Mon. Not. R. Astron. Soc. **324,** 712–716 (2001)

Bumpy power spectra and $\Delta T/T$

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Accepted 2001 January 28. Received 2001 January 23; in original form 2000 November 6

ABSTRACT

With the recent publication of the measurements of the radiation angular power spectrum from the BOOMERanG-98 Antarctic flight, it has become apparent that the currently favoured spatially flat cold dark matter model (matter density parameter $\Omega_{\mathrm{m}} = 0.3$, flatness being restored by a cosmological constant $\Omega_\Lambda = 0.7$, Hubble parameter $h = 0.65$, baryon density parameter $\Omega_{\mathrm{b}}h^2 = 0.02$) no longer provides a good fit to the data. We describe a phenomenological approach to resurrecting this paradigm. We consider a primordial power spectrum which incorporates a bump, arbitrarily placed at k_{b} and characterized by a Gaussian in $\log k$ of standard deviation σ_{b} and amplitude A_{b}, which is superimposed on to a scale-invariant power spectrum. We generate a range of theoretical models that include a bump at scales consistent with cosmic microwave background (CMB) and large-scale structure observations, and perform a simple χ^2 test to compare our models with the *COBE* Differential Microwave Radiometer (DMR) data and the recently published BOOMERanG-98 and MAXIMA-1 data. Unlike models that include a high baryon content, our models predict a low third acoustic peak. We find that low ℓ observations ($20 < \ell < 200$) are a critical discriminant of the bumps because the transfer function has a sharp cut-off on the high ℓ side of the first acoustic peak. Current galaxy redshift survey data suggest that excess power is required at a scale around 100 Mpc, corresponding to $k_{\mathrm{b}} \sim 0.05\,h\,\mathrm{Mpc}^{-1}$. For the concordance model, use of a bump-like feature to account for this excess is not consistent with the constraints imposed by recent CMB data. We note that models with an appropriately chosen break in the power spectrum provide an alternative model that can give distortions similar to those reported in the automated plate measurement (APM) survey as well as consistency with the CMB data. We prefer, however, to discount the APM data in favour of the less biased decorrelated linear power spectrum recently constructed from the Point Source Catalogue Redshift (PSCz) redshift survey. We show that the concordance cosmology can be resurrected using our phenomenological approach and that our best-fitting model is in agreement with the PSCz observations.

Key words: cosmic microwave background – cosmology: theory.

1 INTRODUCTION

The recent BOOMERanG-98 (1998 balloon-borne observations of millimetre extragalactic radiation and geophysics; de Bernardis et al. 2000) and MAXIMA-1 (the first flight of the millimetre-wave anisotropy experiment imaging array; Hanany et al. 2000) measurements of an acoustic peak in the angular power spectrum of the cosmic microwave background (CMB) temperature at $l \approx 200$ (de Bernardis et al. 2000; Hanany et al. 2000) has provided remarkable confirmation that the growth via gravitational instability of primordial adiabatic density fluctuations seeds large-scale structure. One consequence of the location of this peak, arising from the compression of an acoustic wave on first entering the horizon of last scattering, is that the spatial geometry of the universe is flat.

However, the weakness of the second acoustic peak at $l \approx 400$, arising from the subsequent first rarefaction of the acoustic wave on the last scattering horizon, has provoked considerable speculation as to the additional freedom that could be added to the concordance cold dark matter (CDM) model (matter density parameter $\Omega_{\mathrm{m}} = 0.3$, flatness being restored by a cosmological constant $\Omega_\Lambda = 0.7$, Hubble parameter $h = 0.65$, baryon density parameter $\Omega_{\mathrm{b}}h^2 = 0.02$) to accommodate such an effect. Ideas that have been proposed include enhancement of the baryon fraction

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(White, Scott & Pierpaoli 2000; Lange et al. 2001), a large neutrino asymmetry (Lesgourgues & Peloso 2000), delay of recombination (Hu & Peebles 2000), an admixture of a component of cosmological defects (Bouchet et al. 2000) and models employing double inflation in supergravity (Kanazawa et al. 2000).

