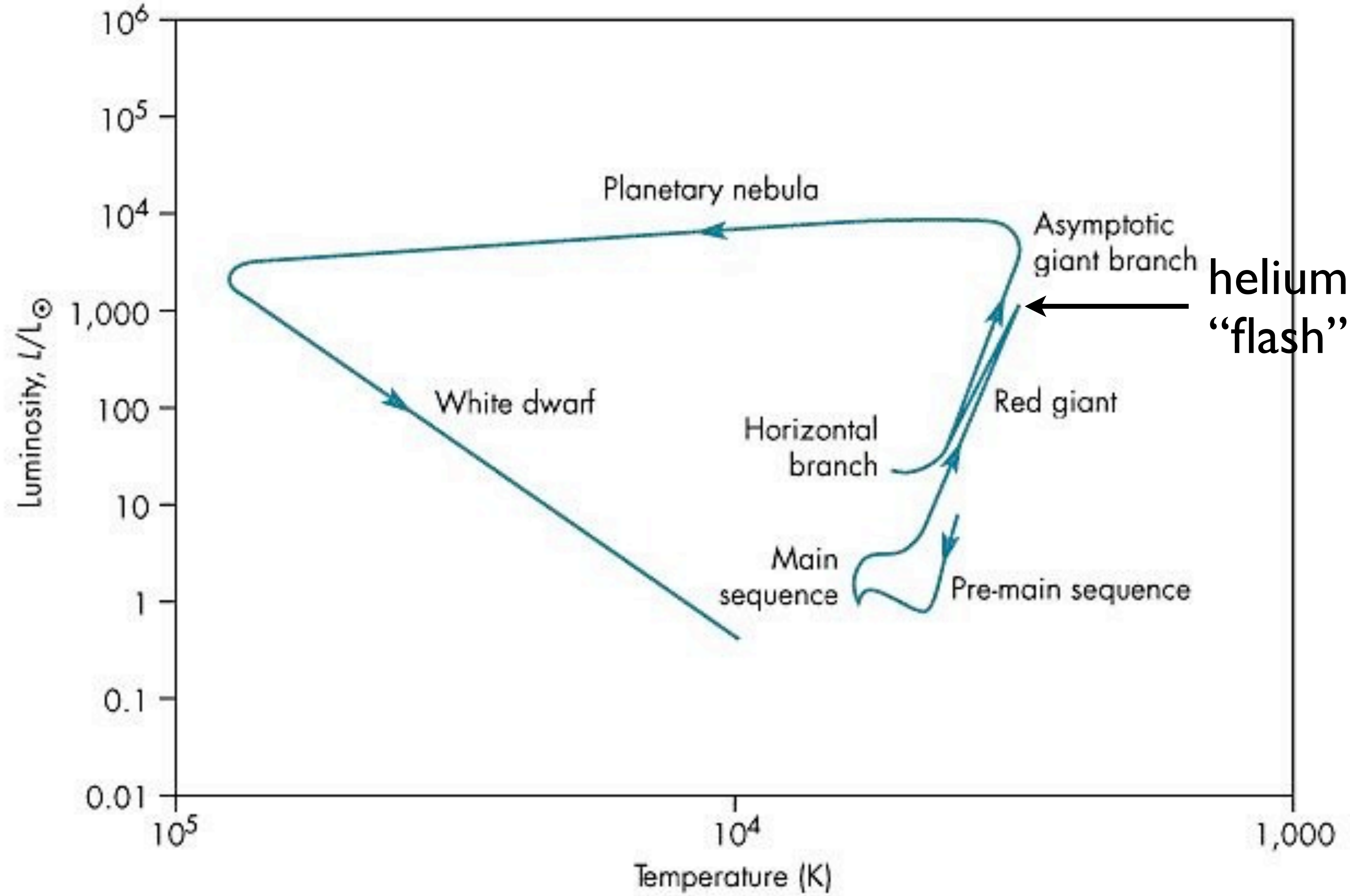

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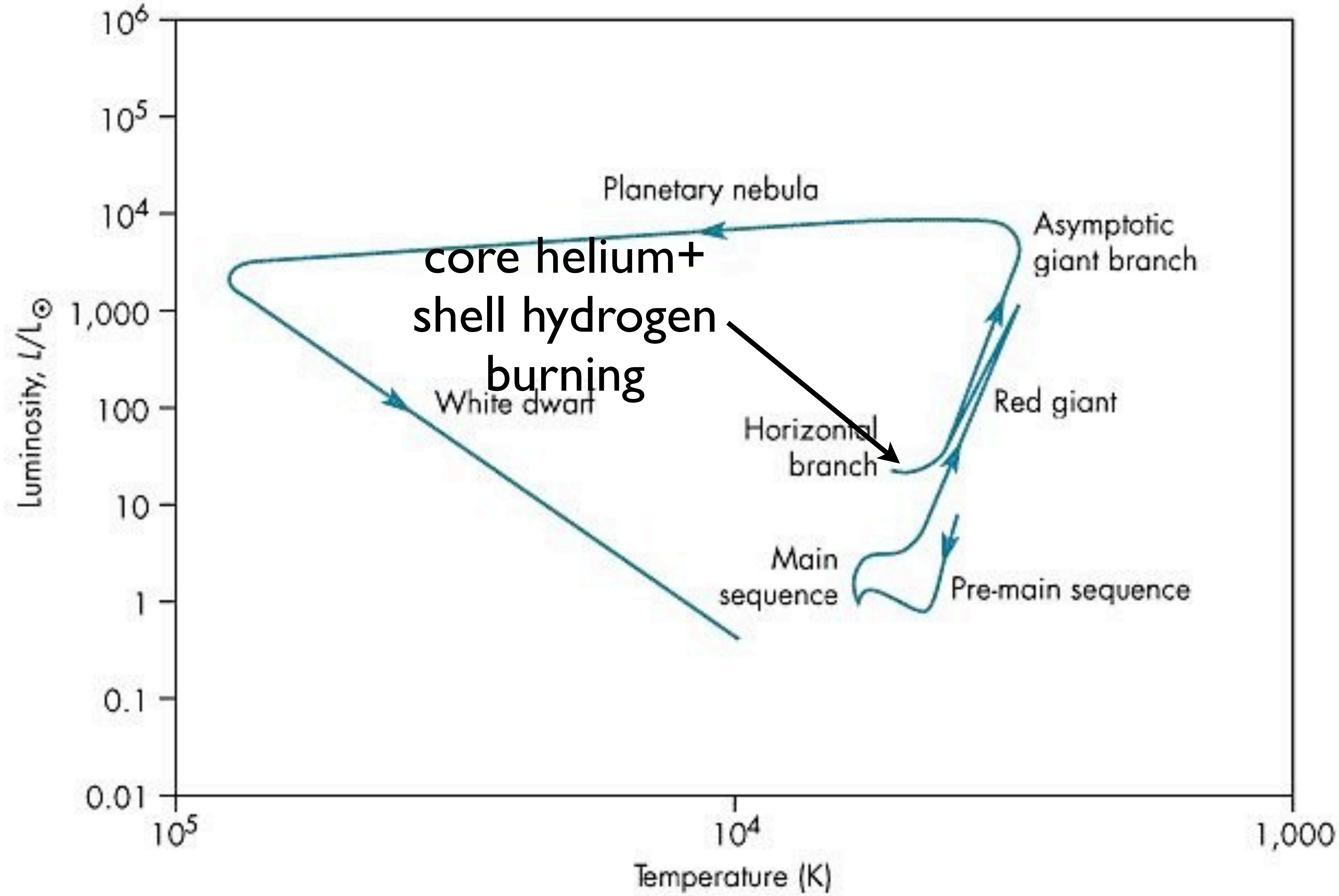
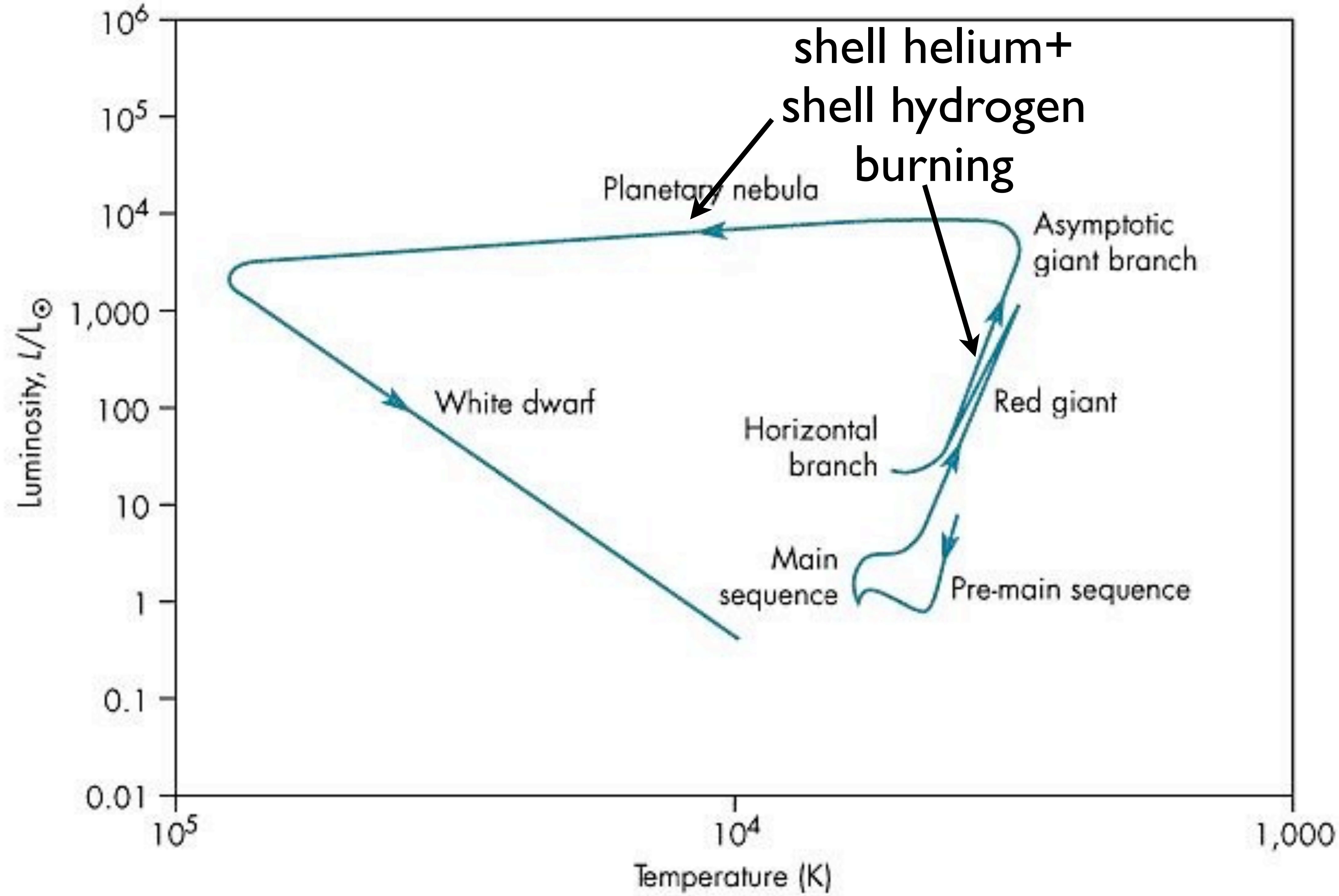
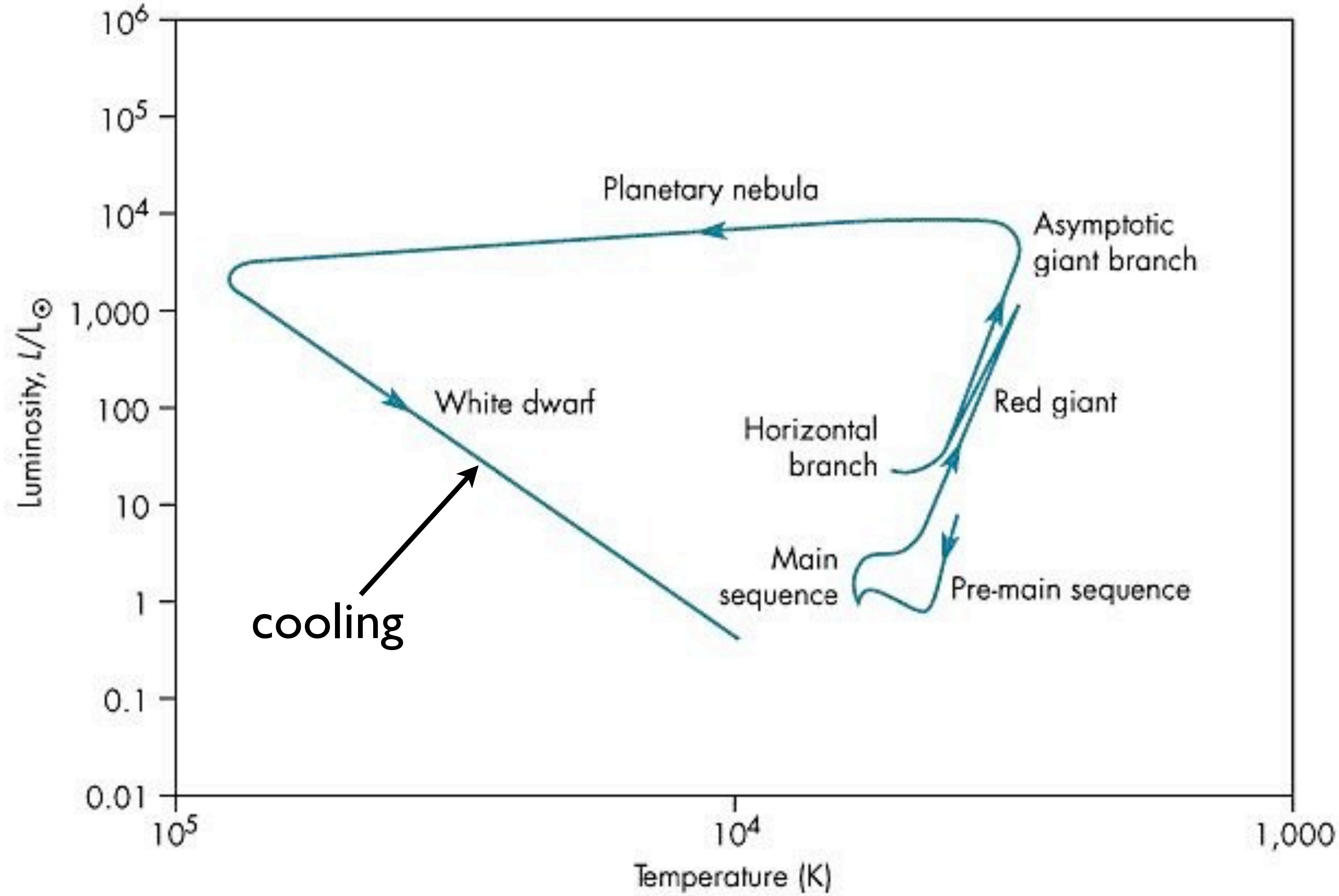


