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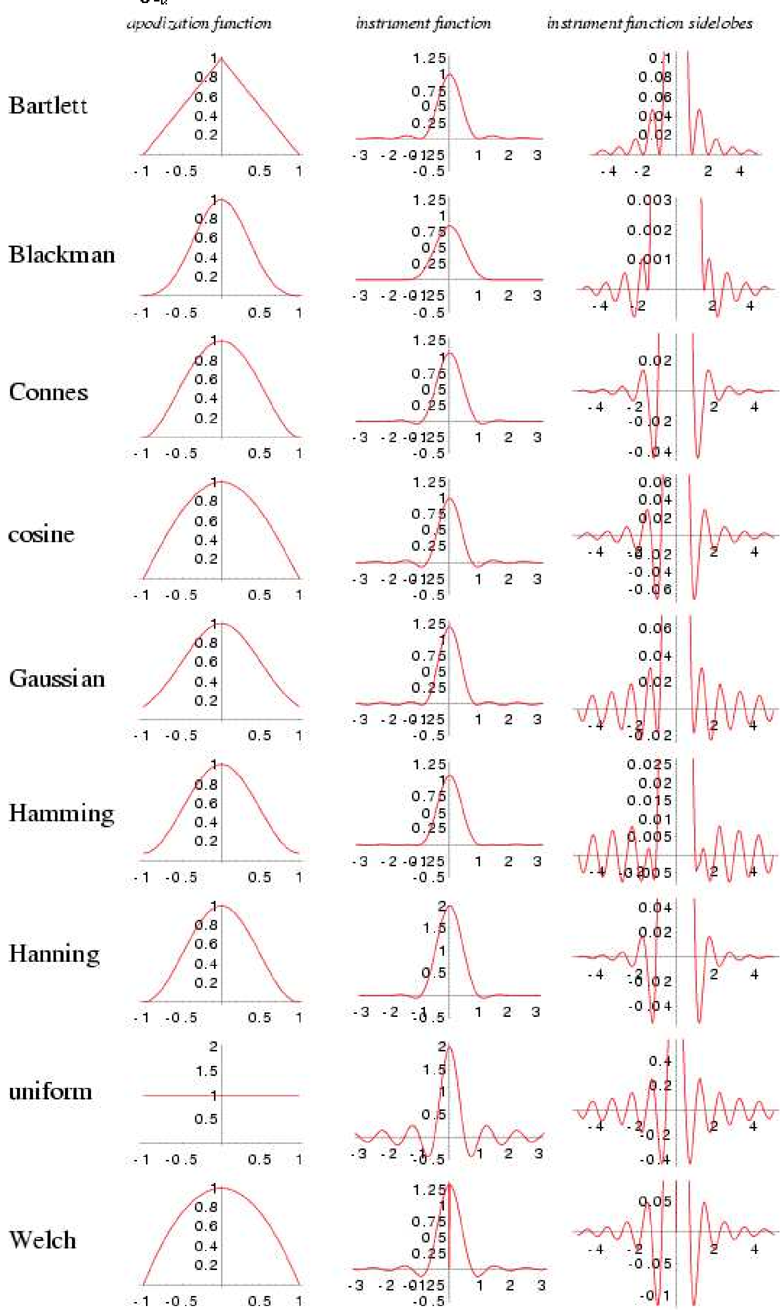
Apodization Function

A function (also called a tapering function) used to bring an interferogram smoothly down to zero at the edges of the sampled region. This suppresses sidelobes which would otherwise be produced, but at the expense of widening the lines and therefore decreasing the resolution.

The following are apodization functions for symmetrical (2-sided) interferograms, together with the instrument functions (or apparatus functions) they produce and a blowup of the instrument function sidelobes. The instrument function $I(k)$ corresponding to a given apodization function $A(x)$ can be computed by taking the finite Fourier cosine transform,

$$I(k) = \int_{-a}^{a} \cos(2\pi k x) A(x)\, d x. \qquad (1)$$



type	apodization function	instrument function
Bartlett	$1-\frac{\lvert x\rvert}{a}$	$a \operatorname{sinc}^2(\pi k a)$
Blackman	$B_A(x)$	$B_I(k)$
Connes	$\left(1-\frac{x^2}{a^2}\right)^2$	$8 a \sqrt{2\pi}\,\frac{J_{5/2}(2\pi k a)}{(2\pi k a)^{5/2}}$
cosine	$\cos\left(\frac{\pi x}{2a}\right)$	$\frac{4 a \cos(2\pi a k)}{\pi(1-16 a^2 k^2)}$
Gaussian	$e^{-x^2/(2\sigma^2)}$	$2\int_0^a \cos(2\pi k x) e^{-x^2/(2\sigma^2)}\,dx$
Hamming	$\mathrm{Hm}_A(x)$	$\mathrm{Hm}_I(k)$
Hanning	$\mathrm{Hn}_A(x)$	$\mathrm{Hn}_I(k)$
uniform	1	$2 a \operatorname{sinc}(2\pi k a)$
Welch	$1-\frac{x^2}{a^2}$	$W_I(k)$

where

$$B_A(x) = \frac{21}{50}+\frac{1}{2}\cos\left(\frac{\pi x}{a}\right)+\frac{2}{25}\cos\left(\frac{2\pi x}{a}\right) \quad (2)$$

$$B_I(k) = \frac{a\left(\frac{21}{25}-\frac{9}{25}a^2 k^2\right)\operatorname{sinc}(2\pi a k)}{(1-a^2k^2)(1-4a^2k^2)} \quad (3)$$

