SCHRIFTELIJK HERTENTAMEN
LARGE SCALE STRUCTURE OF THE UNIVERSE
3^{nd} trimester 2002

Mind you: 4 questions, 3 pages !

Question 1: Mapping the galaxy distribution

Mapping the cosmic spatial galaxy distribution consists of recording the location of galaxies in a well-defined region of space. Its sky position provides 2 coordinates, its distance the 3^{rd} coordinate. As it would be cumbersome to measure distances directly, they are estimated from the redshift z of a galaxy.

a) What is the relation between redshift z and distance (i.e. luminosity distance d_{lum} for $z \ll 1$. Also, what is the relation for higher z, to 2^{nd} order in z (involving deceleration parameter q_0.

b) The redshift of a galaxy is directly related to the velocity v_{gal} of a galaxy, for $z \ll 1$,

$$cz \approx v_{gal}.$$

In a uniformly expanding Universe, this is directly proportional to the distance. In reality, however, there are various systematic distortions. 1) How do these distortions enter into the redshift z (i.e. provide relation between redshift z, distance d and additional terms ...). 2) Describe two important distorting effects. This involves making sketches of the situations.

c) As it is often unfeasiblee to uniformly map the complete galaxy distribution within a large 3-D cosmic volume, a variety of survey strategies have been developed. These produce the galaxy distribution in well defined selective regions, or according to well defined selections of the full galaxy distribution. Describe (at least) 3 of these !

Question 2: the Cosmic Foam

a) Galaxies group into a large range of galaxy concentrations (clumps) and structures. Describe this 'hierarchy' of galaxy groups. Also include typical characteristics of these structures, such as size, mass, typical number of galaxies, if relevant the morphology of its galaxy content, the presence of an important gaseous content and its properties (e.g. temperature), etc.

b) On large, Megaparsec, scales this spatial distribution of galaxies in clumps is not a featureless random one. Instead, it appears to display a distinct and very characteristic pattern, the "Cosmic Foam". 1) List and ddescribe the essential distinct structural elements of the cosmic foam. 2) in this list, also mention typical dimensions, morphology (shape, geometry), content/structure, connection to other elements, etc.

a) Describe the essential idea of the theory of gravitational instability (if necessary, do not fail to use sketches) ?

b) What are the 3 equations of motion for a self-gravitating (and, to keep it simple, pressureless) medium ? Here: write the equations in physical coordinates (regular form), and give their name !

c) For a perturbed self-gravitating medium we work instead of with physical coordinates with (comoving) perturbation quantities. Give the definition of these comoving perturbation quantities for a self-graviting (pressureless) medium.

d) Give the (linearized) equations of motion in comoving coordinates (i.e. skip the higher order nonlinear terms).

e) Combining the above equations of motion yields the equation for the linear evolution of density perturbations δ:

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\,\delta = 4\pi G\bar{\rho}\,\delta\ ,$$

in which $\bar{\rho}(t)$ is the average uniform density of the (FRW) Universe. 1) What is the generic form of the solutions for this linear evolution equation ? Which 2 different modes of (linear) evolution can we distinguish ? (i.e. I also want to learn about things like "growth rate" !!!)

f) What is the specific solution for the density perturbation evolution in an Einstein-de Sitter Universe ($\Omega_0 = 1$). (needed: the specific form of the evolution equation and corresponding derivation of solution).

g) Equivalently for an $\Omega_0 = 0$ (empty) Universe. How can you thus describe/explain physically the contrast between perturbation evolution in a "high-density" Universe and a "low-density" Universe (i.e. explain why you find such a contrast in time evolution).

i) In f) and g) having provided the exact solutions for the density perturbation evolution, make a sketch for the growth rate $D(t)$ as function of cosmic expansion factor $a(t)$ for a range of different FRW Universes, from low-density (e.g. $\Omega_0 \approx 0.01$) to high-density ($\Omega_0 \approx 2 - 10$) FRW universes.

a) For a given linear density perturbation field $\delta(\mathbf{x}, t)$ give the (integral !) expression for the corresponding potential perturbation field $\phi(\mathbf{x}, t)$, i.e. what is the solution of the Poisson equation,

$$\nabla^2 \phi = 4\pi G \bar{\rho} a^2 \delta(\mathbf{x}, t) \ .$$

In this, I want you to replace the average density $\bar{\rho}$ by a cosmologically representation in terms of Ω and Hubble parameter H.

b) 1) For the density growing mode, $D_1(t)$, what is the corresponding growth rate of the potential ϕ (in terms of D_1 and a) ? 2) What then is the potential growth rate specifically for an Einstein-de Sitter Universe ?

c) Given the fact that the temperature perturbations of the Microwave Background on large scales are due to the socalled Sachs-Wolfe effect, reflecting the corresponding potential perturbations on the surface of last scattering as a result of the gravitational redshift/blueshift and time dilation as photons originate in deeper or shallower potential wells, what does this imply for the redshift dependence of the temperature perturbations $\Delta T/T$ in an Einstein-de Sitter Universe ? Note: the Sachs Wolfe temperature perturbations $\Delta T/T$ is given by:

$$\frac{\Delta T}{T} \approx \frac{1}{3} \frac{\phi}{c^2}$$

d) The gravity perturbation $\mathbf{g}$,

$$\mathbf{g} = -\frac{1}{a} \nabla \phi \ ,$$

what is the (integral !) expression for $\mathbf{g}$ in terms of the density perturbation field $\delta(\mathbf{x}, t)$.

e) What then is the growth rate of the peculiar gravity $\mathbf{g}(\mathbf{x}, t)$ in the linear regime ? (in terms of D and a).

f) In the linear regime there is a direct (linear) relation between the induced velocity perturbation $\mathbf{v}(\mathbf{x}, t)$ and the corresponding perturbation field $\mathbf{g}(\mathbf{x}, t)$. 1) Write down the expression for this relation ! 2) write down the definition for the prominent Ω dependent function $f(\Omega)$ in this relation. 3) What is approximately its Ω-dependence for $\Omega < 1$.

g) 1) Using this relation between peculiar velocity $\mathbf{v}$ and peculiar gravity $\mathbf{g}$, what is the (integral) relation between velocity field $\mathbf{v}$ and density field δ ? This relation has figured prominently in attempts to determine Ω from galaxy flows on Megaparsec scales. 2) How does one then proceed (in principle) ? 3) What has formed a major (systematic) obstacle in settling the value of Ω in this fashion ?

SUCCES BIJ DIT TENTAMEN !!!!
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Rien