Cross-Correlation Theorem

Let $f \star g$ denote the cross-correlation of functions $f(t)$ and $g(t)$. Then

$$\begin{aligned}
f \star g &= \int_{-\infty}^{\infty} \bar{f}(\tau)\, g(t+\tau)\, d\tau && (1)\\
&= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} \bar{F}(\nu)\, e^{2\pi i \nu\tau}\, d\nu \int_{-\infty}^{\infty} G(\nu')\, e^{-2\pi i \nu'(t+\tau)}\, d\nu'\right] d\tau && (2)\\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{F}(\nu)\, G(\nu')\, e^{-2\pi i \tau(\nu'-\nu)}\, e^{-2\pi i \nu' t}\, d\tau\, d\nu\, d\nu' && (3)\\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{F}(\nu)\, G(\nu')\, e^{-2\pi i \nu' t}\left[\int_{-\infty}^{\infty} e^{-2\pi i \tau(\nu'-\nu)}\, d\tau\right] d\nu\, d\nu' && (4)\\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{F}(\nu)\, G(\nu')\, e^{-2\pi i \nu' t}\, \delta(\nu'-\nu)\, d\nu'\, d\nu && (5)\\
&= \int_{-\infty}^{\infty} \bar{F}(\nu)\, G(\nu)\, e^{-2\pi i \nu t}\, d\nu && (6)\\
&= \mathcal{F}\left[\bar{F}(\nu)\, G(\nu)\right], && (7)
\end{aligned}$$

where $\mathcal{F}$ denotes the Fourier transform, $\bar{z}$ is the complex conjugate, and

$$\begin{aligned}
f(t) &\equiv \mathcal{F}_\nu[F(\nu)](t) = \int_{-\infty}^{\infty} F(\nu)\, e^{-2\pi i \nu t}\, d\nu && (8)\\
g(t) &\equiv \mathcal{F}_\nu[G(\nu)](t) = \int_{-\infty}^{\infty} G(\nu)\, e^{-2\pi i \nu t}\, d\nu. && (9)
\end{aligned}$$

Applying a Fourier transform on each side gives the cross-correlation theorem,

$$f \star g = \mathcal{F}\left[\bar{F}(\nu)\, G(\nu)\right]. \qquad (10)$$

If $F = G$, then the cross-correlation theorem reduces to the Wiener-Khinchin theorem.

SEE ALSO: Fourier Transform, Wiener-Khinchin Theorem.
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