Envelope Surfaces

Nico Kruithof and Gert Vegter

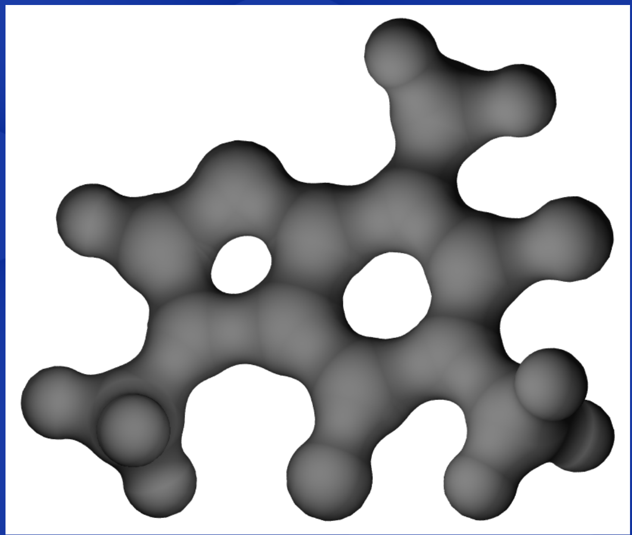
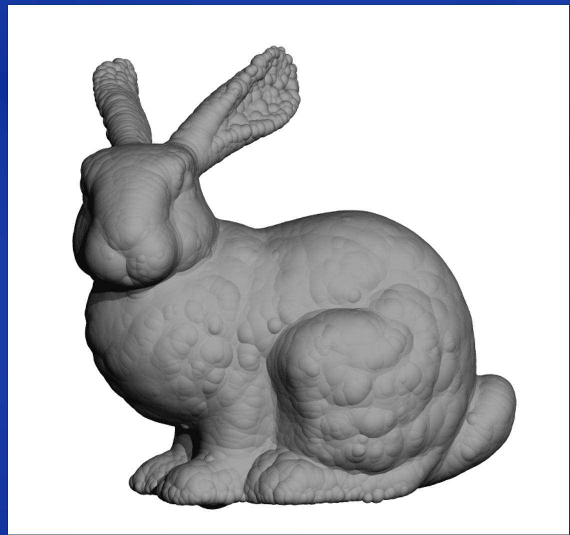
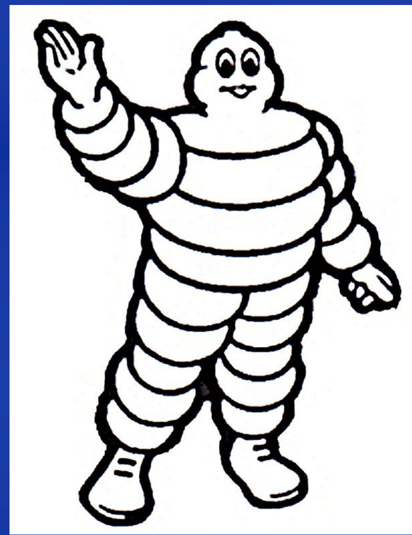
Rijks*universiteit* Groningen

March 6^{st}, 2006

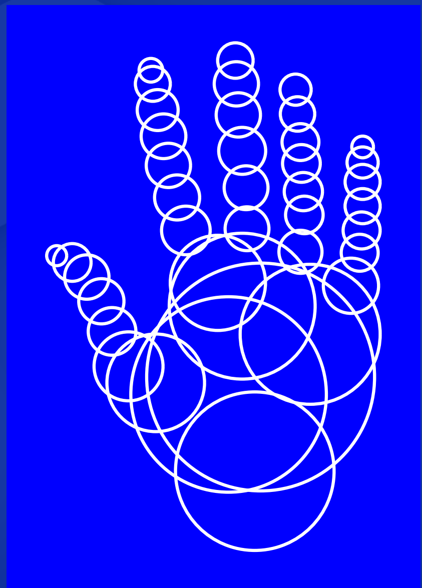
Outline

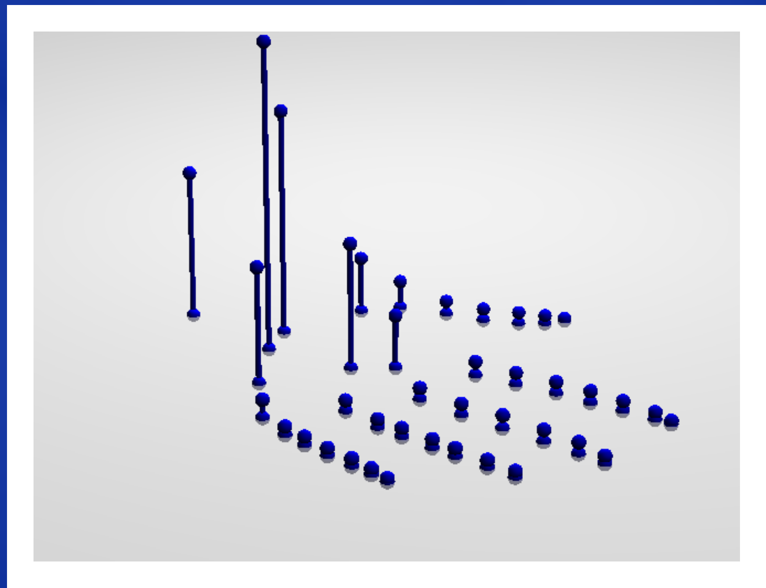
- Medial axis
- Skin surfaces
- Approximation by skin surfaces
- Envelope surfaces

