The Milky Way
Part 3 — Stellar kinematics
Physics of Galaxies 2011
part 8
Stellar motions in the MW disk

- Let’s continue with the rotation of the Galaxy, this time from the point of view of the stars.

- First, we need to introduce the concept of the Local Standard of Rest (LSR).

  - The LSR is the frame rotating with the Galaxy that has zero deviation from the Galactic velocity of circular rotation at the radius of the Solar Neighborhood.
As you saw when looking at the motion of gas in the MW disk, the Galaxy rotates *differentially*:

- stars closer to the center take less time to complete their orbits around the Galaxy than those farther out

- If the Galaxy rotated as a solid (or rigid) body, then all stars in circular motion would take the *same* time to complete their orbits around the Galaxy

How can we tell?
Let’s start with the equation we derived last time for the motion of gas in circular orbits with velocity $v_c$ at some point $R$ away from the GC:

$$v_c = \omega \times R$$

Here, $\omega$ is the angular velocity around the center.

At the Sun,

$$v_0 = \omega_0 \times R_0$$
Recall that the line-of-sight (or radial) velocity when seen from the LSR (~the Sun) is then

\[ v_{\text{los}} = [\omega - \omega_0] R_0 \sin l \]

Using \[ \omega - \omega_0 = \frac{d\omega}{dR} \bigg|_{R_0} (R - R_0) \]
we can write \[ v_{\text{los}} = \frac{d\omega}{dR} \bigg|_{R_0} (R - R_0) R_0 \sin l \]

Now we need the cosine rule: \[ R^2 = R_0^2 + d^2 - 2R_0 d \cos l \]

To first order in distance, \[ R^2 \approx R_0^2 - 2R_0 d \cos l \]

and \[ R^2 - R_0^2 = (R - R_0)(R + R_0) \approx (R - R_0) \times 2R_0 \]
...at least near the Sun!
And so \((R - R_0) \approx -d \cos l\)

Therefore we can write \(v_{\text{los}} = - \left. \frac{d\omega}{dR} \right|_{R_0} R_0 d \sin l \cos l\)

or \(v_{\text{los}} = - \frac{1}{2} \left. \frac{d\omega}{dR} \right|_{R_0} R_0 d \sin 2l = Ad \sin 2l\)

where Oort’s A constant is \(A = - \frac{1}{2} R \left. \frac{d\omega}{dR} \right|_{R_0}\)

or, in terms of the magnitude of the circular velocity,

\[ A = \frac{1}{2} \left( \frac{v_c}{R} - \left. \frac{dv_c}{dR} \right|_{R_0} \right) \]

We’ll come back to its physical meaning shortly!
Now let’s look at the transverse velocity:

\[ v_t = [\omega - \omega_0] R_0 \cos l - \omega d \]

or

\[ v_t = \left. \frac{d\omega}{dR} \right|_{R_0} (R - R_0) R_0 \cos l - \omega d \]

Using the same approximation for \((R - R_0)\) as before and noting that \(\cos^2 l = \frac{1}{2}(\cos 2l + 1)\), we have

\[ v_t = d \left[ -\omega - \frac{1}{2} R_0 \left. \frac{d\omega}{dR} \right|_{R_0} (1 + \cos 2l) \right] \]

\[ = d [B + A \cos 2l] \]
where Oort’s B constant is

\[
B = - \left( \omega + \frac{1}{2} R \frac{d\omega}{dR} \right) \bigg|_{R_0} \\
= - \frac{1}{2} \left( \frac{v_c}{R} + \frac{dv_c}{dR} \right) \bigg|_{R_0}
\]

Note that the proper motion in the \( l \) direction is \( \mu_l = v_t/d \), so \( \mu_l = B + A \cos 2l \)
Combining $A$ and $B$, we see that the magnitude of the circular velocity at $R_0$ ($\sim$Sun) is

$$v_c = R_0(A - B)$$

and the slope of the rotation curve at $R_0$ is

$$\left. \frac{dv_c}{dR} \right|_{R_0} = - (A + B)$$
So what are $A$ and $B$ physically?

