

*Climbing Mount Olympus :  
Geometry  
as Pathway to the Universe*

Rien van de Weygaert

Kapteyn Institute, University Groningen, Ολάνδια

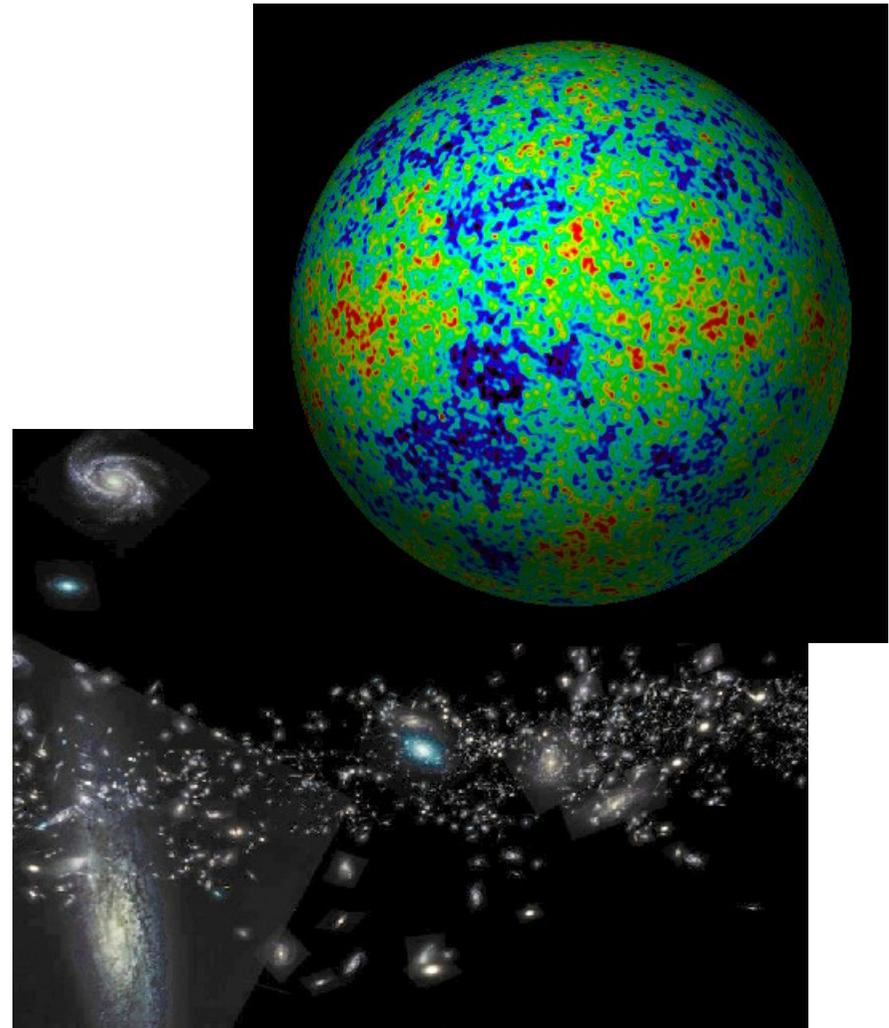
ΙΔΙΣΥΕΕΠ conference, Θεσσαλονικη,

Δεκεμβριου 5-7, 2003

# *Cosmology:* *the Science of the Universe*

## *What is Cosmology?*

- ❖ Formally:  
Astronomical science or theory of the Universe as an ordered unity; study of the structure and evolution of the Universe.
- ❖ Broadest meaning:  
human enterprise, joining science, philosophy, theology and the arts to seek to gain understanding of what unifies and is fundamental to our world
- ❖ Scientific:  
Study of large-scale structure and infrastructure of the Universe



# *Cosmology:*

## *the key questions and issues*

- ❖ *What is the structure of the Universe ?*
- ❖ *How big is the world ? (finite, infinite,...)*
- ❖ *How did the world begin ? When did the world begin ? Did it begin at all ?*
- ❖ *What is the fate of the Universe ?*
- ❖ *What is the world made of ? What does it contain ?*
- ❖ *Are these questions meaningful at all ?*

# *The Universe:*

*... a journey ...*



*... The Stars ...*



*Our Galaxy ...*



*... the galaxies ...*



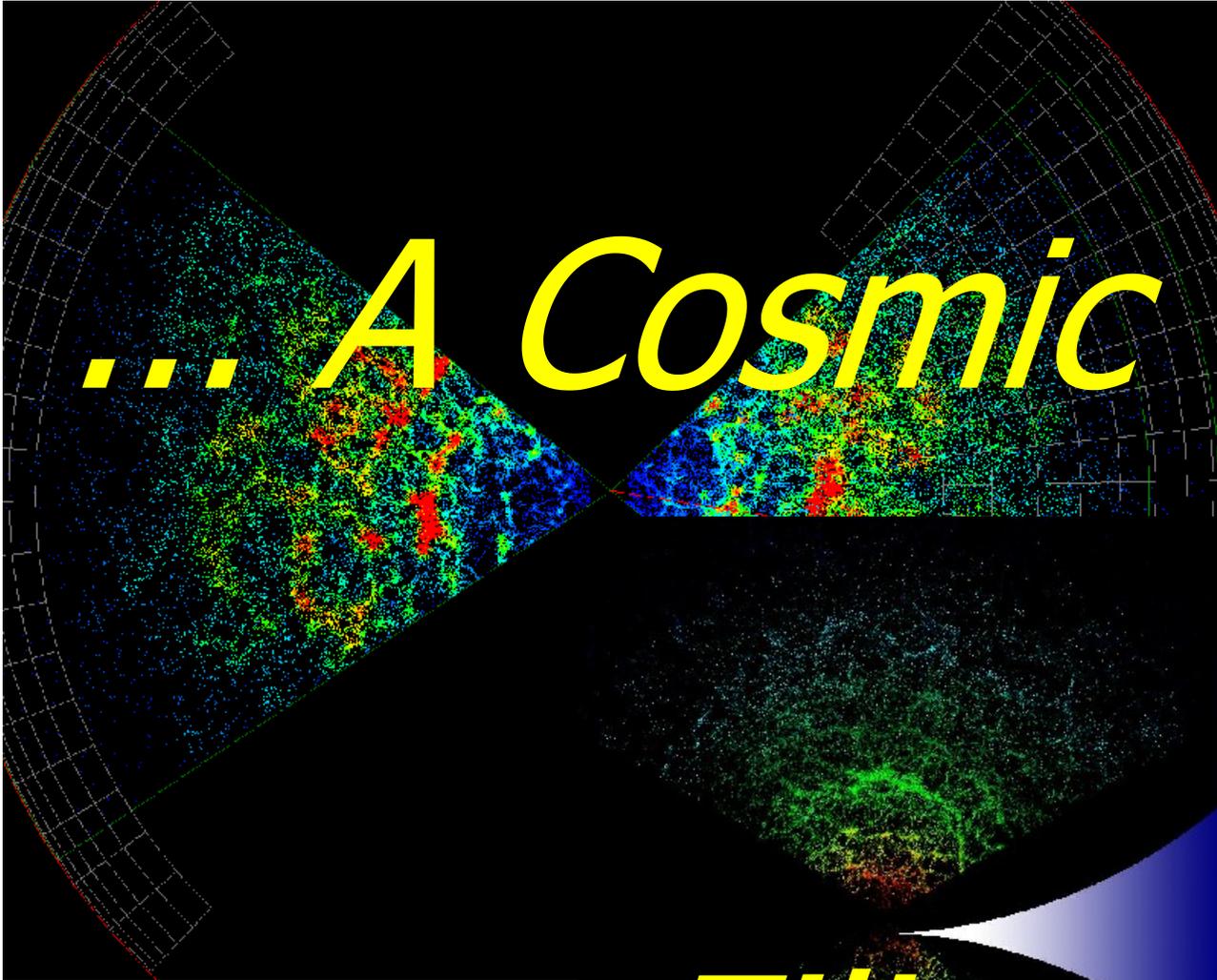


*... galaxy*

*assemblies ...*

*... groups &*

*clusters ...*



... *A Cosmic*

A Cosmic Microwave Background (CMB) fluctuation map, showing temperature variations across the sky. The map is overlaid with a grid of latitude and longitude lines. The color scale ranges from blue (cooler) to red (warmer), with yellow and green in between. The map is split into two hemispheres by a diagonal line.



*Filigreee ...*

A galaxy cluster, showing numerous galaxies of various colors (yellow, orange, red, blue) and sizes, set against a dark background. The cluster is framed by a blue border.



*...Out to the deepest*

*realms of*

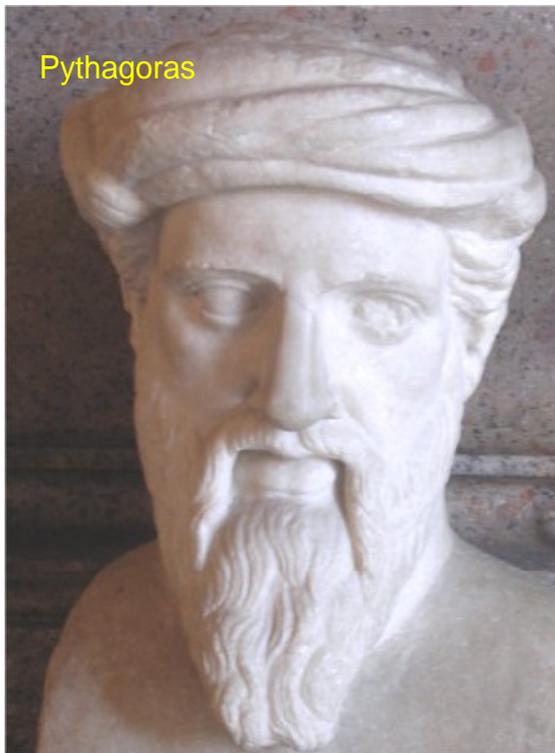
*the visible Universe ...*

T = 2.728 K

# Ionian, 6<sup>th</sup> century B.C.

A phase transition in human history: the mythical world obsolete

... the Ionian coast, 6<sup>th</sup> century B.C.,  
regularities and symmetries in nature  
recognized as keys to the cosmos ...



