Red Arrows in Orbit

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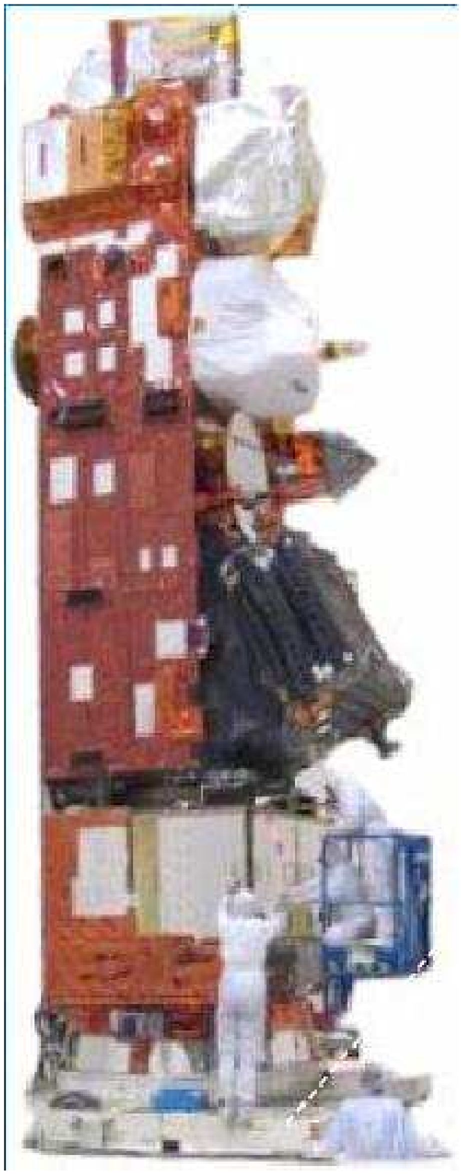
SURREY SPACE CENTRE

Satellites in Orbit

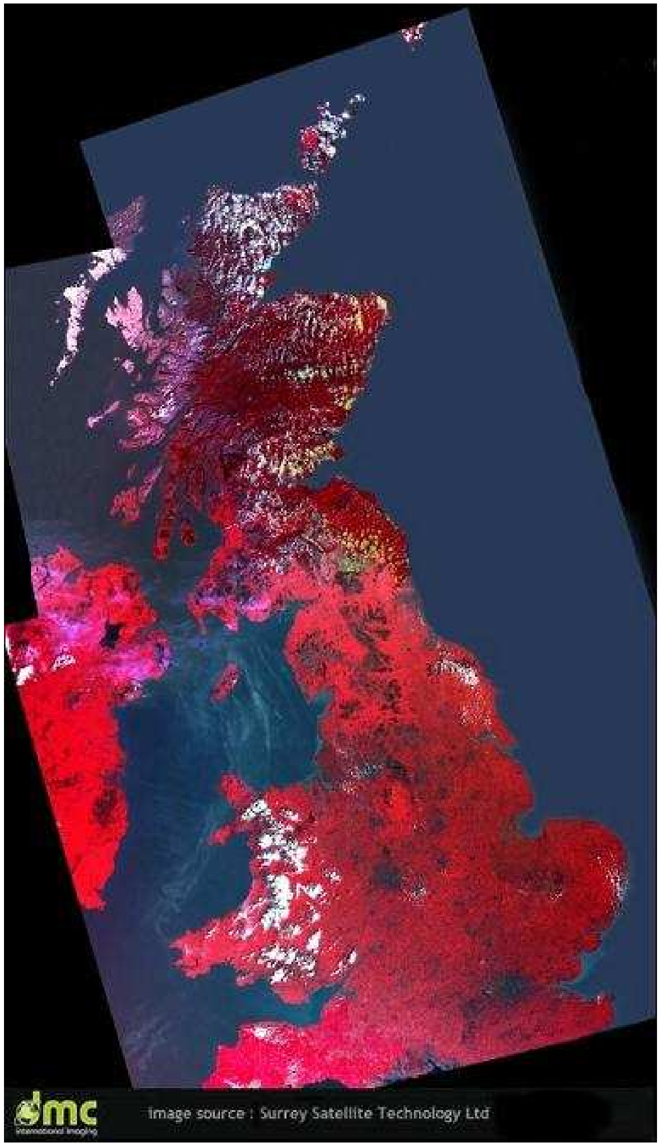
Mission	Year	Launch	Mass	Mission	Year	Launch	Mass
Giove-A	2006	Cosmos	400	CERISE	1995	Ariane	70
TopSat	2005	Cosmos	150	FASat-A	1995	Tsyklon	70
Beijing-1	2005	Cosmos	100	HealthSat-2	1993	Ariane	70
DMC	2003	Cosmos	4x100	PoSat-1	1993	Ariane	70
PICOSat	2001	Athena	70	KITSAT-1	1992	Ariane	70
Tsinghua-1	2000	Cosmos	70	S80/T	1992	Ariane	70
SNAP-1	2000	Cosmos	8	UoSat-5	1991	Ariane	50
UoSat-12	1999	Dnepr	400	UoSat-4	1990	Ariane	50
Clementine	1999	Ariane	70	UoSat-3	1990	Ariane	50
FASat-B	1998	Zenit	70	UoSat-2	1984	Delta	50
Thai-Phatt	1998	Zenit	70	UoSat-1	1981	Delta	50