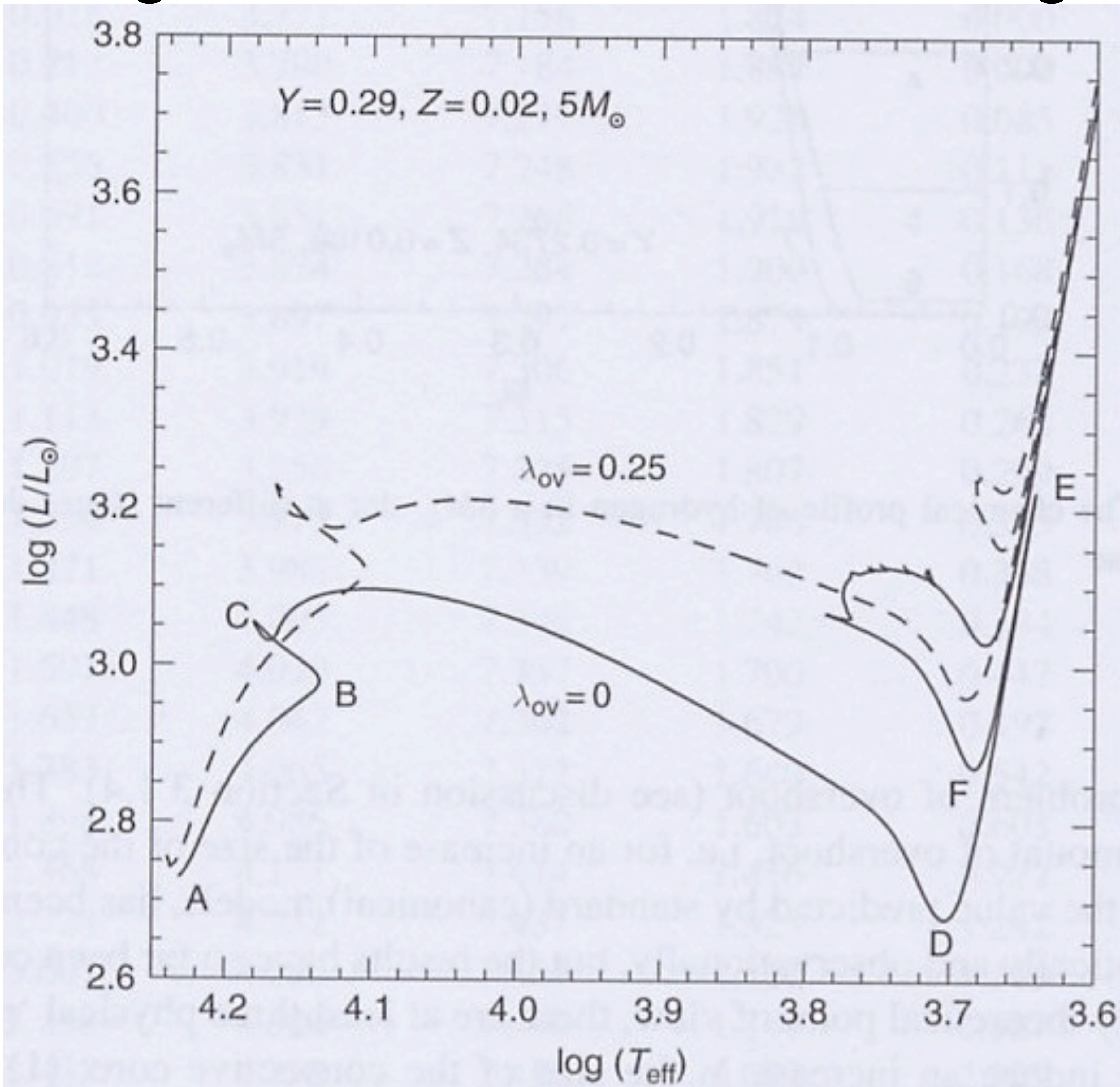
Stellar evolution: low mass stars



Stellar evolution: low mass stars



High-mass stars: H- and He-burning



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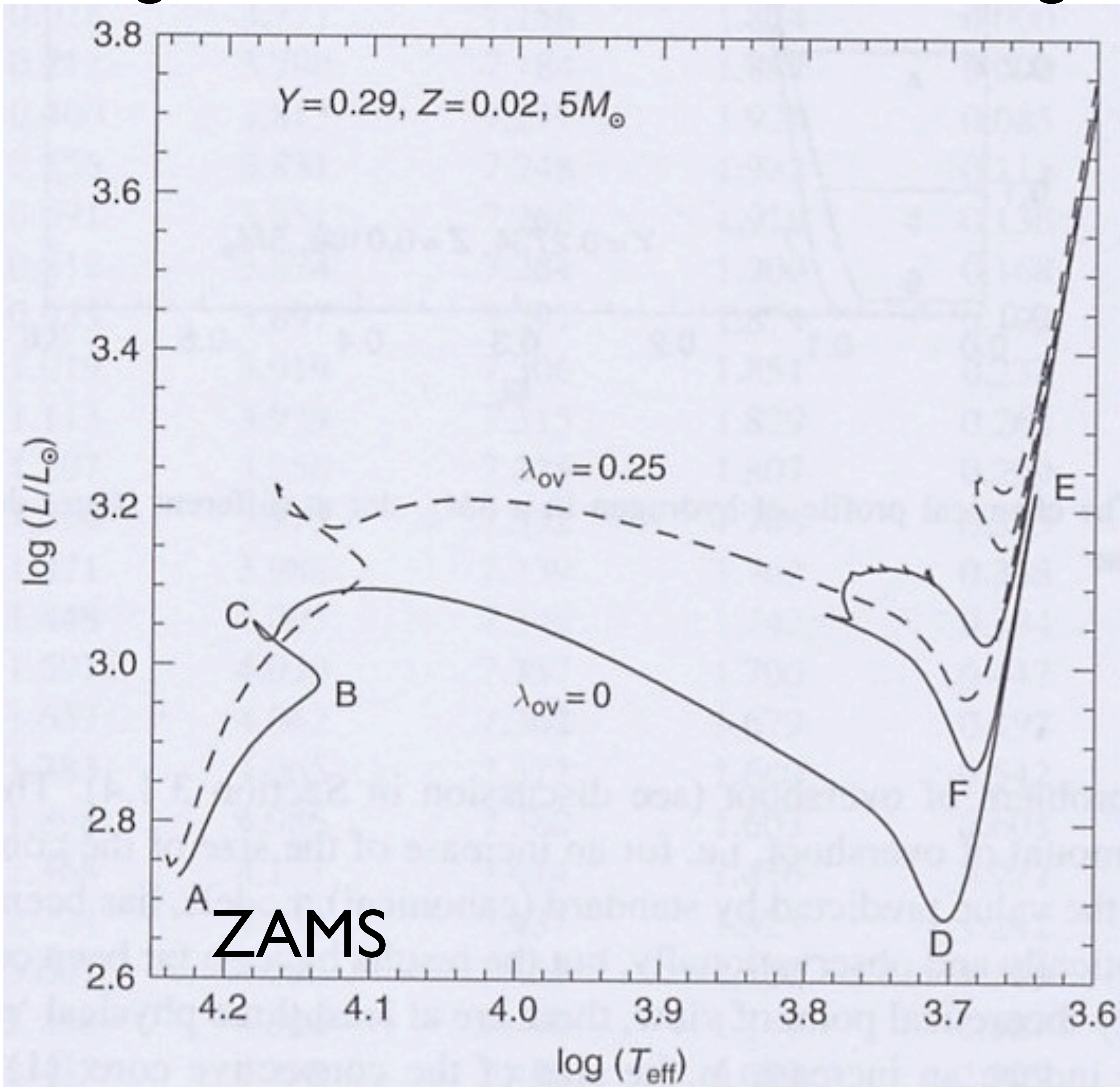

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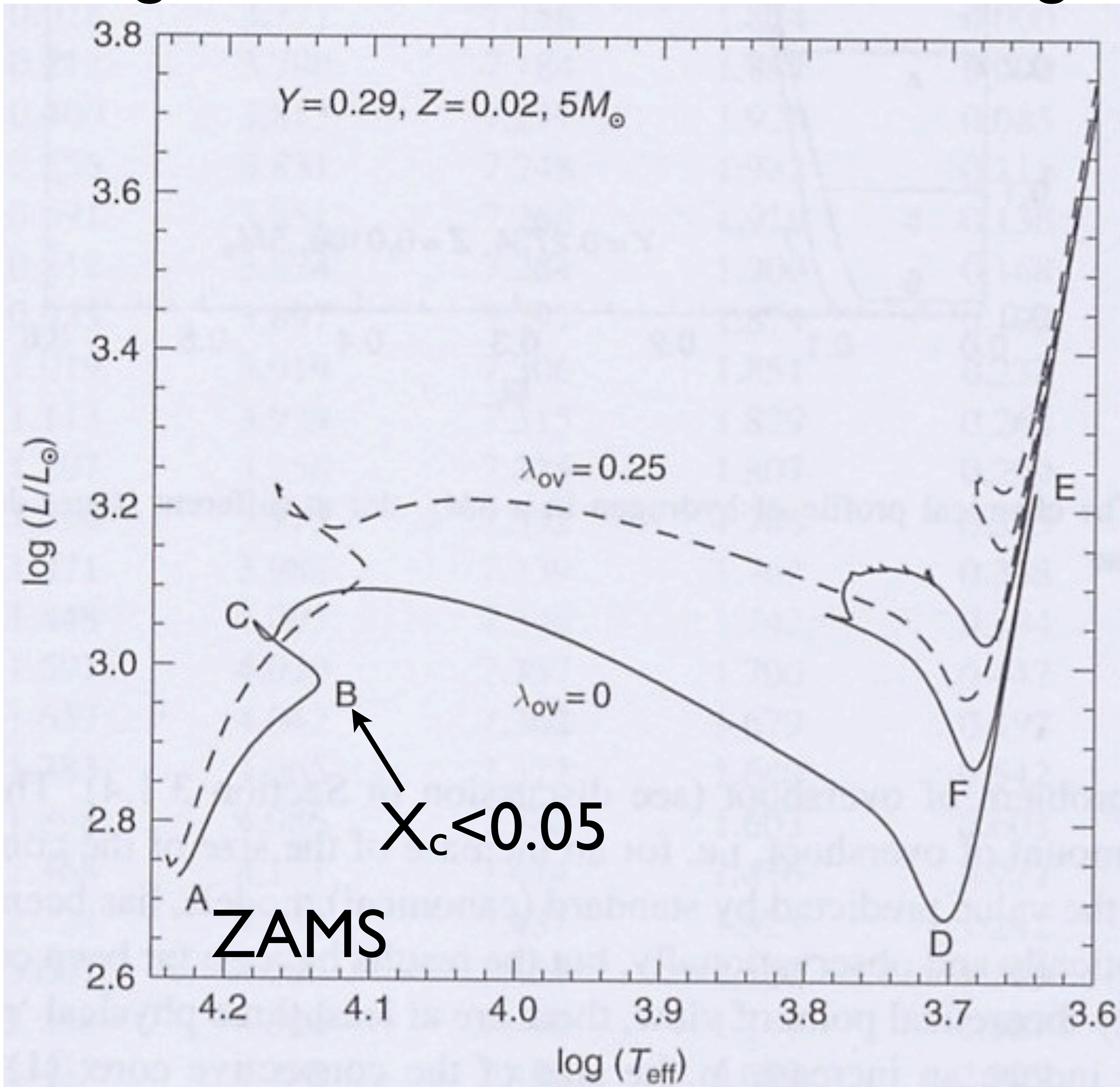