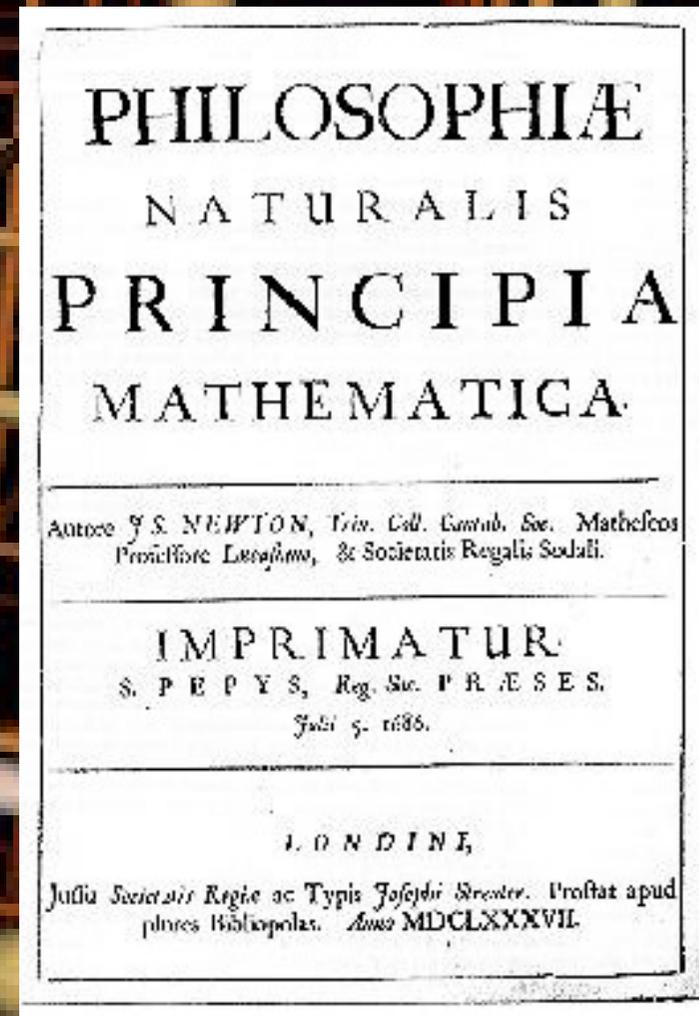
- Mathematics as natural language of cosmos
- Physical cosmos modelled after ideal form, encrypted in concepts of geometry

... Anaximander of Miletus: the Apeiron  
Pythagoras of Samos: music of spheres  
Plato: Platonic solids

# Renaissance of Western Science

In footsteps of Copernicus, Galilei & Kepler,  
*Isaac Newton* (1687) in his *Principia*  
formulated a comprehensive model of the  
world. Cosmologically, it meant

- absolute and uniform time
- space & time independent of matter
- dynamics: - action at distance  
- instantaneous
- Universe edgeless, centerless & infinite
- Cosmological Principle:  
Universe looks the same at every  
place in space, every moment in time
- absolute, static & infinite space



# Einstein's Field Equations

... Spacetime becomes a dynamic continuum,  
integral part of the structure of the cosmos ...  
curved spacetime becomes force of gravity

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = -\frac{8\pi G}{c^4} T^{\alpha\beta}$$

... its geometry rules the world,  
the world rules its geometry...

# *Einstein's Field Equations*

... Spacetime becomes a dynamic continuum,  
integral part of the structure of the cosmos ...  
curved spacetime becomes force of gravity

*But, no longer*

*Euclides suffices ...*

... its geometry rules the world,  
the world rules its geometry...

# Geometry of the Universe: Cosmological Principle

"God is an infinite sphere whose centre is everywhere and its circumference nowhere"  
Empedocles, 5<sup>th</sup> cent BC

## Cosmological Principle:

Describes the symmetries in global appearance of the Universe:

- Homogeneous → The Universe is the same everywhere:  
- physical quantities (density, T,p,...)
- Isotropic → The Universe looks the same in every direction
- Universality → Physical Laws same everywhere
- Uniformly Expanding → The Universe "grows" with same rate in  
- every direction  
- at every location

"all places in the Universe are alike"  
Einstein, 1931

# *Geometry of the Universe*

## Fundamental Tenet

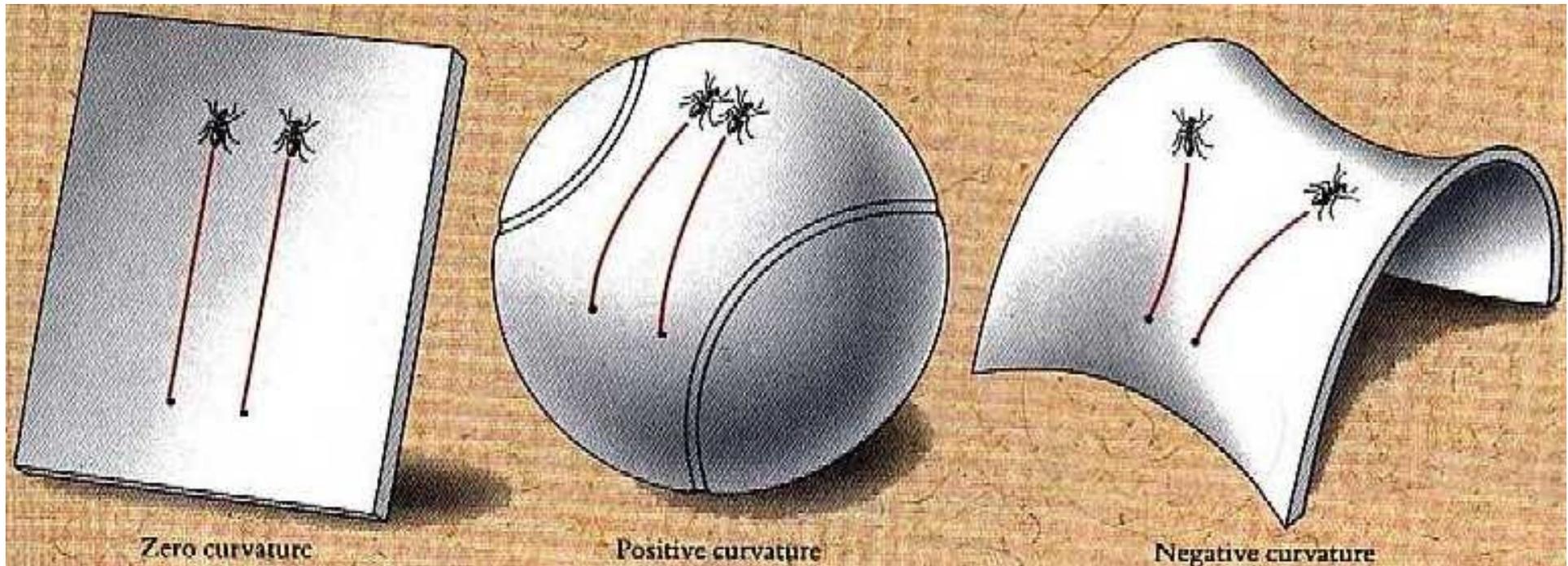
### (Non-Euclidian = Riemannian) Geometry:

- there exist no more than THREE uniform spaces:

- |    |                           |                           |
|----|---------------------------|---------------------------|
| 1) | Euclidian (flat) Geometry | Euclides                  |
| 2) | Hyperbolic Geometry       | Gauß, Lobachevski, Bolyai |
| 3) | Spherical Geometry        | Riemann                   |

uniform=  
homogeneous & isotropic  
(cosmological principle)

# *Geometry of the Universe*



The three possible geometries of the Universe

# *Geometry & Dynamics:*

## *Friedmann-Robertson-Walker-Lemaitre Universe*

Having confined the Universe to the highly symmetric geometries corresponding the Cosmological Principle, the Einstein field equations are reduced tenfold to

- TWO equations, for
- ONE scale factor  $R(t)$ , the uniform growth factor of the Universe

