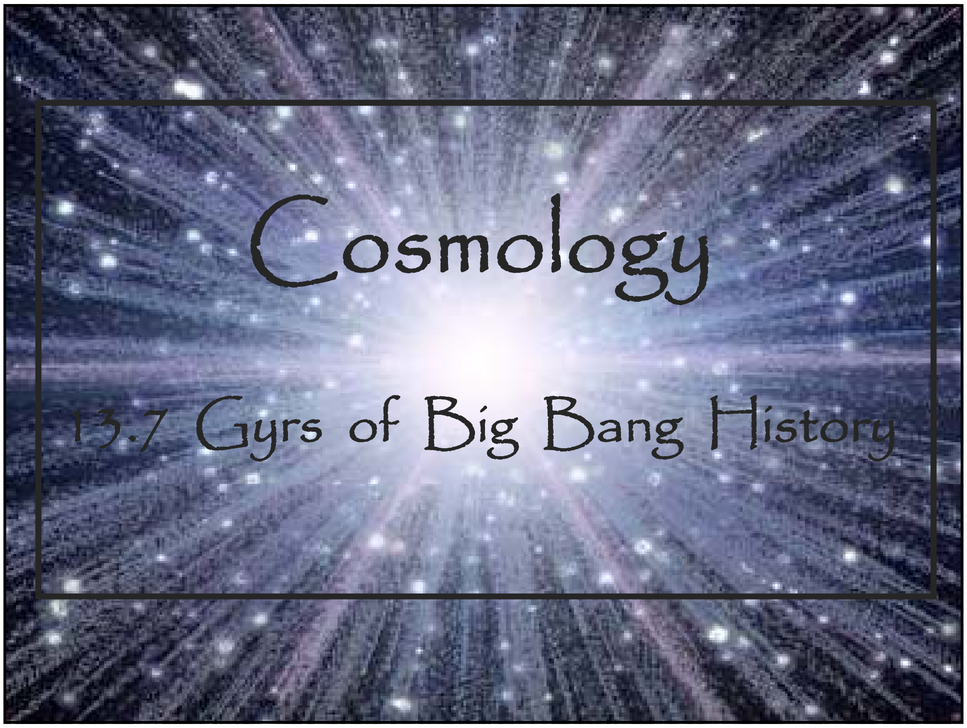
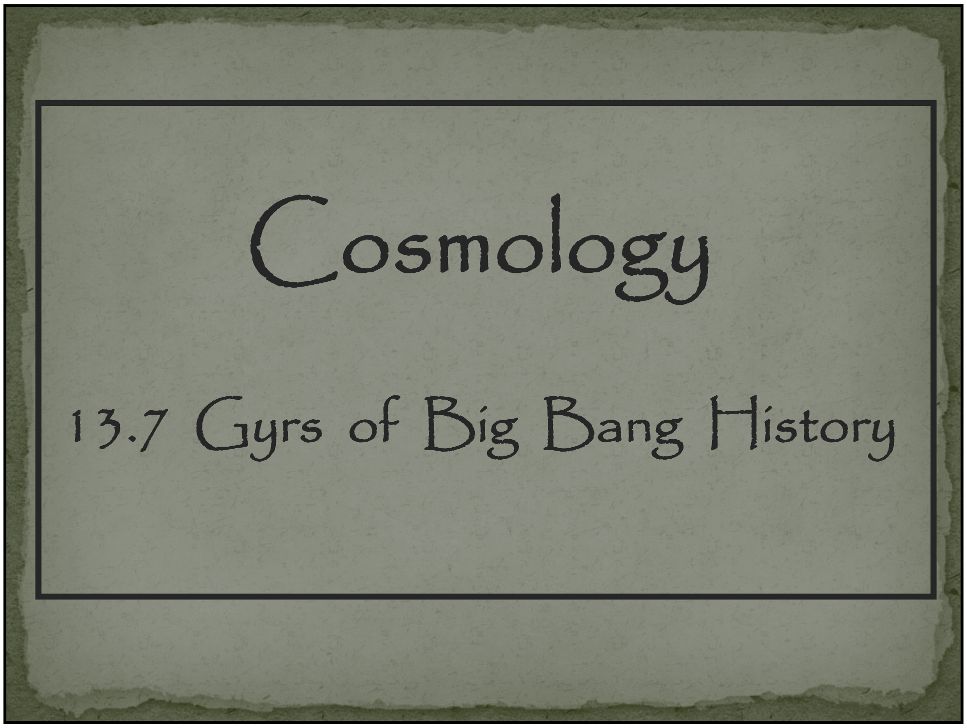


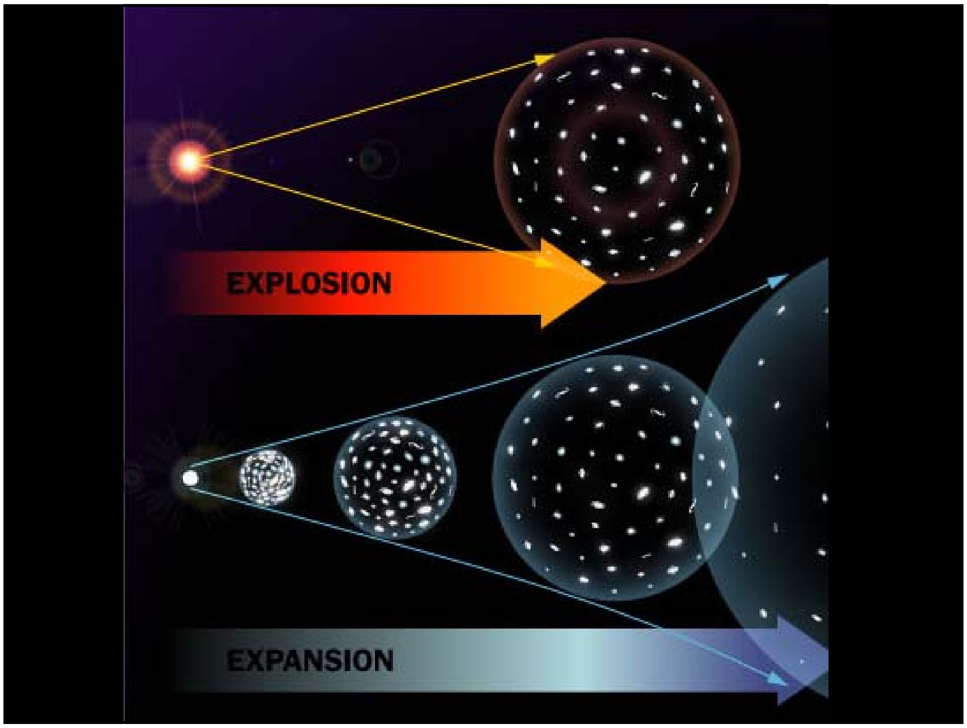
Cosmology

13.7 Gyrs of Big Bang History



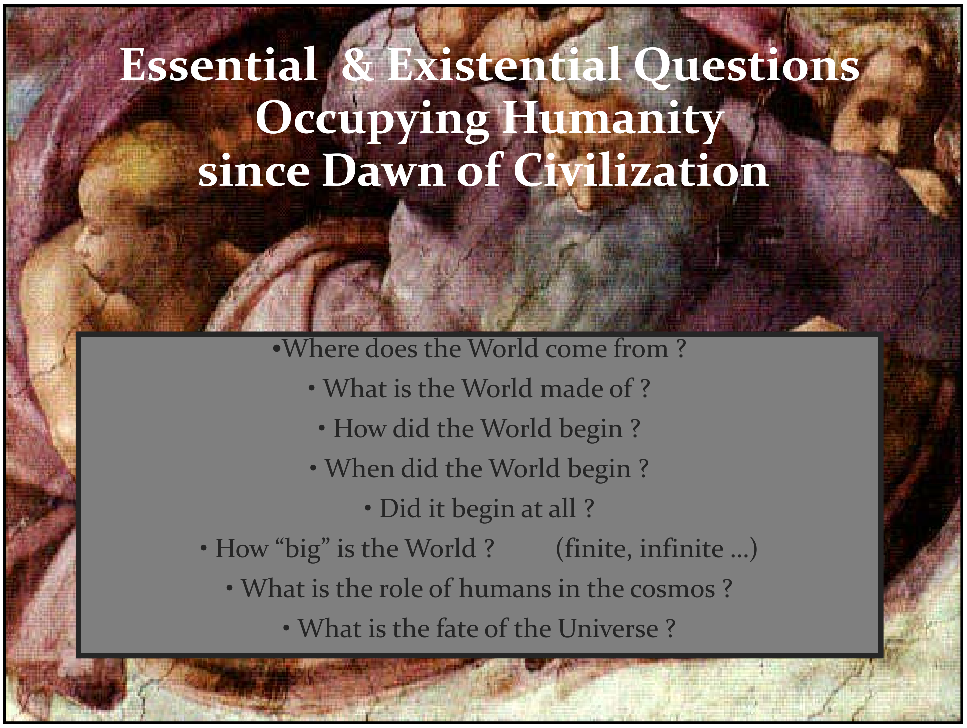
Cosmology

13.7 Gyrs of Big Bang History


EXPLOSION
EXPANSION

Cosmology

Science of the Universe



Essential & Existential Questions Occupying Humanity since Dawn of Civilization

- Where does the World come from ?
- What is the World made of ?
- How did the World begin ?
- When did the World begin ?
- Did it begin at all ?
- How “big” is the World ? (finite, infinite …)
- What is the role of humans in the cosmos ?
- What is the fate of the Universe ?

Cosmic Time:
Origin and Fate ?

- Does the Universe have an origin ?

  If so, how old is it ?

  Or, … did it always exist, infinitely old …

- What is the fate of the Universe ?

  … will it always be there, or is there an end ?

Energy:
Content of the Universe

- What are the components of the Universe ?
- How does each influence the evolution of the Universe ?

  … and …

- How is each influenced by the evolution of the Universe ?

Cosmological Riddles

- Is our Universe unique, or are there many other Universes (multiverse) … ?
- What made the Universe originate ?

Cosmological Riddles

- Why are the physical laws as they are ?
  Do they need to be ?
- How many dimensions does the Universe have?
  More than 1timelike + 3 spacelike ?

Cosmological Riddles

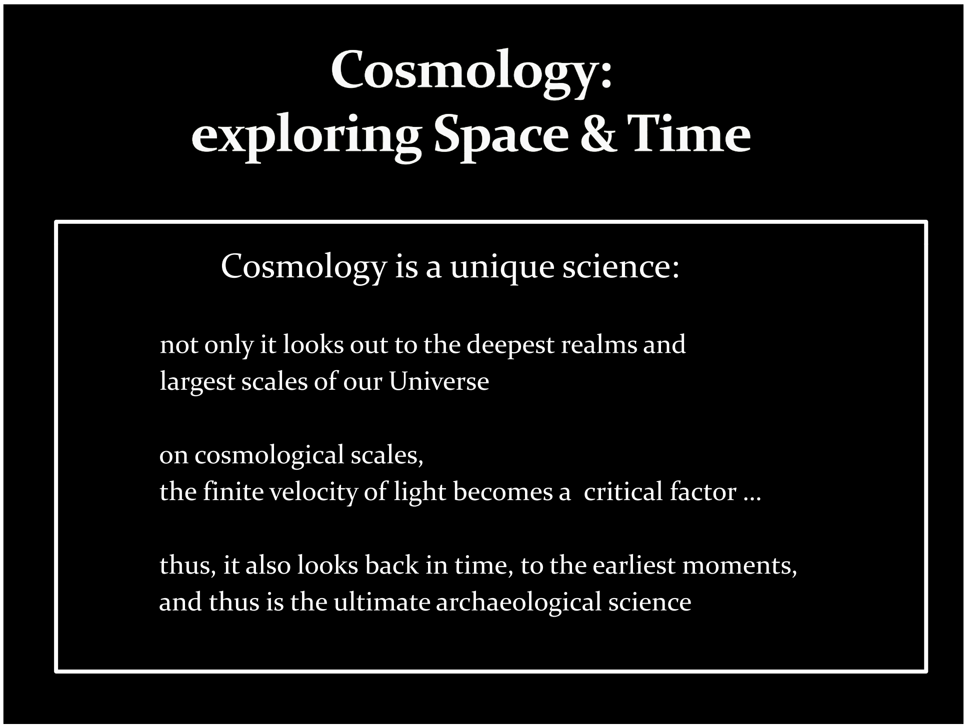
- … and …
- Are our brains sufficiently equipped to understand and answer the ultimate questions … ?

A unique time …

- The past century, since 1915, marks a special epoch
- For the first time in human history, we are able to address the great questions of Cosmology …
- scientifically …

Cosmology

Observing our Universe



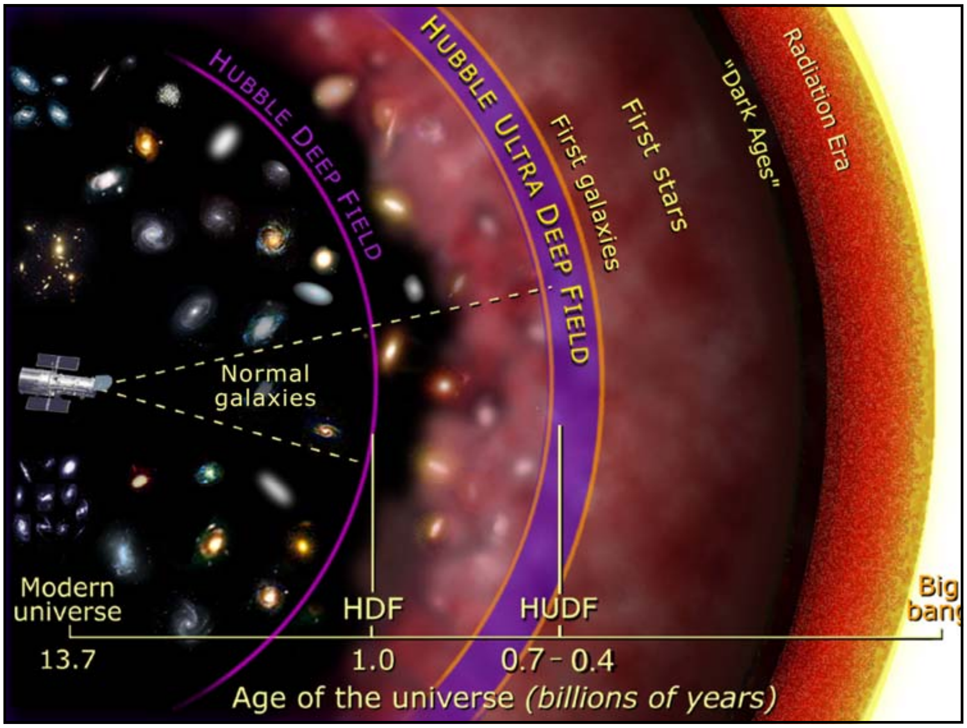
Cosmology:
exploring Space & Time

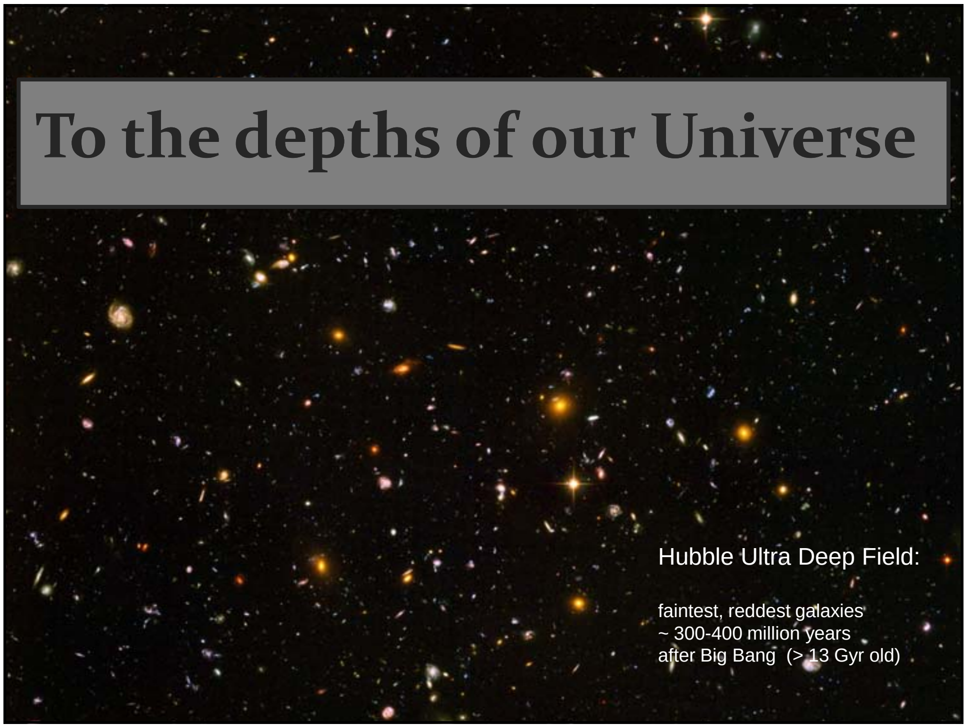
Cosmology is a unique science:

not only it looks out to the deepest realms and
largest scales of our Universe

on cosmological scales,
the finite velocity of light becomes a critical factor …

thus, it also looks back in time, to the earliest moments,
and thus is the ultimate archaeological science

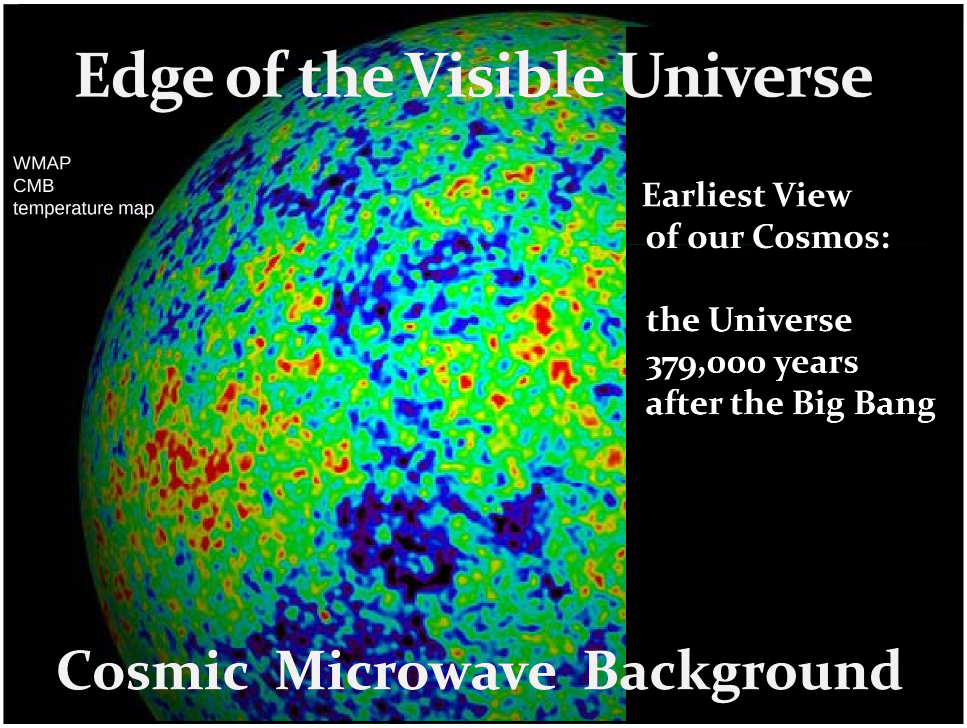




To the depths of our Universe

Hubble Ultra Deep Field:

faintest, reddest galaxies
~ 300-400 million years
after Big Bang (> 13 Gyr old)



Edge of the Visible Universe

WMAP
CMB
temperature map

Earliest View
of our Cosmos:

the Universe
379,000 years
after the Big Bang

Cosmic Microwave Background

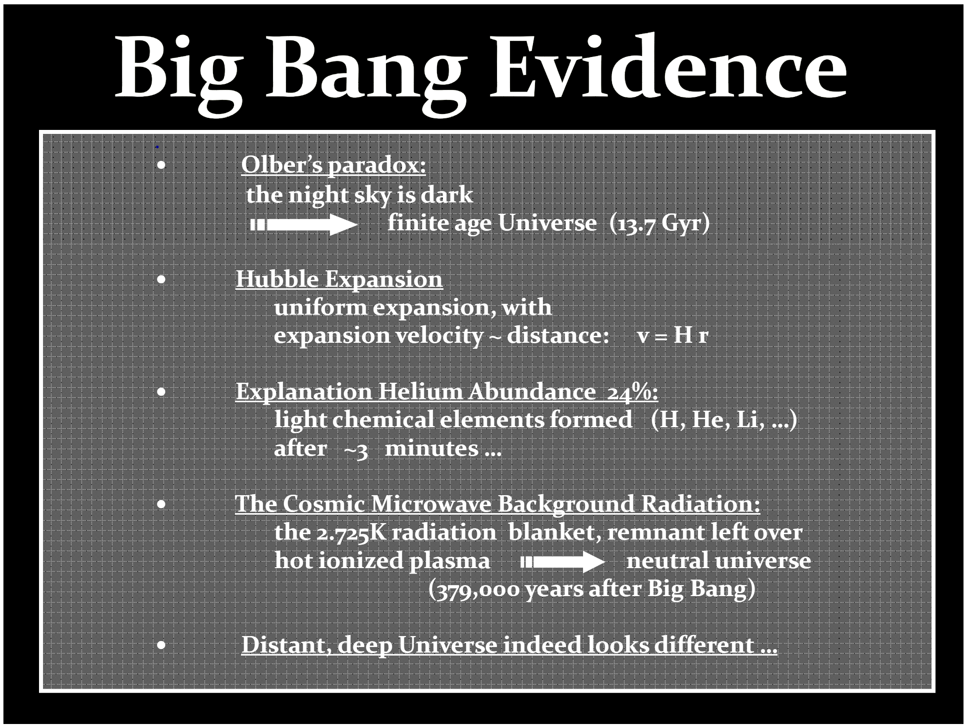
the Universe:

a Unique Astrophysical Object

- There is only one (visible) Universe …
- Finite velocity of light, c:
  … a look in depth = a look back in time …
- c & implications for space-time:
  observational cosmology limited to only
  a minor thin “shell” of all of spacetime …

Hot Big Bang

Key Observations



Big Bang Evidence

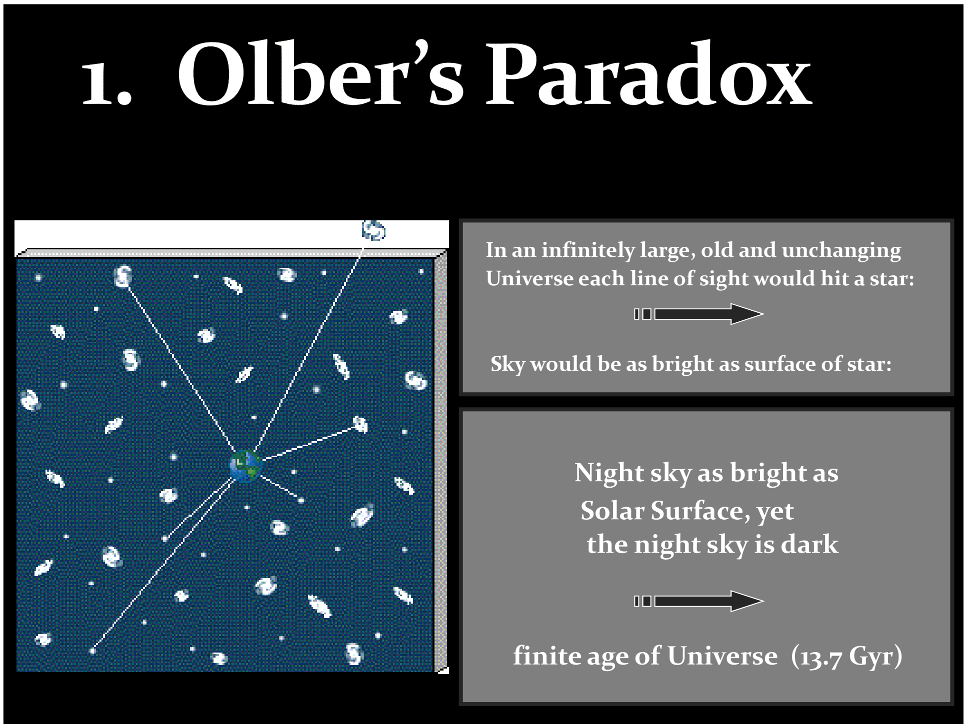
- <u>Olber’s paradox:</u>
  the night sky is dark
  → finite age Universe (13.7 Gyr)

- <u>Hubble Expansion</u>
  uniform expansion, with
  expansion velocity ~ distance: v = H r

- <u>Explanation Helium Abundance 24%:</u>
  light chemical elements formed (H, He, Li, …)
  after ~3 minutes …

- <u>The Cosmic Microwave Background Radiation:</u>
  the 2.725K radiation blanket, remnant left over
  hot ionized plasma → neutral universe
  (379,000 years after Big Bang)

- <u>Distant, deep Universe indeed looks different …</u>



1. Olber's Paradox

In an infinitely large, old and unchanging Universe each line of sight would hit a star:

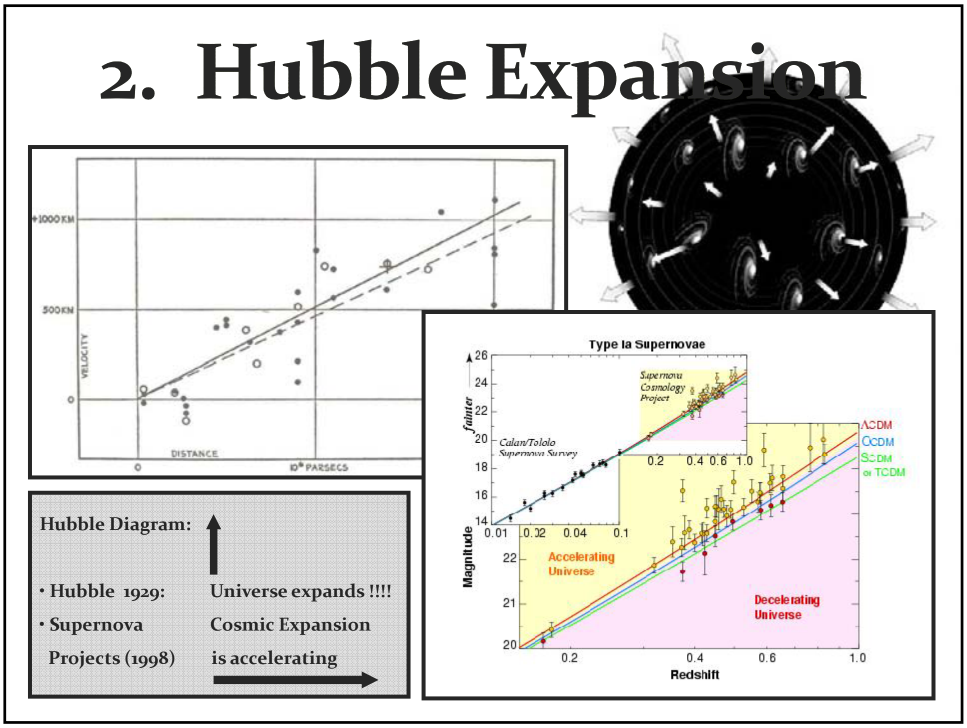
→

Sky would be as bright as surface of star:

Night sky as bright as
Solar Surface, yet
the night sky is dark

→

finite age of Universe (13.7 Gyr)

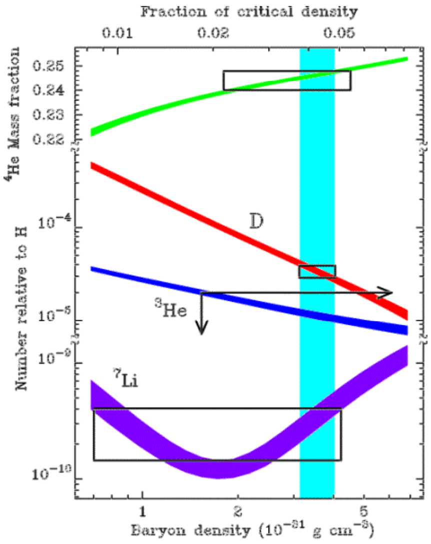


2. Hubble Expansion

Hubble Diagram: ↑

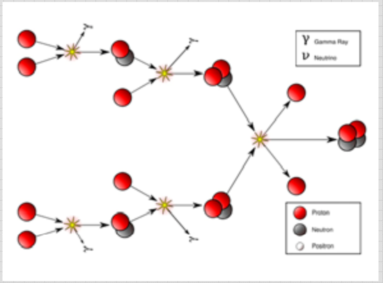
- Hubble 1929: Universe expands !!!!
- Supernova Projects (1998) Cosmic Expansion is accelerating →

