Rijksuniversiteit Groningen

Bachelor Thesis

Theories of Modified Gravity

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Abstract

General Relativity can be modified to explain observations that would otherwise infer the presence of Dark Matter and Dark Energy. Several different modifications have been considered in the past few decades, a few of which will be discussed in this thesis, exploring both the theoretical and the observational aspects. MOND, TeVeS, Quintessence, f(R), Brans-Dicke, Horndeski and Galileon theories have been analyzed and summarized. A short description of screening mechanisms is also included.

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“Physics is like sex:
sure, it may give some practical results, but that’s not why we do it.”

Richard P. Feynman

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Chapter 1

Conventions and Definitions

1.1 Conventions

Throughout this paper Planck units are used ($c = \hbar = G = 1$), unless otherwise specified. This means that $\kappa \equiv 8\pi G = 8\pi$. The Planck mass is defined as $m_{pl} \equiv \sqrt{\frac{\hbar c}{G}}$, and the reduced Planck mass as $M_{pl} \equiv \sqrt{\frac{\hbar c}{8\pi G}} = \frac{m_{pl}}{\sqrt{8\pi}}$. A dot denotes a time derivative: $\dot{A} \equiv \frac{dA}{dt}$, $\ddot{A} \equiv \frac{d^2A}{dt^2}$ etc. A subscript comma denotes a derivative; $G_{,x} \equiv \frac{\partial G}{\partial x}$, $G_{,xx} \equiv \frac{\partial^2 G}{\partial x^2}$; and a subscript semicolon ($G_{;x}$) denotes a covariant derivative; a derivative along tangent vectors on a manifold. Round brackets around multiple indices denotes the symmetrized part of the tensor: $A_{(\mu\nu)} \equiv \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$ and square brackets denotes the antisymmetrized part: $A_{[\mu\nu]} \equiv \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu})$
The mostly plus convention is used for the metric: $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$

Chapter 2

Introduction

2.1 Why Modify Gravity?

In 1687 Newton published the now world famous Philosophiæ Naturalis Principia Mathematica [1]. He presented a mathematical treatment of gravity, resulting in the well known formula for gravitational forces:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{2.1}$$

This however turned out to be only half the story. The theory of general relativity, proposed by Einstein some 250 years later, is the other half. Einstein's theory says that gravitation isn't a force at all, but rather a distortion of the four dimensional space-time in which we live. Newton's theory allowed information to travel at infinite speeds, whereas Einstein showed that nothing can exceed the speed limit of the universe, the speed of light, not even gravity. General relativity accounts for many experimental phenomenon, including gravitational lensing (e.g. by observing stars behind the sun during a solar eclipse), anomalous precession of planet orbits and the behavior of gravity waves (e.g. by observing the decay of a binary pulsar orbit).
However in the last few decades scientists have found several problems which cannot be explained properly with general relativity alone (see next section). Solutions of these problems in the framework of General Relativity demand large amounts of invisible matter and energy, called Dark Matter (DM) and Dark Energy (DE) respectively.

Currently the standard model of cosmology is the ΛCDM model (see next section). Modified gravity theories (MG) seek to find explanations for the problems in the ΛCDM model by modifying General Relativity. In order to test the validity of modified gravity

theories one can compare the predictions of the theory with solar system experiments and observations of, e.g., rotation curves of galaxies, the accelerated expansion of the universe and the spectrum of the CMB.
One of the most famous attempts to modify gravity was MOdified Newtonian Dynamics (MOND), proposed by M. Milgrom in 1986 [2]. Besides MOND multiple different theories have been developed over the past decades. A few examples are Scalar-Tensor-Vector Gravity Theory (STVG) [3], sometimes also referred to as MOdified Gravity (MOG), and Tensor-Vector-Scalar gravity (TeVeS), which is a relativistic extension of MOND [4].

Section 2.2 provides an overview of the basics of cosmological physics. Chapter 3 contains a summary of a few different theories of modified gravity; MOND, TeVeS, Quintessence, f(R) Theories, Brans-Dicke and Horndeski Theory and a short description of Screening Mechanisms.

2.2 The ΛCDM Model

2.2.1 Current Model

The current standard model for cosmology is the ΛCDM model. It is based on the cosmological principle, which states that the universe is isotropic and homogeneous (on large scales), and Einstein's theory of General Relativity (GR). According to the ΛCDM model the universe was created during the Big Bang. The model includes large amounts of Dark Energy (Λ) and (cold) dark matter (CDM). To account for the accelerated expansion one can include a Cosmological Constant (CC). The simplest physical mechanism that causes the CC to exist is DE. DE functions as anti-gravity, pushing everything apart, and can be interpreted as a nonzero vacuum energy density. It has a negative pressure $p = -\rho$ which, according to GR, results in accelerated expansion. The DE density of the universe is estimated to be $\sim 69\%$ of the total mass and energy density based on observations using the Planck satellite [5].

