Star Clusters

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deel 2

Types of star clusters

Open or Galactic Clusters

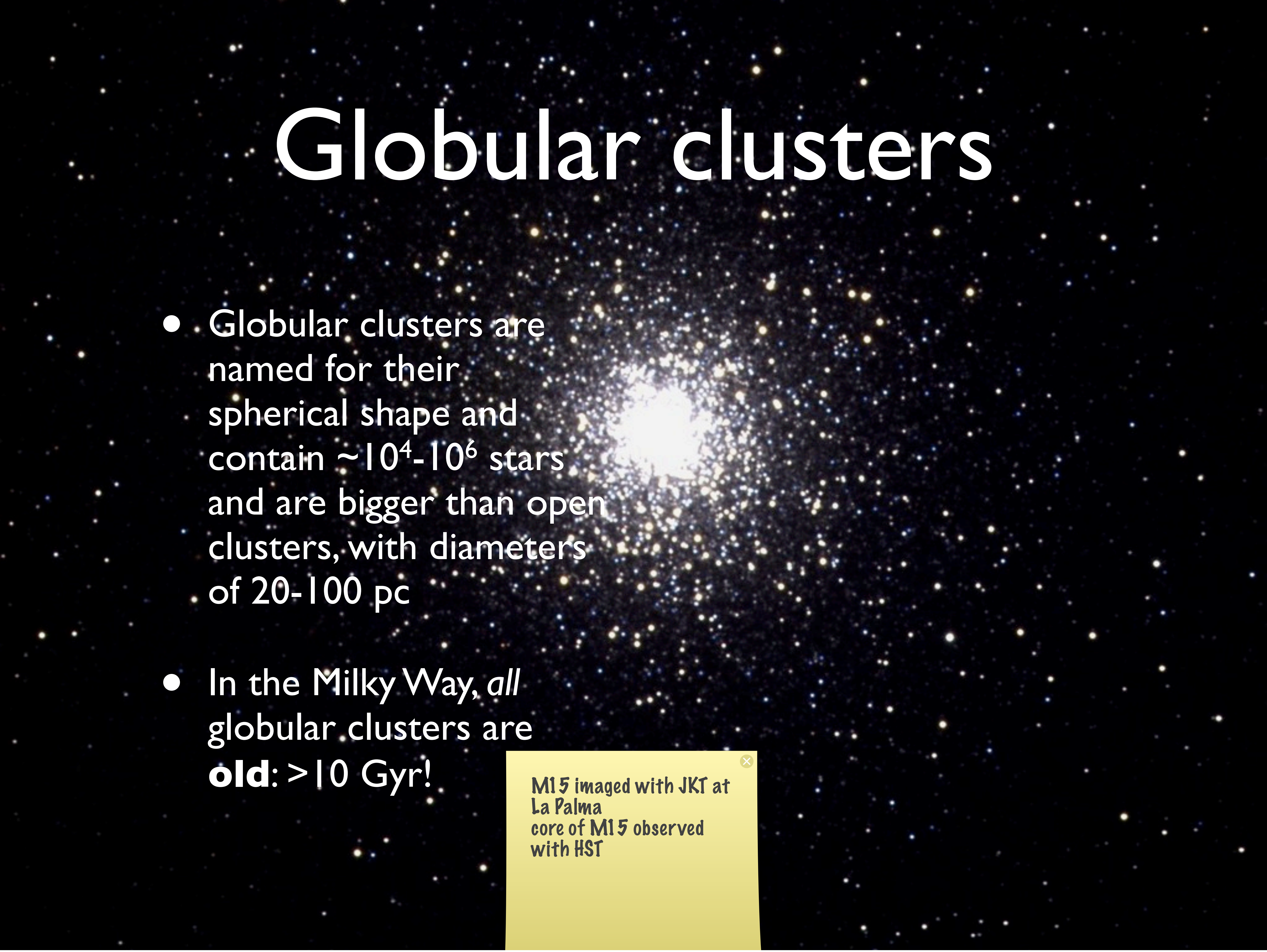
- "Open" or Galactic clusters are low mass, relatively small (~10 pc diameter) clusters of stars in the Galactic disk containing $<10^3$ stars
  - The Pleiades cluster is a good example of an open cluster
    - the "fuzziness" is starlight reflected from insterstellar dust

- Because open clusters live in the disk of our Milky Way, they are subject to strong tides and shearing motions (which we'll discuss later)

- Because they are so small and contain few stars, they also *evaporate* quickly

- Therefore they do not live very long unless they are very massive --- so most of them are quite *young*, as we'll soon see

Globular clusters



- Globular clusters are named for their spherical shape and contain ~10^4-10^6 stars and are bigger than open clusters, with diameters of 20-100 pc

- In the Milky Way, *all* globular clusters are **old**: >10 Gyr!

