Astrophysics A

Problem Set 2

Due: 21st May 2007

1. Two oscillating dipole moments (i.e. radio antennas) $\mathbf{d}_1$ and $\mathbf{d}_2$ are oriented in the vertical direction (along z) and are a horizontal distance L apart. They oscillate in phase at the same frequency ω. Consider radiation at angle θ with respect to the vertical and in the vertical plane containing the two dipoles.
   a. Write an expression for the quantity $< dP/d\Omega >$ (the time-averaged energy per unit time per unit solid angle).
   Hint: you might find useful to define the quantity: $\delta = \omega L \sin\theta / c$
   b. From the previous result, show that when $L << \lambda$, the radiation is the same as from a single oscillating dipole of amplitude $\mathbf{d}_1 + \mathbf{d}_2$.
2. Venus has a thick cloud layer that reflects light very well. At IR wavelengths these clouds are opaque and the temperature measured at this λ is only 225 K. However, radio waves with wavelengths $\lambda > 30$ cm can easily propagate through the atmosphere of Venus (but they are still absorbed and emitted by the soil at the planet's surface). Knowing this, and making motivated approximations if necessary, answer the following questions:
   a. At what frequency does the atmosphere of Venus become optically thin to radio waves? Is it correct to assume that we are in Rayleigh-Jeans regime?
   b. The radio flux received at the Earth from Venus at 30 cm is $F_\nu \approx 0.23$ Jy (the Jansky, $1\ \mathrm{Jy} = 10^{-26}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$) when Venus is at quadrature. Calculate the temperature of Venus at its surface.
   c. If Venus is observed with a 25 m dish radio telescope, with a bandwidth $\Delta\nu = 5$ MHz, what will be the total power detected by the receiver?

3. An absorbing (i.e. black) solar sail spacecraft of $m_{sp} = 1.2 \times 10^4$ kg is launched from the Earth.
   a. How large must be the sail to allow the spacecraft to leave the Solar System?
   b. How large must be the sail to allow the spacecraft to have an (initial) acceleration $a_{sp} = g$?
   c. In the last case, make a zero-order approximation of the time required for the spacecraft to reach a velocity $v_{sp} = 0.01$
   d. How would the previous results change if the sail is now taken to be perfectly reflecting?
4. In complex notation an electrical vector can be described as

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$$

with $\mathbf{E}_0 = (E_1, E_2)$ in an (x, y)-axes system. The complex amplitudes E_1 and E_2 are given by:

$$\begin{aligned} E_1 &= \mathcal{E}_1 e^{i\phi_1} \\ E_2 &= \mathcal{E}_2 e^{i\phi_2} \end{aligned}$$

It is known that $\varepsilon_1 = 0.7$ Vm^{-1}, $\varepsilon_2 = 3.2$ Vm^{-1}, $\phi_1 = 20^o$ $\phi_2 = 37^o$

   a. Calculate the four Stokes Parameters
   b. The following four Stokes parameters are measured

$$\begin{aligned} I &= 513.6\ \mathrm{V^2\,m^{-2}} \\ Q &= 117.2\ \mathrm{V^2\,m^{-2}} \\ U &= 227.8\ \mathrm{V^2\,m^{-2}} \\ V &= 445.3\ \mathrm{V^2\,m^{-2}} \end{aligned}$$

Calculate ε_1, ε_2, ϕ_1 and ϕ_2

c. At an observing frequency of 60 Mhz the following experiment has been carried out. In the optical path of a telescope a polarising plate is placed. This plate only transmits emission with **E** directed along one direction. The results are shown in figure.

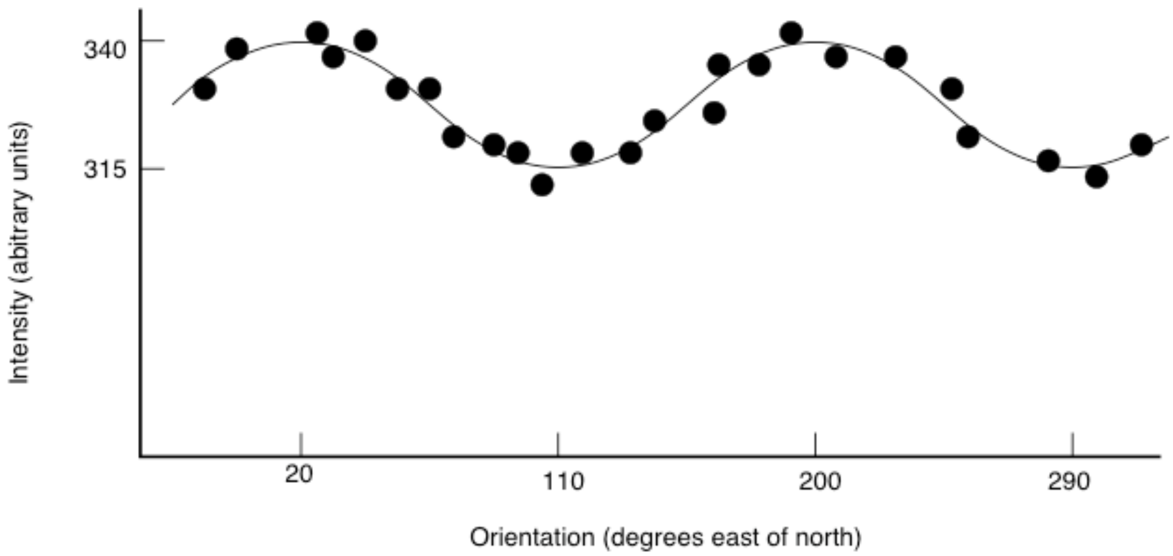

Figure 3.4: Measured intensity at 60 MHz.

Describe what experiment has been carried out to obtain the measurements. What can be deduced about the polarisation state of the observed emission qualitatively and quantitatively?