∇riangulating Radiation

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Sterrewacht
Leiden

3D

∇riangulating Radiation

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Outline

- Transport theory
- Numerical methods
- New method
- Example: *Epoch of Reionization*

Transport Theory

Transport Theory

- *Master Equation*: **transport of probability in some abstract space.**

$$\boxed{\mathbf{D}f = \mathbf{C}f}$$

- *Drift* **and** *collision* **(interaction) terms.**

Electron, neutron & photon transport; gas dynamics; economics; behavioral sciences; chemistry; traffic analysis.

Boltzmann Equation

- Project *ME* on phase space:

$$f(\vec{\mu}) = f(\vec{x}, \vec{n}, E, t)$$

$$\left[\frac{\partial}{\partial t} + \vec{n} \cdot \vec{\nabla}\right] f(\vec{\mu}) = \left.\frac{\partial f(\vec{\mu})}{\partial t}\right|_{\text{coll}}$$

- Describes the transport of particles, which interact with each other, or with a background medium.

- Every *interaction* has its own term σ_i.

Path Length

- Interaction space can be parametrised by *free paths*:

$$p(s) = \frac{e^{-s/\lambda}}{\lambda}$$

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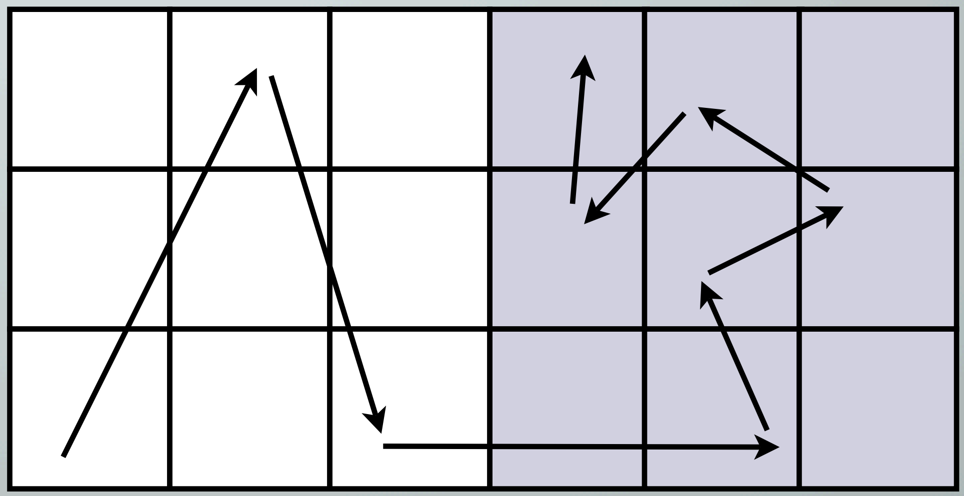
- A *Mean Free Path*:

$$\lambda = \frac{1}{n\sigma}$$

Numerical Method

Monte Carlo

- Model *macroscopic* system by sampling *microscopic* interactions.
- Send out N packets into *random* directions and *sample* the path length d.f.
- Particles move one *mfp* on average => interaction!