Here we suggest a more phenomenological solution, which is motivated by suggestive, although not overwhelming, evidence from galaxy surveys that there is excess power relative to the scale-invariant ($n \approx 1$) fluctuation spectrum of the conventional model near $100\,h^{-1}$ Mpc. The case for excess power has not hitherto been completely convincing because one is probing the limit of current surveys. Nevertheless, several independent data sets have provided such indications (see, e.g. Broadhurst et al. 1990; Landy et al. 1996; Einasto et al. 1997).

In fact the multiple inflationary model of Adams, Ross & Sarkar (1997) predicts the suppression of the second acoustic peak through the generation of features in the primordial power spectrum from phase transitions that occur during inflation. More generally, there are strong theoretical arguments which suggest that arbitrary features can be dialled on to the primordial power spectrum predicted by generic inflationary models (see, e.g., García-Bellido, Linde & Wands 1996; Randall, Soljačić & Guth 1996; Linde & Mukhanov 1997; Lesgourges et al. 1998; Starobinsky 1998; Chung et al. 2000; Martin, Riazuelo & Sakellariadou 2000).

We therefore consider a primordial power spectrum which incorporates a phenomenological bump, arbitrarily placed at k_{b} and characterized by a Gaussian in $\log k$ of standard deviation σ_{b} and amplitude A_{b}, what is superimposed on to a scale-invariant power spectrum as advocated by Silk & Gawiser (1999). We examine the constraint on the bump parameters for the ΛCDM concordance model (Ostriker & Steinhardt 1995) ($\Omega_{\mathrm{m}} = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.65$, $\Omega_{\mathrm{b}} h^2 = 0.02$) imposed by the CMB data, and restrict the choice of bump parameters to the region of parameter space that is consistent with observations of large-scale power and CMB anisotropies.

2 THE THEORETICAL MODELS

In our paper we consider one particular spatially flat CDM model; the ΛCDM concordance model of Ostriker & Steinhardt (1995) ($\Omega_{\mathrm{m}} = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.65$, $\Omega_{\mathrm{b}} h^2 = 0.02$). We assume Gaussian and adiabatic initial conditions with a scale-invariant ($n = 1$) power-law form as predicted by the simplest inflationary models. The radiation angular power spectrum is calculated using the CMBFAST program (Seljak & Zaldarriaga 1996) once the code has been modified to incorporate a bump in the primordial spectrum as advocated by Silk & Gawiser (1999). We model this bump as a Gaussian in $\log k$ with a central location in wavenumber k_{b}, a standard deviation σ_{b}, and an amplitude A_{b}, resulting in the new primordial power spectrum given below, where $P_0(k)$ is the power spectrum of the model without the feature.

$$P(k) = P_0(k)\left\{1 + A_{\mathrm{b}} \exp\left[-\frac{(\log k - \log k_{\mathrm{b}})^2}{2\sigma_{\mathrm{b}}^2}\right]\right\}. \tag{1}$$

We restrict the choice of bump parameters to the region of parameter space that is consistent with large-scale structure and CMB observations. The parameters are varied as follows: $0.05 < \sigma_{\mathrm{b}} < 2.0$, $0.0 < A_{\mathrm{b}} < 3.0$, $0.001 < k_{\mathrm{b}}\, h\,\mathrm{Mpc}^{-1} < 0.140$.

Our focus is directed towards determining whether it is possible to resurrect the concordance model without resorting to the proposed ideas listed in our introduction that may prove to contradict observation. Since the CMB observations indicate that the second acoustic peak is suppressed in relation to the first acoustic peak, it may be that a dip in the primordial power spectrum around the scale of the second peak could also enable the concordance model to fit the data. Theories that predict a bump in the primordial power spectrum have been inspired by hints of such a feature from observations of large-scale structure. We do not investigate a dip in this paper because there is less theoretical motivation for this scenario. We attempt to increase the first-to-second-peak ratio with the incorporation of a bump around the scale of the first peak, then renormalize the radiation angular power spectrum to fit the data.

Table 1. The data used in this study.

