Tutorial V
The Early Universe

Exercise 1: Microwave Background.

a) For an adiabatic expanding medium we know that $TV^{\gamma-1} = cst$, with T the temperature of the medium and V the volume. Derive the temperature change of a uniform radiation field in an expanding Universe as a function of expansion factor a_{exp}.

b) Given the fact that in an expanding Universe the frequency ν of radiation is redshifted, so that the frequency of a photon of current frequency ν_o has a frequency $\nu(z) = \nu_o(1+z)$ at redshift z, show that when the radiation field is blackbody at a particular cosmic epoch, it will remain blackbody ! (hint: combine temperature and frequency evolution of photons). What does this mean for the Cosmic Microwave Background.

c) It is known that to an impressive degree of accuracy the Cosmic Microwave Background is a black body radiation field, with a temperature of $T = 2.725K$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp h\nu/kT - 1}$$

Infer the number density n_γ of photons as a function of the CMB temperature T. When compared to the number of baryons in the Universe, can you tell why the Universe is such an exceptional physical system ?

d) Also infer the energy density u_γ as a function of T. Why do n_γ and u_γ have different dependence on temperature T ?

e) The almost perfect blackbody character of the CMB has very strong implications for cosmology. What is this consequence ? When were most CMB photons created, ie. as a result of which cosmological process at around z 10^9 ? And which three radiation processes were responsible for transforming the spectral energy distribution of these primordial photons into a blackbody spectrum ? Why could simple Thomson scattering, so important at later times, not take care of this ?

f) At around a temperature of $T \approx 3000K$ the Universe, and in particular the blackbody cosmic radiation, undergoes a major transition. This, perhaps most important cosmological transition, includes three closely related processes. Describe each of these, and describe qualitatively what happened. You may use some drawings and sketches.

g) According to a simple equilibrium evaluation on the basis of the Saha equation the transition should have happened at a temperature of $T_\gamma = 3740K$. Why did it take place much later, at $T_\gamma \approx 3000K$?

Exercise 2:
Helium Abundance and Neutrino Species

Radiation consists of 2 particle species: *photons* (γ) and *neutrinos* (ν). Neutrinos are neutral elementary particles that only interact through the weak nuclear force. After photons they are by far the most abundant species in nature. Neutrinos are fermions, spin 1/2 particles, while photons are bosons.

For a long time it has not been clear how many species N_ν of neutrinos there are. Three species are known, *electron neutrino*, *muon neutrino* and *tau neutrino*. However, there could be 4 or more ... If it were not for the helium abundance in the Universe !

a) From the Maxwell-Boltzmann equation, derive an expression for the neutron/proton ratio n/p as a function of temperature.

b) Calculate the number of neutrons and protons at a temperature of $T = 10^{11}$K, $T = 1.5 \times 10^{10}$ K and $T = 10^{10}$ K. How does the equilibrium ratio between neutrons and protons evolve, and can you explain why it shifts so suddenly around $T = 1.5 \times 10^{10}$?

As long as the temperature is high enough, ie. as long as $T > 10^{10}$ K, protons and neutrons are in thermal equilibrium through the weak interaction processes,

$$n + \nu_e \rightleftharpoons p + e^- \,, \tag{1}$$

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$$n + e^+ \rightleftharpoons p + \bar{\nu}_e \,. \tag{2}$$

However, the reaction rate Γ_w of the weak interaction decreases in time because the neutrino density n_ν decreases as a result of the expansion of the Universe, and the interaction cross section σ_w also rapidly declines as $\sigma_w \propto T^2$, resulting in

$$\Gamma_w = n_\nu c \sigma_w \propto t^{-5/2} \,, \tag{3}$$

Because the expansion rate of the Universe, ie. the Hubble parameter $H(t)$, decreases less rapidly,

$$H(t) \propto t^{-1} \,, \tag{4}$$

at a certain moment the characteristic time of the weak interaction, $T_w = 1/\Gamma_w$, will become longer than the Hubble time $t_H \equiv 1/H$). This marks the freeze-out time of the n/p ratio,

$$\Gamma_w(t_{\text{freeze}}) \approx H(t_{\text{freeze}}) \,. \tag{5}$$

c) Write down the Hubble parameter $H(t)$ as a function of radiation energy density $\rho_{rad}(t)$, during the radiation-dominated era at the time of primordial nucleosynthesis.

