Cosmology,

lect. 2

Gravity:
Ruler of the Universe

Four Fundamental Forces of Nature

- **Strong Nuclear Force**

  Responsible for holding particles together inside the nucleus.
  The nuclear strong force carrier particle is called the gluon.
  The nuclear strong interaction has a range of 10^{-15} m (diameter of a proton).

- **Electromagnetic Force**

  Responsible for electric and magnetic interactions, and determines structure of atoms and molecules.
  The electromagnetic force carrier particle is the photon (quantum of light)
  The electromagnetic interaction range is infinite.

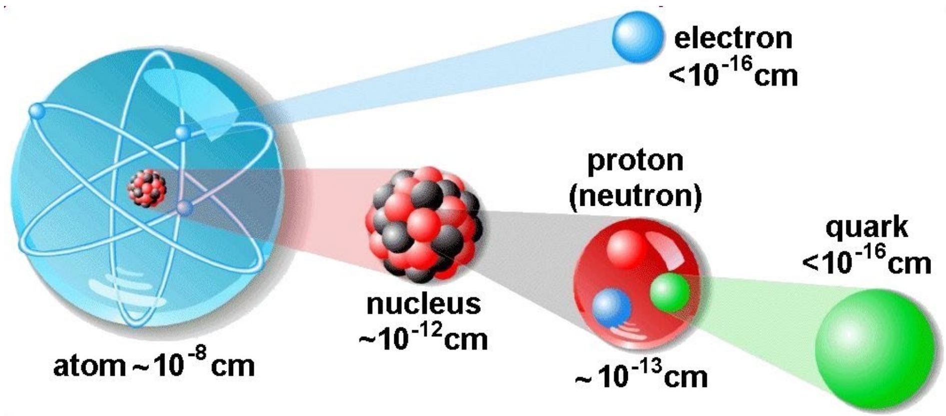
- **Weak Force**

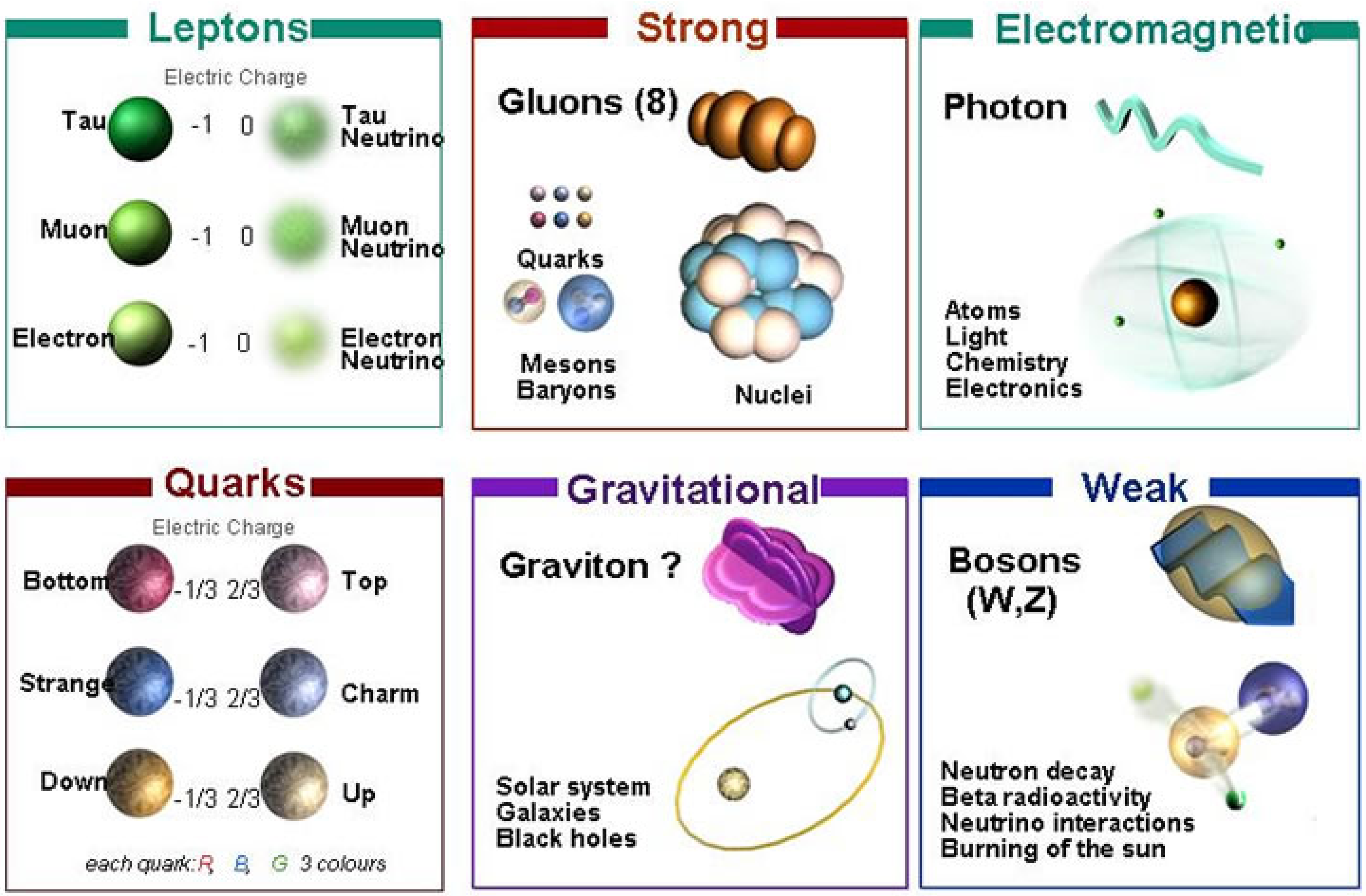
  Responsible for (beta) radioactivity.
  The weak force carrier particles are called weak gauge bosons (Z, W^+, W^-).
  The nuclear weak interaction has a range of 10^{-17} m (1% of proton diameter).

- **Gravity**

  Responsible for the attraction between masses. Although the gravitational force carrier
  The hypothetical (carrier) particle is the graviton.
  The gravitational interaction range is infinite.
  By far the weakest force of nature.

Four Fundamental Forces of Nature





Interaction	Current Theory	Mediators	Relative Strength[1]	Long-Distance Behavior	Range(m)
Strong	Quantum chromodynamics (QCD)	gluons	10^{38}	1 (see discussion below)	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	photons	10^{36}	$\frac{1}{r^2}$	infinite
Weak	Electroweak Theory	W and Z bosons	10^{25}	$\frac{e^{-m_{W,Z}r}}{r}$	10^{-18}
Gravitation	General Relativity (GR)	gravitons	1	$\frac{1}{r^2}$	infinite

The weakest force is Gravity !

$$g = G\frac{m}{r^2}$$

However, note that

- .

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The weakest force is Gravity !

However:

- **its range is infinite, not shielded**
- **it is cumulative as all mass adds,**

  while electromagetic charges can be + or - , cancelling each others effect.

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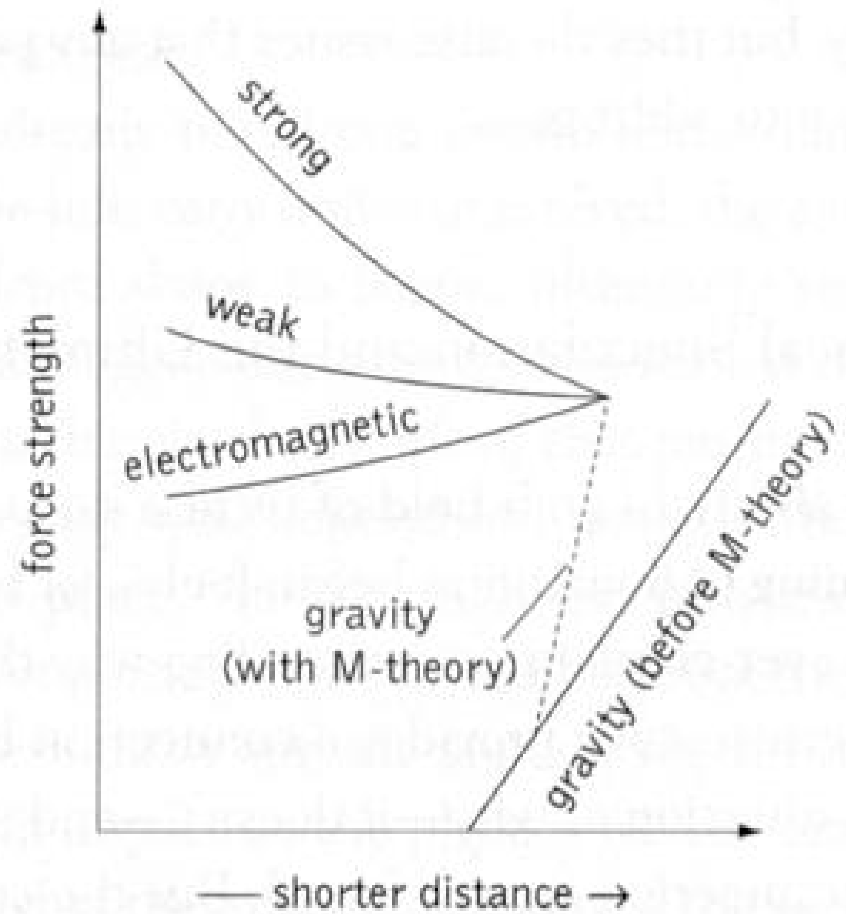
The weakest force, by far, rules the Universe …

Gravity has dominated its evolution, and determines its fate …

Grand Unified Theories (GUT)