Future of Satellites



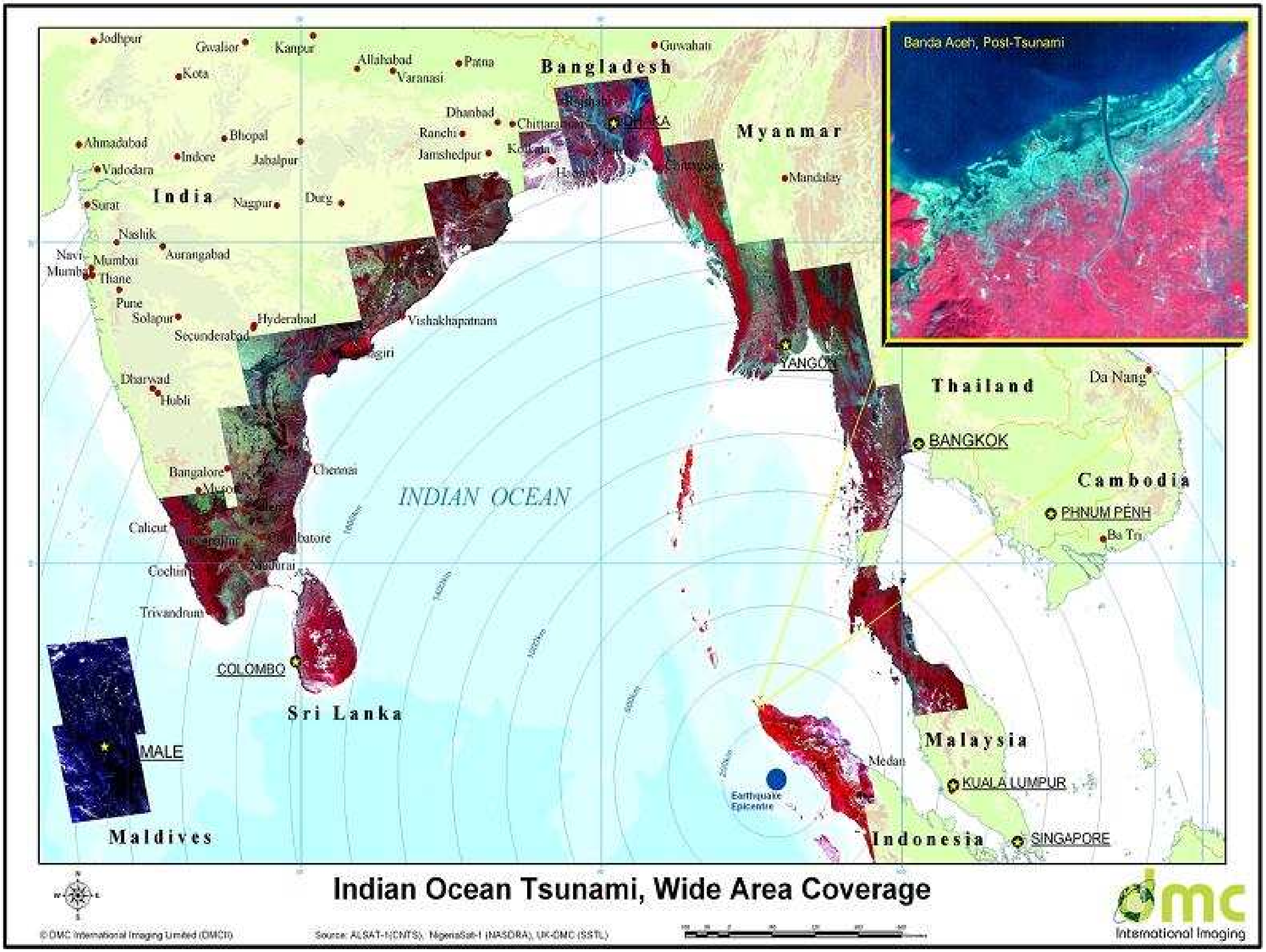
OLD MODEL	FUTURE?
Tonnes	<10 - 100's kg
Multi-sensor	Single or Multi sensor
Expensive (£10^5)	Cheap (£10^4)
Lead time (10-15 yrs)	6-18 months
Few sats	Many sats
Competition for data	High data rates
High risk	Low risk
State of art +10 years	COTS +1 year

