Information about Information about Cosmology from Large-Scale Structure

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Information about Information about Information about Cosmology from Large-Scale Structure

- Cosmological Fisher information in the matter power spectrum as a function of scale from *N*-body simulations
- Same thing from the halo model; physical explanation
- Maybe difficult to extract cosmological information on non-linear scales from matter power spectrum, even under ideal circumstances.
- Other statistics useful? No information in things => information in nothing?

Collaborators

István Szapudi

Chris Rimes

Andrew Hamilton

Fisher Information

- How much information about the depth of a (non-moving) fish?

- Michael Palin takes many measurements, finds fractional variance *Var*

- Info = 1/ *Var*

- What if he adds another depth meter?

Fisher information

- **Twice the info with 2 meters? Some of the variance is covariance, e.g. from water conditions.**

- **Construct covariance matrix.**

$$C_{ij} = <\Delta d_i \Delta d_j> \qquad C = \begin{pmatrix} Var & a \times Var \\ a \times Var & Var \end{pmatrix}$$

$$Info = \sum F_{ij} = \sum (C^{-1})_{ij} = \frac{2}{(1+a) \times Var}$$

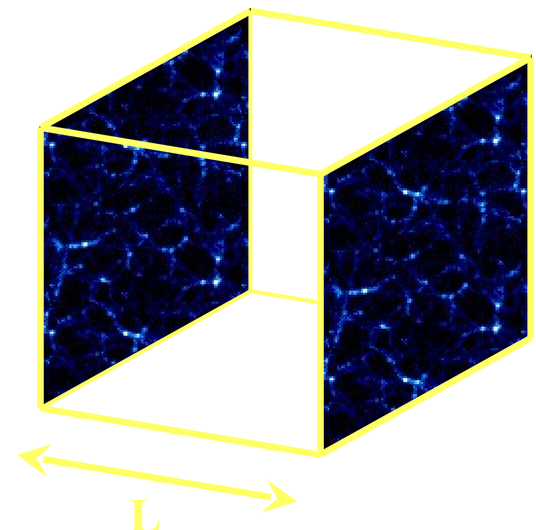
- $a \longrightarrow 1$**: same info as one meter (1/*Var*)**

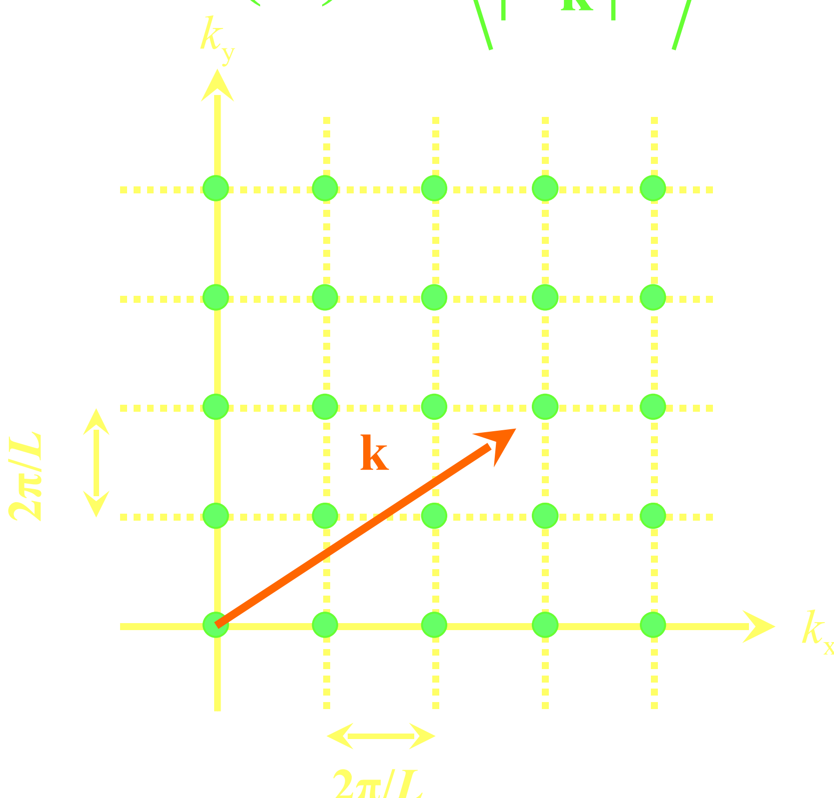
- $a = 0$**: double the info (2/*Var*)**

The power spectrum

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}} = \sum_{\mathbf{k}} \delta_{\mathbf{k}}\, e^{-ik\cdot x}$$

$$P(k) \equiv \left\langle \left|\delta_{\mathbf{k}}\right|^2 \right\rangle$$



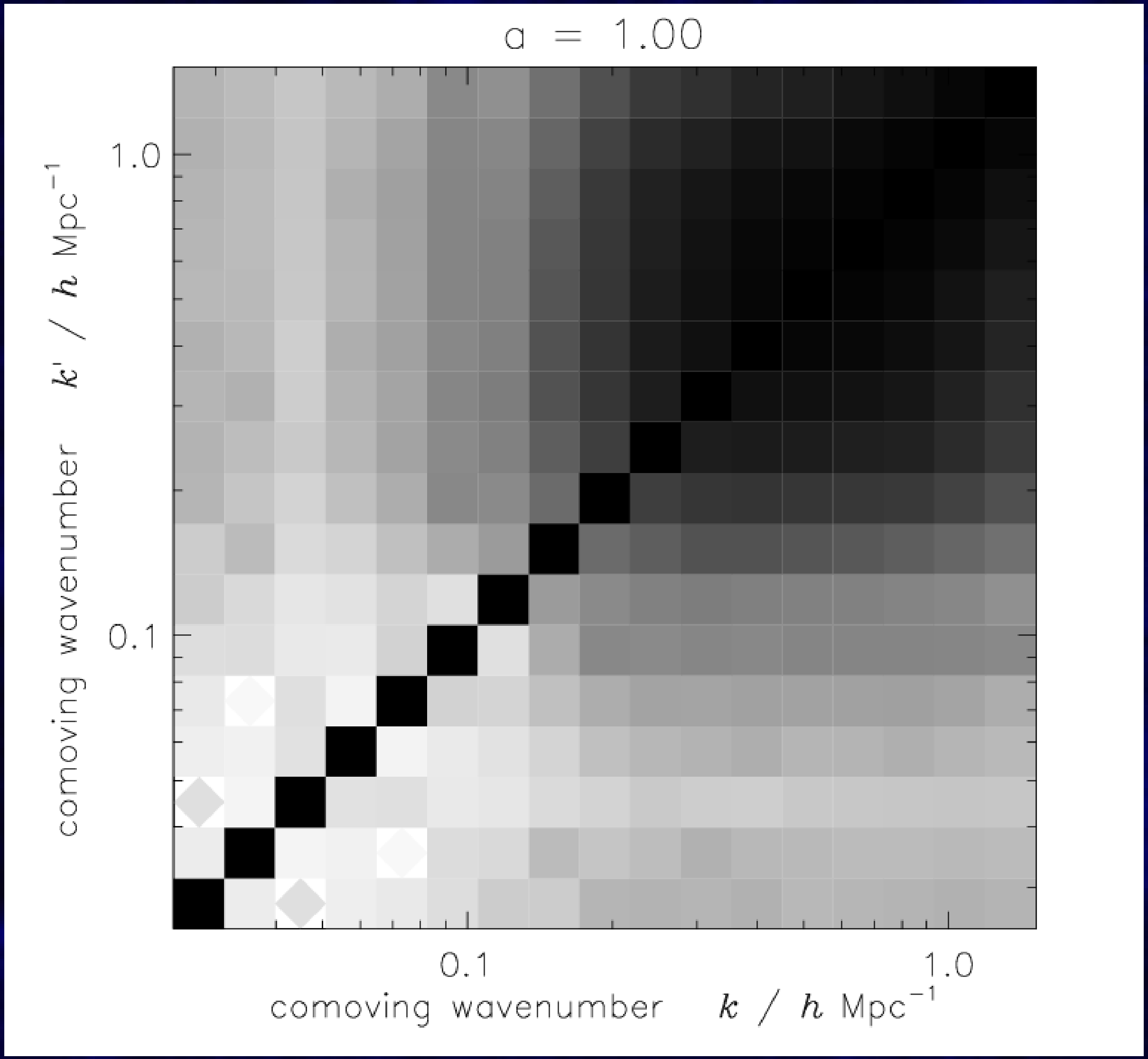


Gaussian field: all initial cosmological information in large-scale structure is in the power spectrum.