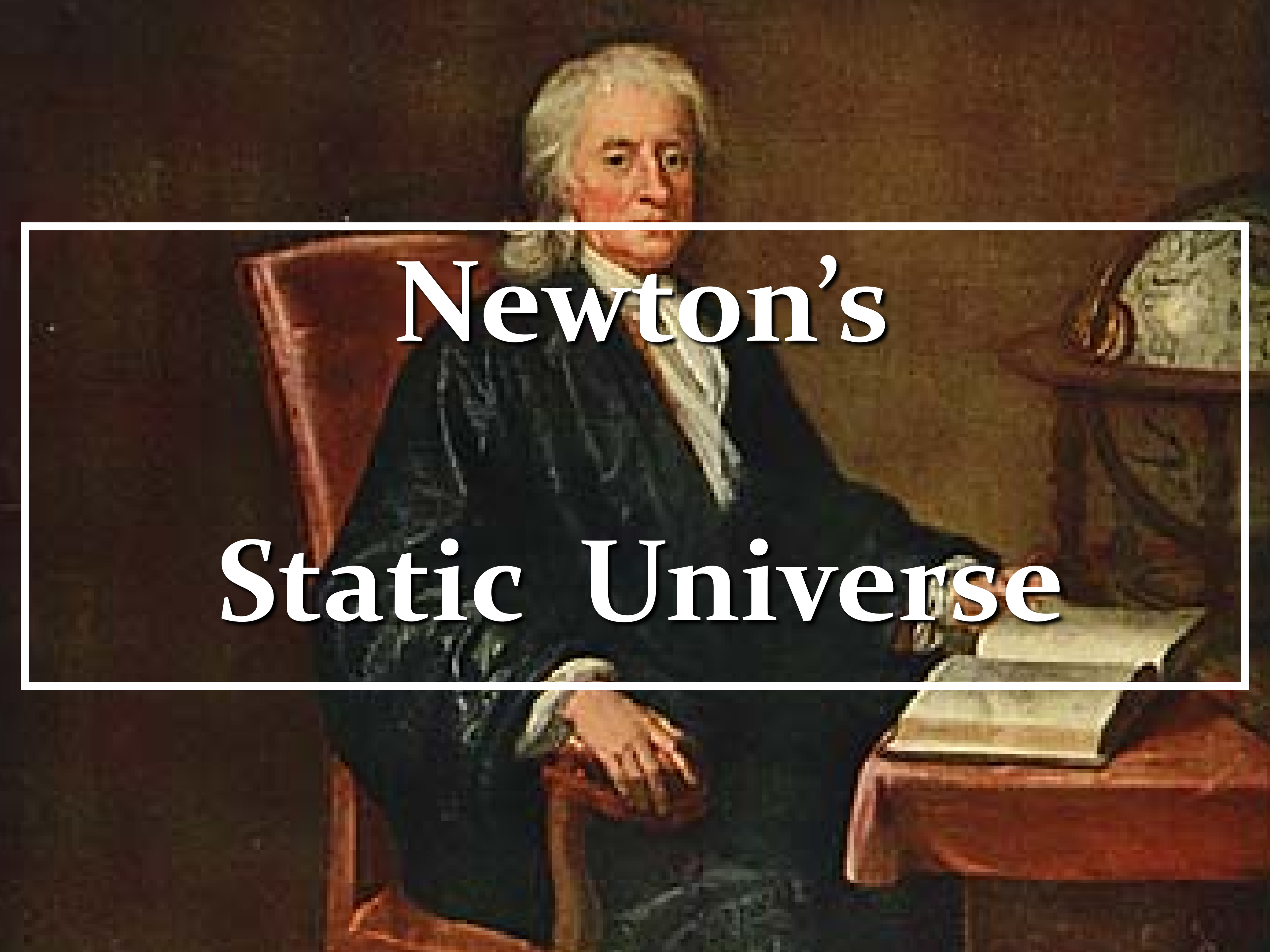
Grand Unified Theories

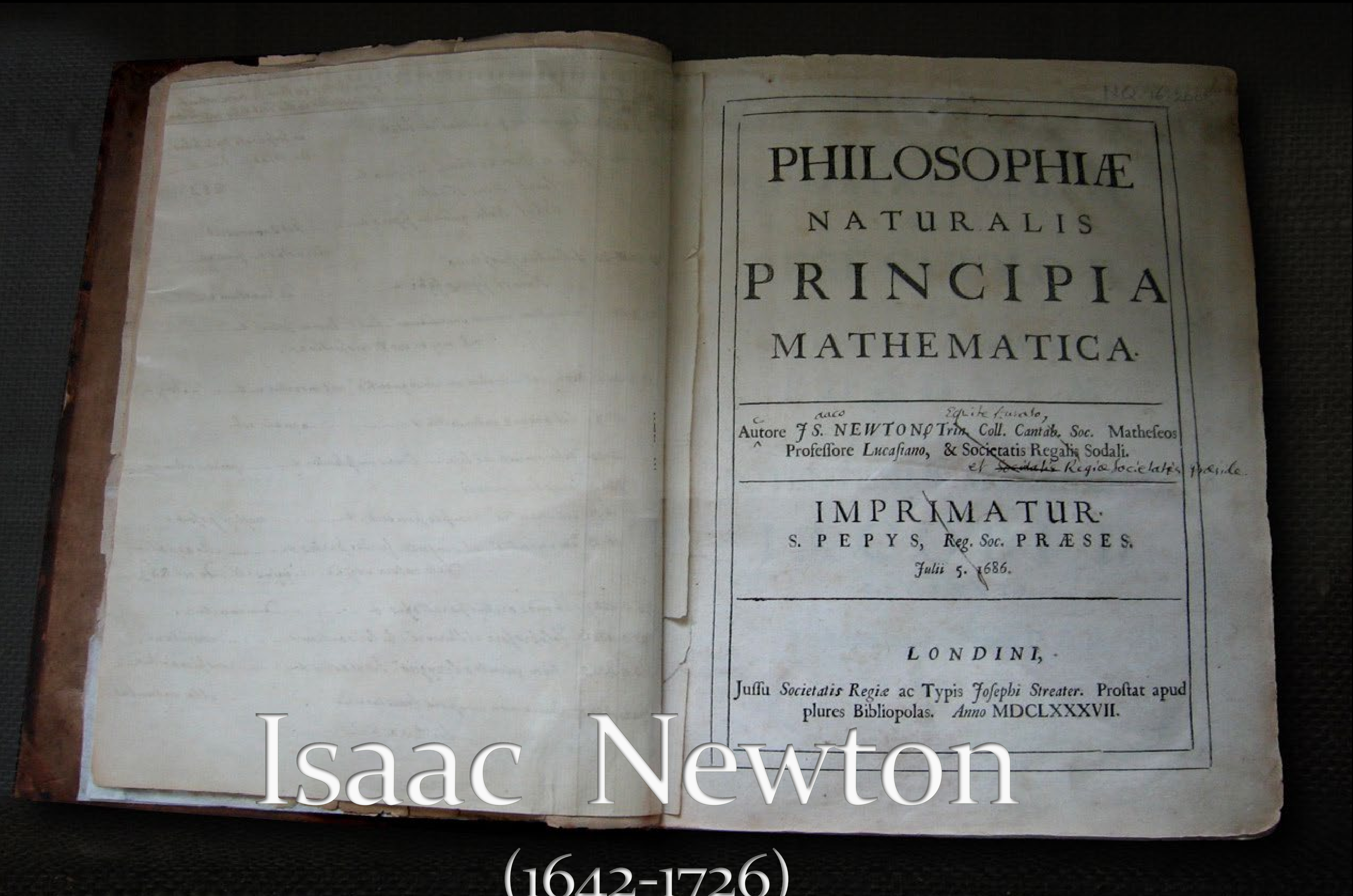
* describe how
  - Strong
  - Weak
  - Electromagnetic

  Forces are manifestations of the same underlying GUT force ...

* This implies the strength of the forces to diverge from their uniform GUT strength

* Interesting to see whether gravity at some very early instant unifies with these forces ???


Newton's
Static Universe



Isaac Newton

(1642-1726)

Newton's Laws of Motion

Newton’s 1st Law:

zero force - body keeps constant velocity

$$\vec{F} = 0 \quad \Rightarrow \quad \vec{v} = cst.$$

Newton’s 2nd Law:

force = acceleration x mass = change of velocity x mass

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

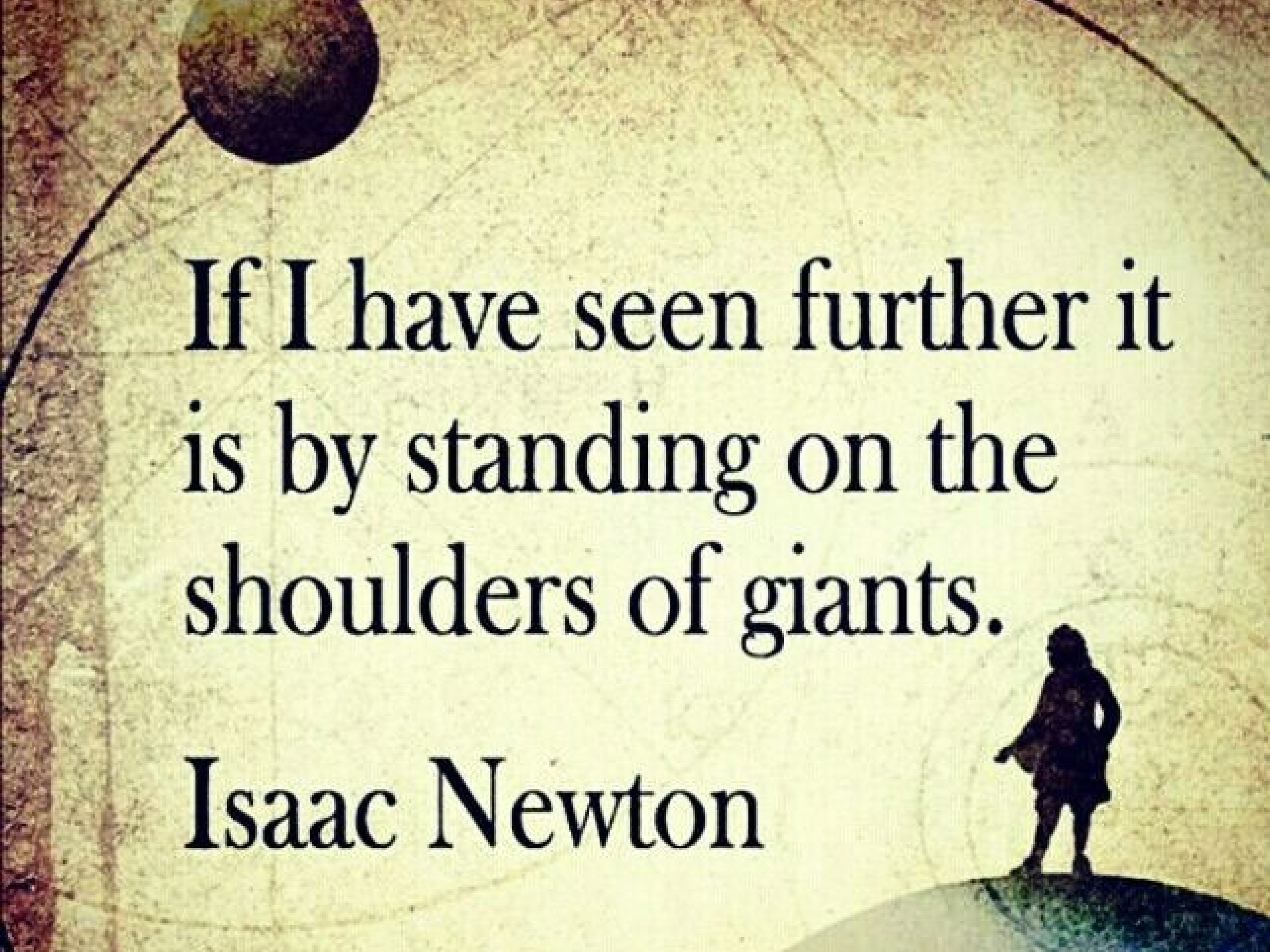
Newton’s 3rd Law:

action = reaction

$$\vec{F}_a = -\vec{F}_b$$

Newton's Gravity

$$\vec{F}_g = -G\frac{mM}{r^2}\vec{e}_r$$


If I have seen further it is by standing on the shoulders of giants.
Isaac Newton

The Unchanging Universe

- In two thousand years of astronomy, no one ever guessed that the universe might be expanding.
- To ancient Greek astronomers and philosophers, the universe was seen as the embodiment of perfection, the heavens were truly heavenly: – unchanging, permanent, and geometrically perfect.
- In the early 1600s, Isaac Newton developed his law of gravity, showing that motion in the heavens obeyed the same laws as motion on Earth.

Newton's Universe

- However, Newton ran into trouble when he tried to apply his theory of gravity to the entire universe.
- Since gravity is always attractive, his law predicted that all the matter in the universe should eventually clump into one big ball.
- Newton knew this was not the case, and assumed that the universe had to be static
- So he conjectured that:

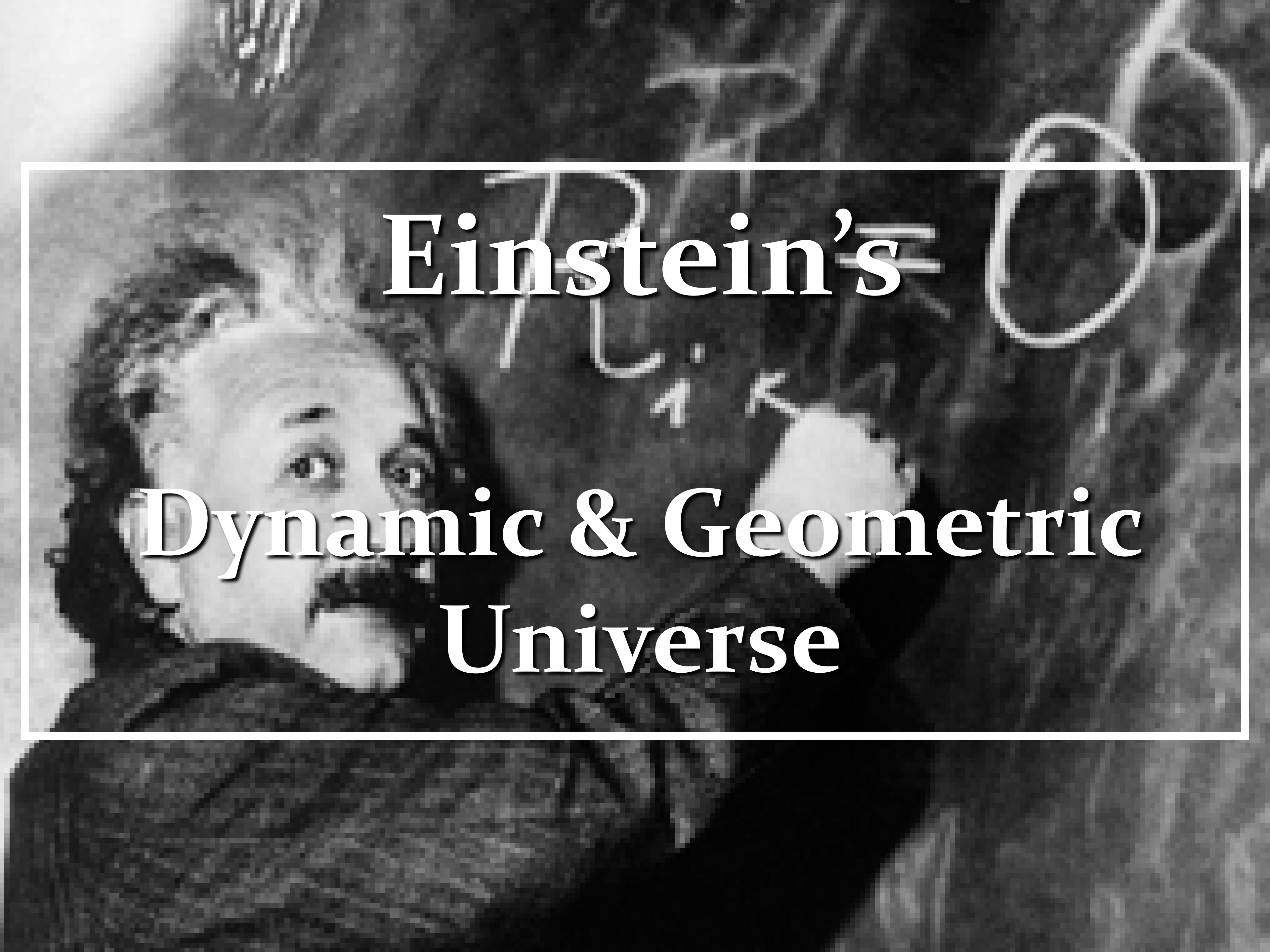
the Creator placed the stars such that they were

``at immense distances from one another."