DMC Images



- DMC Images 600 km swath
- Whole UK in 4 images
- Resolution 32 metres
- Landsat requires 5 years
- SPOT = 30 x DMC Sats

Asian Tsunami



Formation Flying: Pros & Cons

- ✔ Redundancy - robust to single point failure
- ✔ Flexibility - adaptable to changes in mission
- ✔ Synthesise more complex satellites
- ✘ Collision and Contamination risk
- ✘ Complexity of command and control
- ✘ Distributed knowledge - inter-satellite communication

Relative Motion Models

- Hills Equations (Hill, Clohessy & Wiltshire)
  - ✘ valid only over short timescales
- Nonlinear Models (Karlgaard & Lutz)
  - – incorporates curvature of circular orbit
  - ✘ ignores other perturbations
- Inclusion of J_2 (Schweighart & Sedgwick, Alfriend & Schaub)
  - – considers both secular evolution and periodic variations
    - ✘ much greater complexity and valid for few orbital periods
- Eccentric Orbits (Melton et al)
  - – employs variable rotating frame
    - ✘ not time explicit and provides little insight

Formation Assembly

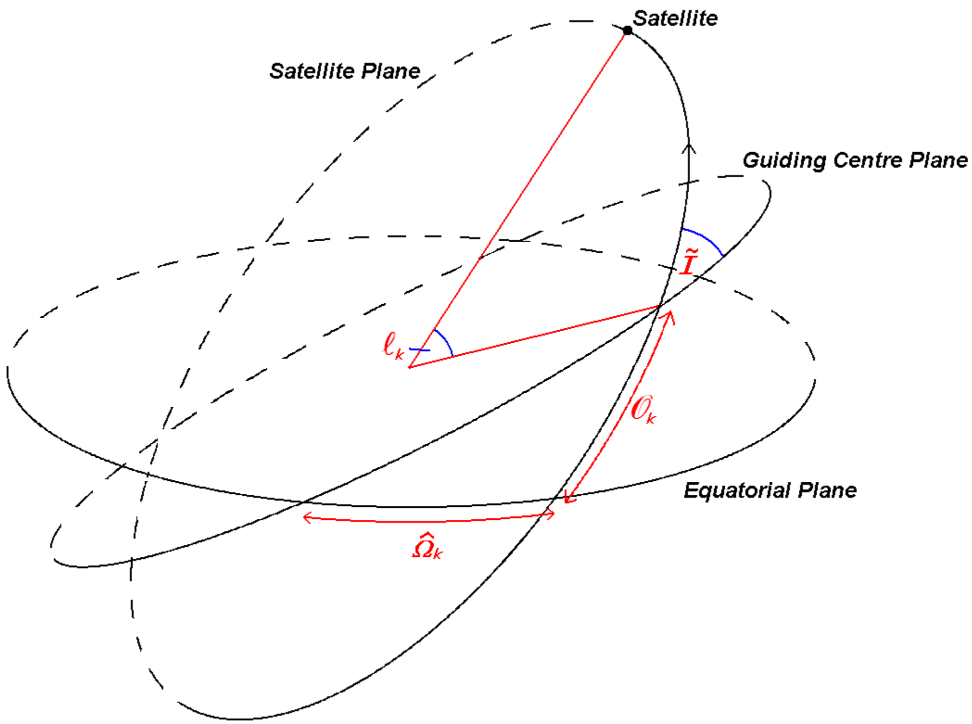
$$\ddot{x} - 2\Omega\dot{y} - 3\Omega^2 x = T_x$$
$$\ddot{y} + 2\Omega\dot{x} = T_y$$
$$\ddot{z} + \Omega^2 z = T_z$$

