

# Modes of Fluctuations

There are 2 different perturbation modes:

## ① Adiabatic (curvature) modes:

"honest-to-god" fluctuations in the energy density ( $\delta\rho \neq 0$ ), equally shared between all components.

radiation  
matter



components = radiation  
matter.

## ② Isocurvature modes



The fluctuations in radiation are such that they compensate for fluctuations in matter density, such that  $\delta\rho = 0$ .

## ③ Isothermal modes



No fluctuations in radiation density, only in matter density:

as  $p_r = aT^4$

Perturbations  
in

function  
of  
state.

The isocurvature and isothermal modes represent fluctuations in the form of the local equation of state:

- **isocurvature** fluctuations accomplish this by fluctuations in the number density of species  $X$  ( $n_x$ ) with respect to the number density of photons ( $n_\gamma$ ):

$$\left. \begin{array}{l} \bullet n_x \\ \bullet n_\gamma \propto s \quad (\propto T^3) \end{array} \right\} \delta\left(\frac{n_x}{s}\right) \neq 0$$

$s$ : entropy density  
(in cosmology fully dominated by relativistic species, photons & neutrinos).

such that they compensate exactly radiation fluctuations  $\delta p_r$ :

$$\bullet \delta p = \delta(p_x + p_r) = 0$$

- **isothermal** fluctuations:

- no fluctuations in radiation energy density:

$$\delta p_r = 0 \Rightarrow \delta p = \delta p_x.$$

$$\left. \begin{array}{l} \bullet n_\gamma \propto T^3 = \text{cst.} \\ \bullet n_x : \end{array} \right\} \delta\left(\frac{n_x}{s}\right) \neq 0.$$

modes fluct., cont'd.

The entropy  $S$  per unit mass, of a fluid composed of matter and radiation has a very high value because of the enormous value of the entropy per baryon:

$$\bullet \frac{S}{n_B} \approx \frac{1}{7} \eta^{-1} \approx 10^9$$

Entropy in volume  $V$ :

$$\bullet S = sV = \frac{\rho_r c^2 + p_r}{T} V = \underline{\underline{\frac{4}{3} \sigma T^3 V}}$$

$$\bullet \left. \begin{aligned} p_r &= \frac{1}{3} \rho_r c^2 \\ \rho_r c^2 &= \frac{\sigma}{c^2} T^4 \end{aligned} \right\}$$

per mass  $M = \rho_m V:$

$$\boxed{S' = \frac{S}{M} = \frac{4}{3} \sigma \frac{T^3}{\rho_m}}$$

$$\propto \frac{\rho_r^{3/4}}{\rho_m}$$

$$\boxed{\frac{\delta S'}{S'} = \frac{3}{4} \frac{\delta p_r}{p_r} - \frac{\delta \rho_m}{\rho_m}}$$

Entropy per mass Perturbation.

$\Rightarrow$  Adiabatic Perturbation:

$$\frac{\delta S'}{S'} = 0: \quad \delta = \frac{\delta \rho_m}{\rho_m} = \frac{\delta T}{T} = \frac{3}{4} \frac{\delta p_r}{p_r} = \frac{3}{4} \delta$$

- adiabatic fluctuations

- Entropy perturbations  $\equiv 0$  :

$$\delta_m = \frac{3}{4} \delta_r$$

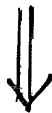
- The distinction between the two modes of fluctuations, adiabatic and isocurvature, is particularly relevant for very large scale perturbations, (nowadays), i.e. for as long as perturbations are outside of horizon.



Once, an isocurvature mode becomes sub-horizon sized, fluctuations in local pressure ( $p_r \propto p_r'$ ) can "push" energy density around and convert isocurvature perturbation  $\rightarrow$  curvature perturbation.



- the photon fluctuations get erased in radiation-dominated phase. Thus, isocurvature fluctuations evolve into isothermal perturbations



- once in matter-dominated regime, the only relevant fluctuations are the matter fluctuations, which then evolve further as their adiabatic equivalents.

# Coupled Perturbations

Assume gravity is only important force in medium containing several distinct components, i.e. neglect pressure forces.

## (1) Continuity Equation

Matter:  $p = 0$ .

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho + \frac{p}{c^2}) \vec{v} = 0:$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \rho_m \vec{v} = 0. \implies \frac{\partial \delta \rho_m}{\partial t} + \nabla \cdot \vec{v} = 0.$$

linear.

Radiation:  $p = \frac{1}{3} \rho_r c^2$

$$\frac{\partial \rho_r}{\partial t} + \nabla \cdot (\rho_r + \frac{p}{c^2}) \vec{v} = 0:$$

$$\frac{\partial \rho_r}{\partial t} + \frac{4}{3} \nabla \cdot \rho_r \vec{v} = 0. \implies \frac{\partial \delta \rho_r}{\partial t} + \frac{4}{3} \nabla \cdot \vec{v} = 0.$$

linear.

$$(2): \quad \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a} \vec{v} = -\frac{1}{a} \nabla \phi$$

$$\downarrow$$

$$\begin{cases} \frac{\partial^2 \delta \rho_m}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta \rho_m}{\partial t} = + \frac{1}{a^2} \nabla^2 \phi = 4\pi G \bar{\rho} (\rho_m + \rho_r + \frac{3p}{c^2}) \\ \frac{\partial^2 \delta \rho_r}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta \rho_r}{\partial t} = \frac{1}{a^2} \nabla^2 \phi = 4\pi G \bar{\rho} (\rho_m + \rho_r + \frac{3p}{c^2}) \end{cases}$$

$$\begin{cases} \frac{\partial^2 \delta_m}{\partial t^2} + 2 \frac{\dot{a}}{a} \delta_m = 4\pi G (\rho_m \delta_m + 2\rho_r \delta_r) \\ \frac{\partial^2 \delta_r}{\partial t^2} + 2 \frac{\dot{a}}{a} \delta_r = 4\pi G \left( \frac{4}{3} \rho_m \delta_m + \frac{8}{3} \rho_r \delta_r \right) \end{cases}$$

$$\mathcal{L} \begin{pmatrix} \delta_m \\ \delta_r \end{pmatrix} = 4\pi G \begin{pmatrix} \rho_m & 2\rho_r \\ \frac{4\rho_m}{3} & \frac{8\rho_r}{3} \end{pmatrix} \begin{pmatrix} \delta_m \\ \delta_r \end{pmatrix}$$

\*  $\mathcal{L} \equiv \frac{\partial^2}{\partial t^2} + 2 \frac{\dot{a}}{a}$  linear operator.

Note: if  $\delta_r = \frac{4}{3} \delta_m$ :

evolution of matter and radiation fully coupled

"Adiabatic Mode":

entropy per baryon:  $S \propto T^3 \propto \rho_r^{3/4}$   
 $\rho_r \propto T^4$

$$\Rightarrow \frac{\delta S}{S} = \frac{3}{4} \delta_r = \delta_m.$$

$$\delta(\text{entropy}) = \delta(\text{matter})$$

Note: Matter-Dark Matter System

$$L \begin{pmatrix} \delta_b \\ \delta_d \end{pmatrix} = \frac{4\pi G \rho}{\Omega} \begin{pmatrix} \Omega_b & \Omega_d \\ \Omega_b & \Omega_d \end{pmatrix} \begin{pmatrix} \delta_b \\ \delta_d \end{pmatrix}$$