Does it stay there?

Growth of correlations

(Rimes & Hamilton)



$$\frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

$$C_{ij} = \langle \Delta P_i \Delta P_j \rangle$$

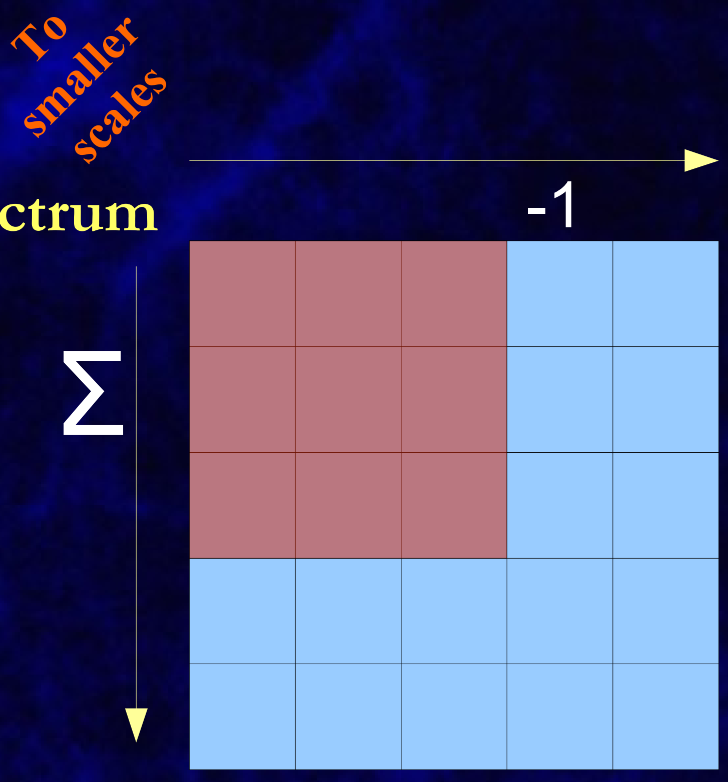
400 realizations of 256 h^{-1} Mpc box

k* increasing, scale *decreasing

Fisher Information

- **The Fisher information in the power spectrum about parameters α and β: concavity of likelihood function**

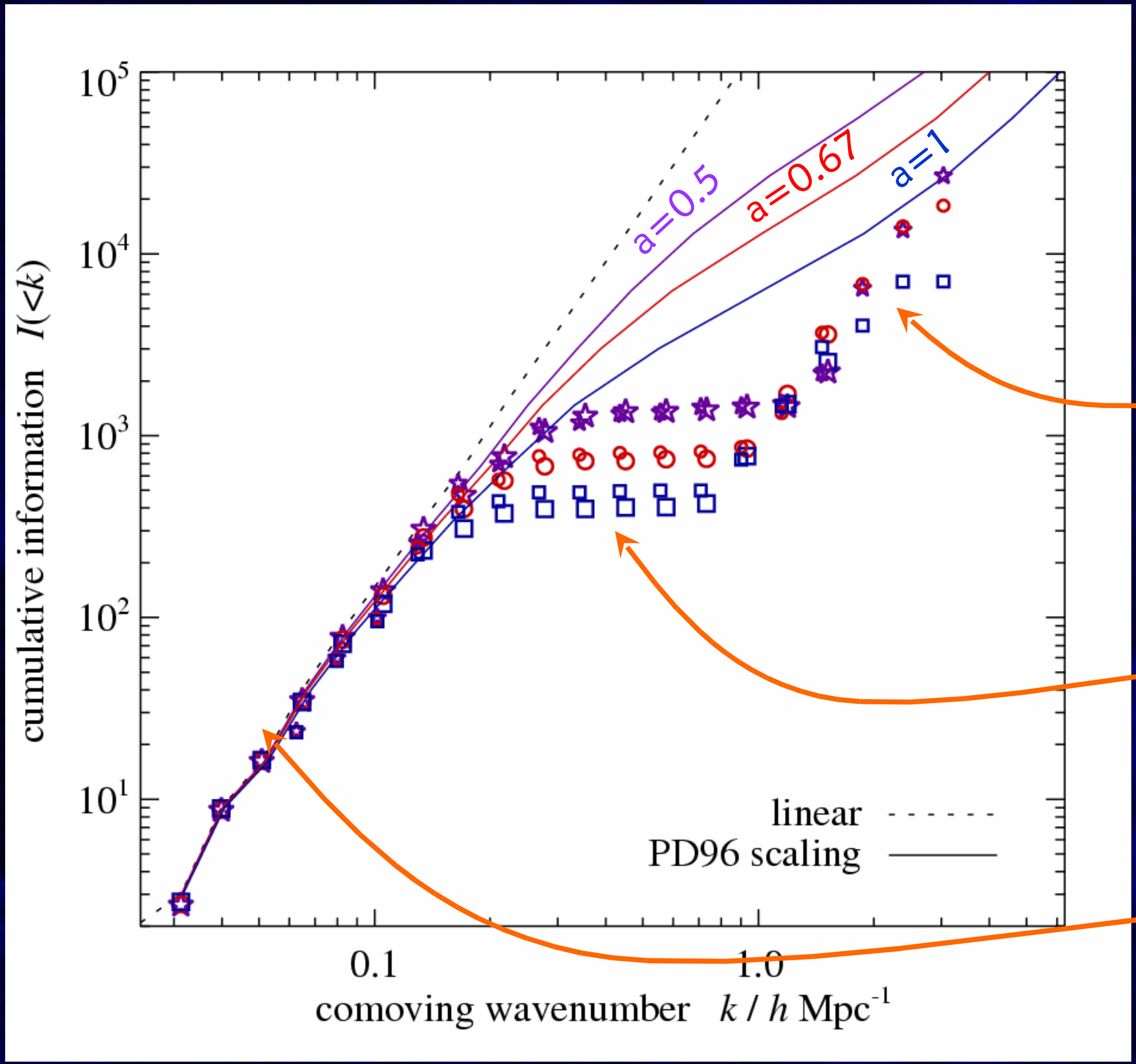
$$F_{\alpha\beta} \equiv -\left\langle \frac{\partial^2 \ln L}{\partial\alpha\,\partial\beta} \right\rangle_{ML}$$



- **Cumulative information over a range of bins *R*:**

$$F_{\alpha\beta}(R) = \sum_{i,j\in R} \frac{\partial \ln P_i}{\partial\alpha} (C_R^{-1})_{ij} \frac{\partial \ln P_j}{\partial\beta}$$