- absolute and uniform time
- space & time independent of matter
- dynamics: - action at distance
  - instantaneous
- Universe edgeless, centerless & infinite
- Cosmological Principle:

  Universe looks the same at every

  place in space, every moment in time
- absolute, static & infinite space



Einstein’s

Dynamic & Geometric Universe

Albert Einstein

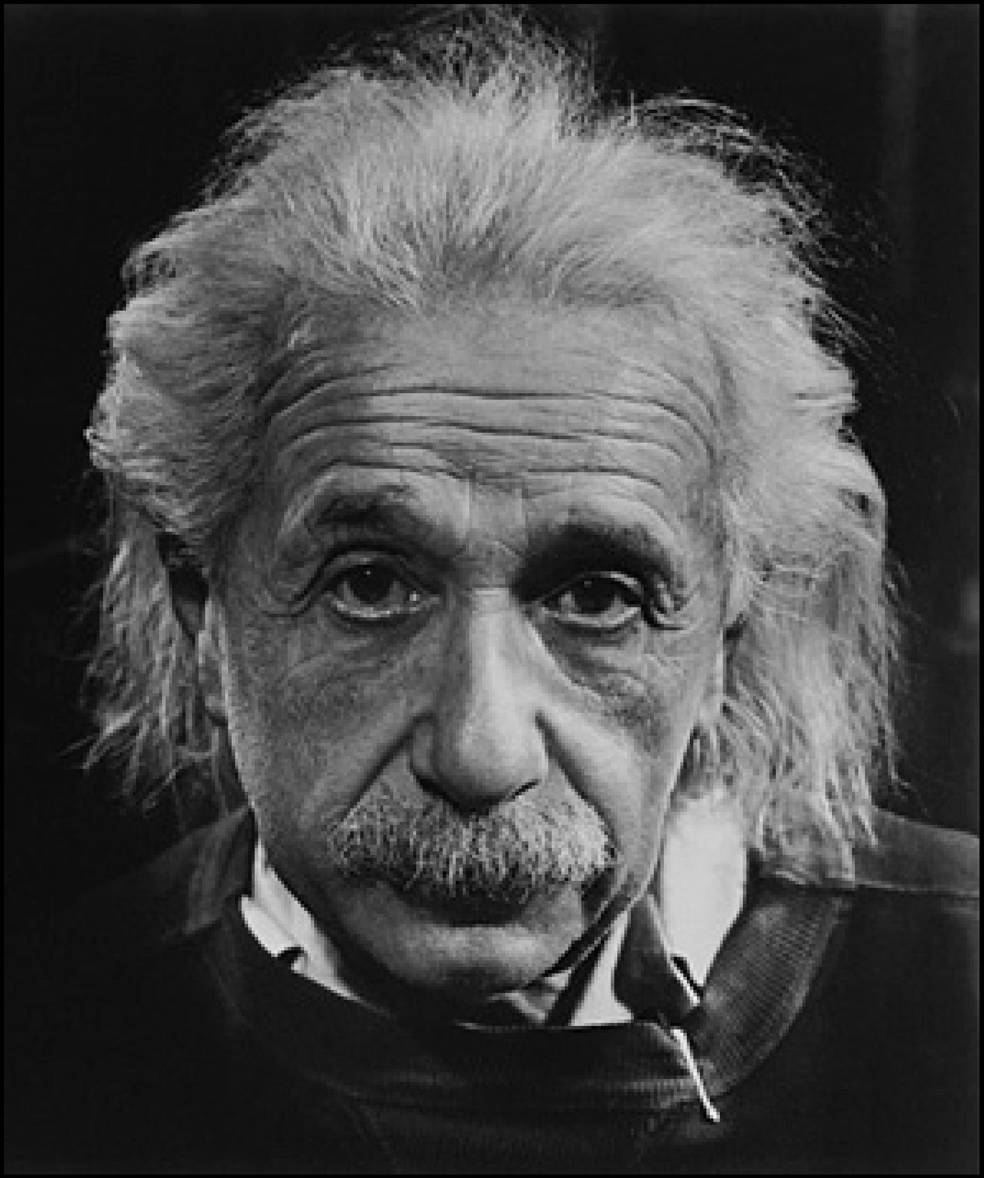
Albert Einstein
(1879-1955; Ulm-Princeton)

father of
General Relativity (1915),

opening the way towards
Physical Cosmology

The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction.

(Albert Einstein, 1954)



Einstein's Universe

In 1915,

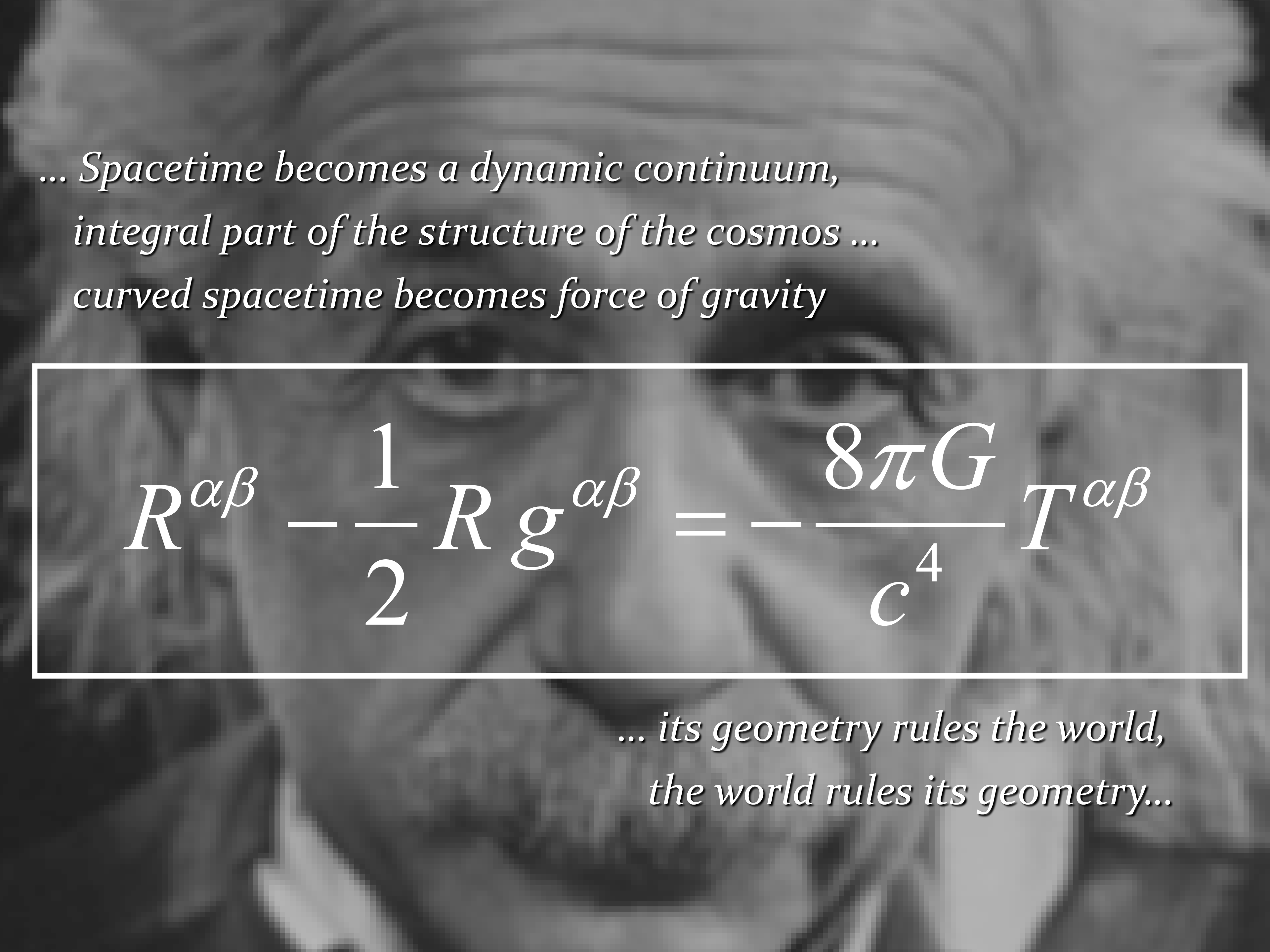
Albert Einstein completed his General Theory of Relativity.

- General Relativity is a "metric theory":
  gravity is a manifestation of the geometry, curvature, of space-time.

- Revolutionized our thinking about the nature of space & time:
  - no longer Newton's static and rigid background,
  - a dynamic medium, intimately coupled to the universe's content of matter and energy.

- All phrased into perhaps
  the most beautiful and impressive scientific equation
  known to humankind, a triumph of human genius,

Einstein Field Equations

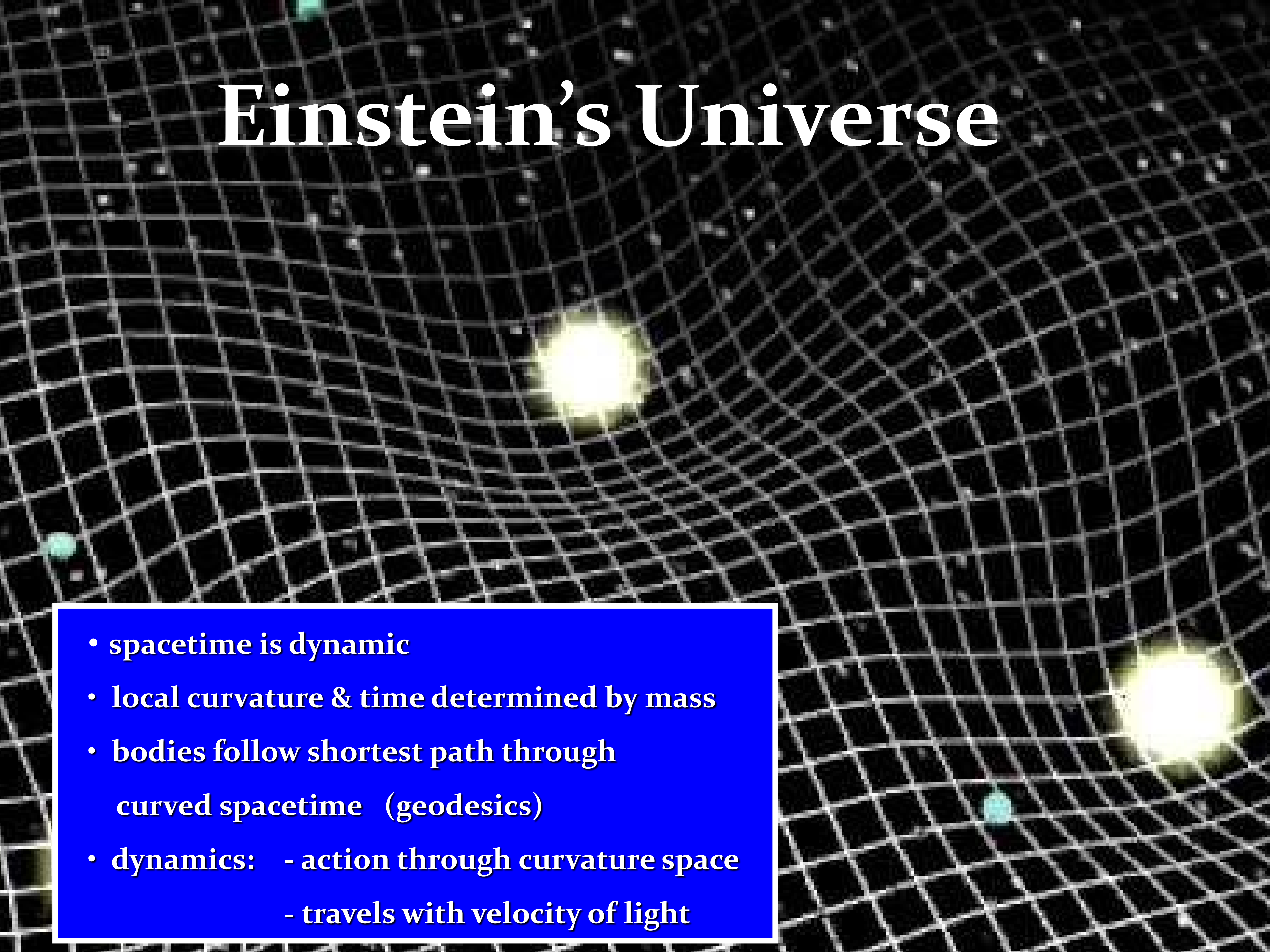


... Spacetime becomes a dynamic continuum,
integral part of the structure of the cosmos ...
curved spacetime becomes force of gravity

$$R^{\alpha\beta} - \frac{1}{2} R\, g^{\alpha\beta} = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

... its geometry rules the world,
the world rules its geometry...