3. Light Element Abundance



Mass Fraction Light Elements

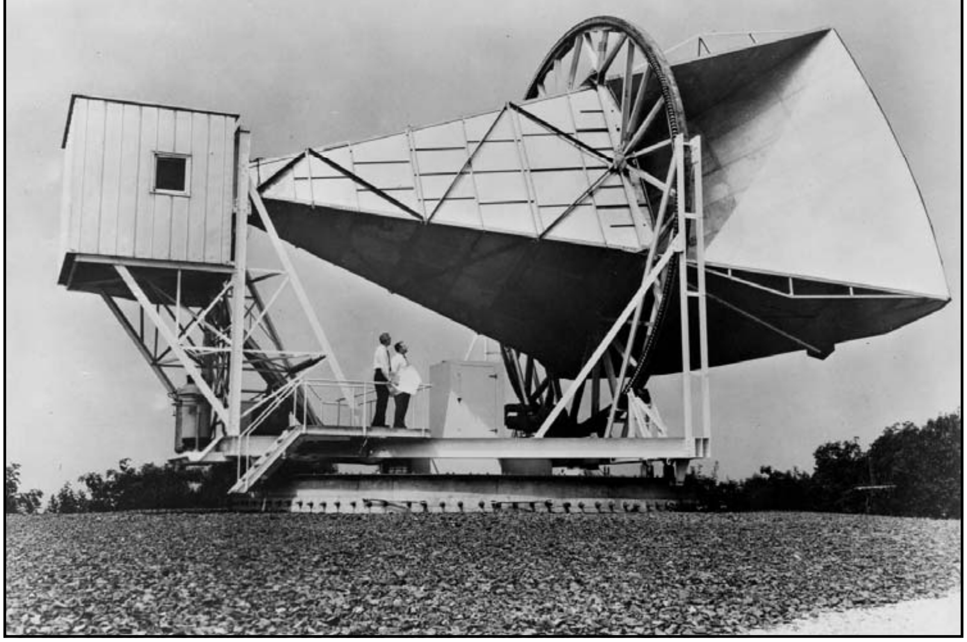
24%	^{4}He	nuclei
traces	D, ^{3}He, ^{7}Li	nuclei
75%	H	nuclei (protons)

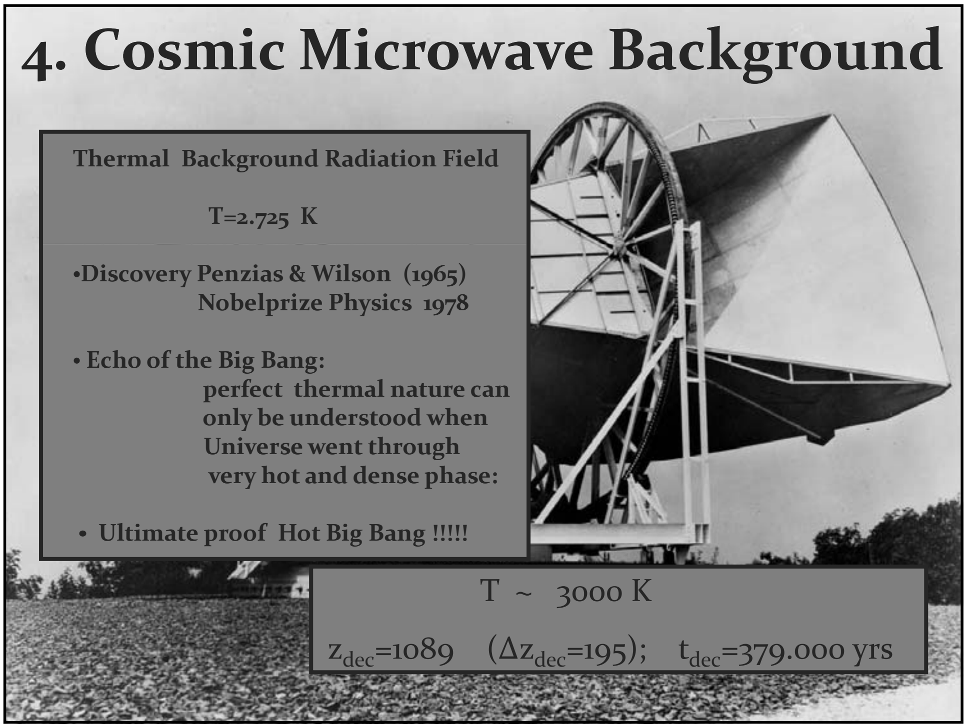


Between 1-200 seconds after Big Bang, temperature dropped to 10^9 K:

Fusion protons & neutrons into light atomic nuclei

4. Cosmic Microwave Background





4. Cosmic Microwave Background

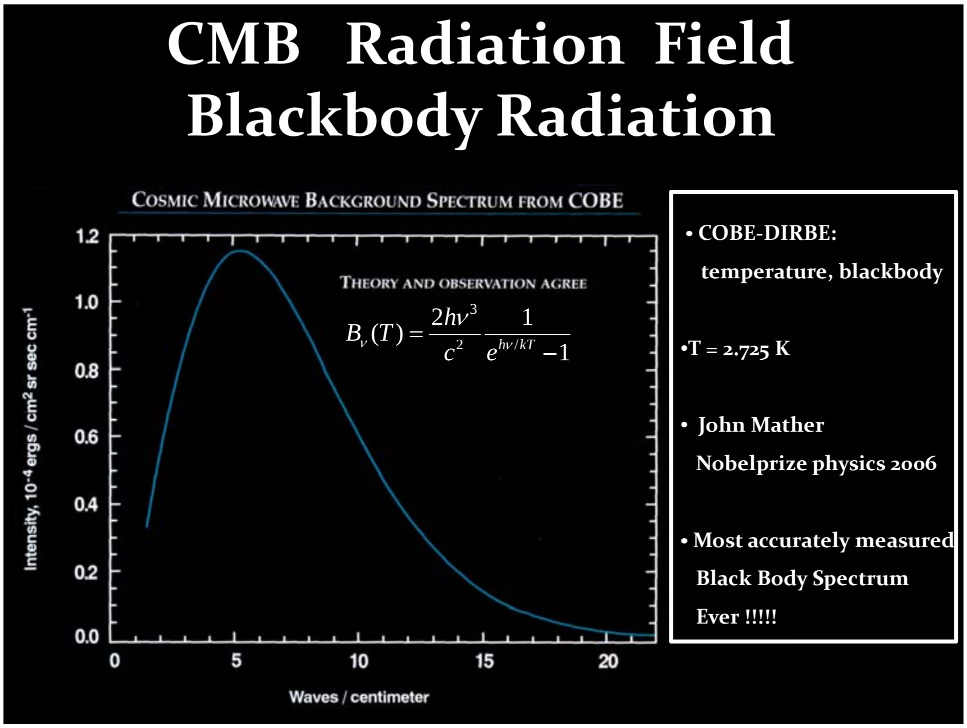
Thermal Background Radiation Field

T=2.725 K

- Discovery Penzias & Wilson (1965)
  Nobelprize Physics 1978
- Echo of the Big Bang:
  perfect thermal nature can only be understood when Universe went through very hot and dense phase:
- Ultimate proof Hot Big Bang !!!!!

T ~ 3000 K

z_{dec}=1089 (Δz_{dec}=195); t_{dec}=379.000 yrs



CMB Radiation Field
Blackbody Radiation

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

- COBE-DIRBE:
  temperature, blackbody
- T = 2.725 K
- John Mather
  Nobelprize physics 2006
- Most accurately measured
  Black Body Spectrum
  Ever !!!!!

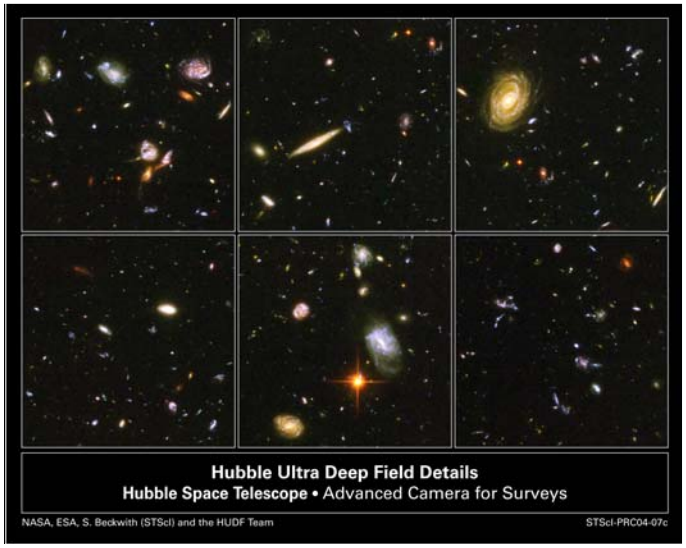
CMB Photons

Note:

far from being an exotic faraway phenomenon, realize that the CMB nowadays is counting for approximately 1% of the noise on your tv set …

Courtesy: W. Hu

5. Changing Universe



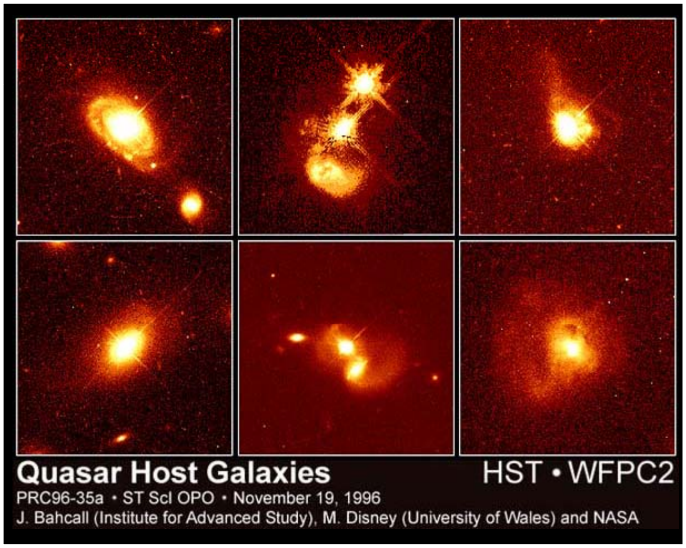
The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time

→

Galaxies in Hubble Ultra Deep Field

5. Changing Universe



Quasar Host Galaxies HST • WFPC2

PRC96-35a • ST ScI OPO • November 19, 1996

J. Bahcall (Institute for Advanced Study), M. Disney (University of Wales) and NASA

The appearance of the Universe does change when looking deeper into the Universe:

Depth=Time

→

Quasars

(very high z)

Gravity:

Ruler of the Universe

Four Fundamental Forces of Nature

- **Strong Nuclear Force**

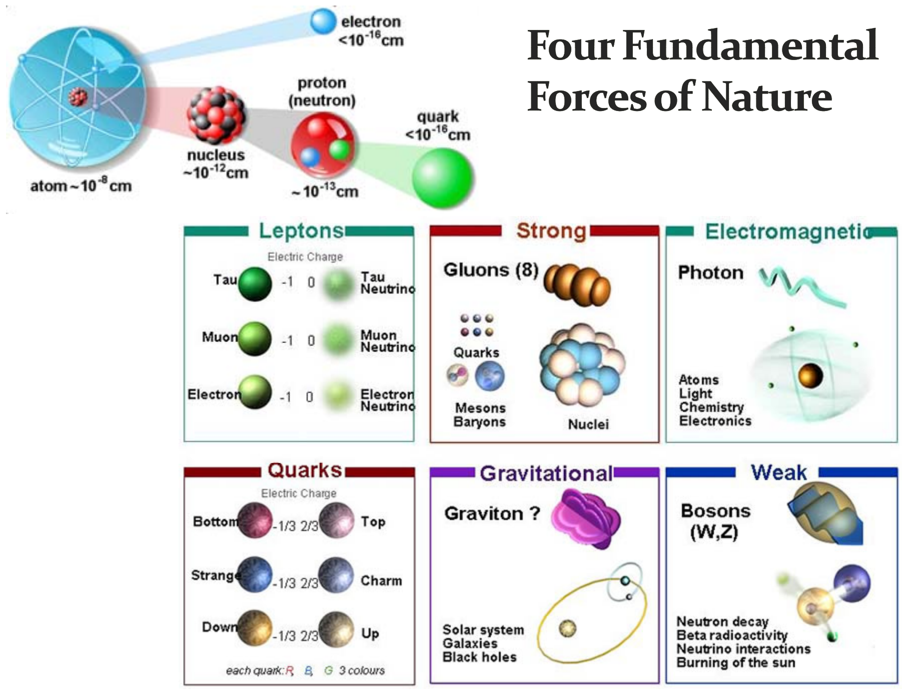
  Responsible for holding particles together inside the nucleus.
  The nuclear strong force carrier particle is called the gluon.
  The nuclear strong interaction has a range of 10^{-15} m (diameter of a proton).
- **Electromagnetic Force**

  Responsible for electric and magnetic interactions, and determines structure of atoms and molecules.
  The electromagnetic force carrier particle is the photon (quantum of light)
  The electromagnetic interaction range is infinite.
- **Weak Force**

  Responsible for (beta) radioactivity.
  The weak force carrier particles are called weak gauge bosons (Z, W^+, W^-).
  The nuclear weak interaction has a range of 10^{-17} m (1% of proton diameter).
- **Gravity**

  Responsible for the attraction between masses. Although the gravitational force carrier
  The hypothetical (carrier) particle is the graviton.
  The gravitational interaction range is infinite.
  By far the weakest force of nature.

Four Fundamental Forces of Nature

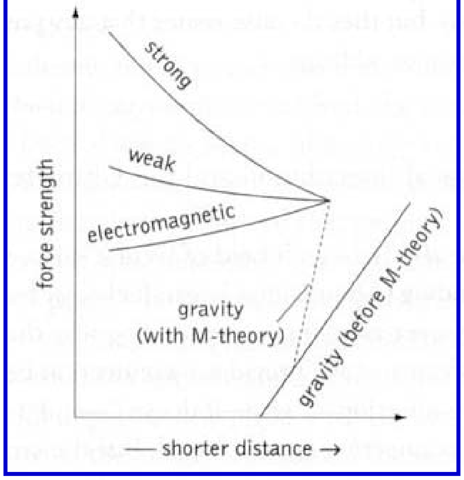


Interaction	Current Theory	Mediators	Relative Strength[1]	Long-Distance Behavior	Range(m)
Strong	Quantum chromodynamics (QCD)	gluons	10^{38}	1 (see discussion below)	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	photons	10^{36}	$\frac{1}{r^2}$	infinite
Weak	Electroweak Theory	W and Z bosons	10^{25}	$\frac{e^{-m_{W,Z}r}}{r}$	10^{-18}
Gravitation	General Relativity (GR)	gravitons	1	$\frac{1}{r^2}$	infinite

The weakest force, by far, rules the Universe …

Gravity has dominated its evolution, and determines its fate …

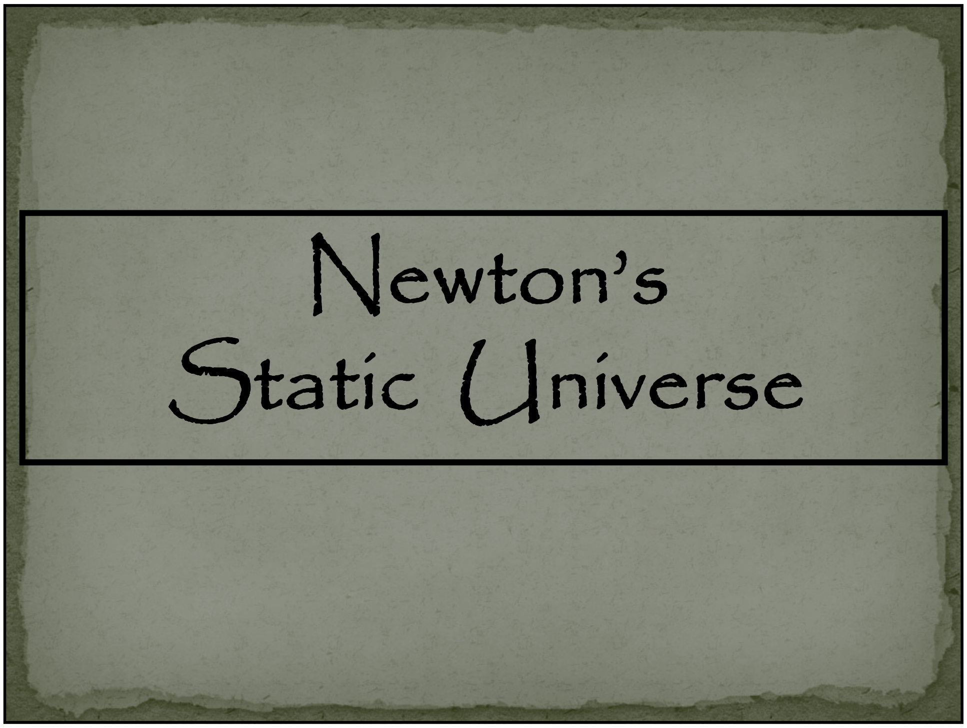
Grand Unified Theories (GUT)



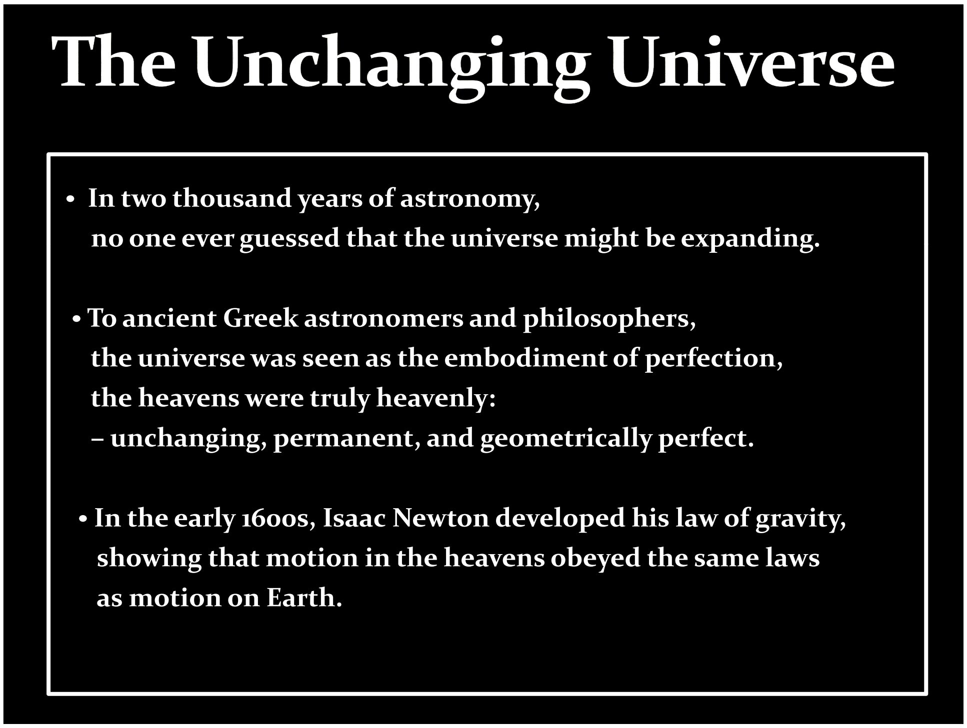
Grand Unified Theories

- * describe how
  - • Strong
  - • Weak
  - • Electromagnetic

  Forces are manifestations of the same underlying GUT force …
- * This implies the strength of the forces to diverge from their uniform GUT strength
- * Interesting to see whether gravity at some very early instant unifies with these forces ???



Newton's
Static Universe



The Unchanging Universe

- In two thousand years of astronomy,
  no one ever guessed that the universe might be expanding.

- To ancient Greek astronomers and philosophers,
  the universe was seen as the embodiment of perfection,
  the heavens were truly heavenly:
  – unchanging, permanent, and geometrically perfect.

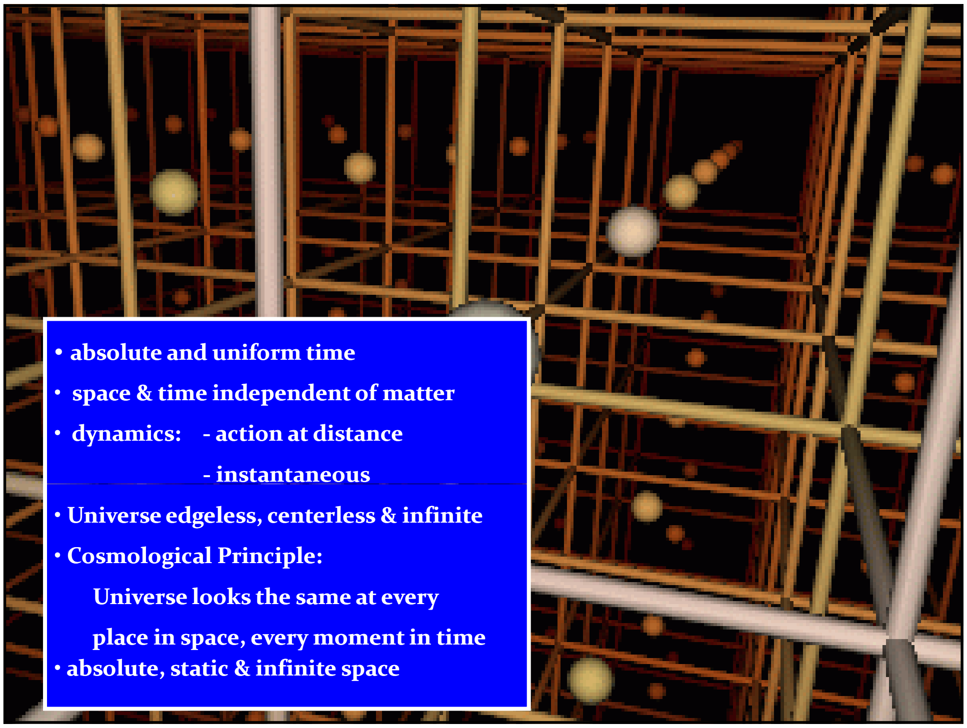
- In the early 1600s, Isaac Newton developed his law of gravity,
  showing that motion in the heavens obeyed the same laws
  as motion on Earth.

Newton's Universe

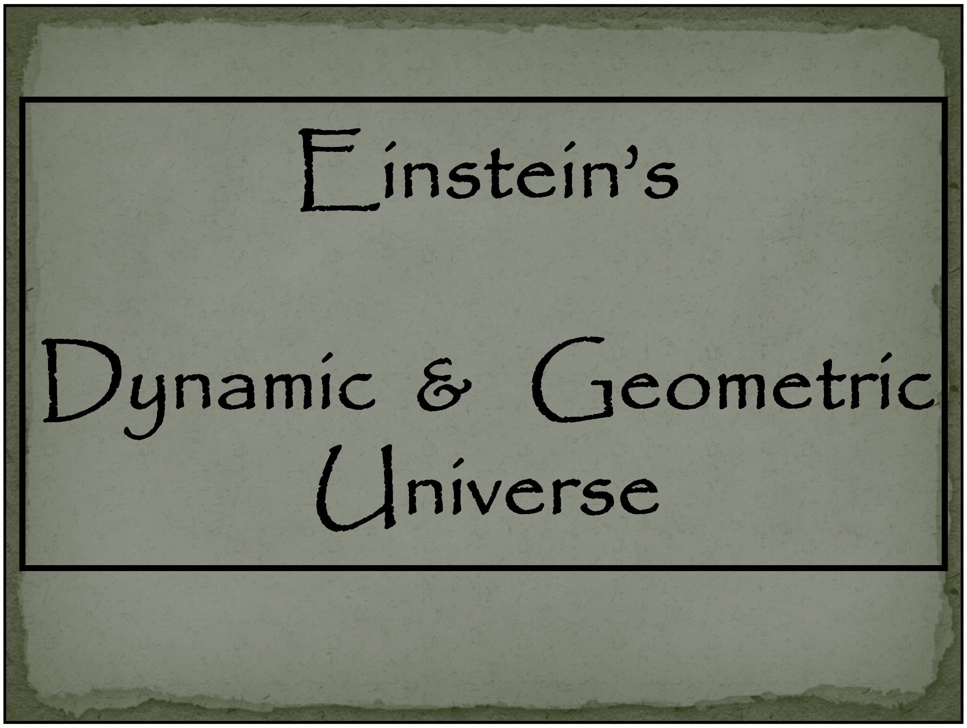
- However, Newton ran into trouble when he tried to apply his theory of gravity to the entire universe.
- Since gravity is always attractive, his law predicted that all the matter in the universe should eventually clump into one big ball.
- Newton knew this was not the case, and assumed that the universe had to be static
- So he conjectured that:

the Creator placed the stars such that they were

``at immense distances from one another.”

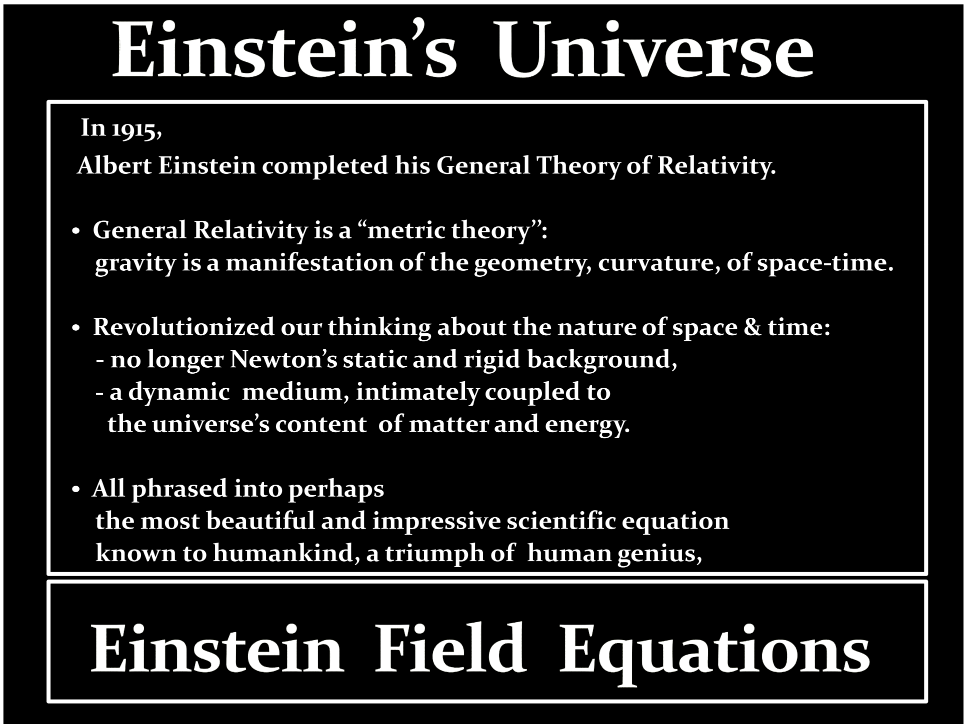


- absolute and uniform time
- space & time independent of matter
- dynamics: - action at distance
  - instantaneous
- Universe edgeless, centerless & infinite
- Cosmological Principle:
  Universe looks the same at every
  place in space, every moment in time
- absolute, static & infinite space



Einstein's

Dynamic & Geometric Universe

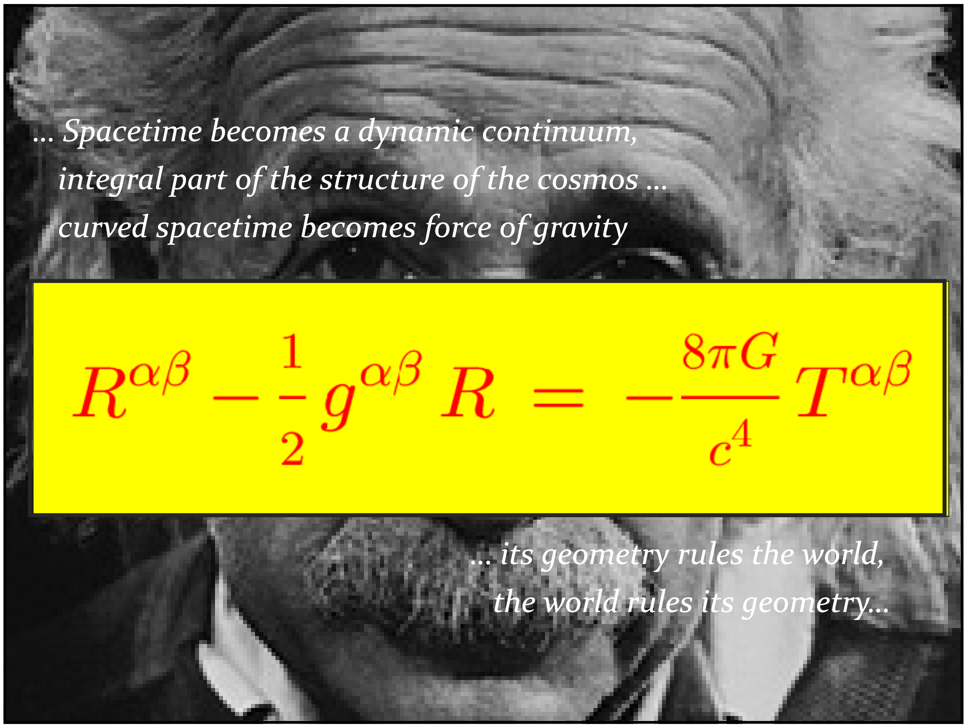


Einstein's Universe

In 1915,
Albert Einstein completed his General Theory of Relativity.

