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Created, developed, and nurtured by Eric Weisstein at Wolfram Research Calculus and Analysis > Integral Transforms > Convolution

Convolution Theorem



Let f(t) and g(t) be arbitrary functions of time t with Fourier transforms. Take

$$f(t) = \mathcal{F}_{v}^{-1} [F(v)](t) = \int_{-\infty}^{\infty} F(v) e^{2\pi i v \tau} dv$$
 (1)

$$g(t) = \mathcal{F}_{v}^{-1} \left[G(v) \right](t) = \int_{-\infty}^{\infty} G(v) e^{2\pi i v t} dv, \tag{2}$$

where $\mathcal{F}_{r}^{-1}(t)$ denotes the inverse Fourier transform (where the transform pair is defined to have constants A=1 and $B=-2\pi$). Then the convolution is

$$f * g \equiv \int_{-\infty}^{\infty} g(t') f(t - t') dt'$$

$$= \int_{-\infty}^{\infty} g(t') \left[\int_{-\infty}^{\infty} F(v) e^{2\pi i v (t - t')} dv \right] dt'.$$
(4)

Interchange the order of integration,

$$f * g = \int_{-\infty}^{\infty} F(v) \left[\int_{-\infty}^{\infty} g(t') e^{-2\pi i v t'} dt' \right] e^{2\pi i v t} dv$$
 (5)

$$=\int_{-\infty}^{\infty} F(v) G(v) e^{2\pi i v \tau} dv$$
 (6)

$$=\mathcal{F}_{v}^{-1}\left[F\left(v\right)G\left(v\right)\right]\left(t\right).\tag{7}$$

So, applying a Fourier transform to each side, we have

$$\mathcal{F}[f * g] = \mathcal{F}[f]\mathcal{F}[g]. \tag{8}$$

The convolution theorem also takes the alternate forms

$$\mathcal{F}[fg] = \mathcal{F}[f] * \mathcal{F}[g] \tag{9}$$

$$\mathcal{F}^{-1}\left(\mathcal{F}\left[f\right]\mathcal{F}\left[g\right]\right) = f * g \tag{10}$$

$$\mathcal{F}^{-1}\left(\mathcal{F}\left[f\right]*\mathcal{F}\left[g\right]\right) = fg. \tag{11}$$

SEE ALSO: Autocorrelation, Convolution, Fourier Transform, Wiener-Khinchin Theorem. [Pages Linking Here]

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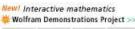
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