Tutorial I Large Scale Structure

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Excercise I, the Metric

Given the Robertson-Walker Metric

$$ds^2 = cdt^2 - a(t)^2[dr^2 + S_k(r)^2 d\Omega^2] \tag{1}$$

1. Explain the basic underlying assumption of the RW-metric
2. Starting from the metric give a simple expression for the proper distance of a galaxy at a given time, with comoving coordinates (r, θ, ϕ)? (Assume that there is no intervening matter between the observer on earth and the galaxy)
3. Can you think of any relativistic effects that could be observed when there is substantial matter in between?
4. A photon is emitted by the galaxy at t_e and observed at t_0, give an integral for the proper distance. Remember the photon always travelled at a Null geodesic, $ds^2 = 0$

Excersize II, Newtonian Cosmology

Assume expanding sphere which is isotropic and homogeneous, and consists of a pressureless gas with density ρ.

1. Give the equations of motions for a test particle at position R?
2. Derive an expression for the 'Hubble parameter' $\frac{\dot{R}}{R}$. (hint muliply the previous with $\frac{dR}{dt}$ and integrate)
3. Describe for the following cases the evolution of the sphere, if the integration constant is zero, positive and negative

Excersize III, Dynamics of the Universe

The Friedmann equation is the general relativistic equivalent of the Newtonian equation of motion:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{\kappa c^2}{R_0^2 a^2} \tag{2}$$

Here a is the expansion factor of the Universe and rho is the density of the Universe. Both are functions of time and change with the redshift of the Universe!

1. If the Hubble parameter is $H_0 = 50\ \frac{km/s}{Mpc}$. Give rough estimates for the age, size and critical density (ρ_c) of the Universe

2. If the density of the Universe is the critical density then curvature is flat, i.e.

   - $\rho = rho_c$ then $\kappa = 0$ flat
   - $\rho < rho_c$ then $\kappa < -1$ negative curvature
   - $\rho > rho_c$ then $\kappa = 1$ positive curvature

   Give an expression for the ρ_c in terms of the Hubble parameter.

3. For the following form of the equation of state: $P = w\rho$
   and the fluid equation in an expanding Universe $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$.
   Solve for $\rho(a)$, if $w = 0$ (matter), $w = 1/3$ (radiation), $w = -1$ (cosmological costant).

4. Solve the friedmann equation for (a) an empty Universe, (b) a flat Universe filled with matter, (c) flat with radiation, and (d) flat cosmological constant

5. For the previous Universes with $H_0 = 50km\ s^{-1}\ Mpc^{-1}$ calculate the ages of the Universes? What are the ages of the Universes at $z = 1$?

6. A galaxy is now ($t = t_0$) observed to have a redshift $z = 7$. If the size of Universe now is L_0, what was the size of the Universe when the photon was emmitted ($t = t_e$).

7. Calculate the proper distance $d_p(t_0)$ to the galaxy for all four cases, and the proper distance when the light was emitted $d_p(t_e)$.