Experiment	ℓ_{eff}	$\delta T^{\mathrm{data}}_{\ell_{\mathrm{eff}}} \pm \sigma^{\mathrm{data}}$ (μK^2)
COBE	2.1	$72.25^{+528.0}_{-72.5}$
COBE	3.1	$784.0^{+476.25}_{-470.71}$
COBE	4.1	$1156.0^{+444.0}_{-437.76}$
COBE	5.6	$630.01^{+294.15}_{-287.76}$
COBE	8	$864.36^{+224.64}_{-224.27}$
COBE	10.9	$767.29^{+231.27}_{-229.05}$
COBE	14.3	$681.21^{+249.04}_{-244.4}$
COBE	19.4	$1089.0^{+324.76}_{-327.24}$
BOOMERanG	50.5	1140 ± 280
BOOMERanG	100.5	3110 ± 490
BOOMERanG	150.5	4160 ± 540
BOOMERanG	200.5	4700 ± 540
BOOMERanG	250.5	4300 ± 460
BOOMERanG	300.5	2640 ± 310
BOOMERanG	350.5	1550 ± 220
BOOMERanG	400.5	1310 ± 220
BOOMERanG	450.5	1360 ± 250
BOOMERanG	500.5	1440 ± 290
BOOMERanG	550.5	1750 ± 370
BOOMERanG	600.5	1540 ± 430
MAXIMA	73	2000^{+680}_{-510}
MAXIMA	148	2960^{+680}_{-550}
MAXIMA	223	6070^{+1040}_{-900}
MAXIMA	298	3720^{+620}_{-540}
MAXIMA	373	2270^{+390}_{-340}
MAXIMA	448	1530^{+310}_{-270}
MAXIMA	523	2340^{+430}_{-380}
MAXIMA	598	1530^{+380}_{-340}
MAXIMA	673	1830^{+490}_{-440}
MAXIMA	748	2180^{+700}_{-620}

3 THE OBSERVATIONAL DATA

Our data sample is listed in Table 1. It consists of the 8 uncorrelated *COBE* differential microwave radiometer (DMR) points from Tegmark & Hamilton (1997), the 12 data points from the BOOMERanG-98 Antarctic flight (de Bernardis et al. 2000) and the 10 recently published MAXIMA-1 data points (Hanany et al. 2000).

4 CONSTRAINING THE MODELS

We use a simple χ^2 goodness-of-fit analysis employing the data in Table 1 along with the corresponding window functions for the uncorrelated *COBE* DMR points (Tegmark & Hamilton 1997) and assuming a top hat window function over the BOOMERanG-98 and MAXIMA-1 bins. The window functions describe how the anisotropies at different ℓ contribute to the observed temperature anisotropies (Lineweaver et al. 1997). For a given theoretical model, they enable us to derive a prediction for the δT for each experiment, to be compared with the observations in Table 1.

It has been noted that the use of the χ^2 test can give a bias in parameter estimation in favour of permitting a lower power spectrum amplitude because in reality there is a tail to high-temperature fluctuations. Other methods have been proposed (Bartlett et al. 1999; Bond, Jaffe & Knox 2000) which give good approximations to the true likelihood, although they require extra information on each experiment which is not yet readily available. We do not use these more sophisticated techniques here.

There are $N_{\rm data} = 30$ data points. Rather than adopting the *COBE* normalization, the theoretical models are normalized to the full observational data set resulting in a hidden parameter. We use the method of Lineweaver & Barbosa (1998) to treat the correlated calibration uncertainty of the 12 BOOMERanG-98 data points and the 10 MAXIMA-1 data points as free parameters with Gaussian distributions about their nominal values of 10 per cent for BOOMERanG-98 and 4 per cent for MAXIMA-1. This results in two further hidden parameters. We do not account for the 10 per cent correlation between the BOOMERanG-98 bins nor that between the MAXIMA-1 bins which would further reduce the degrees of freedom. Accounting for the correlations would provide tighter constraints on the models, so the constraints we impose are conservative.

Because we are measuring absolute goodness-of-fit on a model-by-model basis, with three hidden parameters, the appropriate distribution for the χ^2 statistic has $N_{\rm data} - 3$ degrees of freedom. Nothing further is to be subtracted from this to allow for the number of parameters, as they are not being varied in the fit. To assess whether or not a model is a good fit to the data, we need the confidence levels of this distribution. These are $\chi^2_{27} < 29.87$ at the 68 per cent confidence level, $\chi^2_{27} < 40.11$ at the 95 per cent confidence level and $\chi^2_{27} < 46.96$ at the 99 per cent confidence level. Models which fail these criteria are rejected at the given level.