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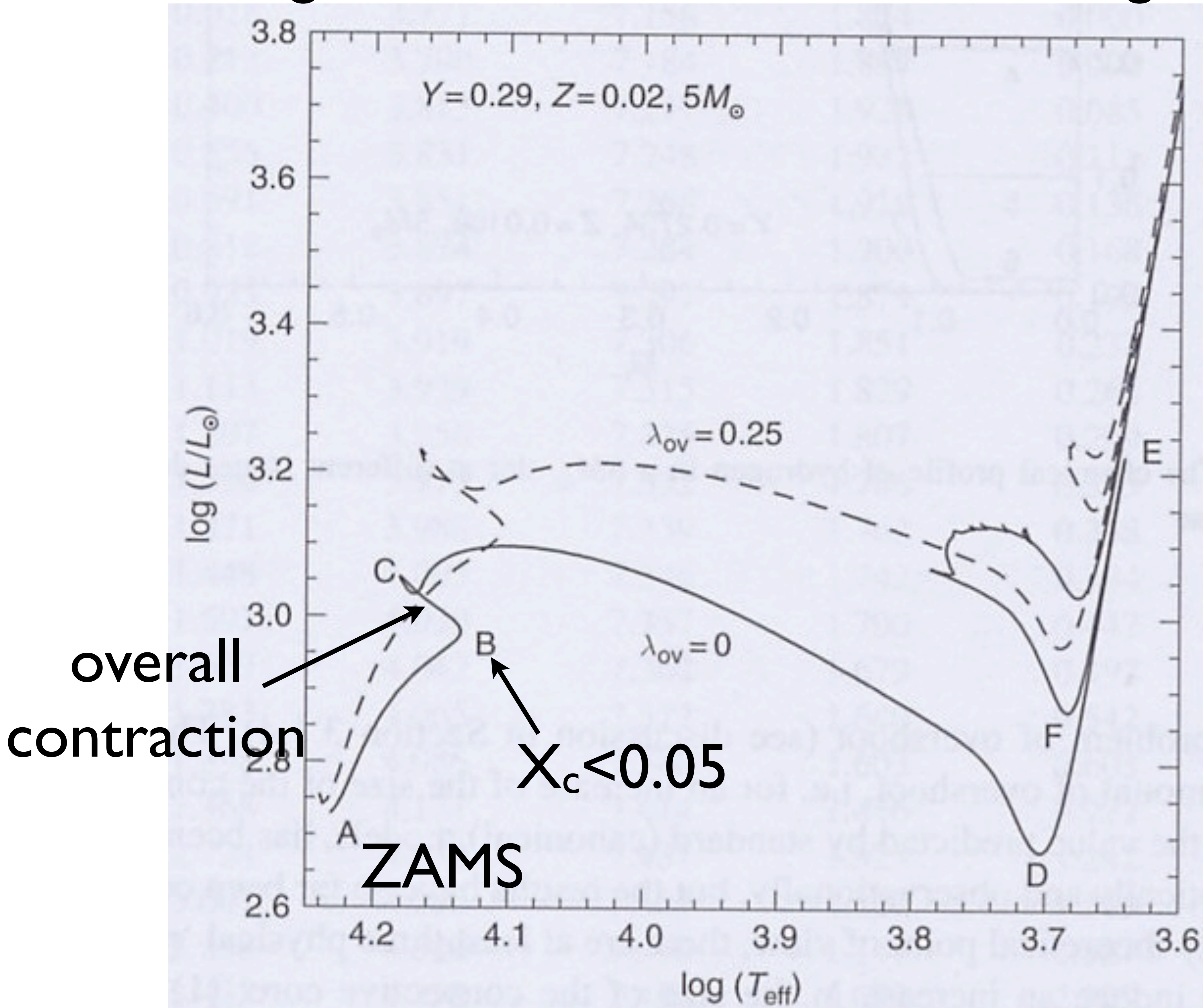

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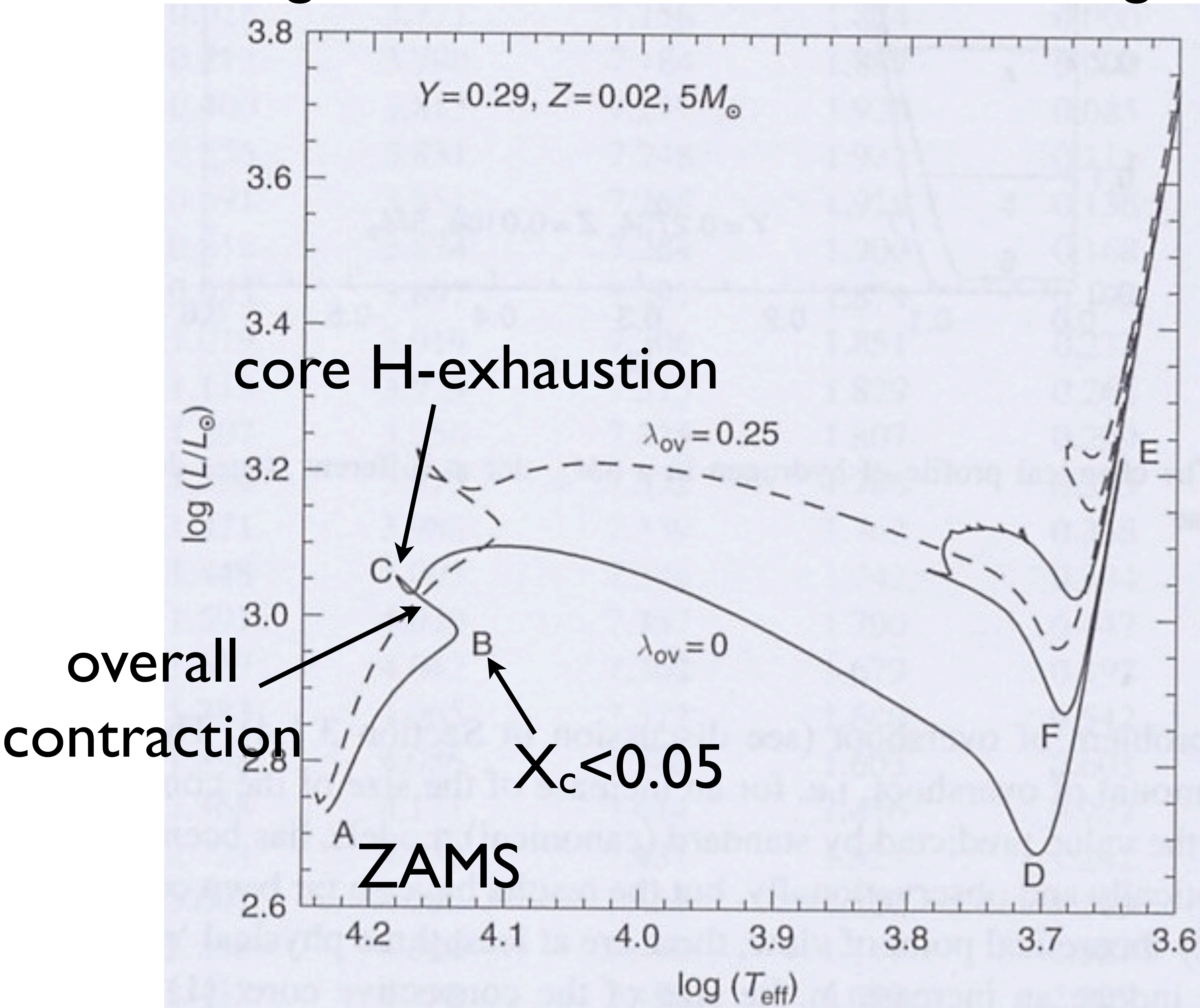

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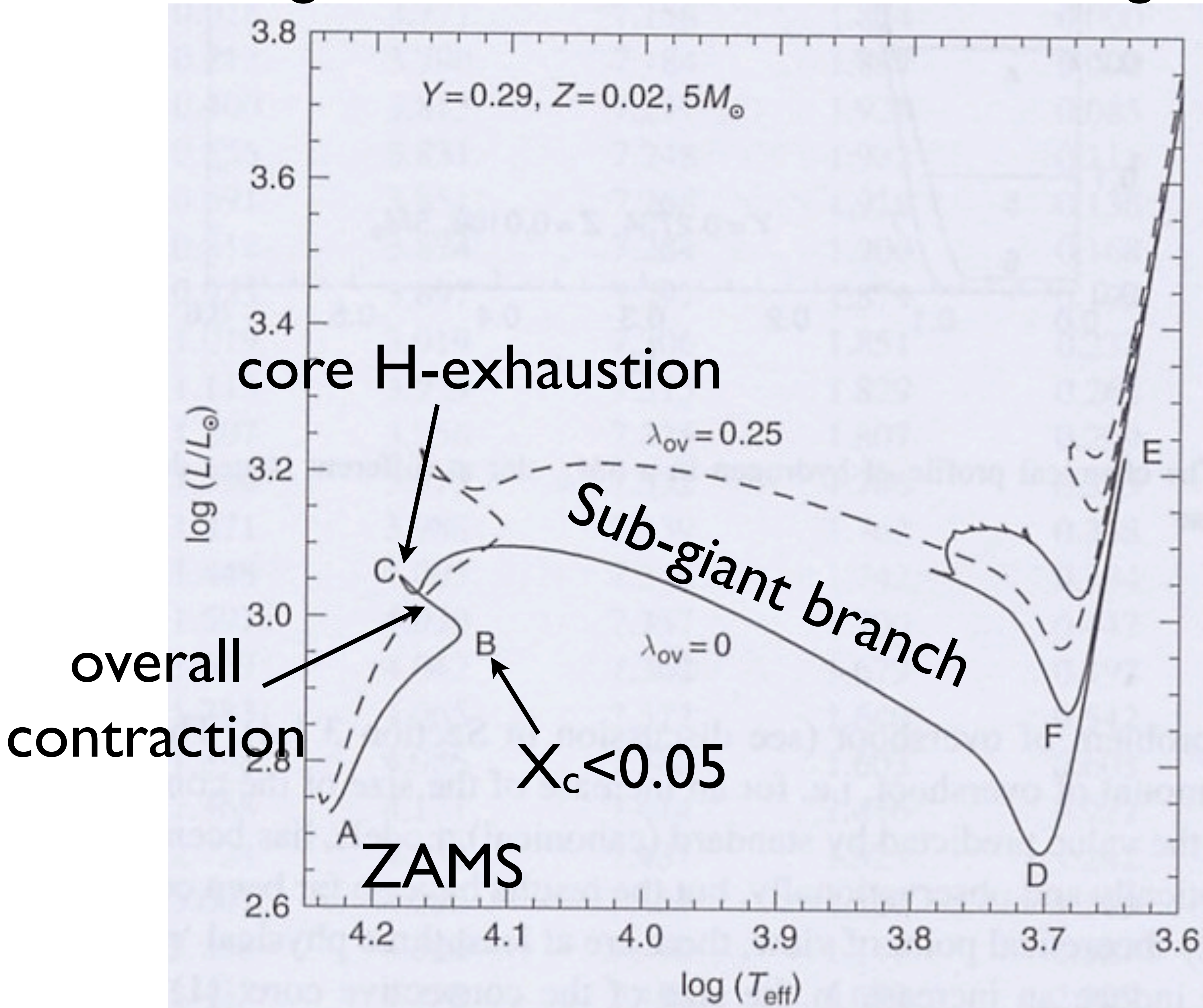