d) Show, on the basis of Fermi-Dirac statistics for neutrinos, that the total radiation energy $\rho_{rad}(t)$ in terms of present-day photon energy density $\rho_{\gamma 0}$ and number of neutrino species is given by

$$\rho_{rad,0} \;=\; (1 \,+\, 0.227 N_\nu)\, \rho_{\gamma 0} \tag{6}$$

where

$$\rho_{\nu 0} \;=\; \frac{7}{8} \,\times\, N_\nu \,\times\, \left(\frac{4}{11}\right)^{4/3} \rho_{\gamma 0} \;=\; 0.227\, N_\nu\, \rho_{\gamma 0}\,. \tag{7}$$

e) How do you expect that the number of neutrino species N_ν will determine the helium abundance in the early Universe ?

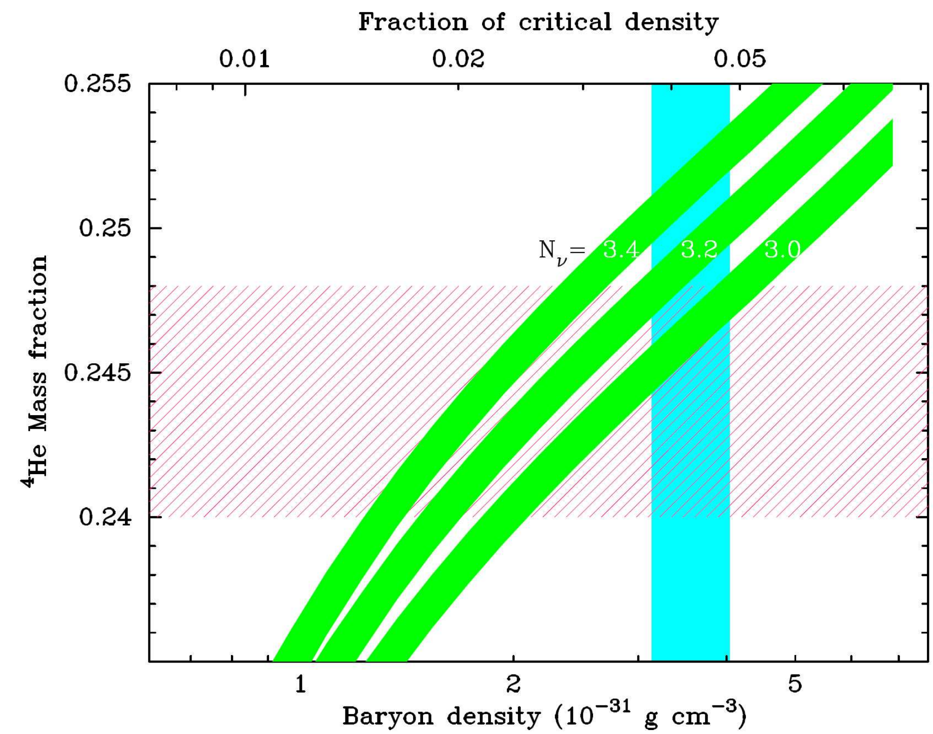

Figure 1: Dependence of helium abundance on number of neutrino species. Figure shows helium abundance (lightgreen) as function of baryon density, for 3 different values of number of neutrino families N_ν. Dashed region: region of measured/estimated cosmic helium abundance. Cyan: cosmic baryon density value in agreement with observations.