Although we are unable to give the overall best-fitting model for currently permitted cosmologies, since this would require varying each of the cosmological parameters as well as those describing the bump, we find that, for the paradigm being considered, the best-fitting model is $k_{\rm b} = 0.004\,h\,{\rm Mpc}^{-1}$, $A_{\rm b} = 0.9$, $\sigma_{\rm b} = 1.05$. This model has a χ^2 of 22.0, which is in good agreement with expectations for a fit to 30 data points with six adjustable parameters (the three bump parameters and the three hidden parameters).

By marginalizing over the bump parameters we are able to determine the 68 per cent confidence-level limits on each parameter. We find that the lower limit on $k_{\rm b}$ extends right to the edge of the region of parameter space that we are investigating. We do not feel it necessary to push this limit further since it extends into the region of greatest observational uncertainty as a result of cosmic variance. The upper limits in both $\sigma_{\rm b}$ and $A_{\rm b}$ also reach the edges of our parameter space, indicating that the CMB data allow a lot of freedom with the amplitude and standard deviation of a bump. We find that at the 68 per cent confidence level $k_{\rm b} \le 0.014\,h\,{\rm Mpc}^{-1}$, $\sigma_{\rm b} \ge 0.15$ and $A_{\rm b} \ge 0.3$.

In Fig. 1 we show the best-fitting model as well as a range of $k_{\rm b}$ models with the same $A_{\rm b}$, $\sigma_{\rm b}$ and normalization to illustrate the effect of varying $k_{\rm b}$. In Fig. 2 we plot the best-fitting model together with models of varying $\sigma_{\rm b}$ and $A_{\rm b}$. From these figures it can be seen that, unlike models incorporating a high baryon content, our model predicts a low third acoustic peak. Also, these figures highlight the fact that low ℓ observations ($20 < \ell < 200$) are a critical discriminant of the bumps, because beyond the first acoustic peak the models become less distinguishable. This because the transfer

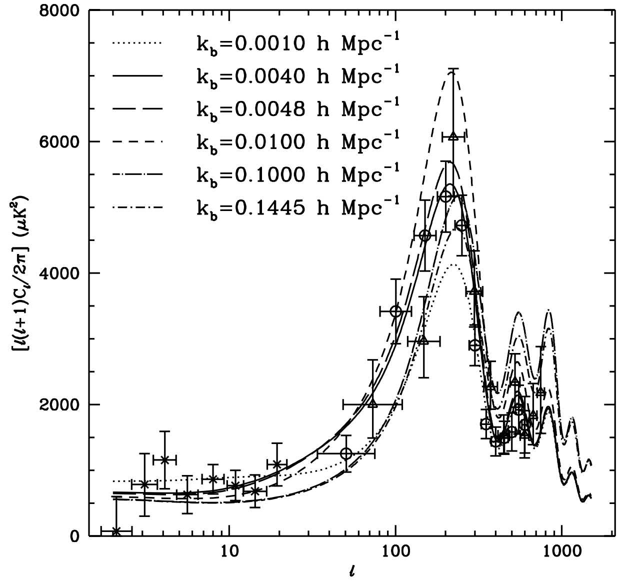

Figure 1. The observational data set of Table 1. The crosses indicate the 8 uncorrelated *COBE* DMR points (Tegmark & Hamilton 1997), the circles indicate the 12 BOOMERanG-98 data points (de Bernardis et al. 2000) and the triangles indicate the 10 MAXIMA-1 data points (Hanany et al. 2000). The solid curve shows the best-fitting model ($k_{\rm b} = 0.004\,h\,{\rm Mpc}^{-1}$, $\sigma_{\rm b} = 1.05$, $A_{\rm b} = 0.9$) normalized to the full observational data set. The remaining curves show the same model with varying $k_{\rm b}$ as indicated. All models are normalized to the best-fitting model.