(4)

$$\mathrm{Hm}_A(x) = \frac{27}{50}+\frac{23}{50}\cos\left(\frac{\pi x}{a}\right) \quad (5)$$

$$\mathrm{Hm}_I(k) = \frac{a\left(\frac{27}{25}-\frac{16}{25}a^2k^2\right)\operatorname{sinc}(2\pi a k)}{1-4a^2k^2} \quad (6)$$

$$\mathrm{Hn}_A(x) = \cos^2\left(\frac{\pi x}{2a}\right) \quad (7)$$

$$= \frac{1}{2}\left[1+\cos\left(\frac{\pi x}{a}\right)\right] \quad (8)$$

$$\mathrm{Hn}_I(k) = \frac{a \operatorname{sinc}(2\pi a k)}{1-4a^2k^2} \quad (9)$$

$$= a\left[\operatorname{sinc}(2\pi k a)+\frac{1}{2}\operatorname{sinc}(2\pi k a-\pi)+\frac{1}{2}\operatorname{sinc}(2\pi k a+\pi)\right] \quad (10)$$

$$W_I(k) = a\,2\sqrt{2\pi}\,\frac{J_{3/2}(2\pi k a)}{(2\pi k a)^{3/2}} \quad (11)$$

$$= a\,\frac{\sin(2\pi k a)-2\pi a k\cos(2\pi a k)}{2a^3k^3\pi^3}. \quad (12)$$

The following table summarizes the widths, peaks, and peak-sidelobe-to-peak (negative and positive) for common apodization functions.

type	instrument function FWHM	IF peak	$\frac{\text{peak }(-)\text{ sidelobe}}{\text{peak}}$	$\frac{\text{peak }(+)\text{ sidelobe}}{\text{peak}}$
Bartlett	1.77179	1	0.00000000	0.0471904
Blackman	2.29880	$\frac{21}{25}$	−0.00106724	0.00124325
Connes	1.90416	$\frac{16}{15}$	−0.0411049	0.0128926
cosine	1.63941	$\frac{4}{\pi}$	−0.0708048	0.0292720
Gaussian	--	1	--	--
Hamming	1.81522	$\frac{27}{25}$	−0.00689132	0.00734934
Hanning	2.00000	1	−0.0267076	0.00843441
uniform	1.20671	2	−0.217234	0.128375
Welch	1.59044	$\frac{4}{3}$	−0.0861713	0.356044

A general symmetric apodization function $A(x)$ can be written as a Fourier series

$$A(x) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{b}\right), \quad (13)$$

where the coefficients satisfy

$$a_0 + 2\sum_{n=1}^{\infty} a_n = 1. \quad (14)$$

The corresponding instrument function is

$$I(t) \equiv \int_{-b}^{b} A(x)\, e^{-2\pi i k x}\, dx \quad (15)$$

$$= 2b\left\{a_0 \operatorname{sinc}(2\pi k b) + \sum_{n=1}^{\infty}\left[\operatorname{sinc}(2\pi k b + n\pi) + \operatorname{sinc}(2\pi k b - n\pi)\right]\right\}. \quad (16)$$

To obtain an apodization function with zero at $k a = 3/4$, use

$$a_0 \operatorname{sinc}\left(\tfrac{3}{2}\pi\right) + a_1\left[\operatorname{sinc}\left(\tfrac{5}{2}\pi\right) + \operatorname{sinc}\left(\tfrac{1}{2}\pi\right)\right] = 0. \quad (17)$$

Plugging in (15),

$$-(1-2a_1)\frac{2}{3\pi} + a_1\left(\frac{2}{5\pi} + \frac{2}{\pi}\right) = -\tfrac{1}{3}(1-2a_1) + a_1\left(\tfrac{1}{5}+1\right) = 0 \quad (18)$$

$$a_1\left(\tfrac{6}{5} + \tfrac{2}{3}\right) = \tfrac{1}{3} \quad (19)$$

$$a_1 = \frac{\frac{1}{3}}{\frac{6}{5}+\frac{2}{3}} = \frac{5}{6\cdot 3 + 2\cdot 5} = \tfrac{5}{28} \quad (20)$$

$$a_0 = 1 - 2a_1 = \frac{28 - 2\cdot 5}{28} = \tfrac{18}{28} = \tfrac{9}{14}. \quad (21)$$

The Hamming function is close to the requirement that the instrument function goes to 0 at $k a = 5/4$, giving

$$a_0 = \tfrac{25}{46} \approx 0.5435 \quad (22)$$

$$a_1 = \tfrac{21}{92} \approx 0.2283. \quad (23)$$

The Blackman function is chosen so that the instrument function goes to 0 at $k a = 5/4$ and $k a = 9/4$, giving

$$a_0 = \frac{3969}{9304} \approx 0.42659 \quad (24)$$

$$a_1 = \frac{1155}{4652} \approx 0.24828 \quad (25)$$

$$a_2 = \frac{715}{18608} \approx 0.38424, \quad (26)$$

SEE ALSO: Bartlett Function, Blackman Function, Connes Function, Cosine Apodization Function, Full Width at Half Maximum, Gaussian Function, Hamming Function, Hanning Function, Mertz Apodization Function, Parzen Apodization Function, Uniform Apodization Function, Welch Apodization Function.
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