Einstein's Universe

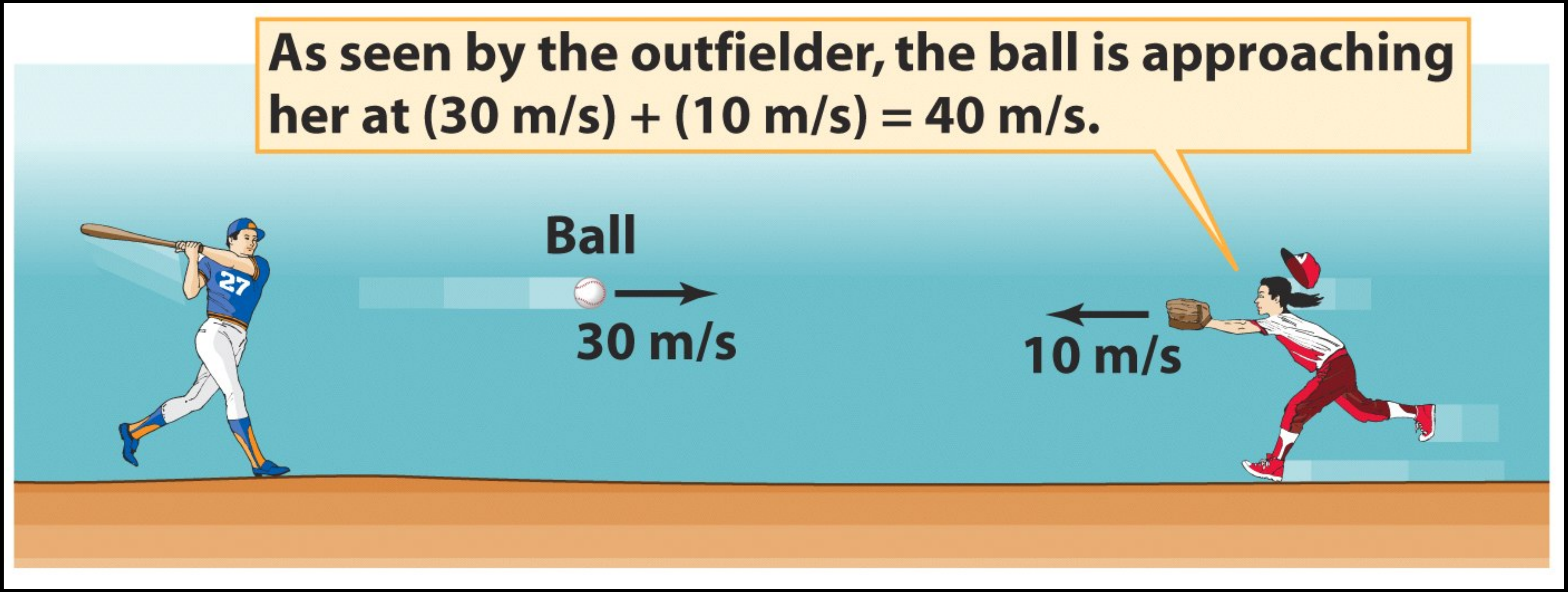


- spacetime is dynamic
- local curvature & time determined by mass
- bodies follow shortest path through curved spacetime (geodesics)
- dynamics: - action through curvature space
  - travels with velocity of light

Relativity:

Space & Time

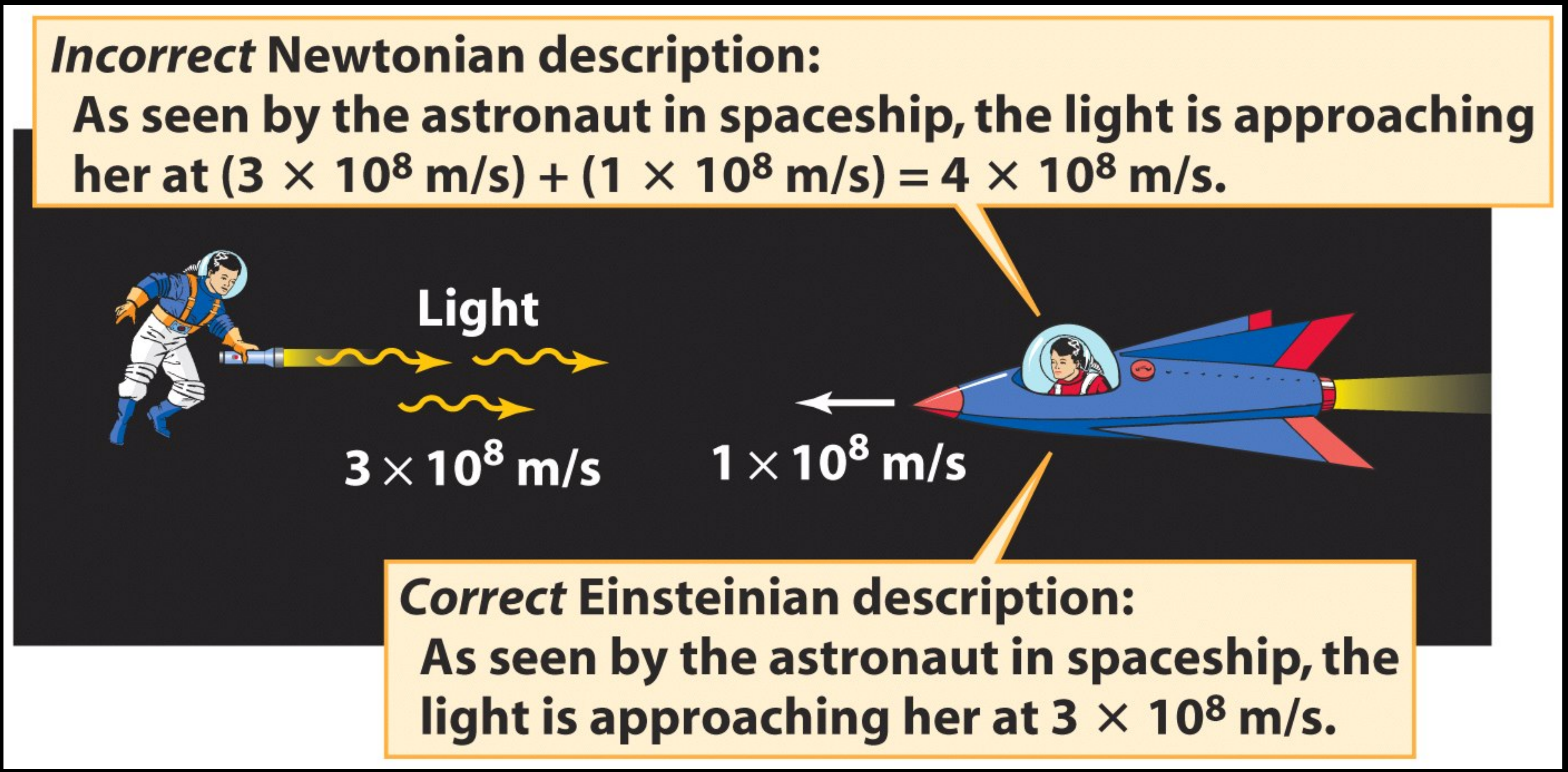
Galilean Relativity



Principle of Relativity

The same laws of electrodynamics and optics are valid for
all frames of reference for which the equations of mechanics hold good.
We will raise the conjecture (the purport of which will hereafter be called
the ‘Principle of Relativity’) to the status of a postulate
and also introduce another postulate which is only apparently irreconcilable
with the former, namely that light is always propagated in empty space
with a definite velocity c which is independent of
the state of motion of the emitting body.

Constant Speed of Light



Fundamental Relativity Tenets

- All **Laws of Nature** are equivalent in **reference frames** in uniform relative motion

- the **(Vacuum) speed of light** is **c** in all such frames

Relativistic Spacetime

- speed of light constant = c = 3 x 10^8 km/s
  in all reference systems:

- only possible if time and space not absolute, but dependent
  on reference system

- Manifestations:

  - Time dilation
  - Length contraction
  - Relativity of Simultaneity

Time Dilation

Time interval in frame train passenger (observer 1)

$$\Delta t_1 = \frac{round\ trip\ distance}{speed\ of\ light} = \frac{2d}{c}$$

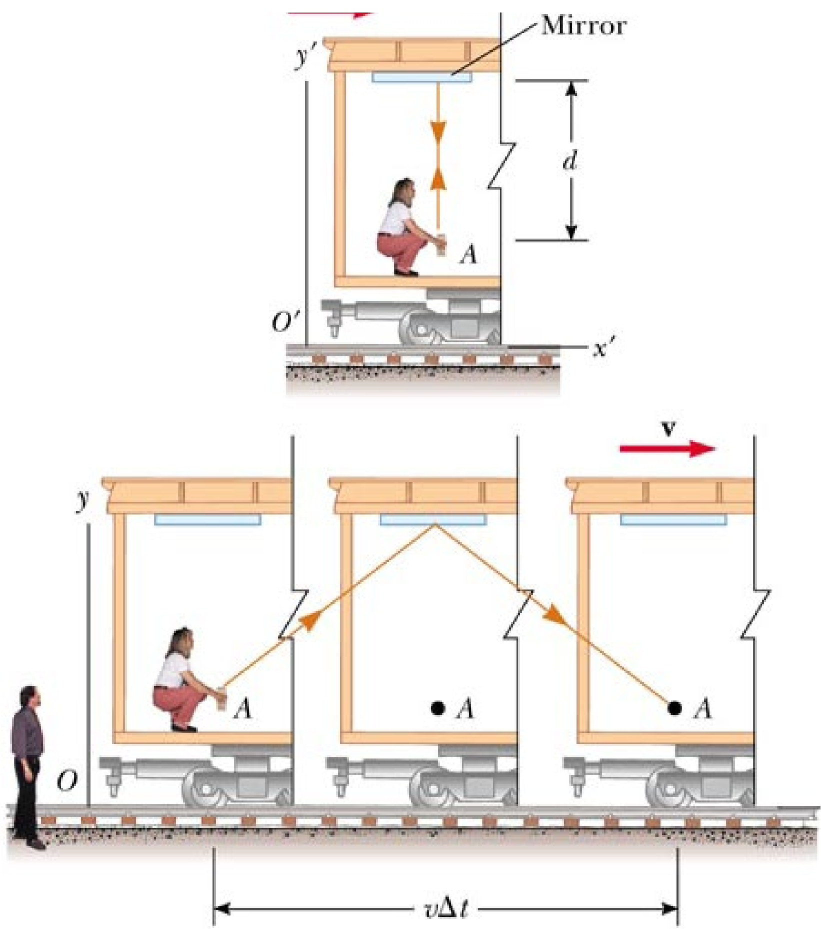
Observer 2 measures a ***longer*** time

$$\Delta t_2 = \frac{2\sqrt{d^2 + (v\Delta t_2 / 2)^2}}{c}$$

$$\Delta t_2 = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{2d}{c}\right) = \gamma \Delta t_p$$

Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} > 1$$

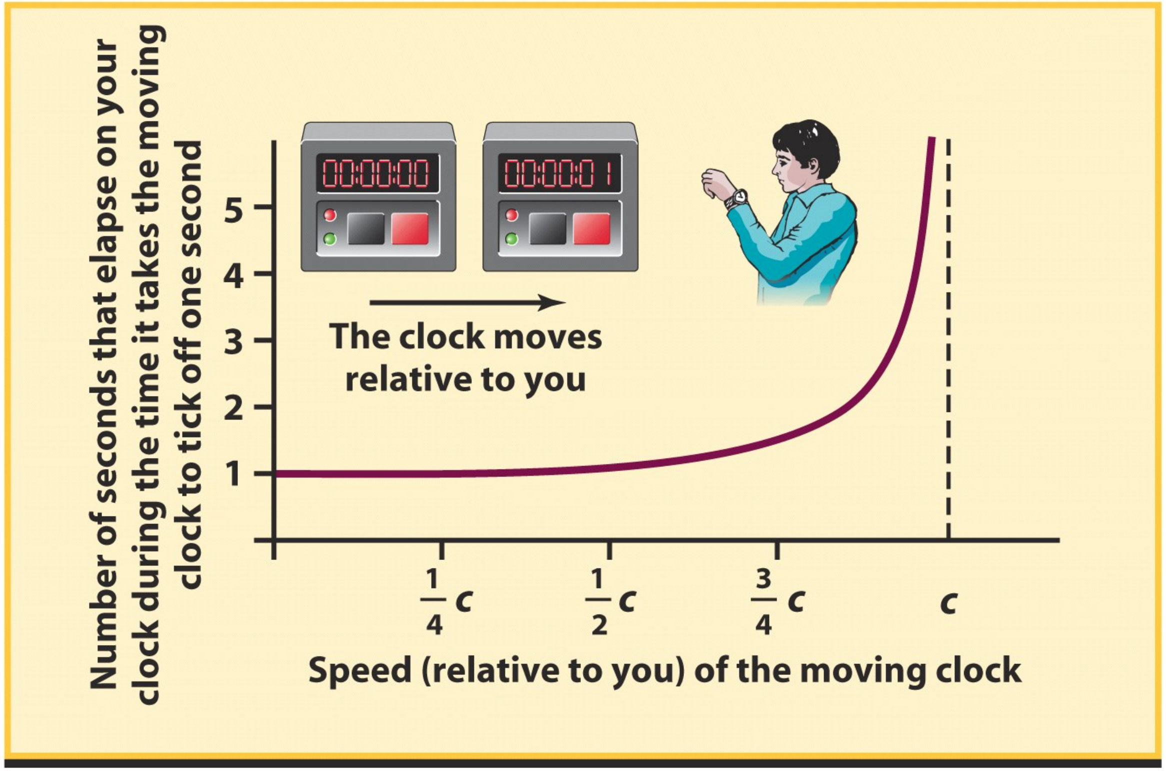


Time Dilation

An observer will note a

- slowing of clocks (time dilation)
- a shortening of rulers (length contraction)