M15 imaged with JKT at La Palma
core of M15 observed with HST

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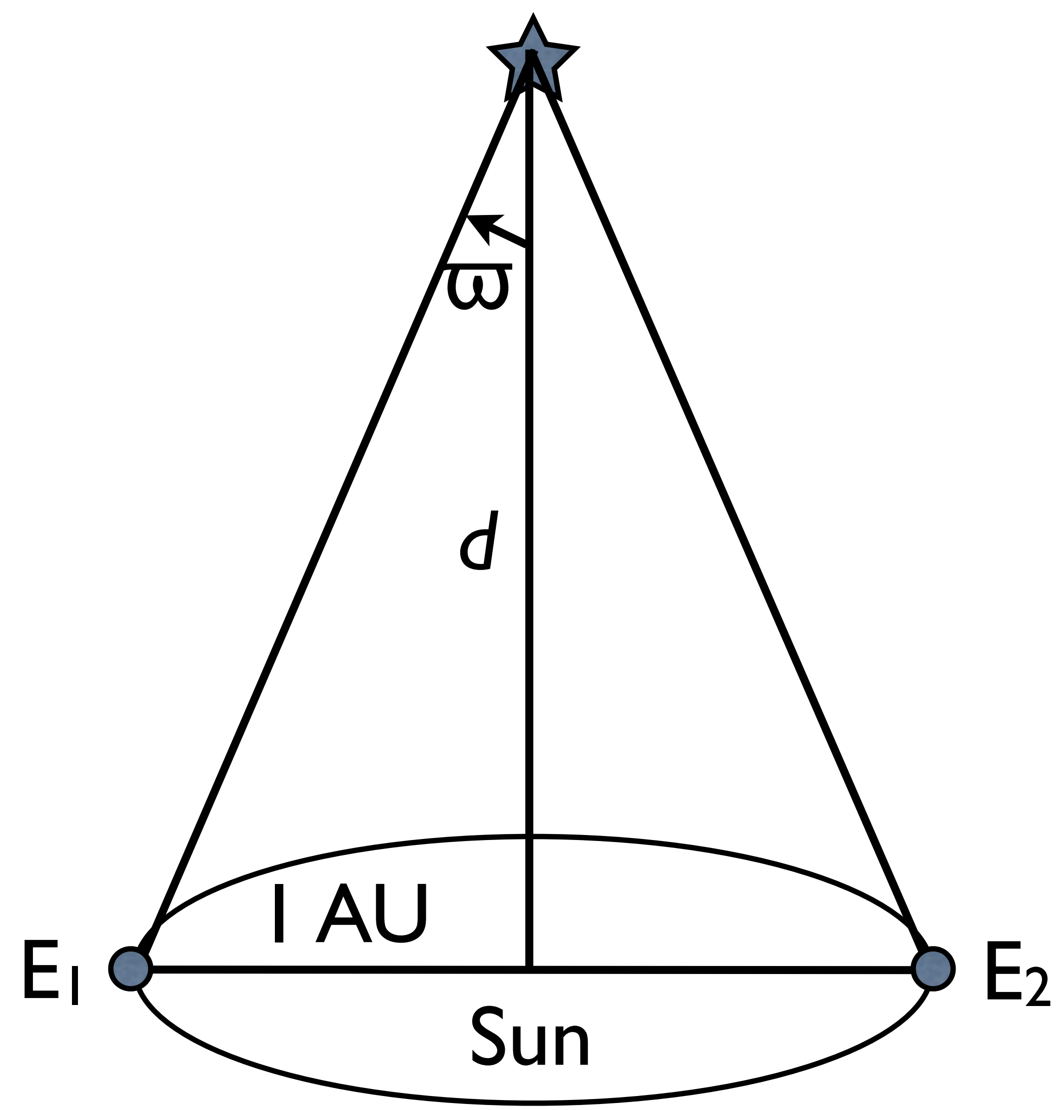
Distances to clusters

- First, a quick review of distances in astronomy...
- The fundamental distance measure in astronomy is based on *parallax*, the change in angle a (stationary) star makes when seen from opposite sides of the Earth's orbit around the Sun

- Consider the Earth, 1 AU away from the Sun at position E_1. Six months later, the Earth is at position E_2, but the star has remained in the same place *relative to the Sun.* Then, as seen from Earth, the star *appears* to have subtended an angle 2ϖ on the sky.



