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Calculus and Analysis > Integral Transforms > Convolution

Cross-Correlation Theorem



Let $f \star g$ denote the [cross-correlation](#) of functions $f(t)$ and $g(t)$. Then

$$f \star g = \int_{-\infty}^{\infty} \bar{f}(\tau) g(t + \tau) d\tau \tag{1}$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \bar{F}(\nu) e^{2\pi i \nu \tau} d\nu \int_{-\infty}^{\infty} G(\nu') e^{-2\pi i \nu' (t+\tau)} d\nu' \right] d\tau \tag{2}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}(\nu) G(\nu') e^{-2\pi i \tau (\nu' - \nu)} e^{-2\pi i \nu' t} d\tau d\nu d\nu' \tag{3}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}(\nu) G(\nu') e^{-2\pi i \nu' t} \left[\int_{-\infty}^{\infty} e^{-2\pi i \tau (\nu' - \nu)} d\tau \right] d\nu d\nu' \tag{4}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{F}(\nu) G(\nu') e^{-2\pi i \nu' t} \delta(\nu' - \nu) d\nu' d\nu \tag{5}$$

$$= \int_{-\infty}^{\infty} \bar{F}(\nu) G(\nu) e^{-2\pi i \nu t} d\nu \tag{6}$$

$$= \mathcal{F}[\bar{F}(\nu) G(\nu)], \tag{7}$$

where \mathcal{F} denotes the [Fourier transform](#), \bar{z} is the [complex conjugate](#), and

$$f(t) \equiv \mathcal{F}_\nu[F(\nu)](t) = \int_{-\infty}^{\infty} F(\nu) e^{-2\pi i \nu t} d\nu \tag{8}$$

$$g(t) \equiv \mathcal{F}_\nu[G(\nu)](t) = \int_{-\infty}^{\infty} G(\nu) e^{-2\pi i \nu t} d\nu. \tag{9}$$

Applying a [Fourier transform](#) on each side gives the cross-correlation theorem,

$$f \star g = \mathcal{F}[\bar{F}(\nu) G(\nu)]. \tag{10}$$

If $F = G$, then the cross-correlation theorem reduces to the [Wiener-Khinchin theorem](#).

SEE ALSO: [Fourier Transform](#), [Wiener-Khinchin Theorem](#).
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