The Earth’s atmosphere: seeing, background, absorption & scattering
Observational Astronomy 2019
Part 8
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Seeing

- All ground-based observatories suffer from a major problem:
  - light from distant objects must pass through the Earth’s atmosphere
  - recall from our discussion of optics that the atmosphere has refractive power

Seeing

- The refraction itself isn’t the problem: it’s the variation in the index of refraction that causes problems
- Wind, convection, and land masses cause turbulence, mixing the layers of different indices of refraction in non-uniform and continuously varying ways

This non-uniform, continuous variation in the index of refraction cause the initially plane-parallel wavefronts to tilt, bend, and corrugate
- This causes the twinkling we see when we look at stars with our eyes
- Any degradation of images by the atmosphere is called seeing
Seeing

To understand seeing, we need to examine the index of refraction and its dependence of temperature and pressure (and humidity, too)

- Cauchy’s formula, extended by Lorentz to account for humidity, gives the excess (over vacuum) index of refraction of air:

\[ n_{\text{air}} - 1 = \frac{77.6 \times 10^{-6}}{T} \left( 1 + \frac{7.52 \times 10^{-3}}{\lambda^2} \right) \left( p + 4810 \frac{v}{T} \right) \]

where \( p \) is the pressure in mbars, \( T \) is the temperature in K, and \( v \) is the water vapor pressure in mbars

The dominant terms are

\[ n_{\text{air}} - 1 = 77.6 \times 10^{-6} p T^{-1} \]

Seeing — perturbations of the transiting wavefront — is caused by changes in \( n_{\text{air}} \) differentially across the wavefront due to turbulence

- Note that water vapor has little effect on \( n \) in the optical (except for fog), but can affect columns of air by causing convection: wet air is lighter than dry air!

Modern observatories are built at very dry sites, so the overall effect of humidity is typically unimportant for seeing (but important for differential refraction)

- Ignoring \( v \) above, then, for an ideal gas and adiabatic conditions,

\[ \frac{dn}{dT} \propto \frac{p}{T^2} \]

Therefore the primary source of index of refraction variations in the atmosphere is thermal variations

- We care (mostly) about small-scale variations in \( T \): thermal turbulence

Thermal turbulence

- Thermal turbulence in the atmosphere is created on a variety of scales by

  convection: air heated by conduction with the warm surface of the Earth becomes buoyant and rises, displacing cooler air

  humid (wet) air also rises and displaces dry air
Thermal turbulence

- **wind shear**: high winds — in particular the jet stream — generate wind shear and eddies at various scales, creating a turbulent interface between other layers in laminar (non-turbulent) flow.

- **disturbances**: large land-form variations — like mountains — can create turbulence.

Thermal turbulence

- Turbulence occurs when **inertial forces** dominate over **viscous forces** in a fluid.
  - This means that turbulent motions — eddies and vortices — dominate over smooth (laminar) flows.
  - This is true for typical wind speeds and length scales.

- Turbulence is created by **eddy transfer**.
  - Kinetic energy is deposited on large scales, which creates **eddies**.
    - These in turn create smaller eddies, and so on...
    - ...until the shears are so large compared to the eddy scale that the viscosity of air takes over and the kinetic energy is turned into heat.
  - This stops the **turbulent cascade** and smooths out the temperature fluctuations.
Thermal turbulence

- Kolmogorov showed that the energy spectrum of such a turbulent cascade has a characteristic shape.
  - The **inertial range** follows a spectrum
    \[ E \propto \kappa^{-5/3} \]
  - where \( \kappa \) is the wavenumber or inverse length.

Thermal turbulence

- When turbulence occurs in an atmospheric layers with a temperature gradient differing from an adiabatic gradient...
  - one in which no heat is transferred (i.e., the entropy does not increase)
  - it mixes air of different temperatures at the same altitude, producing temperature fluctuations following a Kolmogorov spectrum.

Atmospheric layers contributing to turbulence

- There are effectively **four** layers of the atmosphere that affect the seeing, with gradual transitions of properties between them.

