Astrophysics A

Problem Set 3

Due: 29th May 2007

1. A particle of mass m and charge e moves at constant, non-relativistic speed $v_\perp$ in a circle of radius R.
   a. what is the power emitted per unit solid angle in a direction at angle θ to the axis of the circle?
   b. describe qualitatively and quantitatively the polarization of the radiation as a function of the angle θ.
   c. what is the spectrum of the emitted radiation?
   d. suppose that the considered particle is rotating because of the presence of a magnetic field **B** directed along z. Find the frequency of circular motion ω_B and show that the total emitted power is:
   $$P = \frac{2}{3} r_0^2 c \left(\frac{v_\perp}{c}\right)^2 B^2$$
   and is emitted solely at the frequency ω_B.

2. A particle (rest mass m) initially at rest absorbs a photon energy $h\nu$ and converts this into increased internal energy (heat). The particle has thus increased its rest mass to m' and moves with velocity v'.
   a. setting up the conservation of energy and momentum, show that
   $$\frac{m}{m'} = \left(1 + \frac{2h\nu}{mc^2}\right)^{-1/2}$$
   b. by considering the appropriate Lorentz transformations, show that if the particle had been moving initially and absorbed a photon of energy $h\nu$, this same equation for the ratio of the initial and final rest masses holds with ν' replacing ν, where ν' is given by the Doppler formula.

3. The generally accepted model for a pulsar is a rapidly rotating neutron star with a strong magnetic field B_0. A neutron star is formed through the collapse of a massive star at the end of its nuclear burning lifetime; its original magnetic field is frozen in with the plasma, and becomes highly concentrated during the collapse, explaining the large value of B_0. If the magnetic axis of the neutron star is not aligned with the rotation axis, magnetic dipole radiation results from the time varying magnetic dipole $\mathbf{m}(t)$. Can work out the characteristics of the radiation by analogy to electric dipole radiation. If M is the mass of the neutron star and R its radius, the angle between the magnetic and rotation axes is α, and the rotation angular velocity is ω.

a. find an expression for the radiated power P in terms of ω, R, B_0, and α.

b. the rotational energy of the pulsar is the ultimate source of the radiated energy. Therefore, the pulsar inevitably has to slow down. Find an expression of the slow-down time scale τ of the pulsar. Approximate the neutron star as a homogeneous body, for which the rotational energy is given by $E_{rot} = 1/5\, MR^2\omega^2$.

c. For $M = 1M_\odot$, $R = 10^6$ cm, $B_0 = 10^{12}$ Gauss, $\alpha = 90°$, find P and τ for the following rotation rates: $\omega = 10^4$, 10^3, and 10^2 s^{-1}.

4. Prove that the relativistic Larmor's formula is [eq. (4.92) RL]:

$$P = \frac{2q^2}{3c^3}\gamma^4\left(a_\perp^2 + \gamma^2 a_\parallel^2\right)$$

to this purpose

a. Calculate the transformation of the acceleration.
In other words, if a point has an acceleration (a'_x, a'_y, a'_z) in the frame K' which moves with velocity v with respect to the frame K, calculate the acceleration (a_x, a_y, a_z) in the frame K.
Hint: make use of the following definition:

$$\sigma \equiv 1 + \frac{vu'_x}{c^2}$$

b. If K' is the instantaneous rest frame of the particle, show that:

$$a'_\parallel = \gamma^3 a_\parallel \qquad \text{and} \qquad a'_\perp = \gamma^2 a_\perp$$

these are are the components parallel and perpendicular to the direction of v, respectively