Cumulative information $I_A(<k)$



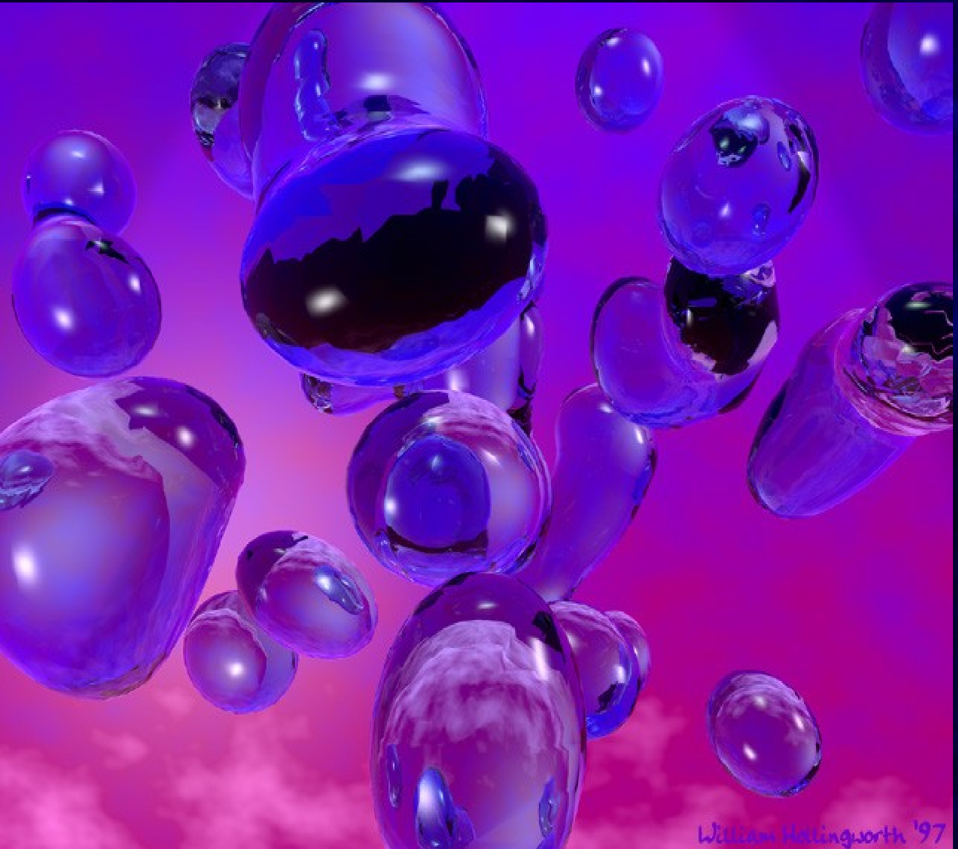
Information rises again at small scales (but more shallowly than k^3)

Very little additional info on "translinear" scales

$I_A(<k) \propto k^3$ in linear regime

Sharply demarcated behavior in different regimes:

A job for the halo model?



• Halo models: virialized dark-matter haloes distributed according to leading-order perturbation theory (not quite right – Smith, Scoccimarro & Sheth)

•Need 4-point function (trispectrum) to describe <$\Delta P_i \Delta P_j$>, the product of two 2-point functions.

$$C_{ij} = \frac{1}{V_{survey}} \left[\underbrace{T_{ij}}_{\text{Non-Gaussian part}} + \underbrace{\delta_{ij} (2\pi)^3 \frac{2P(k_i)^2}{V(k_i)}}_{\text{Gaussian part}} \right]$$

where the trispectrum

$$T_{ij} = \langle T(k_i, -k_i, k_j, -k_j) \rangle_{k_i \cdot k_j}$$

Halo model trispectrum (Cooray & Hu 2001)

$$T = T^{1h} + T^{2h} + T^{3h} + T^{4h}$$

$$T_{ij}^{1h} = I_{ii}^{04}$$

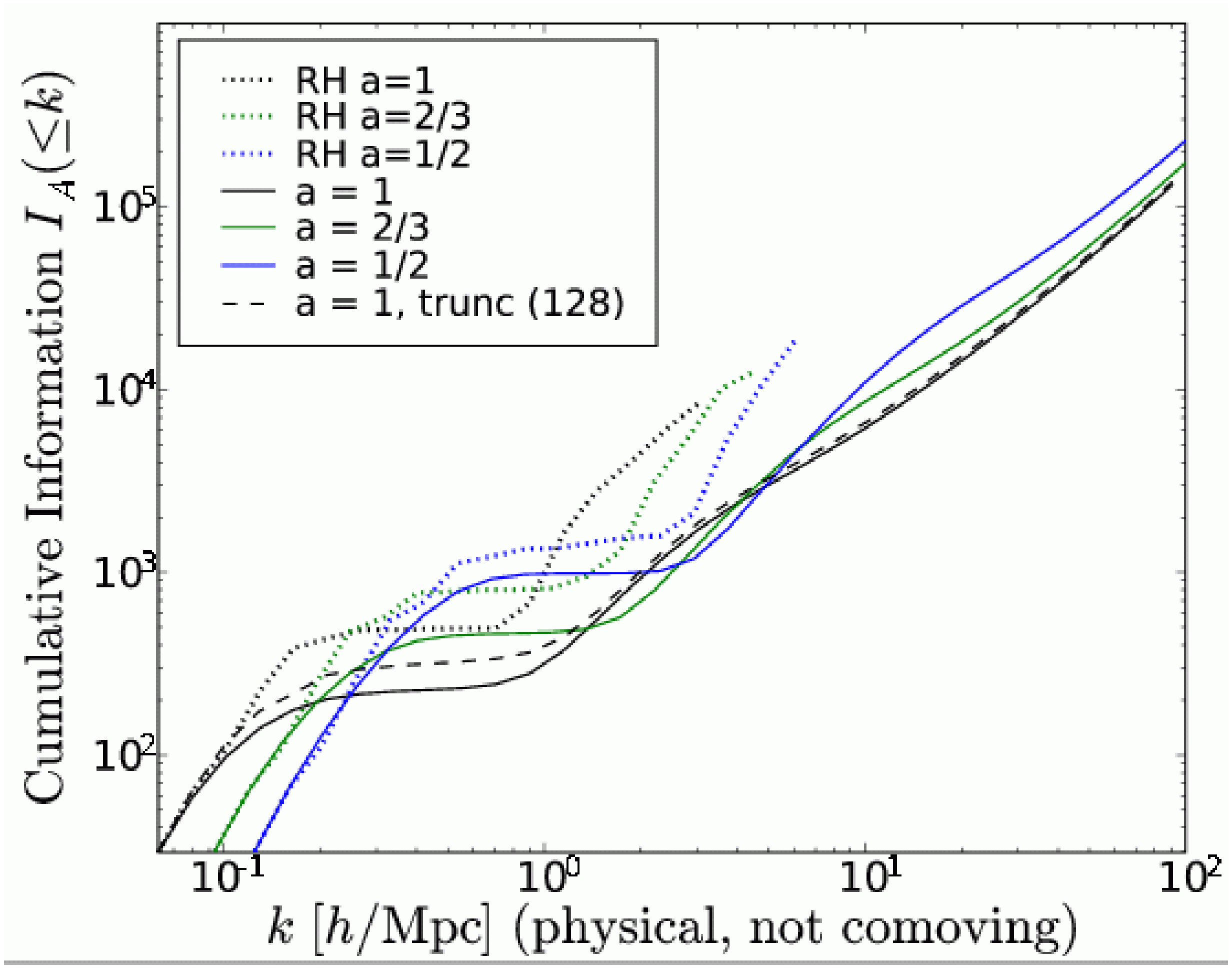
$$T_{ij}^{2h} = \left\langle \left(P_i^{lin} I_i^{11} I_{iij}^{13} + 3\text{ perm.} \right) + \left(P_{ij}^{lin} I_i^{11} I_j^{11} I_{ij}^{22} I_{iij}^{13} + 2\text{ perm.} \right) \right\rangle$$

$$T_{ij}^{3h} = \left\langle \left[B^{pt}(k_i, -k_i, k_j - k_j) I_{jj}^{12} + P_i^{lin} P_i^{lin} I_{jj}^{22} \right] I_i^{11} I_i^{11} + 5\text{ perm.} \right\rangle$$

$$T_{ij}^{4h} = \left(I_i^{11} I_j^{11} \right)^2 \left\langle T^{pt}\left(k_i, -k_i, k_j, -k_j\right) \right\rangle + 2\left(I_i^{11} I_j^{11} P_i^{lin} P_j^{lin} \right)\left(I_i^{21} I_j^{11} P_j^{lin} + I_j^{21} I_i^{11} P_i^{lin} \right)$$