- Then if *r* (=1 AU) is the radius of the Earth's orbit, we find $\frac{r}{d} = \tan\varpi \approx \varpi\,\mathrm{rad}$
  - because ϖ is clearly small; converting to seconds of arc, $\varpi'' = 206265\varpi\,\mathrm{rad}$
- *Defining* 1 AU such that $d = \frac{206265}{\varpi''}\,\mathrm{AU}$
- and 1 **parsec** as the distance at which a star would have a parallax of 1″:

$$1\,\mathrm{pc} = 2062065\,\mathrm{AU} = 3.086 \times 10^{13}\,\mathrm{km} = 3.26\,\mathrm{light\ \ years}$$

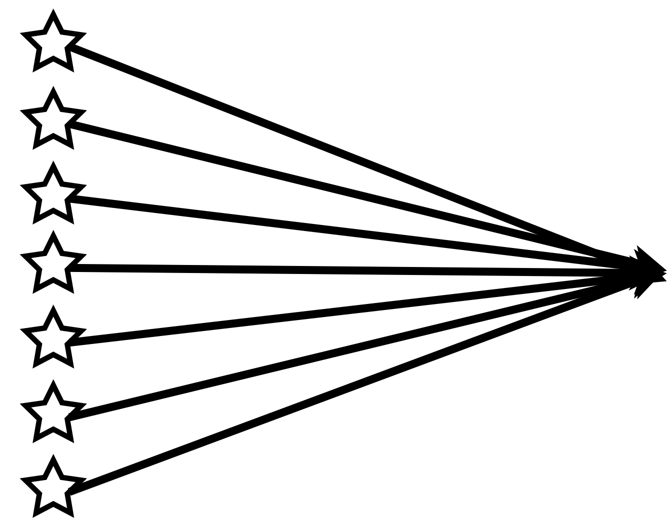
- The distance to a star with observed parallax ϖ″ is then $d = \frac{1}{\varpi''}\,\mathrm{pc}$

- Now, if we have an object with some flux *F* at some distance *D*, then the inverse-square law for flux tells us that the flux *f* we receive at some other distance *d* is $$f = \left(\frac{D}{d}\right)^2 F$$
- Therefore, if we pick a *standard distance D* to refer to all objects, we can define an **absolute magnitude** *M* to be the magnitude object of *apparent magnitude m* would have at that distance:
$$m - M = -2.5 \log(f/F) = 5 \log(d/D)$$
- The standard distance *D* is always taken to be 10 pc, which gives us the *distance modulus*:
$$m - M = 5 \log d - 5$$

The “moving-cluster” method

- Now let’s imagine a cluster receding from the Sun
  - It first occupies a large area on the sky and then slowly shrinks to a *convergent point*
- If the cluster has a constant *physical* diameter *d* and is at a distance *D*, then its *angular diameter* on the sky is

$$\theta = \frac{d}{D}$$



- Let's take the logarithm of both sides:

$$\ln \theta = \ln d - \ln D$$

- Now let's take the derivative with respect to time: $$\frac{d \ln \theta}{dt} = -\frac{d \ln D}{dt}$$

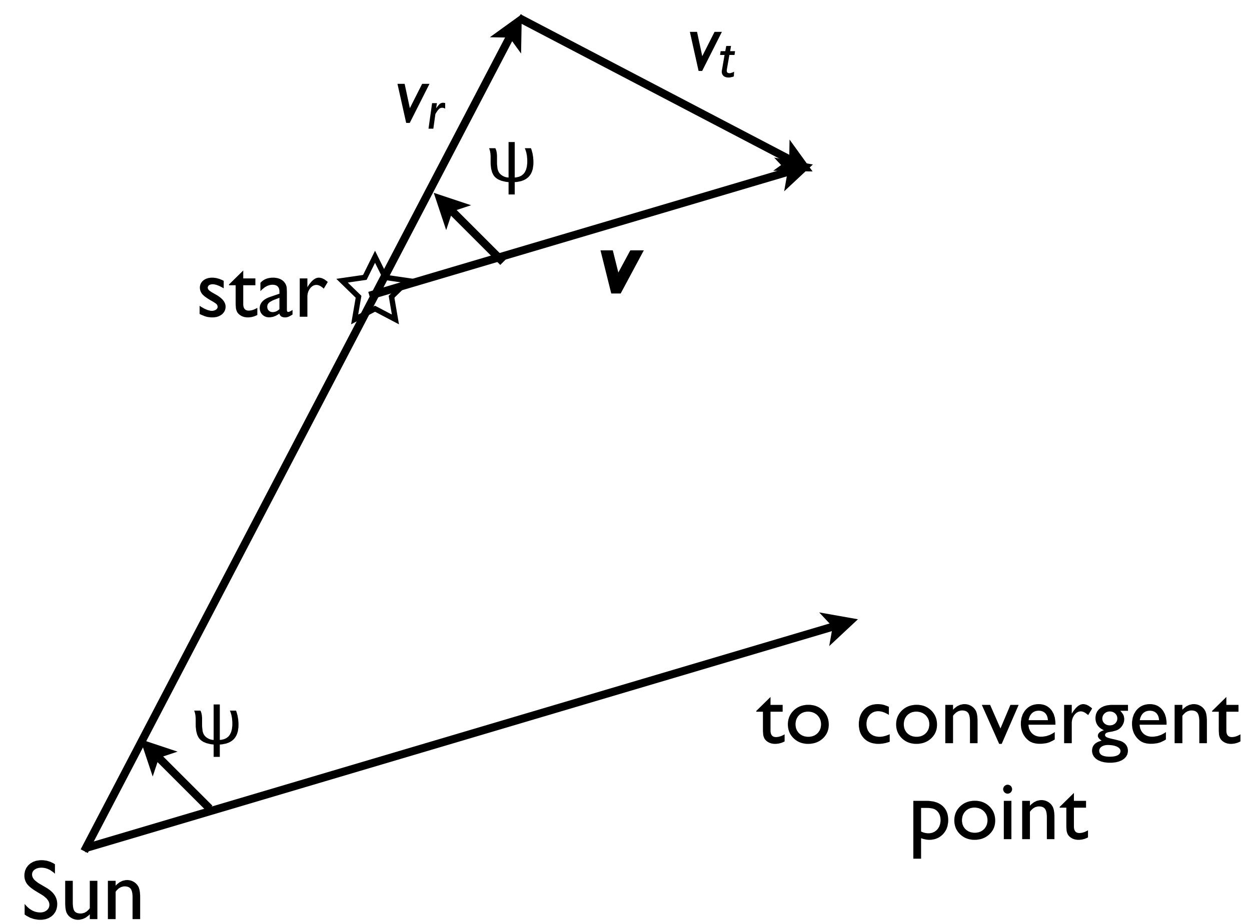
- Rearranging and letting $v_r = dD/dt$ and $\dot{\theta} = d\theta/dt$, we have $$D = -\frac{\theta}{\dot{\theta}} v_r$$