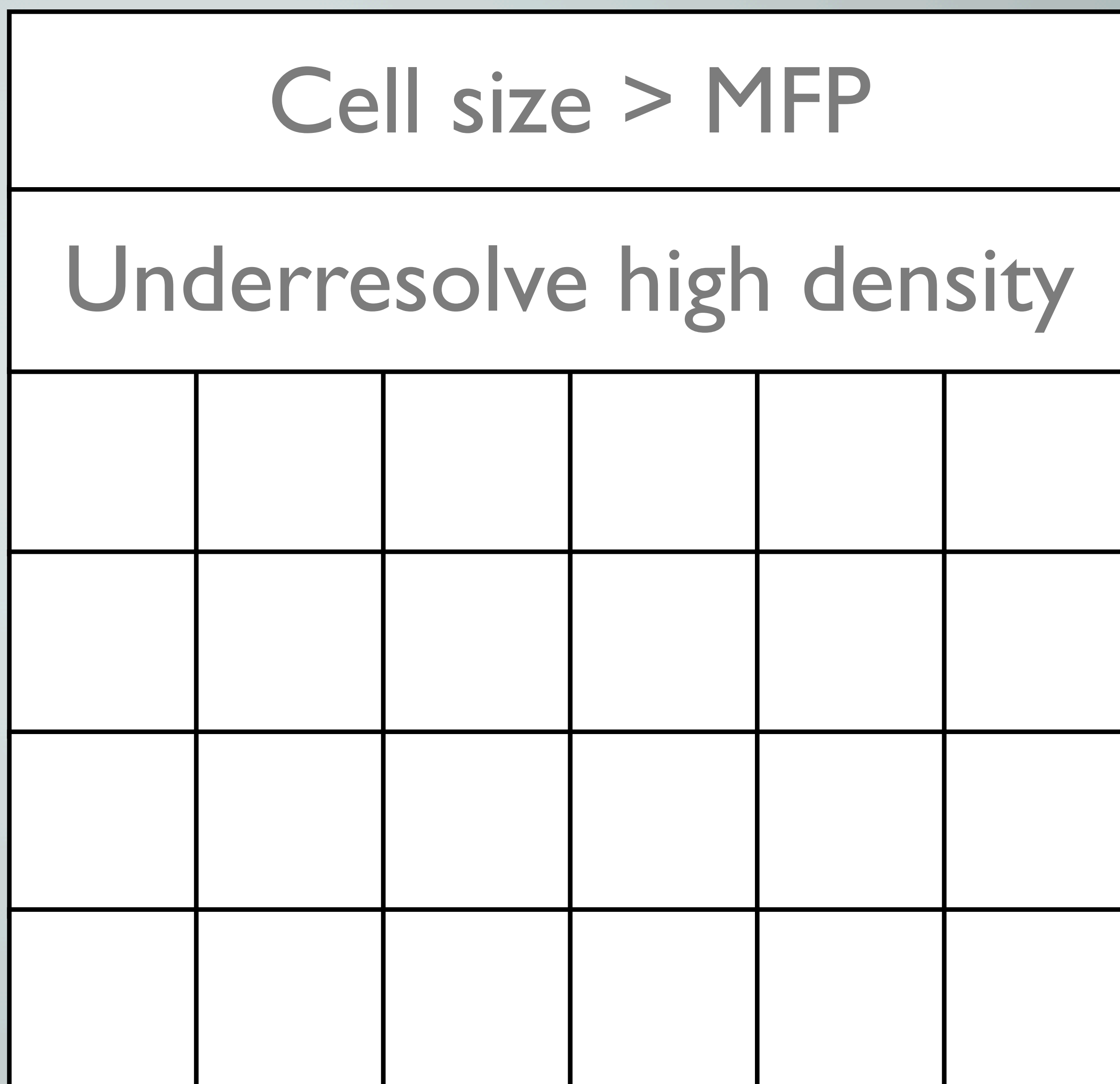


Problems

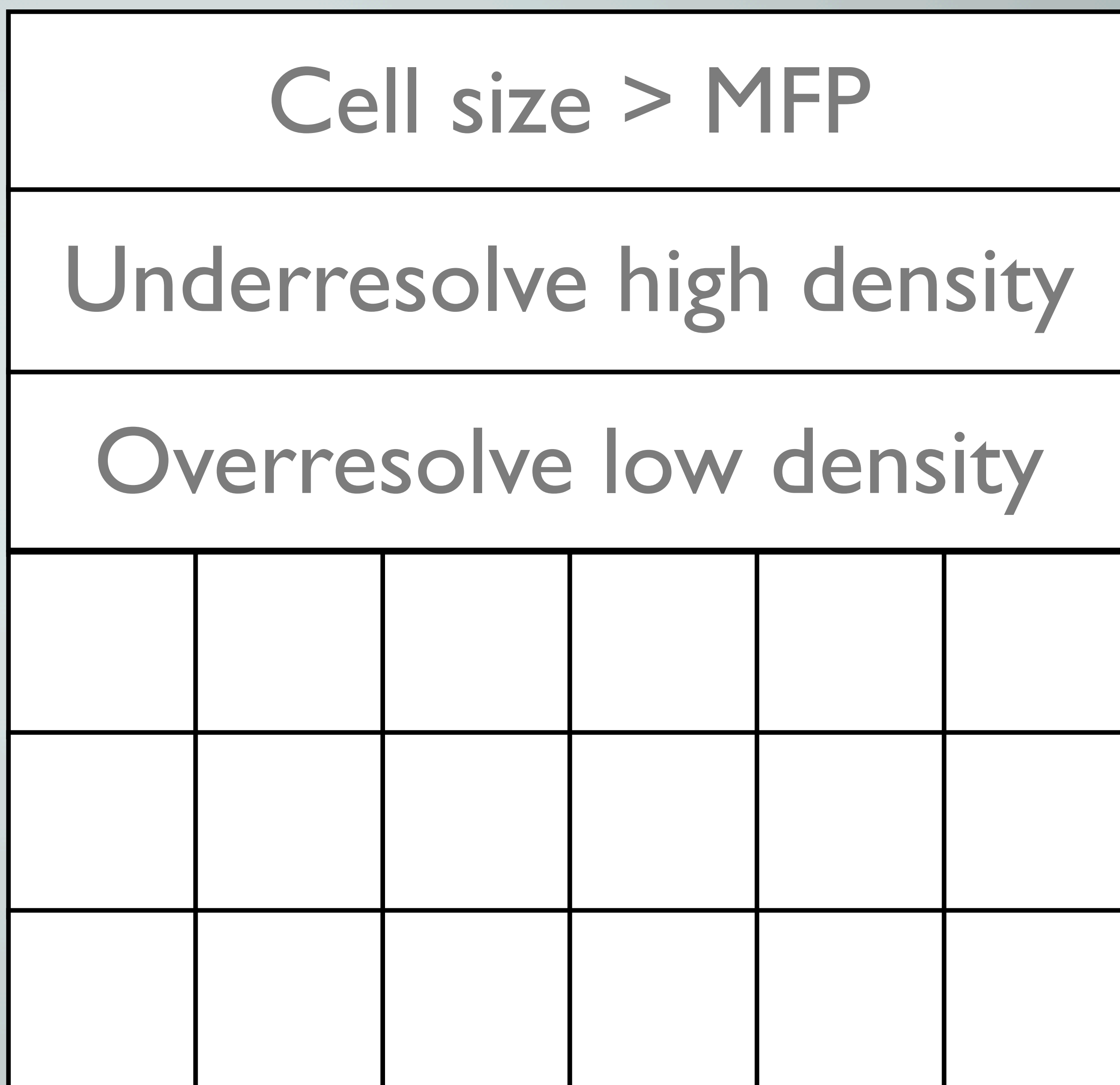
Problems

Cell size > MFP					

Problems



Problems


Cell size > MFP
Underresolve high density
Overresolve low density

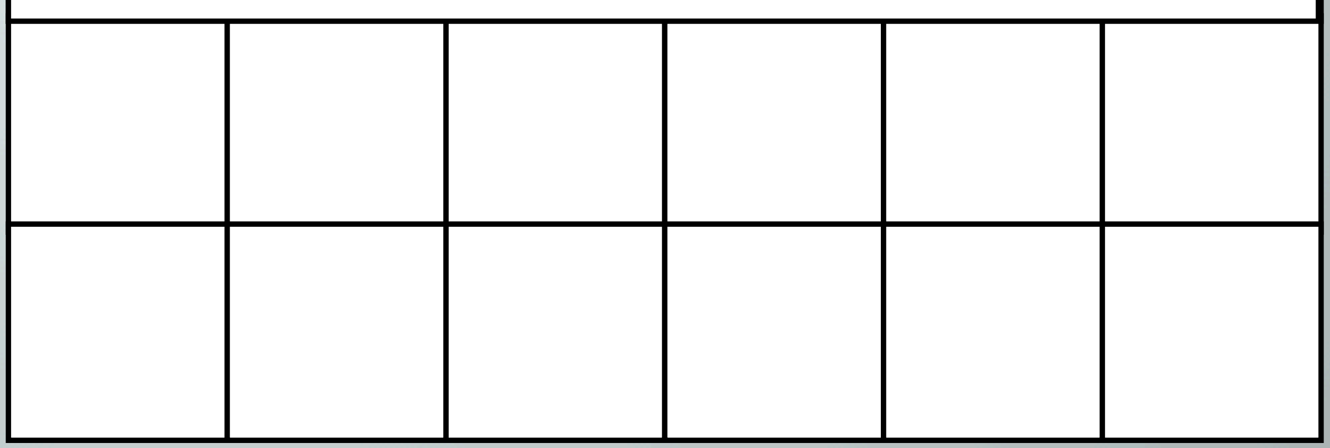
Problems

Cell size > MFP

Underresolve high density

Overresolve low density