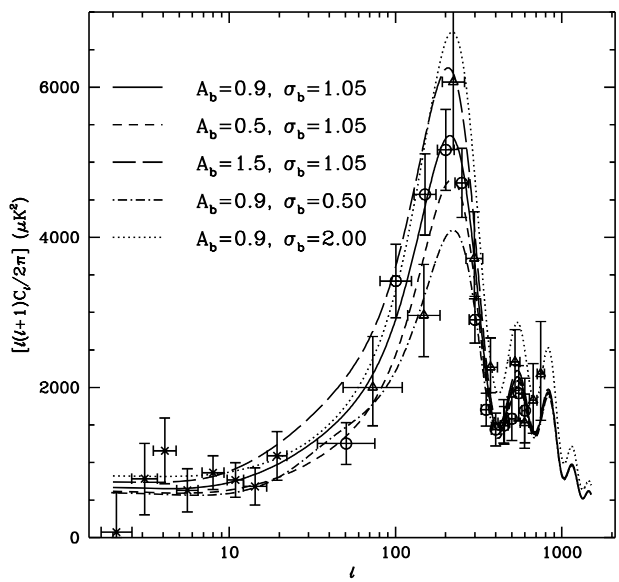

Figure 2. The same data sample as in Fig. 1. The solid curve shows the best-fitting model ($k_{\rm b} = 0.004\,h\,{\rm Mpc}^{-1}$, $\sigma_{\rm b} = 1.05$, $A_{\rm b} = 0.9$) normalized to the full observational data set. The dotted curve shows the same model with $\sigma_{\rm b} = 2.0$, the dotted–dashed curve the same model with $\sigma_{\rm b} = 0.5$ and the dashed curves the same model with $A_{\rm b} = 0.5$ (small dashes) and 1.5. All models are normalized to the best-fitting model.

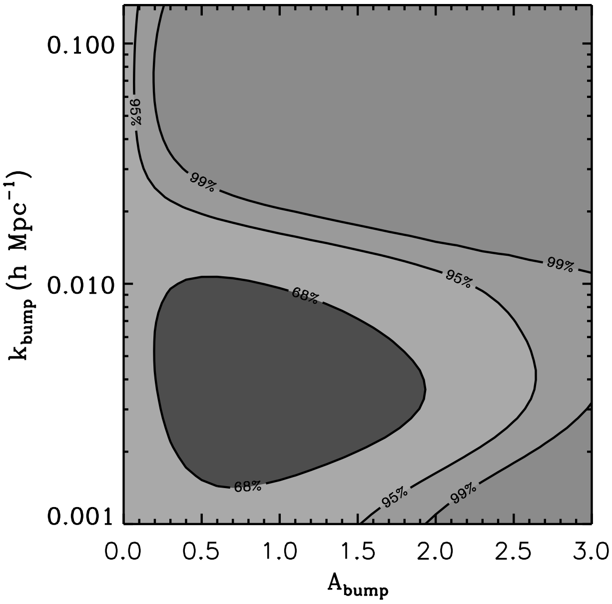

Figure 3. Confidence level contours for the concordance ΛCDM model as a function of k_b and A_b, σ_b fixed at 1.05. The region within the 68 per cent contour line is allowed at the 68 per cent confidence level.