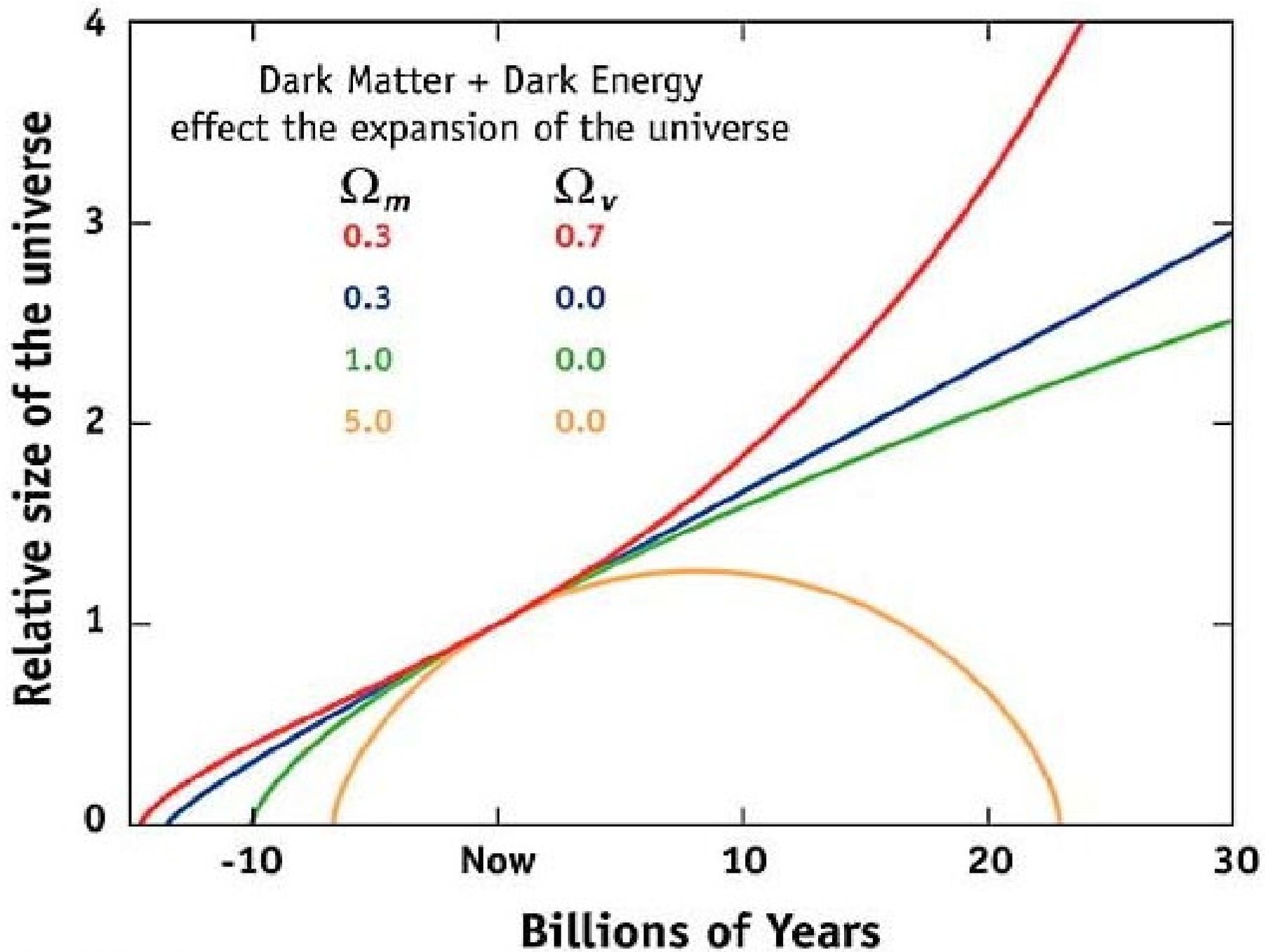
The equations are called the Friedmann-Robertson-Walker-Lemaitre equations, and are the fundamental solutions for dynamics and evolution of the Universe.

# *Geometry & Dynamics:*

*Friedmann-Robertson-Walker-Lemaitre Universe*

$$\ddot{R} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) R + \frac{\Lambda}{3} R$$

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{\Lambda}{3} R^2 - kc^2$$



# Geometry & Dynamics:

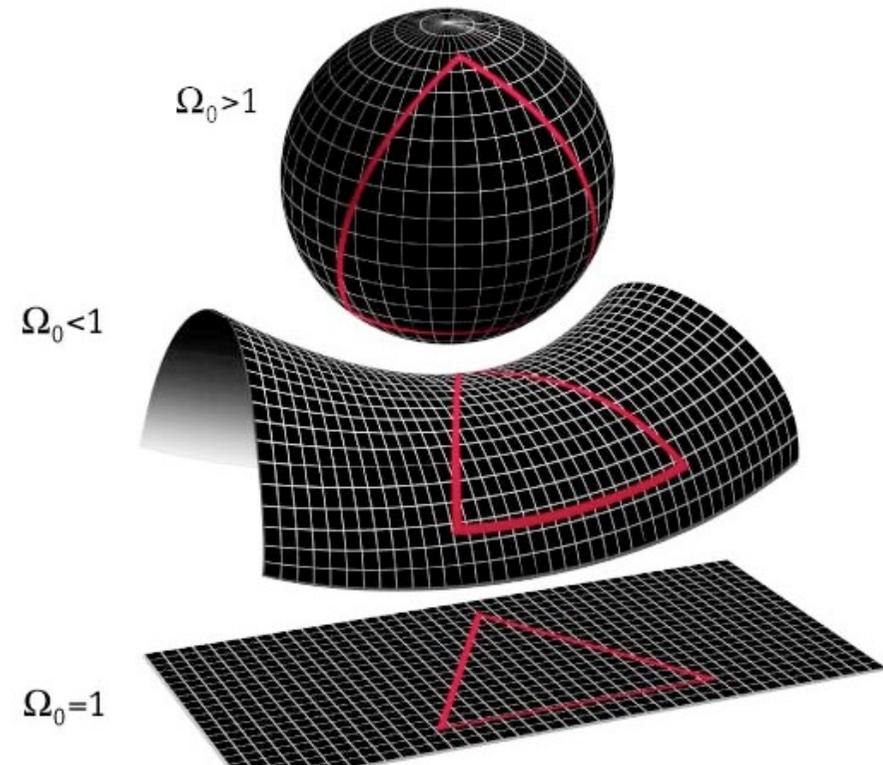
## Friedmann-Robertson-Walker-Lemaitre Universe

... for an expanding Universe with

- matter density  $\rho(t)$
- pressure  $p(t)$
- cosmological constant  $\Lambda(t)$   
[or, the elusive dark energy  $\rho_v$ ]

... whose dynamics is ultimately set by the geometrical curvature term "k" :

$$k = \frac{H_0^2 R_0^2}{c^2} (\Omega_0 - 1) = \begin{cases} 1 & sph : \Omega_0 > 1 \\ 0 & flat : \Omega_0 = 1 \\ -1 & hyp : \Omega_0 < 1 \end{cases}$$

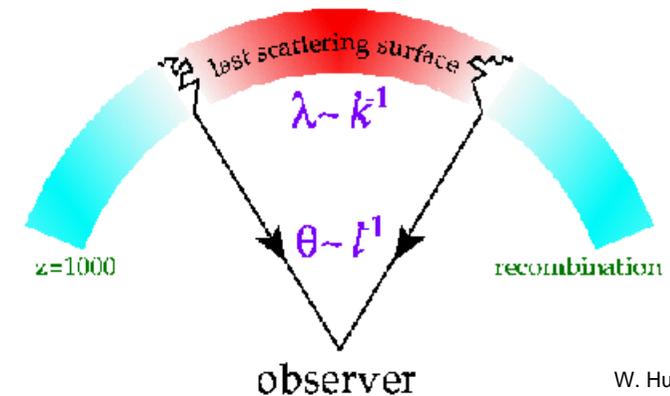
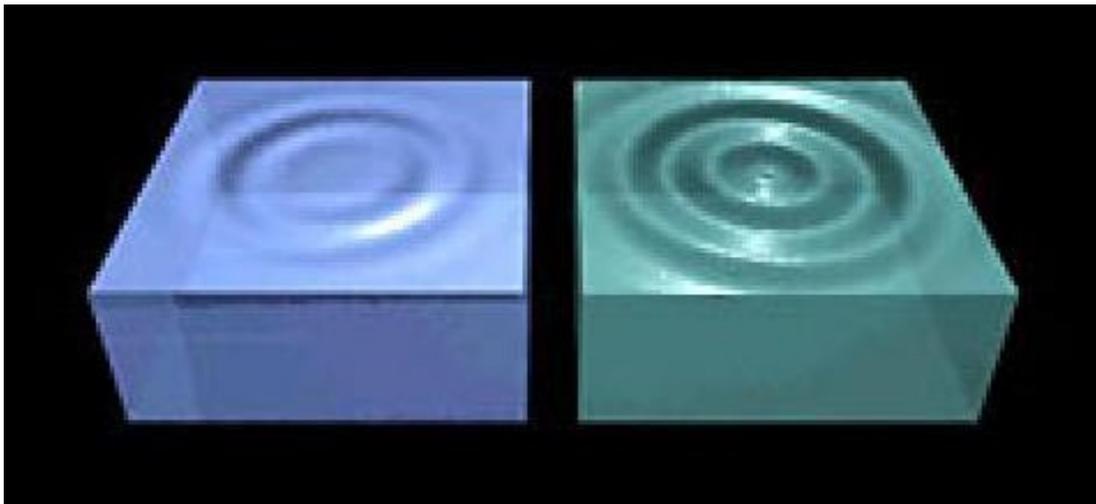


# Geometry of the Universe: Music of the Spheres

## Measuring the Geometry of the Universe:

- Object with known physical size, at large cosmological distance
- Measure angular extent on sky
- Comparison yields light path

➔ Geometry of space

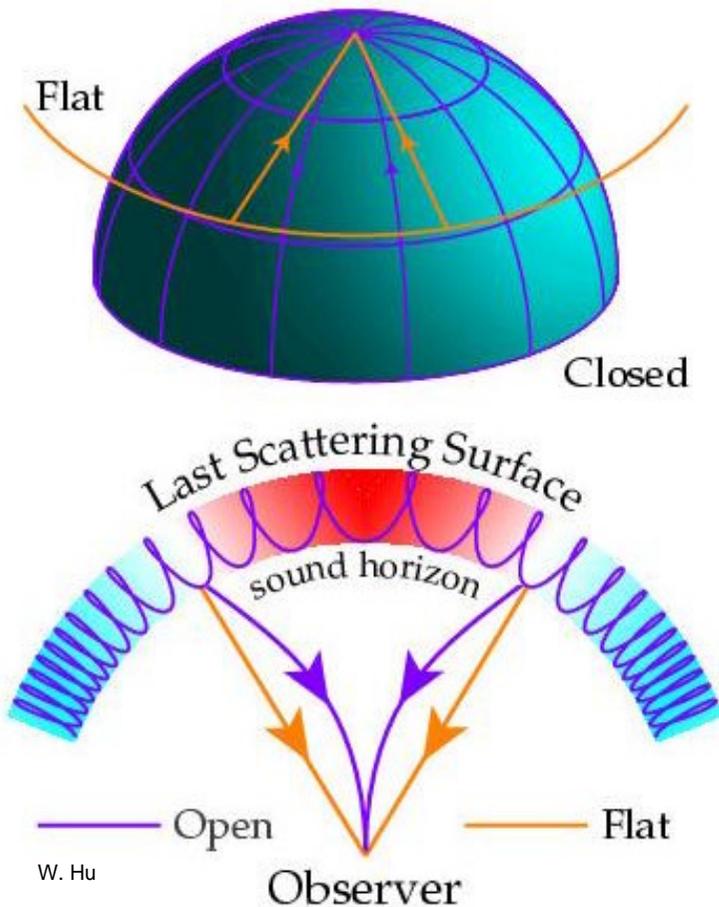


## "Physical Object":

- Sound waves in primordial matter-radiation plasma: wavelength  $\lambda_s$
- observable at surface of epoch recombination, at which photons were last scattered