Adaptive Mesh Refinement



Problems

Cell size > MFP

Underresolve high density

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Adaptive Mesh Refinement

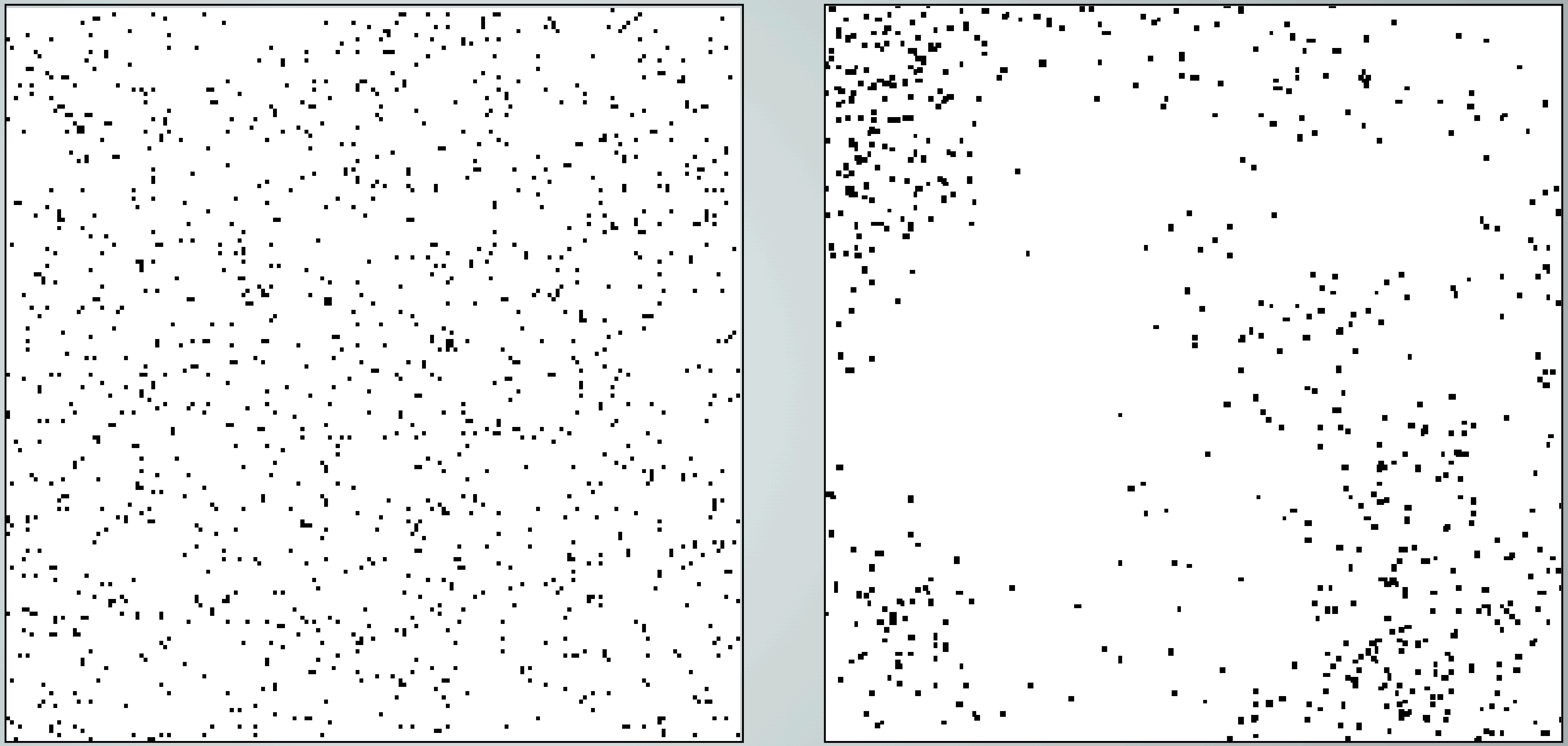
Homogeneity

Problems

Cell size > MFP
Underresolve high density
Overresolve low density
Adaptive Mesh Refinement
Homogeneity
Isotropy