- So if we can measure the *radial velocity* v_r and the rate $-\dot{\theta}/\theta$ at which the cluster appears to be shrinking, we can measure the distance *D* to the cluster

- We can measure the direction to the convergent point by measuring the **proper motions** of the cluster stars: the *transverse* motion of the stars across (i.e., on the plane of) the sky over some amount of time

- Once we know this direction, we can use this to determine the distance to the cluster stars

- The vector ***v*** is in the direction of the convergent point
- Then the star's radial velocity is $v_r = v \cos \psi$
- and its transverse velocity is $v_t = v \sin \psi$
- But $v_t = \mu d$, where μ is the star's proper motion and *d* is its distance
- So $\mu = \dfrac{v_r \tan \psi}{d}$
- and then $d = \dfrac{v_r \tan \psi}{\mu}$



Cluster	Distance (pc)
Hyades	45
Ursa Major group	24
Pleiades	115
Scorpio-Centaurus	170

The Virial Theorem

- A key concept in astronomy, from stars to star clusters to galaxies to galaxy clusters to the formation of galaxies, is the concept of *virialization*
  - gravitationally-bound systems in equilibrium obey the remarkable property that their total energy is *always* one-half of their (time-averaged) potential energy

- First, let's consider, for some system of *N* particles, the quantity $Q = \sum_i \mathbf{p}_i \cdot \mathbf{r}_i$
- where $\mathbf{p}_i$ and $\mathbf{r}_i$ are the linear momentum and position vectors of some particle i, and the sum is taken over all *N* particles

- The time derivative of *Q* is

$$\frac{dQ}{dt} = \sum_i \left(\frac{d\mathbf{p}_i}{dt} \cdot \mathbf{r}_i + \mathbf{p}_i \cdot \frac{d\mathbf{r}_i}{dt} \right) \qquad (1)$$

- and the left-hand side of this expression is

$$\frac{dQ}{dt} = \frac{d}{dt} \sum_i m_i \frac{d\mathbf{r}_i}{dt} \cdot \mathbf{r}_i = \frac{d}{dt} \sum_i \frac{1}{2} \frac{d}{dt} (m_i r_i^2) = \frac{1}{2} \frac{d^2 I}{dt^2}$$

- where the *moment of inertia* is

$$I = \sum_i m_i r_i^2$$

- Substituting this back into (1), we have

$$\frac{1}{2}\frac{d^2I}{dt^2} - \sum_i \mathbf{p}_i \cdot \frac{d\mathbf{r}_i}{dt} = \sum_i \frac{d\mathbf{p}_i}{dt} \cdot \mathbf{r}_i$$

- where the second term on the left-hand side is just twice the negative of the *kinetic energy K* of the system:

$$-\sum_i \mathbf{p}_i \cdot \frac{d\mathbf{r}_i}{dt} = -\sum_i m_i \mathbf{v}_i \cdot \mathbf{v}_i = -2\sum_i \frac{1}{2} m_i v_i^2 = -2K$$

- Newton's second law (F=dp/dt) then allows us to write

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2K = \sum_i \mathbf{F}_i \cdot \mathbf{r}_i \qquad (2)$$

