Problem Set 5

Due: 14^{th} June 2007

Exercise 1

We observe synchrotron radiation emission all along the sky from the Milky Way. According to these observations, the brightness temperature within 3 degrees of the galactic disk, at a frequency 408 MHz, is typically 150 K, and has a spectrum $P(\nu) \propto v^{-1}$ near this frequency. Assume that a typical line of sight goes through a pathlength of 10 kpc of a region with a uniform magnetic field $B = 3\,\mu$G, and assume a number density of relativistic electrons $n(\gamma)d\gamma = C_0\gamma^{-p}d\gamma$, where $C_0 = 3.6 \times 10^{-4}$ cm^3.

(a) What is the value of p of the energy distribution of the electrons?

(b) What is the γ factor of the electrons which contribute most of the emission at the frequency of 408 MHz?

(c) How long does it take for the electrons with this γ to slow down?

(d) What is the energy density in relativistic electrons with γ factor larger than the value computed in part (b)? Compare this energy density to the magnetic field energy density.

(e) Show how to calculate the value of C_0 and verify that you get $C_0 = 3.6 \times 10^{-4}$ cm^3.

Exercise 2

An ultrarelativistic electron emits synchrotron radiation. Show that its energy decreases with time according to:

$$\gamma(t) = \frac{\gamma_0}{1 + A\gamma_0 t}, \qquad \text{where} \qquad A = \frac{2e^4 B_\perp^2}{3\,m^3 c^5}. \tag{1}$$

Here γ_0 is the initial value of γ and $B_\perp = B\sin\alpha$. Show that the time for the electron to lose half its energy is

$$t_{1/2} = \frac{5.1 \times 10^8}{\gamma_0 B_\perp^2} \quad \text{seconds.} \tag{2}$$

Exercise 3

Single pulses from a pulsar are recorded at a radiotelescope with bandwidth $\Delta\nu = 100$ MHz and centre observing frequency $\nu_{\rm m} = 1200$ MHz. The pulse arrival time differs by $\Delta t = 48.7$ ms from one end of the band to the other.

(a) What is the dispersion measure DM of this pulsar?

(b) Assuming that the average electron density in the interstellar medium is $\langle n_e \rangle = 0.03\ \mathrm{cm}^{-3}$, estimate the distance L of the pulsar.

(c) Verify that $\nu \gg \nu_\mathrm{p}$ for the given conditions (where ν_p is the plasma frequency) and therefore the previous derivation is correct.

Exercise 4

The spectrum shown in Fig. (6.14) of Ribicky & Lightman is observed from a point source of unknown distance d. The measured flux peaks at F_0.

- A model for this source is a spherical mass of radius R that is emitting synchrotron radiation in a magnetic field of strength B.
- The space between us and the source is uniformly filled with a thermal bath of hydrogen that emits and absorbs mainly by bound-free transitions, and it is believed that the hydrogen bath is unimportant compared to the synchrotron source at frequencies where the former is optically thin.
- The synchrotron source function can be written as

$$S_\nu = A \left(\frac{B}{B_0} \right)^{-1/2} \left(\frac{\nu}{\nu_0} \right)^{5/2} , \tag{3}$$

  where A has dimensions of erg cm^{-2} s^{-1} Hz^{-1}.

- The absorption coefficient for synchrotron radiation is

$$\alpha_\nu^{\mathrm{sync}} = C \left(\frac{B}{B_0} \right)^{(p+2)/2} \left(\frac{\nu}{\nu_0} \right)^{-(p+4)/2} , \tag{4}$$

  where C has dimensions of cm^{-1}.

- The absorption coefficient for bound-free transition is

$$\alpha_\nu^{\mathrm{bf}} = D \left(\frac{\nu}{\nu_0} \right)^{-3} , \tag{5}$$

  where D has dimensions of cm^{-1}.

- Note that A, B_0, ν_0, C and D are constants and p is the power-law index for the assumed power-law distribution of relativistic electrons in the synchrotron source.

Questions:

(a) Find the size of the source R and the magnetic field strength B in terms of the solid angle $\Omega = \pi(R^2/d^2)$ subtended by the source, of ν_2 and of the mentioned constants.

(b) Now using also D and ν_1, in addition to the previous constants, find the solid angle Ω of the source and its distance d.

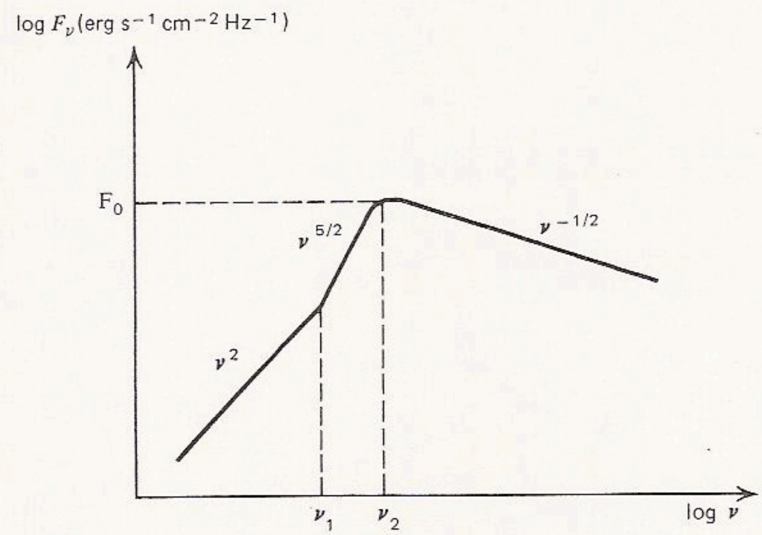

Figure 6.14 Observed spectrum from a point source.

Some quantities which might be useful:

- Stefan-Boltzmann constant: $\sigma_{\rm SB} = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.
- Gravitational constant: $G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}.
- Speed of light (exact): $c = 2.99792458 \times 10^{10}$ cm s^{-1}.
- Boltzmann constant: $k = 1.380658 \times 10^{-16}$ erg K^{-1}.
- Classical electron radius: $r_0 = \frac{e^2}{mc^2} = 2.8179 \times 10^{-13}$ cm.
- Electron mass: $m_e = 9.109 \times 10^{-28}$ g $= 0.511$ MeV/c^2.
- Proton mass: $m_p = 1.673 \times 10^{-24}$ g $= 938.3$ MeV/c^2.
- Electron charge: $e = 1.602 \times 10^{-19}$ C $= 4.803 \times 10^{-10}$ statC.
- Thomson cross section: $\sigma_T = 6.6524 \times 10^{-25}$ cm^2.
- Solar mass: $M_\odot = 1.989 \times 10^{33}$ g $\simeq 2 \times 10^{33}$ g.
- Astronomical unit: 1 AU $= 1.4960 \times 10^{13}$ cm $\simeq 1.5 \times 10^{13}$ cm.
- Parsec: 1 pc $= 3600\frac{180}{\pi}$ AU $= 3.0857 \times 10^{18}$ cm.
- Acceleration due to gravity on the Earth's surface: $g = 9.80665 \times 10^2$ cm s^{-2}.