$$\boxed{\begin{aligned} T_x(t) &= \tfrac{2}{3}T_1 + \tfrac{\Lambda}{2}\sin(\Omega t - \Phi) \\ T_y(t) &= T_0 - T_1(\Omega t) + \Lambda\cos(\Omega t - \Phi) \\ T_z(t) &= \Gamma\cos(\Omega t - \zeta) \end{aligned}}$$

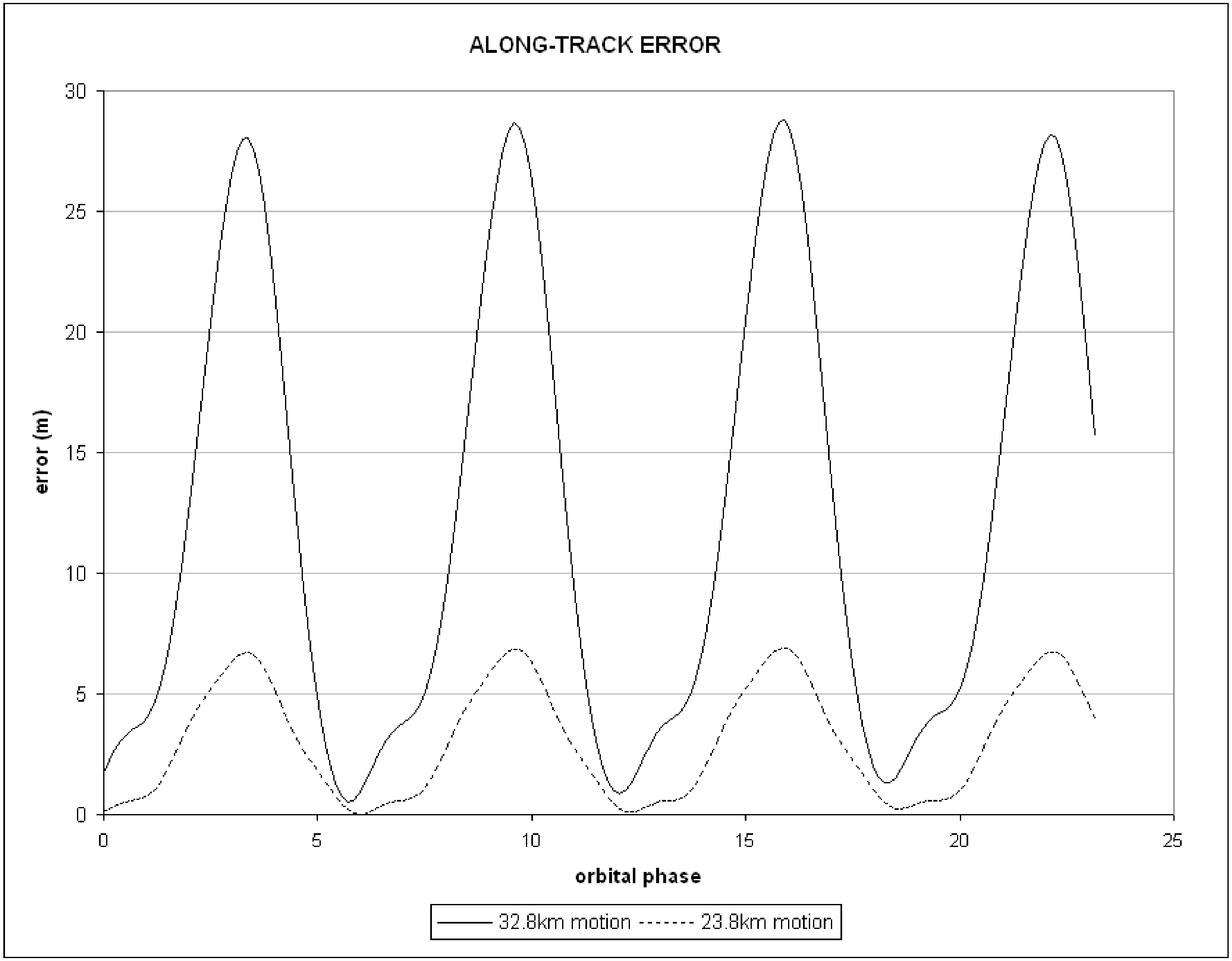
where $(T_0, T_1, \Lambda, \Gamma, \zeta, \Phi)$ are all functions of the boundary conditions and t_F.