Atmospheric layers contributing to turbulence

- **surface** or **ground layer**: few to 10’s of meters — up to few arcminutes seeing!
  - turbulence generated by wind shear due to friction and topographic effects
    - site geometry, trees, boulders, etc.
  - best sites have surface layers <10 m
    - build telescope above this layer.
Atmospheric layers contributing to turbulence

- **Planetary boundary layer**: 
  - "1 km above sea level — 10's of arcseconds seeing
  - both frictional effects and day-night heating-cooling cycles
  - creates convection up to inversion layer
  - most observatories built above this layer

Atmospheric layers contributing to turbulence

- **Atmospheric boundary layer**: above planetary boundary layer
  - not dominated by convection but by abrupt changes in topography

Atmospheric layers contributing to turbulence

- **Free atmosphere**: seeing ~0.4″
  - best sites mostly see this layer
  - unaffected by frictional influence of ground
  - large-scale flows
  - tropical trade winds (at lower altitudes), jet stream (at higher altitudes)
  - mechanical turbulence

Seeing

- To understand turbulence in the context of seeing, we need to translate the Kolmogorov spectrum into a spatial variation
- To do this, we define a **structure function**: in 1D,
  \[ D_T(r) = \langle [T(x+r) - T(x)]^2 \rangle \]
- where \( T(x) \) and \( T(x+r) \) are the temperatures at two points separated by \( r \)
Seeing

- For a Kolmogorov spectrum,
  \[ D_T(r) = C_T^2 r^{2/3} \]
  - where \( C_T^2 \) is the temperature structure constant and gives the intensity of thermal turbulence
  - The variation of the index of refraction (of air) is given by the index of refraction structure constant,
  \[ C_n^2 = C_T^2 \left[ 77.6 \times 10^{-6} (1 + 7.52 \times 10^{-3} \lambda^{-2}) p T^{-2} \right]^2 \]

Seeing

- There is a characteristic transverse linear scale over which we can consider the atmospheric variations to “flatten out” and in which plane-parallel wavefronts are transmitted
  - over these scales, the images are “perfect”
  - This scale is called the Fried parameter \( r_0 \)

Seeing

- The Fried parameter is well-described by integrating \( C_n^2 \) along the line-of-sight through the atmosphere:
  \[ r_0 = \left[ 1.67 \lambda^{-2} (\cos z)^{-1} \int C_n^2(z')dz' \right]^{-3/5} \]
  - where \( z \) is the zenith angle and \( z' \) is the distance along the line of sight
  - A larger Fried parameter is better! Hence the inverse dependence on \( C_n^2 \)
  - note also the dependence on \( \cos z \), so the Fried parameter gets smaller as one looks through more atmosphere

Seeing

- The Fried parameter is inversely proportional to the size of the PSF transmitted by the atmosphere
  - The FWHM of the observed PSF can be predicted if we know the variation of \( C_n^2 \) along the line-of-sight:
  \[ \theta_{\text{FWHM}}'' = 2.59 \times 10^{-5} \lambda^{-1/5} \left[ (\cos z)^{-1} \int C_n^2(z')dz' \right]^{3/5} \]
Seeing

- This is a typical $C_n^2$ profile
- Note that FWHM is **bigger** for larger zenith angles and **smaller** for longer wavelengths

![Graph showing $C_n^2$ profile](image)

Fig. 1. A. Representative profile of the contribution to seeing as a function of altitude. The intensity of the fluctuations is expressed in terms of the index of refraction variance coefficient as defined in Section 1.2. Most of the fluctuations occur near the ground and in relatively thin turbulent layers generated by wind shear. (From Ref. 6.)

Isoplanaticity and the Fried model

- In 1665, Hooke suggested the existence of “small, moving regions of the atmosphere having different refractive powers that act like lenses” to explain **scintillation**, the “twinkling” of stars
- In 1966, Fried showed that this simple model was a reasonable representation of reality...