I: halo profile integrals over mass
B^{pt}: Perturbation theory bispectrum
T^{pt}: Perturbation theory trispectrum

Behavior with redshift.
Expect info up to some small *physical* scale to:
- remain constant (~stable clustering?)
 or
- decrease (info diverted into other statistics?)
- constrain halo-model parameters this way?



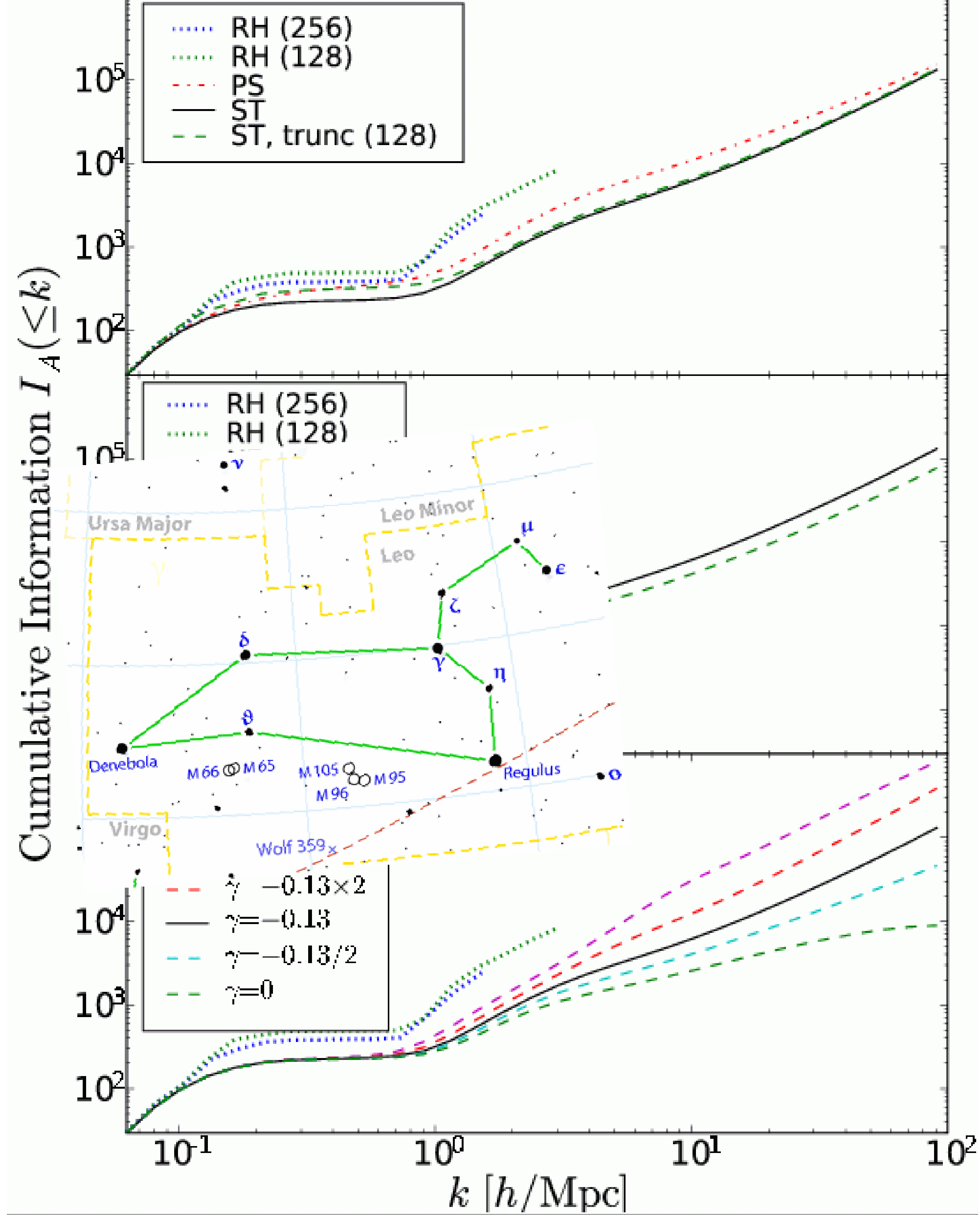
Fiducial (Sheth-Tormen) model: information slowly decreases with time.

Information curve comparison:

- Same behavior as in simulations.

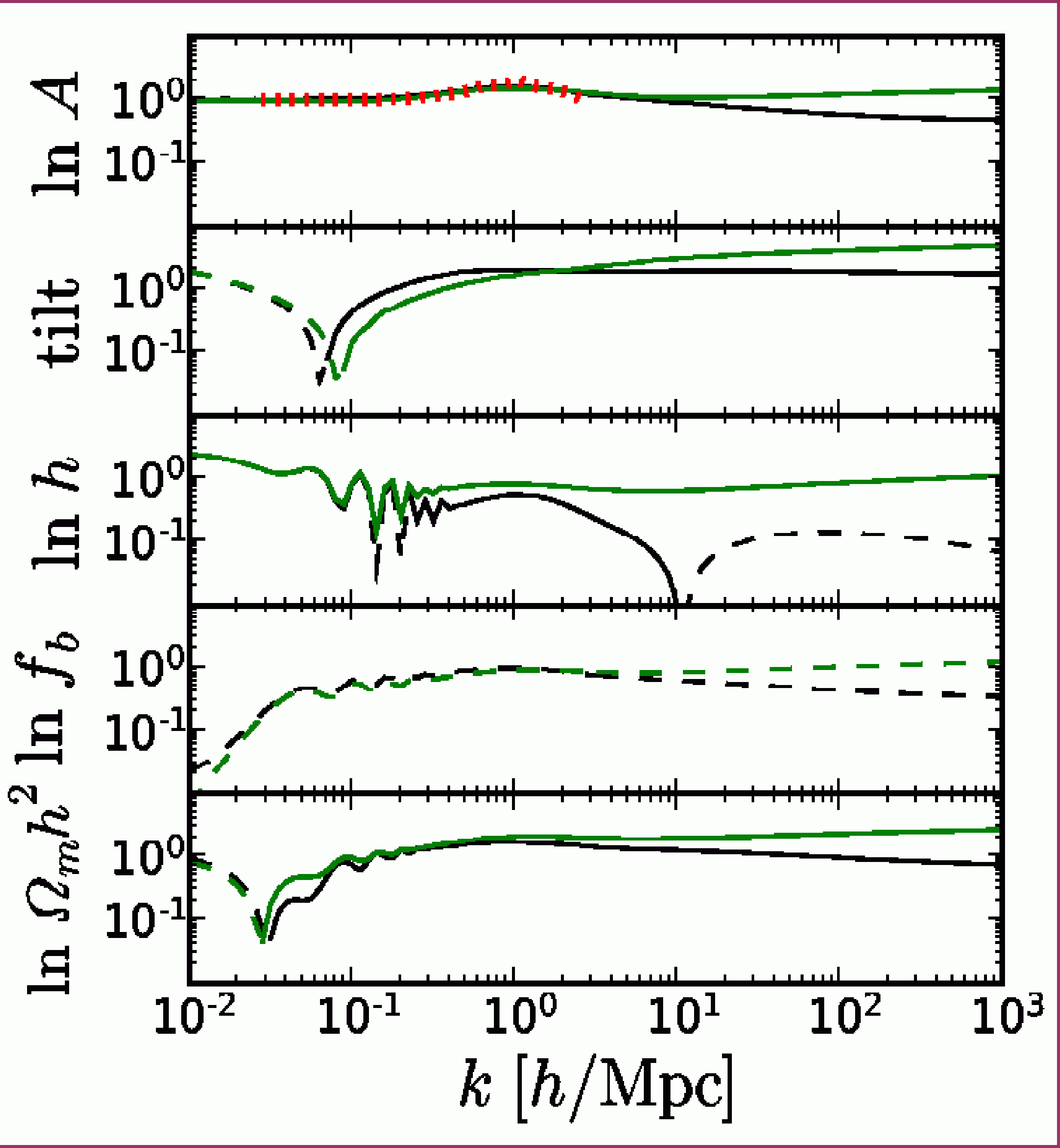
- Hope for cosmological parameter constraints on small scales? Small-scale info encoded in halo distribution?

