Applied Signal Processing – 18/4/2007 13.00-16.00h

Please put your name and student number on all of your answer sheets. The final grade for this exam will be $\frac{9}{10.5}(\textit{total number of points})+1$.

1. Find the z-transform of the following sequences. Wherever convenient, use the properties of the z-transform to make the solution easier:

(a) $x[n]=\left(\frac{1}{2}\right)^n \mu[-n]$ **(0.5)**

(b) $x[n]=\left(\frac{1}{3}\right)^n \mu[n]+4^n \mu[-n-1]$ **(0.5)**

(c) $x[n]=n\left(\frac{1}{2}\right)^n \mu[n+1]$ **(0.5)**

2. Consider the discrete-time LTI system described by the following simple difference equation:

$$y[n]=x[n]-x[n-1]$$

PART I

(a) Determine the impulse response of this system, $h[n]$, and plot $h[n]$.

(b) Determine and write a closed-form expression for the DTFT, $H(e^{i\omega})$, of $h[n]$. $H(e^{i\omega})$ is the frequency response of the system.

(c) Plot the magnitude $|H(e^{i\omega})|$ over the range $-\pi<\omega<\pi$

(d) Plot the phase of $H(e^{i\omega})$ over $-\pi<\omega<\pi$ **((a)-(d) – 1.5 points)**

PART II

Given $x_a(t)=3(1-|t|)\,\{\mu(t+1)-\mu(t-1)\}$, where $\mu(t)=\begin{matrix}1, t\geqslant 0\\ 0, t<0\end{matrix}$.

Observe that $x_a(t)$ has a triangular shape of height 3 and a duration of 2 seconds centered at t=0.

Let $x[n]=x_a(nT_s)$, where $T_s=\frac{1}{3}$. This means that $x[n]$ is obtained by sampling $x_a(t)$ at the rate of 3 samples per second.

(e) Plot $x[n]$

(f) Determine and write a closed-form expression forthe DTFT, $X(e^{i\omega})$, of $x[n]$

(g) Plot the magnitude $|X(e^{i\omega})|$ over $-\pi<\omega<\pi$ **((e)-(g) – 2 points)**

PART III

(h) Determine and plot the output signal $y[n]$ when the sampled signal $x[n]$ is input to the system $y[n]=x[n]-x[n-1]$

(i) Determine and write a closed-form expression for the DTFT, $Y(e^{i\omega})$, of $y[n]$

(j) Plot the magnitude $|Y(e^{i\omega})|$ over $-\pi<\omega<\pi$

(k) Determine the numerical value of $\sum_{n=-\infty}^{n=\infty} y^2[n]$. **((h)-(k) – 1.5 points)**

3. Consider the discrete-time LTI system defined by the transfer function

$$H(z)=\frac{20-24z^{-1}+20z^{-2}}{(2-z^{-1})(2+2z^{-1}+z^{-2})}$$

(a) Draw the pole-zero diagram of $H(z)$. **(0.5)**

(b) Given that the impulse response $h[n]$ of this system is causal, what is the Region of Convergence? **(0.5)**

(c) Draw a block diagram which implements this transfer function in cascade form, using 1st and 2nd order sections. **(1)**

<u>4.</u> Given the second orderband stop filter with transfer function

$$H_{BS}(z) = \frac{\kappa(1-2\beta z^{-1}+z^{-2})}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}$$

where α, β and γ are real constants, with $|\alpha|<1, \ |\beta|<1$

(a) Determine α and β so that filter has a notch at $\omega_0 = 0.3\pi$ and a band width of 0.3π . **(1)**

(b) What is the quality factor of the filter? **(0.5)**

(c) Draw the magnitude response of $H_{BS}(z)$ **(0.5)**