# Music of the Spheres

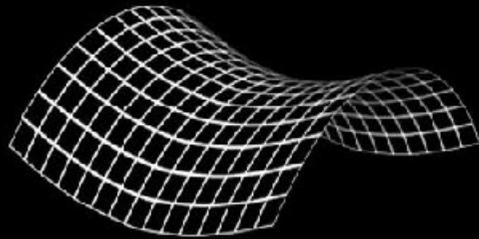
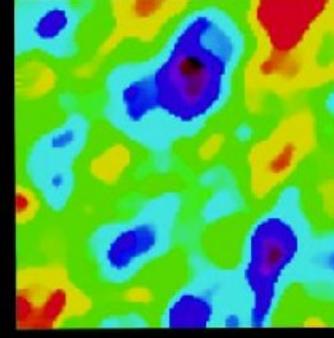
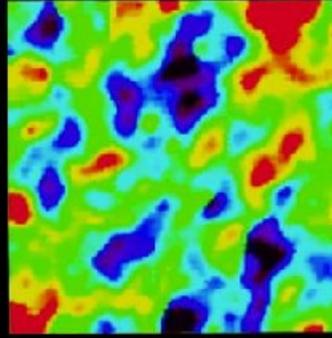
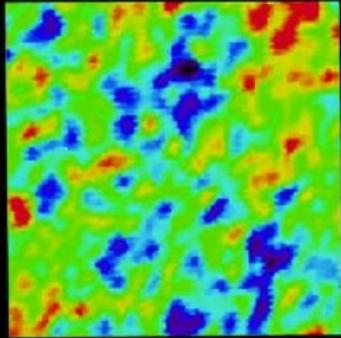
## Primordial Soundwaves



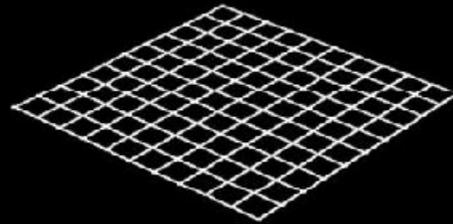
- small ripples in primordial matter & photon distribution
- gravity tries to compress primordial photon gas, photon pressure resists:
- compressions and rarefactions in photon gas:
  - → → sound waves
- sound waves not heard, but seen:
  - compressions: (photon) temperature higher
  - rarefactions: lower
- fundamental mode sound spectrum
  - size of "Instrument":
  - (sound) horizon size Universe last scattering
- Observed, angular size:  $\theta \sim 1^\circ$ 
  - exact scale maximum compression, the "cosmic fundamental mode of music"

→ → → Curvature Universe ← ← ←

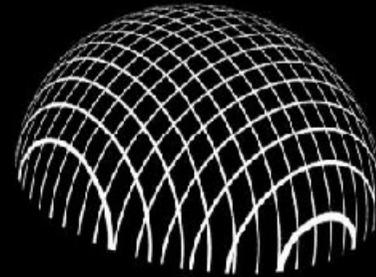
# *Music of the Spheres*



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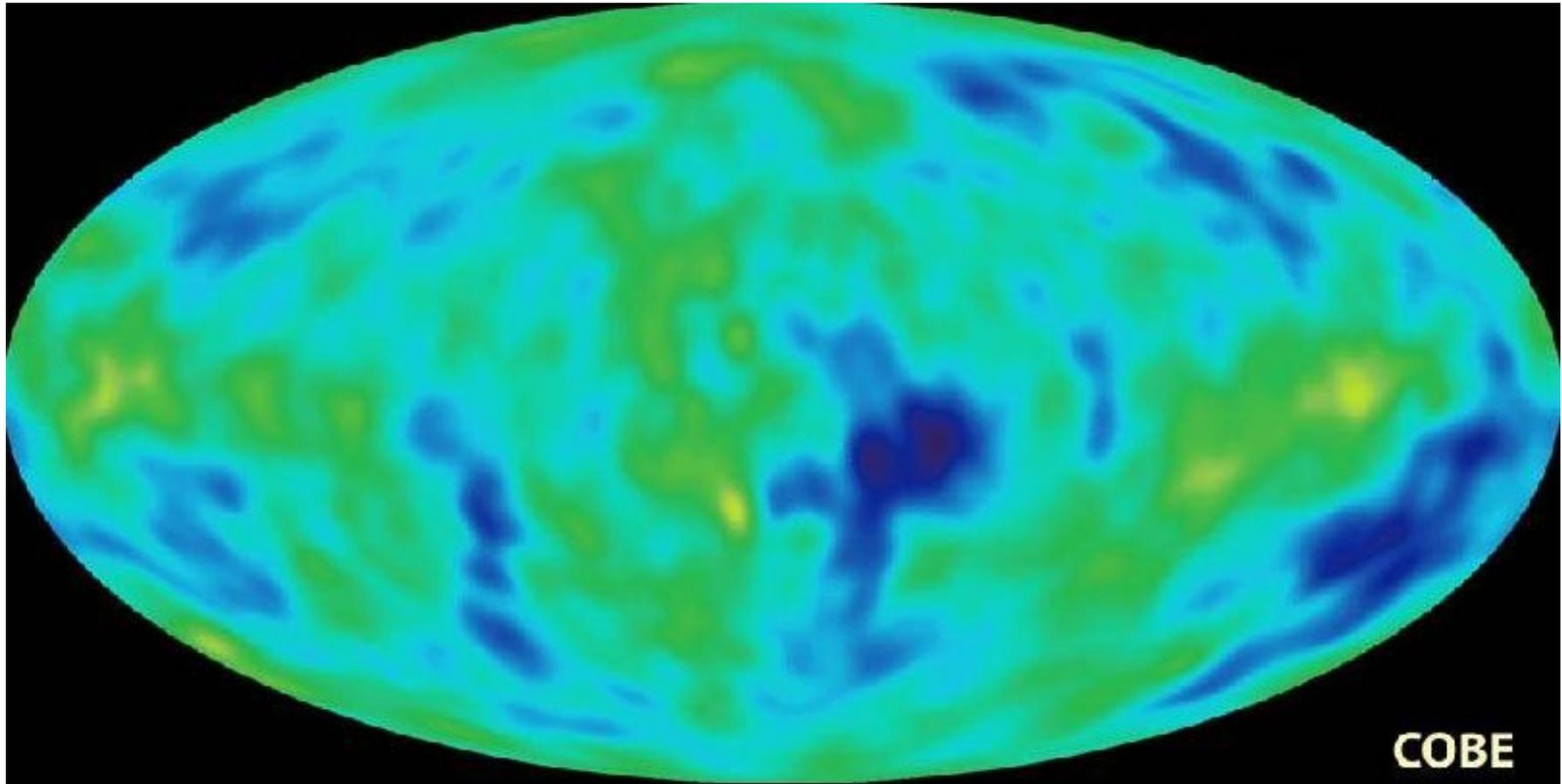


**FLAT**



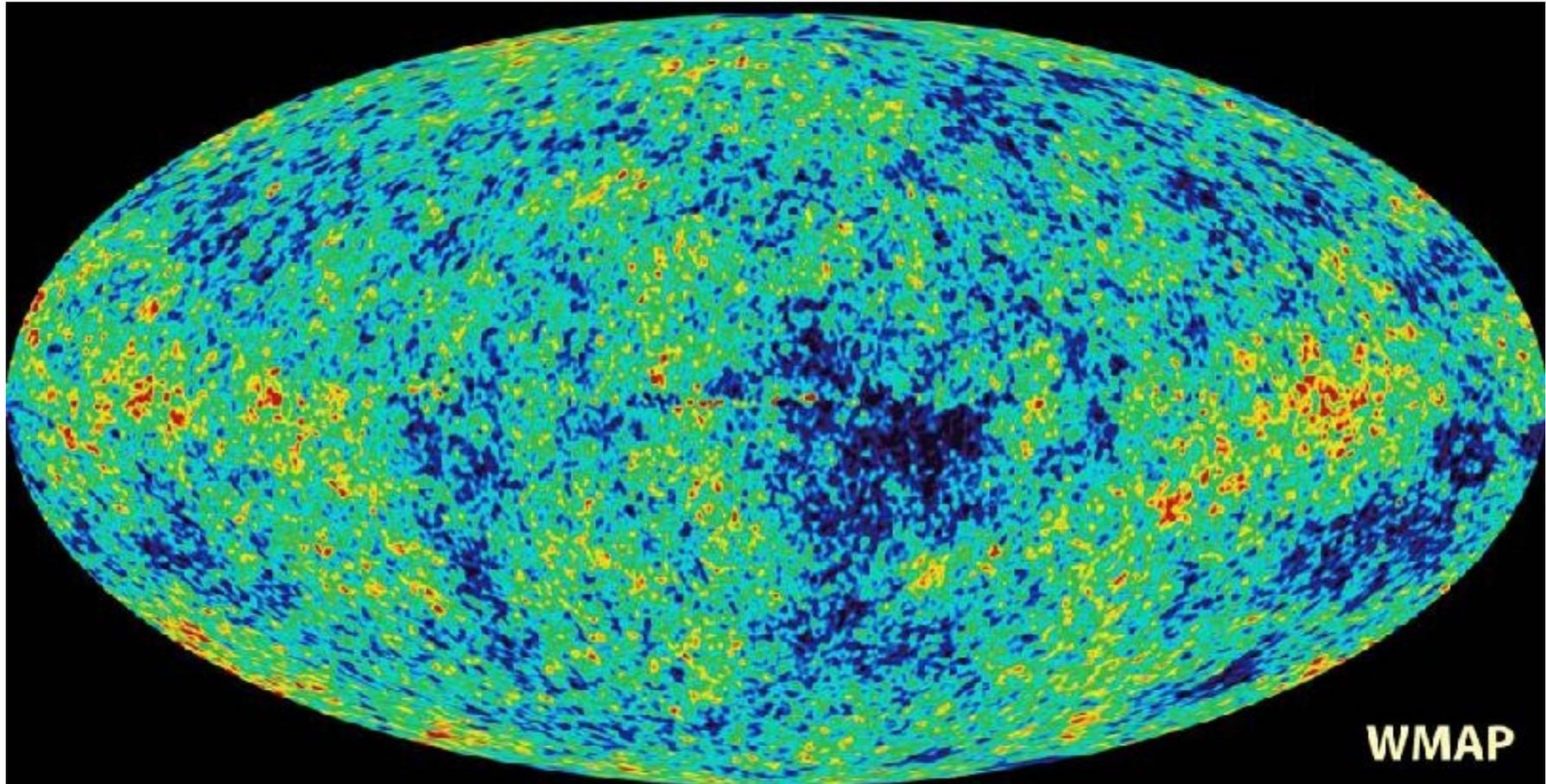
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# *Music of the Spheres*