- $A$ is the *local (azimuthal) shear* in the MW’s rotation curve: the deviation from solid-body rotation
- $B$ is the *local voriticity*, the gradient in the angular momentum in the MW disk locally — the tendency for stars to circulate around a point

Recent results:

- $A=14.8\pm0.8 \text{ km s}^{-1} \text{ kpc}^{-1}$
- $B=-12.4\pm0.6 \text{ km s}^{-1} \text{ kpc}^{-1}$

So the rotation curve appears to be gently falling and $v_c = 218(R_0/8 \text{ kpc}) \text{ km s}^{-1}$
Variation of radial and tangential velocity for stars on circular orbits
• So the MW is in differential rotation

• Question: why did we go through all this, if we can measure the MW’s rotation curve from the gas?

• Historically, the gas velocities couldn’t be measured, and the $A$ and $B$ coefficients are relatively easy to measure if you have enough stars with accurate radial velocities and proper motions — note that near the Sun, distances are not really needed, just velocities as a function of longitude

• The interpretation of $A$ and $B$ in terms of differential rotation was a great breakthrough by Oort in understanding the MW
The Solar motion and the LSR

- Stars in the MW disk do not move in perfectly circular orbits
  - Although this is no surprise, let’s examine how we know this...

- The motion of the Sun with respect to the LSR is called the Solar motion
  - The Solar motion is defined relative to the mean motion of stars with the same stellar type (gK, dM, etc.)
This means that the LSR is defined to be the *mean motion of stars in the Solar Neighborhood*.

In the LSR frame, the velocity components of a star are

\[ (U, V, W) = (v_R, v_\phi - \Theta_{LSR}, v_z) \]
• Let’s examine the motions of nearby disk stars, using Hipparcos proper motions and measured radial velocities.

• Their motions should be a reflection of the Solar motion:
  \[ \langle v \rangle = -\langle v_\odot \rangle \]

• The \( U \) and \( W \) components are independent of color (spectral type):
  \[ U_\odot = 10.0 \pm 0.4 \text{ km s}^{-1} \]
  \[ W_\odot = 7.2 \pm 0.4 \text{ km s}^{-1} \]
- On the other hand, the $V$ motion is strongly dependent on color for $B-V<0.6$

- It turns out that the $V$ motion is coupled to the random motions in the disk, due to scattering processes

- So we need to find the $V$ motion under the condition of no random motions:
  \[ V_\odot = 5.2 \pm 0.6 \text{ km s}^{-1} \]
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- So we need to find the $V$ motion under the condition of no random motions:

$$V_\odot = 5.2 \pm 0.6 \text{ km s}^{-1}$$
- So the magnitude of the Solar motion is $|v_\odot|=13.4 \text{ km s}^{-1}$, in towards the Galactic Center, up towards the North Galactic Pole, and away from the plane.

- The trend of $V_\odot$ with $S^2$ is called asymmetric drift, the tendency of the mean rotation velocity (of some population of stars) to lag behind that of the LSR more and more with increasing random motion of the population.

- Here we define the magnitude of any random motion in direction $i$ as $\sigma_i = \langle (v_i - \langle v_i \rangle)^2 \rangle^{1/2}$.
- All velocity dispersions of MW disk stars in the Solar Neighborhood increase with color until $B-V\sim0.6$
- Why this color? Stars with $B-V<0.6$ all have ages $<10$ Gyr
- Random motions are clearly driven by processes that take time
- scattering by GMCs, spiral arms, graininess in MW potential, etc.
The Schwarzschild distribution