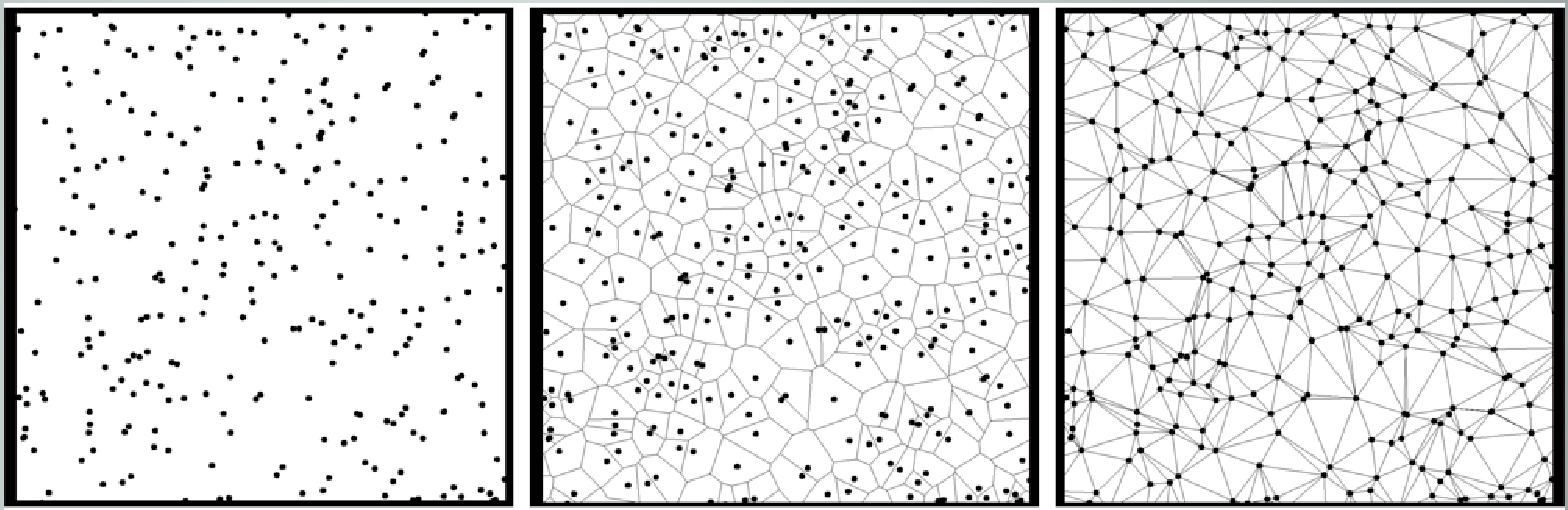
New Method

New Method



$$n_{\mathrm{p}}(\vec{x}) = \Phi * f\left(n(\vec{x})\right)$$

The Jigsaw!



- **Using Euclidean recipe => lattice** *isotropic*!

  Lattice QCD (Christ, Friedberg & Lee 1982)
  SUSY (Kaku 1983)
  Lattice Boltzmann

Adaptive

Edge Length

$$\left\langle L^k \right\rangle \propto n_{\mathrm{p}}^{-k/d}$$

Free Path

$$\left\langle s^k \right\rangle \propto \lambda^k = n^{-k}$$

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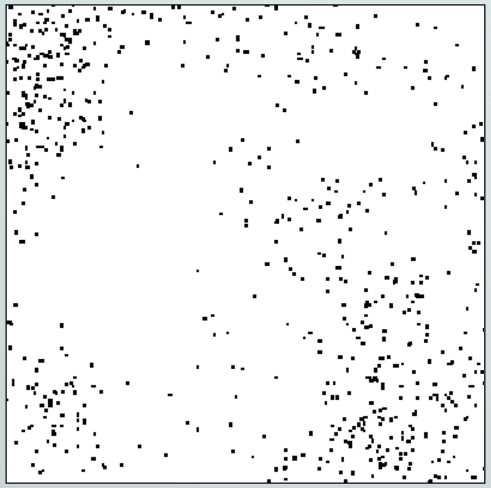

$$n_{\mathrm{p}}(\vec{x}) \propto n^d(\vec{x})$$

Adaptive

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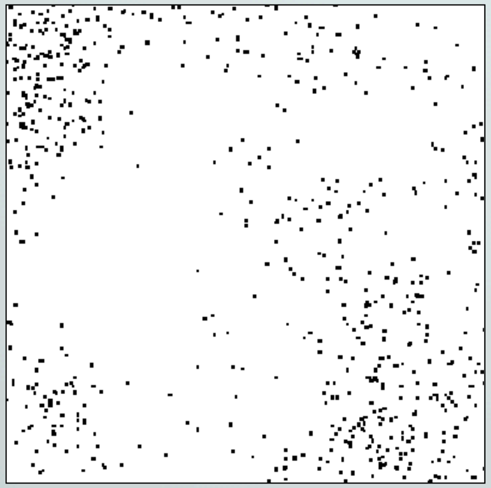


$$n_{\mathrm{p}}(\vec{x}) \propto n^d(\vec{x})$$

Global correlation **=>**

$$\langle L^k \rangle = c(k)\lambda^k$$

Transport on Graph

Monte Carlo

- Fixed grid;
- *Stochastic* particle movement.

New Method

- *Stochastic* grid;
- Deterministic particle movement.

Transport on Graph

Monte Carlo

- Fixed grid;
- *Stochastic* particle movement.

New Method

- *Stochastic* grid;
- Deterministic particle movement.

Particles move one MFP!