The Cosmic Microwave Background Temperature Anisotropies:  
the Embryonic Universe

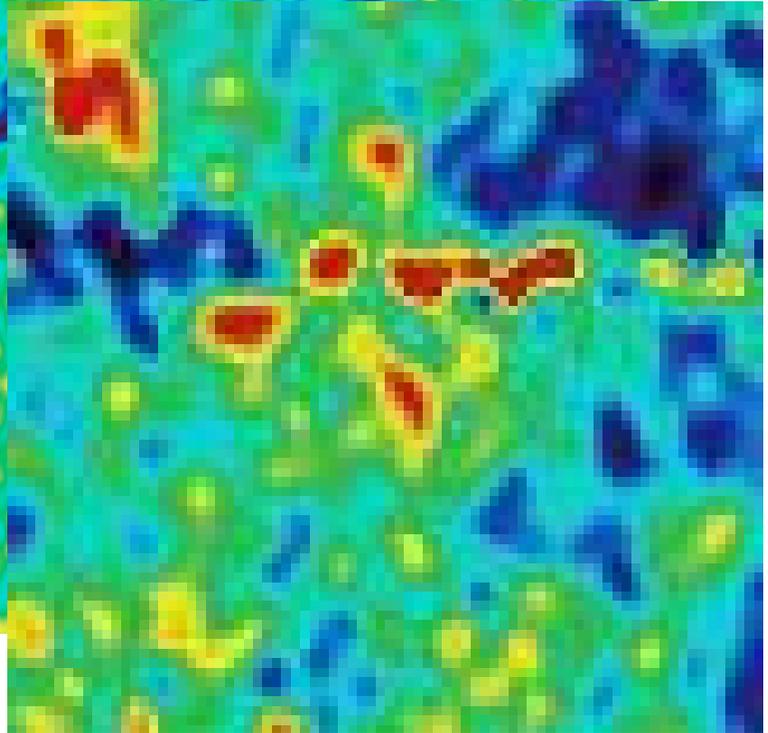
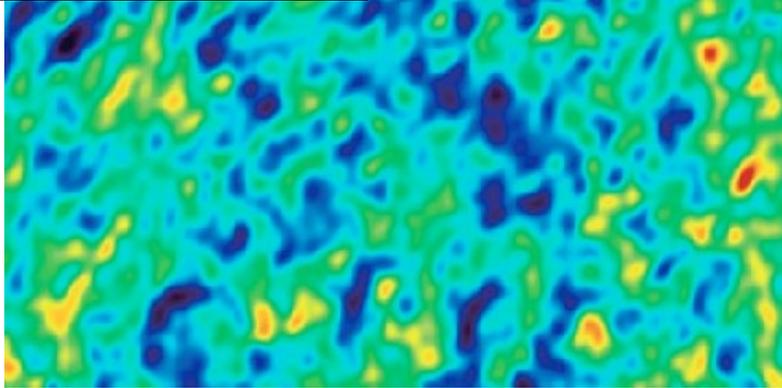
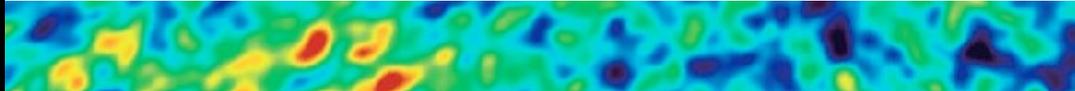
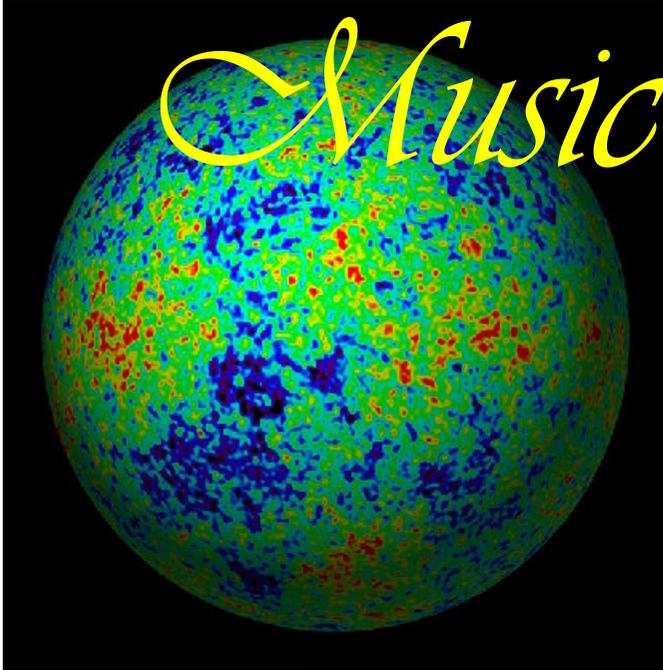
# *Music of the Spheres*



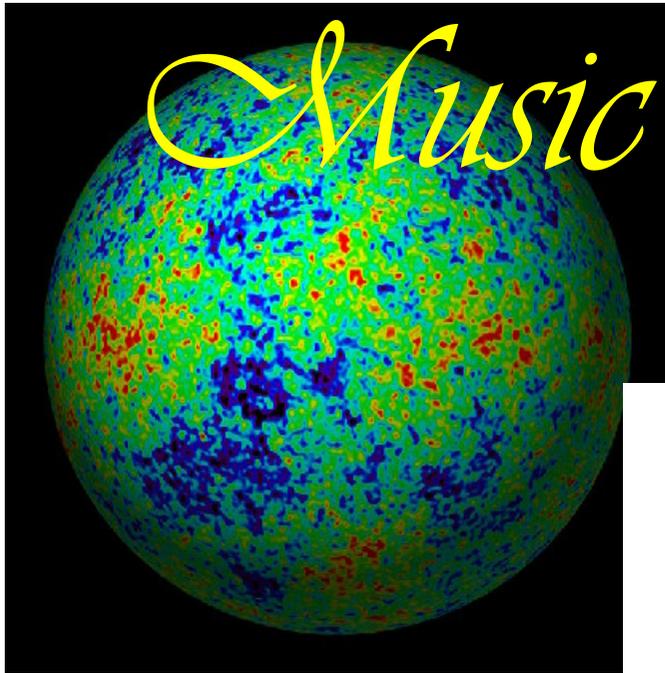
The Cosmic Microwave Background Temperature Anisotropies:  
the Embryonic Universe

*Music*

*of the Spheres*



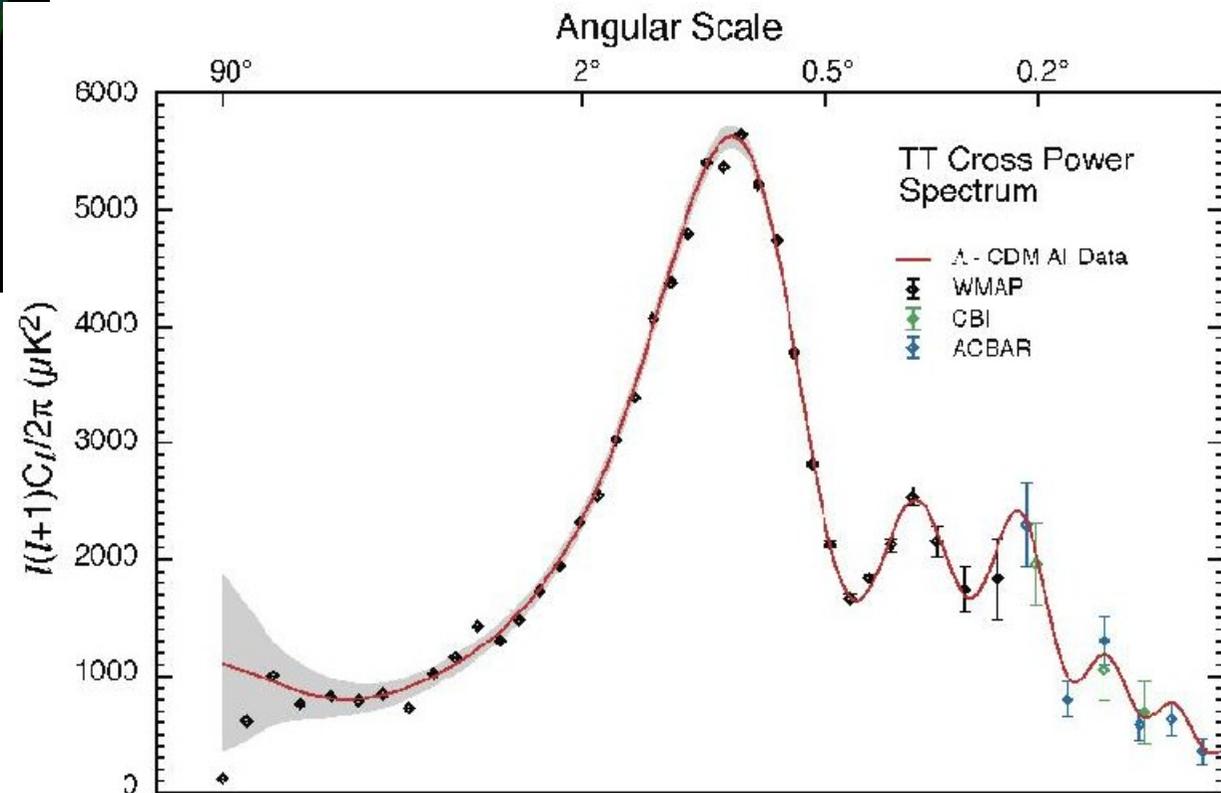
The Cosmic Microwave Background Temperature Anisotropies:  
the Embryonic Universe