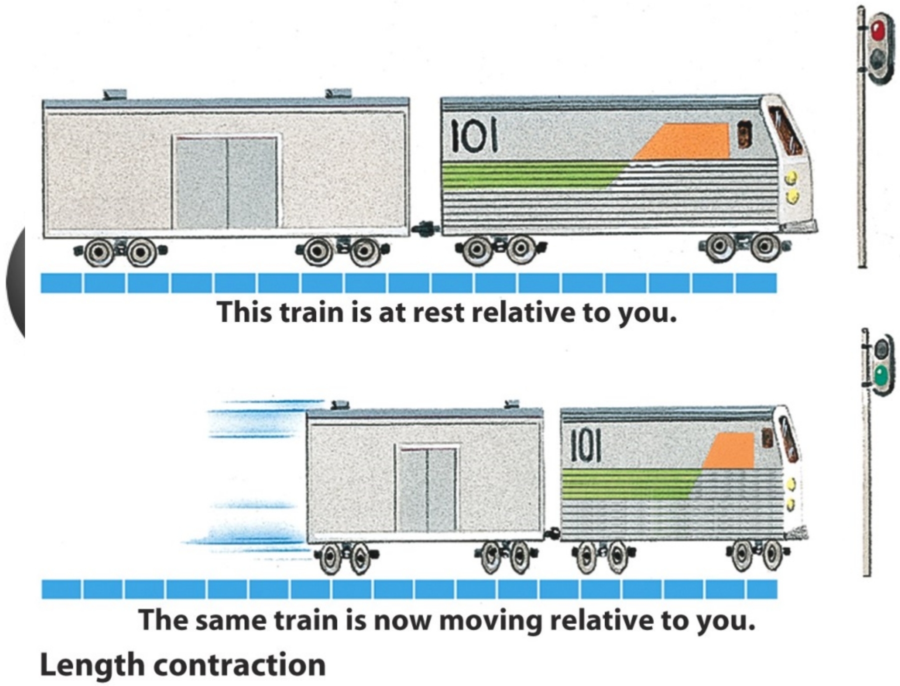
that are moving with respect to the observer



Time dilation

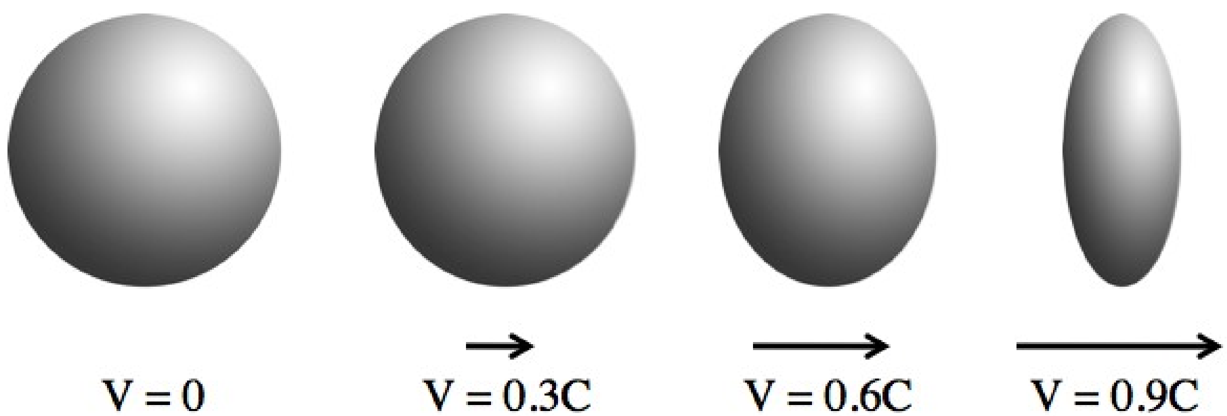
This effect becomes significant only if the clock or ruler is moving at a substantial fraction of the speed of light

Length Contraction



at the same time, we notice the effect on the other component of spacetime,

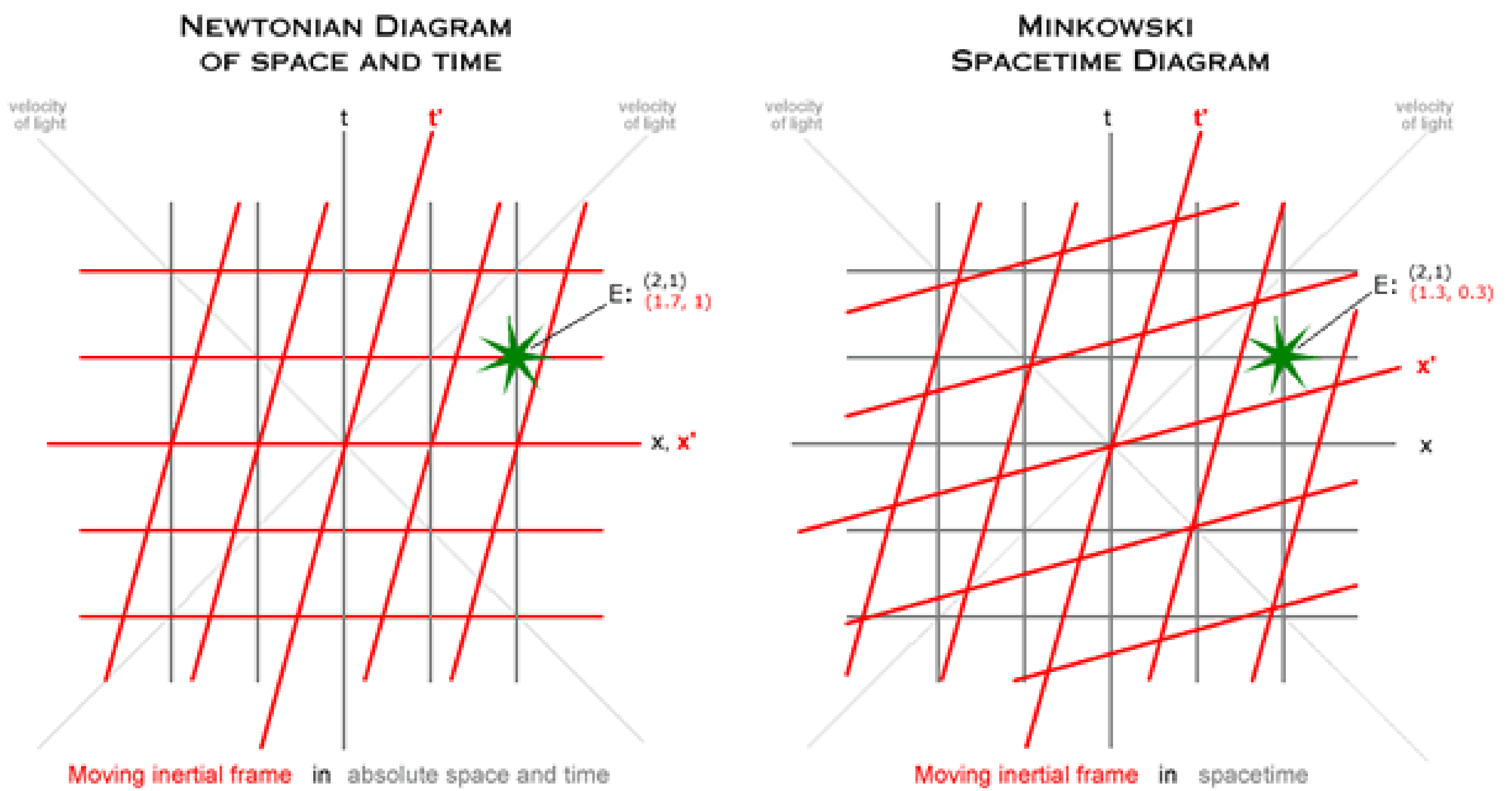
$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$



Relativistic
Time Dilation & Length Contraction

Carl Sagan
Cosmos

Space Time Diagrams



Note: relativity of time intervals –
time interval in restframe different from moving frame !

Minkowski spacetime

Point in 4D spacetime:

$$x^{\mu} = (ct, x, y, z)$$

$$x^0 = ct,$$
$$x^1 = x,\ x^2 = y,\ x^3 = z$$

Distances in flat (Minkowski) spacetime

$$s^2 = \eta_{\mu\nu} x^{\mu} x^{\nu} = c^2 t^2 - x^2 - y^2 - z^2$$

Note: with Einstein summation convention, and

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with $\eta_{\mu\nu}$ the **metric tensor** for Minkowski space.

Lorentz Transformation

$$x' = \gamma_v \left(x - \frac{v}{c} ct \right)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma_v \left(ct - \frac{v}{c} x \right)$$

$$\gamma_v = \frac{1}{\sqrt{1-(v/c)^2}} > 1$$

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}$$

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

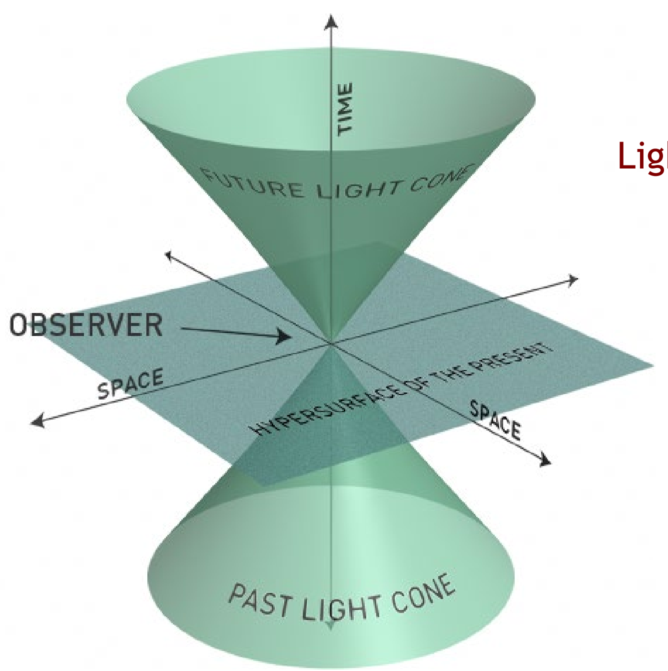
with $\Lambda^{\mu}{}_{\nu}$ the **Lorentz transform tensor**.

$$c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)$$

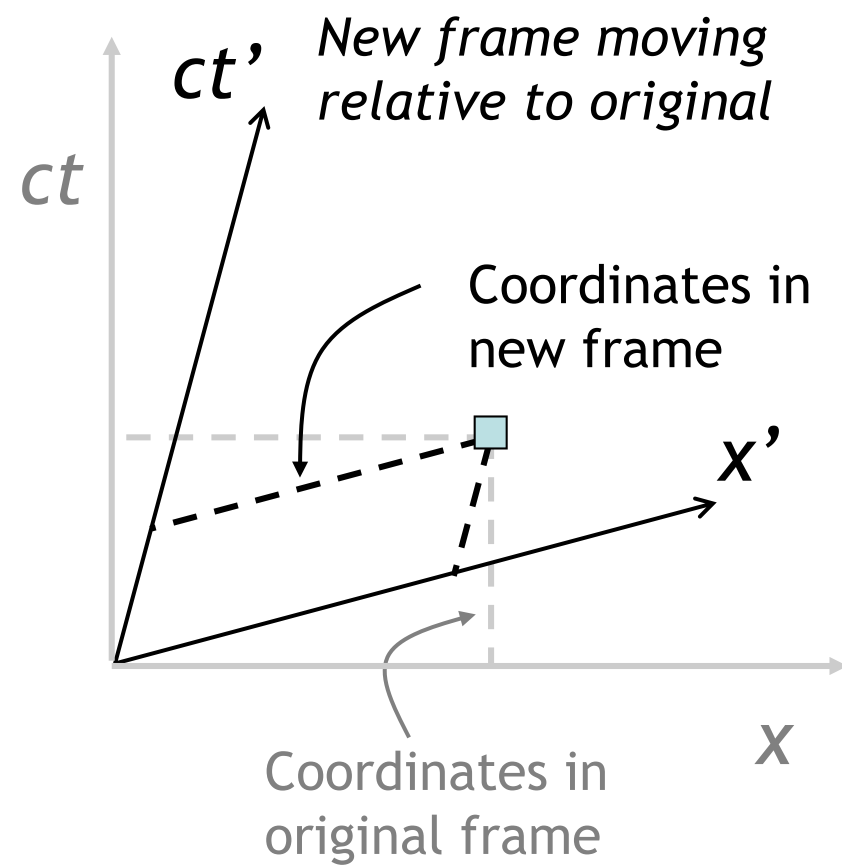
interval invariant under Lorentz transform

Frames of Reference

- In relativity events look different in reference frame moving at some velocity
- The new reference frame can be represented as same events along different coordinate axes
- A graphical way of showing that length and time are contracted or expanded.



