# Correlation Function and Spherical Collapse Model

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## 1 Correlation Function

Given a spatial random field (isotropic and homogeneous) with the following Power Spectum.

$$P(k) = \begin{cases} 64k^{-1} & k < k_{max} \\ 0 & k > k_{max} \end{cases}$$

Here k is given in units of the inverse $Mpc\ h^{-1}$. The maximum wavelenght and thus the minimum radius is $16 Mpc\ h^{-1}$.

1. Sketch the Power Spectrum
2. Calculate the correlation function, $\xi(r)$, Sketch it.
3. Calculate the variance of the one point probability function
4. Using a sharp K-filter give the variance as function of Mass

Assume the following Einstein-de Sitter Cosmology and the growth factor is given by the following expression

$$D(z) = \frac{5}{2} H_0^2\ H(z) \int_z^\infty \frac{(1+z')}{H(z')^3} dz' \tag{1}$$

5 Calculate the growth factor $D(z)$

6 Calculate the variance at a=0.3 (Remember the convention is that spectra, variances are written in present day values)

## 2 Spherical Collapse Model

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} \qquad \text{and} \qquad \frac{1}{2}\left(\frac{dR}{dt}\right)^2 = \frac{GM}{R} + E \tag{2}$$

$$M = \frac{4\pi R_i^3}{3}\bar{\rho}_i(1+\Delta_i) \qquad \text{and} \qquad \Delta_i = \frac{\int_0^{R_i} dr 4\pi r^2 \delta_i(r)}{4\pi R_i^3/3} \tag{3}$$

The total energy within a certain shell is assumed to be negative. Very important note: $R$ is the physical coordinate, not the comoving one!!!

1. Prove the following relation

$$R\frac{d}{dt}\left(R\frac{dR}{dr}\right) = GM + 2ER \tag{4}$$

2. Substitute the above with $t = \frac{R}{\sqrt{-2E}}\theta$
3. Solve the above equation for $R(\theta)$ and $t(\theta)$

We can now use the following initial conditions. We may assume that the overdensity of the sphere was very small in the beginning i.e. $\Delta_i \ll 1$. And we assume that the initial velocity is given by the following equation:

$$v_i = \sqrt{1-\alpha_i}H_iR_i \tag{5}$$

Here $\alpha$ is factor that determines the deviation from the Hubble flow initially. If it is zero then the sphere just moves along with the expansion of the Universe.

4 Give an expression for the initial kinetic energy $K_i$

5 Show that the initial potential energy is given by

$$W_i = -\Omega_i(1+\Delta_i)\frac{(H_iR_i)^2}{2} \tag{6}$$

6 Give an expression for $R_{max}/R_i$ in terms of $\Omega_i$,$\alpha_i$ and $\Delta_i$ ($R_{max}$ is the maximum physical radius attained by the sphere, $\frac{dR}{dt}$ is then zero.)

7 Assuming EdS Universe give an expression for the overdensity as function of $\theta$

8 Give the overdensity at the maximum radius.

The linear growing mode in an EdS Universe is give according to

$$\Delta = \frac{3}{5}\Delta_i\left(\frac{t}{t_i}\right) \tag{7}$$

9 Give the linear extrapolated density at turnaround

10 Same as [9] but now when the sphere has collapsed