The WMAP CMB temperature power spectrum

# Music of the Spheres

## The Cosmic Tonal Ladder



The Cosmic Microwave Background Temperature Anisotropies:  
the Embryonic Universe

# *Music of the Spheres*

*and, indeed ...*

*the Universe appears to be flat,*

*perfectly so ...*

*Euclid was right after all ...*

## *Euclid's Universe*

# *Music of the Spheres*



*Euclid's Universe*

# Music of the Spheres

Old Universe - New Numbers

$$\Omega_{\text{tot}} = 1.02^{+0.02}_{-0.02}$$

$$w < -0.78 \text{ (95\% CL)}$$

$$\Omega_{\Lambda} = 0.73^{+0.04}_{-0.04}$$

$$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$$

$$\Omega_b = 0.044^{+0.004}_{-0.004}$$

$$n_b = 2.5 \times 10^{10} \text{ cm}^{-3}$$

$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_m = 0.27^{+0.04}_{-0.04}$$

$$\Omega_v h^2 < 0.0076 \text{ (95\% CL)}$$

$$m_\nu < 0.23 \text{ eV (95\% CL)}$$

$$T_{\text{cmb}} = 2.725^{+0.002}_{-0.002} \text{ K}$$

$$n_\gamma = 410.4^{+0.9}_{-0.9} \text{ cm}^{-3}$$

$$\eta = 6.1 \times 10^{-10} \text{ }^{+0.3 \times 10^{-10}}_{-0.2 \times 10^{-10}}$$

$$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$$

$$\sigma_8 = 0.84^{+0.04}_{-0.04} \text{ Mpc}$$

$$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$$

$$A = 0.833^{+0.086}_{-0.083}$$

$$n_s = 0.93^{+0.03}_{-0.03}$$

$$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$$

$$r < 0.71 \text{ (95\% CL)}$$

$$z_{\text{dec}} = 1089^{+1}_{-1}$$

$$\Delta z_{\text{dec}} = 195^{+2}_{-2}$$

$$t_0 = 13.7^{+0.2}_{-0.2} \text{ Gyr}$$

$$t_{\text{dec}} = 379^{+8}_{-7} \text{ kyr}$$

$$t_r = 180^{+20}_{-80} \text{ Myr (95\% CL)}$$

$$\Delta t_{\text{dec}} = 118^{+3}_{-2} \text{ kyr}$$

$$z_{\text{eq}} = 3233^{+194}_{-210}$$

$$\tau = 0.17^{+0.04}_{-0.04}$$

$$z_r = 20^{+10}_{-9} \text{ (95\% CL)}$$

$$\theta = 0.598^{+0.002}_{-0.002}$$

$$d_A = 14.0^{+0.2}_{-0.3} \text{ Gpc}$$

$$l_A = 301^{+1}_{-1}$$

$$r_s = 147^{+2}_{-2} \text{ Mpc}$$

and at high precision,

we have its numbers...

# The Apeiron

With the measured flatness of the Universe,  
an intriguing issue surfaces:

Curvature  $\longleftrightarrow$  Matter-Energy

$$k c^2 = H_0^2 R_0^2 (\Omega_{tot} - 1) \approx 0$$

$\Rightarrow$

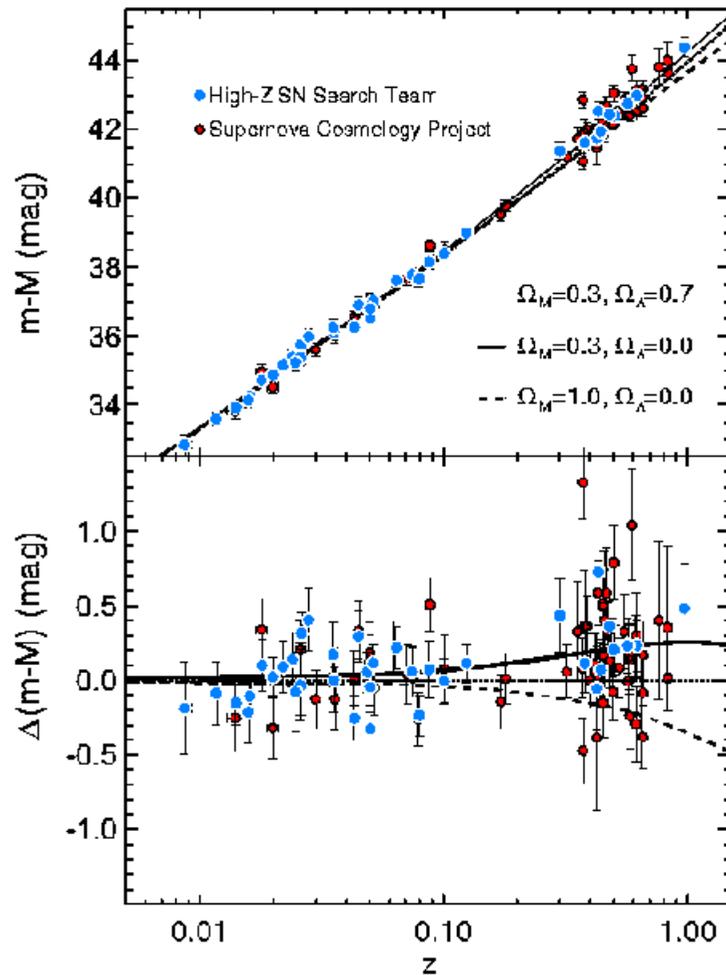
$$\Omega_{tot} = \Omega_{matter} + \Omega_{rad} + \Omega_{\Lambda} \approx 1$$

$$\Omega_{rad} \approx 10^{-5}$$

$$\Omega_{matter} \approx 0.3$$

$$\Rightarrow \Omega_{\Lambda} \approx 0.7 \quad \leftarrow$$

# The Apeiron



Measuring the brightness of supernova explosions, and their apparent dimming due to distance:

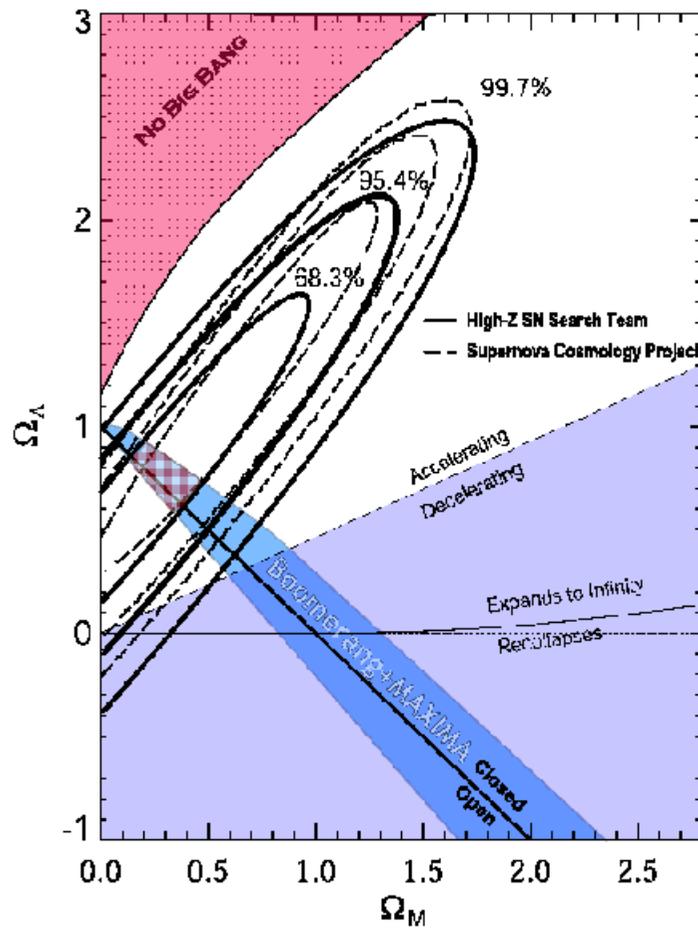
distance dependent on cosmology

$$\begin{aligned}\Omega_{rad} &\approx 10^{-5} \\ \Omega_{matter} &\approx 0.3 \\ \Rightarrow \Omega_{\Lambda} &\approx 0.7 \quad \Leftarrow\end{aligned}$$