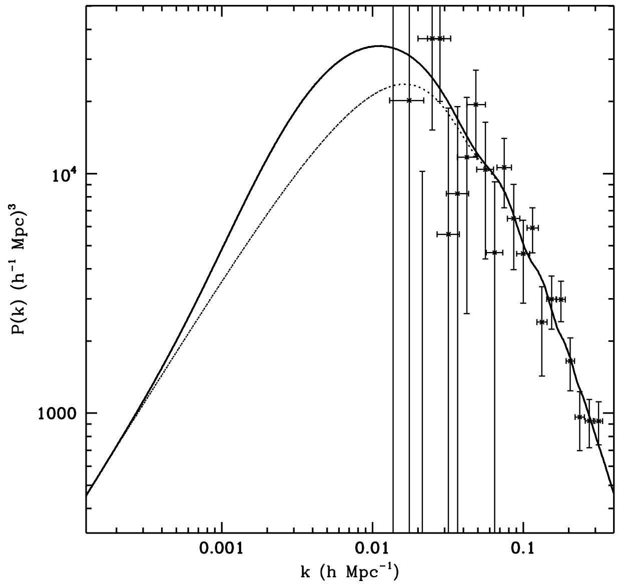

Figure 4. The PSCz decorrelated linear primordial power spectrum (Hamilton & Tegmark 2000). The solid curve shows the standard spatially flat cosmological model with a bump at $k_b = 0.004\,h\,\mathrm{Mpc}^{-1}$, normalized to the CMB data sample with a bias factor of 1.07. The dotted curve shows the standard model without the bump, normalized to the CMB data sample with a bias factor of 1.16.

function has a much sharper cut-off on the high ℓ side of the first peak, relative to the cut-off on the low ℓ side.

A confidence level con tour map of k_b versus A_b for the cosmology of interest with σ_b fixed at 1.05 is shown in Fig. 3. This indicates that, for the chosen cosmology, the scale at which a bump can appear in the primordial power spectrum is quite constrained by the CMB data sample. At the 68 per cent confidence level, we are limited to models with a bump at scales $k_b \leq 0.010\,h\,\mathrm{Mpc}^{-1}$ for this value of σ_b, although we have rather more freedom with the amplitude of the bump.

5 FROM CMB TO GALAXY SURVEYS

Since our incorporation of a bump into the primordial power spectrum of density perturbations was inspired by observation of large-scale structure, it is interesting to ask how our modified model compares with the decorrelated linear power spectrum that was recently generated from the Point Source Catalogue Redshift (PSCz; Hamilton & Tegmark 2000). We treat the 22 decorrelated PSCz data points as uncorrelated so that the theoretical model that we find to be the best fit to the CMB observational data set can be compared with the galaxy survey observations using a χ^2 test. The theoretical model is renormalized to the observational data set resulting in a hidden parameter. This allows for a bias factor b where

$$P(k)_{\mathrm{PSCz}} = b^2 P(k)_{\mathrm{CMB}}. \qquad (2)$$

We note that non-linearity corrections to the data are omitted in our comparison. In ΛCDM models, the effects of non-linearity in the matter–power spectrum are partially cancelled by galaxy-to-mass antibias, so that the PSCz power spectrum is close to linear all the way to $k = 0.3\,h\,\mathrm{Mpc}^{-1}$ [Hamilton, private communication].

Fig. 4 plots our best-fitting CMB normalized standard low-density cosmological model, with and without the bump, against the PSCz decorrelated linear power spectrum. Our best-fitting model has been renormalized to the PSCz data set with a bias parameter of 1.07 and the model without the bump takes a bias

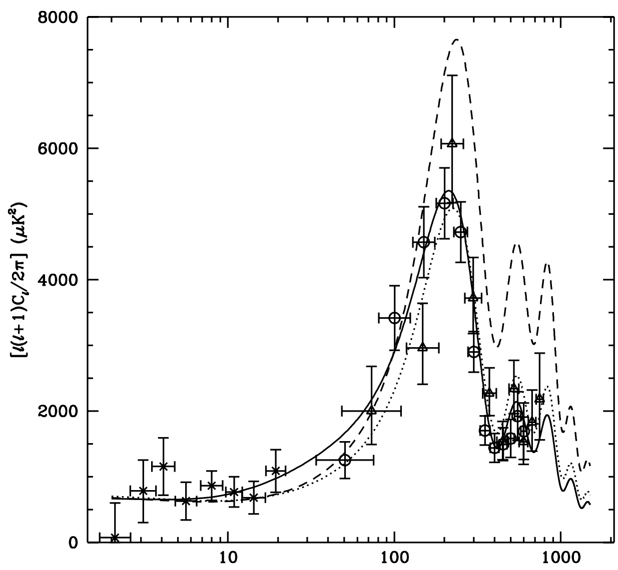

Figure 5. The same data sample as in Fig. 1. The solid curve shows the best-fitting model ($k_b = 0.004\,h\,\mathrm{Mpc}^{-1}$, $\sigma_b = 1.05$, $A_b = 0.9$), the dotted curve shows the standard ΛCDM model without a bump and the dashed curve shows the same model with a bump at $k_b = 0.052\,h\,\mathrm{Mpc}^{-1}$, $\sigma_b = 1.05$, $A_b = 0.9$. Each model is independently normalized to the full CMB observational data set.