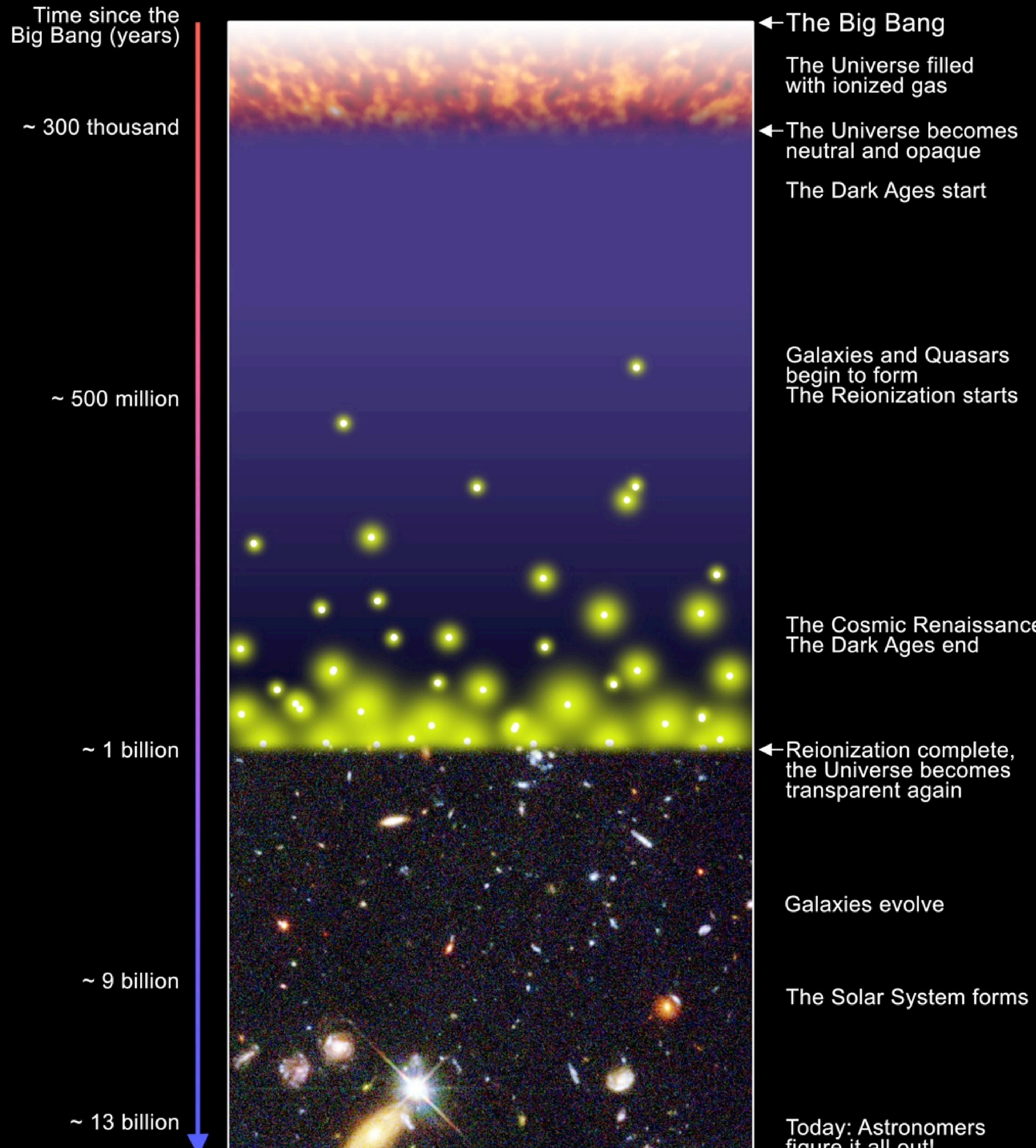
Radiation Transport

in the

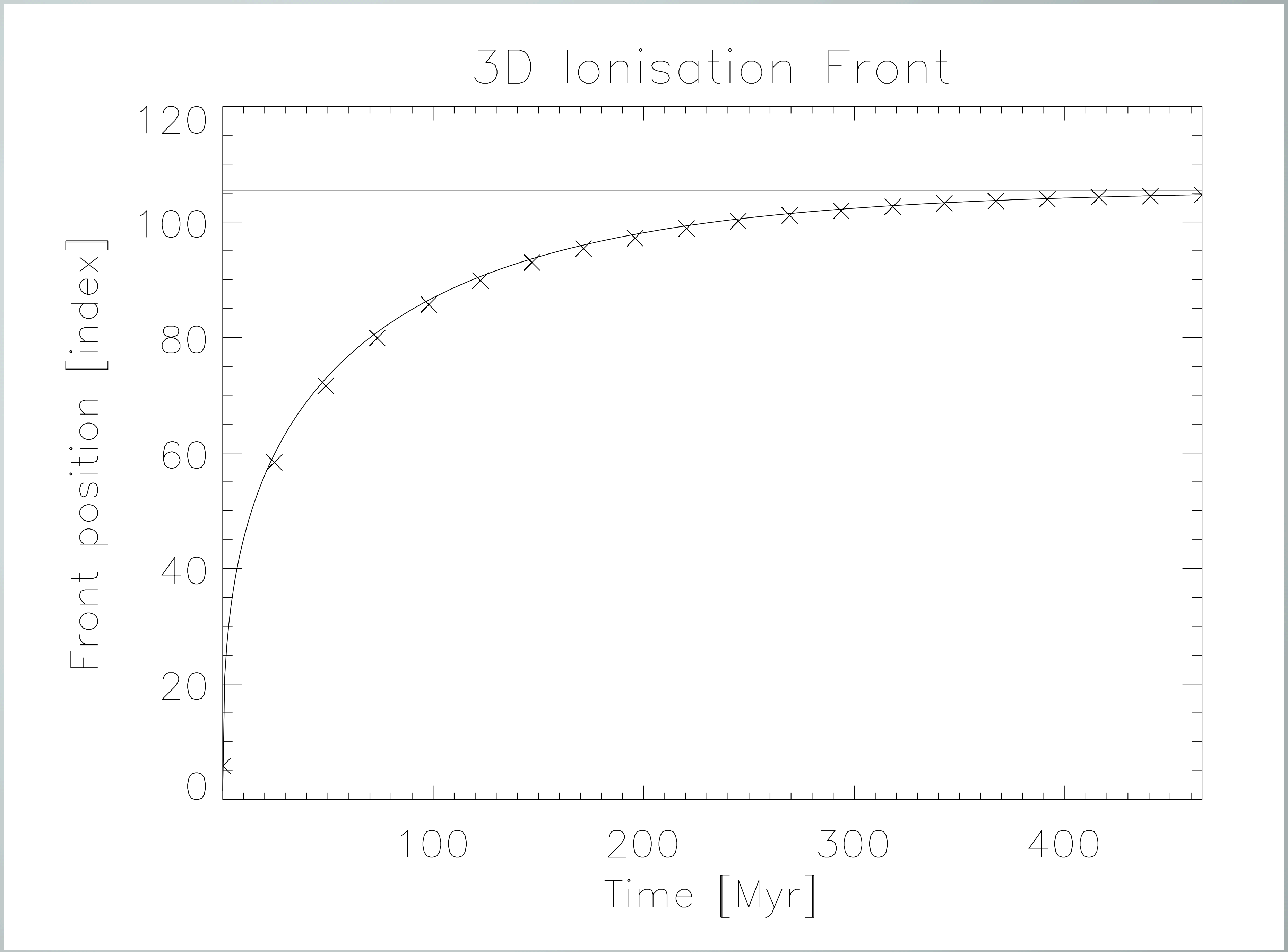
Early Universe