- Now let $\mathbf{F}_{ij}$ be the force on particle i due to particle j, then, considering all possible forces acting on i,

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = \sum_i \left(\sum_{j, j \neq i} \mathbf{F}_{ij} \right) \cdot \mathbf{r}_i$$

- Rewriting the position vector of i as

$$\mathbf{r}_i = (\mathbf{r}_i + \mathbf{r}_j)/2 + (\mathbf{r}_i - \mathbf{r}_j)/2$$

- we find

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = \frac{1}{2} \sum_i \left(\sum_{j, j \neq i} \mathbf{F}_{ij} \right) \cdot (\mathbf{r}_i + \mathbf{r}_j) + \frac{1}{2} \sum_i \left(\sum_{j, j \neq i} \mathbf{F}_{ij} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j)$$

- where Newton's third law, $\mathbf{F}_{ij}=-\mathbf{F}_{ji}$, means the first term on the right-hand side is zero by symmetry

- So we can now write
$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = \frac{1}{2} \sum_i \left(\sum_{j, j \neq i} \mathbf{F}_{ij} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j) \qquad (3)$$
- If only gravitational forces between particles with mass are at work in the system, then $\mathbf{F}_{ij} = G \frac{m_i m_j}{r_{ij}^2} \hat{\mathbf{r}}_{ij}$
- where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the separation between particles i and j, and $\hat{\mathbf{r}}_{ij}$ is the unit vector from i to j: $\hat{\mathbf{r}}_{ij} = \frac{\mathbf{r}_j - \mathbf{r}_i}{r_{ij}}$

- Substituting the gravitational force into (3), we have
$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = -\frac{1}{2}\sum_i \sum_{j,j\neq i} G\frac{m_i m_j}{r_{ij}^3}(\mathbf{r}_i - \mathbf{r}_j)^2$$
$$= -\frac{1}{2}\sum_i \sum_{j,j\neq i} G\frac{m_i m_j}{r_{ij}}$$
- where the potential energy between particles i and j is $U = -G\frac{m_i m_j}{r_{ij}} = -G\frac{m_j m_i}{r_{ji}}$
- and so the potential energy has been double-counted

- Taking this into account, we have

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i = -\frac{1}{2}\sum_i \sum_{j,j\neq i} G\frac{m_i m_j}{r_{ij}} = \frac{1}{2}\sum_i \sum_{j,j\neq i} U_{ij} = U$$

- Finally substituting this into (2) and taking the average with respect to time we have

$$\frac{1}{2}\left\langle \frac{d^2 I}{dt^2} \right\rangle - 2\langle K \rangle = \langle U \rangle$$

- In an equilibrium system over a long-enough period of time, the first term on the left-hand side is zero, so we have the *virial theorem for a system in gravitational equilibrium*: $-2\langle K \rangle = \langle U \rangle$

- Since $\langle E \rangle = \langle K \rangle + \langle U \rangle,$
- we can also write the virial theorem as

$$\boxed{\langle E \rangle = \langle U \rangle / 2}$$

- Note that the virial theorem **applies to a wide variety of systems**, from an *ideal gas* to a *cluster of galaxies* to a *star in equilibrium*!

Cluster dynamics

- In a cluster, stars will orbit around the centre of mass of the cluster
- Pairs of stars can exchange energy and momentum via gravitational encounters, which we call *collisions*
  - But these are not **true** collisions: the distance between stars even in dense clusters is much larger than their radii

- If their are enough collisions (encounters), the cluster will reach an equilibrium velocity distribution
  - Note that this is a *distribution*: not all stars will be moving at the same speed
    - some will be moving faster, some will be moving slower than the *mean* velocity

- Recall from our discussion of stars that the potential energy of a uniform-density sphere of mass *M* and radius *R* is

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

  - For a typical globular cluster with 10^6 stars of typical mass 0.5 $M_\odot$ and core radius 5 pc, $U=-2.5\times10^{51}$ erg

- If the cluster has *N* stars each with mass *m*, the kinetic energy of the random motion of its stars is

$$K = \sum_{i=1}^{N} m v_i^2/2 = \frac{m}{2}\sum_{i=1}^{N} v_i^2$$

- The total mass of the cluster is *M*=*Nm*, so

$$K = \frac{Nm}{2}\sum_{i=1}^{N}\frac{v_i^2}{N} = \frac{M\langle v^2\rangle}{2}$$

- where the *mean square velocity* is

$$\langle v^2\rangle = \sum_{i=1}^{N}\frac{v_i^2}{N}$$

- Now we can use the virial theorem to write

$$M\langle v^2\rangle = \frac{3}{5}\frac{GM^2}{R}$$

- Solving for the mean squared velocity we find

$$\langle v^2\rangle = \frac{3}{5}\frac{GM}{R}$$

- For the globular cluster discussed earlier, the *root mean square (RMS) velocity* is

$$v_{\rm RMS} = \sqrt{\langle v^2\rangle} = \sqrt{-U/M} = 16\,{\rm km/s}$$

  - ...the typical velocity of star in the cluster

- How fast does a star have to be moving to escape the cluster?

