Abundances and chemical evolution models
Physics of Galaxies 2011
part 6
Chemical evolution

- Why do we care?

- **All** elements heavier than Li were formed by **nucleosynthesis in stars**

- The differences in the compositions in stars tell us about *when* and *how* the stars were made
The life cycle of gas and stars in galaxies

- Gas is turned into stars (somehow)
- Stars burn H into He and then He into C and then, if massive enough, C into heavier elements, all the way to Fe in the most massive stars
- These elements are returned to the interstellar medium at the end of the stars’ lives through winds or supernovae
Therefore the chemical abundance of the gas – and the next generation of stars – should increase as a function of time...

...in the absence of gas flowing into the system...

“Metallicity” is thus a kind of clock... sort of!
The build-up of metals in a stellar population

- “Galactic Chemical Evolution” (GCE) models

- The simplest model for chemical enrichment in a galaxy is the **closed-box model**, in which the galaxy is considered to a single “box” (“one zone”) **with no inflow or outflow**

- **Assume:**
  - Gas is well-mixed
  - Metals are returned to gas faster than the star-formation timescale (“instantaneous recycling”)
• Definitions:

• \( M_{\text{TOT}} = M_g(t) + M_s(t) \), where \( M_g(t) \) is the mass of gas at time \( t \) and \( M_s(t) \) is the mass in unevolved stars at \( t \)

• \( M_Z(t) \) is the mass of metals in the gas

• \( Z = Z_{\text{gas}} = \frac{M_Z(t)}{M_g(t)} \) is the metallicity if the gas \( (\text{the mass fraction in metals}; Z_\odot \approx 0.019) \) and \( Z = 0 \) at \( t = 0 \)

• \( \delta M_s \) is the net flow of gas into stars at each new generation of stars

• \( y \delta M_s \) is the mass of new metals released back into the ISM after each generation, so \( y \) is the yield
The mass fraction of metals locked up in the low-mass stars (and remnants) is $Z\delta M_s$

Now, in each generation, $\delta M_Z = y\delta M_s - Z\delta M_s$

so $\delta Z = \delta \left( \frac{M_Z}{M_g} \right) = \frac{\delta M_Z}{M_g} - \frac{M_Z}{M_g^2} \delta M_g$

or $\delta Z = \frac{1}{M_g} (\delta M_Z - Z\delta M_g)$

now, note that $\delta M_s = -\delta M_g$, since $M_{TOT}$ is constant, so

$$\delta M_Z = -y\delta M_g + Z\delta M_g$$
Substituting, we find 

$$\delta Z = \frac{1}{M_g}(-y\delta M_g + Z\delta M_g - Z\delta M)$$

And so 

$$\delta Z = -y\frac{\delta M_g}{M_g}$$

Finally, we have

$$Z(t) = -y \ln \left[ \frac{M_g(t)}{M_{\text{tot}}} \right]$$

if $y$ does not depend on time

- We often write $M_{g}/M_{\text{TOT}} = \mu$, the gas fraction

- Note that this is unphysical: $Z \to \infty$ as $M_g \to 0$

- In a closed box, metallicity grows with time
How do the metallicities of the stars evolve in a closed box?

The mass of stars $M_s(t)$ formed before time $t$, and so with metallicity $< Z(t)$ is just $M_g(0) - M_g(t)$ and therefore

$$M_s(< Z) = M_g(0) \left[ 1 - \exp(-Z/y) \right]$$

Note the lack of an explicit time here! So when the gas density is high relative to the number of stars formed, the abundance of metals is low.

Once the gas is all consumed, the mass of stars with metallicities in $(Z, Z+dZ)$ is just

$$dM_s(< Z) \propto \exp(-Z/y) dZ$$
Comparing a closed-box model with the distribution of metallicities in the bulge, a very good fit can be found with $y=Z_\odot$ and $Z(t)=0$. 
The closed-box model and the thin disk

- Let’s derive the yield for the Solar neighborhood in the closed box model: $Z(t) = -y \ln \left[ \frac{M_g(t)}{M_{tot}} \right]$

- For $t=$today, $Z \sim 0.7Z_\odot$

- The total mass is $M_g(\text{today})+M_s(\text{today}) \sim 10 \ M_\odot/\text{pc}^2 + 40 \ M_\odot/\text{pc}^2 = 50 \ M_\odot/\text{pc}^2$

