Statistical Signal Processing
Assignment 5

Submission deadline: 6th, January, 2016

Note: **C**: Computer Assignment, **W**: Workout Problem. For computer assignments, please submit both the code (properly commented) and the results in form of plots (properly labelled).

1. (**W**) For the signal model,

$$x[n] = s[n] + w[n], \text{ where } s[n] = \sum_{i=1}^{p} A_i \cos(2\pi f_i n), \quad n = [0, 1, ..., N-1]$$

   where frequencies f_i are known and the amplitudes A_i are to be estimated. $w[n]$ is White Gaussian Noise (WGN) with distribution $\mathcal{N}(0, \sigma^2)$.

   (a) If the frequencies are known to be $f_i = i/N$, find the Least Squares Estimator (LSE).
   (Hint: Write down $s[n]$ as $\mathbf{s} = \mathbf{H}\theta$. For the given frequencies, the columns of $\mathbf{H}$ are orthogonal.)

   (b) Determine the PDF of LSE by calculating it's Expectation and Variance.

2. (**C+W**) Consider the two parameter model for a straight line fit given by

$$x_i = A + Bi + w_i, \quad i = [1, 2, ...N]$$

   where A, and B are parameters to be estimated, and w_i are IID random variables drawn from $\mathcal{N}(0, \sigma^2)$.

   (a) Compute the LSE for A and B.

   (b) Generate mock data with $A = 3,\ B = 5,\ \sigma^2 = 1$, and $N = 20$. Use the estimators derived in (a) and calculate $\hat{A}$ and $\hat{B}$ for 10000 different noise realizations. Compute and plot(colormap preferably) the joint PDF of $\hat{A}$ and $\hat{B}$.

3. (**W**) The data $x_i = A + w_i$ for $i = [1, 2, .., N]$ are observed. The unknown parameter A is assumed to have the prior PDF

$$p(A) = \begin{cases} \lambda e^{-\lambda A}, & A > 0 \\ 0, & A < 0 \end{cases}$$

   where $\lambda > 0$ and w_i is WGN with variance σ^2 and independent of A. Find the Maximum A Posteriori (MAP) estimator of A.

4. (**C+W**) Workout the mathematics of Sequential Least Squares (see section 8.7 in Steven M. Kay book) and generate the Variance, Gain and Estimate evolution plots (fig. 8.9 in the book) for the simple model to estimate the DC level $x_i = A + w_i$. Use $\sigma^2 = 1$ for the noise.