- For this to happen, the particle must be moving fast enough to be *unbound* from the cluster, so its total energy must be zero or greater when it is launched

- The escape velocity v_e is defined as the velocity of a particle that has zero total energy at the edge of the system: K+U=0

- At the edge of the cluster (radius R, mass M), the potential energy of the star is

$$U = -G\frac{mM}{R}$$

- and the star's kinetic energy is

$$K = mv_e^2/2$$

- Therefore the escape velocity is

$$v_e = \sqrt{2GM/R}$$

- ...a bit more than $\sqrt{3}$ times the RMS velocity for the system

- As the highest-velocity stars escape the cluster, the other stars must adjust their velocities to re-establish an equilibrium velocity distribution

- How long does this take?

    - We need to compute the amount of time for a star to suffer one strong encounter and therefore adjust its velocity: this will define the *relaxation time* of the cluster

- Let's suppose that our star has a "sphere of influence" with a cross-sectional area of πr^2
  - If any other star enters this sphere, it could be said to have experienced a strong encounter

- If the velocity of the star is *v* and the number density (number per unit volume) of stars is *n*, then we want the volume swept out by the star in time t_{relax} to contain one other star

- This volume is a cylinder with area πr^2 and length $v t_{\text{relax}}$

- Therefore we want $(\pi r^2 v t_{\text{relax}}) n = 1$

- which defines t_{relax} to be

$$t_{\text{relax}} = 1/(\pi n r^2 v)$$

- But we haven't actually chosen *r* yet!
- A sensible choice would be the radius at which the gravitational potential energy of the pair of stars is equal to the typical kinetic energy of each star:

$$\frac{Gm^2}{r} = \frac{mv^2}{2}$$

- So $r = \frac{2Gm}{v^2}$
- Then t_{relax} is $t_{\mathrm{relax}} = \frac{v^3}{4\pi G^2 m^2 n}$

- A more careful analysis shows that this relation needs to be reduced by a factor of $1/[\ln(N/2)]$, where N is the number of stars in the cluster

- Now, the number of stars per unit volume is just

$$n = \frac{N}{(4/3)\pi R^3}$$

- Then the relaxation time is

$$t_{\rm relax} = \frac{R^3 v^3}{3G^2 m^2 N \ln(N/2)}$$

- Let's use the virial theorem now: if the average separation between any two stars in the cluster is *R*, then the gravitational potential energy between these two stars is

$$U_{\rm pair} = -Gm^2/R$$

- and there are *N*(*N*-1)/2 possible pairs of stars in the cluster, so

$$U = -N(N-1)Gm^2/2R$$

- The total kinetic energy of the cluster is

$$K = Nmv^2/2$$

- Since virial theorem tells us that K=-U/2, we can write

$$Nmv^2/2 = N(N-1)Gm^2/4R$$

- and then, if *N* is sufficiently large,

$$v^2 = \frac{GNm}{2R}$$

- Finally, we can write the relaxation time as

$$t_{\rm relax} = \frac{2R}{v}\frac{N}{24\ln(N/2)}$$

- where $2R/v$ is the *crossing time* of the cluster: the typical time it takes for a star to cross from one side of the cluster to the other

- For our example globular cluster, the crossing time is t_{cross}=2x10^{13} s=6x10^{5} yrs
- ...and the relaxation time is $N/[24\ln(N/2)]$ times this, so with N=10^{6}, t_{relax}=6x10^{16} s = 2x10^{9} yrs = 2 Gyr
  - comfortably within the age of the Universe!

- The *evaporation time* is the time is takes for the cluster to lose more than half of its stars: t_{evap}~100 t_{relax}
- For a globular cluster, this is significantly longer than the current age of the Universe
- But for open clusters, t_{evap}~3×10^9 yr=3 Gyr --- which is the primary reason we only see the richest, most compact old open clusters

- For a dynamically-relaxed cluster, we can use the virial theorem to estimate the **cluster's mass** if we can measure the cluster radius and the *line-of-sight velocity dispersion*
- For our spherical cluster of mass M and radius R with N stars each of mass m, the total gravitational potential energy is

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

- If the stars have a mean square velocity of $\langle v^2 \rangle$, the total kinetic energy is

$$K = \frac{M\langle v^2\rangle}{2}$$

- Then the virial theorem gives

$$M = \frac{5}{3}\frac{\langle v^2 \rangle R}{G}$$

- We can measure *R*, but how do we measure $\langle v^2 \rangle$?