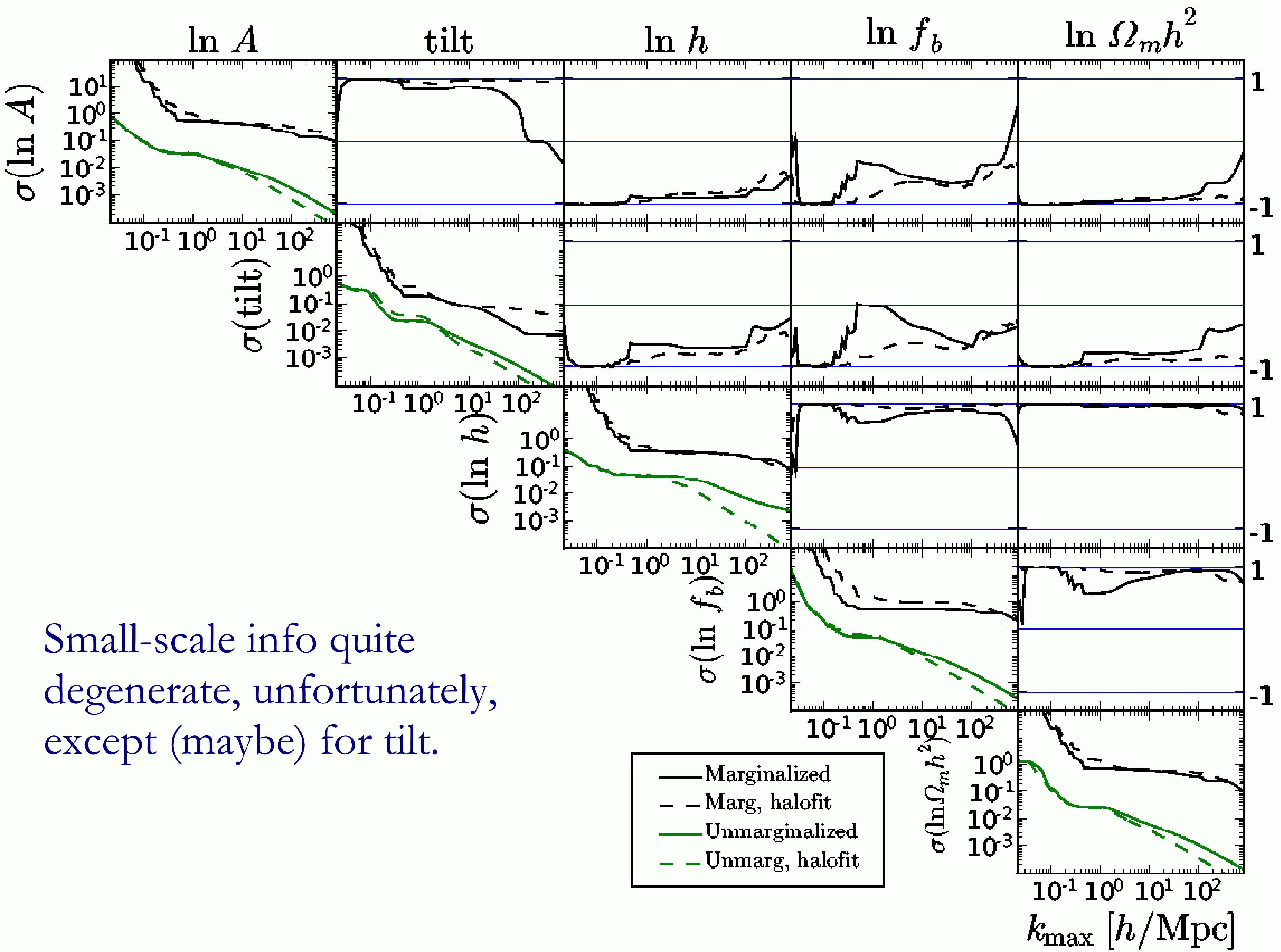
- What about multiple parameters?



To investigate degeneracy of parameters, we need derivatives of $P(k)$ wrt some parameters:

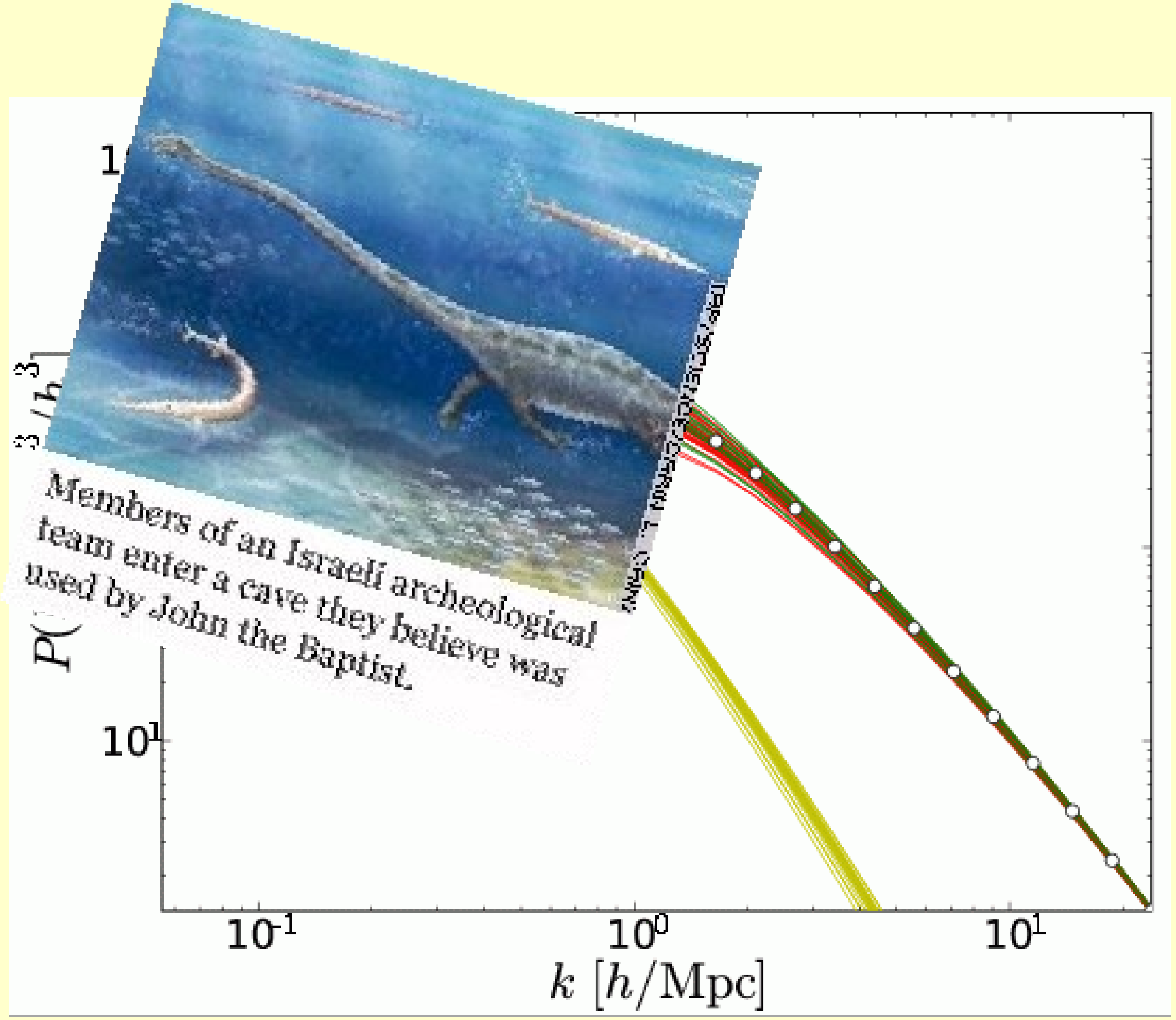
-Tried two methods: our halo model code, HALOFIT





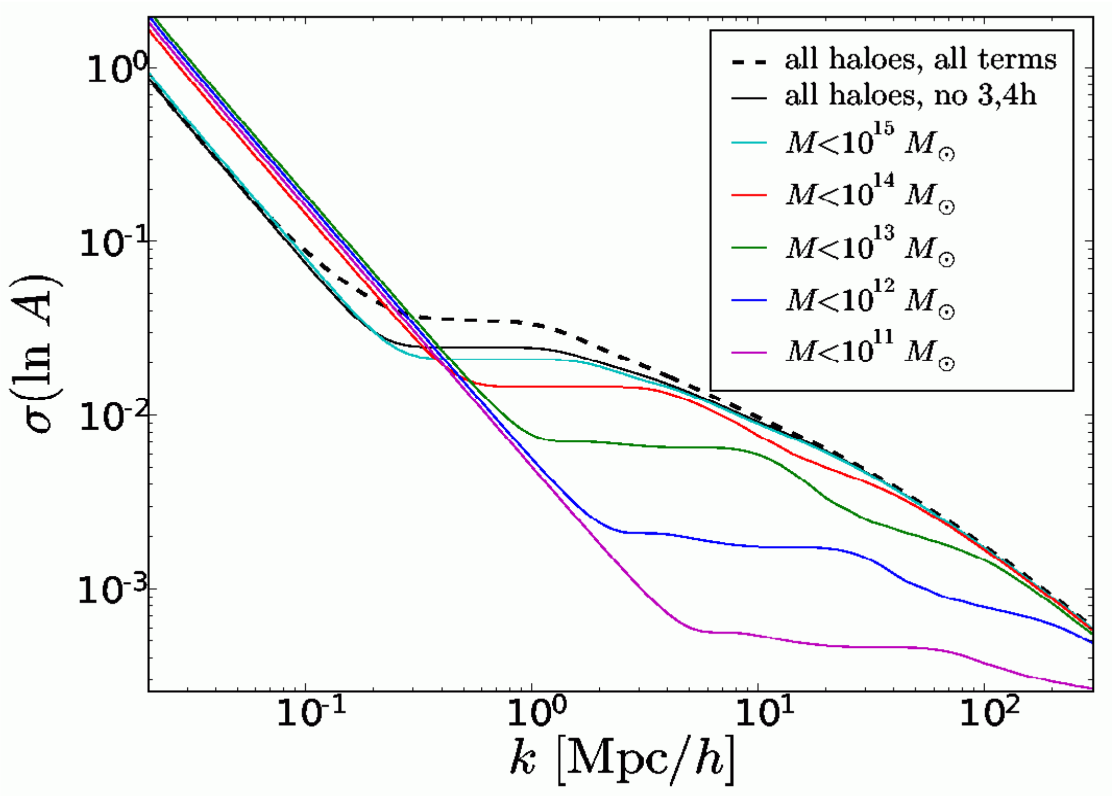
Small-scale info quite degenerate, unfortunately, except (maybe) for tilt.

Physical explanation: halo number fluctuations in a finite volume?



In a finite volume *V*, the number of haloes in each mass bin is not $Vn(m)dm$ (likely fractional), but is an integer drawn from a distribution with that mean. Most of the (co)variance in the halo model is from this effect.

Look at the power spectrum of matter in rural areas of the Universe?



Caveat: need to know halo-halo power spectrum P^{hh} and its covariance; even P^{hh} itself is rather elusive on translinear scales (Smith, Scoccimarro, & Sheth 2006).

If dark matter is collisionless, cosmological info should not be lost. Possibilities:

-- Info is transferred to smaller scales in the power spectrum

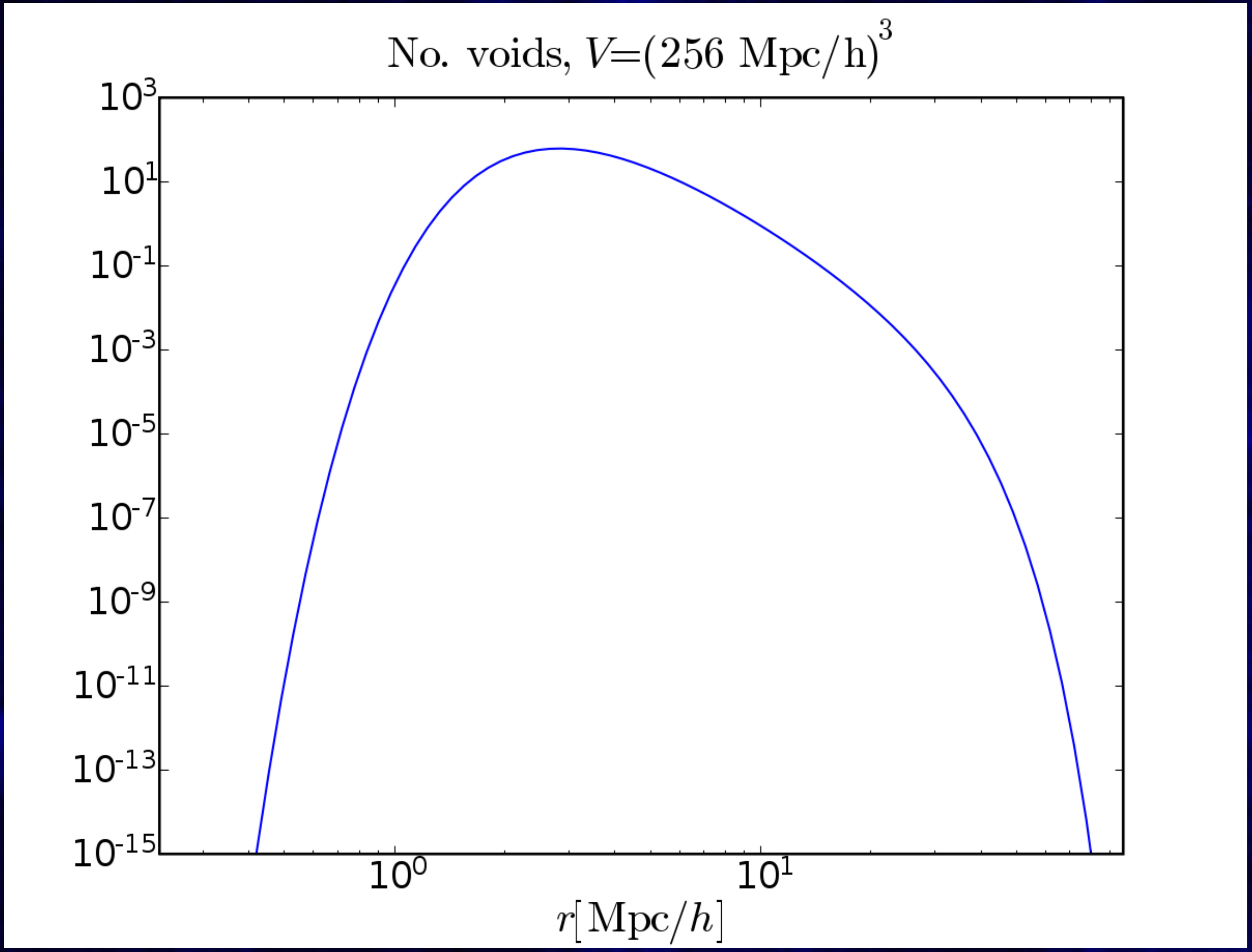
-- Info is transferred to low higher-order statistics

-- Info is transferred to statistics involving all higher-order statistics, e.g. void stats (our eyes perceive information there)

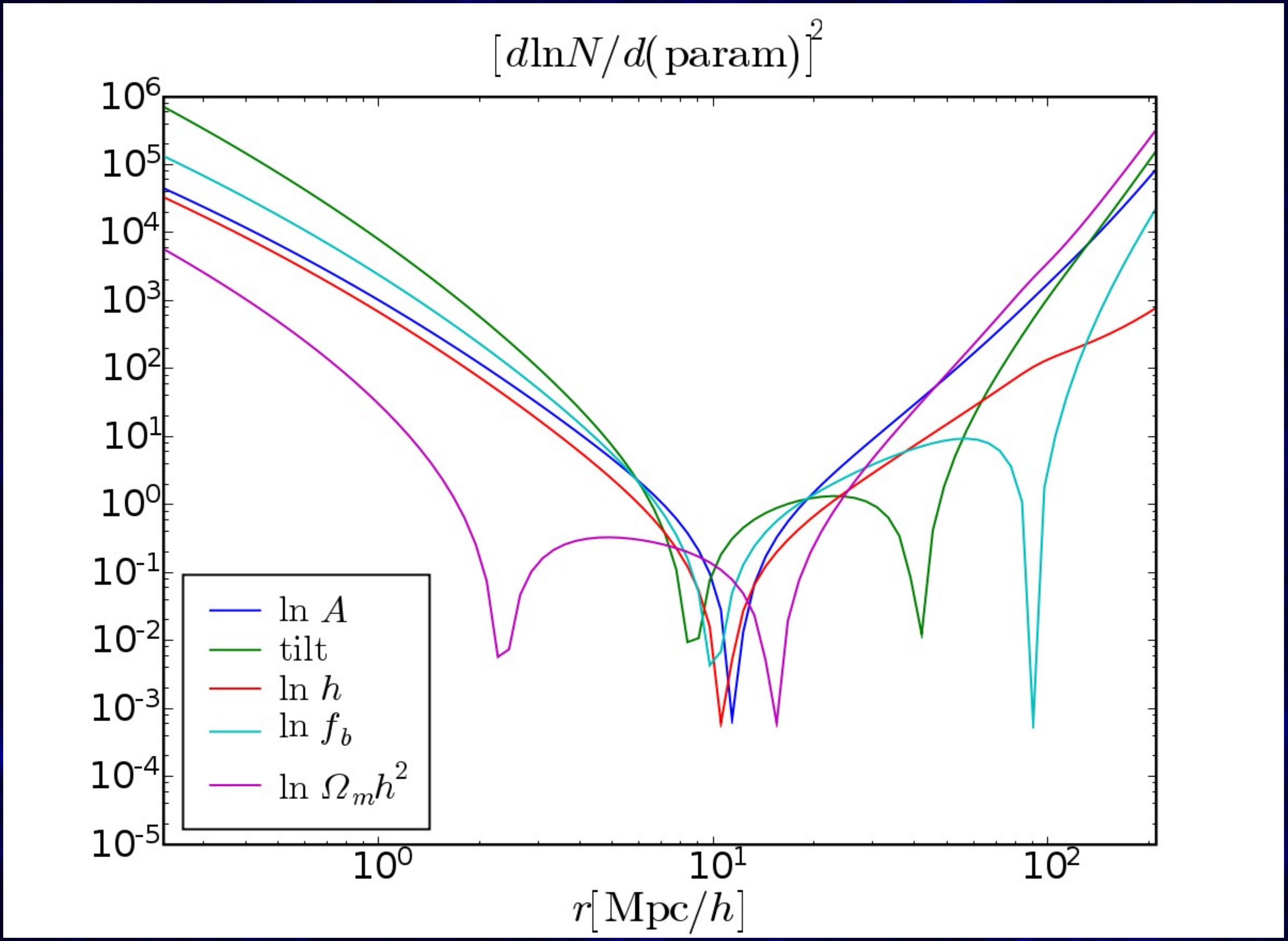
-- Chaotic system => info is there, but virtually impossible to extract.

Try voids (preliminary)