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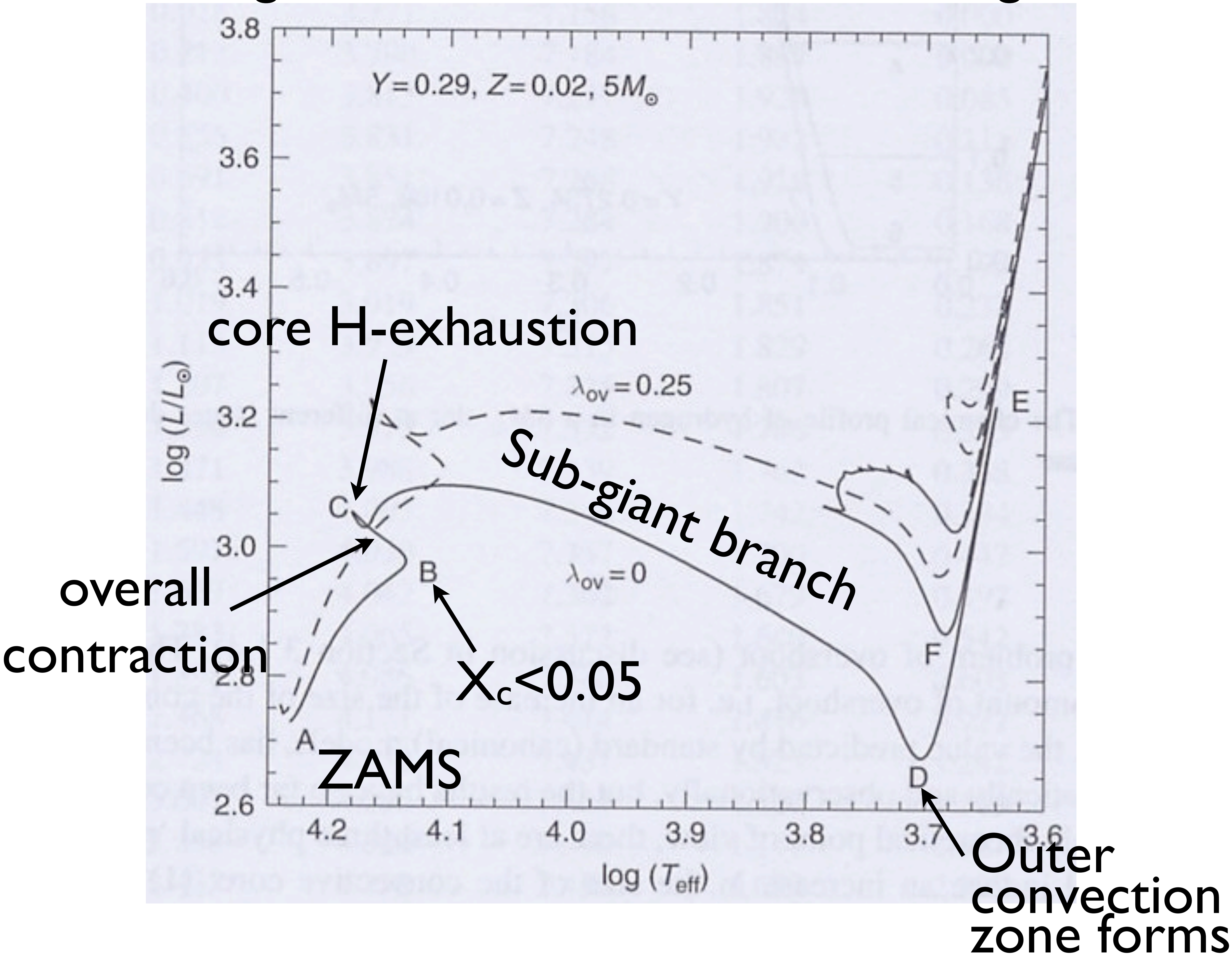
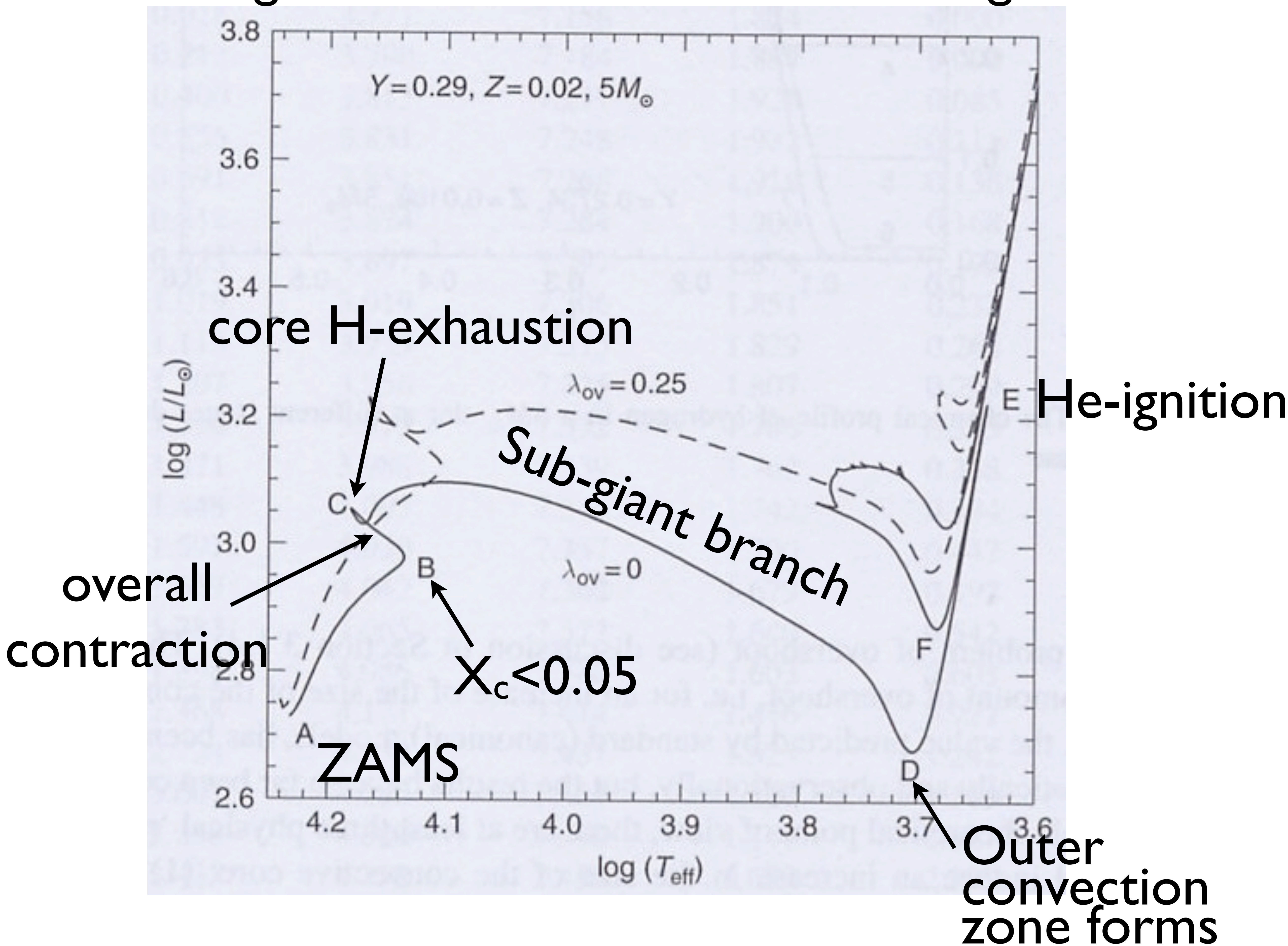
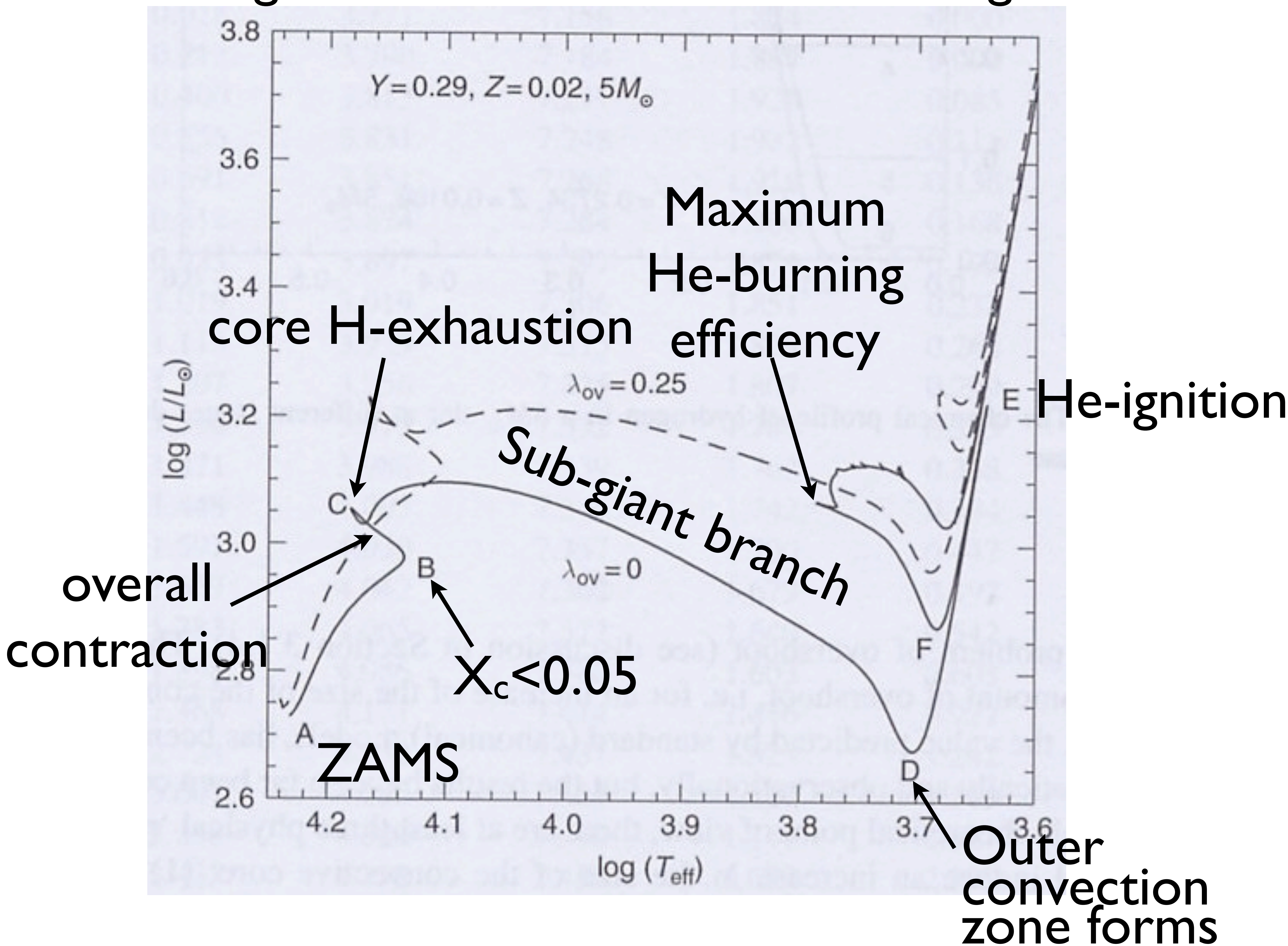