Formation Design Tool

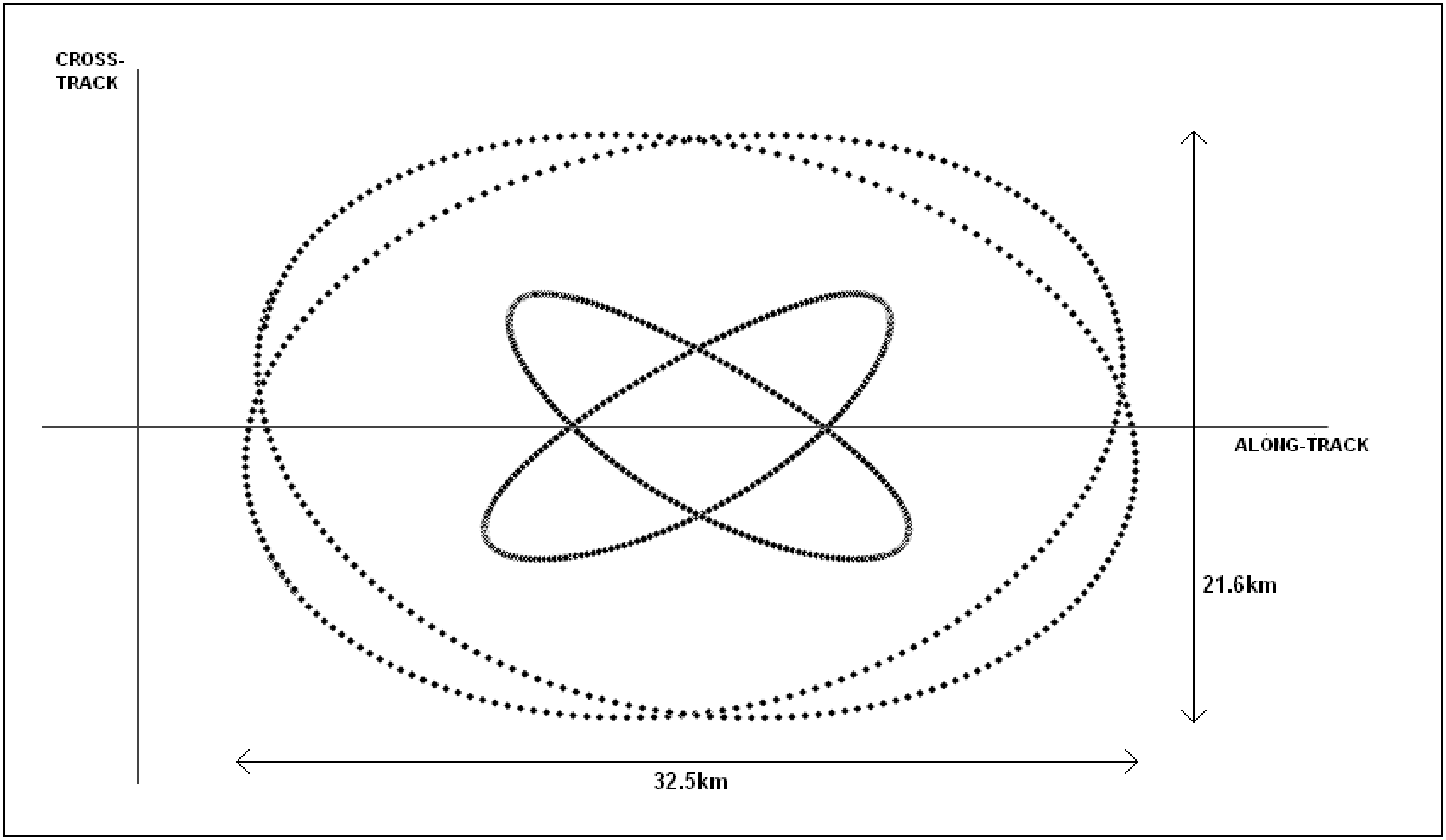
$$r = a(1+\rho) - ae\cos(\alpha - \alpha_P) + a\chi\sin\alpha + \Delta r_2^1 \cos 2\alpha$$
$$\lambda = \alpha(1+\kappa) + 2e\left[\sin(\alpha-\alpha_P) + \sin\alpha_P\right] - 2\chi\left[1-\cos\alpha\right] + \Delta\lambda_2^1 \sin 2\alpha$$
$$I = I_0 + \Delta I_2^1(1-\cos 2\alpha)$$
$$\Omega = \Omega_0 + \theta\alpha + \Delta\Omega_2^1 \sin 2\alpha$$
$$\alpha = n(t - t_e)$$