Isoplanaticity and the Fried model

- Assume that at any given moment, the atmosphere acts like an array of small, contiguous, wedge-shaped refracting cells
- These locally **tilt** the wavefront randomly over the size scale of the cells
  - each cell imposes its own tilt on the wavefront...
  - ...creating **isoplanatic patches** in which the wave is fairly smooth and has little curvature

Isoplanaticity and the Fried model

- Thus, each isoplanatic patch transmits a quality image of the source
- The size of this isoplanatic patch is the Fried parameter $r_0$, roughly the diameter (not the radius!) of the patch
  - This characterizes the seeing
  - The more turbulence there is, the smaller $r_0$ is
Isoplanaticity and the Fried model

- Typically $r_0 \approx 10$ cm in the optical and varies as $\lambda^{6/5}$
- Seeing is always better at longer wavelengths!
- The angular size of the isoplanatic patch is $\theta_0 \sim 0.6 r_0 / h$
- where $h$ is the height of the primary turbulence layer above the telescope

Seeing effects

- There are primary effects of seeing:
  - Scintillation ("twinkling")
  - Image wander ("agitation")
  - Image blurring ("smearing")

Seeing effects

- Scintillation ("twinkling") is the result of a varying amount of energy being received over time
  - intensity changes
  - Variations in the shape of the turbulent layer result in moments when it acts like a (net) concave lens, focussing the light, and other moments when it acts like a (net) convex lens, defocussing the light
  - This is only obvious when the aperture is $< r_0$, like for the human eye
  - For larger apertures, these effects average out
  - Note that planets don't twinkle, because they're much bigger than $\theta_0$
  - Because it is an interference effect, scintillation is highly chromatic

Seeing effects

- Image wander ("agitation") is the motion of an image in the focal plane due to changes in the average tilt of the wavefront
  - Because the wavefront is locally plane-parallel, the image can actually be diffraction-limited if the aperture is $< r_0$
  - Then the Airy disk is imaged, but it will wander
  - Larger apertures average over more isoplanatic patches, so this effect is also only present for small apertures
Seeing effects

- **Image blurring** (smearing) dominates for $D > r_0$
- Each isoplanatic patch gives rise to its own Airy disk (FWHM $\sim \lambda/D$): these are called **speckles**
- Seen together, they have FWHM $\sim \lambda/r_0$ and give rise to a shimmering blur: [https://www.youtube.com/watch?v=vD9qoMqg5U](https://www.youtube.com/watch?v=vD9qoMqg5U)
- If integrating for long times, they will have this size for the **long-exposure PSF**
- In general, the larger the telescope, the more photons it gathers — but the PSF is limited to $\lambda/r_0$

   - need **adaptive optics** (AO) or **speckle interferometry**
   - Remember that FWHM $\sim \lambda^{-1.5}$, so seeing is always better in the red

Seeing: summary

- The size of the PSF at the detector is caused by the seeing (for long integrations)
- The seeing limits the size of the PSF to FWHM $\sim \lambda/r_0$ *instead of* the resolution limit FWHM $\sim \lambda/D$
- since $r_0 = 10$ cm in the optical, if $D > 10$ cm, then for a big telescope the “resolution” is the same as a 10 cm telescope
- this is why we are so interested in adaptive optics (AO)!

Backgrounds, absorption, scattering, and extinction

- In order to get accurate results, we need to correct for the background and for light removed by the atmosphere through absorption and scattering
- Many background sources come from the Earth (its inhabitants and its atmosphere)
- But perhaps the most important background source is the sunlight reflected from the moon: **moonlight**

Sky brightness at CTIO

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- **Airglow** is emission from molecules and some atoms (like O, Na, H) in the atmosphere
  - this is the source of “sky (emission) lines” we see in spectra taken from ground-based sites

- Scattered sunlight during **twilight** is also a problem at the beginning and end of the night
  - although very useful for flat fields!
  - We define three “official” twilights according to the Sun’s position with respect to the horizon:
    - **6 degree twilight**: “civil twilight” — when you need to turn your headlights on
    - **12 degree twilight**: “nautical twilight” — sailors can still see the horizon but also see enough stars to fix a position with a sextant
    - **18 degree twilight**: “astronomical twilight” — fully dark sky

- **Zodiacal light** is sunlight scattered off of Solar System dust, very near the plane of the ecliptic
  - this is the limiting background on moonless nights at a very dark site (and even in space) away from the Galactic plane

- **Unresolved galaxy and starlight** — like in the plane of the Milky Way — can also cause a significant background
  - In the near- to mid-IR (>2.4 µm), **thermal emission** from the Earth and the telescope dominate the background