High-mass stars: H- and He-burning



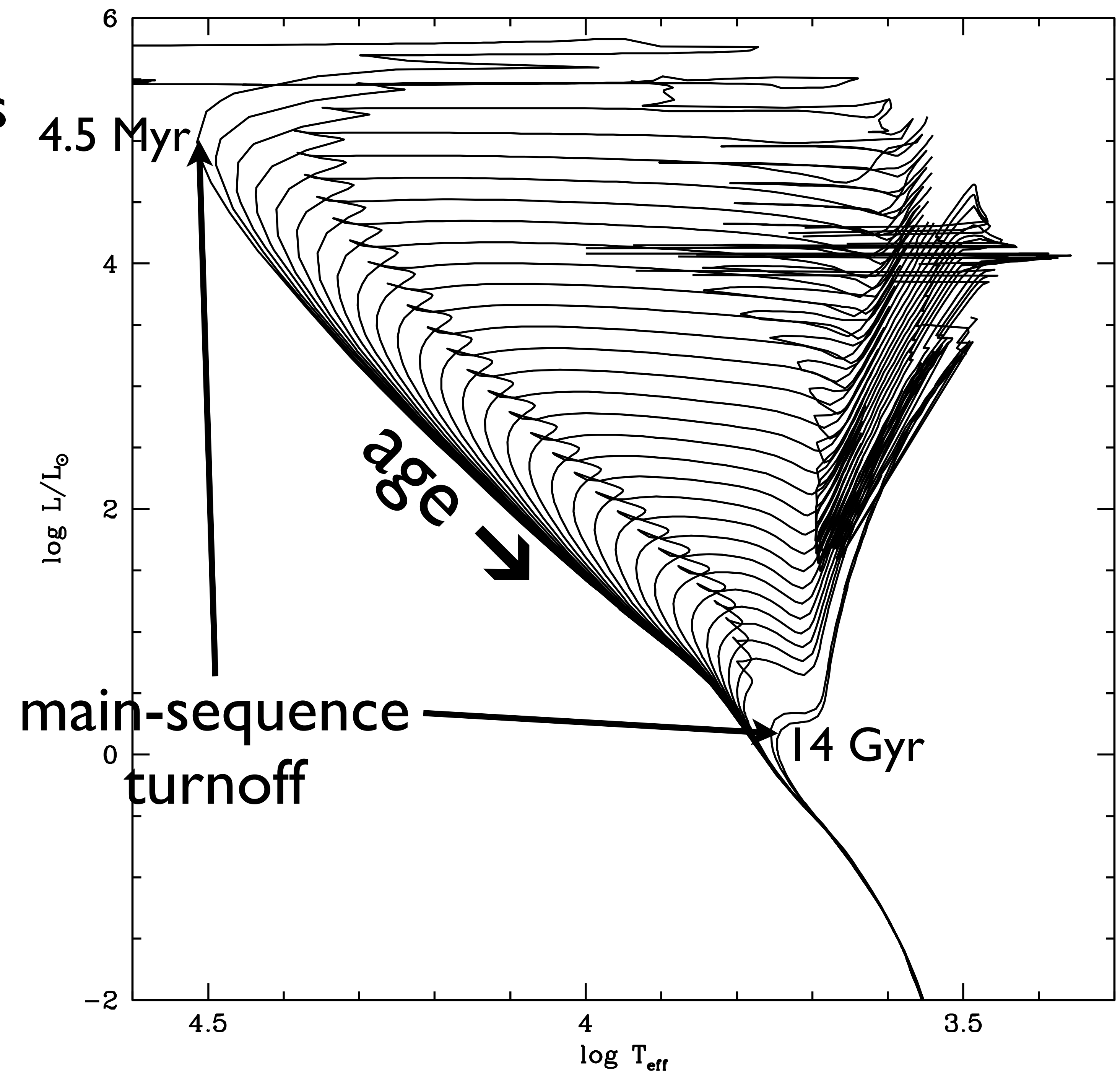
High-mass stars: H- and He-burning



- If we make *many* stellar evolution models of stars of different masses (but the same composition) and then plot all the living stars of the same age, we produce an *isochrone* ("same age")

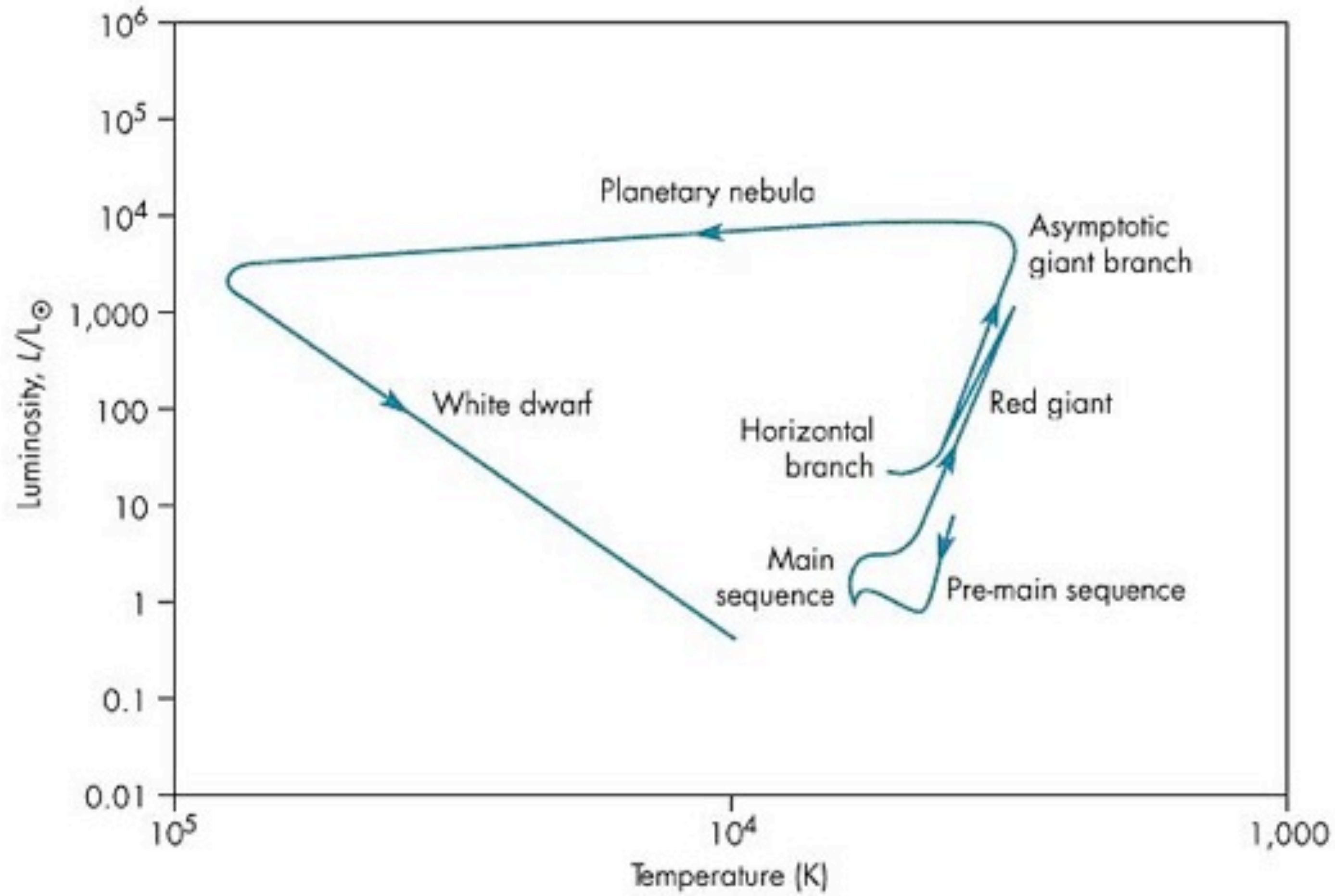
- It is clear that the older the isochrone, the *fainter and cooler* the *main-sequence turnoff* of the stars

Solar-metallicity isochrones from Marigo et al. 2008



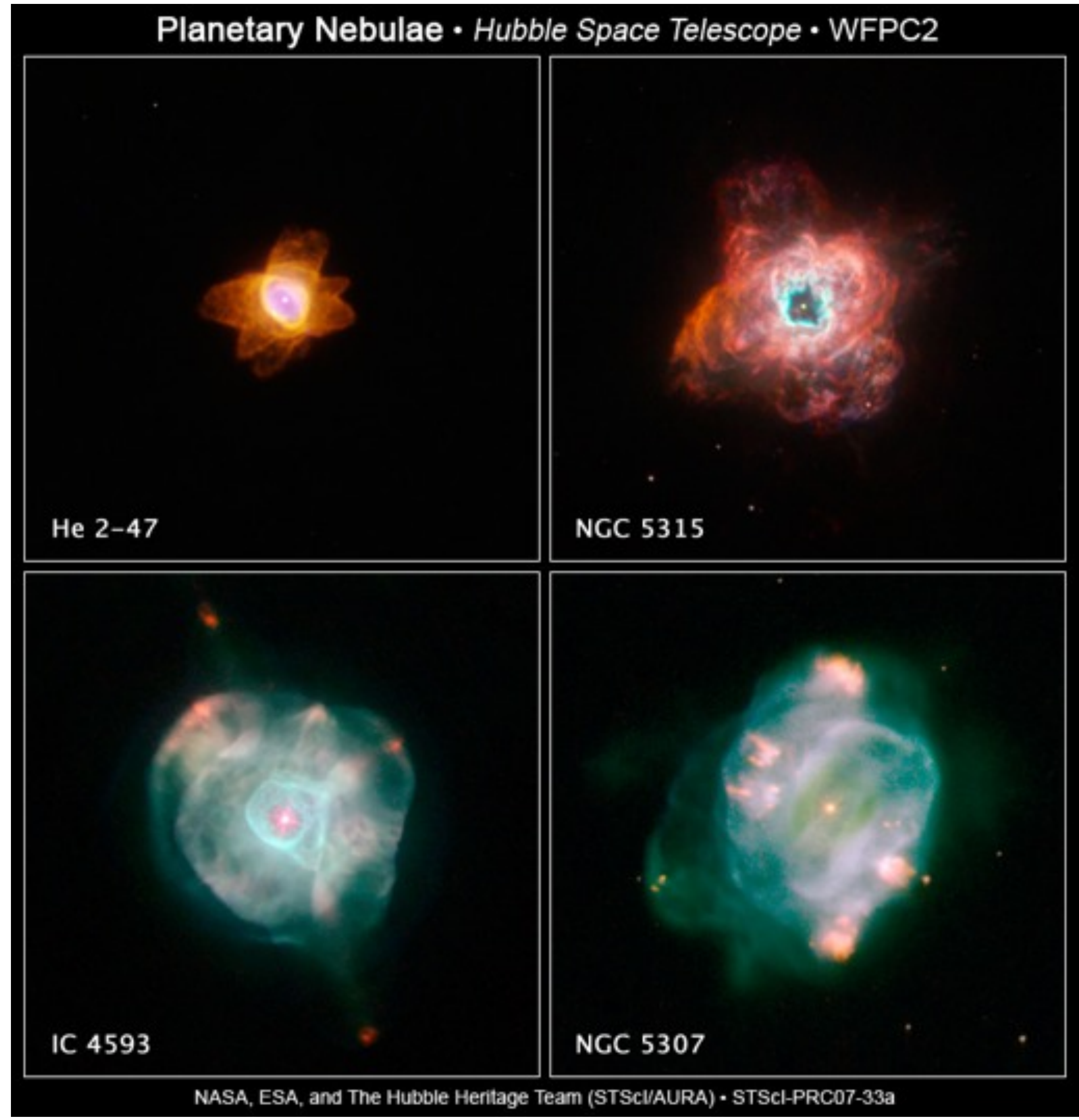
Stellar death: low-mass stars

- When a low-mass star finally exhausts its fuel, it's just a cooling cinder of carbon and oxygen: a **white dwarf**



- It first passes through the **planetary nebula** phase, which is just a white dwarf surrounded by expanding, cooling gas that used to be the star's envelope

- It then sheds its envelope completely and becomes a white dwarf



White dwarfs: quantum physics on the sky

- White dwarfs have no internal energy to heat them, so the gas pressure is not high enough to prevent gravitational collapse
  - this collapse would continue forever...
  - ...except that quantum mechanics steps in (again) to save us: the *Pauli exclusion principle* prevents complete collapse

- The Pauli exclusion principle states that
    - no two fermions --- like electrons or protons or neutrons --- can occupy the same quantum state
        - like the same orbit number, the same orbital angular momentum or the same spin
- If the matter is dense enough, the Pauli exclusion principle forces the electrons into many quantum states and gives rise to *degeneracy pressure*

- We can see this in the following rough calculation:

  - A particle with uncertainty in its position Δx has an uncertainty in its momentum Δp given by the Heisenberg uncertainty principle: $\Delta x \Delta p \geq h/2\pi$

  - so the uncertainty in its momentum is

$$\Delta p \geq \frac{h}{2\pi \Delta x}$$

- When the density is very high, the spacing between electrons is very small, so the momentum must be large

- The pressure of a gas of electrons with density n moving with velocity v against the wall of a box is $P = nvp$

- If we have n_e electrons per unit volume, we have **one** electron per $1/n_e$, and therefore the average spacing between electrons is

$$\Delta x = (1/n_e)^{1/3}$$

- The typical momentum of each electron must be at least that given by the uncertainty principle:

$$p = \frac{h}{2\pi\Delta x} = \frac{h}{2\pi} n_e^{1/3}$$

- and the velocity of each electron is then its momentum divided by its mass: $v = p/m_e$

- So the pressure of the degenerate gas is

$$P = \left(\frac{h}{2\pi}\right) \frac{n_e^{5/3}}{m_e}$$

note that this is about a factor of 2 too low compared to a more accurate calculation, and it only applies to non-relativistic electrons

- Converting this from electron density to mass density, we have

$$P \propto \rho^{5/3}$$

- This is the *equation of state for non-relativistic degenerate matter*

- Now we can integrate the equation of hydrostatic equilibrium, $\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho(r)$
- from the outside of the white dwarf to its centre, assuming (not quite correctly!) constant density to find $P_0 = \frac{2}{3}\pi G \rho^2 R_{\mathrm{WD}}^2$
- We then substitute our equation for degeneracy pressure to find $\rho^{5/3} \propto \rho^2 R_{\mathrm{WD}}^2$

- Now, since $\rho = M_{\rm WD} / \frac{4\pi}{3} R_{\rm WD}^3$
- We have that $\frac{M_{\rm WD}^{5/3}}{R_{\rm WD}^5} \propto \frac{M_{\rm WD}^2}{R_{\rm WD}^4}$
- or, collecting terms, $R_{\rm WD} \propto M_{\rm WD}^{1/3}$
- and thus $M_{\rm WD} V_{\rm WD} = \mathrm{constant}$
- Therefore, as white dwarfs get more massive, they get *smaller*!

- When the star gets so dense the electrons are moving at near the speed of light, the degeneracy pressure becomes $P \propto \rho^{4/3}$

- and therefore, following a similar derivation, the mass of this white dwarf is **constant!**

- The maximum mass it can attain is called the *Chandrasekhar mass*, and for a pure-helium white dwarf, M_{Ch}=1.44 $M_{\odot}$

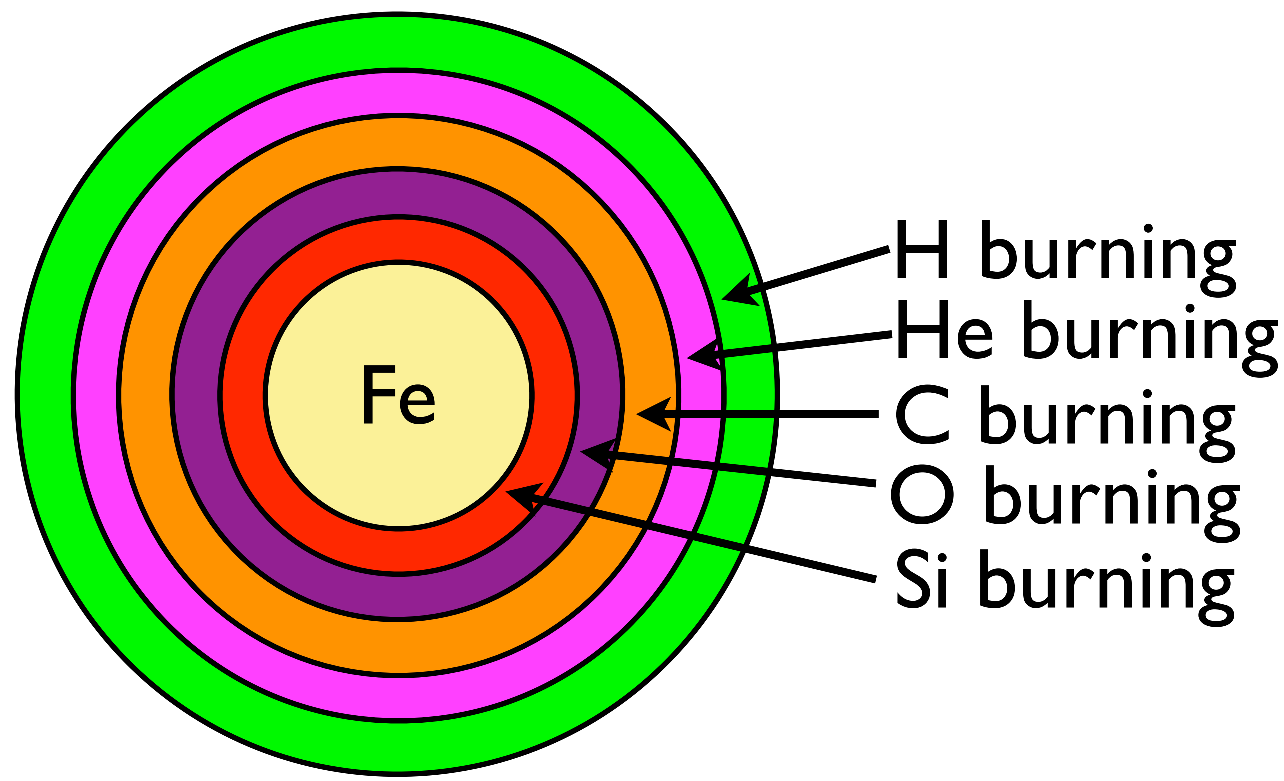
- Note that most *known* white dwarfs have masses <1 $M_\odot$, so the Chandrasekhar limit isn't violated for these objects

- There is a kind of supernova, the *type Ia supernova*, that is very likely the explosion of a white dwarf in a binary system that accretes enough material from its companion to come near or exceed the Chandrasekhar limit, releasing $\sim 10^{53}$ erg of energy and lots of elements in the periodic table around iron (the "iron-peak" elements)

Stellar death: high-mass stars

- The fate of the most massive stars depends on a number of parameters:
  - rotation
  - mass loss during evolution
  - details of convection

- Near the end of the life of any high-mass star, the core is an “onion skin” of burning layers surrounding an iron (Fe) core
    - iron has the highest binding energy and is thus the *most stable nucleus* in the periodic table: very hard to “burn”
    - every layer has exhausted fuel of layer above



- As the core gets hotter and hotter, *photodisintegration* breaks up nuclei, especially iron and helium:

$$
\begin{aligned}
{}^{56}_{26}\mathrm{Fe} + \gamma &\rightarrow 13\,{}^{4}_{2}\mathrm{He} + 4n \\
{}^{4}_{2}\mathrm{He} + \gamma &\rightarrow 2p + 2n
\end{aligned}
$$

- Photodisintegration is highly *endothermic* and removes thermal energy from the gas that could have been used to support the core

- At the same time, the protons suffer inverse β-decay and become neutrons:

$$p + e^- \rightarrow n + \nu$$

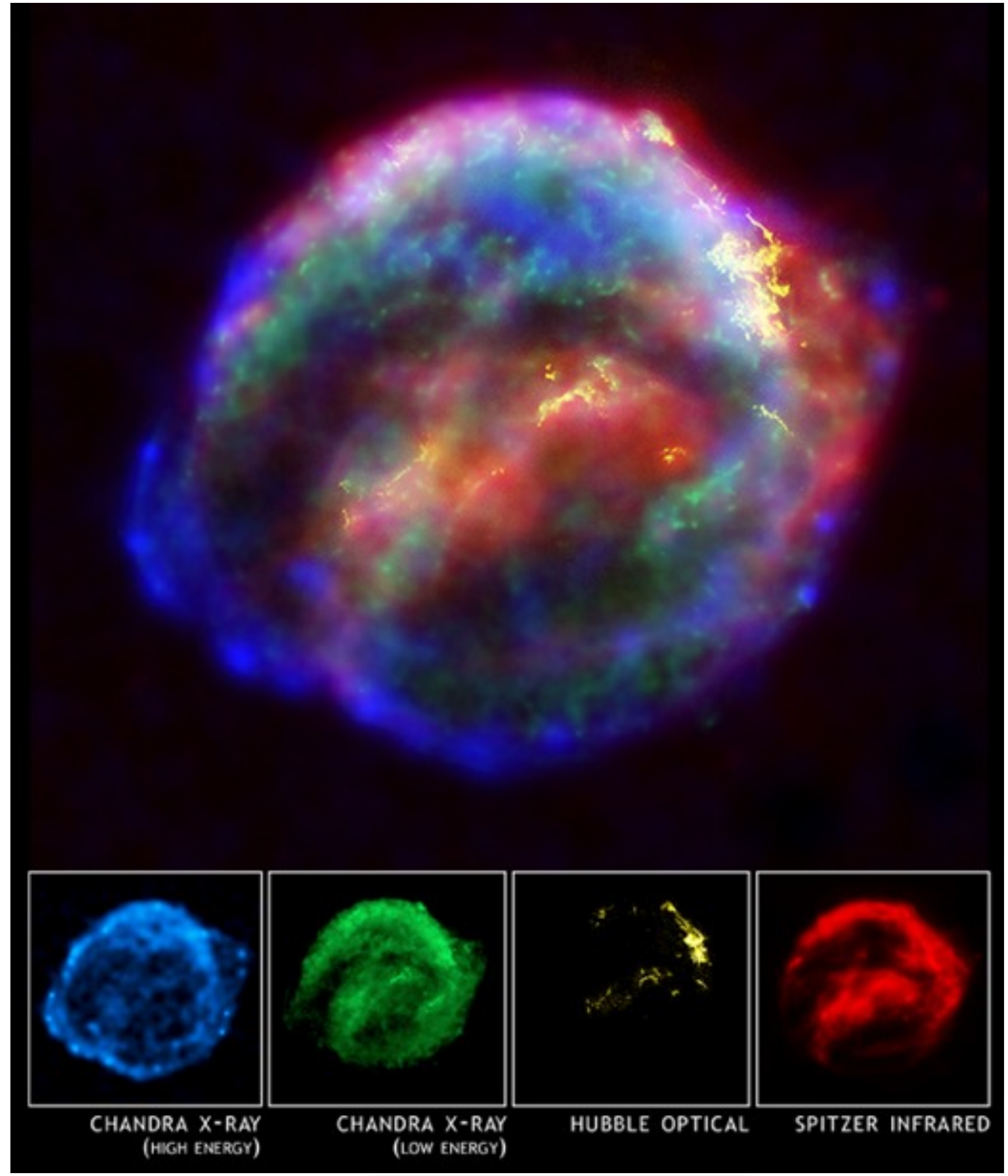
- The released neutrinos stream away from the core, carrying a large amount of energy and the *core collapses* extremely rapidly, while the envelope sits there as if nothing has happened...

- ...until the core reaches a density high enough to cause the neutrons to become degenerate...
- ...which causes a shock wave to propagate out through the envelope, which is ejected at high speed as a **supernova explosion**

- A supernova explosion can be *brighter than an entire galaxy*: a *core-collapse* supernova explosion releases 10^{53} erg of energy!
- During the supernova explosion, temperatures and pressures in the expanding envelope become so high that many elements *much heavier* than iron can be formed

- These *supernova remnants* spread the newly-formed elements into the *interstellar medium*, the space between the stars in a galaxy (which we'll discuss soon)
- The strong magnetic fields in supernova remnants cause high-energy electrons to travel on *helical* (spiral) paths, therefore accelerating and giving off *synchrotron radiation*, which is how we see these remnants

Kepler's Supernova Remnant



Kepler's SNR: all these images are of synchrotron radiation!

- If the mass of the core is not too high, the core remains as a *neutron star*, like a white dwarf but with neutrons providing the degeneracy pressure...
- ...or else, for higher masses, the core collapses into a *black hole*, releasing even more energy

Neutron stars

- The leftover core of the massive star (if it's not too massive) will become a *neutron star*, a star made up *only* of degenerate neutrons
- Following our discussion of white dwarfs, these (non-relativistic) neutrons have a degeneracy pressure of $P \propto n_n^{5/3}/m_n$
- But the star is *all* neutrons, so $\rho = n_n m_n$
- And therefore $P \propto \rho^{5/3}/m_n^{8/3}$

- Therefore there is a mass-radius (or mass-volume) relation like for white dwarfs
- For a 1 $M_\odot$ neutron star, the radius is ~15 km!