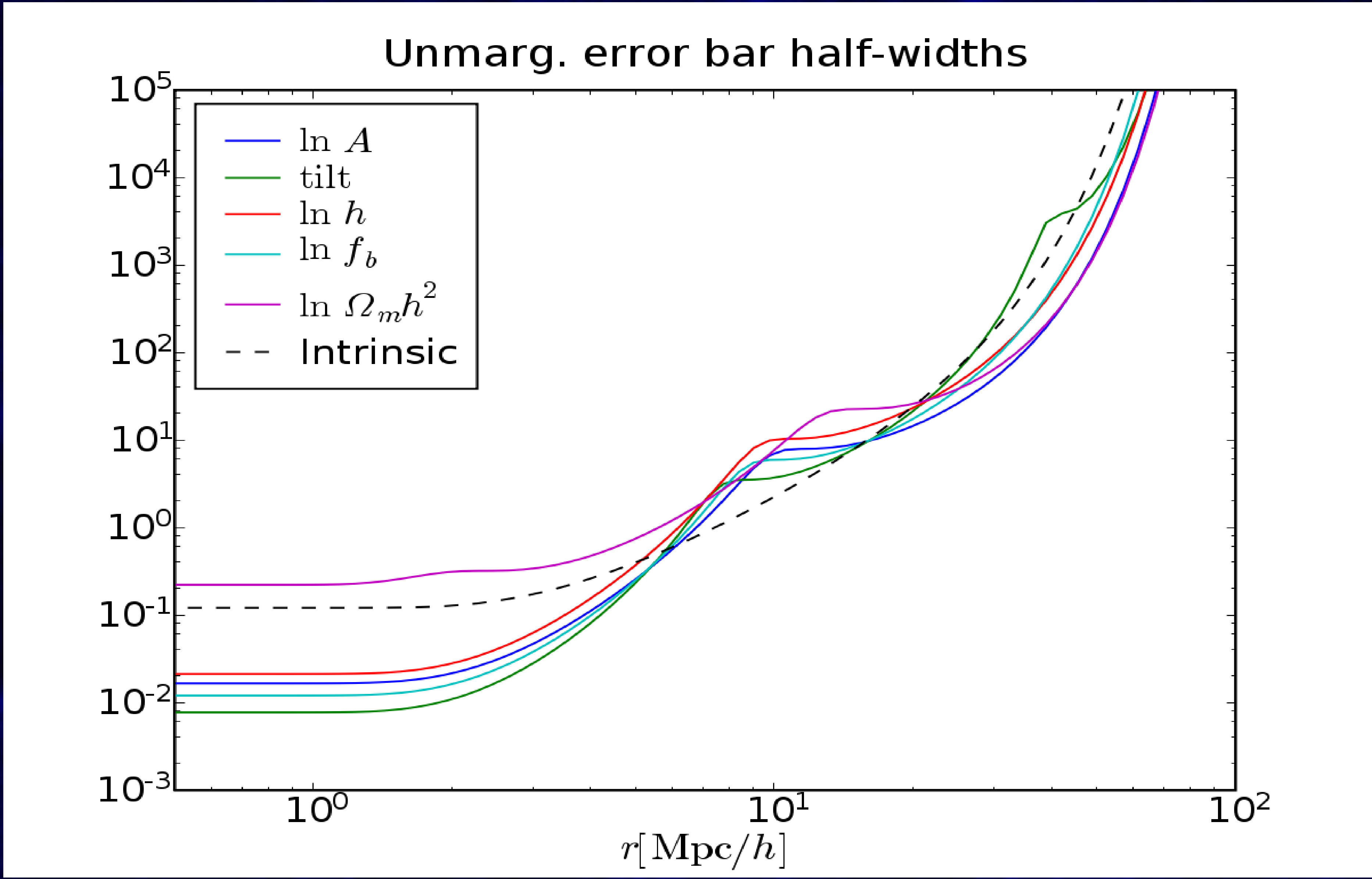
Sheth & van de Weygaert: void mass spectrum has small-scale cut-off. (probably not observable; oh well)



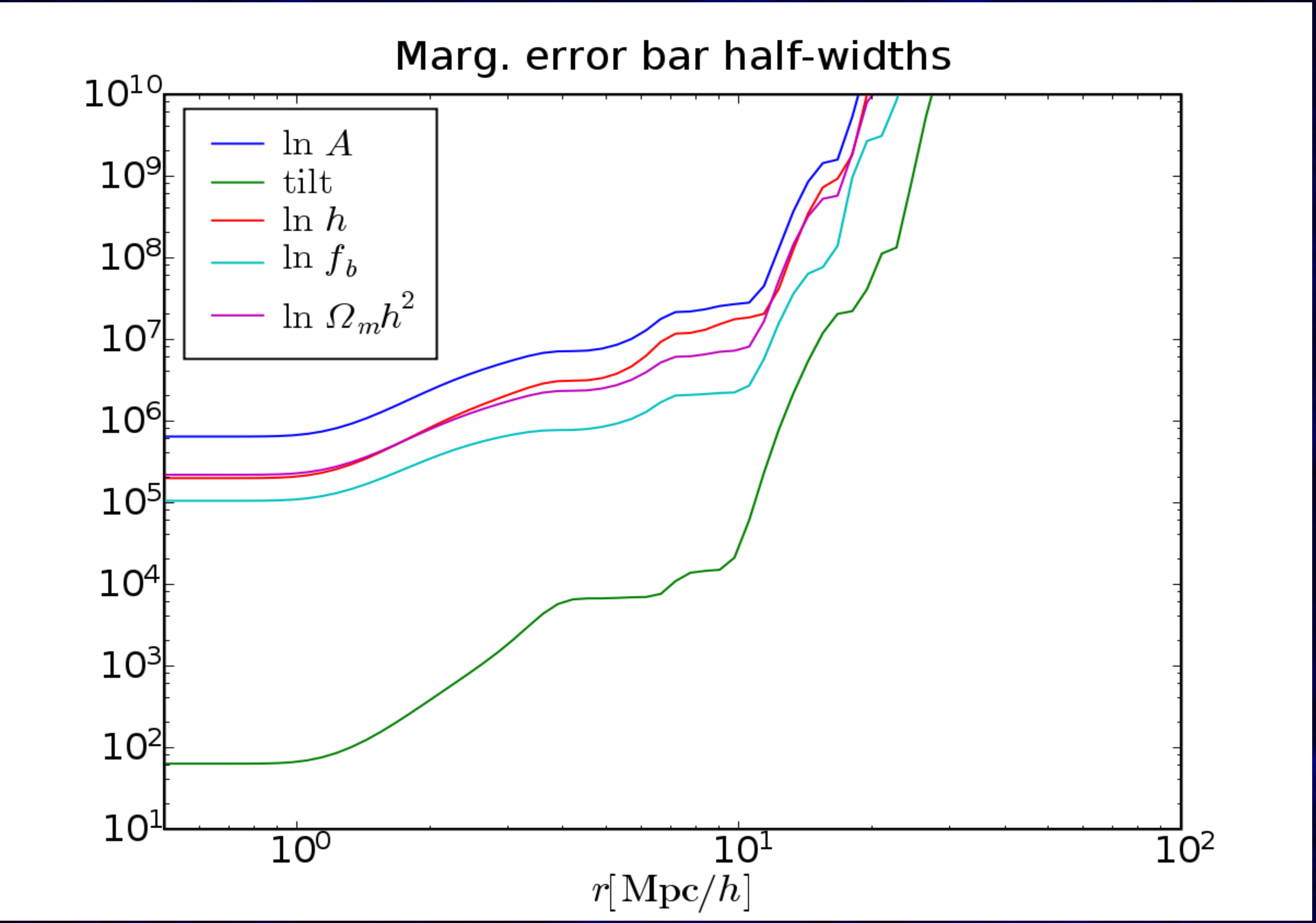
Some dependence of void radius function on cosmological parameters:



Each parameter varied alone (assume Poisson variance, no covariance):



Marginalized; not so good.



Conclusions

- The matter power spectrum on small, non-linear scales is disappointing from the point of view of statistical cosmological parameter estimation, even if you can measure and model it well.
- Hope: measure matter, galaxy power spectrum outside of clusters? Use halo (cluster?) power spectrum itself?
- Information in other statistics, more sensitive to non-linearity?

Physical explanation: halo number fluctuations in a finite volume?

It can be proven that the covariance from the 1h term of the power spectrum is the 1h halo-model covariance term.

This term dominates the halo-model covariance.

Why? Consider a product of integrals over Poisson-fluctuated mass functions in a volume V. It can be shown analytically that

$$\left\langle \int f(m)\tilde{n}(m)\,dm \int g(m)\tilde{n}(m)\,dm \right\rangle =$$

$$\int f(m)n(m)\int f(m)n(m)\,dm + \frac{1}{V}\int f(m)g(m)n(m)\,dm$$

Maybe alarming, since the 2h term of the power spectrum contains such a product. However, the $1/V$ term comes from considering the same halo twice. Something from just one halo cannot enter the 2h term, but it can enter the product of 1h terms.