△riangulating Radiation

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\(\nabla\)riangulating
Radiation

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Outline

- Transport theory
- Numerical methods
- New method
- Example: Epoch of Reionization
Transport Theory

- **Master Equation**: transport of probability in some abstract space.
- **Drift** and **collision** (interaction) terms.

\[ Df = Cf \]

Electron, neutron & photon transport; gas dynamics; economics; behavioral sciences; chemistry; traffic analysis.
Boltzmann Equation

- Project $ME$ on phase space:
  \[
  f(\vec{\mu}) = f(\vec{x}, n, E, t)
  \]

\[
\left[ \frac{\partial}{\partial t} + \vec{n} \cdot \vec{\nabla} \right] f(\vec{\mu}) = \frac{\partial f(\vec{\mu})}{\partial t} \bigg|_{\text{coll}}
\]

- Describes the transport of particles, which interact with each other, or with a background medium.

- Every interaction has its own term $\sigma_i$. 
Path Length

• Interaction space can be parametrised by *free paths*:

\[ p(s) = \frac{e^{-s/\lambda}}{\lambda} \]
Path Length

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- The d.f. has moments:
  \[ \langle s^k \rangle = k!\lambda^k \]
Path Length

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\[ p(s) = \frac{e^{-s/\lambda}}{\lambda} \]

• The d.f. has moments:

\[ \langle s^k \rangle = k! \lambda^k \]

• A Mean Free Path:

\[ \lambda = \frac{1}{n\sigma} \]
Numerical Method
Monte Carlo

- Model *macroscopic* system by sampling *microscopic* interactions.
- Send out $N$ packets into *random* directions and *sample* the path length d.f.
- Particles move one *mfp* on average => interaction!
Problems
Problems

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- Cell size $>$ MFP
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- Adaptive Mesh Refinement
- Homogeneity
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New Method
New Method

\[ n_p(\vec{x}) = \Phi \ast f(n(\vec{x})) \]
The Jigsaw!

Using Euclidean recipe => lattice isotropic!

Lattice QCD (Christ, Friedberg & Lee 1982)
SUSY (Kaku 1983)
Lattice Boltzmann
Adaptive

Edge Length

\[ \langle L^k \rangle \propto n_p^{-k/d} \]

Free Path

\[ \langle s^k \rangle \propto \lambda^k = n^{-k} \]
Adaptive

Edge Length

\[ \langle L^k \rangle \propto n_p^{-k/d} \]

Free Path

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\[ n_p(\vec{x}) \propto n^d(\vec{x}) \]
Adaptive

Edge Length

\[ \langle L^k \rangle \propto n_p^{-k/d} \]

Free Path

\[ \langle s^k \rangle \propto \lambda^k = n^{-k} \]

Global correlation \( \Rightarrow \)

\[ n_p(\vec{x}) \propto n^d(\vec{x}) \]

\[ \langle L^k \rangle = c(k)\lambda^k \]
Monte Carlo

- Fixed grid;
- *Stochastic* particle movement.

New Method

- *Stochastic* grid;
- Deterministic particle movement.
Transport on Graph

Monte Carlo
• Fixed grid;
• Stochastic particle movement.

New Method
• Stochastic grid;
• Deterministic particle movement.

Particles move one MFP!
Radiation Transport
in the Early Universe
What is the Reionization Era?
A Schematic Outline of the Cosmic History

- The Big Bang
  The Universe filled with ionized gas

- The Universe becomes neutral and opaque
  The Dark Ages start

 Galaxies and Quasars begin to form
 The Reionization starts

 The Cosmic Renaissance
 The Dark Ages end

 Reionization complete, the Universe becomes transparent again

 Galaxies evolve

 The Solar System forms

 Today: Astronomers figure it all out!
Blowing Bubbles

3D Ionisation Front

Front position [index]

Time [Myr]
Conclusion

New method:
- Dispenses with regular grids;
- Uses adaptive point process.

Resultant Delaunay graph has edge lengths that correlate linearly with mean free paths.

Transport reduced to walk on adaptive random lattice.

=> Fast, physical, and flexible.
Split into d ‘most’ straightforward.
Conserve momentum on the average.
Interaction

\[ \langle L^k \rangle = c(k) \lambda^k \]

\[ I_{\text{out}} = I_{\text{in}} e^{-c} \]