- General Relativity is a “metric theory”:
  gravity is a manifestation of the geometry, curvature, of space-time.
- Revolutionized our thinking about the nature of space & time:
  - no longer Newton's static and rigid background,
  - a dynamic medium, intimately coupled to
    the universe's content of matter and energy.
- All phrased into perhaps
  the most beautiful and impressive scientific equation
  known to humankind, a triumph of human genius,

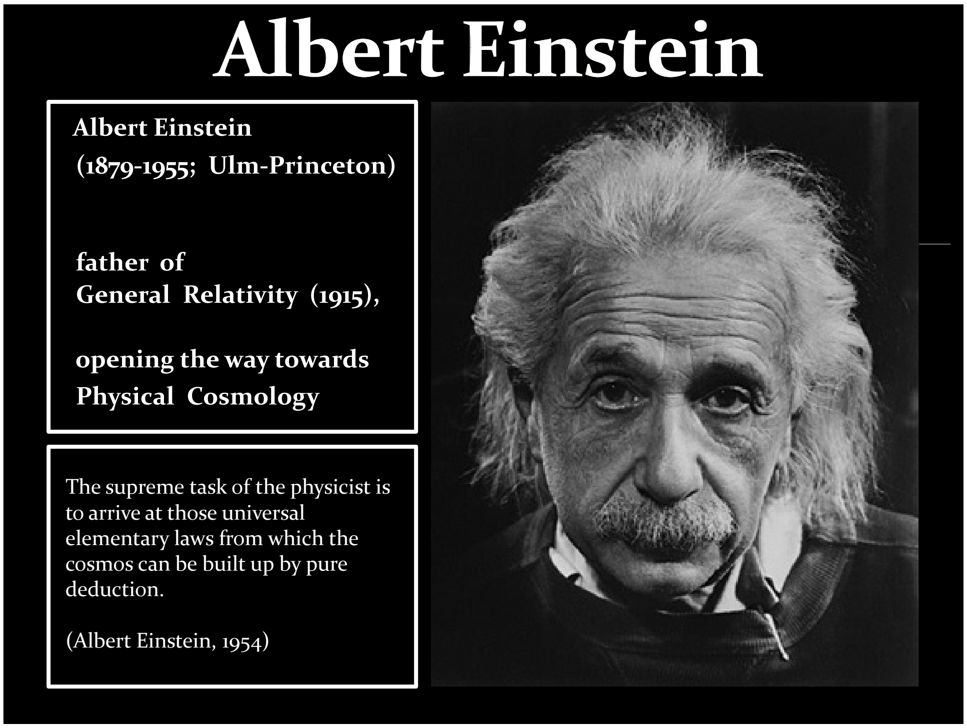
Einstein Field Equations



… Spacetime becomes a dynamic continuum,
integral part of the structure of the cosmos …
curved spacetime becomes force of gravity

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

… its geometry rules the world,
the world rules its geometry…



Albert Einstein

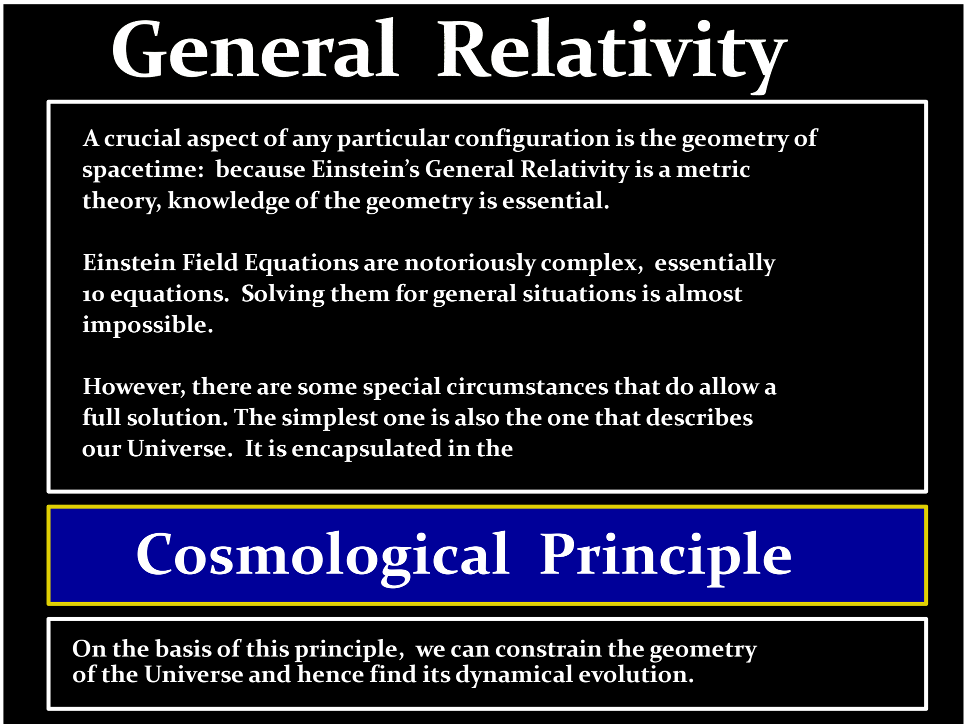
Albert Einstein
(1879-1955; Ulm-Princeton)

father of
General Relativity (1915),

opening the way towards
Physical Cosmology

The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction.

(Albert Einstein, 1954)



General Relativity

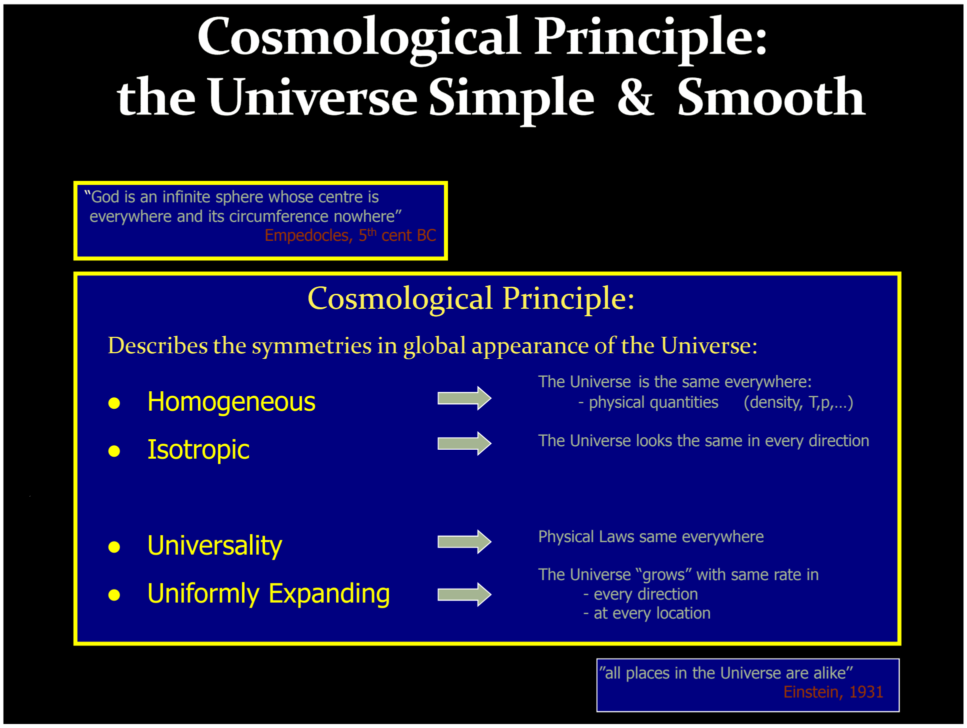
A crucial aspect of any particular configuration is the geometry of spacetime: because Einstein's General Relativity is a metric theory, knowledge of the geometry is essential.

Einstein Field Equations are notoriously complex, essentially 10 equations. Solving them for general situations is almost impossible.

However, there are some special circumstances that do allow a full solution. The simplest one is also the one that describes our Universe. It is encapsulated in the

Cosmological Principle

On the basis of this principle, we can constrain the geometry of the Universe and hence find its dynamical evolution.



Cosmological Principle: the Universe Simple & Smooth

> "God is an infinite sphere whose centre is everywhere and its circumference nowhere"
> Empedocles, 5th cent BC

Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- Homogeneous → The Universe is the same everywhere:
  - physical quantities (density, T,p,…)
- Isotropic → The Universe looks the same in every direction
- Universality → Physical Laws same everywhere
- Uniformly Expanding → The Universe "grows" with same rate in
  - every direction
  - at every location

> "all places in the Universe are alike"
> Einstein, 1931

Geometry of the Universe

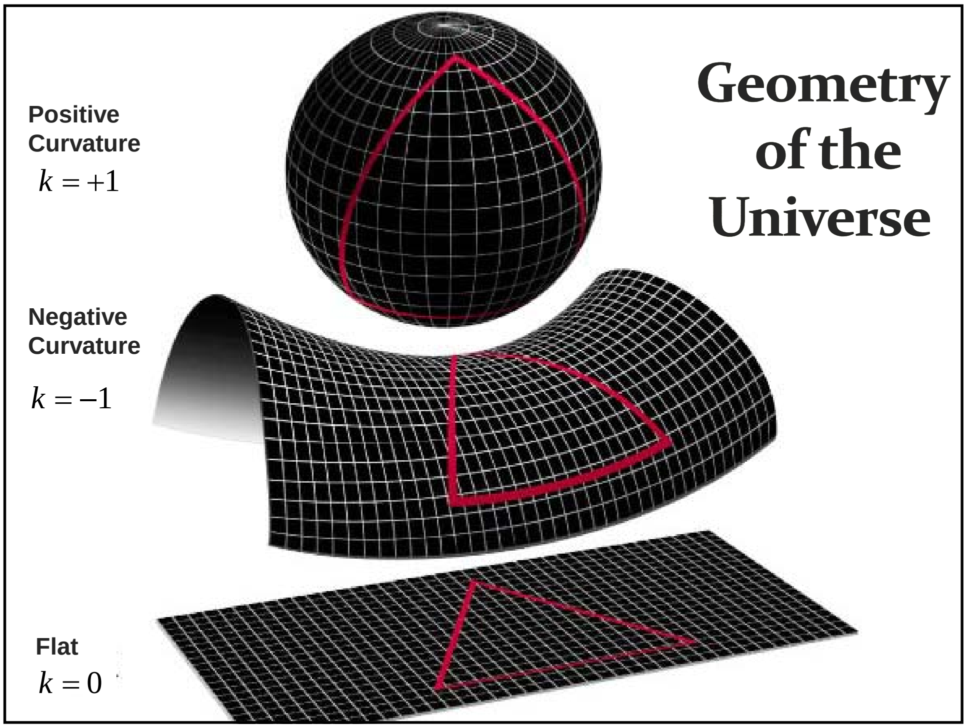
Fundamental Tenet

of (Non-Euclidian = Riemannian) Geometry

There exist no more than THREE uniform spaces:

1)	Euclidian (flat) Geometry	Euclides
2)	Hyperbolic Geometry	Gauß, Lobachevski, Bolyai
3)	Spherical Geometry	Riemann

uniform=
homogeneous & isotropic
(cosmological principle)



Geometry of the Universe

Positive Curvature

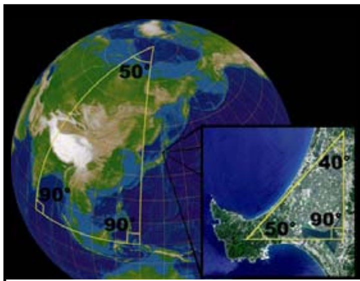
$k = +1$

Negative Curvature

$k = -1$

Flat

$k = 0$



Uniform Spaces:

Geometric Characteristics

	Parallel Lines	Triangular Angles $\alpha+\beta+\gamma$	Circumference Circle $x \equiv \frac{S}{2r}$	Curvature k	Extent	Boundary
Flat Space	parallels: 1 never intersects	π	π	0	open: infinite	unbounded
Spherical Space	parallels: ∞ along great circles, all intersect	$> \pi$	$< \pi$	$1/R^2$ > 0	closed: finite	unbounded
Hyperbolic Space	parallels: ∞ diverge & never intersect	$< \pi$	$> \pi$	$-1/R^2$ < 0	open: infinite	unbounded

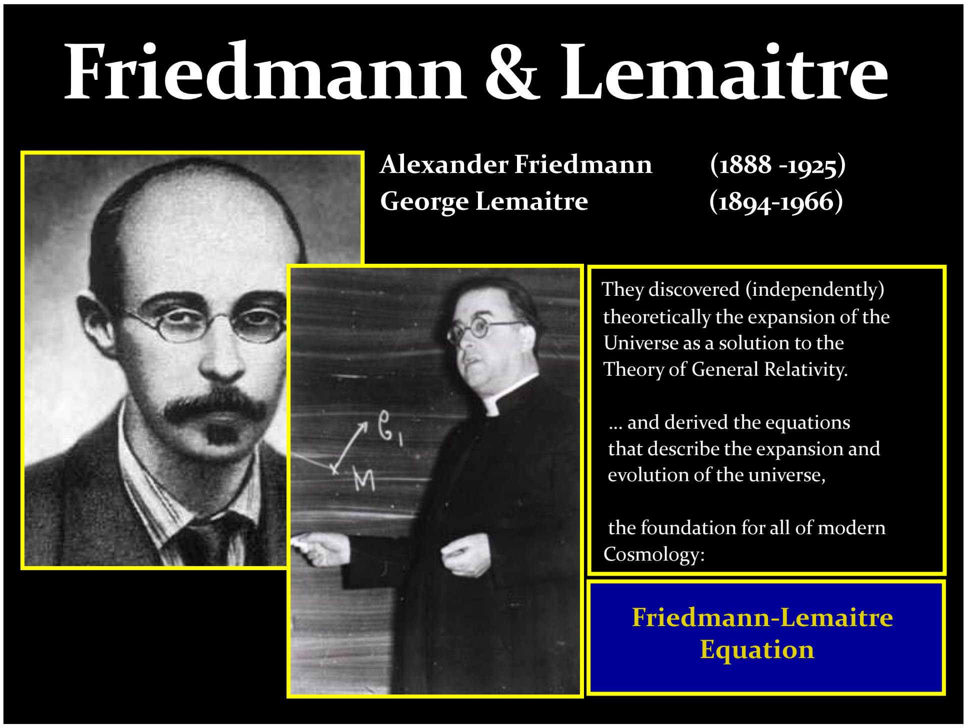
Robertson-Walker Metric

Distances in a uniformly curved spacetime is specified in terms of the Robertson-Walker metric. The spacetime distance of a point at coordinate (r,θ,ϕ) is:

$$ds^2 = c^2dt^2 - a(t)^2\left\{dr^2 + R_c{}^2 S_k{}^2\left(\frac{r}{R_c}\right)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]\right\}$$

where the function $S_k(r/R_c)$ specifies the effect of curvature on the distances between points in spacetime

$$S_k\left(\frac{r}{R_c}\right) = \begin{cases} \sin\left(\frac{r}{R_c}\right) & k = +1 \\ \frac{r}{R_c} & k = 0 \\ \sinh\left(\frac{r}{R_c}\right) & k = -1 \end{cases}$$



Friedmann & Lemaitre

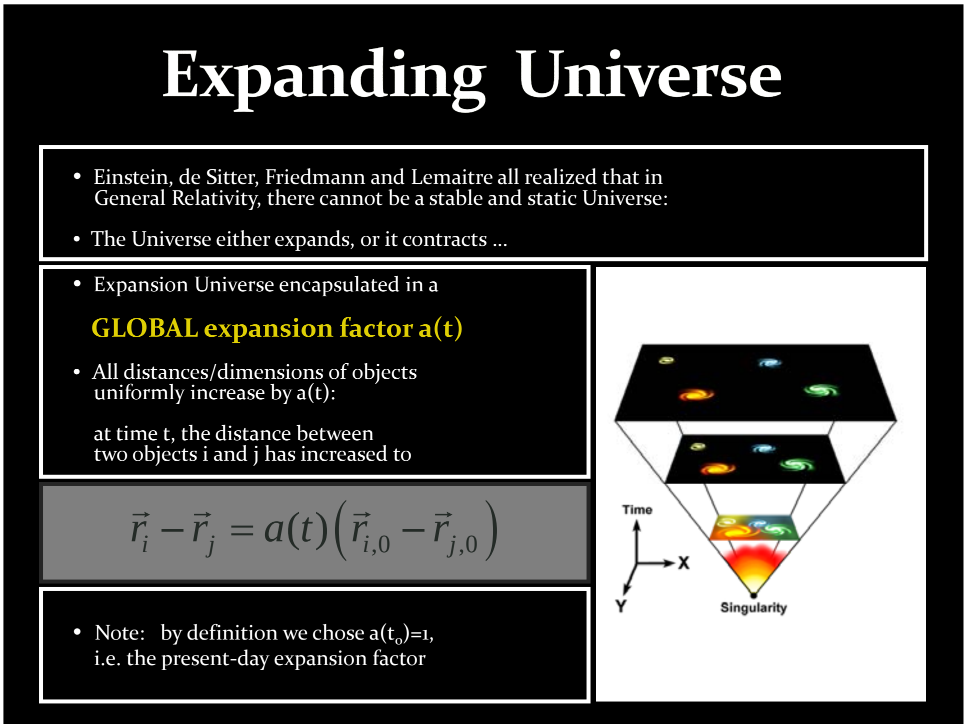
Alexander Friedmann (1888 -1925)
George Lemaitre (1894-1966)

They discovered (independently) theoretically the expansion of the Universe as a solution to the Theory of General Relativity.

... and derived the equations that describe the expansion and evolution of the universe,

the foundation for all of modern Cosmology:

Friedmann-Lemaitre Equation



Expanding Universe

- Einstein, de Sitter, Friedmann and Lemaitre all realized that in General Relativity, there cannot be a stable and static Universe:
- The Universe either expands, or it contracts ...

- Expansion Universe encapsulated in a

GLOBAL expansion factor a(t)

- All distances/dimensions of objects uniformly increase by a(t):

at time t, the distance between two objects i and j has increased to

$$\vec{r}_i - \vec{r}_j = a(t)\left(\vec{r}_{i,0} - \vec{r}_{j,0}\right)$$

- Note: by definition we chose $a(t_o)=1$, i.e. the present-day expansion factor

Friedmann-Robertson-Walker-Lemaitre Universe

Because of General Relativity, the evolution of the Universe is determined by four factors:

• density	$\rho(t)$	
• pressure	$p(t)$	
• curvature	$kc^2 / R_0{}^2$	$k = 0, +1, -1$ R_0: present curvature radius
• cosmological constant	Λ	

• Density & Pressure: - in relativity, energy & momentum need to be seen as one physical quantity (four-vector)
- pressure = momentum flux

• Curvature: - gravity is a manifestation of geometry spacetime

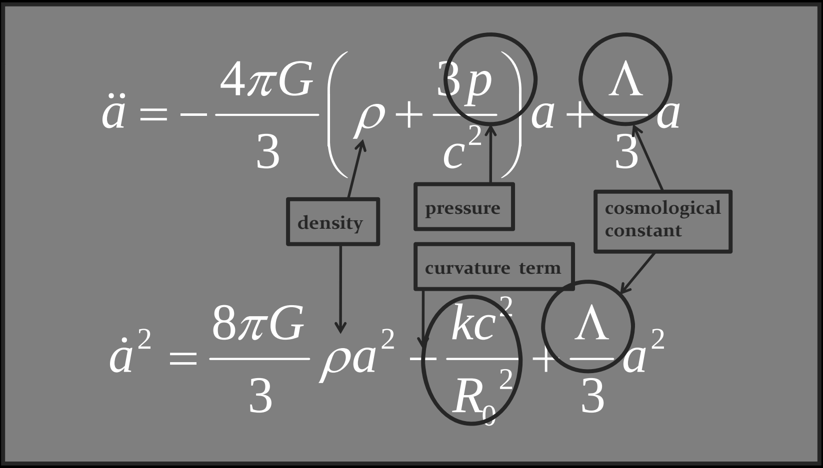
• Cosmological Constant: - free parameter in General Relativity
- Einstein's "biggest blunder"
- mysteriously, since 1998 we know it dominates the Universe

Friedmann-Robertson-Walker-Lemaitre Universe

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0{}^2} + \frac{\Lambda}{3}a^2$$

Friedmann-Robertson-Walker-Lemaitre Universe



$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

density

pressure

cosmological constant

curvature term

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2$$

Friedmann-Robertson-Walker-Lemaitre Universe

Relativistic Cosmology

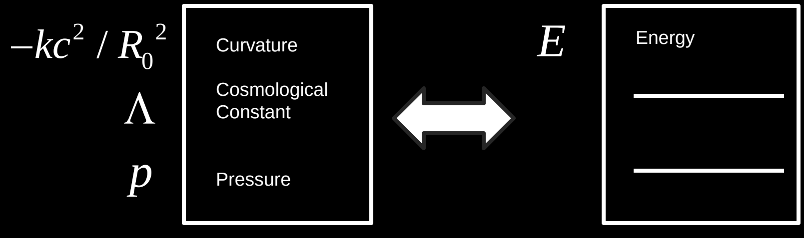
$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a^2$$

Newtonian Cosmology

$$\ddot{a} = -\frac{4\pi G}{3}\rho a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 + E$$

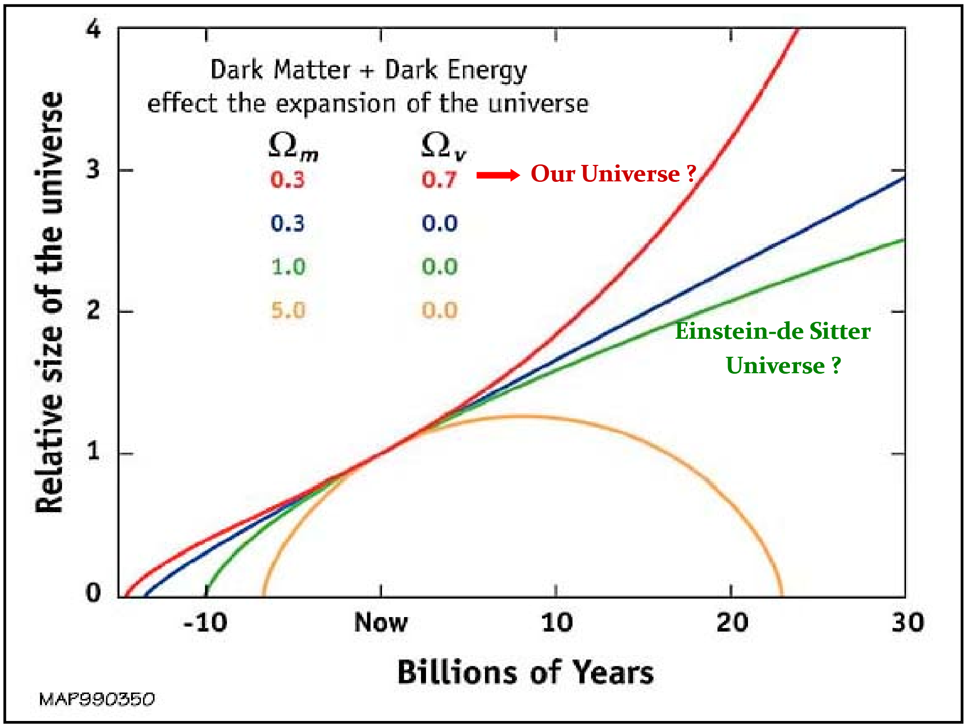


$-kc^2 / R_0^{\ 2}$ Curvature

Λ Cosmological Constant

p Pressure

E Energy



Evolution & Fate Friedmann-Robertson-Walker-Lemaitre Universe

Fully determined by three factors:

- energy content of the Universe
  (density & pressure)
- geometry of the Universe
  (curvature term)
- cosmological constant

Observing

Cosmic Expansion:

Redshift

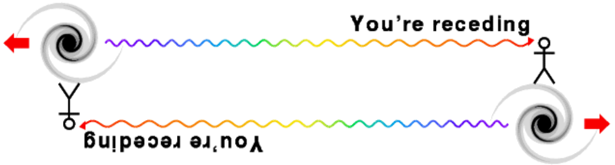
Redshift & Cosmic Expansion

- As a result of the expansion of the Universe, not only distances get stretched:
- also the wavelength of light stretches along with the cosmic expansion
- Cosmic Redshift z:
  directly related to the expansion factor a(t) at which light gets emitted
- As a result, redshift z can be directly translated into:
  - distance of observed object
  - via its 1-1 relation with expansion factor a(t), alternative indication cosmic time t

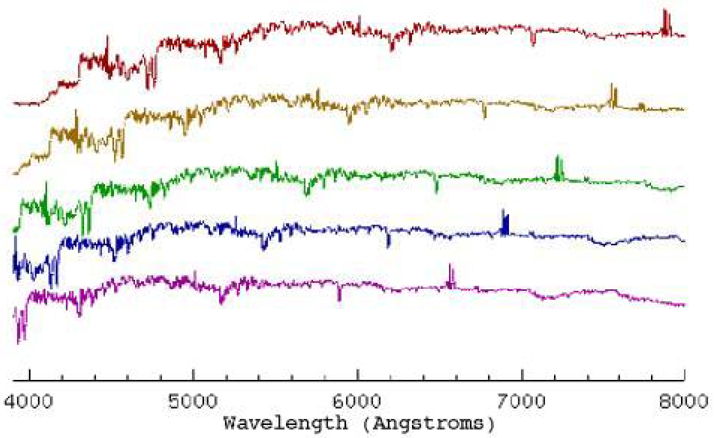
Cosmic Redshift

$$1 + z = \frac{1}{a} \Longleftarrow \begin{cases} \lambda_{em} = \lambda_0 \\ \lambda_{obs} = \dfrac{a(t_{obs})}{a(t_{em})} \lambda_0 \end{cases}$$

$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$



Redshift & Galaxy Spectra

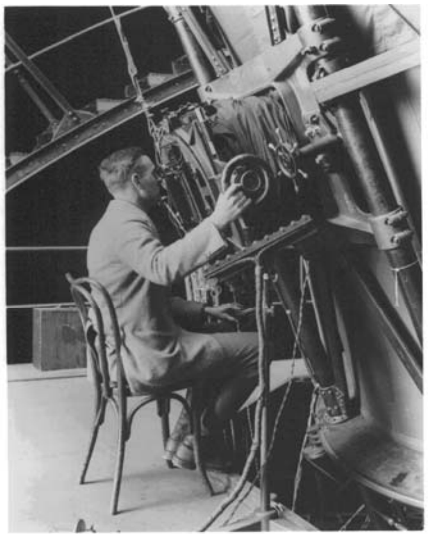


Examples of redshifted galaxy spectra

Hubble's

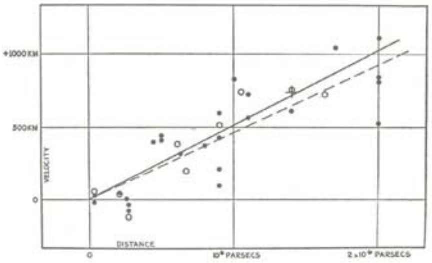
Expanding Universe

Hubble Expansion



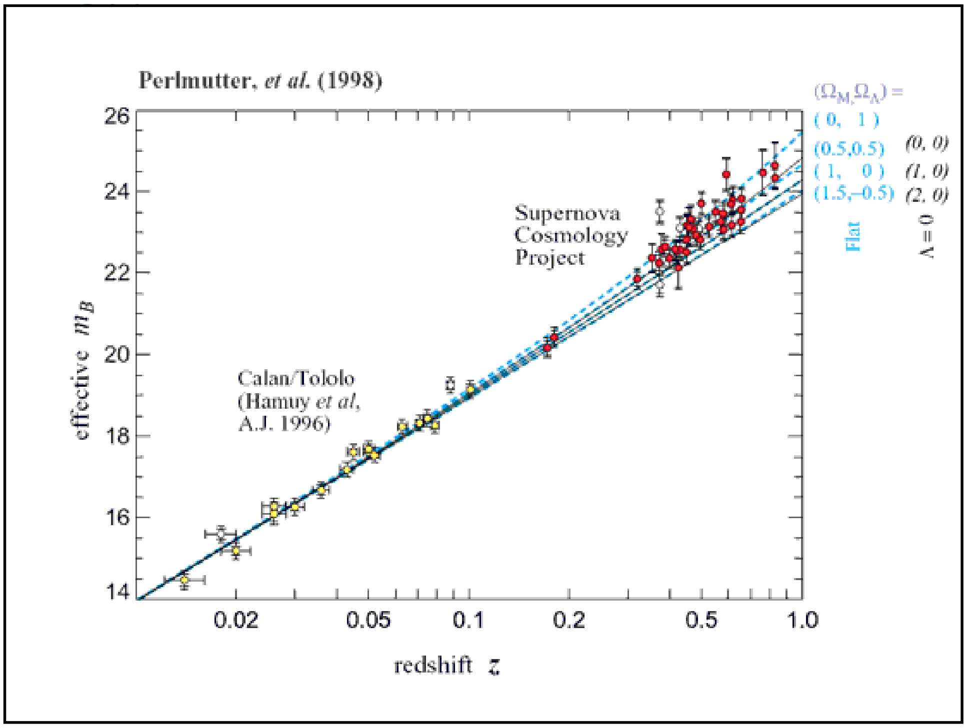
Edwin Hubble

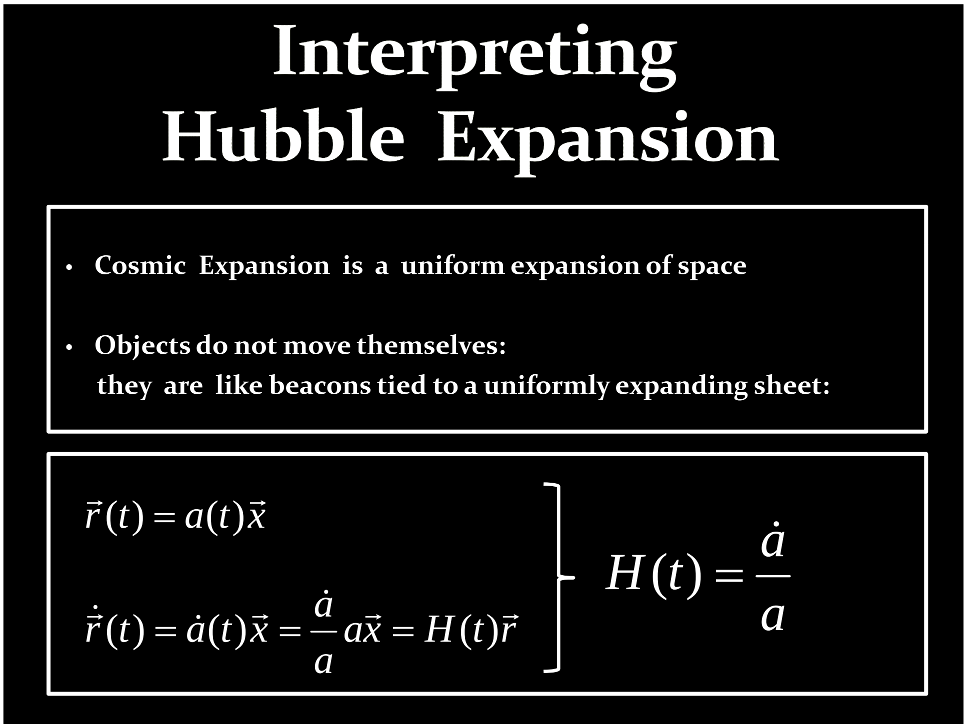
(1889-1953)