- Consider a population of, say, oxygen or nitrogen molecules in air at room temperature.
  - Each component of their velocity distribution will have a Gaussian probability distribution, independent of direction.
- Schwarzschild (1907) proposed that the same might be true for stars, if the Gaussians were dependent on direction: the probability of a star having a velocity in the volume \( d^3v = dv_1 dv_2 dv_3 \) is

\[
P(v)d^3v = \frac{d^3v}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \exp \left[-\sum_{i=1}^{3} \frac{v_i^2}{2\sigma_i^2}\right]
\]
The Schwarzschild distribution implies that stars move on velocity ellipsoids

\( f(\mathbf{v}) \) is constant on ellipsoidal surfaces in velocity space
- When examining the velocity distributions of old stars (dK, dM) it is clear that the $U$ and $W$ velocities are well-described by the Schwarzschild distribution, but the $V$ velocities are not.

- The $V$ distribution is skewed towards negative velocities, with a sharp cutoff on the positive side and a long tail to the negative side.
Why is this?

- Stars with $V<0$ have orbits at smaller radii
- Consider a star on an elliptical orbit inside the Solar circle, with pericenter $R_1<R_0$
- At $R_1$, its velocity is purely tangential and larger than the circular velocity at that point, because it needs to reach larger radii:
  $$\Theta(R_1) > \Theta_c(R_1)$$
• At apocenter $R_0$, its velocity is again purely tangential, but now it must be smaller than the circular velocity, because it needs to reach a smaller radius:

$$\Theta(R_0 = R_{\text{max}}) < \Theta_c(R_0)$$

• There are more stars with $V<0$ than $V>0$, because there are more exponentially more stars as you move closer to the center of the Galaxy.

• The probability that a star from $R<R_0$ visits the Solar neighborhood is higher than those from $R>R_0$, because the velocity dispersion grows exponentially towards the center.
Star streams

- However, the distribution of stars in velocity space is not smooth
- Projections in $U$ and $V$ space show clear substructure: moving groups/streams
- Groups of stars born together
- Dynamical perturbations (due to bar, spiral structure)
Kinematics of the thick disk

If the thick disk is selected by abundances, the thick disk appears to lag the LSR by \(\sim 40-50 \text{ km s}^{-1}\), with higher velocity dispersions but lower rotational velocity

- \(\sigma_R = 61 \text{ km s}^{-1}\)
- \(\sigma_\phi = 58 \text{ km s}^{-1}\)
- \(\sigma_z = 39 \text{ km s}^{-1}\)
- \(V_\phi \sim 160 \text{ km s}^{-1}\)
Kinematics of halo stars

- The halo stars (selected by abundance) show a very-nearly Schwarzschild distribution in velocities
- principal axes closely aligned to \((R, \Theta, Z)\) directions
- The halo does not rotate (at least the majority of the nearby halo) and has velocity dispersions \((140, 105, 95)\) km s\(^{-1}\)
The velocity ellipsoid is aligned along the radial direction

- Halo stars move on very eccentric orbits

- Halo stars have very distinct motions from disk stars (known at least since Oort 1926) and are easy to pick out in proper motion surveys: very large velocities with respect to the Sun

- Large speeds means they must travel very far away from the Galactic Center

- Can be used to estimate escape velocity and thus mass of MW
Kinematics of the Galactic Bulge

- Kinematically, the bulge is not an extension of the halo
- ...bulge stars also have higher metallicities, closer to the disk, although the abundance pattern is similar to the halo
- The bulge rotates at ~100 km s\(^{-1}\)
- Velocity dispersion is lower than the disk:
  \[ \sigma_{\text{los}} \sim 60 - 110 \text{ km s}^{-1} \]
How do we select components of the MW in surveys?

- Often, we are interested in the properties of a single component of the Galaxy.
- We can select from a number of criteria:
  - Spatial distribution
  - Color
  - Kinematics
  - Abundances
How do we select components of the MW in surveys?

- However, no criterion is perfect
  - Overlap
  - Contamination
- ...and selecting one criterion may bias the results that you’re seeking
- E.g., there is a huge fight in the literature right now about whether the thick disk is really a separate entity or just an extension of the thin disk
- selecting by abundance makes this more difficult!