Balls in surface design

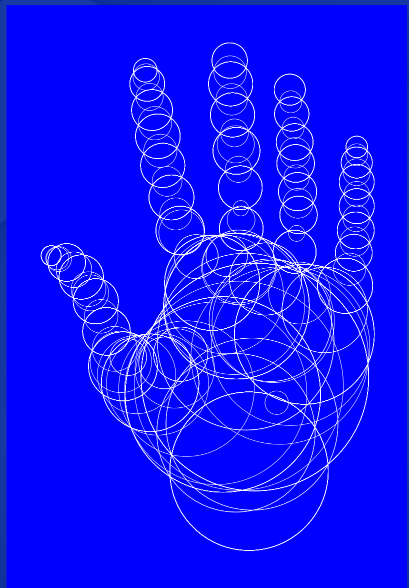
Envelope surfaces

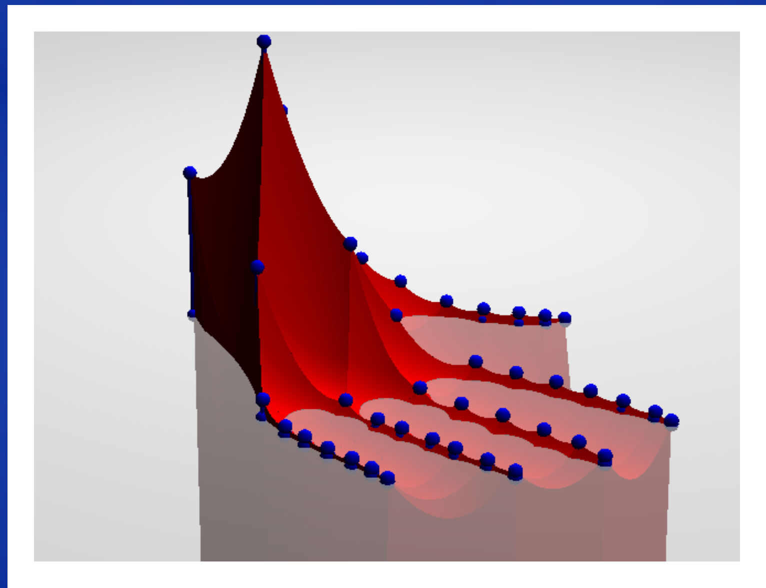




- Weight function: radius-squared
- Set of balls: discrete sample of a weight function
- Goal: construct an interpolating weight function