$v = H\ r$

Hubble Expansion


Perlmutter, *et al.* (1998)
$(\Omega_M, \Omega_\Lambda) =$
(0, 1)
(0.5,0.5)
(1, 0)
(1.5,–0.5)
Flat
(0, 0)
(1, 0)
(2, 0)
$\Lambda = 0$
Supernova Cosmology Project
Calan/Tololo (Hamuy *et al*, A.J. 1996)
effective m_B
redshift z
26
24
22
20
18
16
14
0.02
0.05
0.1
0.2
0.5
1.0



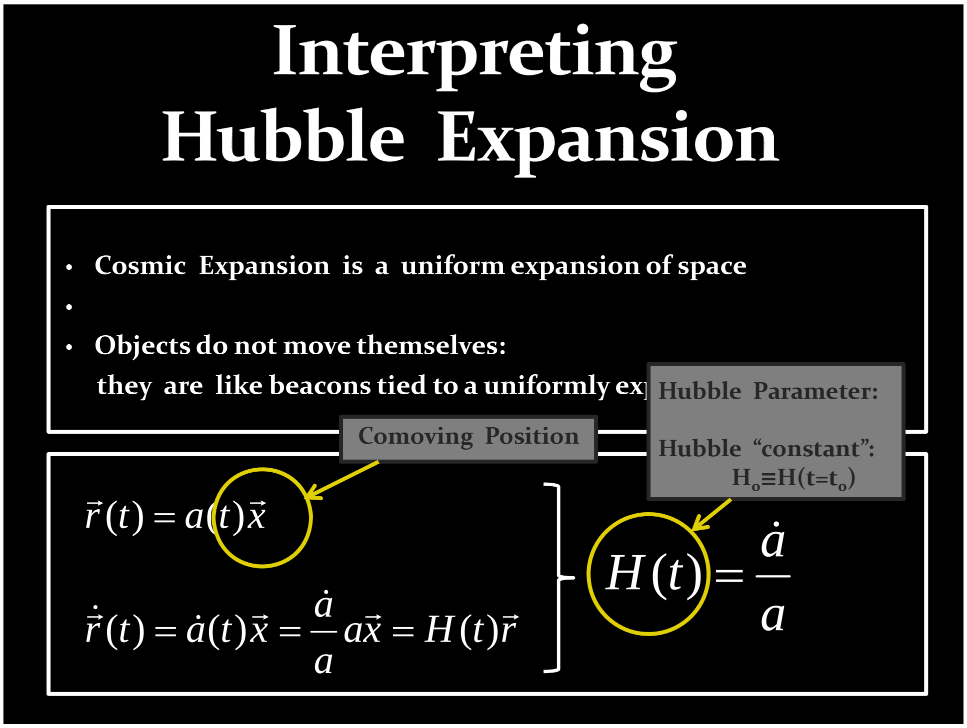
Interpreting Hubble Expansion

- Cosmic Expansion is a uniform expansion of space
- Objects do not move themselves:
  they are like beacons tied to a uniformly expanding sheet:

$$\vec{r}(t) = a(t)\vec{x}$$

$$\dot{\vec{r}}(t) = \dot{a}(t)\vec{x} = \frac{\dot{a}}{a}a\vec{x} = H(t)\vec{r}$$

$$\left.\begin{matrix} \\ \end{matrix}\right\} H(t) = \frac{\dot{a}}{a}$$



Interpreting Hubble Expansion

- Cosmic Expansion is a uniform expansion of space
-
- Objects do not move themselves:
  they are like beacons tied to a uniformly ex

Comoving Position

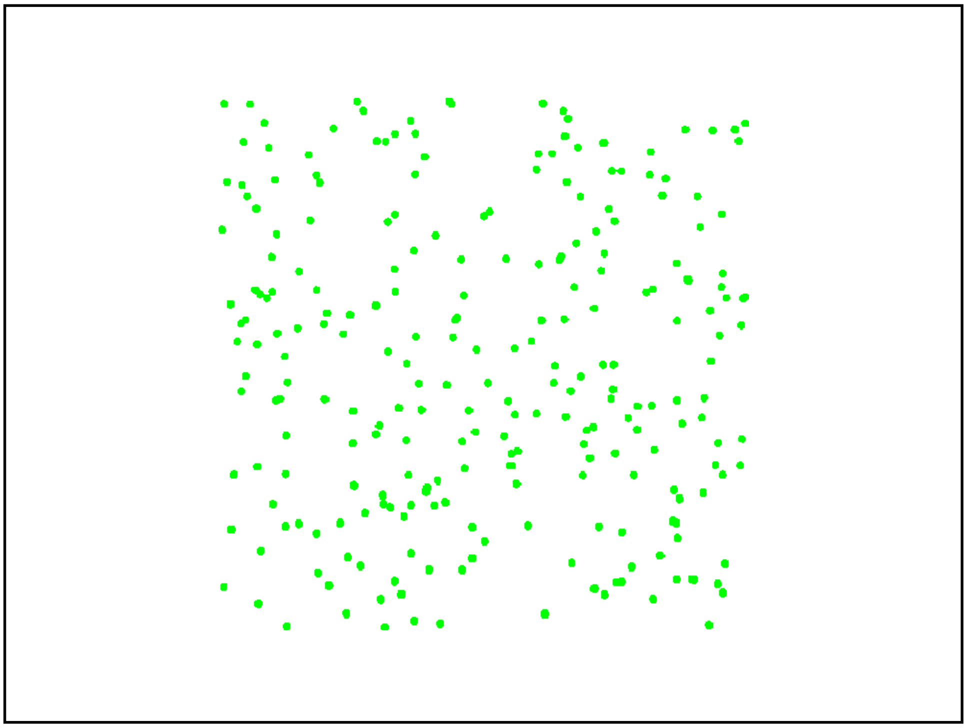
Hubble Parameter:

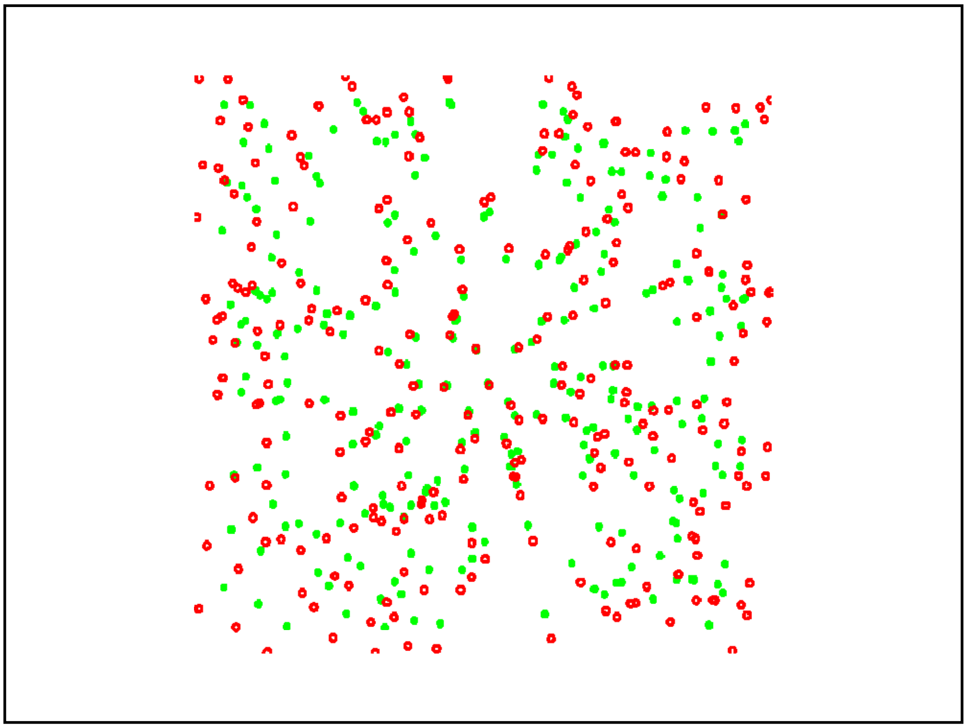
Hubble "constant":
$H_o \equiv H(t=t_o)$

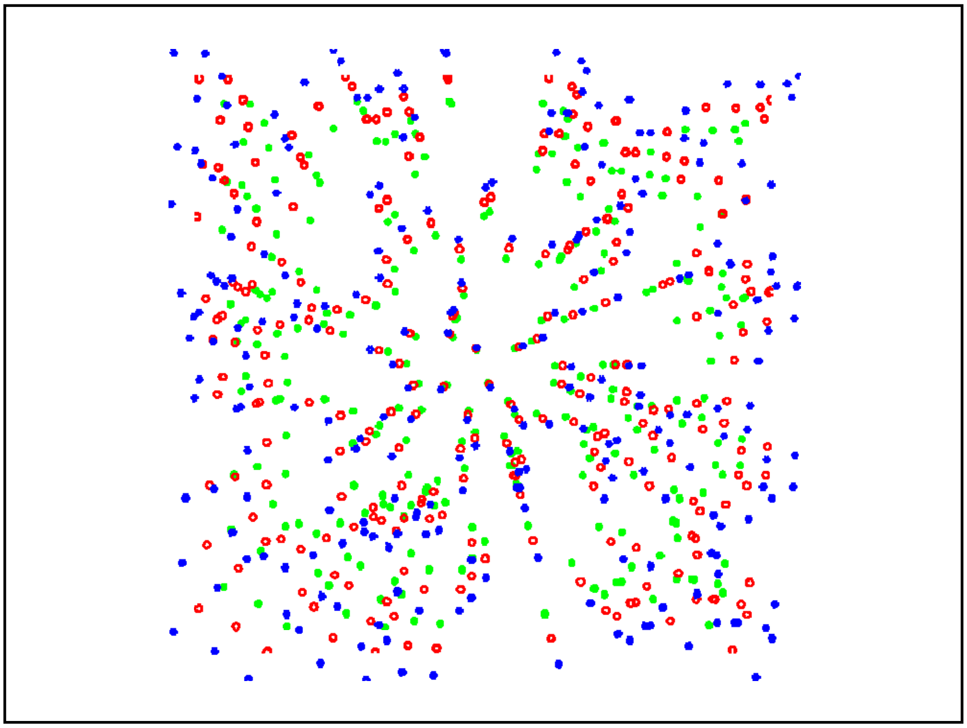
$$\vec{r}(t) = a(t)\vec{x}$$

$$\dot{\vec{r}}(t) = \dot{a}(t)\vec{x} = \frac{\dot{a}}{a}a\vec{x} = H(t)\vec{r}$$

$$\left.\begin{matrix} \\ \end{matrix}\right\} H(t) = \frac{\dot{a}}{a}$$

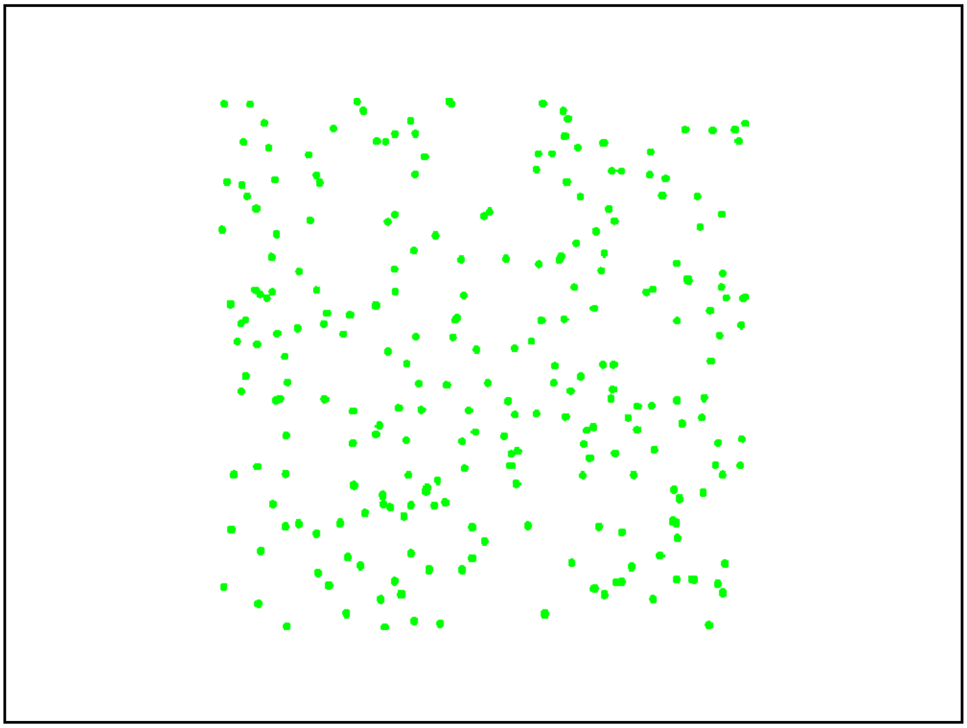


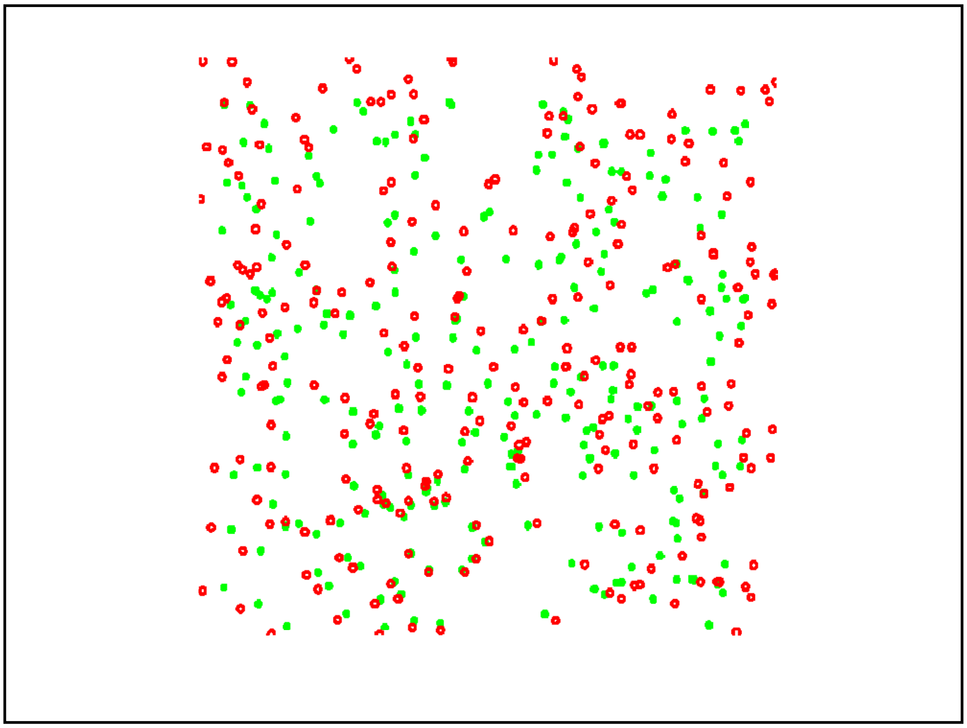


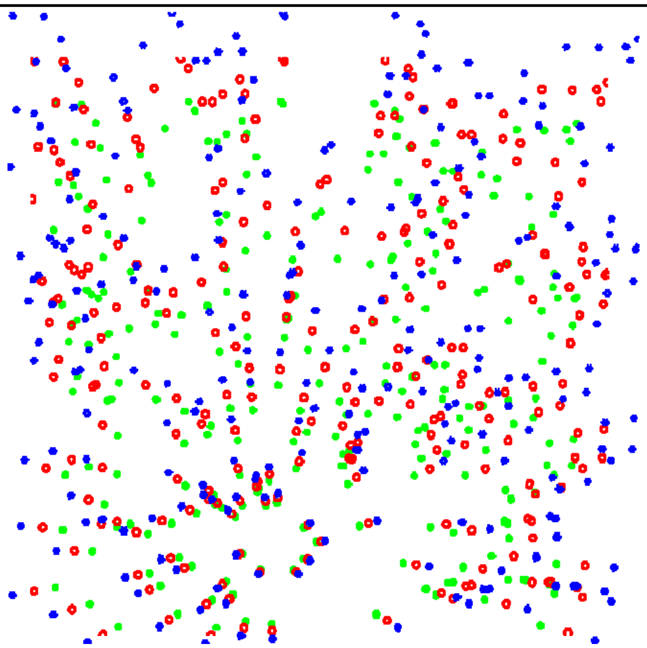


Interpreting Hubble Expansion

- Cosmic Expansion manifests itself in the in a recession velocity which linearly increases with distance
- this is the same for any galaxy within the Universe !
- There is no centre of the Universe: would be in conflict with the Cosmological Principle







Hubble Parameter

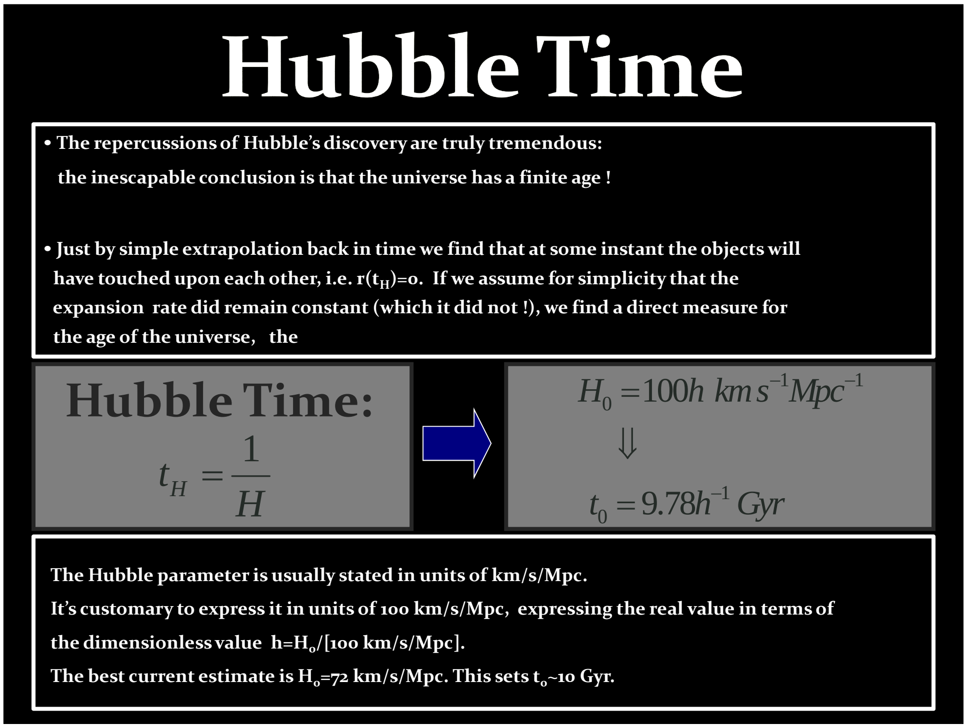
- For a long time, the correct value of the Hubble constant H_o was a major unsettled issue:

H_o = 50 km s^{-1} Mpc^{-1} ⟷ H_o = 100 km s^{-1} Mpc^{-1}

- This meant distances and timescales in the Universe had to deal with uncertainties of a factor 2 !!!

- Following major programs, such as Hubble Key Project, the Supernova key projects and the WMAP CMB measurements,

$$H_0 = 71.9^{+2.6}_{-2.7}\, km\, s^{-1} Mpc^{-1}$$



Hubble Time

- The repercussions of Hubble's discovery are truly tremendous: the inescapable conclusion is that the universe has a finite age !
- Just by simple extrapolation back in time we find that at some instant the objects will have touched upon each other, i.e. $r(t_H)=0$. If we assume for simplicity that the expansion rate did remain constant (which it did not !), we find a direct measure for the age of the universe, the

Hubble Time:

$$t_H = \frac{1}{H}$$

$$H_0 = 100h\ km\,s^{-1} Mpc^{-1}$$

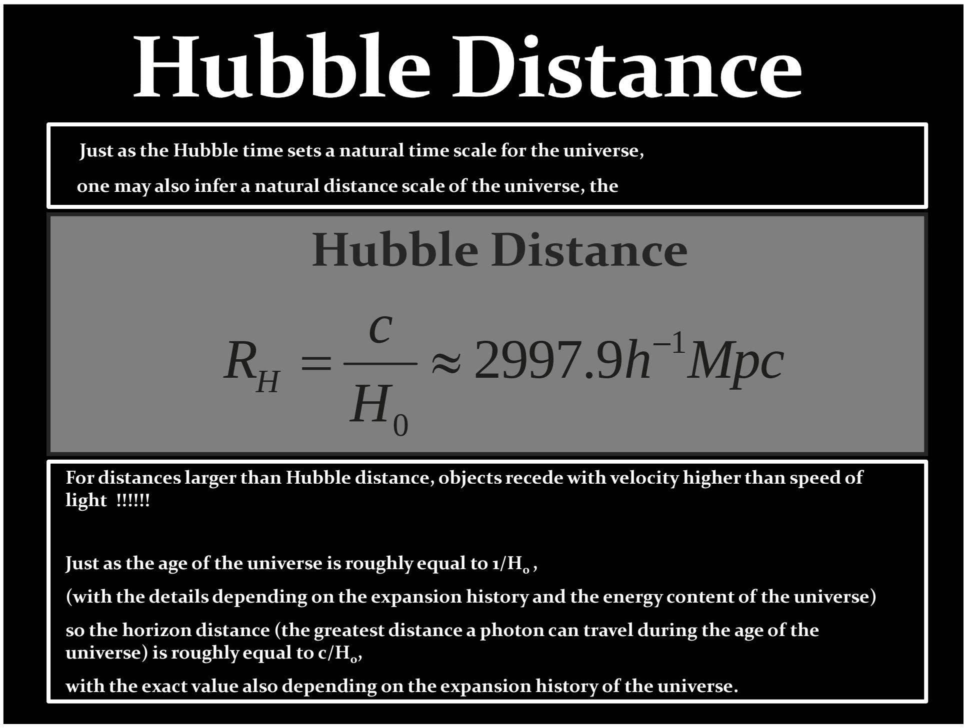
$$\Downarrow$$

$$t_0 = 9.78 h^{-1}\ Gyr$$

The Hubble parameter is usually stated in units of km/s/Mpc.

It's customary to express it in units of 100 km/s/Mpc, expressing the real value in terms of the dimensionless value $h=H_o/[100\ km/s/Mpc]$.

The best current estimate is H_o=72 km/s/Mpc. This sets t_o~10 Gyr.



Hubble Distance

Just as the Hubble time sets a natural time scale for the universe, one may also infer a natural distance scale of the universe, the

Hubble Distance

$$R_H = \frac{c}{H_0} \approx 2997.9 h^{-1} Mpc$$

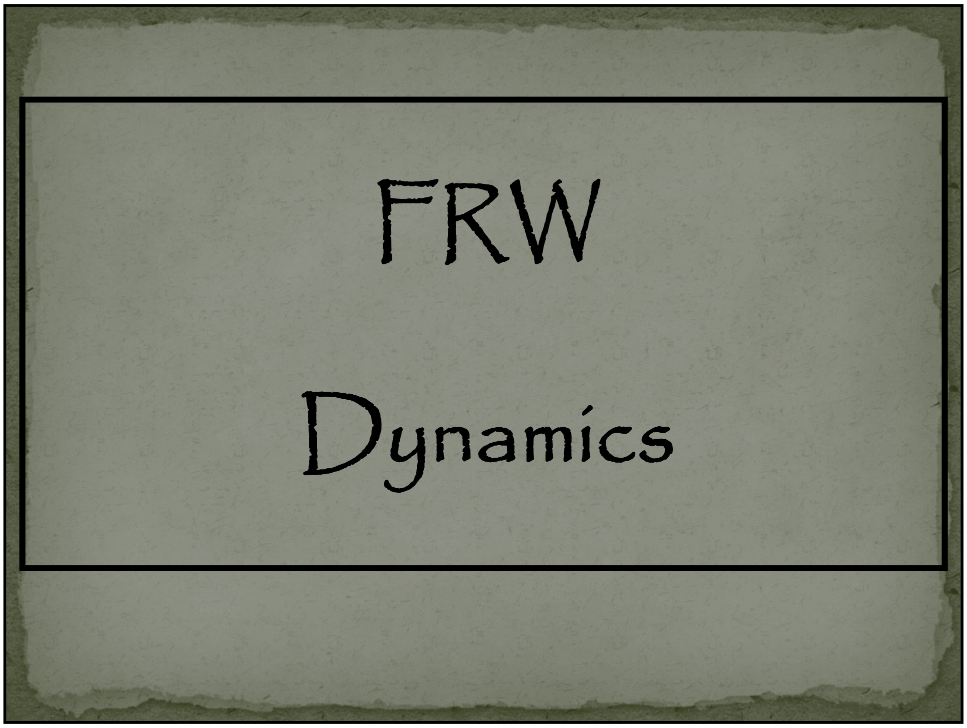
For distances larger than Hubble distance, objects recede with velocity higher than speed of light !!!!!!

Just as the age of the universe is roughly equal to $1/H_o$,

(with the details depending on the expansion history and the energy content of the universe)

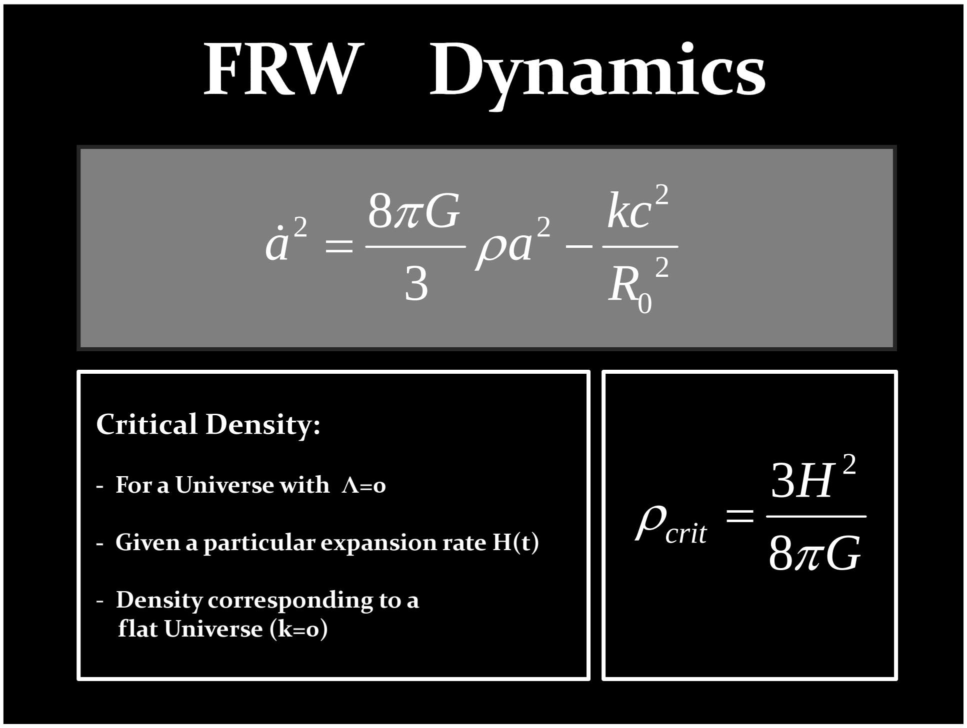
so the horizon distance (the greatest distance a photon can travel during the age of the universe) is roughly equal to c/H_o,

with the exact value also depending on the expansion history of the universe.



FRW

Dynamics



FRW Dynamics

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0{}^2}$$

Critical Density:

- For a Universe with Λ=o
- Given a particular expansion rate H(t)
- Density corresponding to a flat Universe (k=o)

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

FRW Dynamics

In a FRW Universe,
densities are in the order of the critical density,

$$\rho_{crit} = \frac{3H_0^{\ 2}}{8\pi G} = 1.8791 h^2 \times 10^{-29}\ g\ cm^{-3}$$

$$\begin{aligned}\rho_0 &= 1.8791 \times 10^{-29}\ \Omega h^2\ \ g\ cm^{-3} \\ &= 2.78 \times 10^{11}\ \Omega h^2 \quad\ M_{\odot} Mpc^{-3}\end{aligned}$$

FRW Dynamics

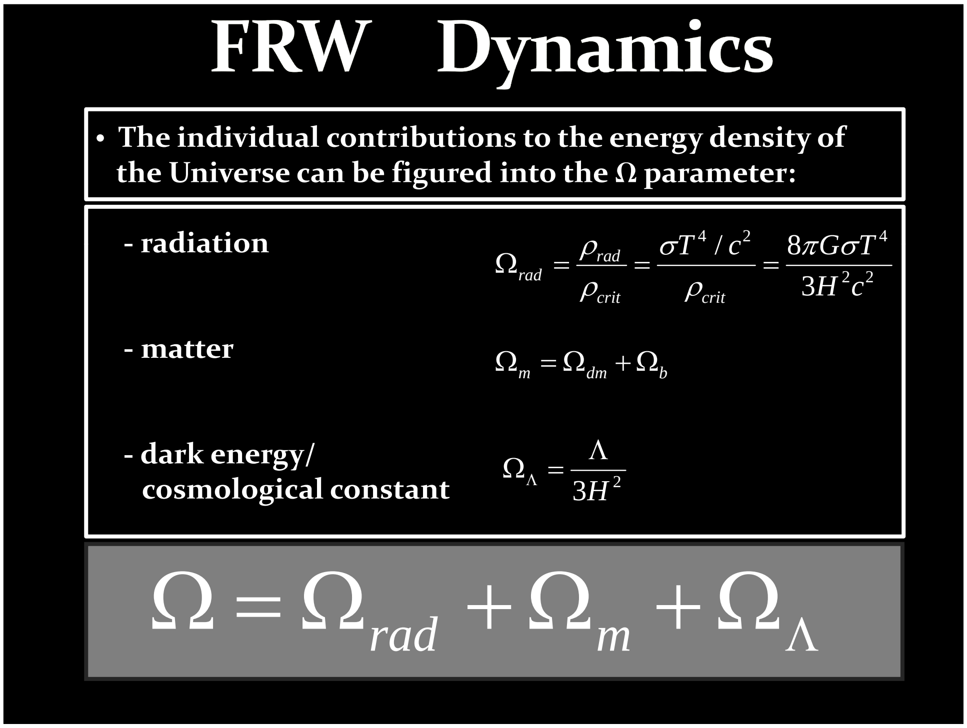
In a matter-dominated Universe,
the evolution and fate of the Universe entirely determined
by the (energy) density in units of critical density:

$$\Omega \equiv \frac{\rho}{\rho_{crit}} = \frac{8\pi G \rho}{3H^2}$$

Arguably, Ω is the most important parameter of cosmology !!!