Model Accuracy



Square Formation



Relative Keplerian Motion

The difference of the Hamiltonians for two satellites at coordinates $(\mathbf{r} \pm \frac{1}{2}\delta\mathbf{r}, \mathbf{v} \pm \frac{1}{2}\delta\mathbf{v})$ expanded around $(\mathbf{r}, \mathbf{v})$:

$$H_R = H_1 - H_2 = \mathbf{v}.\delta\mathbf{v} + \frac{\mu}{|\mathbf{r}|^3}(\mathbf{r}.\delta\mathbf{r})$$

Equations of motion from Hamilton's extended equations:

$$\dot{\mathbf{r}} = \frac{\partial H_R}{\partial \delta\mathbf{v}} \qquad \dot{\mathbf{v}} = -\frac{\partial H_R}{\partial \delta\mathbf{r}} \tag{1}$$

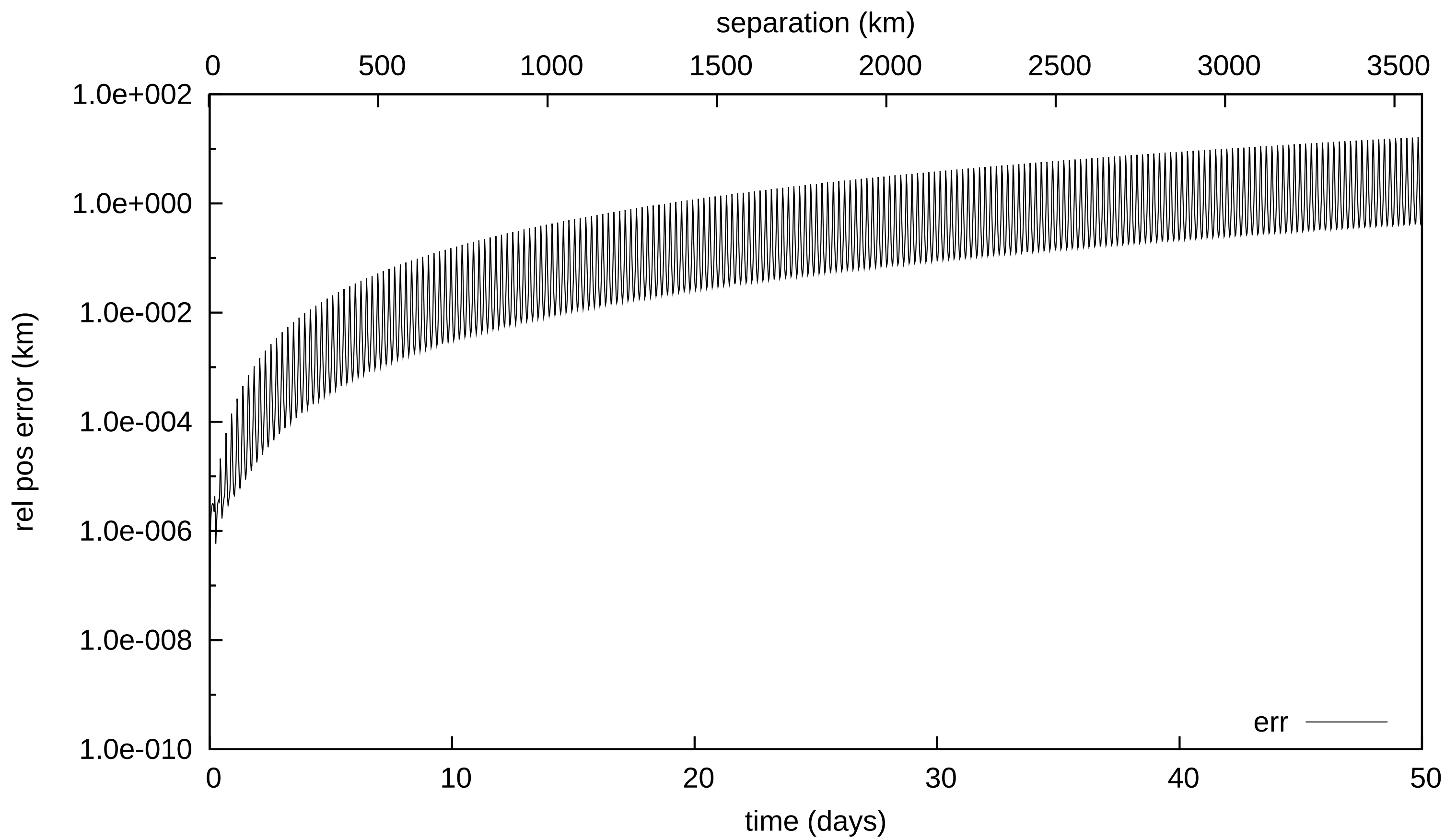
$$\delta\dot{\mathbf{r}} = \frac{\partial H_R}{\partial \mathbf{v}} \qquad \delta\dot{\mathbf{v}} = -\frac{\partial H_R}{\partial \mathbf{r}} \tag{2}$$

The secular drift due to energy differences can be separated:

$$\delta\mathbf{r} = \delta\mathbf{r}_p - \frac{H_R}{H}(\mathbf{r} - \frac{3}{2}\mathbf{v}t)$$