I is the rotational inertia

- How fast might this neutron star be rotating?

- The angular momentum of an object is $J = I\omega$
- For a uniform sphere, this is $J = \frac{2}{5} M R^2 \omega$
- If the NS once had the angular momentum of the Sun, then $\omega/\omega_\odot = (R_\odot/R)^2 = 2 \times 10^9$

- The Sun rotates once every 30 days, so the period of the NS is then
$P = (2.6 \times 10^6\,\mathrm{s})/(2 \times 10^9) = 1.3 \times 10^{-3}\mathrm{s}$

- That's more than 1000 times per second!

- We can perform a similar calculation, conserving *magnetic flux*: then BR^2=constant, where B is the magnetic field

  - If we conserve the Sun's field, the magnetic field of a neutron star could be (in excess of!)

$$
\begin{aligned}
B_{\rm NS} &\sim B_\odot (R_{\rm NS}/R_\odot)^2 = 2\,{\rm G} \times 2 \times 10^9 \\
&= 4 \times 10^9\,{\rm G}
\end{aligned}
$$

note that typical NS magnetic fields have B~10^12!

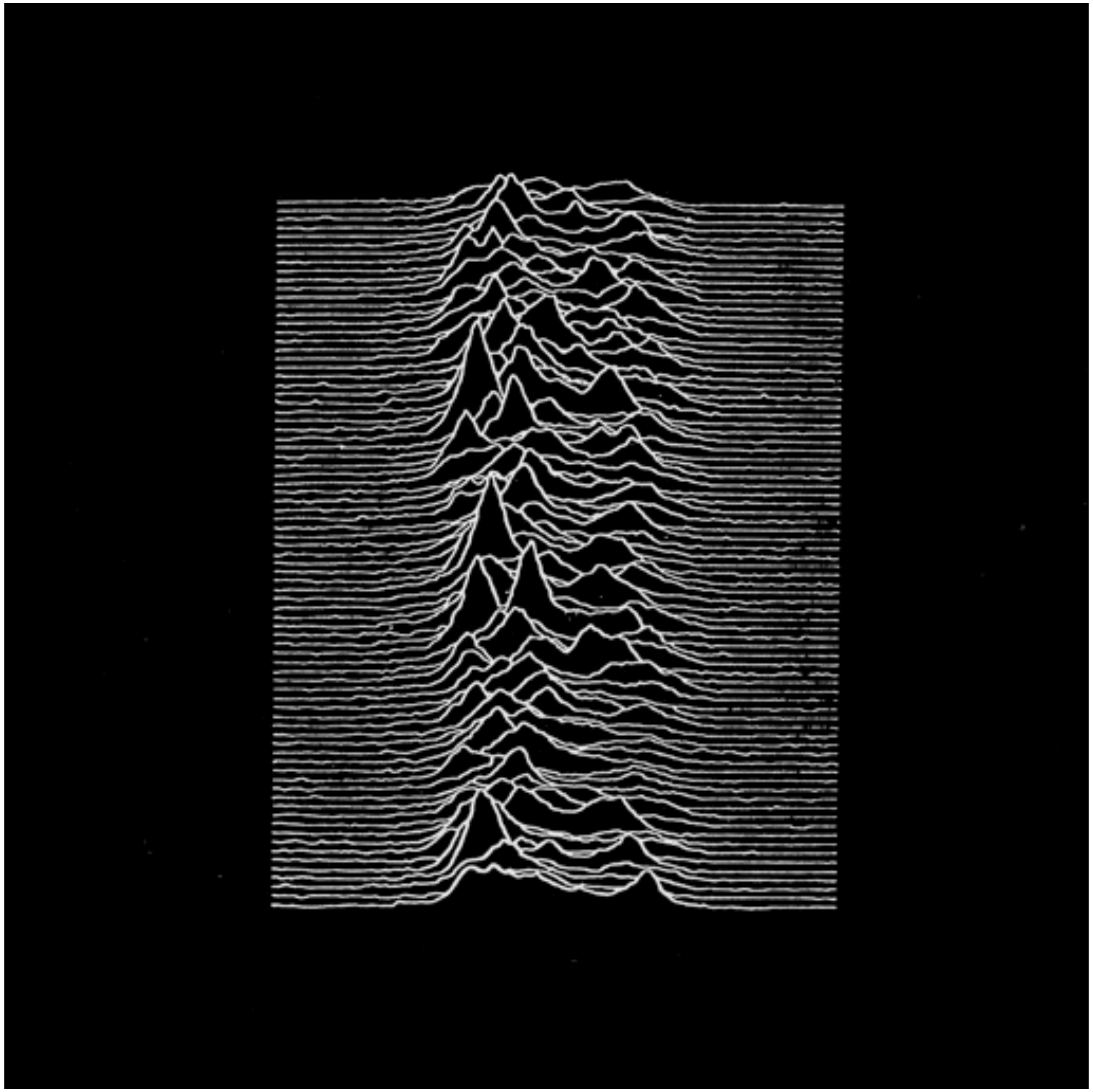
Pulsars

there are only 7 or 8 known "optical counterparts" of isolated neutron stars

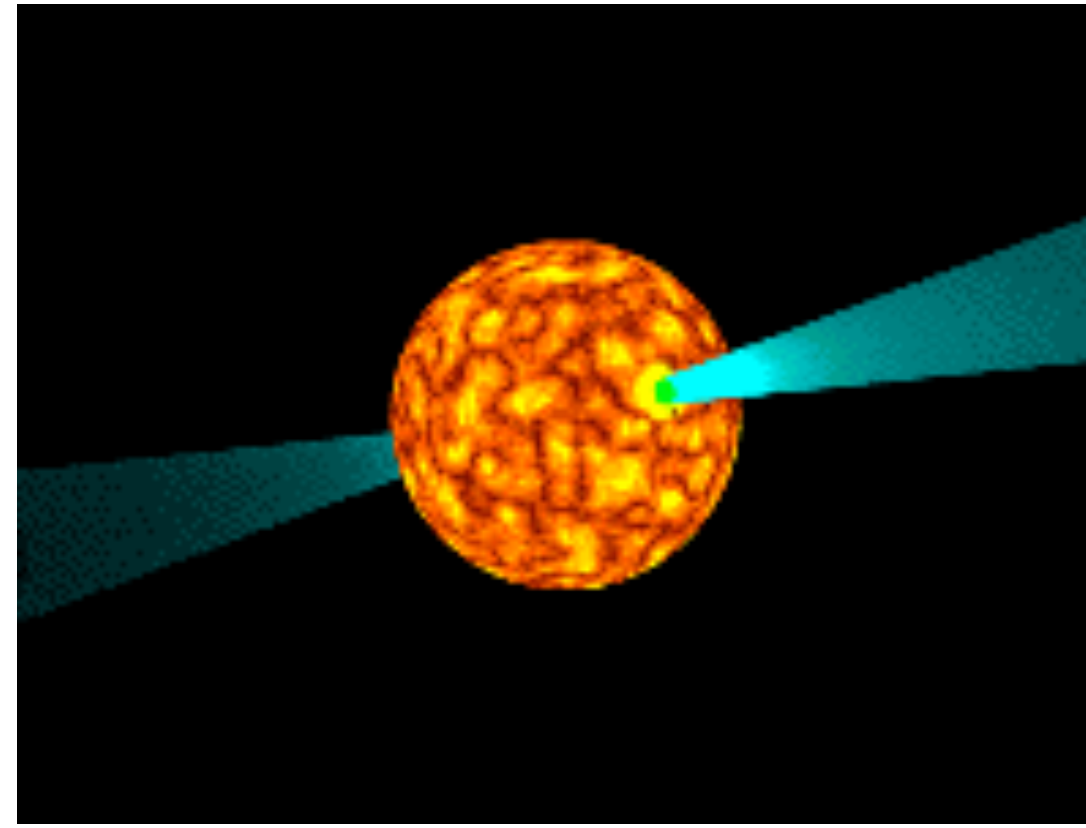
- Now why did we just go through all of this?
- Because we (usually) *cannot* see neutron stars in the optical region of the spectrum!
- Instead, we detect these stars either through their effects on a companion star, when they are part of a binary system, or as *pulsars*

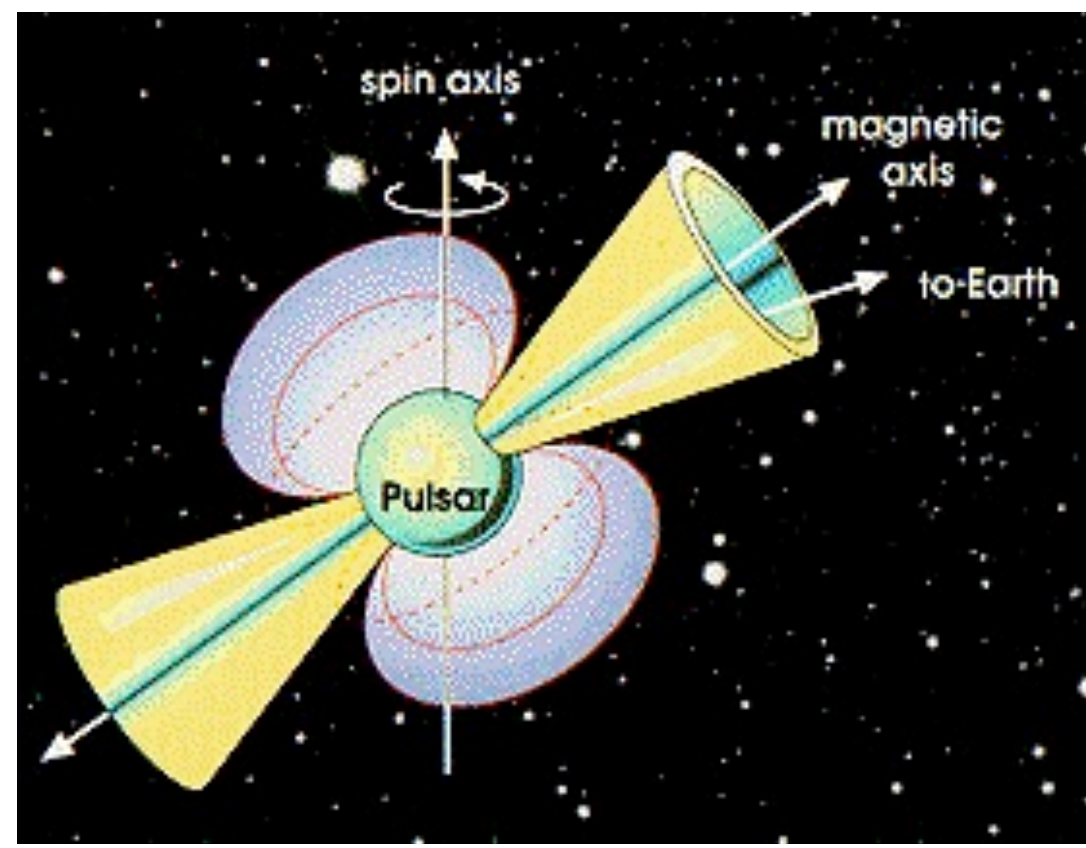
- In 1967, Jocelyn Bell Burnell and her PhD supervisor, Antony Hewish, were looking for the *scintillation* of radio sources caused by charged particles in the interstellar medium

- Instead, they found the first *pulsar*, a regularly-pulsating radio signal with a period of 1.3373011 seconds



- After much thinking, it was determined that the only sensible mechanism for powering these pulsars was a magnetised, spinning neutron star
- If the magnetic field is *not* aligned with the rotation axis of the NS, then charged particles trapped along the field lines emit synchrotron radiation that acts like a "lighthouse": a beam of radiation that can be seen if we look (nearly) along the magnetic axis