Present-day
Cosmic Density:

$$\boxed{\begin{aligned}\rho_0 &= 1.8791 \times 10^{-29}\ \Omega h^2\ \ g\ cm^{-3} \\ &= 2.78 \times 10^{11}\ \Omega h^2 \quad\ M_{\odot} Mpc^{-3}\end{aligned}}$$



FRW Dynamics

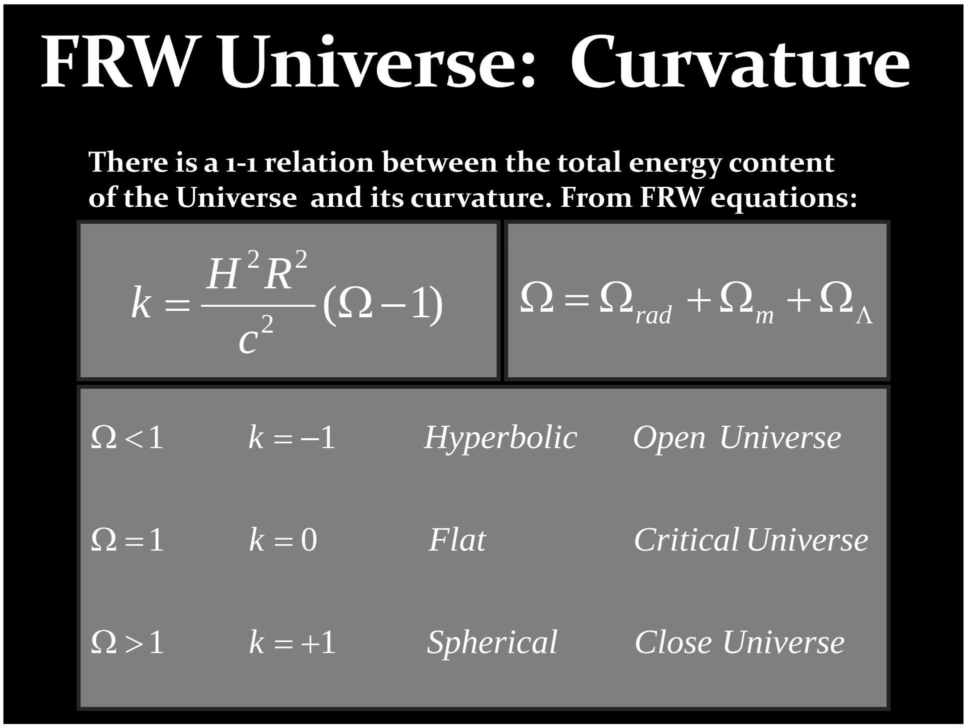
- The individual contributions to the energy density of the Universe can be figured into the Ω parameter:

- radiation $\Omega_{rad} = \frac{\rho_{rad}}{\rho_{crit}} = \frac{\sigma T^4 / c^2}{\rho_{crit}} = \frac{8\pi G \sigma T^4}{3H^2 c^2}$

- matter $\Omega_m = \Omega_{dm} + \Omega_b$

- dark energy/ cosmological constant $\Omega_\Lambda = \frac{\Lambda}{3H^2}$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$



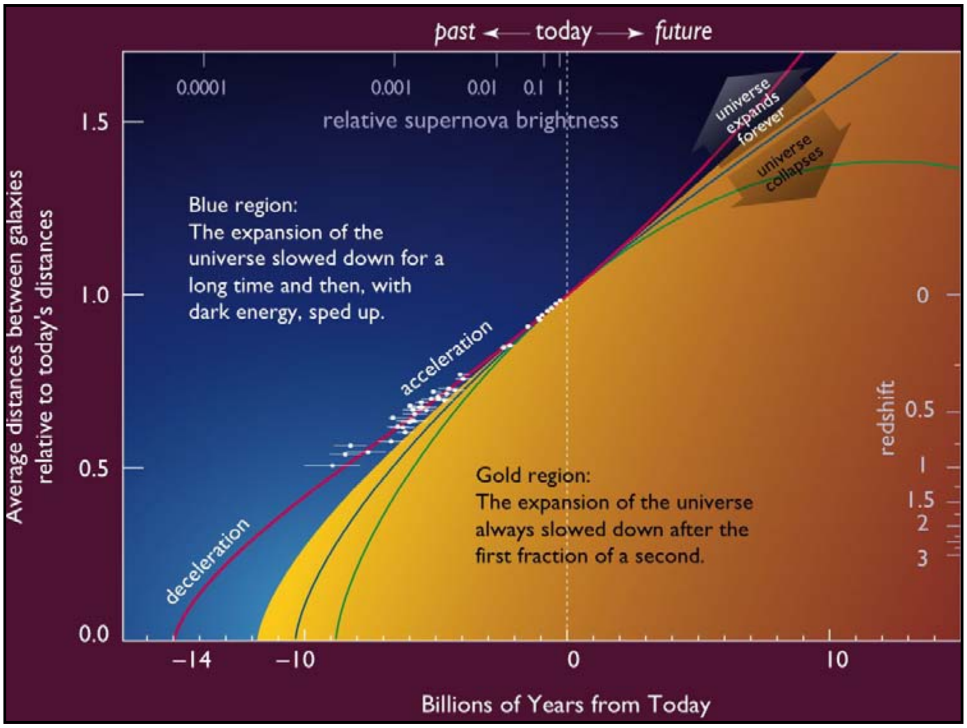
FRW Universe: Curvature

There is a 1-1 relation between the total energy content of the Universe and its curvature. From FRW equations:

$$k = \frac{H^2 R^2}{c^2}(\Omega - 1)$$

$$\Omega = \Omega_{rad} + \Omega_m + \Omega_\Lambda$$

$\Omega < 1$	$k = -1$	*Hyperbolic*	*Open Universe*
$\Omega = 1$	$k = 0$	*Flat*	*Critical Universe*
$\Omega > 1$	$k = +1$	*Spherical*	*Close Universe*



FRW Dynamics:
Cosmic Acceleration

Cosmic acceleration quantified
by means of dimensionless deceleration parameter q(t):

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

$$q = \frac{\Omega_m}{2} + \Omega_{rad} - \Omega_\Lambda$$

$$q \approx \frac{\Omega_m}{2} - \Omega_\Lambda$$

Examples:

$\Omega_m = 1; \quad \Omega_\Lambda = 0;$
$q = 0.5$

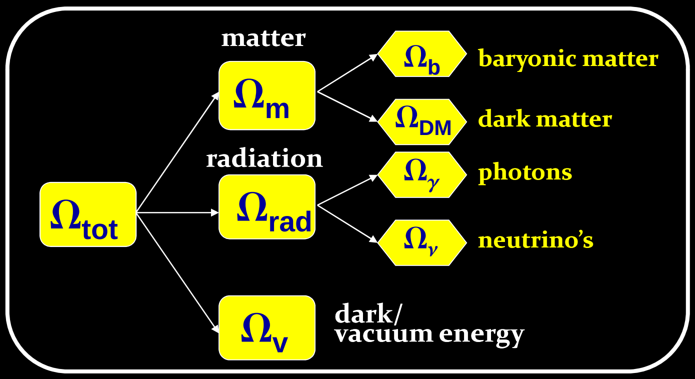
$\Omega_m = 0.3; \ \Omega_\Lambda = 0.7;$
$q = -0.65$

The Elements:

What does
the Universe consist of ?

Cosmic Constituents

The total energy content of Universe made up by various constituents, principal ones:



Ω_{tot}

matter: Ω_m
- Ω_b baryonic matter
- Ω_{DM} dark matter

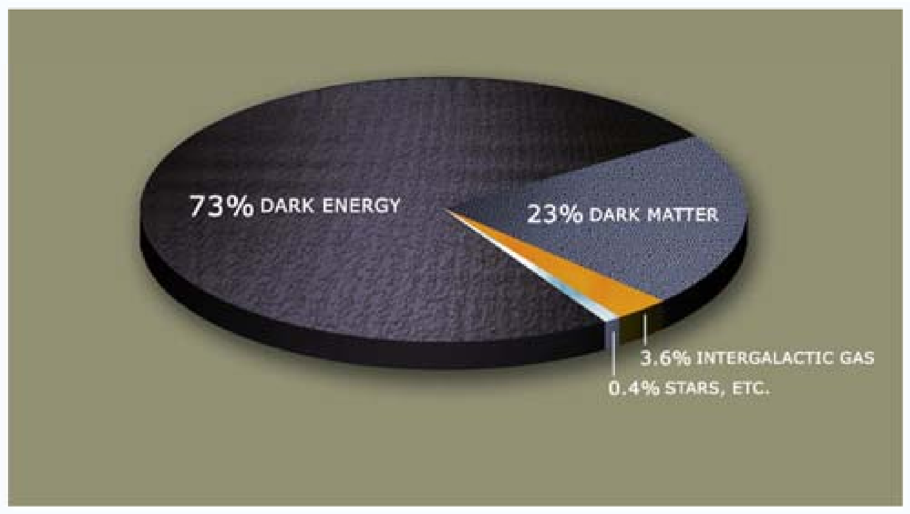
radiation: Ω_{rad}
- Ω_γ photons
- Ω_ν neutrino's

Ω_v dark/ vacuum energy

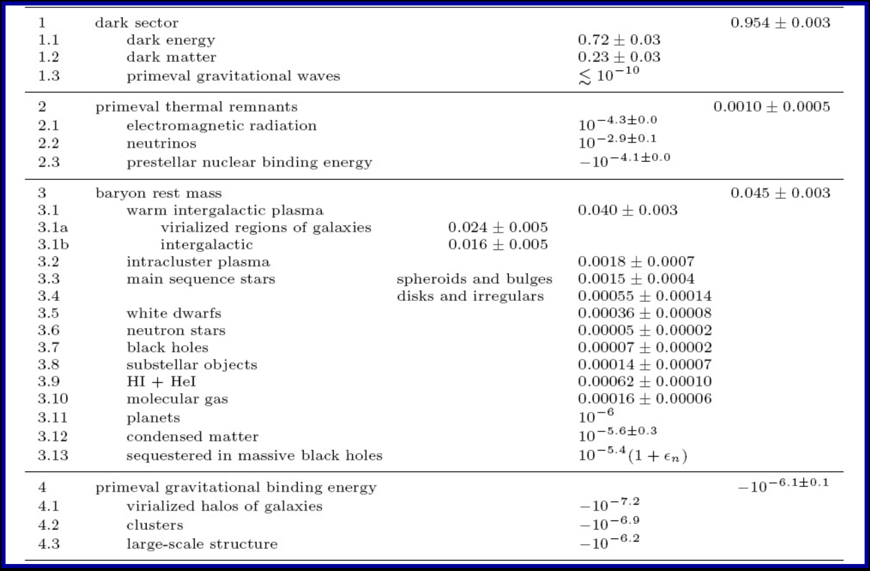
In addition, contributions by
- gravitational waves
- magnetic fields,
- cosmic rays ...

Poor constraints on their contribution: henceforth we will not take them into account !

Cosmic Constituents



Cosmic Energy Inventory



1	dark sector			0.954 ± 0.003
1.1	dark energy		0.72 ± 0.03	
1.2	dark matter		0.23 ± 0.03	
1.3	primeval gravitational waves		$\lesssim 10^{-10}$	
2	primeval thermal remnants			0.0010 ± 0.0005
2.1	electromagnetic radiation		$10^{-4.3\pm0.0}$	
2.2	neutrinos		$10^{-2.9\pm0.1}$	
2.3	prestellar nuclear binding energy		$-10^{-4.1\pm0.0}$	
3	baryon rest mass			0.045 ± 0.003
3.1	warm intergalactic plasma		0.040 ± 0.003	
3.1a	virialized regions of galaxies	0.024 ± 0.005		
3.1b	intergalactic	0.016 ± 0.005		
3.2	intracluster plasma		0.0018 ± 0.0007	
3.3	main sequence stars	spheroids and bulges	0.0015 ± 0.0004	
3.4		disks and irregulars	0.00055 ± 0.00014	
3.5	white dwarfs		0.00036 ± 0.00008	
3.6	neutron stars		0.00005 ± 0.00002	
3.7	black holes		0.00007 ± 0.00002	
3.8	substellar objects		0.00014 ± 0.00007	
3.9	HI + HeI		0.00062 ± 0.00010	
3.10	molecular gas		0.00016 ± 0.00006	
3.11	planets		10^{-6}	
3.12	condensed matter		$10^{-5.6\pm0.3}$	
3.13	sequestered in massive black holes		$10^{-5.4}(1+\epsilon_n)$	
4	primeval gravitational binding energy			$-10^{-6.1\pm0.1}$
4.1	virialized halos of galaxies		$-10^{-7.2}$	
4.2	clusters		$-10^{-6.9}$	
4.3	large-scale structure		$-10^{-6.2}$	

Fukugita & Peebles 2004

Cosmic Constituents:

Equation of State

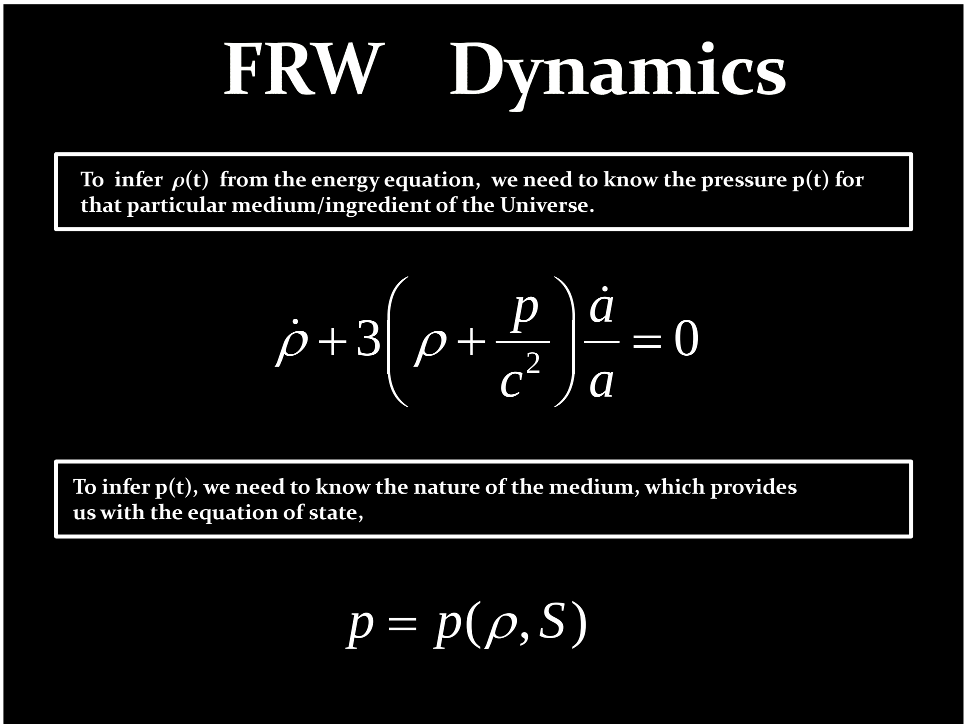
The equations of state for the three classes of <u>cosmologically</u> relevant constituents:

$$
\begin{array}{rlllll}
\text{radiation} & p(\rho) & = \frac{1}{3}\rho c^2 & \Rightarrow & w = \frac{1}{3} & \\
\text{matter} & p(\rho) & = 0 & \Rightarrow & w = 0 & \\
\text{dark energy} & p(\rho) & = -\rho c^2 & \Rightarrow & w = -1 & (\Lambda) \\
 & & = w \rho c^2 & \Rightarrow & -1 < w < -1/3 & (\mathit{general}) \\
 & & = w \rho c^2 & \Rightarrow & w < -1 & (\mathit{phantom})
\end{array}
$$

excluding 1) gravitational waves, 2) magnetic fields, 3) …

Expansion

of FRW Universes



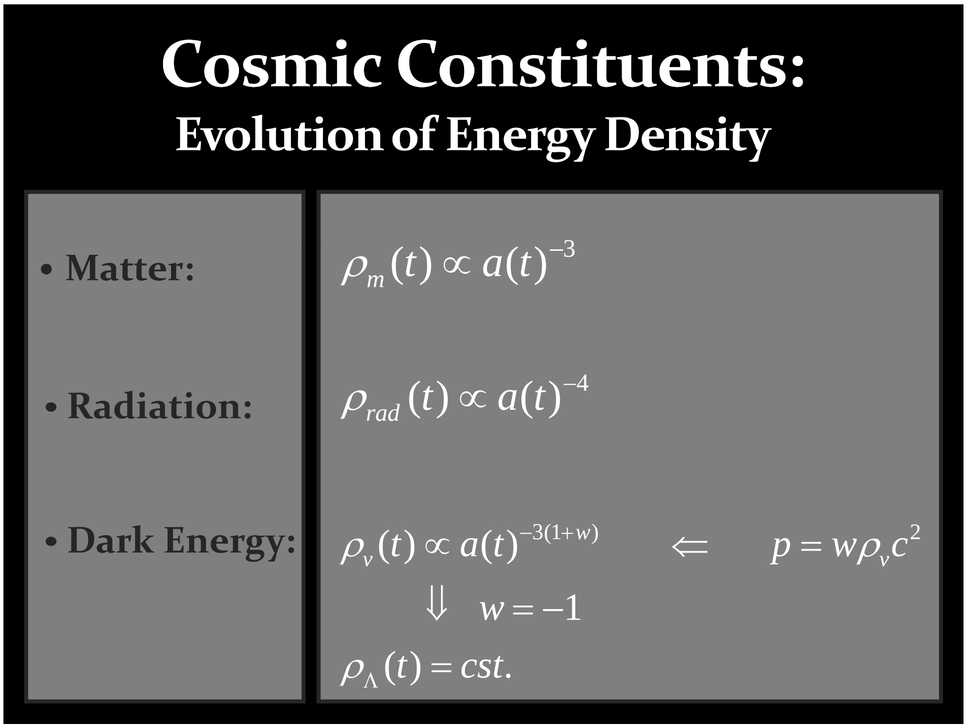
FRW Dynamics

To infer $\rho(t)$ from the energy equation, we need to know the pressure p(t) for that particular medium/ingredient of the Universe.

$$\dot{\rho}+3\left(\rho+\frac{p}{c^2}\right)\frac{\dot{a}}{a}=0$$

To infer p(t), we need to know the nature of the medium, which provides us with the equation of state,

$$p=p(\rho,S)$$



Cosmic Constituents:

Evolution of Energy Density

• **Matter:**	$\rho_m(t)\propto a(t)^{-3}$
• **Radiation:**	$\rho_{rad}(t)\propto a(t)^{-4}$
• **Dark Energy:**	$\rho_v(t)\propto a(t)^{-3(1+w)} \quad \Leftarrow \quad p=w\rho_v c^2$ $\Downarrow \quad w=-1$ $\rho_\Lambda(t)=cst.$

General Solution
Expanding FRW Universe

From the FRW equations:

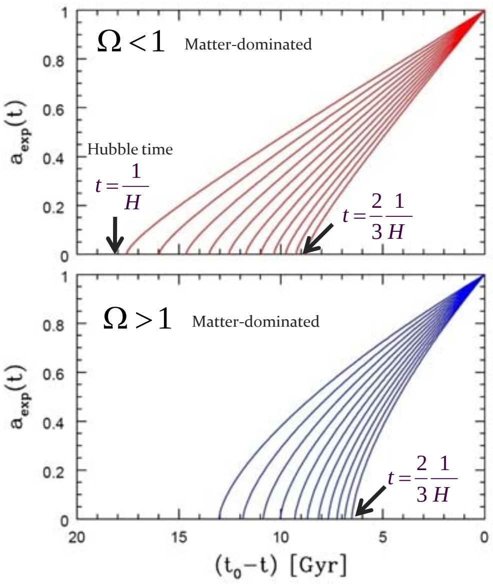
$$\frac{H(t)^2}{H_0{}^2} = \frac{\Omega_{rad,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$\Downarrow \quad a(t)$$

Expansion history
Universe

$$H_0 t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0} a^2 + (1-\Omega_0)}}$$

Age of the Universe



Age of a FRW universe at
Expansion factor a(t)

$$H\,t = \int_0^a \frac{da}{\sqrt{\frac{\Omega_{rad}}{a^2} + \frac{\Omega_m}{a} + \Omega_\Lambda a^2 + (1-\Omega\ )}}$$

Specific Solutions
FRW Universe

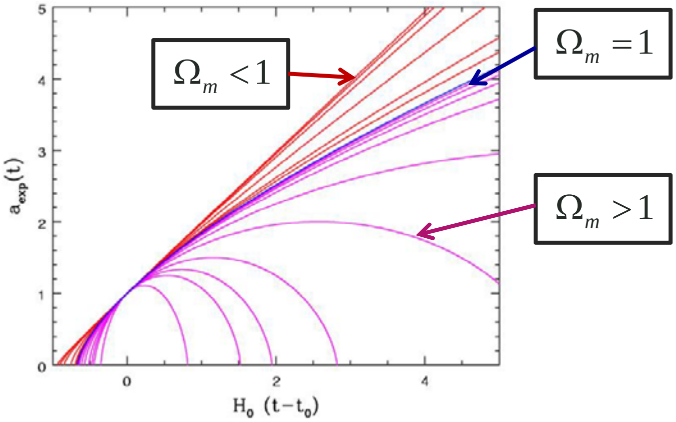
While general solutions to the FRW equations is only possible by numerical integration, analytical solutions may be found for particular classes of cosmologies:

- Single-component Universes:
  - empty Universe
  - flat Universes, with only radiation, matter or dark energy
- Matter-dominated Universes
- Matter+Dark Energy flat Universe

Matter-Dominated Universes

- Assume radiation contribution is negligible:
- Zero cosmological constant:
- Matter-dominated, including curvature

$$\Omega_{rad,0} \approx 5\times10^{-5}$$
$$\Omega_{\Lambda} = 0$$



$\Omega_m < 1$

$\Omega_m = 1$

$\Omega_m > 1$

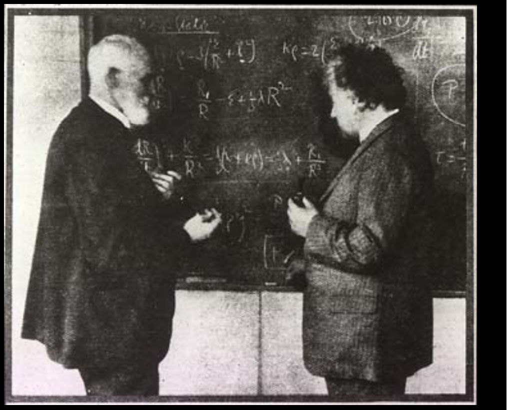
Einstein-de Sitter Universe

$$\left.\begin{array}{l}\Omega_m = 1\\ \Omega_\Lambda = 0\end{array}\right\} k = 0$$

FRW: $\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 = \frac{8\pi G\rho_0}{3}\frac{1}{a}$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Age
EdS Universe: $t_0 = \frac{2}{3}\frac{1}{H_0}$



Albert Einstein and Willem de Sitter discussing the Universe. In 1932 they published a paper together on the Einstein-de Sitter universe, which is a model with flat geometry containing matter as the only significant substance.

Free Expanding "Milne" Universe

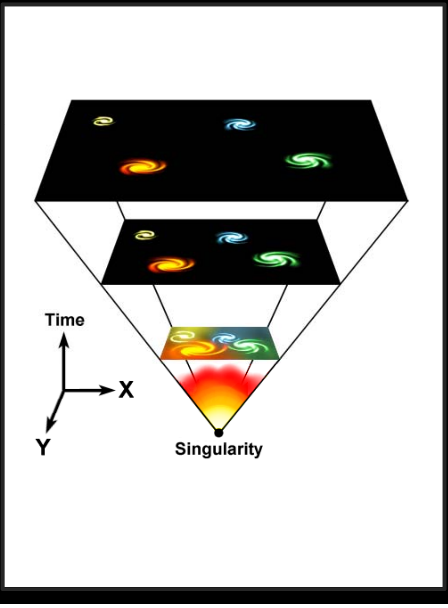
$$\left.\begin{array}{l}\Omega_m = 0\\ \Omega_\Lambda = 0\end{array}\right\} k = -1$$

Empty space is curved

FRW: $\dot{a}^2 = -\frac{kc^2}{R_0^2} = cst.$

$$a(t) = \left(\frac{t}{t_0}\right)$$

Age
Empty Universe: $t_0 = \frac{1}{H_0}$



De Sitter Expansion

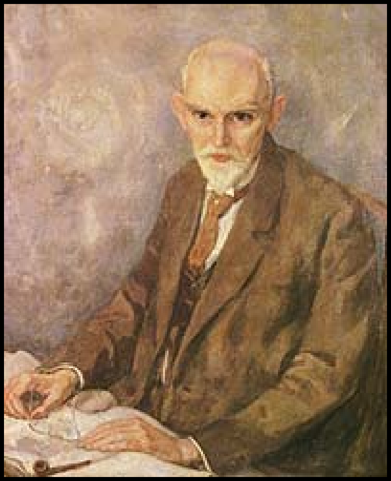
$$\left.\begin{array}{l}\Omega_m = 0\\ \Omega_\Lambda = 1\end{array}\right\} k = 0$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^{\,2}} \quad \Rightarrow \quad H_0 = \sqrt{\frac{\Lambda}{3}}$$

FRW: $\dot{a}^2 = \frac{\Lambda}{3}a^2 \quad \Rightarrow \quad \dot{a} = H_0\,a$

$$\boxed{a(t) = e^{H_0(t-t_0)}}$$

Age
De Sitter Universe: infinitely old



Willem de Sitter (1872-1934; Sneek-Leiden)
director Leiden Observatory
alma mater: Groningen University

Expansion
Radiation-dominated Universe

$$\left.\begin{array}{l}\Omega_{rad} = 1\\ \Omega_m = 0\\ \Omega_\Lambda = 0\end{array}\right\} k = 0$$

In the very early Universe, the energy density is completely dominated by radiation. The dynamics of the very early Universe is therefore fully determined by the evolution of the radiation energy density:

FRW: $\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 = \frac{8\pi G \rho_0}{3}\frac{1}{a^2} \quad \longleftarrow \quad \rho_{rad}(a) \propto \frac{1}{a^4}$

$$\boxed{a(t) = \left(\frac{t}{t_0}\right)^{1/2}}$$

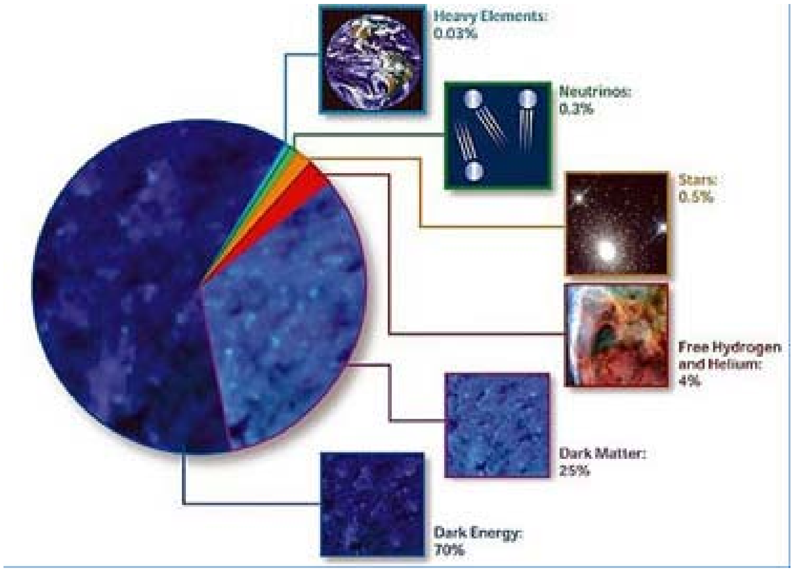
Age
Radiation
Universe:

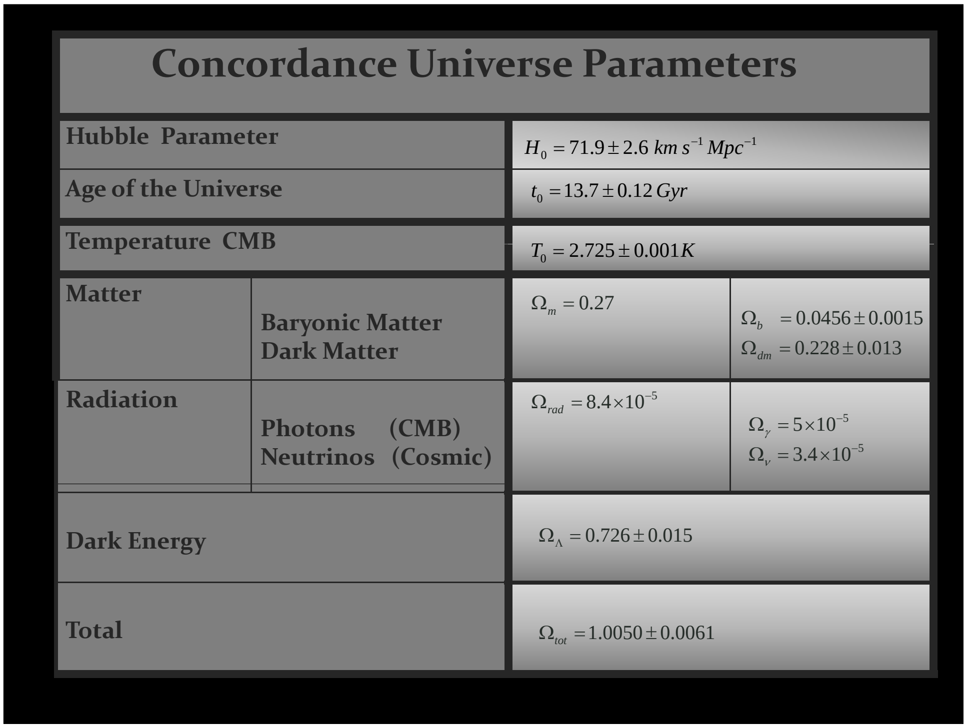
$$\boxed{t_0 = \frac{1}{2}\frac{1}{H_0}}$$

Our Universe

Concordance Cosmology

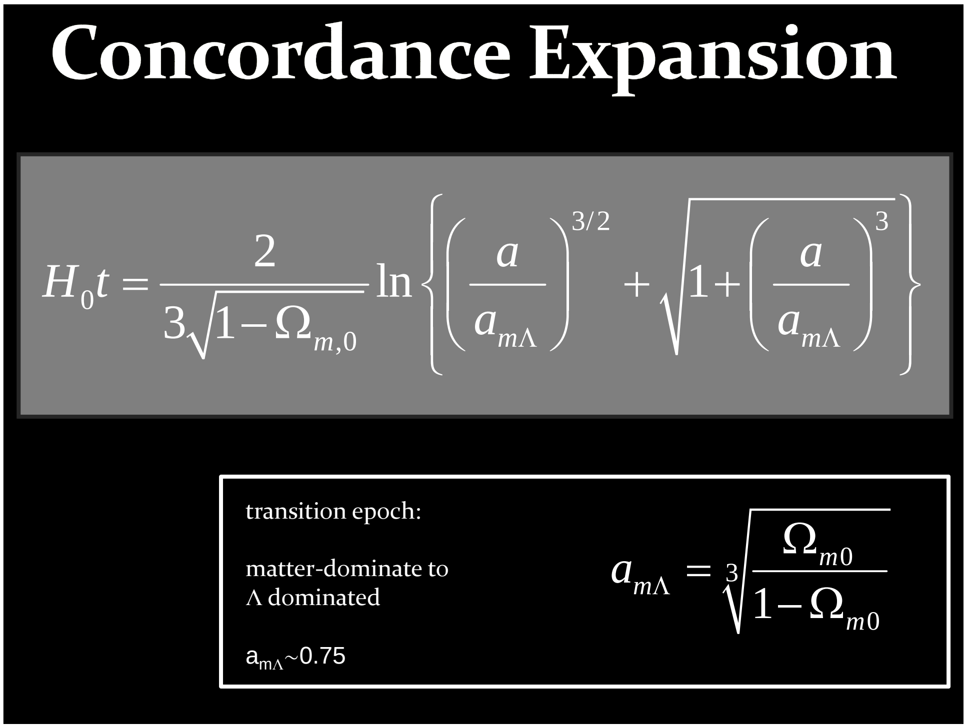
Cosmic Constituents





Concordance Universe Parameters

Hubble Parameter		$H_0 = 71.9 \pm 2.6\ km\ s^{-1}\ Mpc^{-1}$	
Age of the Universe		$t_0 = 13.7 \pm 0.12\ Gyr$	
Temperature CMB		$T_0 = 2.725 \pm 0.001 K$	
Matter	**Baryonic Matter** **Dark Matter**	$\Omega_m = 0.27$	$\Omega_b = 0.0456 \pm 0.0015$ $\Omega_{dm} = 0.228 \pm 0.013$
Radiation	**Photons (CMB)** **Neutrinos (Cosmic)**	$\Omega_{rad} = 8.4 \times 10^{-5}$	$\Omega_\gamma = 5 \times 10^{-5}$ $\Omega_\nu = 3.4 \times 10^{-5}$
Dark Energy		$\Omega_\Lambda = 0.726 \pm 0.015$	
Total		$\Omega_{tot} = 1.0050 \pm 0.0061$	



Concordance Expansion

$$H_0 t = \frac{2}{3\sqrt{1-\Omega_{m,0}}} \ln\left\{\left(\frac{a}{a_{m\Lambda}}\right)^{3/2} + \sqrt{1+\left(\frac{a}{a_{m\Lambda}}\right)^{3}}\right\}$$

transition epoch:

matter-dominate to
Λ dominated

$a_{m\Lambda}$~0.75

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m0}}{1-\Omega_{m0}}}$$

Concordance Expansion

We can recognize two extreme regimes:

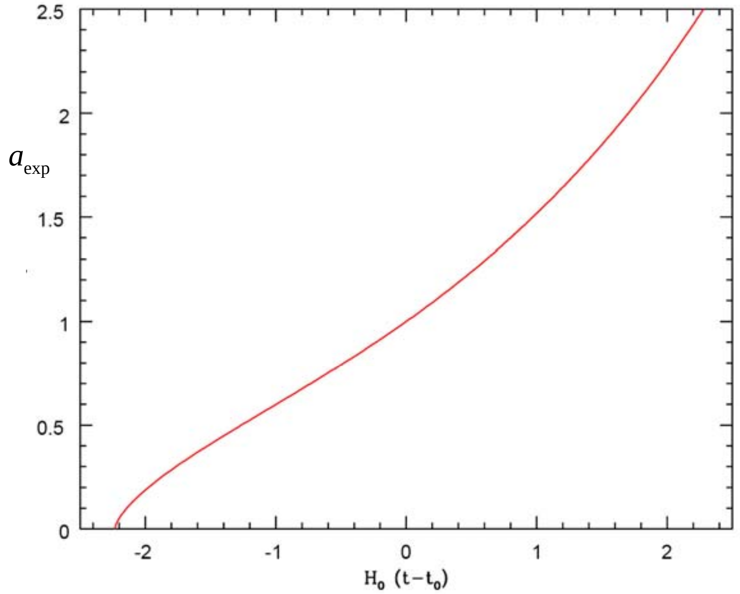
- $a \ll a_{m\Lambda}$ very early times
  matter dominates the expansion, and $\Omega_m \approx 1$: Einstein-de Sitter expansion,

$$a(t) \approx \left(\frac{3}{2} \sqrt{\Omega_{m,0}} H_0 t \right)^{2/3}$$

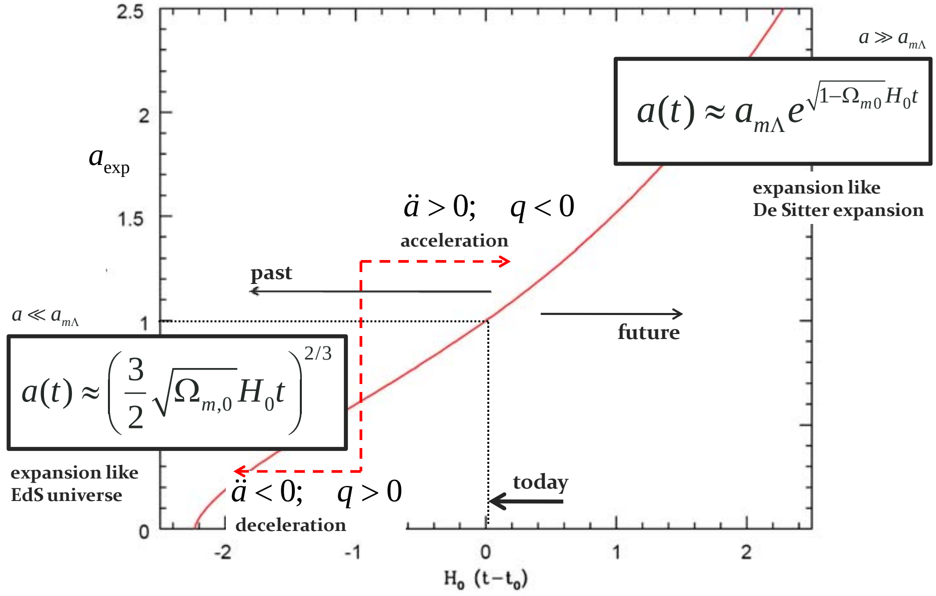
- $a \gg a_{m\Lambda}$ very late times
  matter has diluted to oblivion, and $\Omega_m \approx 0$: de Sitter expansion driven by dark energy

$$a(t) \approx a_{m\Lambda} e^{\sqrt{1-\Omega_{m0}} H_0 t}$$

Concordance Expansion



Concordance Expansion



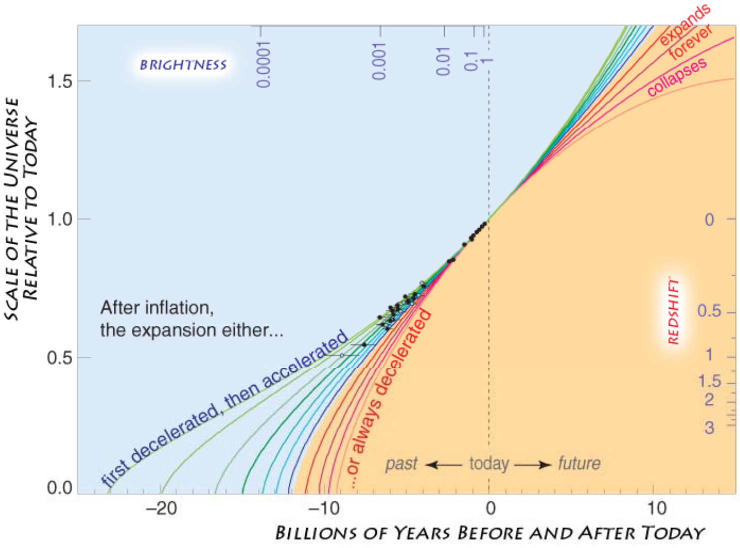
$$a(t) \approx a_{m\Lambda} e^{\sqrt{1-\Omega_{m0}}H_0 t}$$

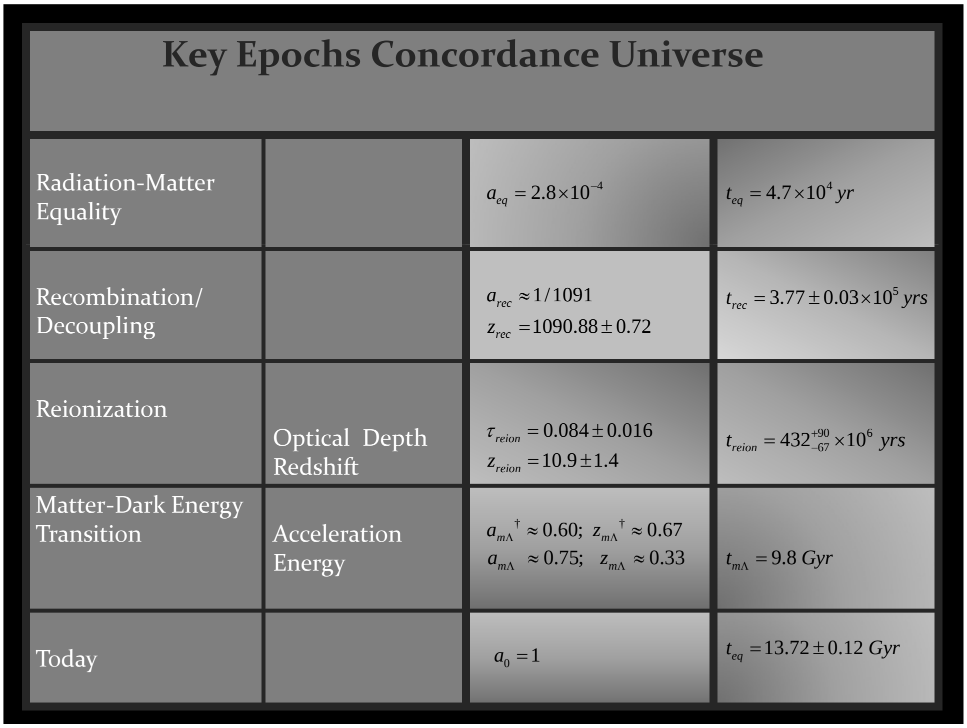
expansion like De Sitter expansion

$$a(t) \approx \left(\frac{3}{2}\sqrt{\Omega_{m,0}}H_0 t\right)^{2/3}$$

expansion like EdS universe

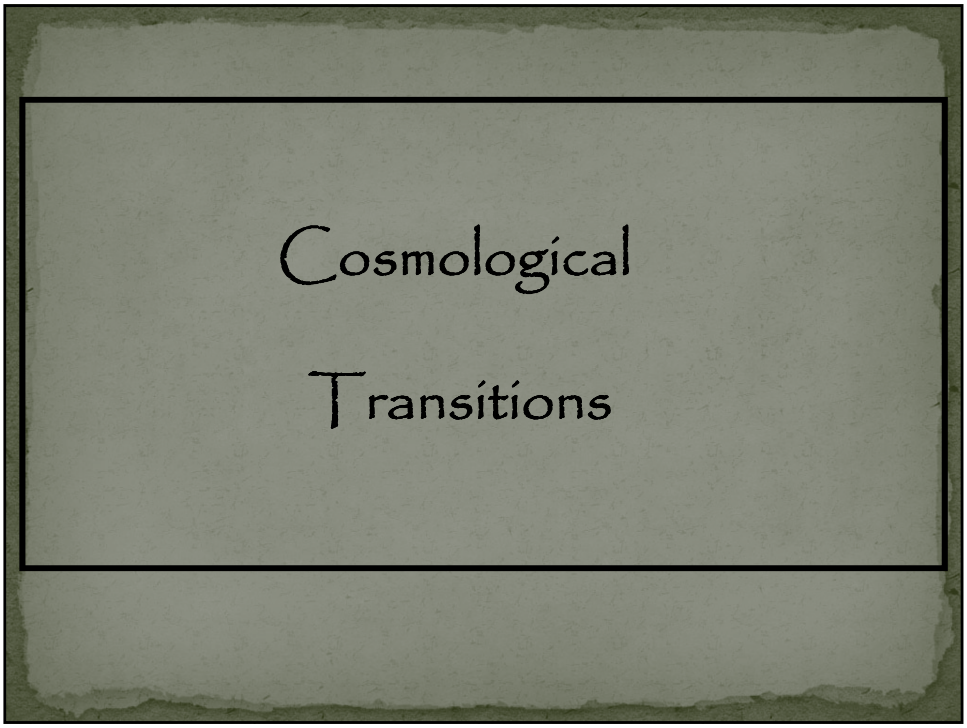
Concordance Expansion





Key Epochs Concordance Universe

Radiation-Matter Equality		$a_{eq} = 2.8 \times 10^{-4}$	$t_{eq} = 4.7 \times 10^4 \, yr$
Recombination/ Decoupling		$a_{rec} \approx 1/1091$ $z_{rec} = 1090.88 \pm 0.72$	$t_{rec} = 3.77 \pm 0.03 \times 10^5 \, yrs$
Reionization	Optical Depth Redshift	$\tau_{reion} = 0.084 \pm 0.016$ $z_{reion} = 10.9 \pm 1.4$	$t_{reion} = 432^{+90}_{-67} \times 10^6 \; yrs$
Matter-Dark Energy Transition	Acceleration Energy	$a_{m\Lambda}{}^{\dagger} \approx 0.60; \; z_{m\Lambda}{}^{\dagger} \approx 0.67$ $a_{m\Lambda} \approx 0.75; \; z_{m\Lambda} \approx 0.33$	$t_{m\Lambda} = 9.8 \, Gyr$
Today		$a_0 = 1$	$t_{eq} = 13.72 \pm 0.12 \, Gyr$



Cosmological Transitions

Dynamical Transitions

Because radiation, matter, dark energy (and curvature) of the Universe evolve differently as the Universe expands, at different epochs the energy density of the Universe is alternately dominated by these different ingredients.

As the Universe is dominated by either radiation, matter, curvature or dark energy, the cosmic expansion a(t) proceeds differently.

We therefore recognize the following epochs:

- radiation-dominated era
- matter-dominated era
- curvature-dominated expansion
- dark energy dominated epoch

The different cosmic expansions at these eras have a huge effect on relevant physical processes

---

Dynamical Transitions

- Radiation Density Evolution

$$\rho_{rad}(t) = \frac{1}{a^4}\,\rho_{rad,0}$$

- Matter Density Evolution

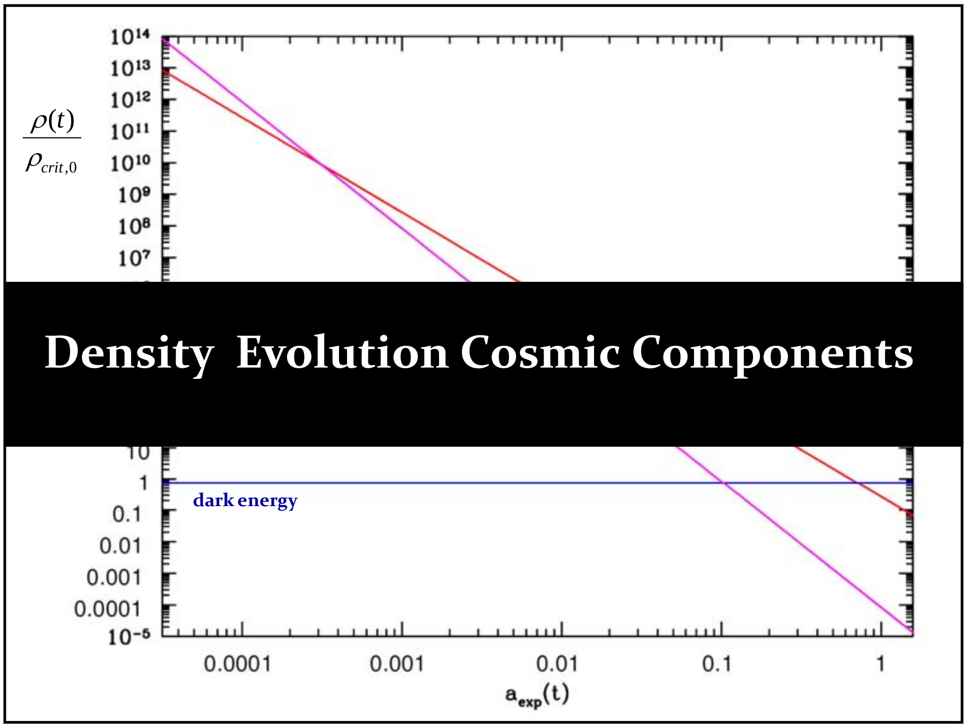
$$\rho_m(t) = \frac{1}{a^3}\,\rho_{m,0}$$

- Curvature Evolution

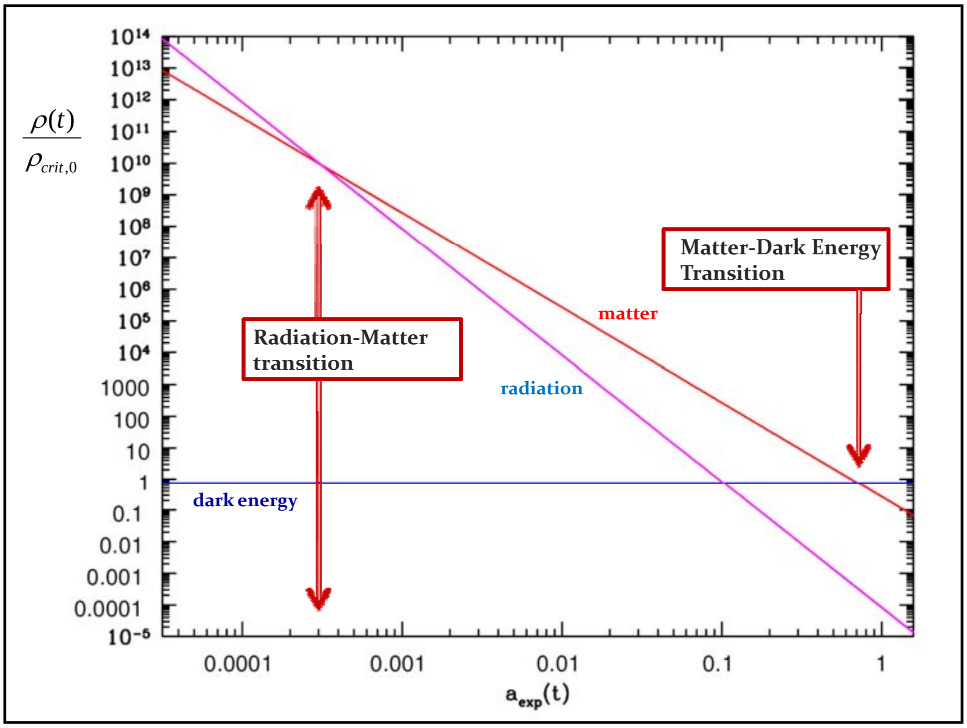
$$\frac{kc^2}{R(t)^2} = \frac{1}{a^2}\frac{kc^2}{R_0^{\,2}} = \frac{1}{a^2}\left(1-\Omega_0\right)$$

- Dark Energy (Cosmological Constant) Evolution

$$\rho_\Lambda(t) = cst. = \rho_{\Lambda 0}$$

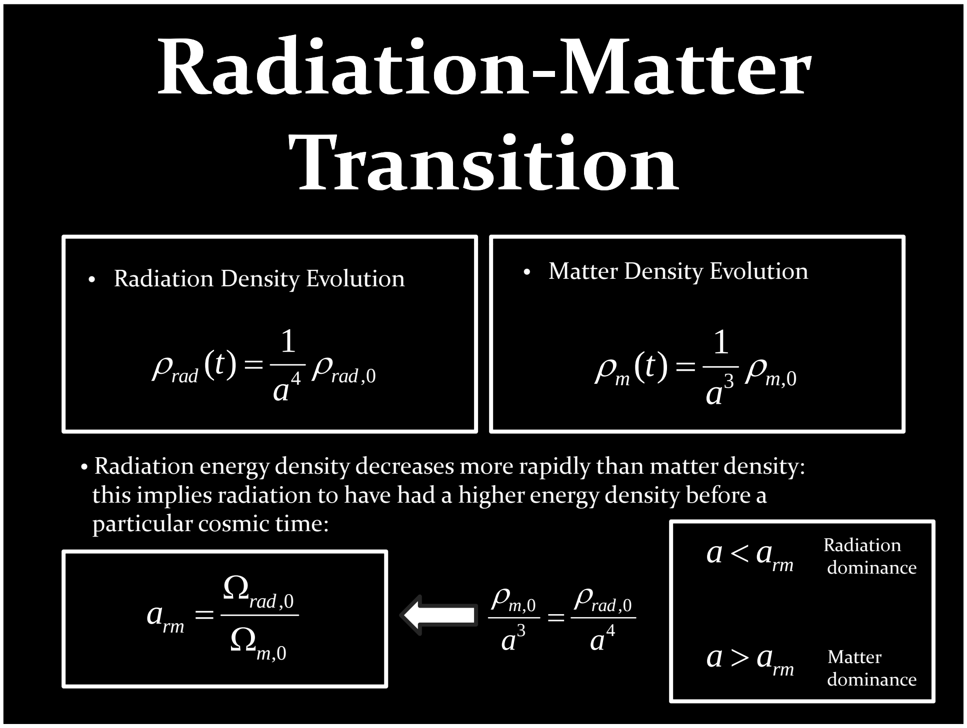


Density Evolution Cosmic Components



Matter-Dark Energy Transition

Radiation-Matter transition



Radiation-Matter Transition

- Radiation Density Evolution

$$\rho_{rad}(t)=\frac{1}{a^4}\rho_{rad,0}$$

- Matter Density Evolution

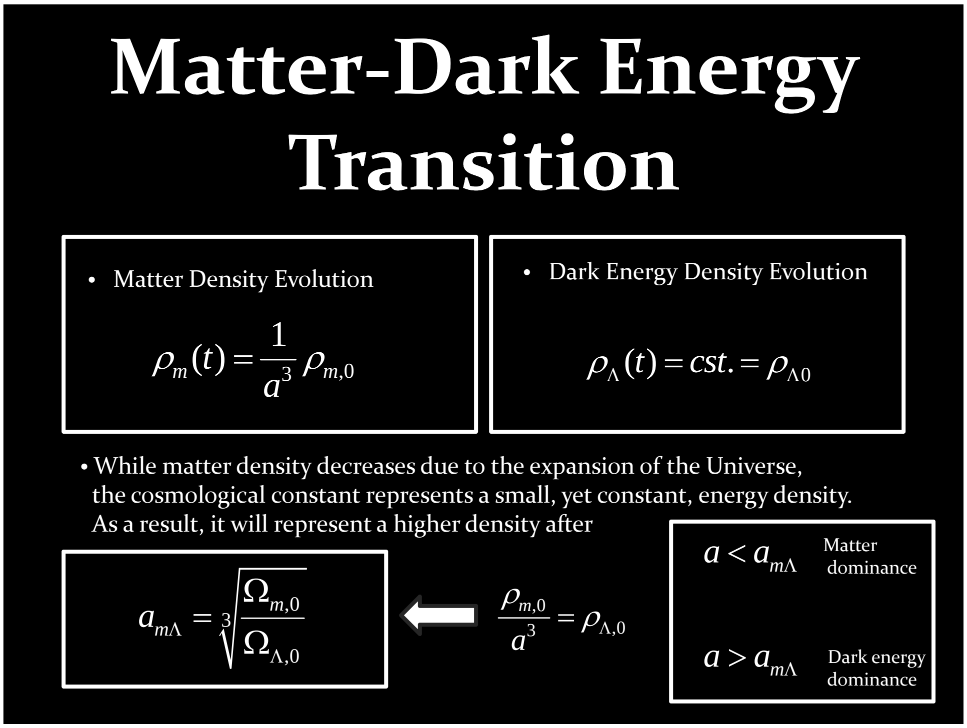
$$\rho_m(t)=\frac{1}{a^3}\rho_{m,0}$$

- Radiation energy density decreases more rapidly than matter density: this implies radiation to have had a higher energy density before a particular cosmic time:

$$a_{rm}=\frac{\Omega_{rad,0}}{\Omega_{m,0}} \Longleftarrow \frac{\rho_{m,0}}{a^3}=\frac{\rho_{rad,0}}{a^4}$$

$a<a_{rm}$ Radiation dominance

$a>a_{rm}$ Matter dominance



Matter-Dark Energy Transition

- Matter Density Evolution

$$\rho_m(t)=\frac{1}{a^3}\rho_{m,0}$$

- Dark Energy Density Evolution

$$\rho_\Lambda(t)=cst.=\rho_{\Lambda 0}$$

- While matter density decreases due to the expansion of the Universe, the cosmological constant represents a small, yet constant, energy density. As a result, it will represent a higher density after

$$a_{m\Lambda}=\sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}} \Longleftarrow \frac{\rho_{m,0}}{a^3}=\rho_{\Lambda,0}$$

$a<a_{m\Lambda}$ Matter dominance

$a>a_{m\Lambda}$ Dark energy dominance

Matter-Dark Energy Transition

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}} \quad \Longrightarrow \quad \left.\begin{array}{l} \Omega_{\Lambda,0} = 0.27 \\ \\ \Omega_{m,0} = 0.73 \end{array}\right\} \quad \begin{array}{l} a_{m\Lambda} = 0.72 \\ \\ a_{m\Lambda}{}^{\dagger} = 0.57 \end{array}$$

$\Big\Downarrow$ Flat Universe

$$a_{m\Lambda} = \sqrt[3]{\frac{\Omega_{m,0}}{1-\Omega_{m,0}}}$$

Note: a more appropriate characteristic transition is that at which the deceleration turns into acceleration:

$$a_{m\Lambda}{}^{\dagger} = \sqrt[3]{\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}} = \sqrt[3]{\frac{\Omega_{m,0}}{2(1-\Omega_{m,0})}}$$

Evolution

Cosmological Density Parameter

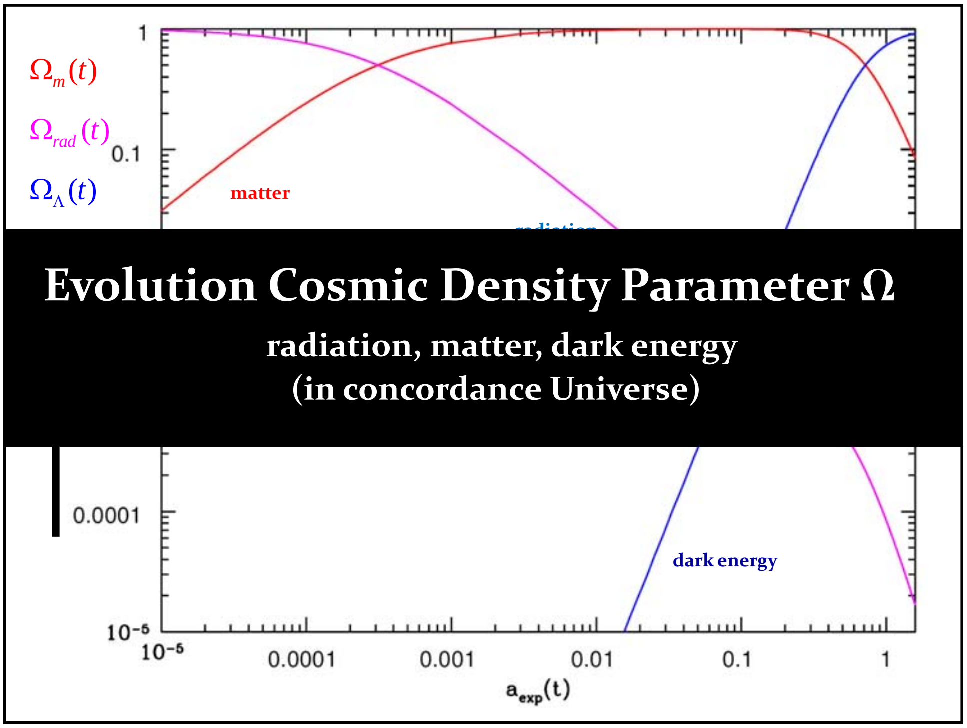
Limiting ourselves to a flat Universe
(and discarding the contribution by and evolution of curvature):

to appreciate the dominance of radiation, matter and dark energy in the subsequent cosmological eras, it is most illuminating to look at the evolution of the cosmological density parameter of these cosmological components:

$$\Omega_{rad}(t) \longleftrightarrow \Omega_m(t) \longleftrightarrow \Omega_\Lambda(t)$$

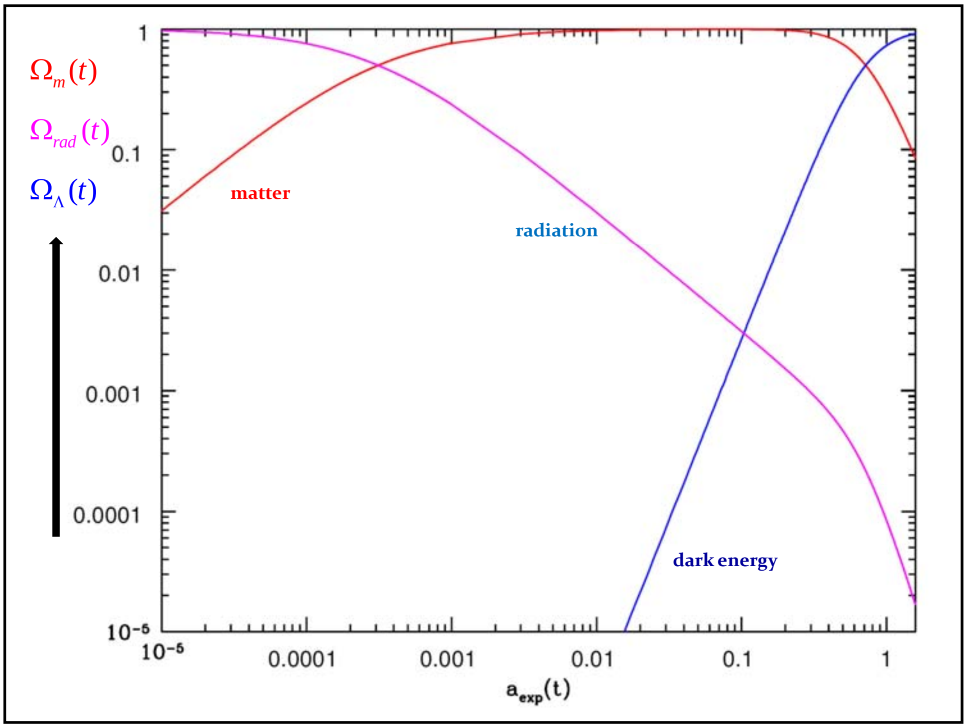
e.g.