Envelope surfaces

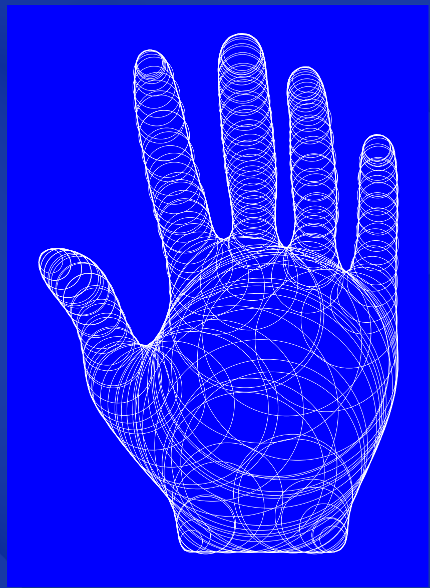




Weight function $W : D \to \mathbb{R}$ (radius-squared), with

- W continuous
- D convex and compact

Envelope surfaces

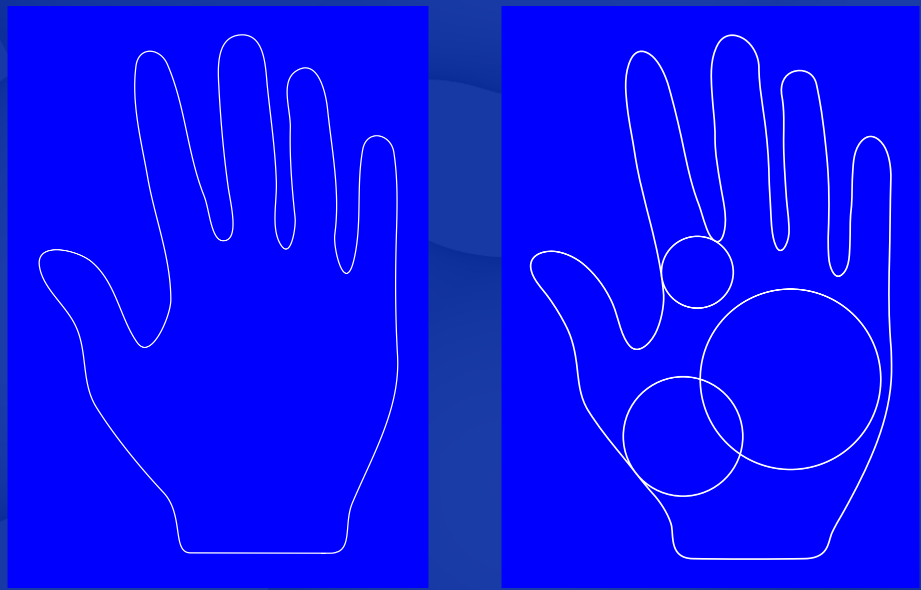


- Envelope of spheres: Boundary of the union of the spheres

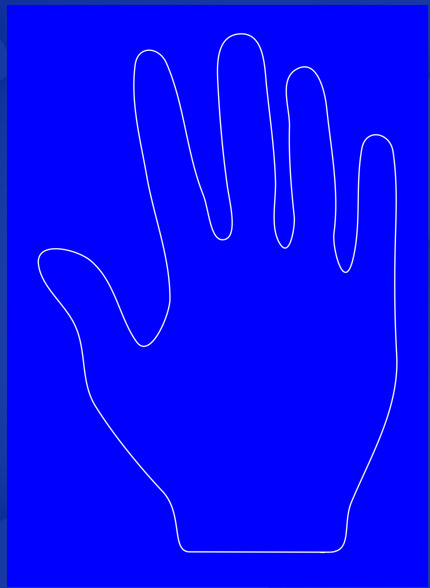
Medial axis transform

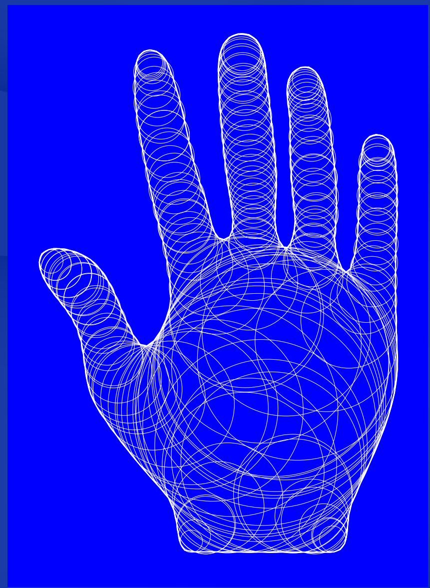
A surface representation using balls

Medial axis



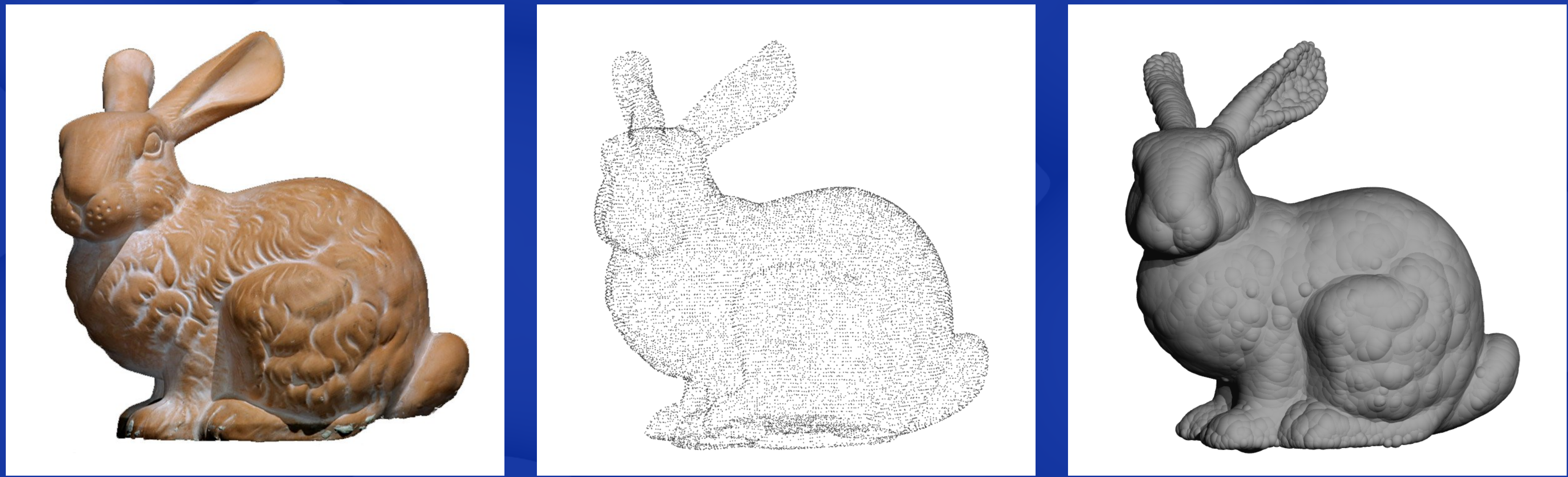
Medial axis





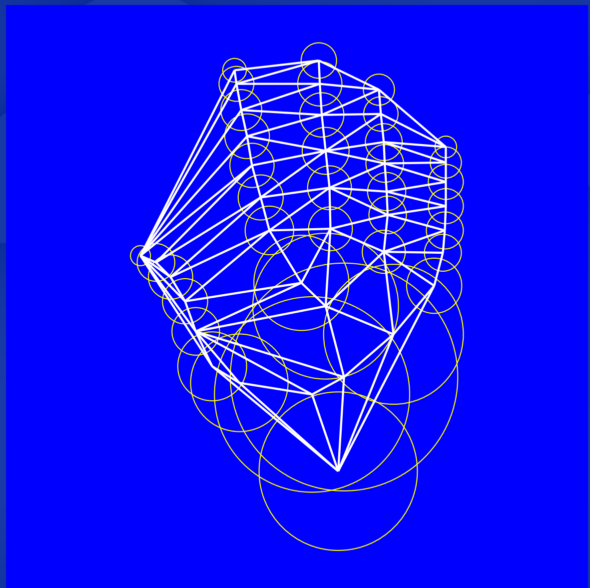
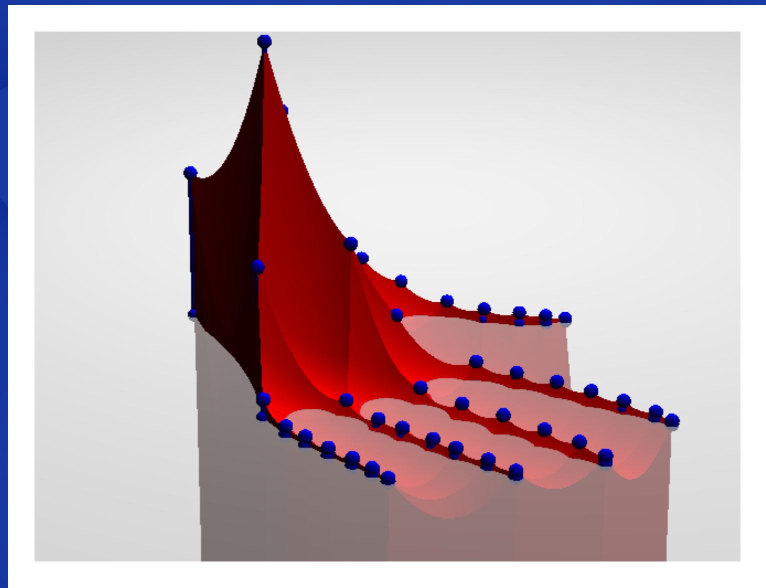
- Envelope of medial axis transform is the curve or surface

Medial axis



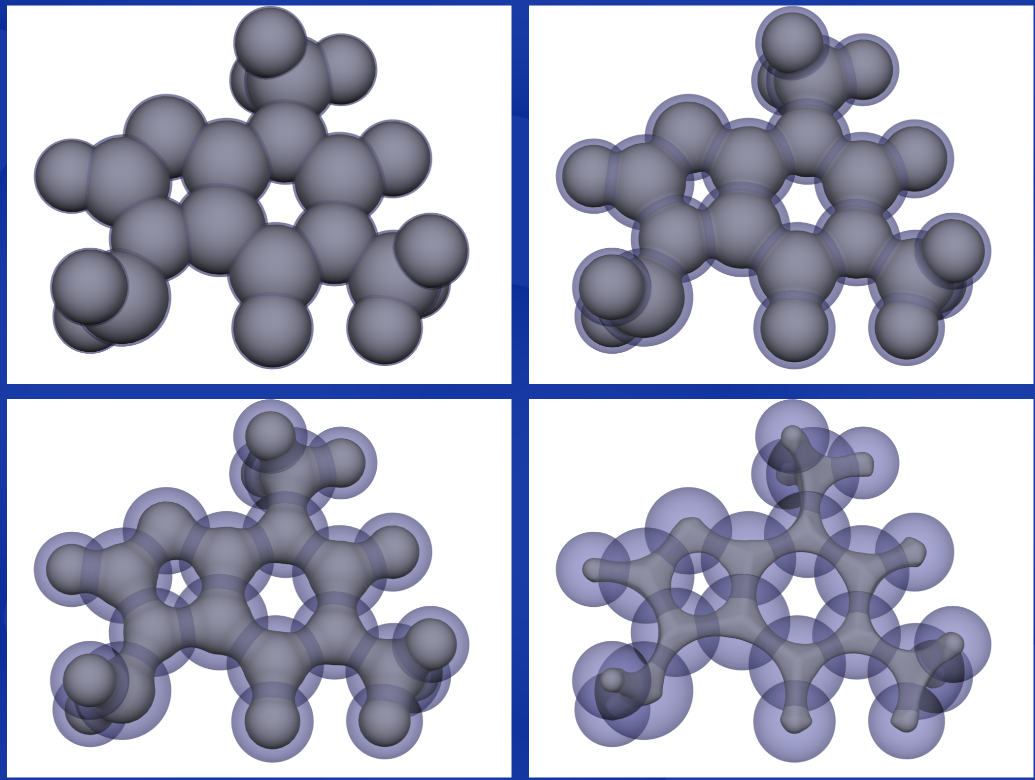
- *The power crust, unions of balls, and the medial axis transform*, N. Amenta and S. Choi and R.K. Kolluri
- *Approximate medial axis as a voronoi subcomplex*, T.K. Dey and W. Zhao