  - What we **can** measure is the *radial velocity* of the stars in the cluster, which gives the component of the motion along the line-of-sight through the cluster, so we can measure $\langle v_x{}^2 \rangle$ if *x* refers to this direction

- Let's resolve the motion of a star in the cluster into rectangular (*x,y,z*) coordinates:

$$\mathbf{v} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}$$

- where $(\hat{\mathrm{x}}, \hat{\mathrm{y}}, \hat{\mathrm{z}})$ are the unit vectors in the three directions

- Then $\mathbf{v} \cdot \mathbf{v} = v^2 = v_x^2 + v_y^2 + v_z^2$

- and the average value $\langle v^2 \rangle$ is

$$\langle v \rangle^2 = \langle v_x \rangle^2 + \langle v_y \rangle^2 + \langle v_z \rangle^2$$

- But if the motions are random, as we might expect in a cluster, then

$$\langle v_x \rangle^2 = \langle v_y \rangle^2 = \langle v_z \rangle^2$$

- so $$\langle v^2 \rangle = 3\langle v_x \rangle^2$$

- and therefore the mass can be estimated from

$$M = \frac{5\langle v_x^2 \rangle R}{G}$$

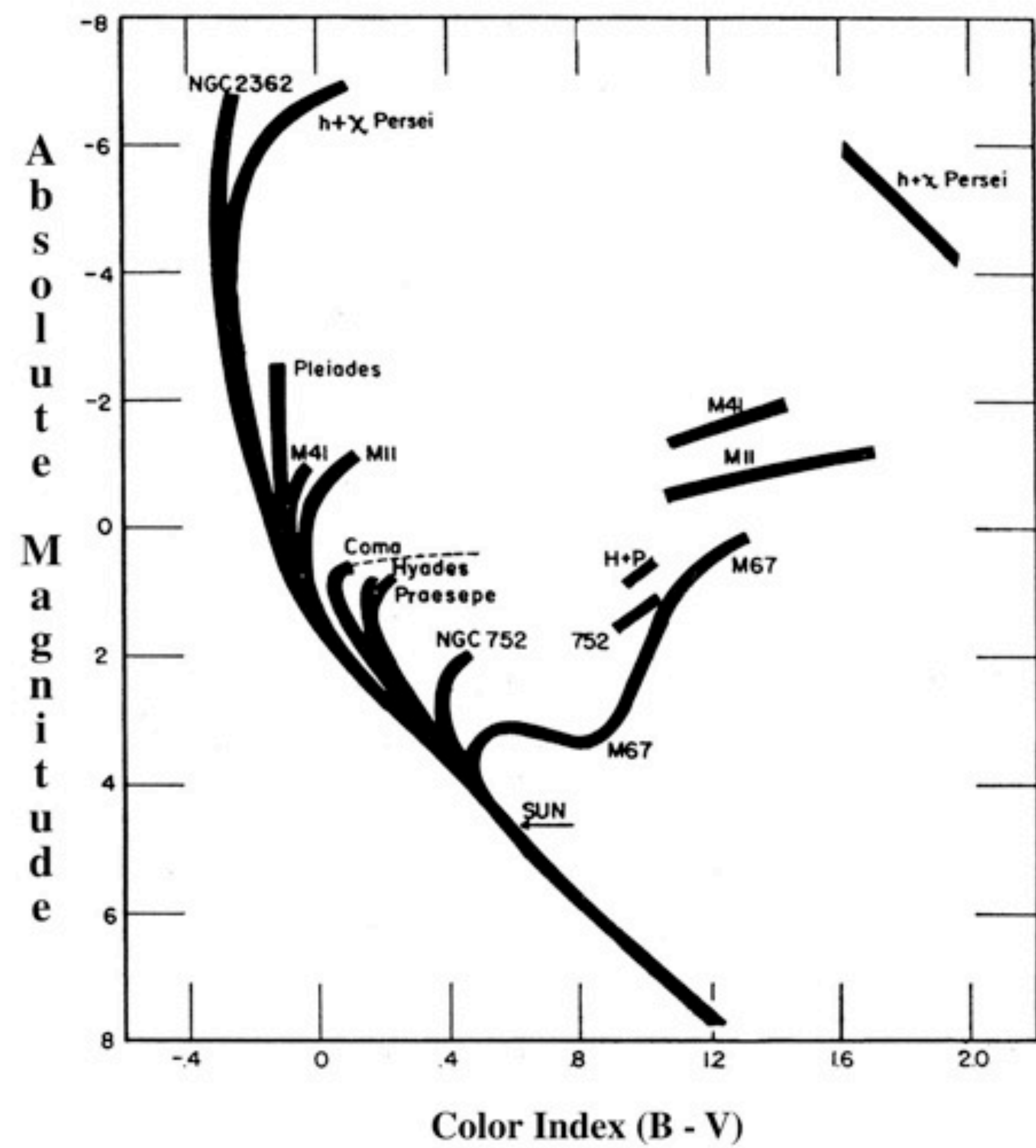
- Our test cluster then has an estimated mass of M=6x10^5 $M_\odot$

