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## Convolution Theorem



Let  $f(t)$  and  $g(t)$  be arbitrary functions of time  $t$  with [Fourier transforms](#). Take

$$f(t) = \mathcal{F}_v^{-1} [F(v)](t) = \int_{-\infty}^{\infty} F(v) e^{2\pi i v t} dv \quad (1)$$

$$g(t) = \mathcal{F}_v^{-1} [G(v)](t) = \int_{-\infty}^{\infty} G(v) e^{2\pi i v t} dv, \quad (2)$$

where  $\mathcal{F}_v^{-1}(t)$  denotes the inverse [Fourier transform](#) (where the transform pair is defined to have constants  $A = 1$  and  $B = -2\pi$ ). Then the [convolution](#) is

$$f * g \equiv \int_{-\infty}^{\infty} g(t') f(t - t') dt' \quad (3)$$

$$= \int_{-\infty}^{\infty} g(t') \left[ \int_{-\infty}^{\infty} F(v) e^{2\pi i v (t - t')} dv \right] dt', \quad (4)$$

Interchange the order of integration,

$$f * g = \int_{-\infty}^{\infty} F(v) \left[ \int_{-\infty}^{\infty} g(t') e^{-2\pi i v t'} dt' \right] e^{2\pi i v t} dv \quad (5)$$

$$= \int_{-\infty}^{\infty} F(v) G(v) e^{2\pi i v t} dv \quad (6)$$

$$= \mathcal{F}_v^{-1} [F(v) G(v)](t). \quad (7)$$

So, applying a [Fourier transform](#) to each side, we have

$$\mathcal{F} [f * g] = \mathcal{F} [f] \mathcal{F} [g]. \quad (8)$$

The convolution theorem also takes the alternate forms

$$\mathcal{F} [f g] = \mathcal{F} [f] * \mathcal{F} [g] \quad (9)$$

$$\mathcal{F}^{-1} (\mathcal{F} [f] \mathcal{F} [g]) = f * g \quad (10)$$

$$\mathcal{F}^{-1} (\mathcal{F} [f] * \mathcal{F} [g]) = f g. \quad (11)$$

**SEE ALSO:** [Autocorrelation](#), [Convolution](#), [Fourier Transform](#), [Wiener-Khinchin Theorem](#). [[Pages Linking Here](#)]

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