Light Cone



$$d\tau = dt\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

Proper Time

Relativistic Mechanics

Relativity is concerned with formulating physical laws and relations in a **coordinate-free form**

Point in 4D spacetime, four-vector:

$$x^\mu = (ct, x, y, z)$$

Four-vectors transform with Lorentz transform between inertial frames

$$x^0 = ct,$$
$$x^1 = x,\ x^2 = y,\ x^3 = z$$

Four-velocity, four-vector: $$U^\mu = \frac{dx^\mu}{d\tau}$$

$$U^\mu = \gamma \begin{pmatrix} c \\ \vec{u} \end{pmatrix}$$

$$U^\mu U_\mu = -c^2$$

Four-momentum

$$P^\mu = m_0 U^\mu = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix}$$

$$P^\mu P_\mu = -\frac{E^2}{c^2} + |\vec{p}|^2$$

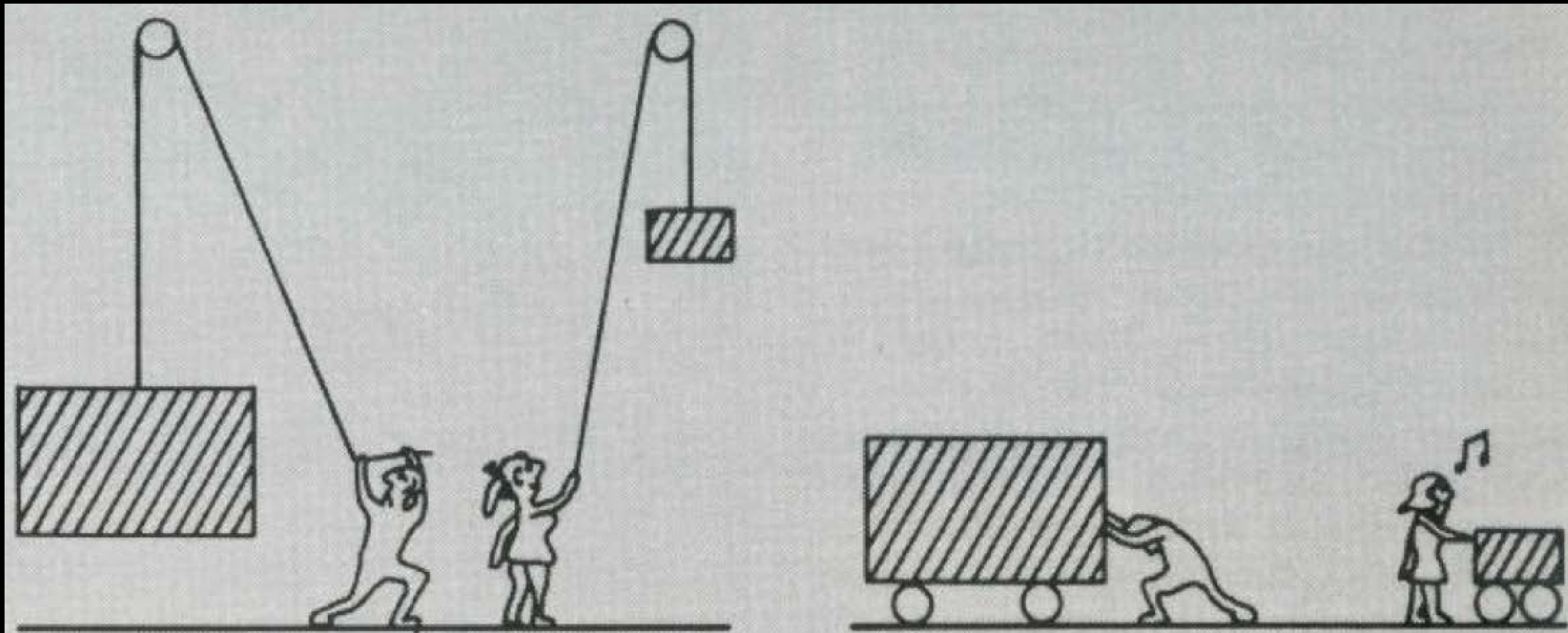
particle at rest:

$$E = m_0 c^2$$

Relativity:

Curved Space

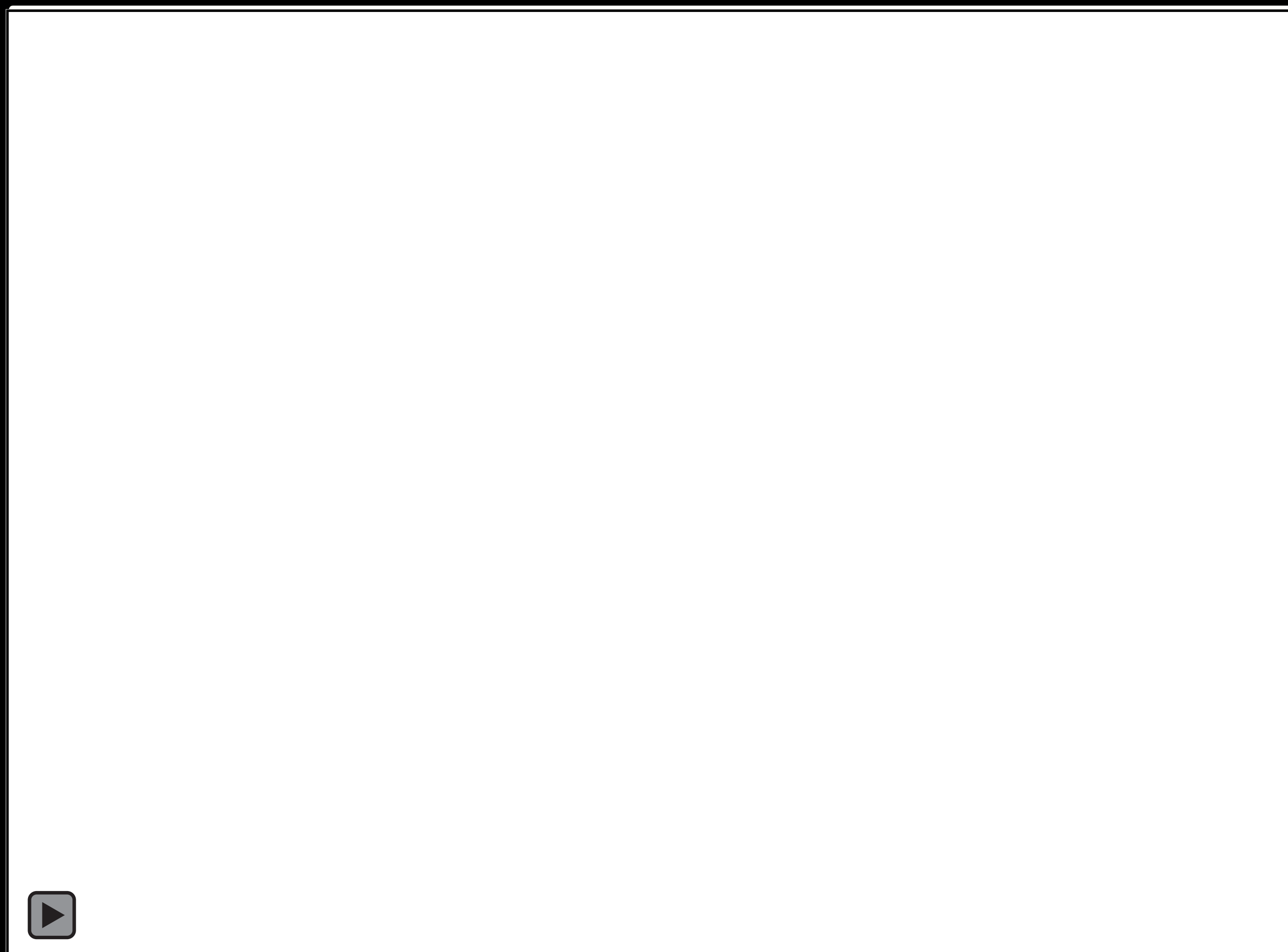
Inertial vs Gravitational Mass



- a larger mass experiences a stronger gravitational force than a light mass — gravitational mass
- a larger mass is more difficult to get moving than a light mass — inertial mass
- As a result, a heavy mass falls equally fast as the light mass:

Gravitational Mass = Inertial Mass

Inertial vs Gravitational Mass: Hammer vs. Feather



David Scott
Apollo 15
1971

Simon Stevin & Galilei

1586: Simon Stevin,
Nieuwe Kerk, Delft

1589 ???? - Galileo Galilei,
leaning tower of Pisa

de Beghinselen der Weeghconst

- *Laet nemen (soo den hoochgheleerden H. IAN CORNETS DE GROOT vlietichste ondersoucker der Naturens verborghentheden, ende ick ghedaen hebben) twee loyen clooten d'een thienmael grooter en swaerder als d'ander, die laet t'samen vallen van 30 voeten hooch, op een bart oft yet daer sy merckelick gheluyt tegen gheven, ende sal blijcken, dat de lichste gheen thienmael langher op wech en blijft dan de swaerste, maer datse t'samen so ghelijck opt bart vallen, dat haer beyde gheluyden een selve clop schijnt te wesen. S'ghelijcx bevint hem daetlick oock also, met twee evegroote lichamen in thienvoudighe reden der swaerheyt, daerom Aristoteles voornomde everedenheyt is onrecht.*

- In: Simon Stevin: De Beghinselen der Weeghconst, 1586.

de Beghinselen der Weeghconst

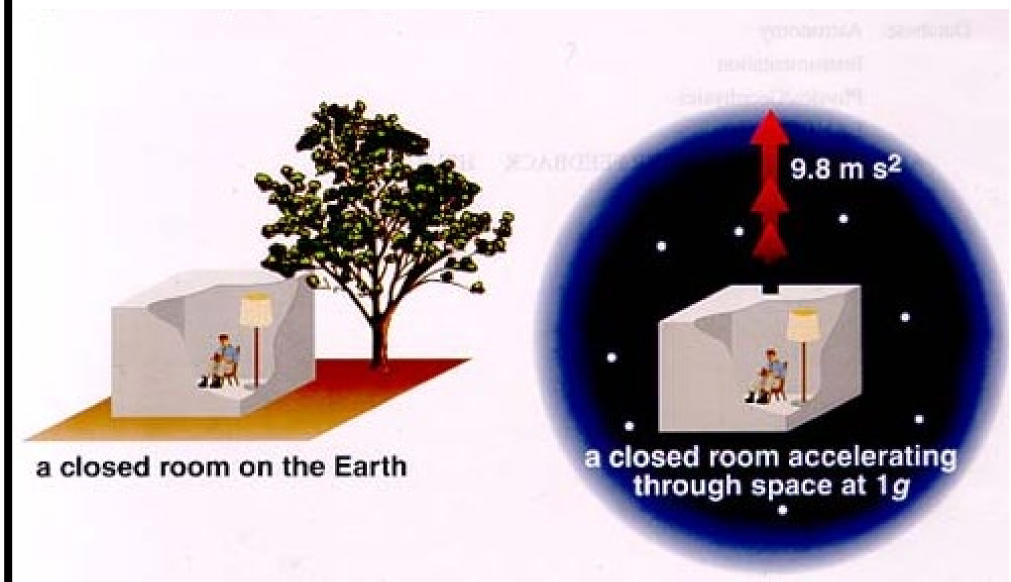
- Let us take (as the highly educated Jan Cornets de Groot, the diligent researcher of the mysteries of Nature, and I have done) two balls of lead, the one ten times bigger and heavier than the other, and let them drop together from 30 feet high, and it will show, that the lightest ball is not ten times longer under way than the heaviest, but they fall together at the same time on the ground. (...) This proves that Aristotle is wrong.'

- In: Simon Stevin: De Beghinselen der Weeghconst, 1586.

Equivalence Principle

Einstein's "happiest thought" came from the realization of

the equivalence principle



Einstein reasoned that:

There is no experiment that can distinguish between

uniform acceleration and

a uniform gravitational field.