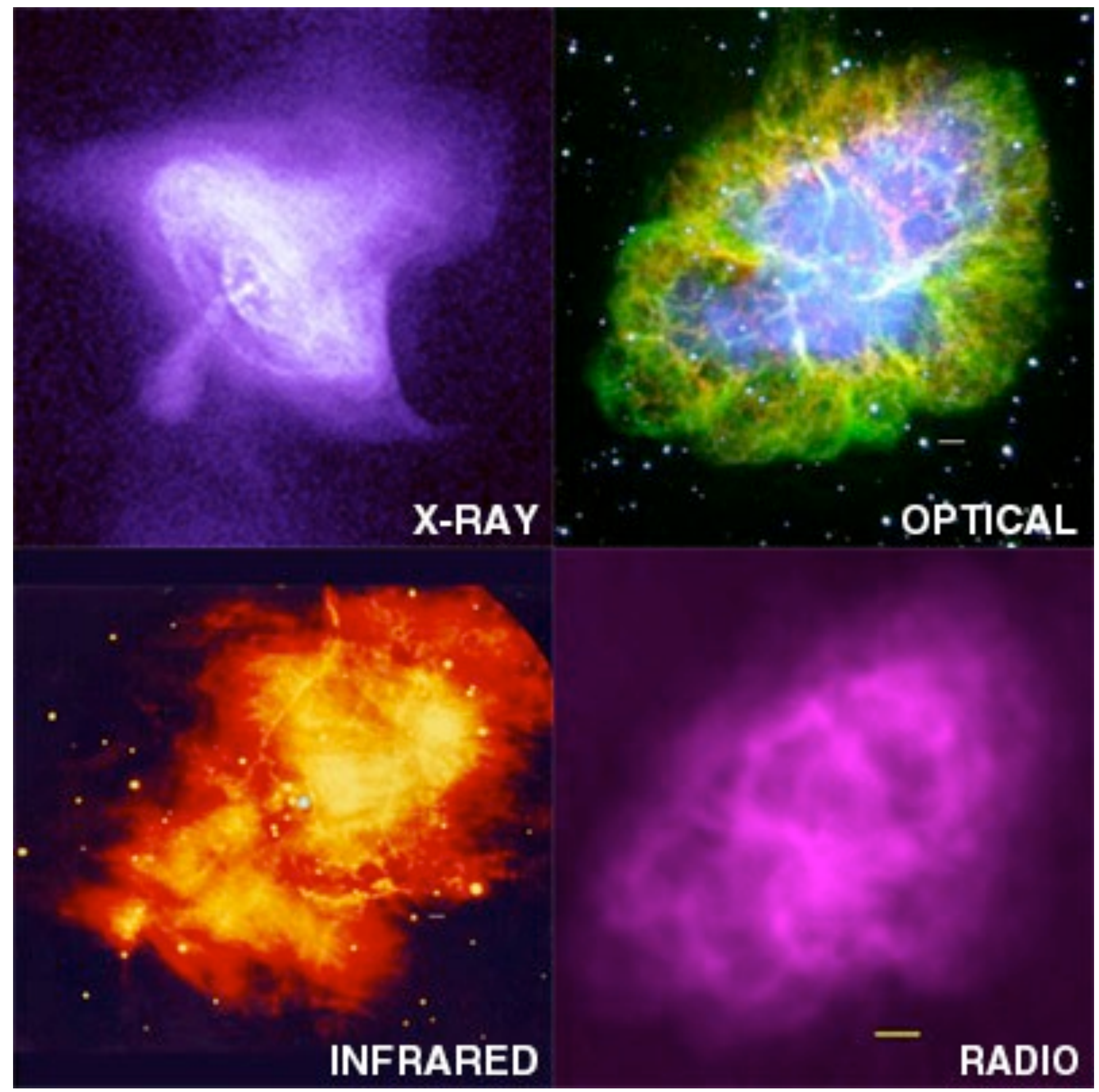


- Because of this “lighthouse” effect, we only see ~20% of all the pulsars on the sky
- Since we know >1000 pulsars already, there should be at least 5 times as many that we haven’t seen

- As pulsars emit radiation, they must be losing energy
- This means that they must be *slowing down*, so the periods get *longer*
- Therefore, *the fastest pulsars are the youngest*

- As an example of this process, the rate at which the Crab Nebula is losing energy via synchrotron radiation is *equal to the rate at which the Crab pulsar is losing energy due to its pulses*

- Supernova remnants are generally powered by pulsars!



The Crab Nebula

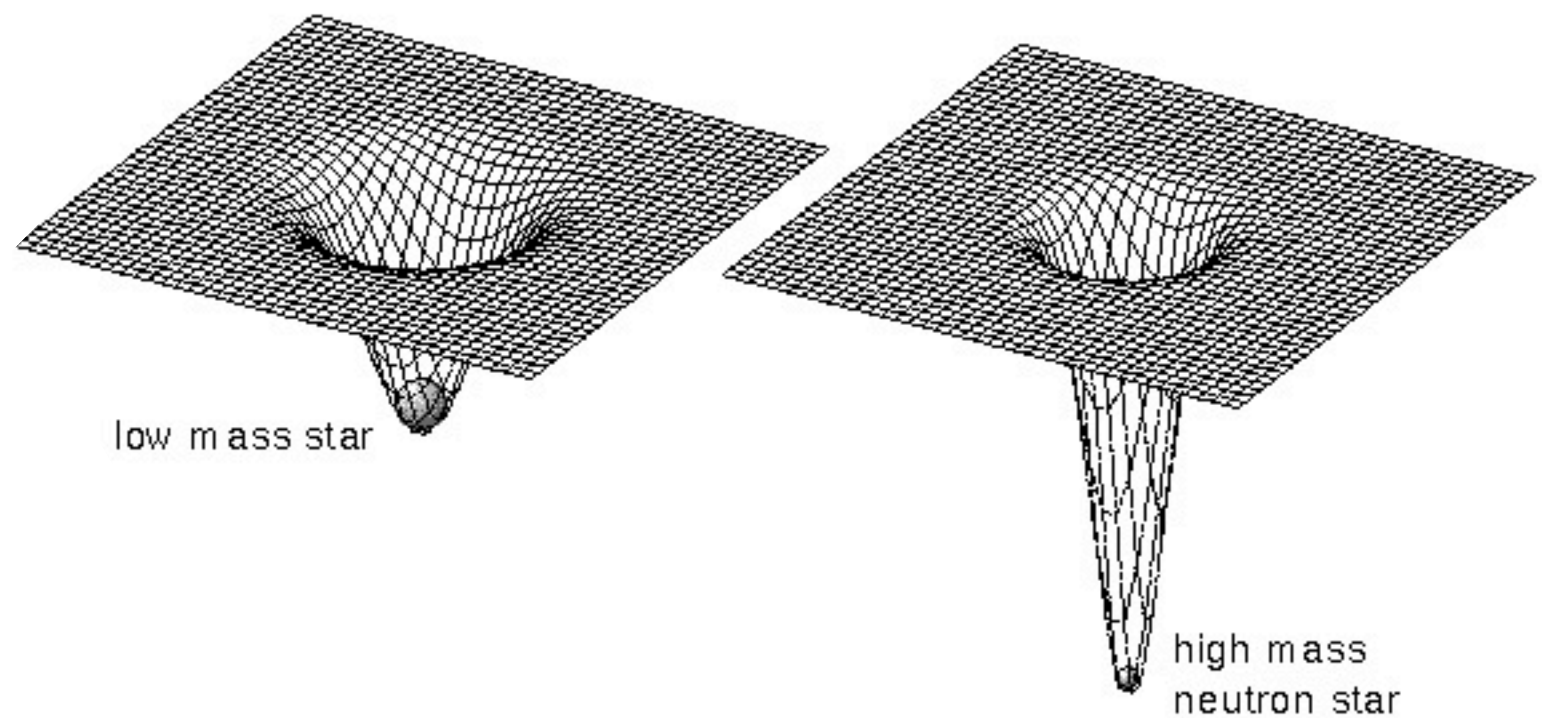
See Kutner, Chapter 8

Black holes

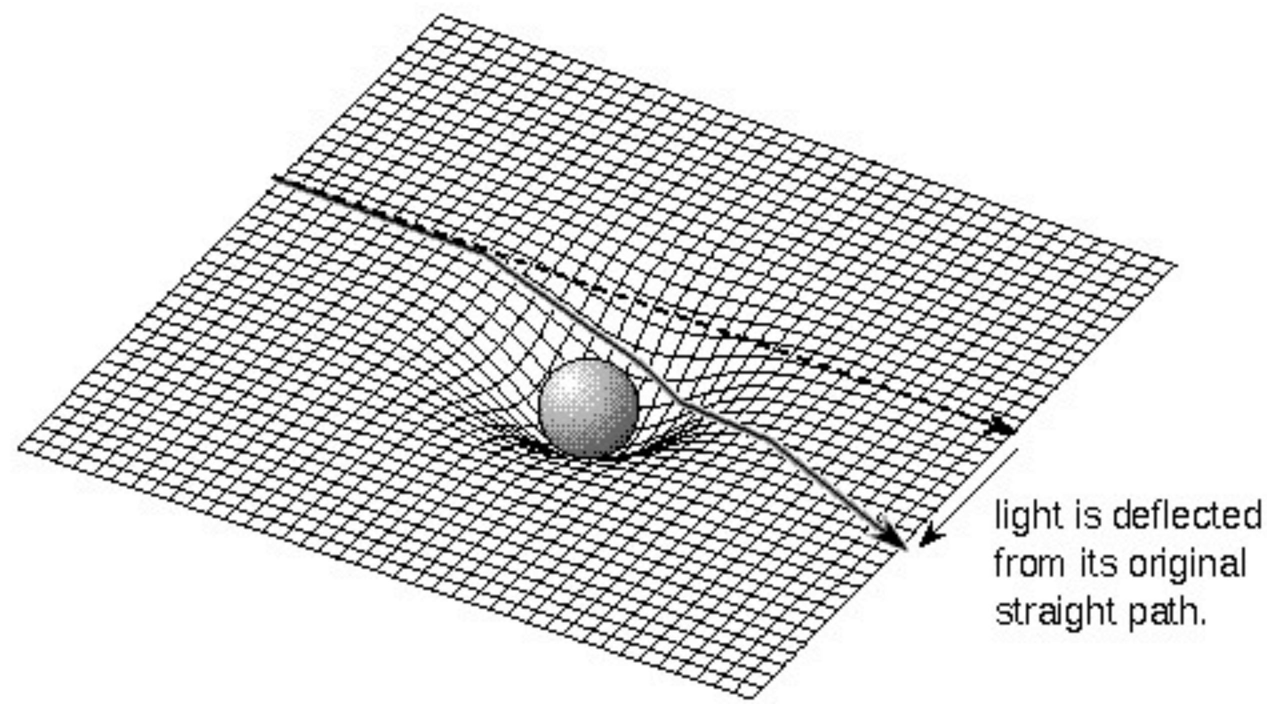
- Just as there is an upper limit to the mass of a white dwarf, there is an upper limit to the mass of a neutron star: 3-8 $M_\odot$
  - This limit is very uncertain because the equation of state of neutron matter is not well-understood
- If the mass of the neutron star exceeds this limit, we know of no other pressure source to stop its collapse

- To understand the basic idea behind a black hole, we need to quickly review *general relativity*
- General relativity begins with the *equivalence principle*: a uniform gravitational field in one direction is *indistinguishable* from a constant acceleration in the opposite direction
    - This can be thought of also as "gravitational and inertial mass are the same for any object"

- The implication of the equivalence principle allowed Einstein to write down an equation that describes gravity as the bending of spacetime by mass



- Now, because photons (light) follows the *shortest path* (the "geodesic") *in spacetime*, mass causes light to *bend*



- A clock traveling on this photon would slow down as the space becomes more curved

- So the photon actually changes it frequency to an outside observed: it slows down and therefore becomes *redder*

- This is known as *gravitational redshift* (and it has been tested many times!)

- Let's consider a photon emitted from an object of mass M, first at a distance r_1 and second at a distance r_2

- Conservation of energy in general relativity then says that the ratio of wavelengths received at these two distances is

$$\frac{\lambda_2}{\lambda_1} = \left[\frac{1 - \frac{2GM}{r_2 c^2}}{1 - \frac{2GM}{r_1 c^2}} \right]^{1/2}$$

- Now we see something funny here!
- If $r_1 < \frac{2GM}{c^2} = R_S$, then λ_2 is **infinite**, *no matter what r_2 is!*
- This defines the *Schwarzschild radius*: no light (i.e., no information) can escape from any object with radius $R<R_S$ -- there is a singularity inside that radius
- We call such an object a **black hole**

- We cannot detect isolated black holes directly, because they give off no radiation
- Rather, we detect them by their effects on matter around them
  - For stellar-mass black holes in binary systems, we detect them via their gravitational effect on their companion
  - For black holes of all masses, we can also detect the radiation from their *accretion disks*, matter that has been ripped from nearby stars and radiates due to liberated gravitational potential energy
- We will discuss *supermassive black holes* later, when we talk about *Active Galactic Nuclei*