Envelope surfaces

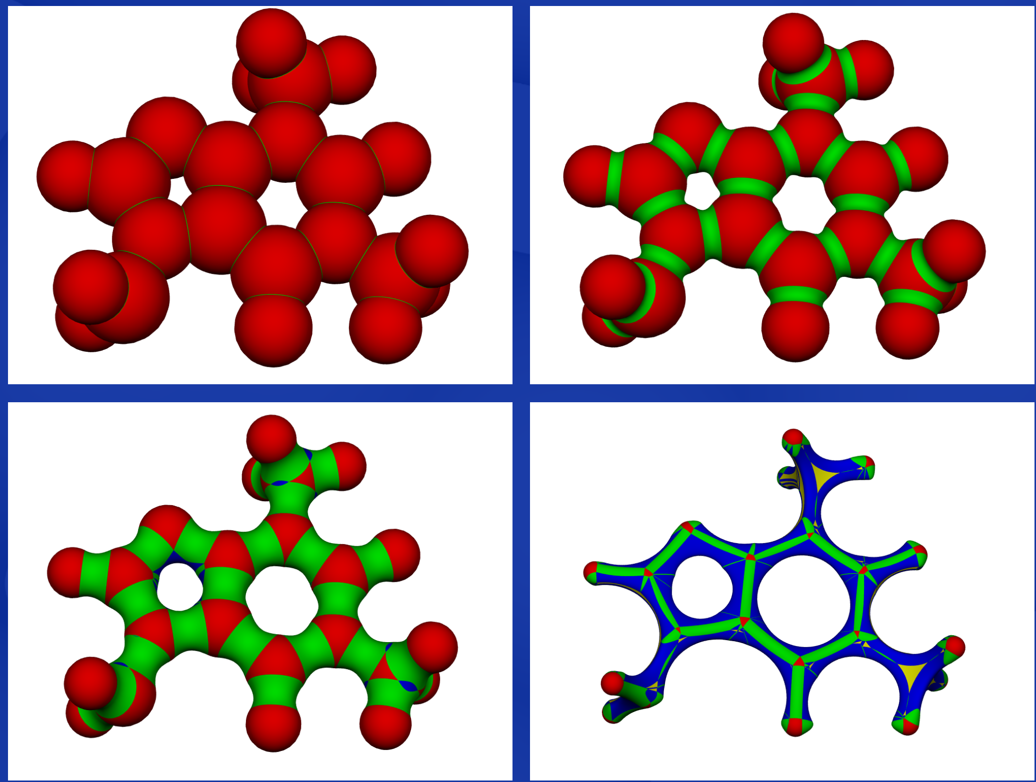
- Initial weight function: all balls are contained inside the union of the input balls
- Envelope is not tangent continuous

Skin surfaces



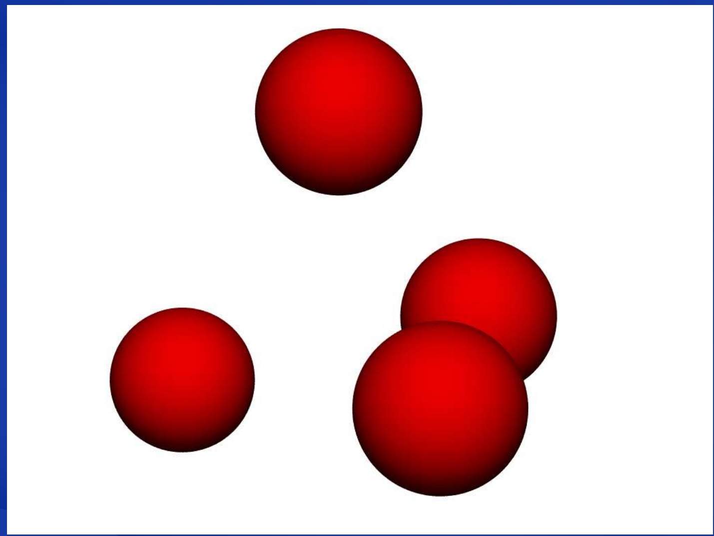
- *Deformable smooth surface design*, H. Edelsbrunner
- Multiply the initial weight function with the shrink factor $s \in (0, 1)$.

Skin surfaces



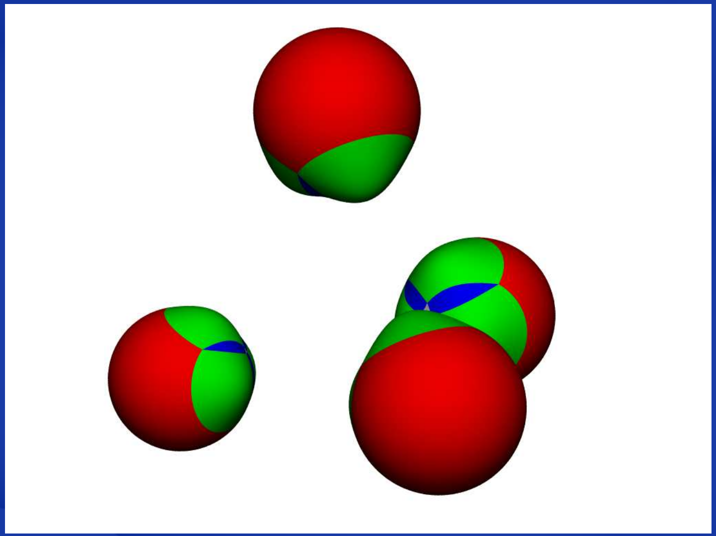
- Decomposition into pieces of quadrics.