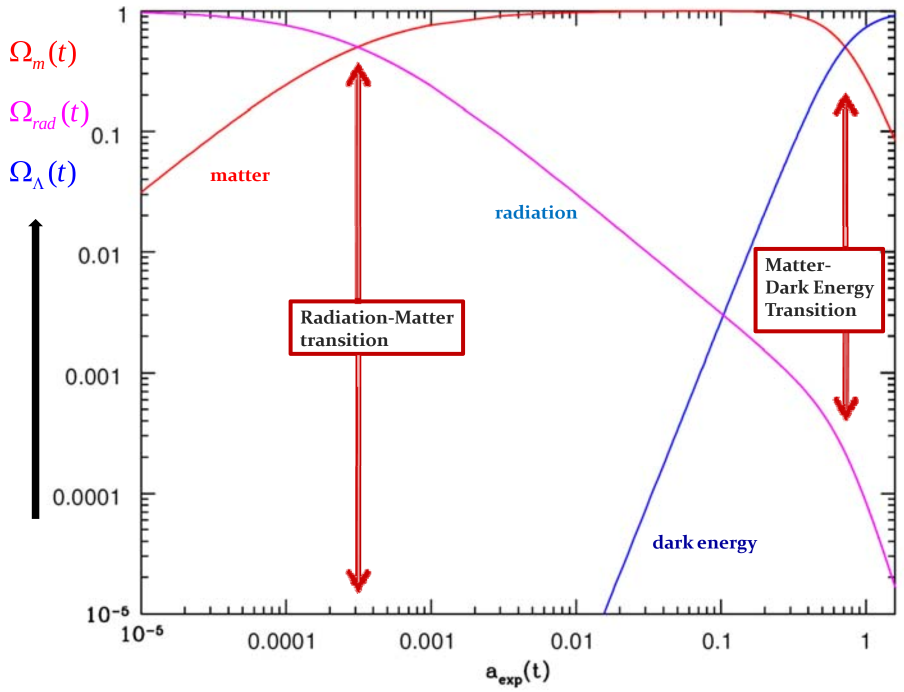
$$\Omega_m(t) = \frac{\Omega_{m,0}\,a^4}{\Omega_{rad,0} + \Omega_{m,0}\,a + \Omega_{\Lambda,0}\,a^4}$$



Evolution Cosmic Density Parameter Ω

radiation, matter, dark energy
(in concordance Universe)





Radiation-Matter transition

Matter-Dark Energy Transition

Standard Big Bang

What it cannot explain

- **Flatness Problem**
  the Universe is remarkably flat, and was even (much) flatter in the past
- **Horizon Problem**
  the Universe is nearly perfectly isotropic and homogeneous, much more so in the past
- **Monopole Problem:**
  There are hardly any magnetic monopoles in our Universe
- **Fluctuations, seeds of structure**
  Structure in the Universe: origin

Flatness Problem

Flatness Problem

FRW Dynamical Evolution:

Going back in time, we find that the Universe was much flatter than it is at the present.

Reversely, that means that any small deviation from flatness in the early Universe would have been strongly amplified nowadays …

We would therefore expect to live in a Universe that would either be almost
$\Omega=0$ or $\Omega\sim\infty$;

Yet, we find ourselves to live in a Universe that is almost perfectly flat … $\Omega_{tot}\sim 1$

How can this be ?

Evolution Ω

From the FRW equations, one can infer that the evolution of Ω goes like
(for simplicity, assume matter-dominated Universe),

$$\left(\frac{1}{\Omega}-1\right)=a(t)\left(\frac{1}{\Omega_0}-1\right) \quad \Longleftrightarrow \quad \Omega(z)=\frac{\Omega_0(1+z)}{1+\Omega_0 z}$$

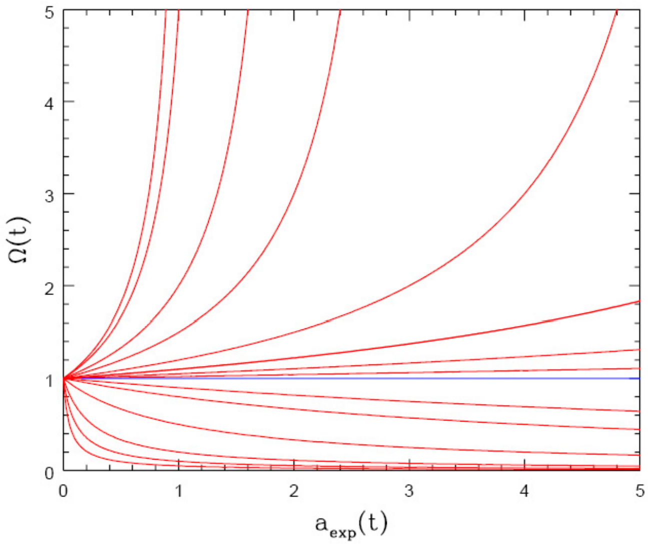
These equations directly show that

$$a\downarrow 0 \quad \Longrightarrow \quad \Omega\rightarrow 1$$

$$k=\frac{H^2R^2}{c^2}(\Omega-1)$$

implying that the early Universe was very nearly flat …

Flatness Evolution



$$\left(\frac{1}{\Omega}-1\right)=a(t)\left(\frac{1}{\Omega_0}-1\right)$$

- At radiation-matter equiv.

  $|1-\Omega_{rm}|\le 2\times10^{-4}$

- Big Bang nucleosynthesis

  $a_{nuc}\sim3.6\times10^{-8}$

  $|1-\Omega_{nucl}|\le 3\times10^{-14}$

- Planck time

  $|1-\Omega_{P}|\le 1\times10^{-60}$

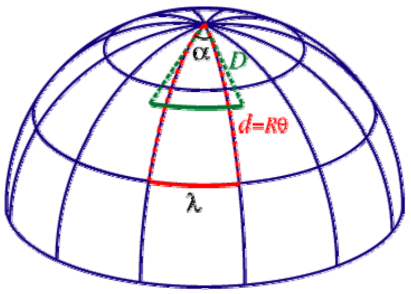
Measuring Curvature

Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path, and from this the curvature of space

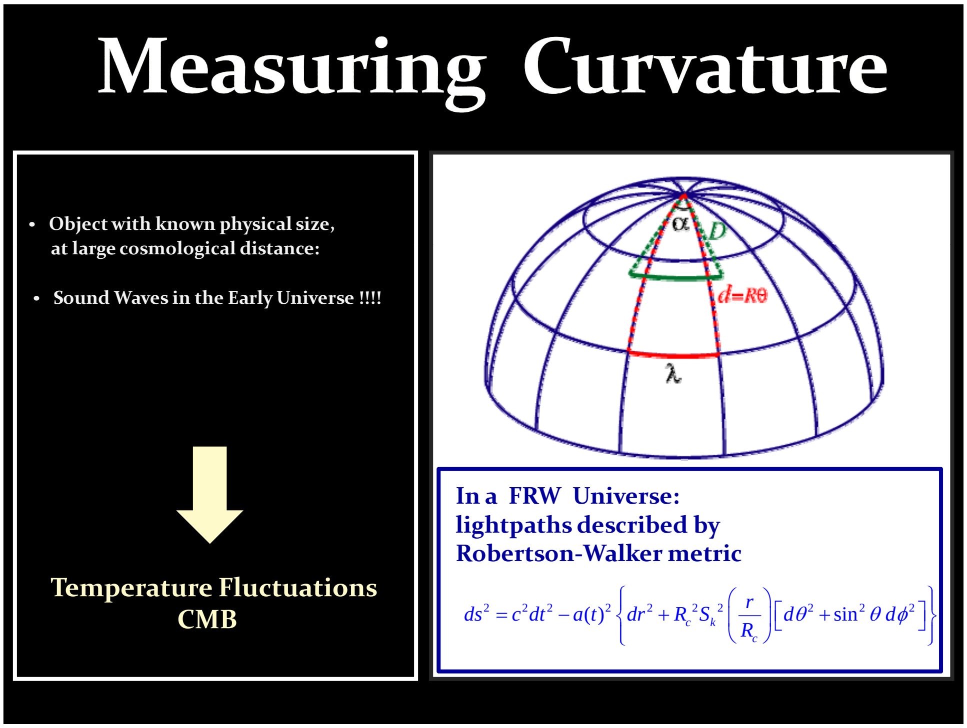
↓

Geometry of Space



In a FRW Universe: lightpaths described by Robertson-Walker metric

$$ds^2=c^2dt^2-a(t)^2\left\{dr^2+R_c^2S_k^2\left(\frac{r}{R_c}\right)\left[d\theta^2+\sin^2\theta\, d\phi^2\right]\right\}$$



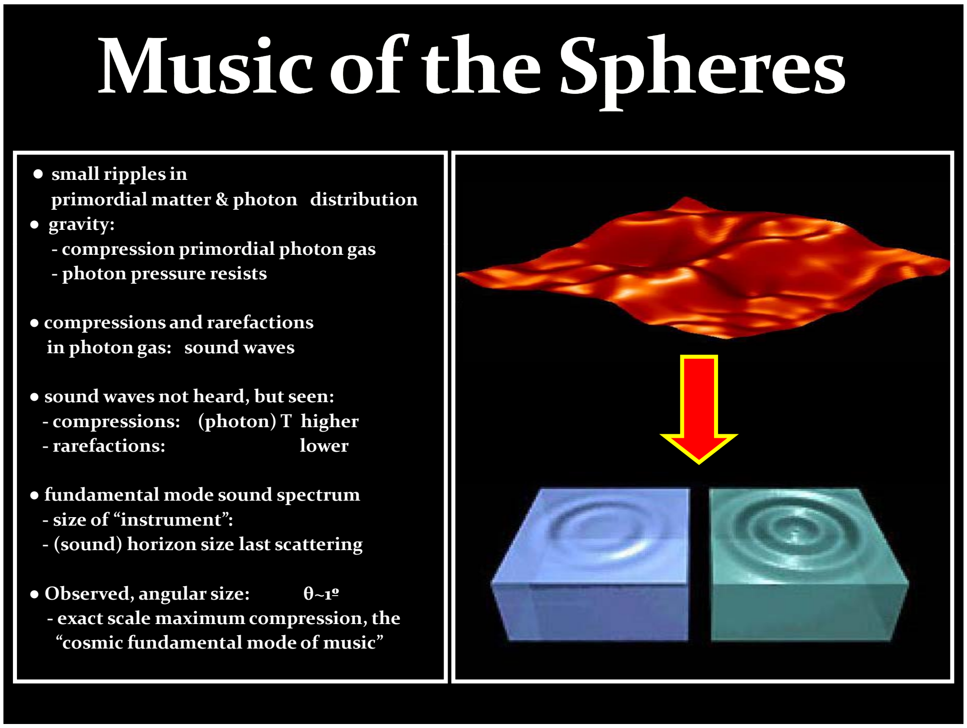
Measuring Curvature

- Object with known physical size, at large cosmological distance:
- Sound Waves in the Early Universe !!!!

Temperature Fluctuations
CMB

In a FRW Universe:
lightpaths described by
Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left\{ dr^2 + R_c{}^2 S_k{}^2 \left(\frac{r}{R_c} \right) \left[d\theta^2 + \sin^2 \theta \, d\phi^2 \right] \right\}$$



Music of the Spheres

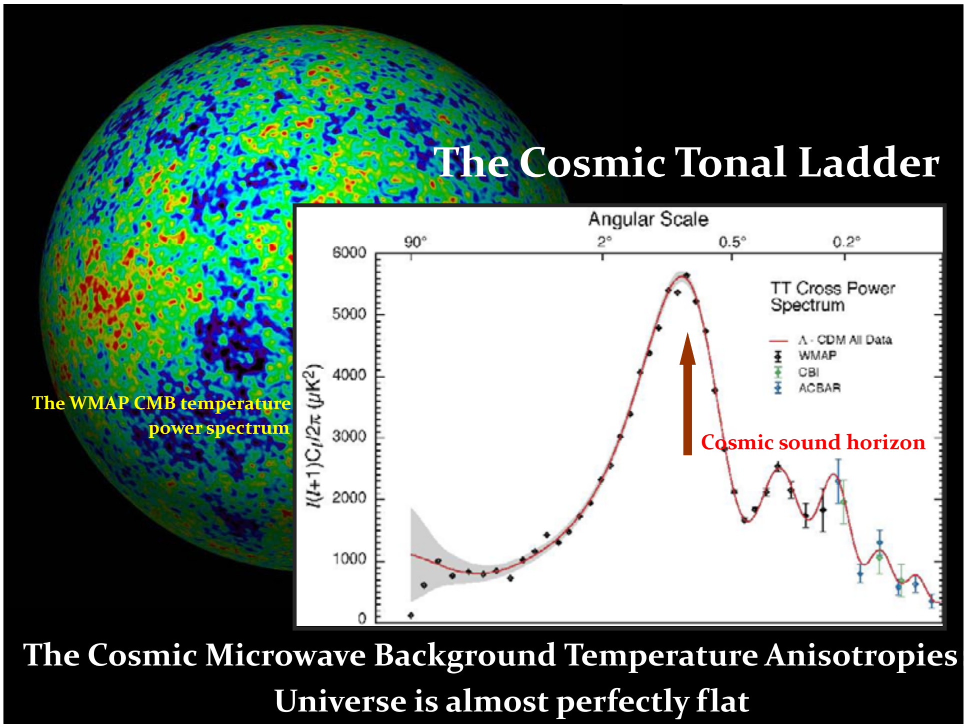
- small ripples in primordial matter & photon distribution
- gravity:
  - compression primordial photon gas
  - photon pressure resists

- compressions and rarefactions in photon gas: sound waves

- sound waves not heard, but seen:
  - compressions: (photon) T higher
  - rarefactions: lower

- fundamental mode sound spectrum
  - size of "instrument":
  - (sound) horizon size last scattering

- Observed, angular size: θ~1º
  - exact scale maximum compression, the "cosmic fundamental mode of music"

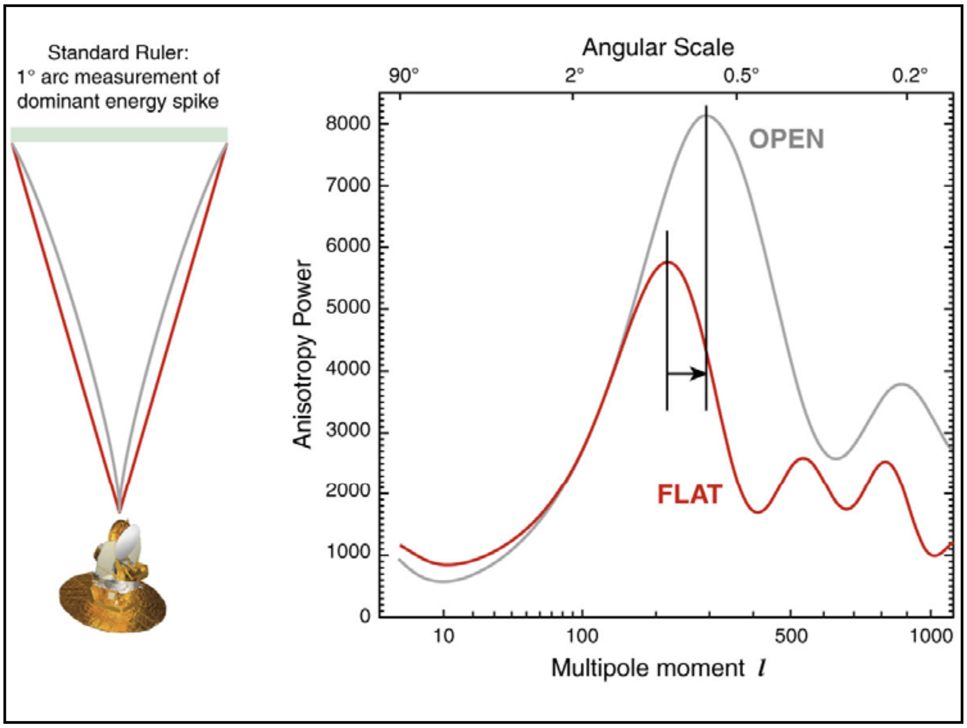


The Cosmic Tonal Ladder

The WMAP CMB temperature power spectrum

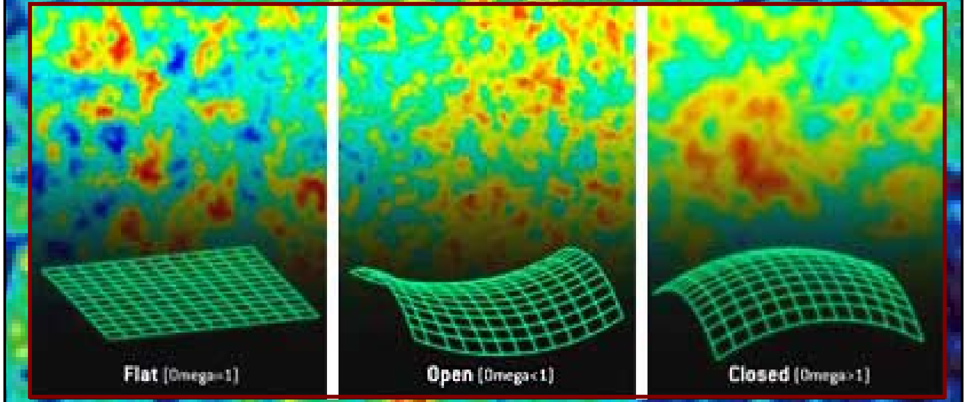
Cosmic sound horizon

The Cosmic Microwave Background Temperature Anisotropies
Universe is almost perfectly flat



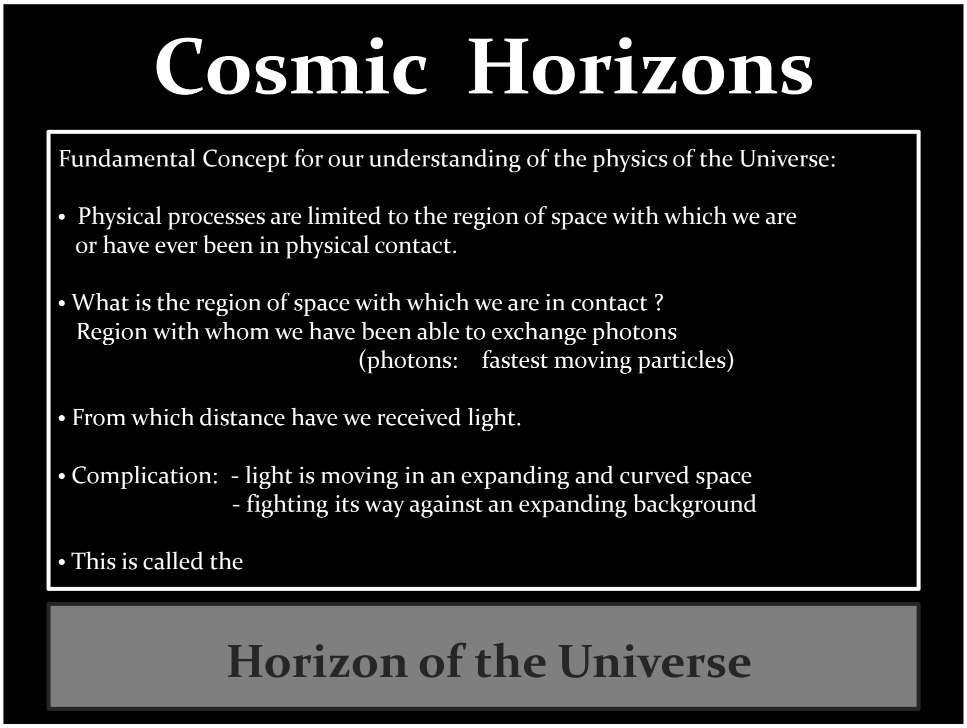
Standard Ruler:
1° arc measurement of dominant energy spike

Angular CMB temperature fluctuations



CMB: Universe almost perfectly Flat

Horizon Problem



Cosmic Horizons

Fundamental Concept for our understanding of the physics of the Universe:

- Physical processes are limited to the region of space with which we are or have ever been in physical contact.

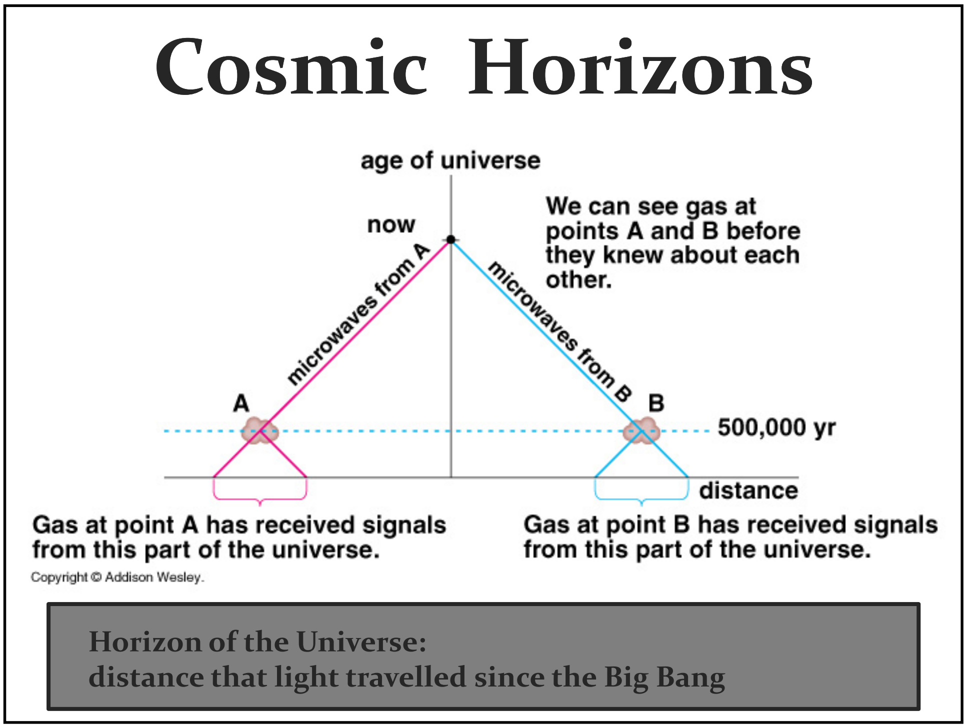
- What is the region of space with which we are in contact ?
  Region with whom we have been able to exchange photons
  (photons: fastest moving particles)

- From which distance have we received light.

- Complication: - light is moving in an expanding and curved space
  - fighting its way against an expanding background

- This is called the

Horizon of the Universe



Cosmic Horizons

We can see gas at points A and B before they knew about each other.

Gas at point A has received signals from this part of the universe.

Gas at point B has received signals from this part of the universe.

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

Light travel in an expanding Universe:

- **Robertson-Walker metric:** $ds^2 = c^2 dt^2 - a(t)^2 dr^2$
- **Light:** $ds^2 = 0$

$$d_{Hor} = \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in comoving space

⇔

$$R_{Hor} = a(t) \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

$$R_{Hor} = a(t) \int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

⇒

$$R_{Hor} = 3ct$$

In an Einstein-de Sitter Universe

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

$$R_{Hor} = a(t)\int_0^t \frac{c\,dt'}{a(t')}$$

Horizon distance in physical space

$$R_{Hor} = 3ct$$

In an Einstein-de Sitter Universe

The horizon distance at recombination/decoupling (CMB),

angular size on the sky:

$$\theta_{Hor} \approx 1.74^\circ\, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100}\right)^{-1/2}$$

Horizon of the Universe:
distance that light travelled since the Big Bang

Cosmic Horizons

The horizon distance at recombination/decoupling (CMB),

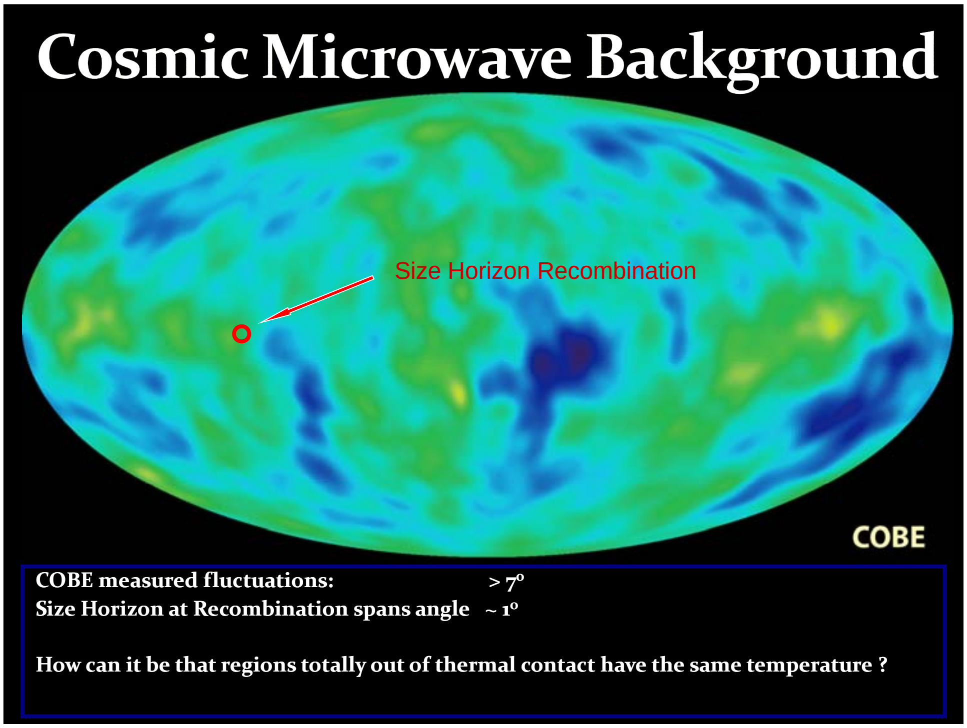
angular size on the sky:

$$\theta_{Hor} \approx 1.74^\circ\, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100}\right)^{-1/2}$$

$\theta \gg 1^\circ$ Large angular scales:
NOT in physical contact

$\theta \ll 1^\circ$ Small angular scales:
In physical (thus, also thermal) contact

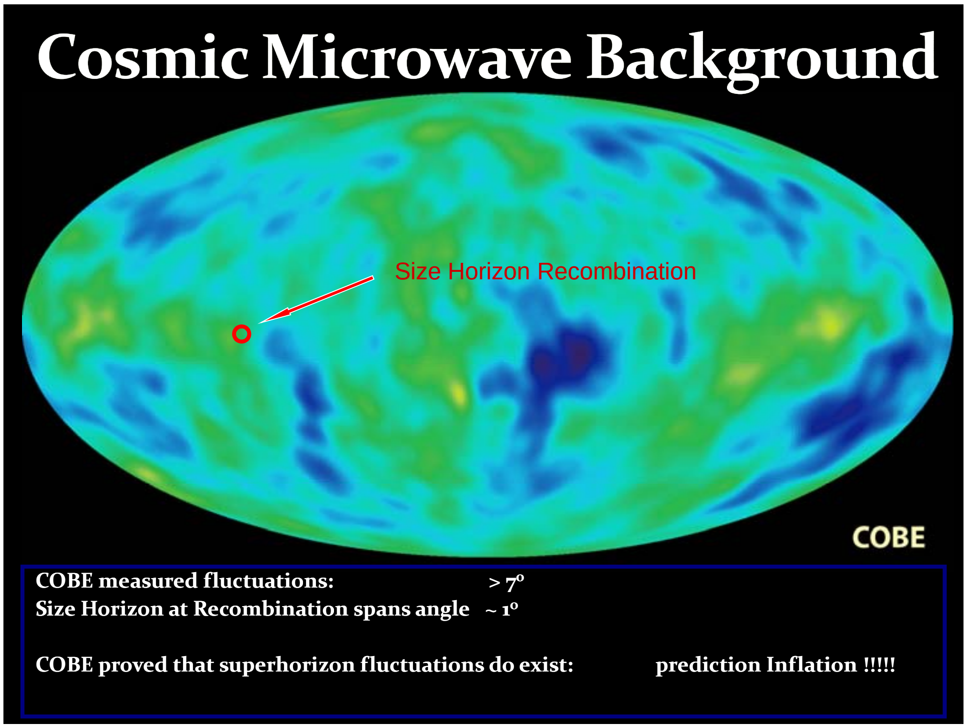
Horizon of the Universe:
distance that light travelled since the Big Bang



Cosmic Microwave Background

COBE measured fluctuations: > 7°
Size Horizon at Recombination spans angle ~ 1°

How can it be that regions totally out of thermal contact have the same temperature ?