# The Apeiron

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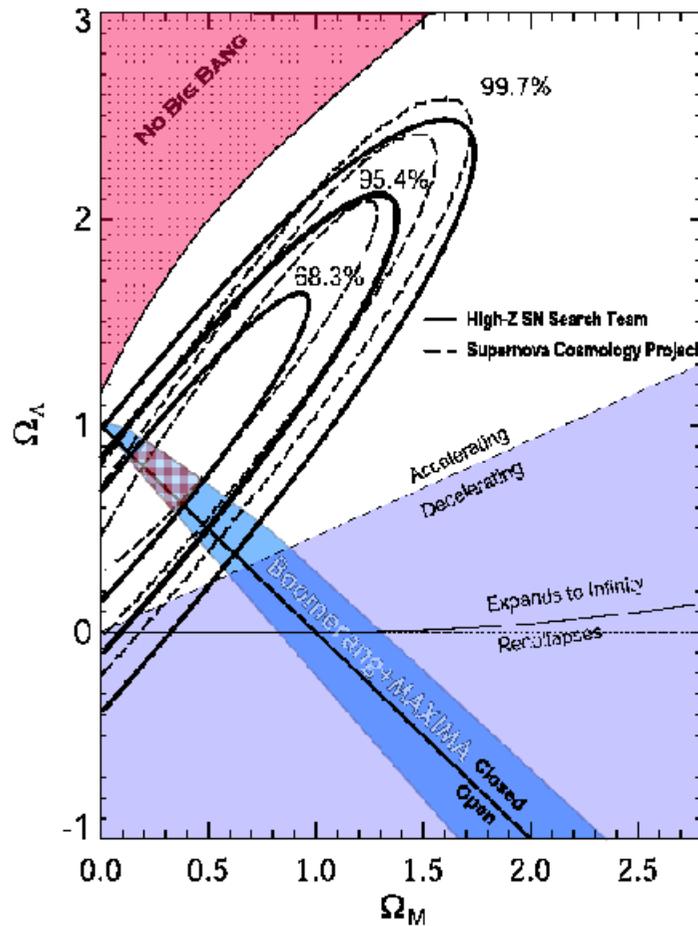


$$\Omega_{rad} \approx 10^{-5}$$

$$\Omega_{matter} \approx 0.3$$

$$\Rightarrow \Omega_\Lambda \approx 0.7 \quad \Leftarrow$$

# The Apeiron

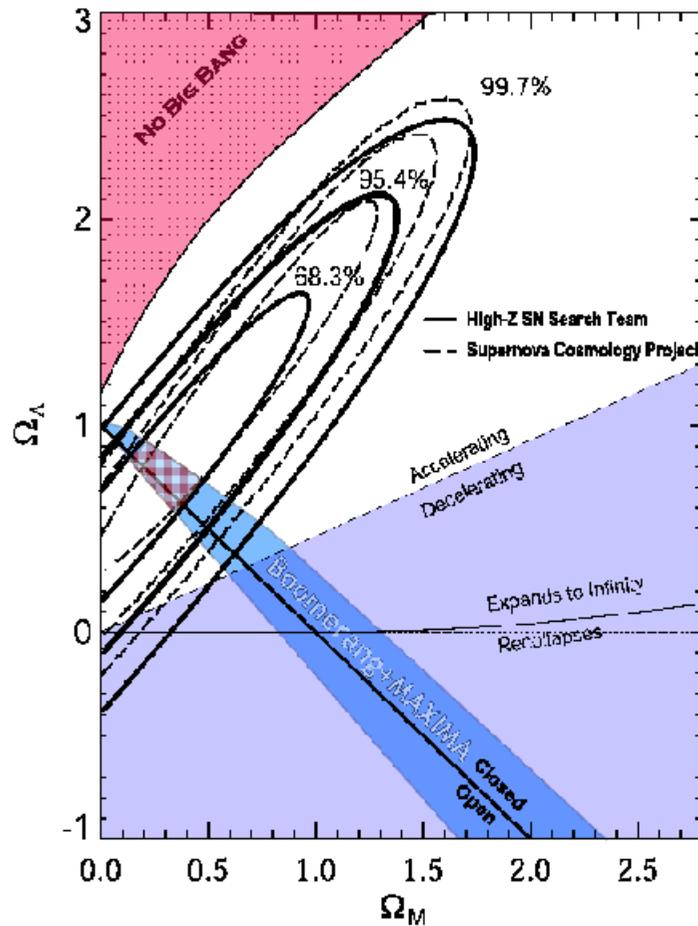


## Conclusion, seemingly inescapable:

- Expansion Universe accelerating !!!  
→ Cosmos forever
- Dynamics Universe dominated by:  
elusive vacuum energy  $\rho_v$  :  
= Cosmological Constant  $\Lambda$   
= Dark Energy
- Totally unclear what it is ...

$$\begin{aligned} \Omega_{rad} &\approx 10^{-5} \\ \Omega_{matter} &\approx 0.3 \\ \Rightarrow \Omega_\Lambda &\approx 0.7 \quad \Leftarrow \end{aligned}$$

# The Apeiron



Is this Anaximander ?

The Apeiron ...  
from which the elements  
[are formed]  
is something that is different

$$\begin{aligned} \Omega_{rad} &\approx 10^{-5} \\ \Omega_{matter} &\approx 0.3 \\ \Rightarrow \Omega_\Lambda &\approx 0.7 \quad \Leftarrow \end{aligned}$$

# Anaximander of Miletus



Anaximander

Miletus, 610 BC-546/545 BC

Founder of astronomy and cosmology as science

- prized symmetry  
introduced geometry and mathematical proportions  
to map and understand heavens
- Origin of the World: the Apeiron
  - nonperceptible substance preceding  
"separation" into contrasting qualities,  
such as hot, cold, wet, dry ...
  - primitive unity all phenomena

"The Apeiron, from which the elements  
[are formed], is something that is different"

# *Famous last words: finite or infinite ?*

Most cosmologists think that the Universe is infinite in all directions ...

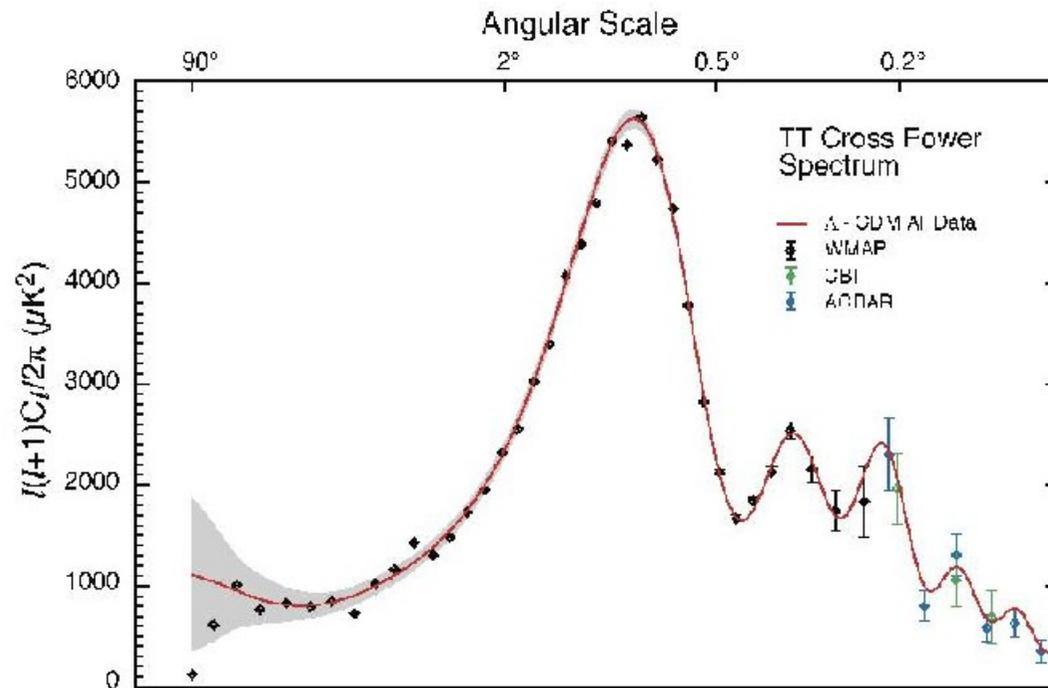
... yet, recent work by Weeks, Luminet et al. (2003), suggests

- it may be finite
- and 12-sides, a dodecahedron

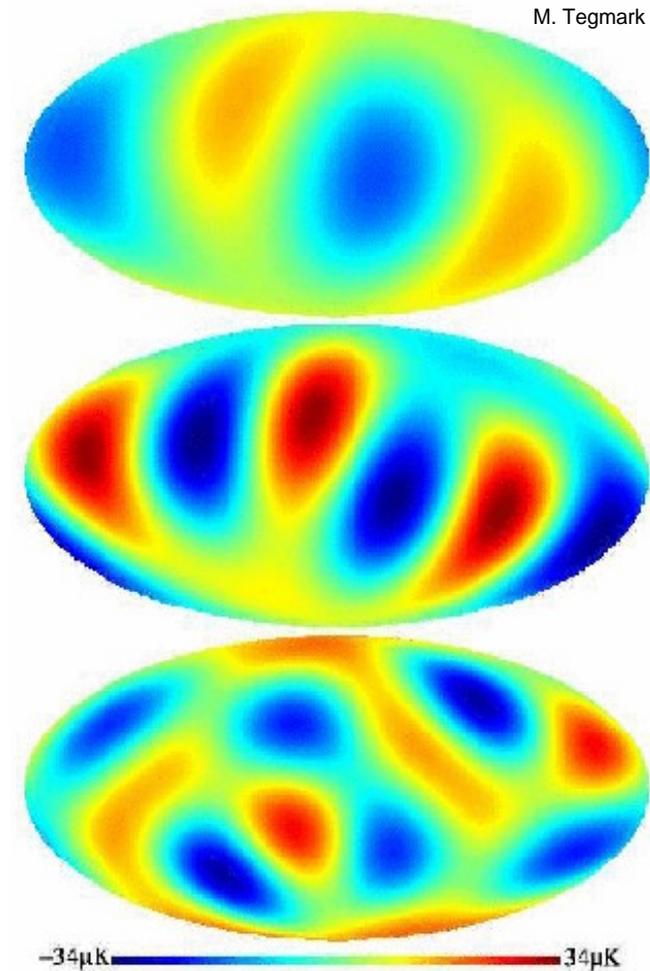
Three major questions for any cosmological model:

- What is its curvature ? (positive, flat, negative)
- Is it finite or infinite ? (open: infinite amount of matter  
closed: finite amount of matter)
- What is its topology ? (multiply connected ?)

# The Cosmos: a Dodecahedron?



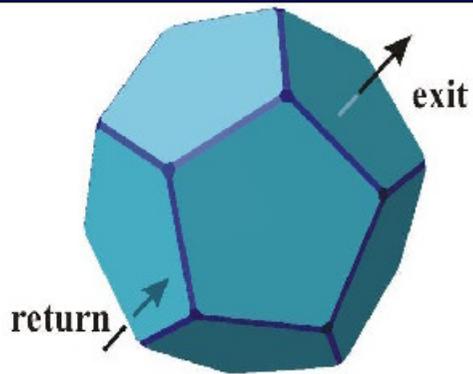
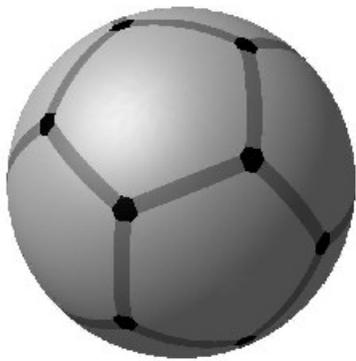
Major problem:  
large-scale CMB quadrupole anisotropy  
has very low amplitude ... too low ...