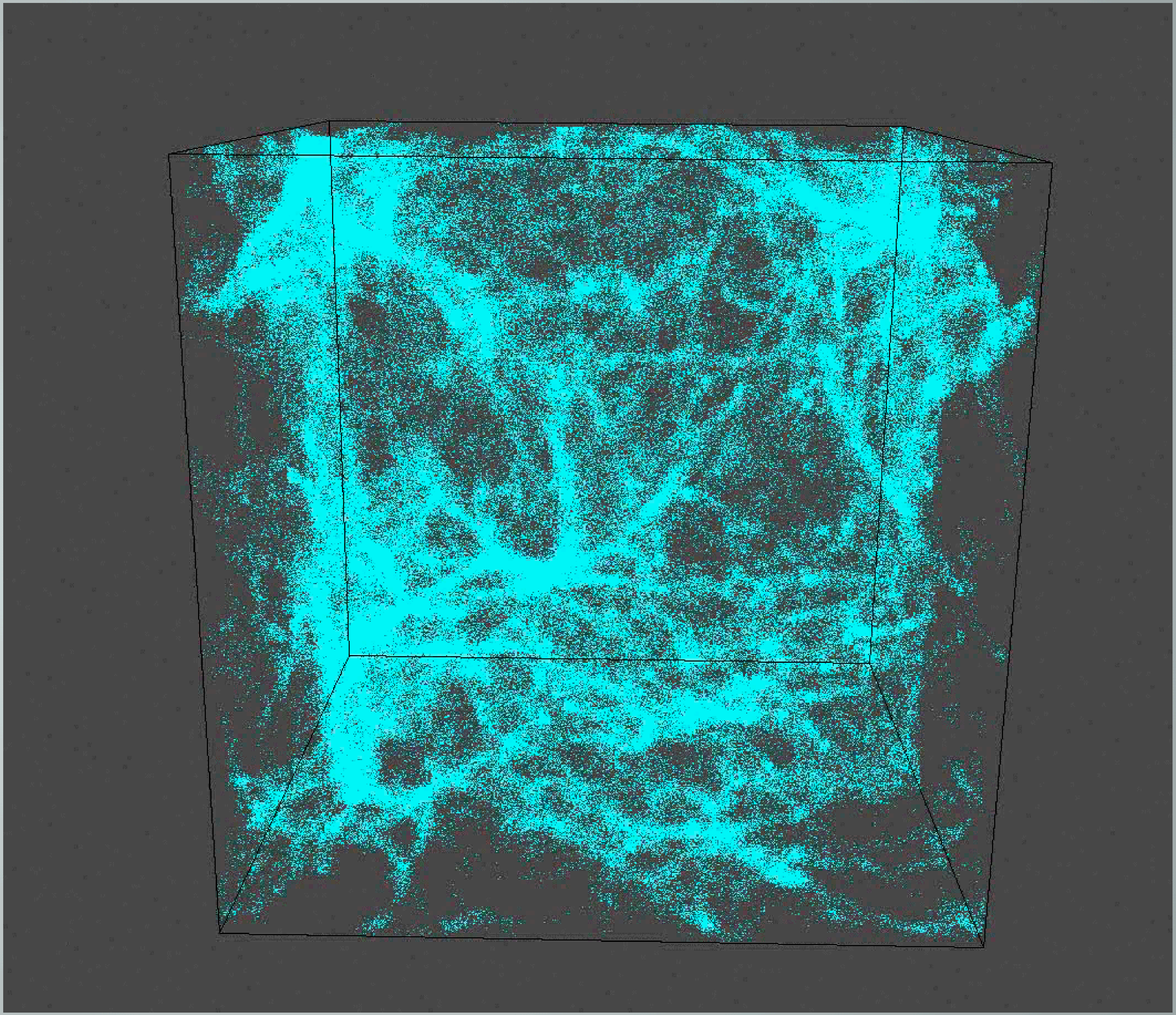
What is the Reionization Era?

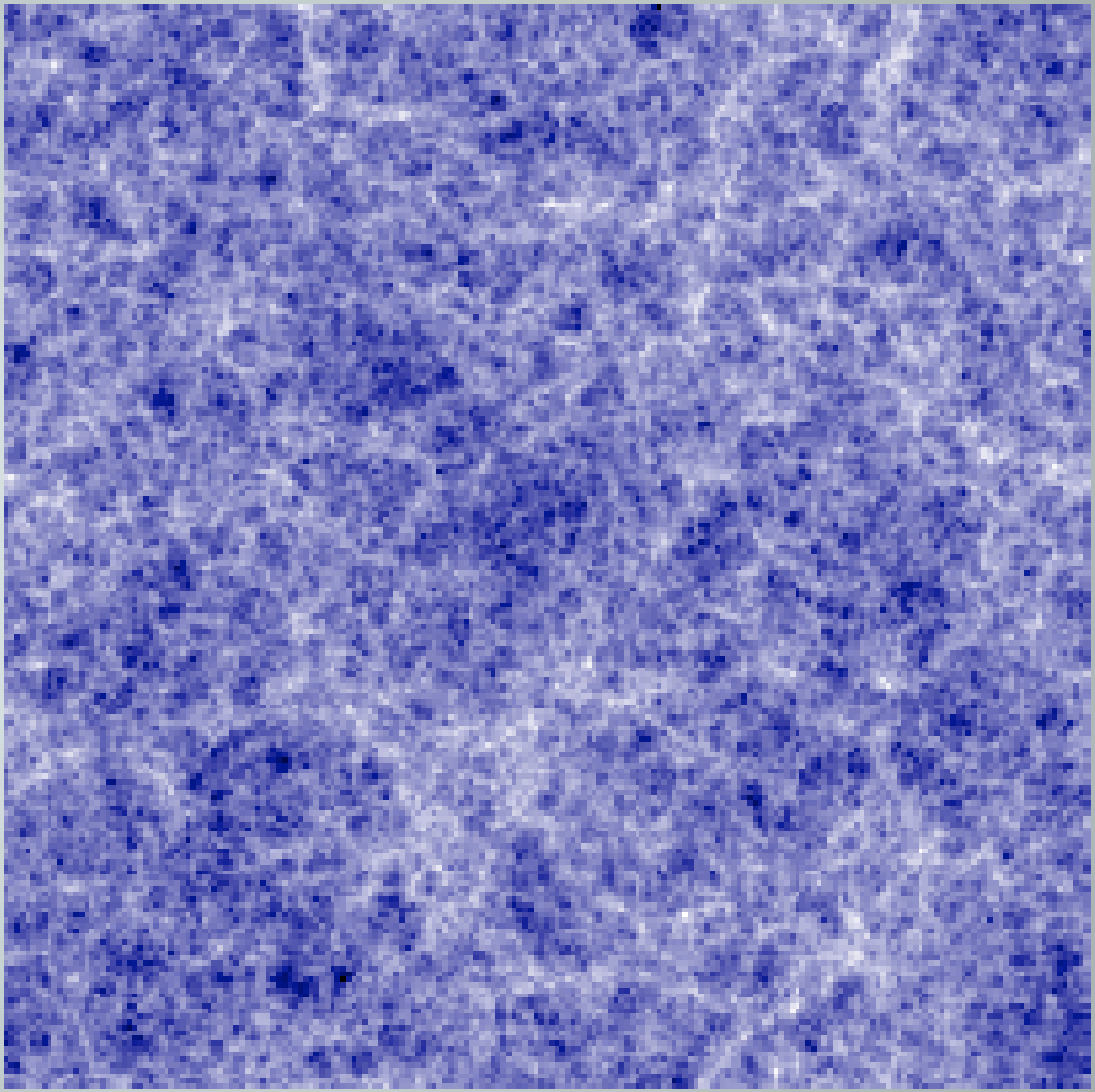
A Schematic Outline of the Cosmic History