Skin surfaces



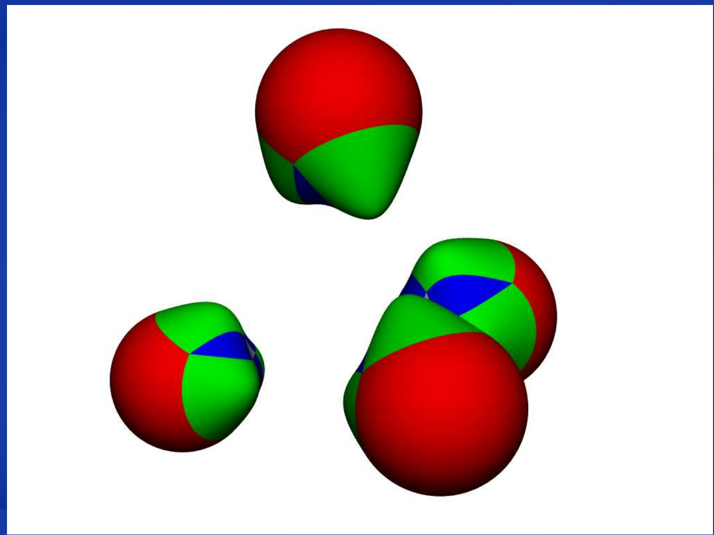
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



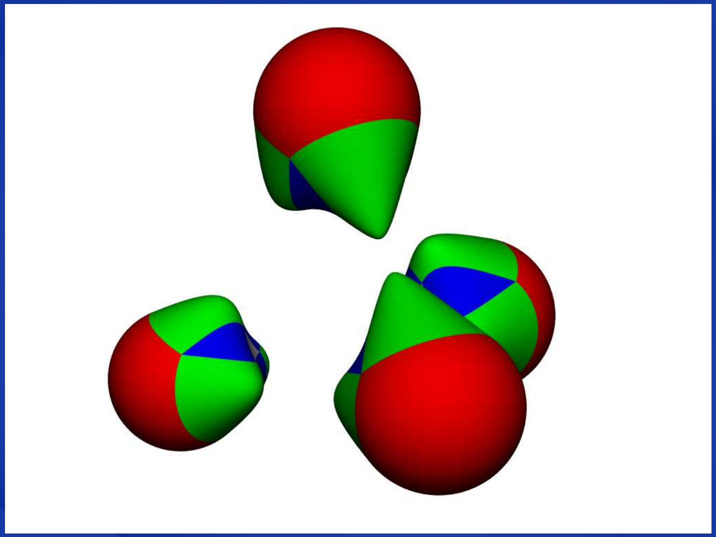
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



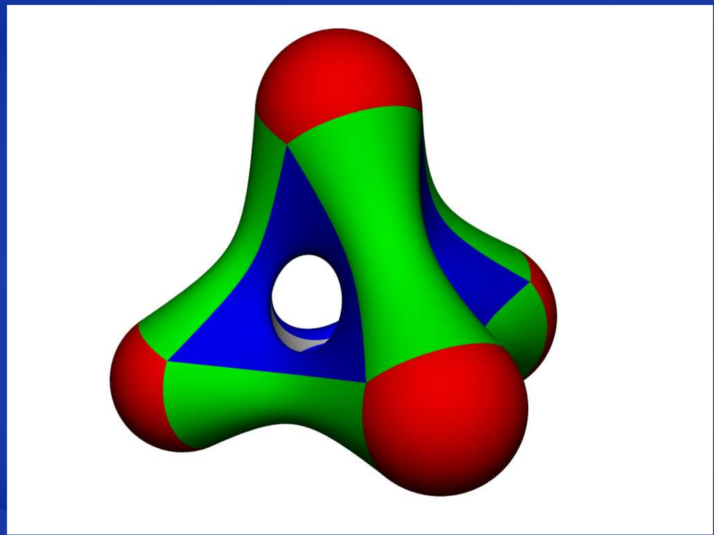
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



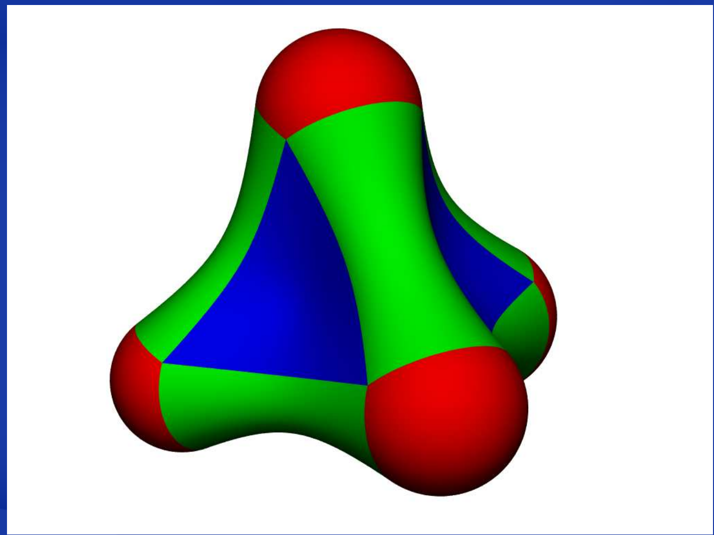
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



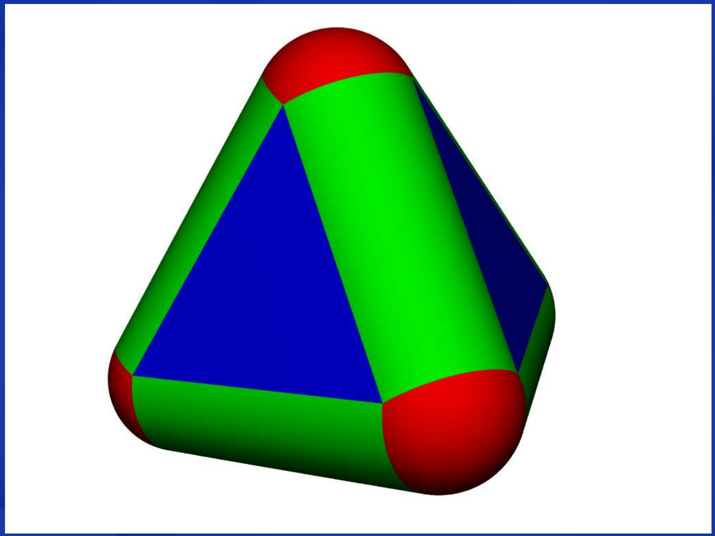
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



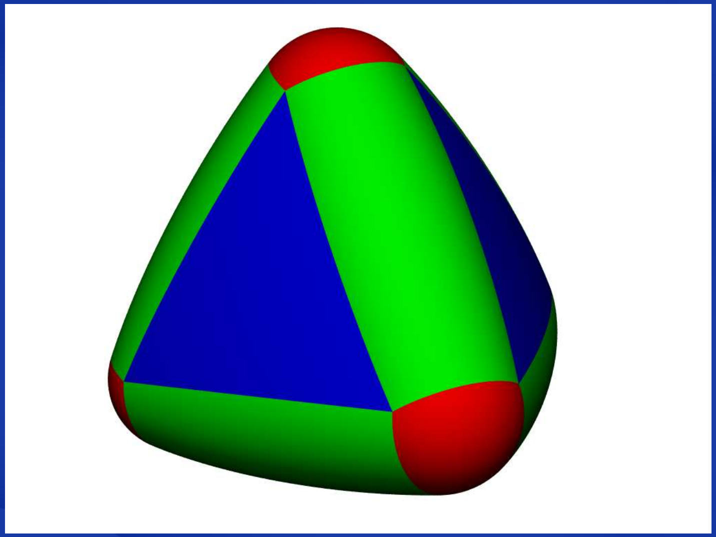
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