Equivalence Principle

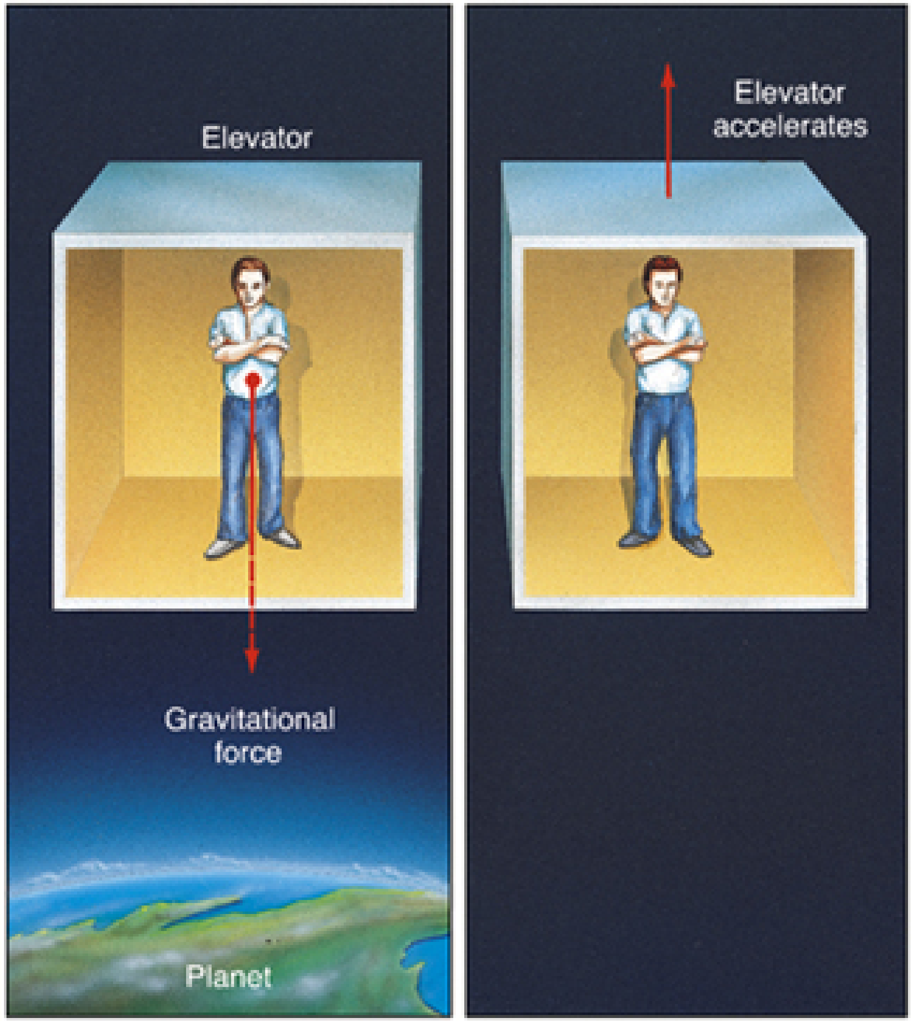
being in

an accelerating frame

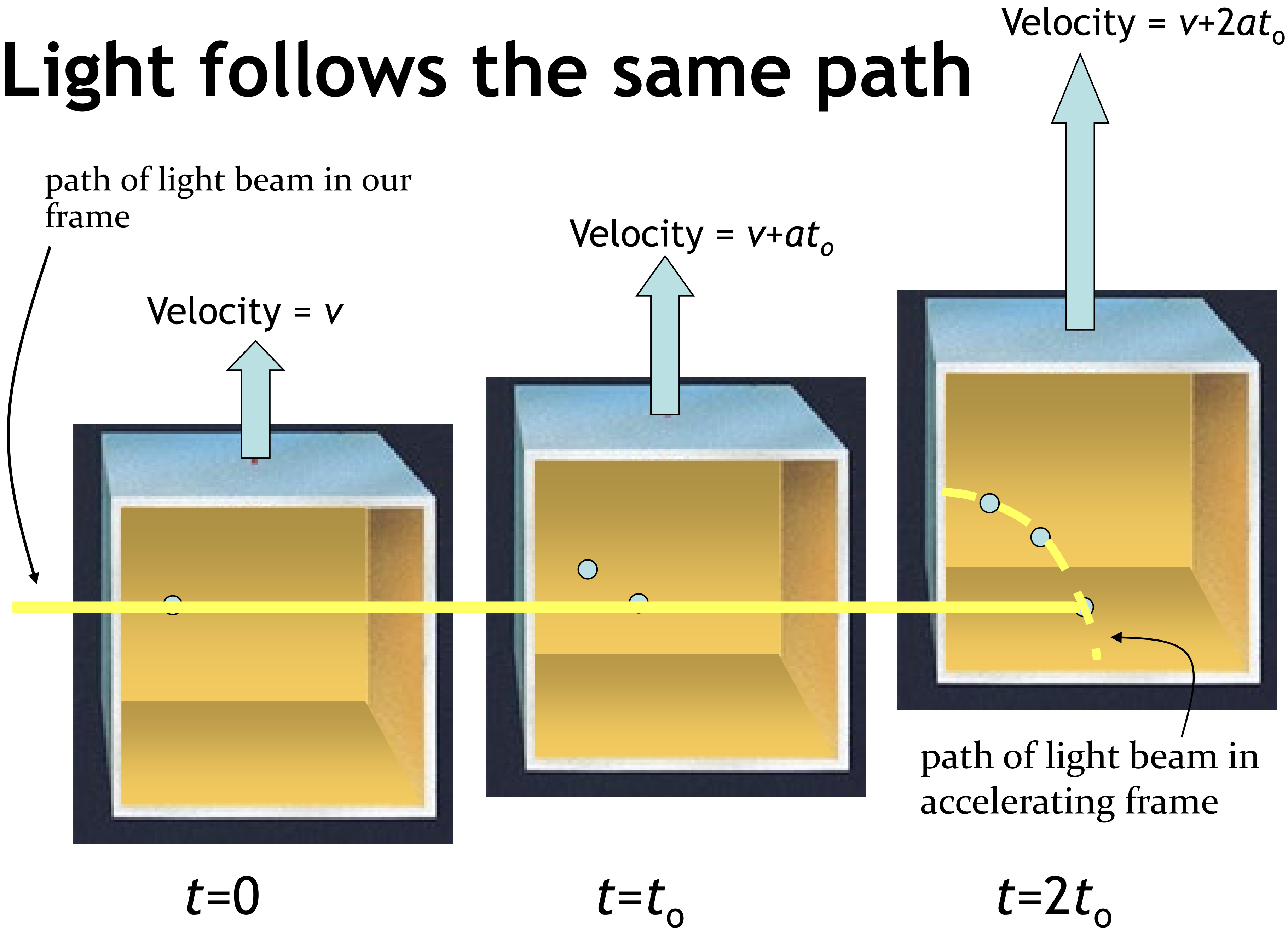
indistinguishable

from being in

a gravitational field



Light follows the same path



Equivalence Principle

The physics in the frame of a freely falling body
is equivalent to that of
an inertial frame in Special Relativity.

⇓

Free-falling bodies follow straight worldlines
- geodesics –
in curved spacetime

Curvature & Metric

Point in 4D spacetime:

$$x^{\mu} = (ct, x, y, z)$$

Distances in curved spacetime:

$$ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}$$

with $g_{\alpha\beta}$ the **metric tensor** for curved spacetime:

metric specifies distance recepy.

Proper time:

$$d\tau = dt \sqrt{1 + \frac{2\phi}{c^2}}$$

General Relativistic Equation of Motion

From Equivalence Principle, one may derive the equation of motion:

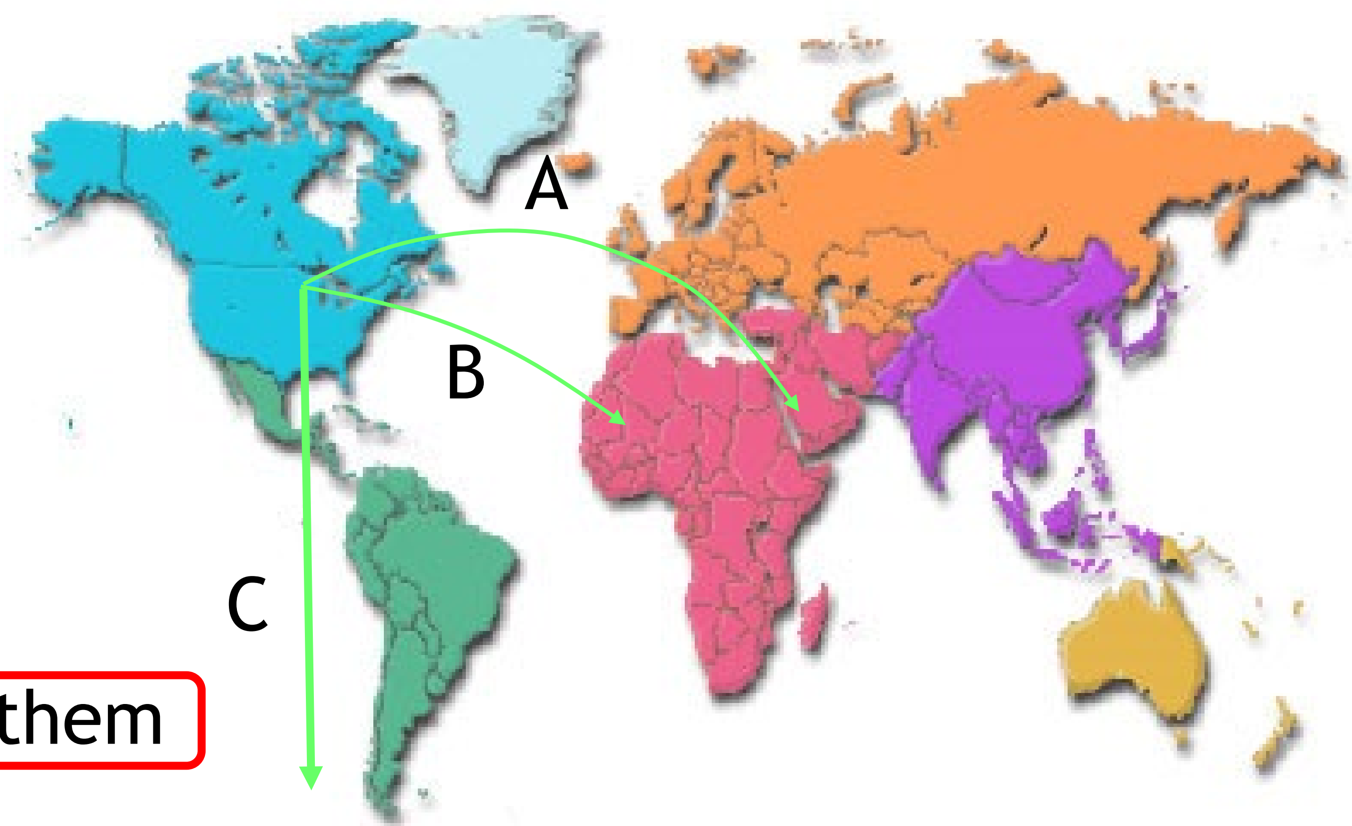
$$\frac{d^2 x^\beta}{d\tau^2} + \Gamma^\beta{}_{\lambda\nu} \frac{dx^\lambda}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

with **Christoffel symbol**/connection

$$\Gamma^\alpha{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\nu} \left\{ \frac{\partial g_{\gamma\nu}}{\partial x^\beta} + \frac{\partial g_{\beta\nu}}{\partial x^\gamma} - \frac{\partial g_{\gamma\beta}}{\partial x^\nu} \right\}$$

This is actually exactly the same equation as that for shortest paths in general curved spaces (in 4D spacetime), the GEODESIC equation.

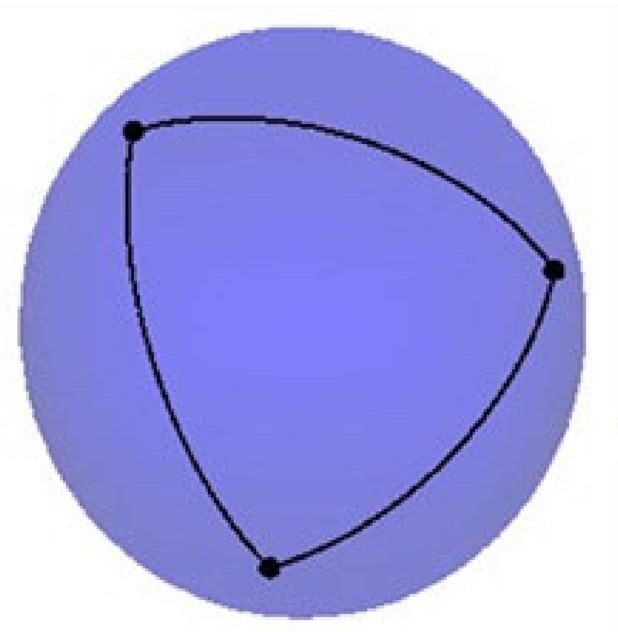
which of these is a straight line?



