Statistical Signal Processing
Assignment 6a

Note: **C**: Computer Assignment, **W**: Workout Problem. For computer assignments, please submit both the code (properly commented) and the results in form of plots (properly labelled).

1. (**C**) Load Image.npy in Python/Matlab. The file contains a 2D array describing a grayscale image.

   (a) Compute the singular value decomposition (SVD) of this image and reconstruct the image using 1, 2, 5, 10, 50, 100 and 200 singular values found using SVD and plot the singular values $diag(\mathbf{W})$ vs. index.

   (b) Now add gaussian random noise ($\sigma = 10,\ 50$ & 100) to the image and plot the singular values found using SVD. What do you observe ? Interpret the results (you can also compare the reconstructed images using 10,50 & 100 singular values).

2. (**C**) Consider the following functions: $f(x) = |0.3x^3{+}0.6x^2|$, $g(x) = 2\sin^2(3x/2)$, $h(x) = e^{-2x^2} + e^{x/2}$.

   (a) Use these functions and their combinations, $f(x),\ g(x),\ h(x),\ \sqrt{f(x)+g(x)}$, $\sqrt{f(x)g(x)},\ \sqrt{f(x)+h(x)},\ \sqrt{f(x)h(x)},\ g(x){+}h(x),\ \sqrt{f(x)+g(x)+h(x)}$, $2\sqrt{f(x)g(x)h(x)}$, to produce 2D array (10×400) for $-2 \leq x \leq 2$. Plot these functions.

   (b) Calculate weighted (use uniform random samples from 0 to 9 for the weights) mean of the above functions and add gaussian random noise to the mean. Generate 1000 different realizations using $\sigma = 1.0$ & 0.1 to produce a 1000×400 matrix (G).

   (c) Calculate the Covariance matrix for the realizations, $Cov(\mathbf{G}) = \mathbf{G}^T\mathbf{G}$. $Cov(\mathbf{G})$ will be a 400×400 matrix. Perform SVD on $Cov(\mathbf{G})$ to calculate $U, W,$ & V.

   (d) Plot $Cov(\mathbf{G})$, singular values $diag(\mathbf{W})$ vs. index (use same plot for both variances) and 1st 10 singular eigenvectors (first 10 columns of U) for both noise realizations.