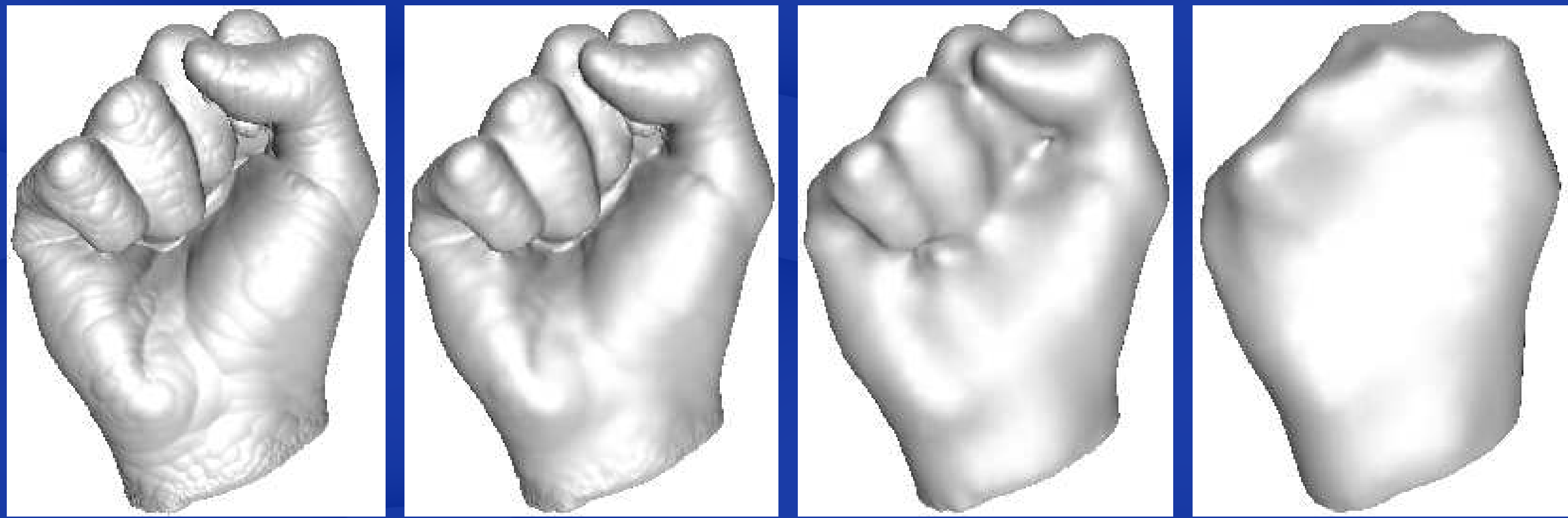
- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



- For interpolation grow the input balls (multiply the weight with $1/s$)

Skin surfaces



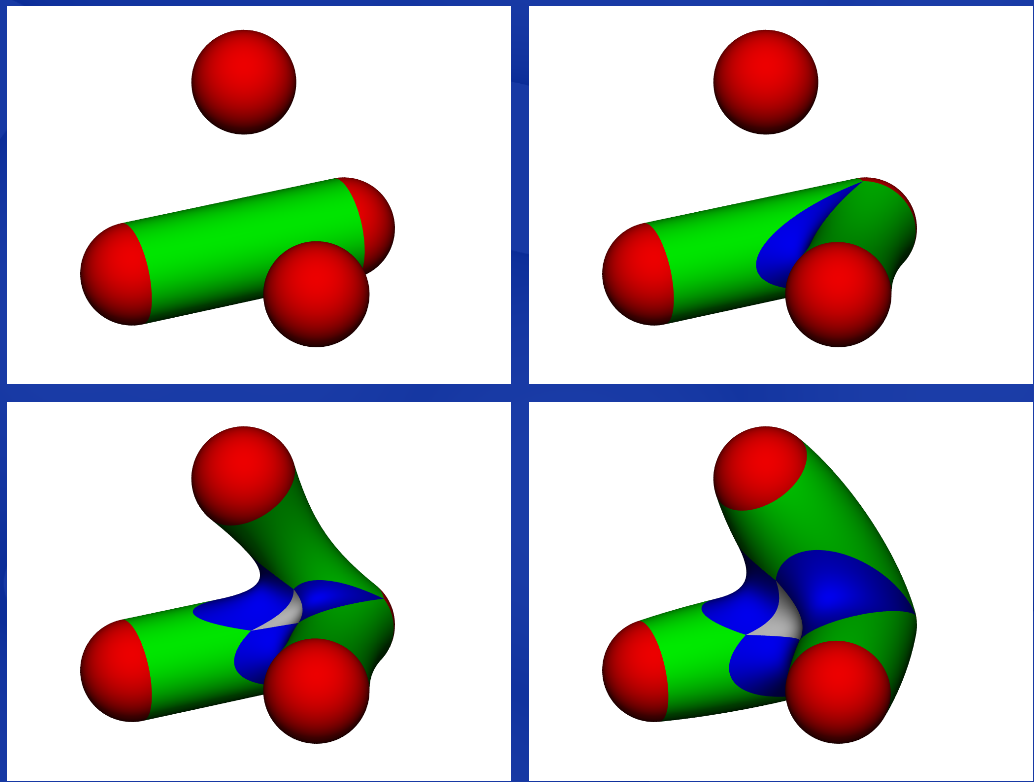
By carefully chosing the shrink factor we can guarantee that the skin surface and the approximated surface:

- have Hausdorff distance at most $\epsilon > 0$;
- have the same topology;
- have the same input balls as maximal balls.