- **Clouds** will reflect light back from the Earth, Sun, and Moon

- Finally, **light pollution** from poorly-regulated human settlements is a serious — and growing — problem for astronomy
World-wide light pollution

Absorption and scattering

- Atmospheric absorption in the optical is minimal
  - at $\lambda<3100$ Å, O$_3$ (ozone) cuts off the "optical window"
  - small but annoying regions at ~6800 Å and ~7500 Å from O$_2$ absorption
  - at $\lambda>8000$ Å, H$_2$O vapor causes absorption, defining "windows" at ~1 µm and beyond

Absorption and scattering

- In the near- to mid-IR, H$_2$O and CO$_2$ (along with a little NO$_2$ and CH$_4$) make life difficult, except in certain windows
Absorption and scattering

- At longer wavelengths, the far-IR is reachable only from space (or very high-flying balloons or airplanes).
- The spectrum opens up again at $\lambda > 850 \mu m - 1 \text{ mm}$ (depending on water vapor), becoming almost totally transparent at cm–mm wavelengths.

Absorption and scattering

- Wavelengths shorter than $3000 \text{ Å}$ are only visible from space (or very high-flying balloons or airplanes).

Absorption and scattering

- Scattering arises from two major regimes of particle-photon scattering:
  - **Molecular** scattering, where the scattering radius $a < \lambda$.
  - **Rayleigh scattering** is dominant, with a cross-section
    $$\sigma_R(\lambda) = \frac{8\pi^3}{3} \left(1 - \frac{n^2}{3}\right)^2 \propto N^{-2} \lambda^{-4}$$
    - where $N$ is the particle density and is a function of temperature and pressure, and therefore altitude, etc.
Absorption and scattering

- Rayleigh scattering is the main cause of blue extinction and is responsible for the blue sky!

Absorption and scattering

- Aerosol scattering, where the scattering radius $\alpha \gtrsim \lambda/10$
  
  - This is Mie scattering, which has (roughly)
    
    $$\sigma \propto \alpha^2 \lambda^{-1}$$
  
  - and so is the main cause of red extinction

Absorption and scattering

- Aerosols are highly variable from night to night, as they come from
  - air pollution
  - “haze” (dust)
  - volcanic ash
  - dust storms
  - etc.

Absorption and scattering

- In the summer at La Palma, wind-blown dust (“calima”) from Saharan dust storms is particularly annoying, causing a significant increase in optical extinction
Airmass

- We define the airmass between the top of the telescope and the top of the atmosphere at the zenith as one airmass or \( X = 1 \).

\[ X = \frac{1}{\cos z} = \sec z \]

Airmass

- Define \( z \) as the zenith angle (or zenith distance), where
- \( z = 90^\circ \) – altitude
- Then in a plane-parallel approximation to the Earth's atmosphere,
  \[ X = \frac{1}{\cos z} = \sec z \]
Airmass

- The plane-parallel approximation is good for \( z < 60^\circ \).
- For larger zenith angles, a spherical-shell description is required:

\[
X = \sec z - 0.0018167 (\sec z - 1) - 0.002875 (\sec z - 1)^2 - 0.0008083 (\sec z - 1)^3
\]

Atmospheric extinction and photometry

- Although this topic actually belongs with our discussion of photometry, we can now discuss the effect of atmospheric extinction on photometry, the measurement of magnitudes.

Atmospheric extinction and photometry

- Because atmospheric absorption and scattering is a radiative transfer problem, we can characterize it as an optical depth problem.
Atmospheric extinction and photometry

Then we can write that the flux remaining after passing through the atmosphere is

\[ f = f_0 e^{-X/\tau} \]

where \( f_0 \) is the original flux of the object above the atmosphere, \( X \) is the airmass, and \( \tau \) is the optical depth.

Therefore, in magnitudes, the loss of light per airmass is linear, with a proportionality constant \( k_{\lambda} = -2.5 \log_{10} e / \tau \)

called the **extinction coefficient**

Atmospheric extinction and photometry

We will return to the process of determining \( k_{\lambda} \) later...

In the meantime, for the sky over La Palma, on average, the extinction per unit airmass is given to the right...

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Clearly, we want to observe objects in the U band as close to overhead as possible!