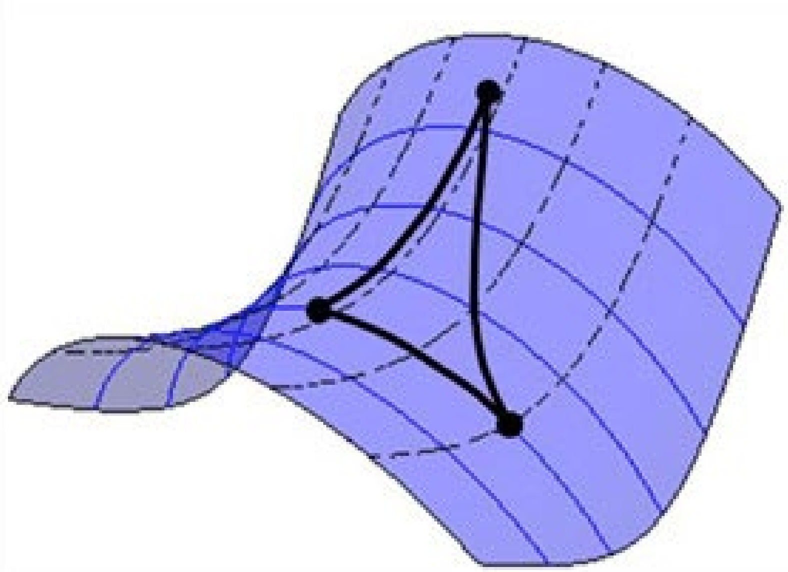
A. A
B. B
C. C
D. All of them

Curved Space:

Positive vs. Negative



positively curved space
sphere



negatively curved space
saddle

Triangle angles >180 degrees

Circle circumference < $2\pi r$

Triangle angles <180 degrees

Circle circumference > $2\pi r$

Einstein’s
Metric theory of Gravity:

how Gravity = Curved Space

Gravity & Curved Spacetime

- Equivalence of acceleration of a frame &
  location in gravitational field

  ⇨ in gravity field, light follows a curved path

- Curved paths:

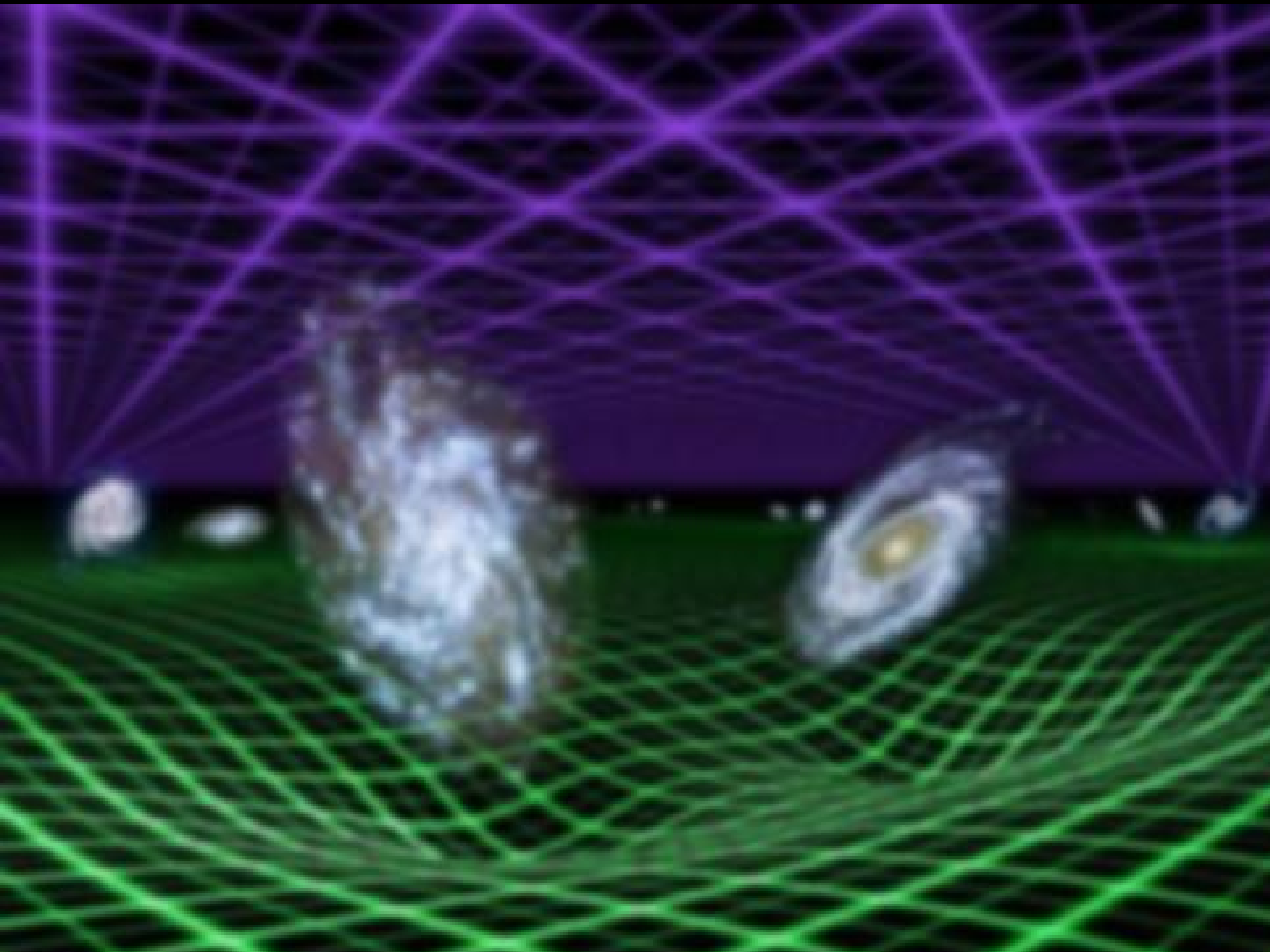
  straight lines in curved spacetime: ***Geodesics***
  (cf. flightpaths airplanes over surface Earth)

- Fundamental tenet of ***General Relativity***:

  !!!!!!!! **Gravity is the effect of curved spacetime** !!!!!!!!

- E.g.: relation between metric component g_{oo} and gravitational potential

$$g_{00} = \frac{2\phi}{c^2}$$



Einstein's
Theory of Gravity:

Source: Energy & Momentum

Source of Gravity: Energy-Momentum Tensor

Energy and **Momentum** are intimately linked physical quantities:
both components of the **energy-momentum** four-vector P_μ,

$$P^\mu = m_0 U^\mu = m_0 \gamma \begin{pmatrix} c \\ \vec{u} \end{pmatrix} \approx m_0 \begin{pmatrix} c \\ \vec{u} \end{pmatrix}$$

Relativity is concerned with formulating physical laws and relations in a **coordinate-free form.**

This is called formulation in **covariant form:**

Looking for tensor equations that are valid in any reference frame:

For the energy-momentum four-vector, we look for the equivalent of Newtonian fluid equations, ie. the equations expression concervation of mass, energy and momentum:

$$T^{\mu\nu}{}_{;\nu} = 0$$

with energy momentum-tensor (specifying energy & momentum content Universe:

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) U^\mu U^\nu - p g^{\mu\nu}$$

Source of Gravity: Energy-Momentum Tensor

Energy momentum-tensor (specifying energy & momentum content Universe:

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

In restframe:

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Notice the presence of the **Pressure** term:

1) pressure is the flux of momentum
2) In relativity momentum is coupled to energy in energy-momentum four-vector

In cosmology this becomes of KEY significance:

source term of gravity:

Energy & Momentum, hence Energy & Pressure appear in dynamics

$$\nabla^2 \phi = 4\pi g \left(\rho + \frac{3p}{c^2}\right)$$

Relativity:

Spacetime is Dynamic

Einstein Field Equation

Einstein Tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_{\mu\nu;\nu} = T_{\mu\nu;\nu} = 0$$

Einstein Tensor only rank 2 tensor for which this holds:

$$G_{\mu\nu} \propto T_{\mu\nu}$$

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Metric tensor: $g_{\mu\nu}$

Energy-Momentum tensor: $T_{\mu\nu}$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) U^{\mu} U^{\nu} - p g^{\mu\nu}$$

SPACETIME REACTS TO CONTENT OF THE UNIVERSE

$\Longleftrightarrow$

CONTENT OF UNIVERSE REACTS TO CURVATURE

Einstein Field Equation

also: $$g_{\mu\nu;\nu} = 0$$

Freedom to add a multiple of metric tensor to Einstein tensor:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Λ : Cosmological Constant

Einstein Field Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Dark Energy

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}\left(T^{\mu\nu} + T^{\mu\nu}{}_{vac}\right)$$

$$T^{\mu\nu}{}_{vac} \equiv \frac{\Lambda c^4}{8\pi G} g^{\mu\nu}$$

Einstein Field Equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Relativistic

vs.

Newtonian Cosmology

Relativistic Cosmology	Newtonian Cosmology
10 field equations	1 field equation
10 potentials	1 potential
nonlinear equations	linear equations
intrinsically geometric	absolute space and time
can cope with ∞ space	requires finite space
all energies gravitate	gravitation mass-density only
pressure gravitates	gravitation mass-density only
Cosmological Constant feasible	repulsive action gravity impossible
Hyperbolic propagation	instantaneous propagations
Singularities spacetime	Singularities space
Horizons & Black Holes	No Horizons & Black Holes
Gravitational Waves	No Gravitational Waves

Uniform Universe:

the

Cosmological Principle

General Relativity

A crucial aspect of any particular configuration is the geometry of spacetime: because Einstein's General Relativity is a metric theory, knowledge of the geometry is essential.

Einstein Field Equations are notoriously complex, essentially 10 equations. Solving them for general situations is almost impossible.

However, there are some special circumstances that do allow a full solution. The simplest one is also the one that describes our Universe. It is encapsulated in the

Cosmological Principle

On the basis of this principle, we can constrain the geometry of the Universe and hence find its dynamical evolution.

Cosmological Principle: the Universe Simple & Smooth

"God is an infinite sphere whose centre is everywhere and its circumference nowhere"
Empedocles, 5th cent BC

Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- **Homogeneous** → The Universe is the same everywhere:
  - physical quantities (density, T,p,…)
- **Isotropic** → The Universe looks the same in every direction
- **Universality** → Physical Laws same everywhere
- **Uniformly Expanding** → The Universe "grows" with same rate in
  - every direction
  - at every location

"all places in the Universe are alike"
Einstein, 1931

Geometry of the Universe

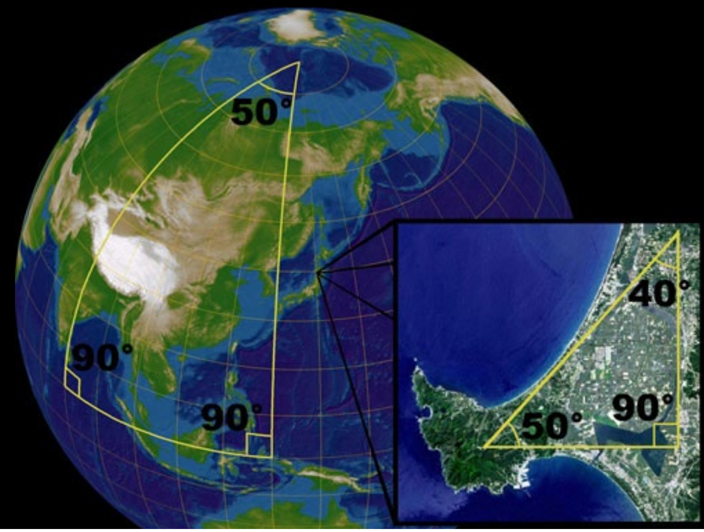
Fundamental Tenet

of (Non-Euclidian = Riemannian) Geometry

There exist no more than THREE uniform spaces:

1)	Euclidian (flat) Geometry	Euclides
2)	Hyperbolic Geometry	Gauß, Lobachevski, Bolyai
3)	Spherical Geometry	Riemann

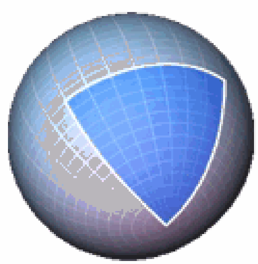
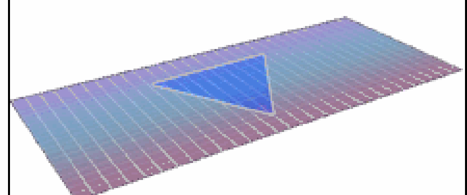
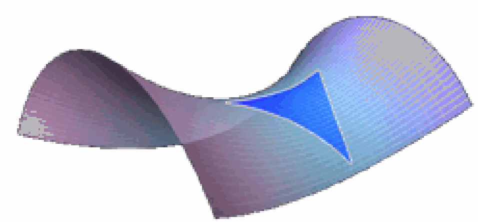
uniform=
homogeneous & isotropic
(cosmological principle)



Uniform Spaces:

Geometric Characteristics

	Parallel Lines	Triangular Angles	Circumference Circle	Curvature	Extent	Boundary
		$\alpha + \beta + \gamma$	$x \equiv \frac{S}{2r}$	k		
Flat Space	parallels: 1 never intersects	π	π	0	open: infinite	unbounded
Spherical Space	parallels: ∞ along great circles, all intersect	$> \pi$	$< \pi$	$1/R^2$ > 0	closed: finite	unbounded
Hyperbolic Space	parallels: ∞ diverge & never intersect	$< \pi$	$> \pi$	$-1/R^2$ < 0	open: infinite	unbounded

Property	Closed	Euclidean	Open
Spatial Curvature	Positive	Zero	Negative
Circle Circumference	$< 2\pi R$	$2\pi R$	$> 2\pi R$
Sphere Area	$< 4\pi R^2$	$4\pi R^2$	$> 4\pi R^2$
Sphere Volume	$< {}^4/_3\ \pi R^3$	${}^4/_3\ \pi R^3$	$> {}^4/_3\ \pi R^3$
Triangle Angle Sum	$> 180°$	$180°$	$< 180°$
Total Volume	Finite ($2\pi^2 R^3$)	Infinite	Infinite
Surface Analog	Sphere 	Plane 	Saddle 

Light Paths in Uniform Curved Spaces