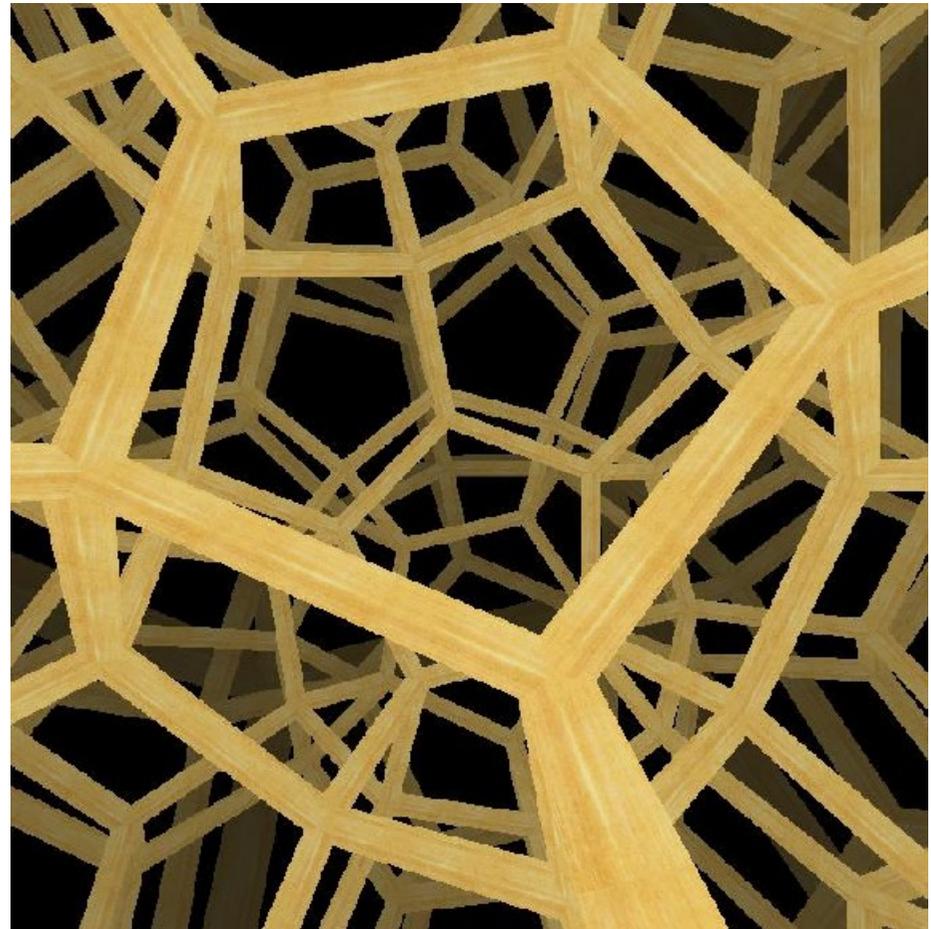
# The Cosmos: a Dodecahedron?

Suggestion Luminet, Weeks et al.:

- Universe finite in extent ...
- and shape of dodecahedron ...



The cosmic soundwaves  
would resonate in cosmic  
cavity at lower amplitude



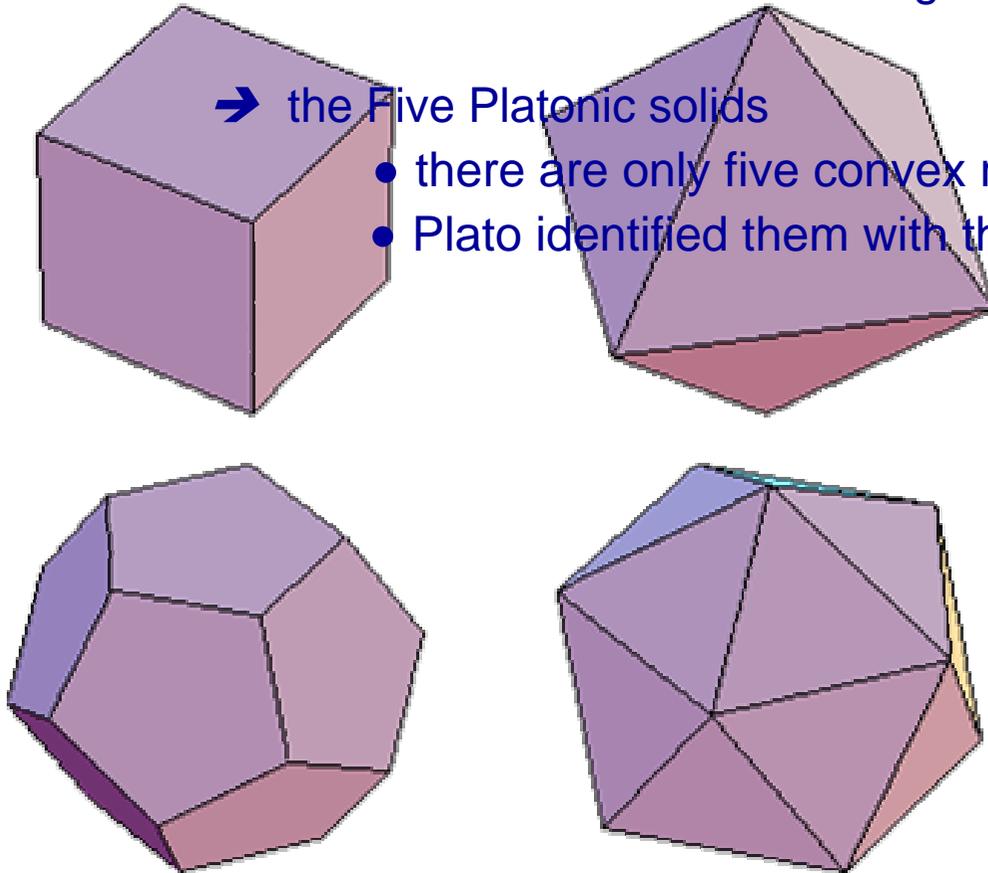
# The Platonic Solids

## Plato's Cosmic Scheme (Timaeus):

- Demiurge, divine craftsman, is a mathematician:
- Universe constructed according to geometric principles

→ the Five Platonic solids

- there are only five convex regular polyhedra !
- Plato identified them with the cosmos and its constituents



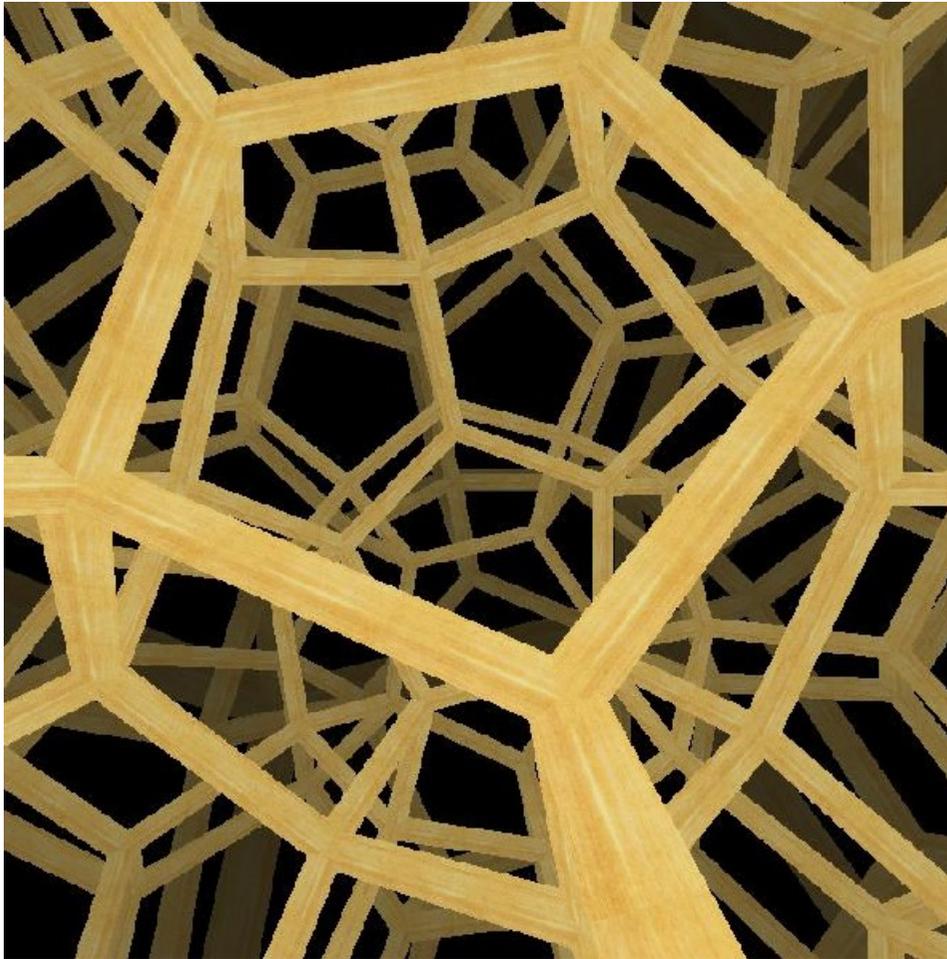
### • Four basic constituents of nature:

- fire                      Pyramid
- air                         Octahedron
- earth                      Cube
- water                      Icosahedron

### • The Cosmos itself:

- the stuff used for `embroidering the constellations on the heavens'  
Dodecahedron

# *The Cosmos: a Dodecahedron ?*



*Plato:*

*said so all along in Timaeus*

*"Let no one unversed in geometry  
enter here"*



**Academia of Plato, Athens**

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