parameter of 1.16. Both models are a very good fit to these up-to-date large-scale structure observations ($\chi^2_{\text{best-fit}} = 17.20$, $\chi^2_{\text{no bump}} = 16.35$, $\chi^2_{21} < 23.46$ at the 68 per cent confidence level), but it is clear that the current data do not probe the scales that are critical to discriminating between the models.

As stated in our introduction, several independent large-scale structure data sets provide suggestive, although not overwhelming, evidence that there is excess power relative to the scale-invariant ($n \approx 1$) fluctuation spectrum of the conventional model near $100\,h^{-1}$ Mpc, corresponding to $k_{\rm b} \sim 0.05\,h\,{\rm Mpc}^{-1}$ (see, e.g., Broadhurst et al. 1990; Landy et al. 1996; Einasto et al. 1997). Fig. 5 shows our best-fitting bump model, together with the standard ΛCDM model without a bump and the same model with a bump at $k_{\rm b} = 0.05\,h\,{\rm Mpc}^{-1}$, each model independently taking its optimal normalization to the full CMB data set. It is interesting to note that a bump in the primordial power spectrum of any amplitude or standard deviation at $k_{\rm b} = 0.05\,h\,{\rm Mpc}^{-1}$ is ruled out at the 95 per cent confidence level by the CMB observational data for the ΛCDM concordance model.

6 SUMMARY

We describe a toy model for a bump to be included in the primordial density power spectrum with the hope of reviving the standard model without resorting to revising fundamental cosmological theories. We have confronted our theoretical models with the recent BOOMERanG-98 and MAXIMA-1 data and have shown that it is indeed possible to resurrect the standard model, although we are somewhat restricted with regard to where we place our additional feature. We find that our model, unlike models that include a high baryon content, predicts a low third acoustic peak.

There are two CMB measurements that will help to discriminate between such a bump and cosmological alternatives for suppressing the second peak. One is, of course, the detection of the third peak. In addition, although low ℓ (20–100) observations have received relatively little attention and hence are currently a poor constraint on cosmological parameters, we have found that the low ℓ power is potentially a critical discriminant for the possible bump feature. This is because the transfer function has a much sharper cut-off on the high ℓ side of the first peak, relative to the cut-off on the low ℓ side.

Current galaxy redshift survey data suggest that excess power is required at a scale around 100 Mpc, corresponding to $k_{\rm b} \sim 0.05\,h\,{\rm Mpc}^{-1}$ (see, e.g., Broadhurst et al. 1990; Landy et al. 1996; Einasto et al. 1997). For the concordance paradigm, use of a bump-like feature to account for this excess is not consistent with the constraints from the CMB data. We note that models with an appropriately chosen break in the power spectrum provide an alternative model that can give distortions similar to those reported in the APM survey as well as consistency with the CMB data (Atrio-Barandela et al. 2000; Barriga et al. 2000). We prefer, however, to discount the APM data in favour of the less-biased decorrelated linear power spectrum recently constructed from the PSCz redshift survey (Hamilton & Tegmark 2000).

The incorporation of a bump in the primordial spectrum at a scale of $k_{\rm b} = 0.004\,h\,{\rm Mpc}^{-1}$, as the CMB data prefers, is in good agreement with the PSCz power spectrum with a bias parameter of 1.07. Future surveys such as 2DF and SDSS should be able to probe the large-scale structure power spectrum at the depths required to further test our conjecture. Large-scale velocity field data are useful only at higher k as a discriminant of bump-like features, and we will address this issue in a later paper.

ACKNOWLEDGMENTS

LMG would like to thank Pedro Ferreira and Andrew Liddle for providing productive insights into the analysis and understanding of this problem and Alessandro Melchiorri and Bill Ballinger for useful discussions. We acknowledge use of the Starlink computer system at the University of Oxford and use of the CMBFAST code of Seljak & Zaldarriaga (1996).

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