- Thus we find $y = 0.43 \ Z_\odot$
The G dwarf problem

- Now let's ask how many long-lived stars we should see with $Z < 0.25 \ Z_\odot$:

$$\frac{M_s(< 0.25 Z_\odot)}{M_s(< 0.7 Z_\odot)} = \frac{1 - \exp(-0.25 Z_\odot/y)}{1 - \exp(-0.7 Z_\odot/y)} \approx 0.55$$

- so **55%** of the stars in the Solar neighborhood should have $Z < Z_\odot/4$

- but we only see $\sim 2\%$ of the local F and G stars with these metallicities!
So the simple closed-box model is clearly wrong for the Solar neighborhood!

Possible solutions:

- No G dwarfs made at early times (too extreme?)
- Yield decreases with increasing Z (disagrees with SN yields)
- Disk pre-enrichment: disk is polluted by spheroid
- Disk infall and enrichment

Note lack of metal-poor G dwarfs w.r.t. model!
- Pre-enrichment does a reasonable job: if $Z(0) \approx 0.15 \, Z_\odot$, then G dwarf problem is mostly resolved.

- The G dwarf problem means that the Solar neighborhood was not always like it is now!

- **Something** was very different – nucleosynthesis gives us an important clue to galaxy evolution!
We know that many galaxies have strong outflows of gas, generally caused by intense star formation or AGN.

What effect does this have on the chemical evolution?
Accretion-box model: the effects of inflow

- At the same time, it has become clear that galaxies also accrete gas.
- What affect does this inflow have on the chemical evolution?
Generalizing the one-zone model

- First, let’s define the **inflowing** gas to have a mass $\delta M_{\text{in}} = f \delta M_s$ with metallicity $Z_{\text{in}}$ and the **outflowing** gas to have a mass $\delta M_{\text{out}} = g \delta M_s$ with metallicity $Z_{\text{out}}$.

- Then it is possible to show that

$$\delta Z = \frac{1}{M_g} [(y + f Z_{\text{in}} - Z - g Z_{\text{out}}) \delta M_{\text{TOT}} - (y + f Z_{\text{in}} - g Z_{\text{out}}) \delta M_g]$$

- In general, this equation can only be solved **only** if $M_{\text{TOT}}$ and $M_g$ are specified functions of time.

- But there are two special cases of interest...
The leaky-box model

- In this case, $\delta M_{\text{TOT}} \approx 0$, and $f=0$, $g \neq 0$, and $Z_{\text{out}} = Z_{\text{SN}}$, so that the SN eject leave directly before mixing into the ISM.

- Note that this prescription violates $M_{\text{TOT}} \sim \text{constant}$, but effect is small at early times.

- Then $Z(t) = -(y - gZ_{\text{out}}) \ln\left[\frac{M_g(t)}{M_{\text{tot}}}\right]$

- Compared to the closed-box model, the only thing that happens is that the yield is reduced by $-gZ_{\text{out}}$

- Explains low apparent yield in dwarf galaxies.
The accreting-box model

- When the infall is small and balances star formation, $\delta M_s \approx 0$; also assume that $Z_{\text{in}} = \text{constant}$ and $Z_{\text{out}} = Z$ (well mixed)

- As $t \to \infty$, $M_{\text{TOT}} \gg M_g$ and

$$Z \to \frac{y + fZ_{\text{in}}}{1 + g}$$

- If $Z_{\text{in}} = 0$ ("pristine" gas), $g = 0$, and $f = 1$, then

$$Z \to y$$
The mass in stars more metal-poor than $Z$ is $M_s(<Z) = M_{TOT}(Z) - M_g$ which is

$$M_s(< Z) = -M_g \ln \left(1 - \frac{Z}{y}\right)$$

Then the yield can be determined from the metallicity: in the Solar neighborhood, $Z=0.7Z_\odot$, then $y \approx 0.71Z_\odot$, so the fraction of stars with $Z<Z_\odot/4$ is

$$M_s(< Z_\odot/4) = 0.43M_g \approx 0.09M_{tot} \approx 0.11M_s$$

So about 11% of the dwarf stars should be more metal-poor than $0.25Z_\odot$, in better agreement with the observations!
Finally, don’t forget about the thick disk!

- Lots of metal-poor dwarfs there — good way to help solve the G dwarf problem...