

- Algebra
- Applied Mathematics
- Calculus and Analysis
- Discrete Mathematics
- Foundations of Mathematics
- Geometry
- History and Terminology
- Number Theory
- Probability and Statistics
- Recreational Mathematics
- Topology

- Alphabetical Index
- Interactive Entries
- Random Entry
- New in MathWorld

MathWorld Classroom

- About MathWorld
- Send a Message to the Team

Order book from Amazon

Last updated:
12,751 entries
Sun Jan 20 2008

Created, developed, and
nurtured by Eric Weisstein
at Wolfram Research

Calculus and Analysis > Integral Transforms > Convolution
Recreational Mathematics > Interactive Entries > Animated GIFs

Convolution

- COMMENT On this Page
- EXPLORE THE TOPIC IN The MathWorld Classroom
- DOWNLOAD Mathematica Notebook

A convolution is an integral that expresses the amount of overlap of one function g as it is shifted over another function f . It therefore "blends" one function with another. For example, in synthesis imaging, the measured dirty map is a convolution of the "true" CLEAN map with the dirty beam (the Fourier transform of the sampling distribution). The convolution is sometimes also known by its German name, *faltung* ("folding").

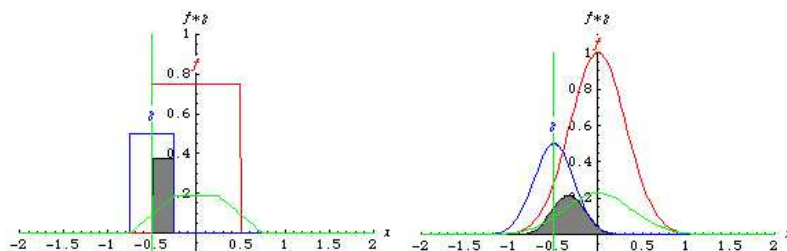
Abstractly, a convolution is defined as a product of functions f and g that are objects in the algebra of Schwartz functions in \mathbb{R}^n . Convolution of two functions f and g over a finite range $[0, t]$ is given by

$$f * g \equiv \int_0^t f(\tau) g(t - \tau) d\tau, \tag{1}$$

where the symbol $f * g$ (occasionally also written as $f \otimes g$) denotes convolution of f and g . Convolution is more often taken over an infinite range,

$$f * g \equiv \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau = \int_{-\infty}^{\infty} g(\tau) f(t - \tau) d\tau \tag{2}$$

(Bracewell 1999, p. 25).



The animations above graphically illustrate the convolution of two rectangle functions (left) and two Gaussians (right). In the plots, the green curve shows the convolution of the blue and red curves as a function of t , the position indicated by the vertical green line. The gray region indicates the product $g(\tau) f(t - \tau)$ as a function of t , so its area as a function of t is precisely the convolution.

The convolution of two rectangle functions $f = \Pi_{t_1, t_2}(t)$ and $g = \Pi_{u_1, u_2}(t)$ has the particularly simple form

$$f * g = [(t - t_1 - u_1) \Pi(t - t_1 - u_1) - (t - t_2 - u_1) \Pi(t - t_2 - u_1) - (t - t_2 - u_1) \Pi(t - t_1 - u_2) + (t - t_2 - u_2) \Pi(t - t_2 - u_2)]. \tag{3}$$

Even more amazingly, the convolution of two Gaussians $f = e^{-t^2 / (2\sigma_1^2)} / (\sigma_1 \sqrt{2\pi})$ and $g = e^{-t^2 / (2\sigma_2^2)} / (\sigma_2 \sqrt{2\pi})$ is another Gaussian

$$f * g = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-t^2 / (2(\sigma_1^2 + \sigma_2^2))}. \tag{4}$$

Let f , g , and h be arbitrary functions and α a constant. Convolution satisfies the properties

$$f * g = g * f \tag{5}$$

$$f * (g * h) = (f * g) * h \tag{6}$$

$$f * (g + h) = (f * g) + (f * h) \tag{7}$$

(Bracewell 1999, p. 27), as well as

$$\alpha(f * g) = (\alpha f) * g = f * (\alpha g). \tag{8}$$

Taking the derivative of a convolution gives

$$\frac{d}{dx} (f * g) = \frac{df}{dx} * g + f * \frac{dg}{dx}. \tag{9}$$

The area under a convolution is the product of areas under the factors,

$$\int_{-\infty}^{\infty} (f * g) dx = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(u) g(x - u) du \right] dx \tag{10}$$

$$= \int_{-\infty}^{\infty} f(u) \left[\int_{-\infty}^{\infty} g(x - u) dx \right] du \tag{11}$$

Other Wolfram Sites:

- Wolfram Research
- Demonstrations Site
- Integrator
- Tones
- Functions Site
- Wolfram Science
- more...

Download Mathematica Player >>

Complete Mathematica Documentation >>

Show off your math savvy with a [MathWorld T-shirt](#).

Introducing a computing revolution... **MATHEMATICA REINVENTED** >>

New! Interactive mathematics
Wolfram Demonstrations Project >>



Mathematica availability information for [Rijksuniversiteit Groningen](#).

$$= \left[\int_{-\infty}^{\infty} f(u) \, du \right] \left[\int_{-\infty}^{\infty} g(x) \, dx \right]. \quad (12)$$

The horizontal [function centroids](#) add

$$\langle x (f * g) \rangle = \langle x f \rangle + \langle x g \rangle, \quad (13)$$

as do the [variances](#)

$$\langle x^2 (f * g) \rangle = \langle x^2 f \rangle + \langle x^2 g \rangle, \quad (14)$$

where

$$\langle x^n f \rangle \equiv \frac{\int_{-\infty}^{\infty} x^n f(x) \, dx}{\int_{-\infty}^{\infty} f(x) \, dx}. \quad (15)$$

There is also a definition of the convolution which arises in probability theory and is given by

$$F(t) * G(t) = \int F(t-x) \, dG(x), \quad (16)$$

where $\int F(t-x) \, dG(x)$ is a [Stieltjes integral](#).

SEE ALSO: [Autocorrelation](#), [Cauchy Product](#), [Convolution Theorem](#), [Cross-Correlation](#), [Recurrence Plot](#), [Wiener-Khinchin Theorem](#). [[Pages Linking Here](#)]

REFERENCES:

Bracewell, R. "Convolution" and "Two-Dimensional Convolution." Ch. 3 in *The Fourier Transform and Its Applications*, 3rd ed. New York: McGraw-Hill, pp. 25-50 and 243-244, 1999.

Hirschman, I. I. and Widder, D. V. *The Convolution Transform*. Princeton, NJ: Princeton University Press, 1955.

Morse, P. M. and Feshbach, H. *Methods of Theoretical Physics, Part I*. New York: McGraw-Hill, pp. 464-465, 1953.

Press, W. H.; Flannery, B. P.; Teukolsky, S. A.; and Vetterling, W. T. "Convolution and Deconvolution Using the FFT." §13.1 in *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, 2nd ed. Cambridge, England: Cambridge University Press, pp. 531-537, 1992.

Weisstein, E. W. "Books about Convolution."

<http://www.ericweisstein.com/encyclopedias/books/Convolution.html>.

LAST MODIFIED: October 13, 2003

CITE THIS AS:

Weisstein, Eric W. "Convolution." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/Convolution.html>

© 1999 CRC Press LLC, © 1999-2008 Wolfram Research, Inc. | [Terms of Use](#)