- Note that this only works for *spherical, relaxed* systems *with uniform density*

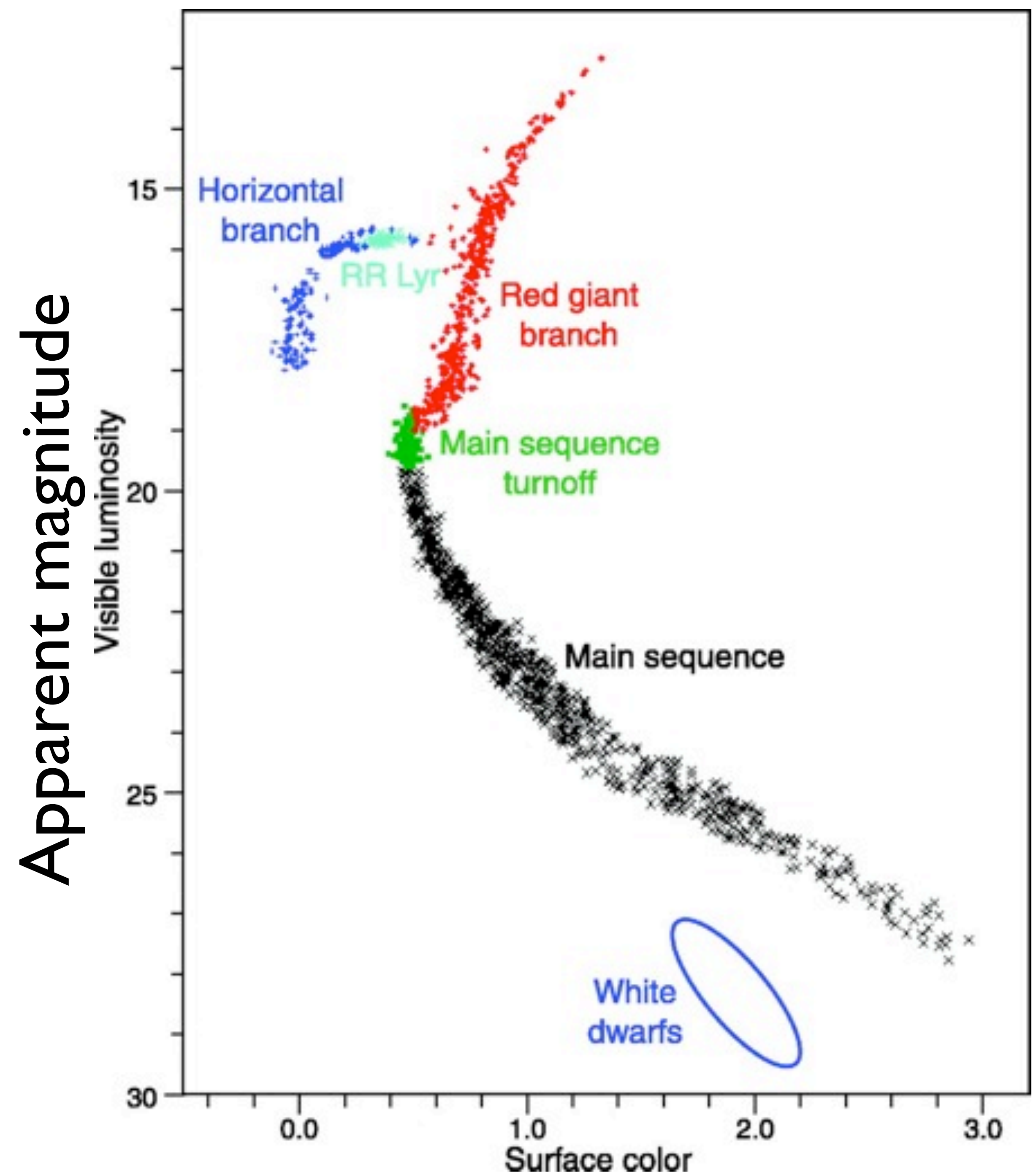
  - If the system is out of equilibrium or is elliptical, or has a density gradient, it won't be strictly correct --- but it's *always* a good first estimate!

Colour-magnitude diagrams of clusters

- As mentioned earlier, open clusters are typically young due to the effect of tidal shear in the Galactic disk and cluster evaporation
- This is reflected in their colour-magnitude diagrams
  - remember that *cooler and fainter* main-sequence turnoffs are *older*



- Globular clusters, on the other hand, are very old (>10 Gyr), at least in the Milky Way



- Globular clusters in the Milky Way are generally very old (>10 Gyr), with only a few younger, metal-rich globular clusters
    - It isn't clear *why* these clusters are younger, but they are the clusters that are closest to evaporation