Envelope surfaces

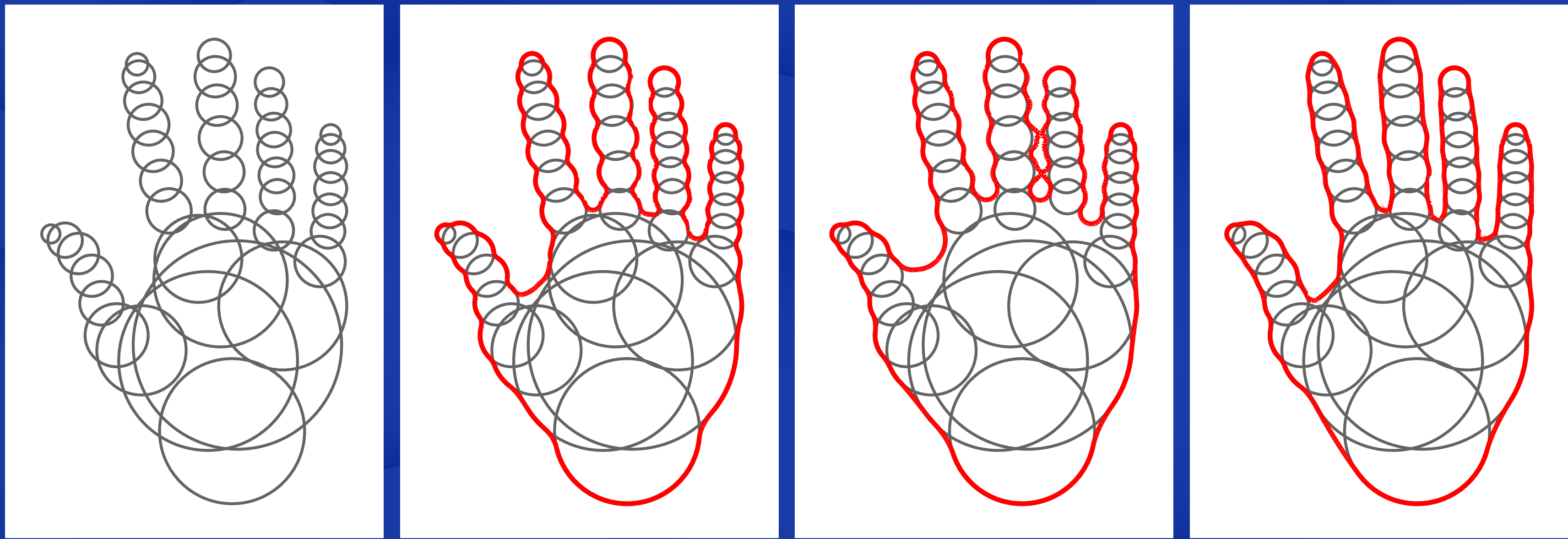
Making the interpolation adaptive

Envelope surfaces

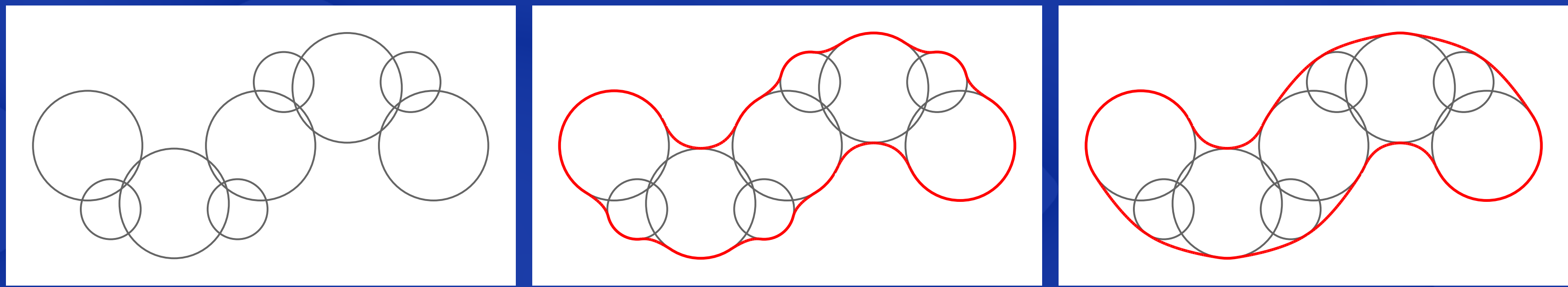


- Envelope surfaces allow for local control over the surface

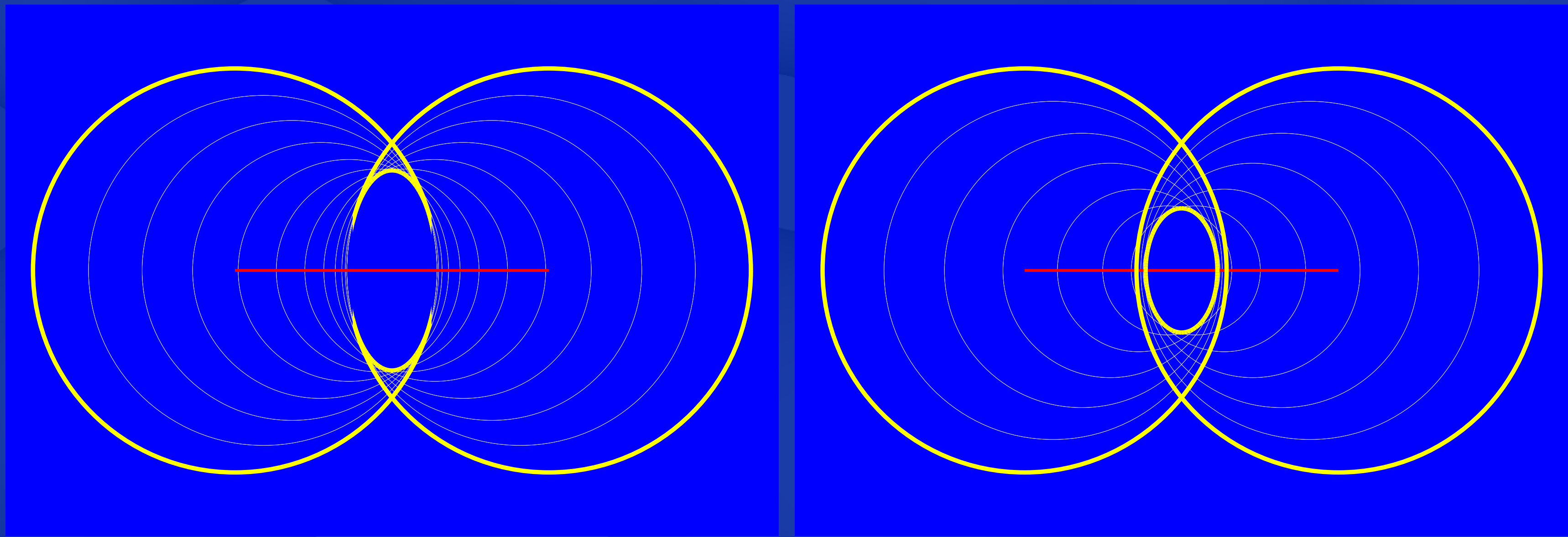
Envelope surfaces



Envelope surfaces

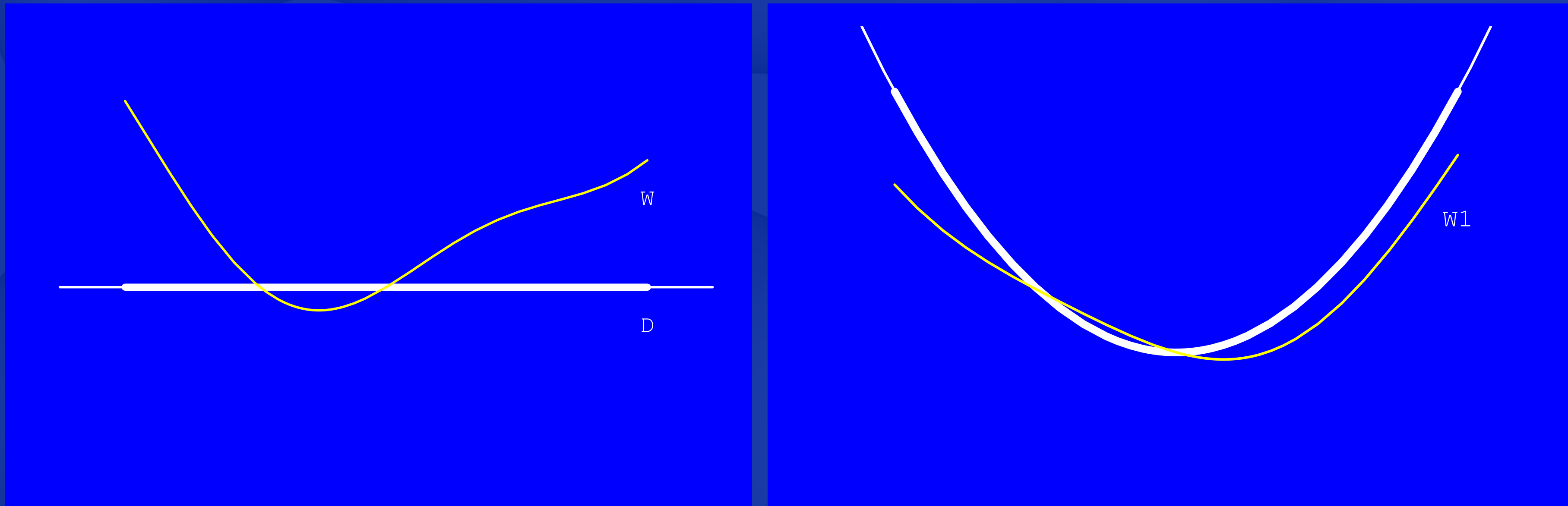


Envelope surfaces



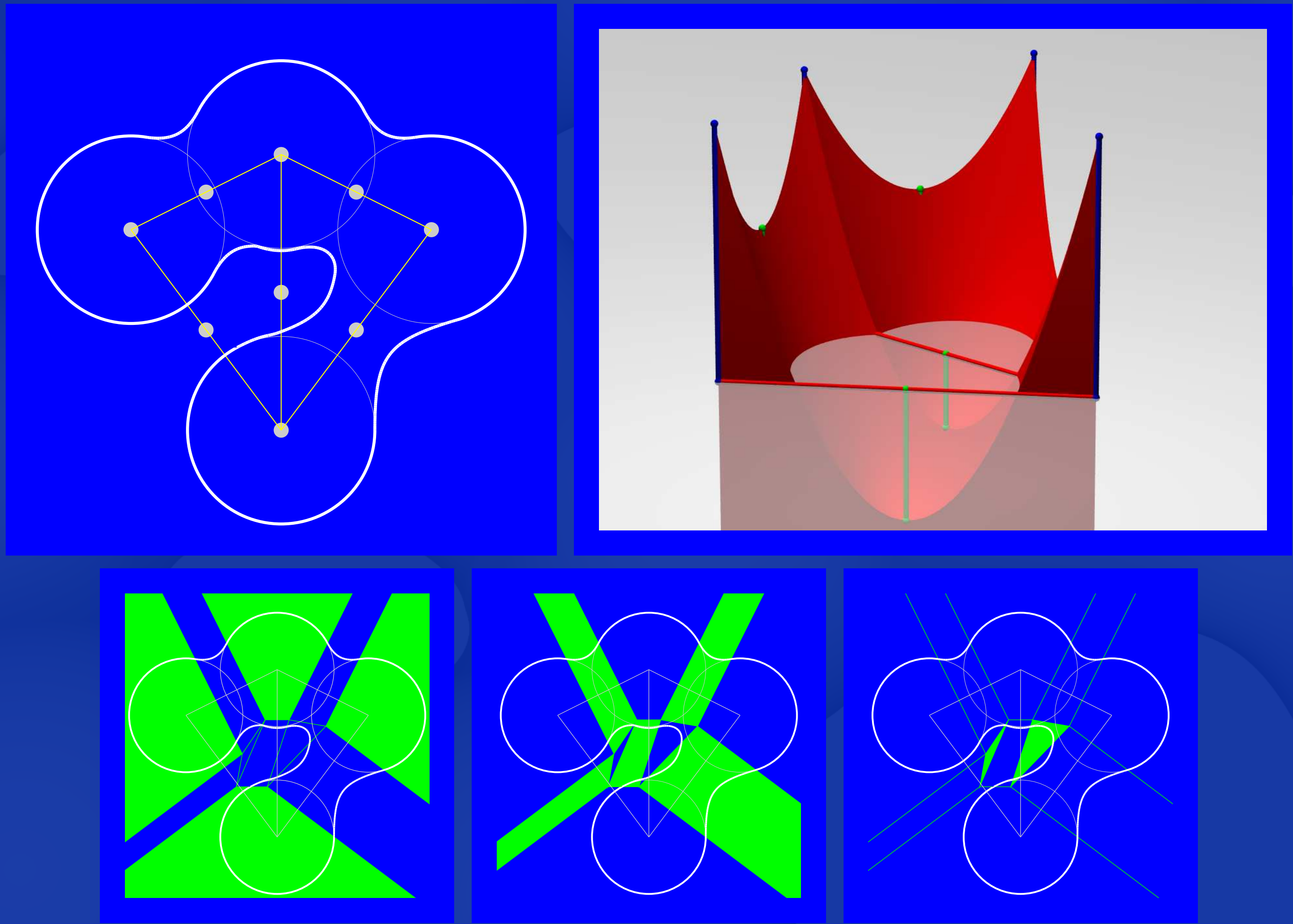
- The envelope surface is tangent discontinuous if the weight changes too much.

Envelope surfaces



- Associated weight: $W_1(p) = \|p\|^2 - W(p)$
- **Theorem:** Envelope surface is C^1 if associated weight function is continuous and strictly convex
- Proof uses the Legendre-transform from convexity theory.

Envelope surfaces



Conclusions

- Envelopes of spheres are well suited for modeling
- Envelope surfaces form a useful extension of skin surfaces
- Piecewise quadratic weight functions yield piecewise quadratic envelope surfaces

Open problems:

- How to control the topology of envelope surfaces