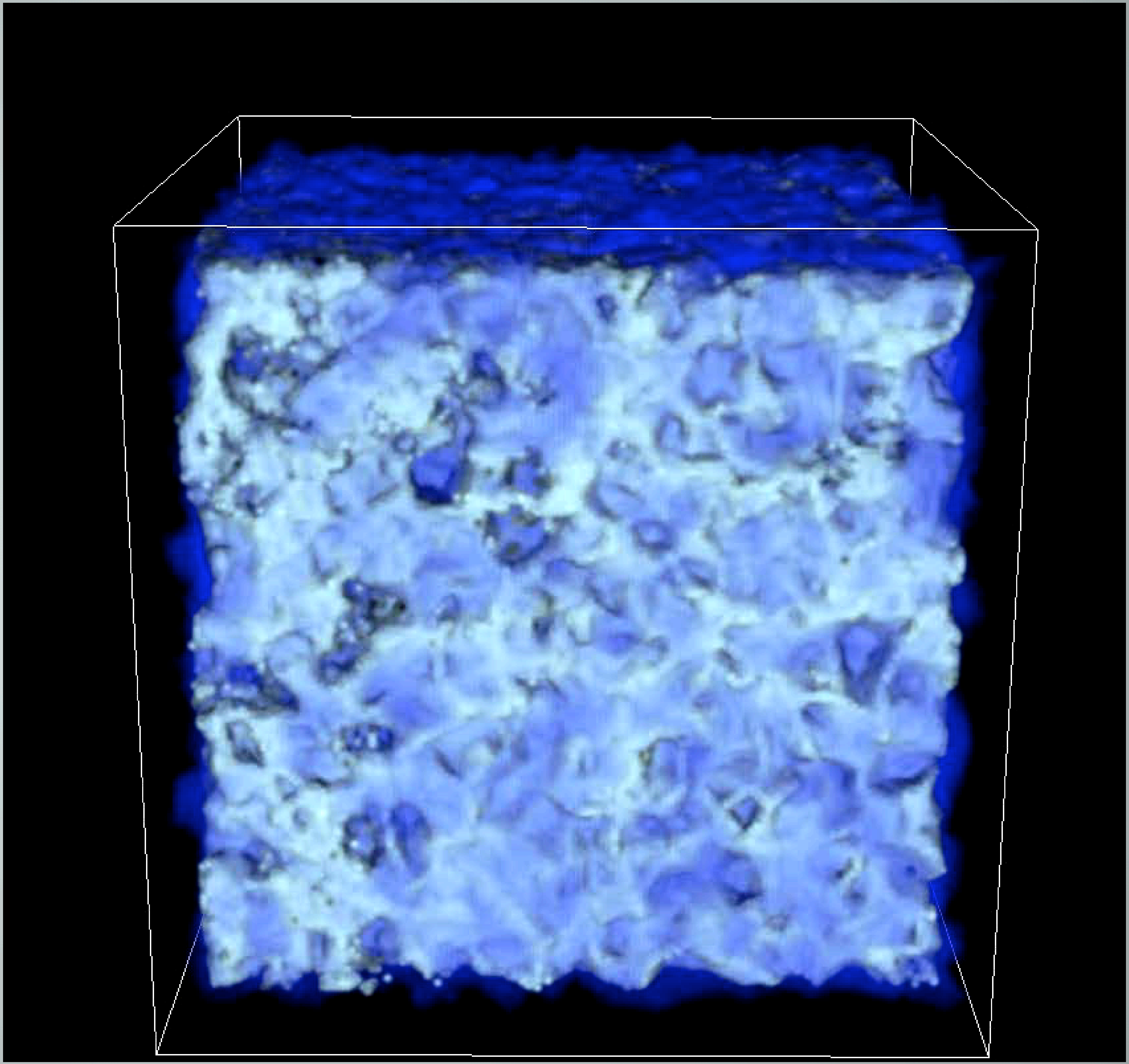


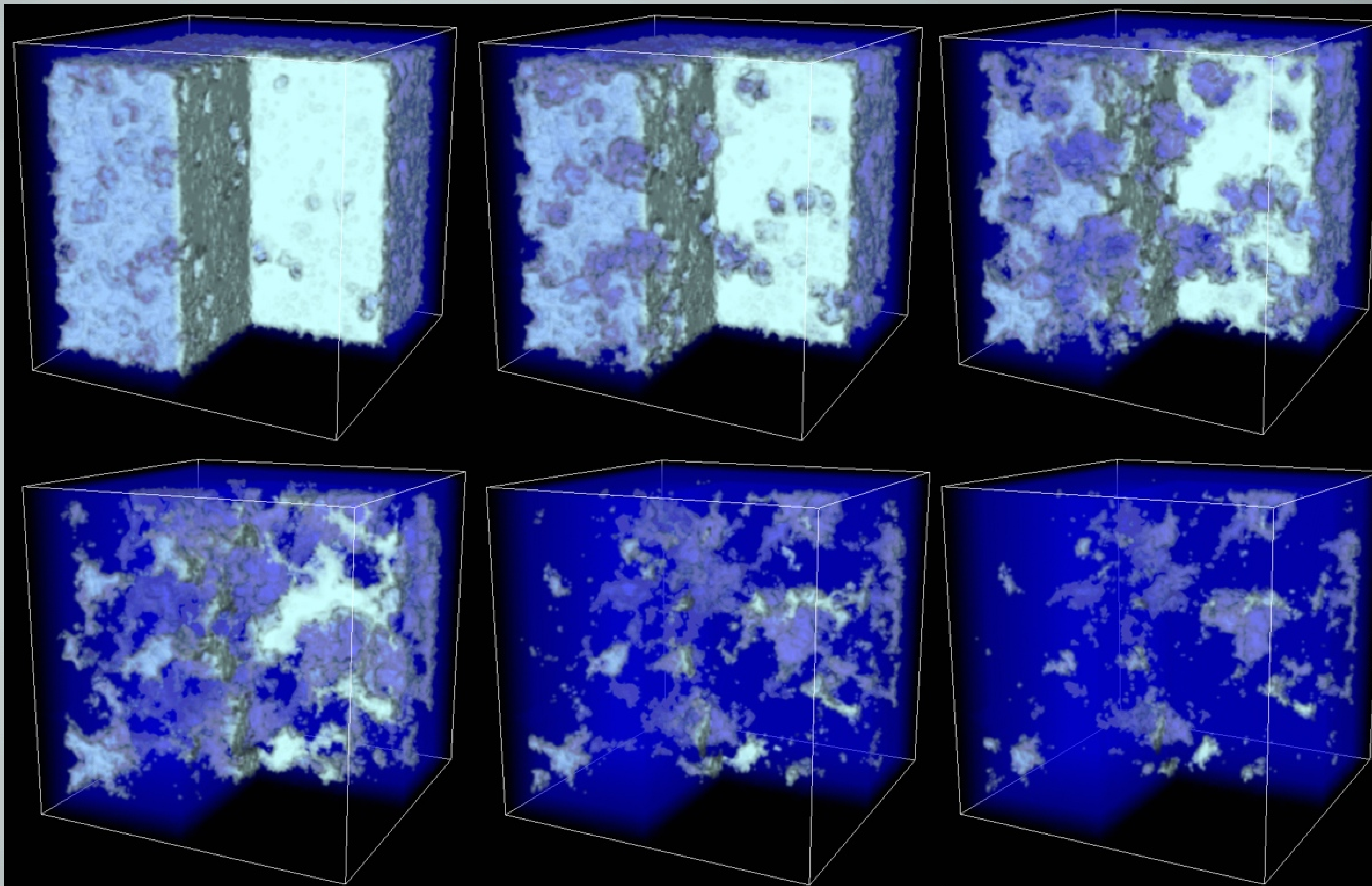
Blowing Bubbles









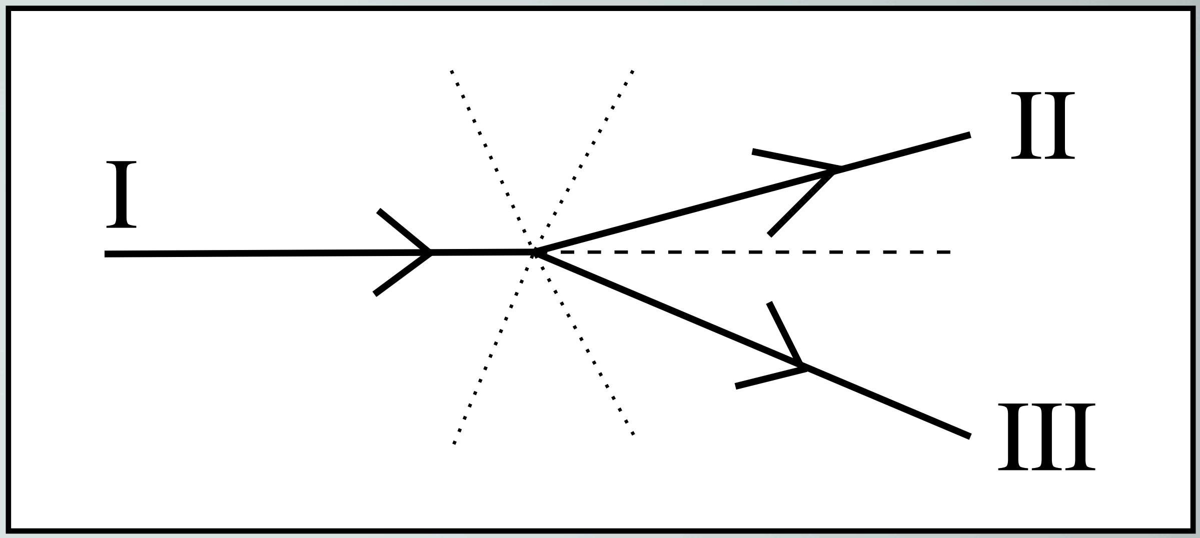


Conclusion

- **New method:**
  - Dispenses with regular grids;
  - Uses adaptive point process.
- **Resultant Delaunay graph has edge lengths that correlate linearly with mean free paths.**