Cosmic Microwave Background

COBE measured fluctuations: > 7°
Size Horizon at Recombination spans angle ~ 1°

COBE proved that superhorizon fluctuations do exist: prediction Inflation !!!!!

Horizon Problem

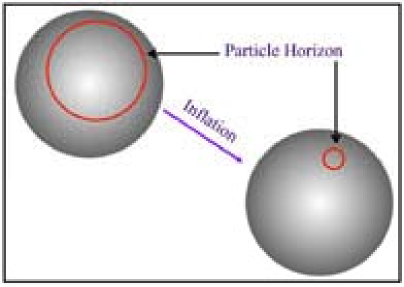
The horizon distance at recombination/decoupling (CMB),

angular size on the sky:

$$\theta_{Hor} \approx 1.74^\circ \, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100} \right)^{-1/2}$$

Angular scales: $\theta_{Hor} \approx 1^\circ$

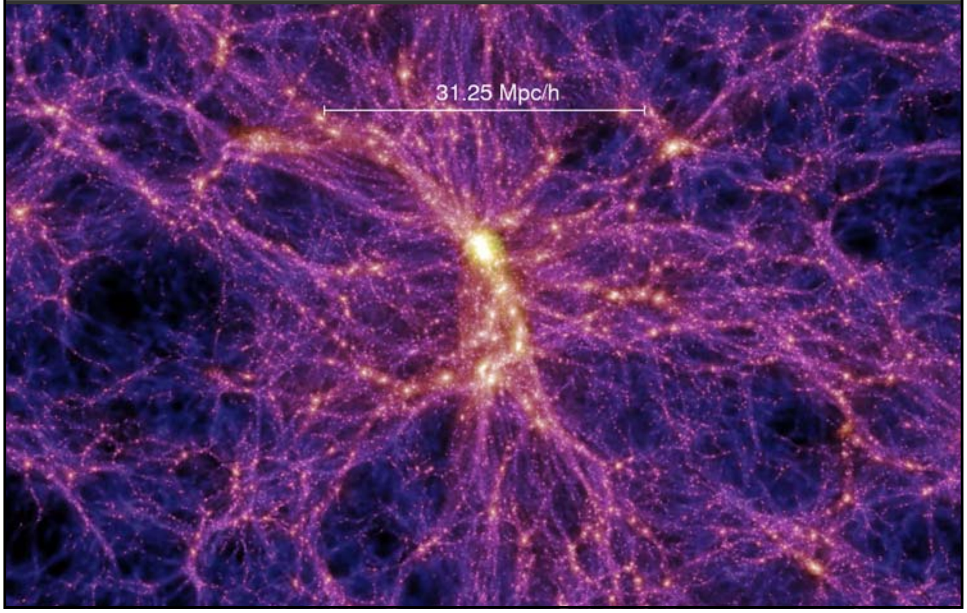
How can it be that regions that were never in thermal still have almost exactly the same temperature T~2.725 K



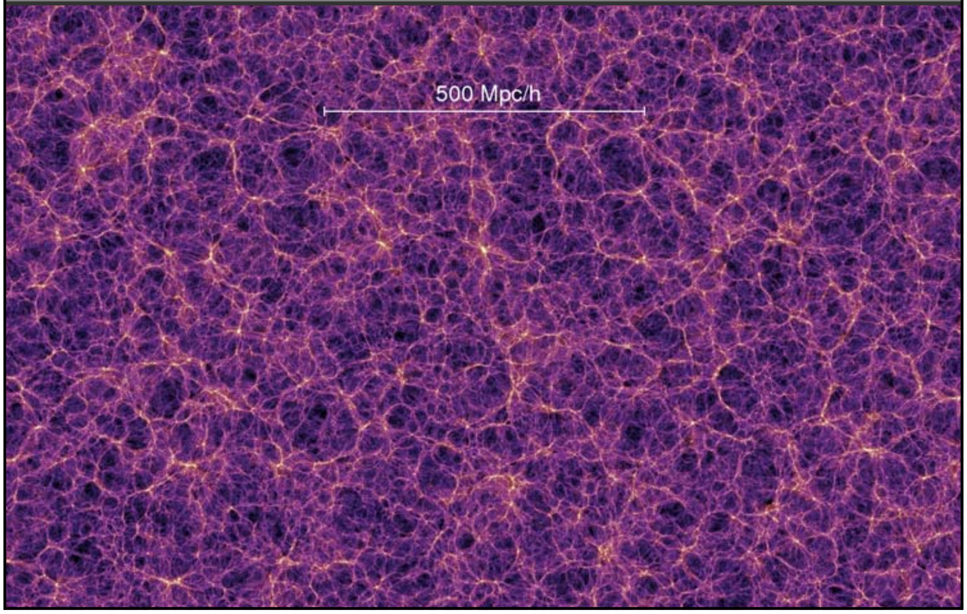
Horizon of the Universe:
distance that light travelled since the Big Bang

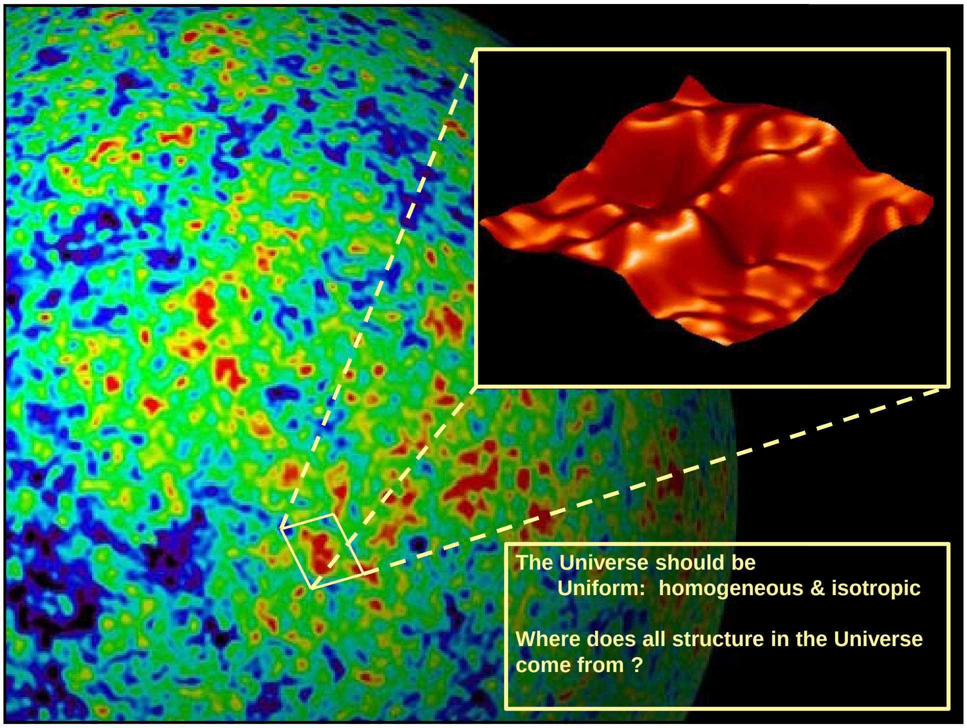
Structure Problem

Millennium Simulation



Millennium Simulation





The Universe should be
Uniform: homogeneous & isotropic

Where does all structure in the Universe come from ?

Monopole Problem

Inflationary Universe

FRW Big Bang extended: Inflationary Universe

Essential
Ingredient/Extension
Standard Cosmology

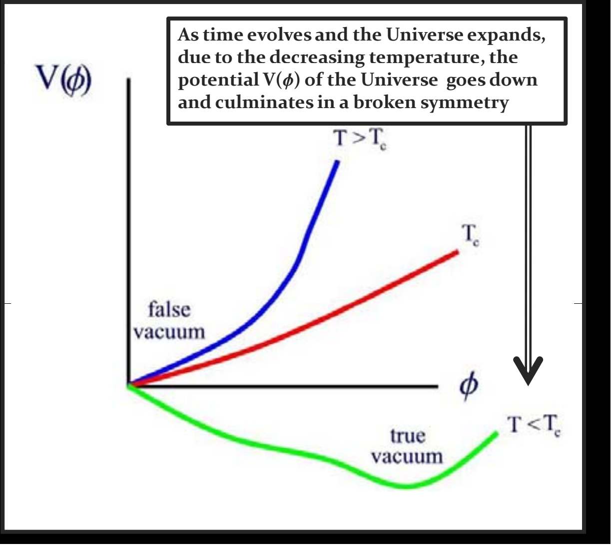
Inflationary Universe

Phase transition
Early Universe

- GUT transition :
  $t \sim 10^{-36}$ sec ??

- (false) vacuum potential
  induces exponential
  (de Sitter) expansion

Universe blows up by factor
$N > 10^{60}$



As time evolves and the Universe expands, due to the decreasing temperature, the potential $V(\phi)$ of the Universe goes down and culminates in a broken symmetry

FRW Big Bang extended: Inflationary Universe

Essential
Ingredient/Extension
Standard Cosmology

Inflationary Universe

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Early Universe

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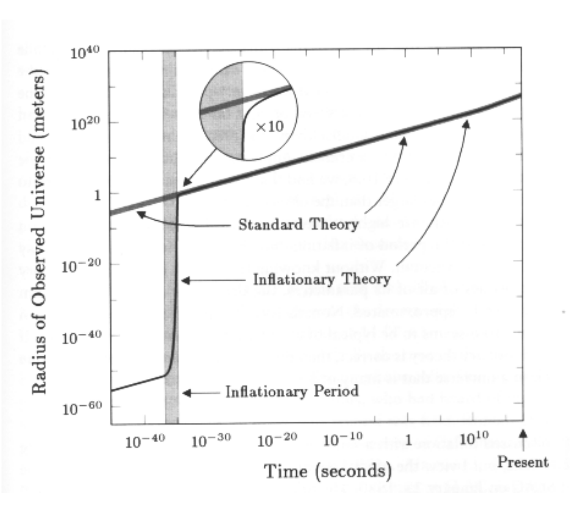

FRW Big Bang extended: Inflationary Universe

Essential
Ingredient/Extension
Standard Cosmology

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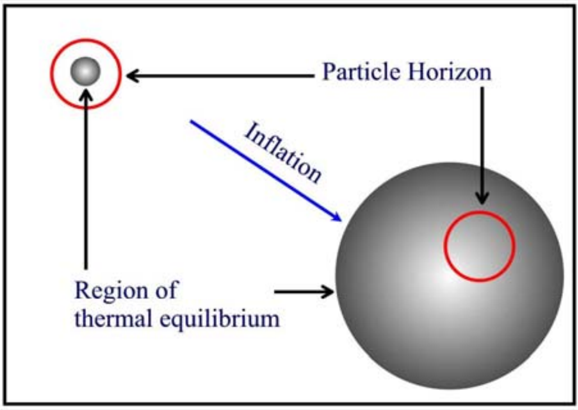
Universe blows up by factor
N > 10^{60}



Inflation explains the horizon problem in a natural fashion:
a region that initially was much smaller than the pre-inflation horizon, rapidly blew to a superhorizon size: any region on the CMB sky was in thermal contact before inflation !

FRW Big Bang extended: Inflationary Universe

Essential
Ingredient/Extension
Standard Cosmology

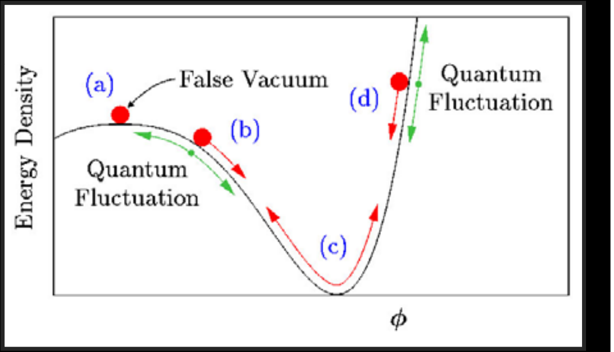
Inflationary Universe

Phase transition
Early Universe

- GUT transition :
$t \sim 10^{-36}$ sec ??

- (false) vacuum potential
induces exponential
(de Sitter) expansion

Universe blows up by factor
$N > 10^{60}$



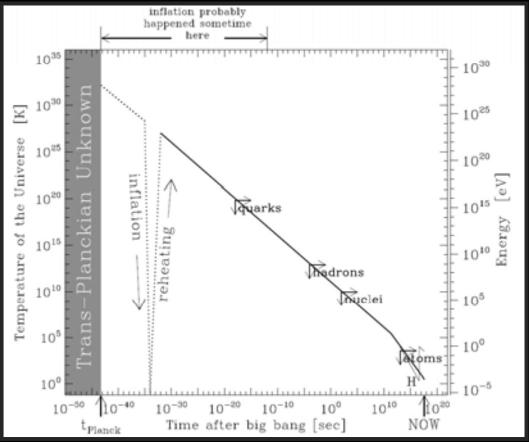
Fluctuation Generation during Inflation:
Quantum fluctuations around the potential get magnified to macroscopic scales.

FRW Big Bang extended: Inflationary Universe

Inflationary Universe

Explains:
- Horizon Problem
- Flatness Problem
- Monopole Problem

And …
- Origin of Structure



Towards the end of inflation, the Universe converts the tremendous amount of vacuum energy that drives inflation into a surge of newly created radiation and particles (“latent heat”). This is the Reheating Phase of inflation.

Early Universe

the first 379,000 years

FRW

Thermodynamics

FRW Dynamics

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a + \frac{\Lambda}{3}a$$

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - \frac{kc^2}{R_0^{\ 2}} + \frac{\Lambda}{3}a^2$$

To find solutions a(t) for the expansion history of the Universe, for a particular FRW Universe ,

one needs to know how the density ρ(t) and pressure p(t) evolve as function of a(t)

FRW equations are implicitly equivalent to a third Einstein equation, the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

FRW Dynamics: Adiabatic Cosmic Expansion

Important observation:
the energy equation,

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = 0$$

is equivalent to stating that the change in internal energy

$$U = \rho c^2 V$$

of a specific co-expanding volume V(t) of the Universe, is due to work by pressure:

$$dU = -p\,dV$$

Friedmann-Robertson-Walker-Lemaitre expansion of the Universe is

➡ **Adiabatic Expansion** ⬅

FRW Dynamics: Thermal Evolution

Adiabatic Expansion of the Universe:

- Implication for Thermal History
- Temperature Evolution of cosmic components

For a medium with adiabatic index γ:

$$TV^{\gamma-1} = cst$$

Radiation (Photons)	$\gamma = \frac{4}{3}$	$T = \frac{T_0}{a}$
Monatomic Gas (hydrogen)	$\gamma = \frac{5}{3}$	$T = \frac{T_0}{a^2}$

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Radiation
&
Matter

Cosmic Radiation

The Universe is filled with thermal radiation, the photons that were created in The Big Bang and that we now observe as the Cosmic Microwave Background (CMB).

The CMB photons represent the most abundant species in the Universe, by far !

The CMB radiation field is PERFECTLY thermalized, with their energy distribution representing the most perfect blackbody spectrum we know in nature. The energy density $u_\nu(T)$ is therefore given by the Planck spectral distribution,

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

At present, the temperature T of the cosmic radiation field is known to impressive precision,

$$T_0 = 2.725 \pm 0.001 K$$

Cosmic Radiation

With the energy density $u_\nu(T)$ of CMB photons with energy hν given, we know the number density $n_\nu(T)$ of such photons:

$$n_\nu(T) = \frac{u_\nu(T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

The total number density $n_\gamma(T)$ of photons in the Universe can be assessed by integrating the number density $n_\nu(T)$ of photons with frequency ν over all frequencies,

$$n_\gamma(T) = \int_0^\infty n_\nu(T)\,d\nu =$$

$$= \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}\,d\nu = 60.4\left(\frac{kT}{hc}\right)^3$$

$T = 2.725\,K$

⬇

$n_\gamma(T) = 412\ cm^{-3}$

Baryon-Photon Ratio

Having determined the number density of photons, we may compare this with the number density of baryons, $n_b(T)$. That is, we wish to know the PHOTON-BARYON ratio,

$$\eta \equiv \frac{n_\gamma}{n_B}$$

$$n_b = \frac{\rho_B}{m_p} = \frac{\Omega_B\,\rho_{crit}}{m_p}$$

The baryon number density is inferred from the baryon mass density. here, for simplicity, we have assumed that baryons (protons and neutrons) have the same mass, the proton mass $m_p \sim 1.672 \times 10^{-24}$ g. At present we therefore find

$$n_b = 1.12\times10^{-5}\ \Omega_b h^2\ g\,cm^{-3}$$

⬇

$$\eta_0 = \frac{n_\gamma}{n_B} \approx 3.65\times10^7\,\frac{1}{\Omega_b h^2}\ g\,cm^{-3}$$

We know that Ω_b~0.044 and h~0.72:

$$\eta_0 = \frac{n_\gamma}{n_b} \approx 1.60\times10^9$$

Baryon-Photon Ratio

From simple thermodynamic arguments, we find that the number of photons is vastly larger than that of baryons in the Universe.

$$\eta_0 = \frac{n_\gamma}{n_b} \approx 1.60 \times 10^9$$

In this, the Universe is a unique physical system, with tremendous repercussions for the thermal history of the Universe. We may in fact easily find that the cosmic photon-baryon ratio remains constant during the expansion of the Universe,

$$n_b(t) = \frac{n_{b,0}}{a^3}$$

$$n_\gamma(t) \propto T(t)^3 \propto \frac{1}{a^3} \Rightarrow n_\gamma(t) = \frac{n_{\gamma 0}}{a^3}$$

➡

$$\eta = \frac{n_\gamma(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0$$

Entropy of the Universe

The photon-baryon ratio in the Universe remains constant during the expansion of the Universe, and has the large value of

$$\eta = \frac{n_\gamma(t)}{n_b(t)} = \frac{n_{\gamma,0}}{n_{b,0}} = \eta_0 = 1.60 \times 10^9$$

This quantity is one of the key parameters of the Big Bang. The baryon-photon ratio quantifies the ENTROPY of the Universe, and it remains to be explained why the Universe has produced such a system of extremely large entropy !!!!!

The key to this lies in the very earliest instants of our Universe !

Adiabatic Expansion
&
Thermal History
of the Universe

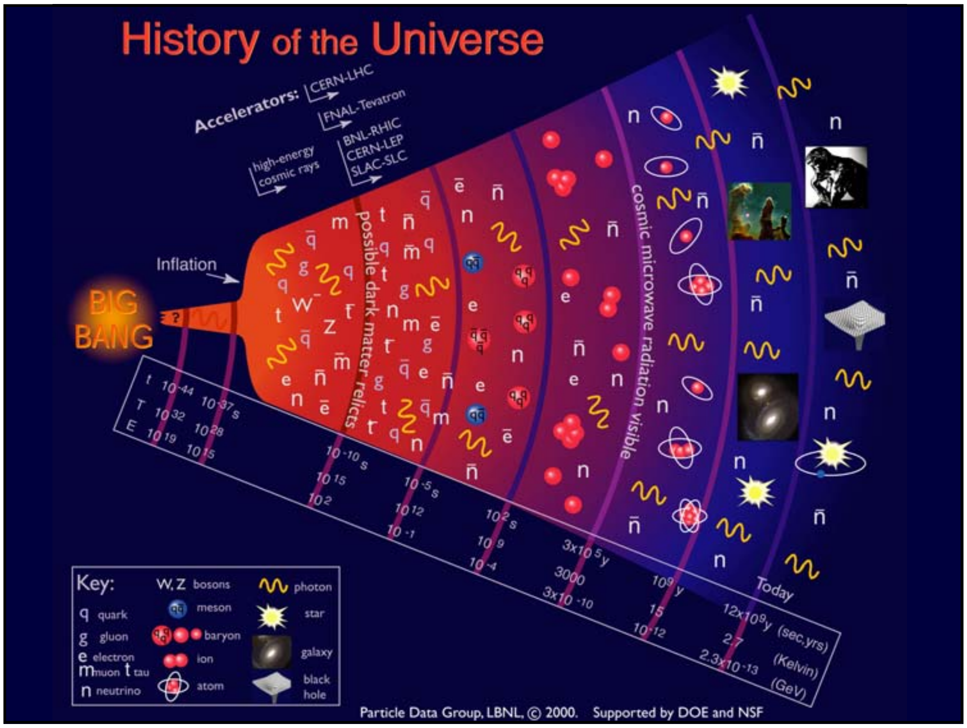
Adiabatic Expansion

$$P \propto \rho^{\gamma} \quad \Rightarrow \quad TV^{\gamma-1} = cst.$$

$$\rightarrow \begin{cases} \gamma = \frac{5}{3} & \Rightarrow \quad T_b \propto V^{-2/3} \propto a^{-2} \\ \gamma = \frac{4}{3} & \Rightarrow \quad T_b \propto V^{-1/3} \propto a^{-1} \end{cases}$$

Cosmic expansion is Adiabatic: **Temperature History**

Hot Big Bang



History of the Universe in Four Episodes: I

On the basis of the
1) complexity of the involved physics
2) our knowledge of the physical processes
we may broadly distinguish four cosmic episodes:

(I)

Origin universe

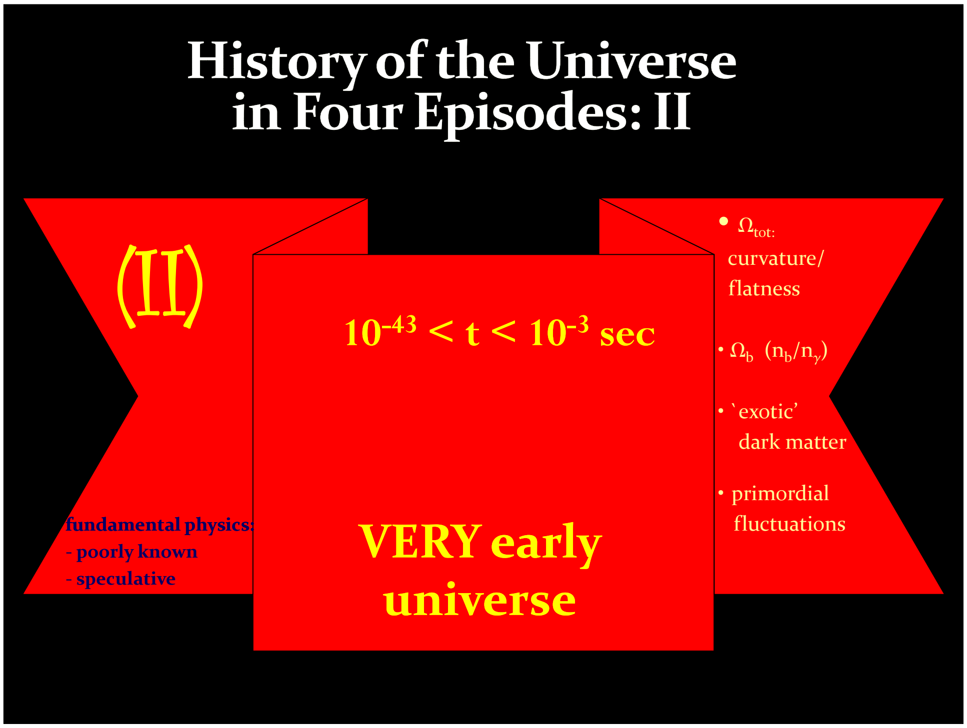
???

$t < 10^{-43}$ sec

fundamental physics:

- totally unknown

Planck Era



History of the Universe in Four Episodes: II

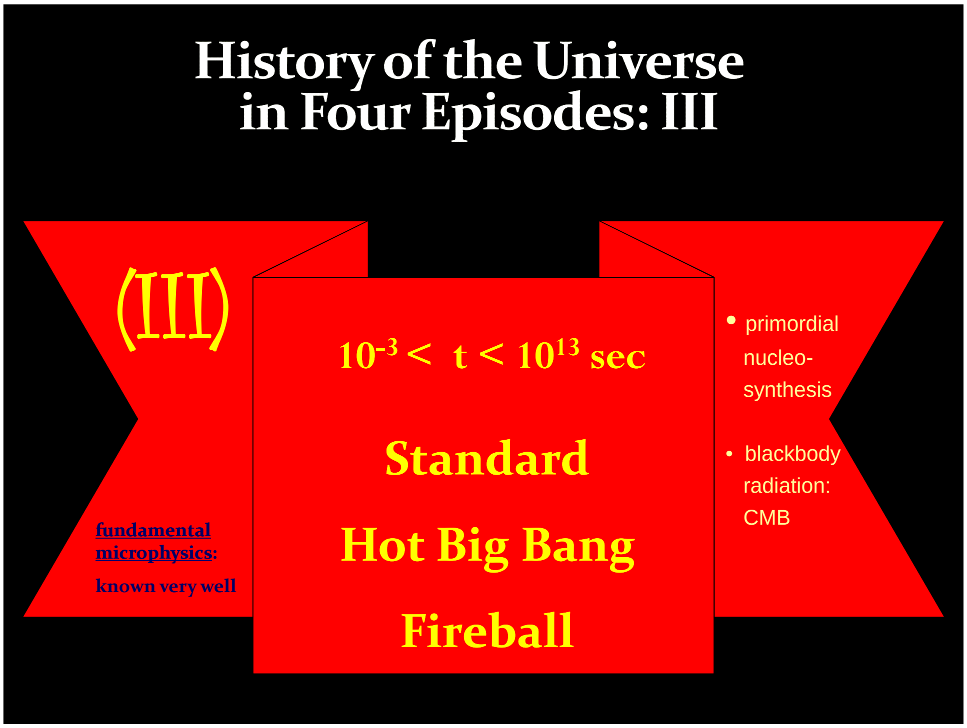
(II)

$10^{-43} < t < 10^{-3}$ sec

VERY early universe

fundamental physics:
- **poorly known**
- **speculative**

- Ω_{tot}: curvature/ flatness
- Ω_b (n_b/n_γ)
- \`exotic' dark matter
- primordial fluctuations



History of the Universe in Four Episodes: III

(III)

$10^{-3} < t < 10^{13}$ sec

Standard

Hot Big Bang

Fireball

fundamental microphysics:

known very well

- primordial nucleo-synthesis
- blackbody radiation: CMB

History of the Universe in Four Episodes: IV

(IV)

t > 10^{13} sec

Post

(Re)Combination

universe

complex macrophysics:

-Fundamentals known

- complex interplay

- structure formation:

stars,
galaxies
clusters
…

Episodes Thermal History

Planck Epoch		t < 10^{-43} sec
Phase Transition Era	GUT transition electroweak transition quark-hadron transition	10^{-43} sec < t < 10^{5}sec
Hadron Era		t ~ 10^{-5} sec
Lepton Era	muon annihilation neutrino decoupling electron-positron annihilation primordial nucleosynthesis	10^{-5} sec < t < 1 min
Radiation Era	radiation-matter equivalence recombination & decoupling	1 min < t <379,000 yrs
Post-Recombination Era	Structure & Galaxy formation Dark Ages Reionization Matter-Dark Energy transition	t > 379,000 yrs