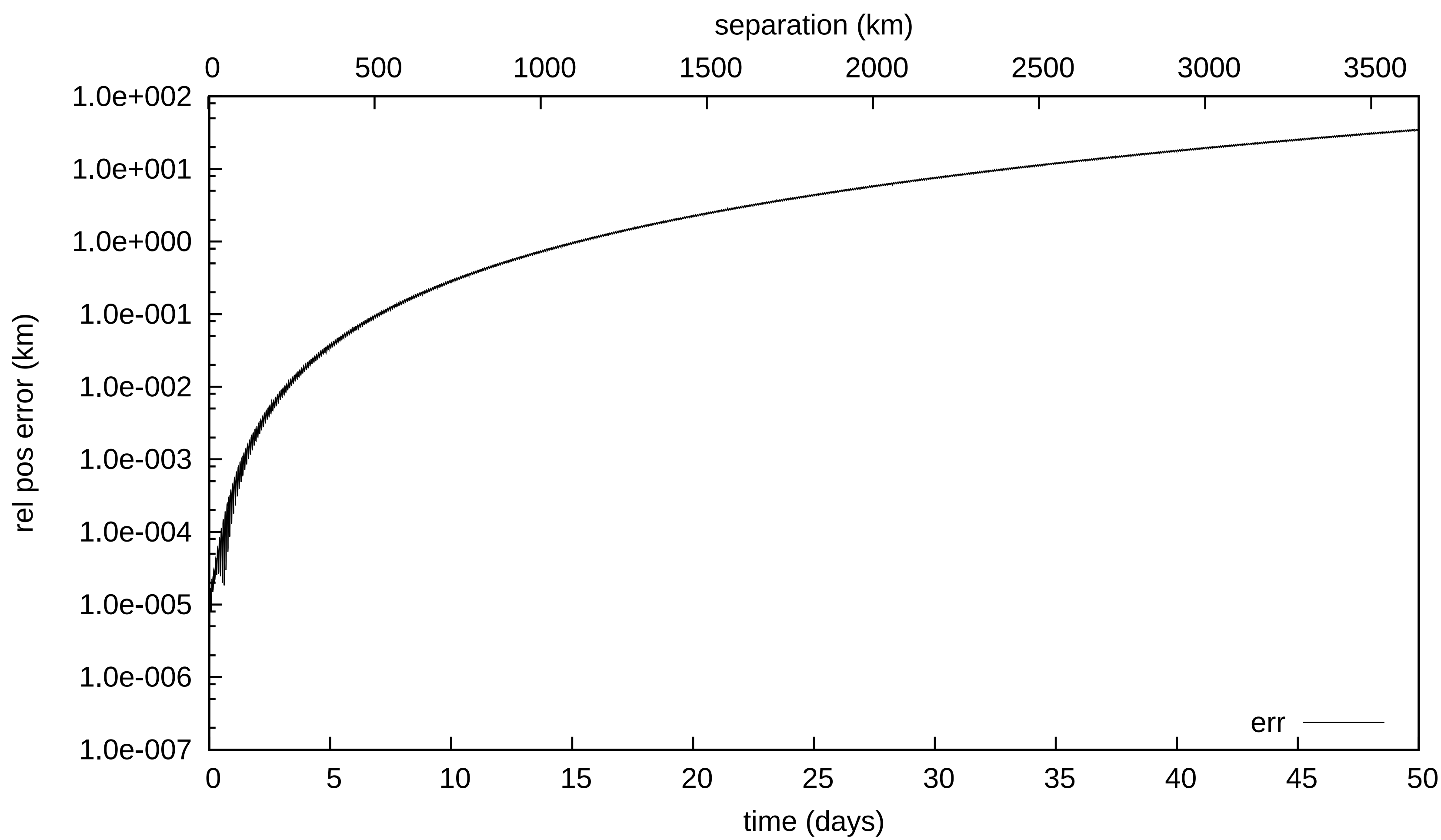
Solving the Relative Motion

	a(km)	e	I	Ω	ω	θ	H	L
Sat1	15945.8	0.45	60	40.03	20	73	-0.199994	1.481152
Sat2	-1.15	0.0001	0.03	0.03	-0.05	0.05	-1.881e-6	-6.605e-5



Positional Accuracy

$a = 7065.3$ km, $\delta a = 1200$ m, $e = 0.055$ include 36×36 model.



Conclusions

- ✔ Formation Acquisition
  - ✔ optimal Hill's Solutions for continuous thrust
- ✔ Describe relative motion for near circular formations
  - ✔ exploits analytic solutions to equations of motion
  - ✔ separates secular, rigid formation motion and inter-satellite motions
  - ✔ accurate to approx 1 metre.
- ✔ Formation Design
  - ✔ Invert from required relative motion into orbital elements
  - ✔ design multiple baslines
- ✔ Symplectic Formation Propagation
  - ✔ long term evolution accurate to $< 2\%$ over 1 month
  - ✔ conserves relative energy and angular momentum exactly