- **Transport reduced to walk on adaptive random lattice.**

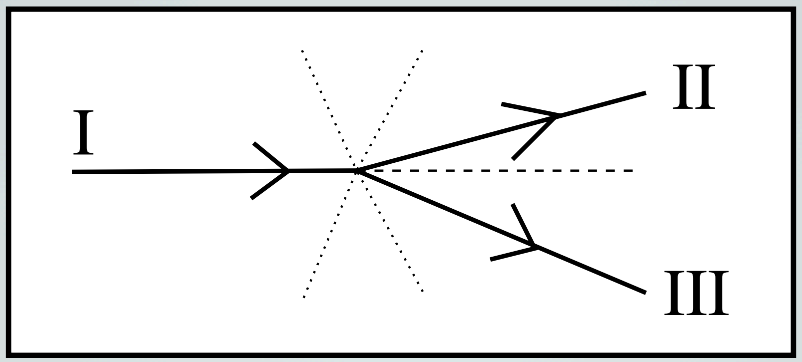
=> Fast, physical, and flexible.

Transport



- Split into d ‘most’ straightforward.
- Conserve momentum on the average.

Interaction



$$\langle L^k \rangle = c(k)\lambda^k$$

$$I_{\mathrm{out}} = I_{\mathrm{in}} \mathrm{e}^{-c}$$