Cold DM is some form of non-baryonic (i.e. not consisting of neutrons and protons), dissipationless (not emitting light to cool) matter distributed throughout the universe. Because it only interacts through gravity (and possibly through the weak force) it cannot be seen directly and its presence is therefore inferred indirectly through observations that are hard to explain without assuming large amounts of invisible mass and energy (e.g. flat rotation curves, galaxy cluster dynamics, gravitational lensing, etc.). Cold DM is called cold because its velocity (at the end of the radiation-matter equality epoch) was much smaller than the speed of light. The Planck data estimates the DM density of the universe to be $\sim$ 27% of the total matter and energy density of the universe. The remaining $\sim$ 4% is visible baryonic matter. There is also a negligible amount of curvature (the universe is flat) and radiation [5].

2.2.2 The Einstein Field Equations

The ΛCDM model incorporates the Einstein Field Equations (EFE) from GR:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2.2}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the metric tensor, R is the scalar curvature (the trace of $R_{\mu\nu}$ with respect to $g_{\mu\nu}$), Λ is the Cosmological Constant (CC), G is the gravitational constant, c is the speed of light, and $T_{\mu\nu}$ is the stress–energy tensor. The EFE can be written more compactly (using Planck units; $G = c = 1$) as

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi T_{\mu\nu} \tag{2.3}$$

where $G_{\mu\nu}$ is the Einstein Tensor, defined as $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. The EFE describe gravity resulting from the curvature of space-time by matter and energy. The left hand side of (2.3) represents the curvature of space-time (dependent on the metric $g_{\mu\nu}$. The relevant parameter is usually taken to be the metric $g_{\mu\nu}$ since $R_{\mu\nu}$ is a (non-linear) function of $g_{\mu\nu}$. Einstein introduced the CC to ensure that the universe was static (not expanding or collapsing). This however turned out to be wrong, as the universe is in fact expanding. The CC is still useful as it fulfills the purpose exactly opposite to its initial function; it describes the expansion of the universe. The EFE are non-linear and an exact general solution does not exist.

2.2.3 Friedman-Lemaître-Robertson-Walker Metric

The most general metric following from the EFE is

$$ds^2 = g_{00}dt^2 + 2g_{0i}dx^i dt + \sigma_{ij}dx^i dx^j \tag{2.4}$$

A metric obeying the EFE for an expanding or collapsing (3+1) dimensional universe where the Cosmological Principle holds is the Friedman-Lemaître-Robertson-Walker Metric (FLRW Metric). It was found independently by Friedman, Lemaître, Robertson and Walker and it follows from (2.4) after imposing the Cosmological Principle (homogeneity and isotropy). The FLRW Metric is given by

$$ds^2 = -dt^2 + \frac{a^2(t)}{1+kr^2/4}[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{2.5}$$

or, equivalently, in Cartesian coordinates

$$ds^2 = -dt^2 + \frac{a^2(t)}{1+k(x^2+y^2+z^2)/4}[dx^2 + dy^2 + dz^2] \tag{2.6}$$

with $k = 0, \pm 1$, depending on whether the universe is open ($k = 1$), closed ($k = -1$) or flat ($k = 0$). From the FLRW Metric the Friedmann equations can be derived:

$$H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi}{3}\rho \tag{2.7}$$

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} - \Lambda = -8\pi p \tag{2.8}$$

where $H \equiv \frac{\dot{a}}{a}$. They are connected through the conservation equation:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{2.9}$$

The density when $k = 0$ is called the critical density and is given by

$$\rho = \frac{3H^2}{8\pi} \equiv \rho_c \tag{2.10}$$

This allows a density parameter Ω_m to be defined:

$$\Omega_m = \frac{\rho_m}{\rho_c}\ . \tag{2.11}$$

Similarly one can define Ω_r, Ω_Λ and Ω_k, representing the radiation density, DE density and the curvature of the universe respectively. The expansion history of the universe is determined by the sum of these parameters:

$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1 \tag{2.12}$$

This can then be combined with (2.7), (2.8) and (2.9) giving

$$H^2 = H_0^2(\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda + \Omega_k(1+z)^2) \tag{2.13}$$

This equation indicates that the mass density $\rho_m \propto a^{-3} = (1+z)$, the radiation density $\rho_r \propto a^{-4} = (1+z)^4$, Ω_Λ is the result of the DE density and the last term represents the amount of curvature.

DE can be interpreted as the energy density of vacuum. Lorentz invariance of the stress-energy tensor of the vacuum means that all observers see the same vacuum:

$$T^{\Lambda}_{\mu\nu} = \rho_\Lambda g_{\mu\nu} \tag{2.14}$$

Putting this in the EFE gives

$$G_{\mu\nu} = 8\pi(T_{\mu\nu} + \rho_\Lambda g_{\mu\nu}) \tag{2.15}$$

where $T_{\mu\nu} = \text{diag}(\rho, p, p, p)$ (for a perfect fluid, assuming homogeneity and isotropy). From this follows that

$$p_\Lambda = -\rho_\Lambda \tag{2.16}$$

DE thus behaves like an ideal fluid with negative pressure. ρ_Λ is related to the cosmological constant Λ through $\rho_\Lambda \equiv \frac{\Lambda}{8\pi}$.

2.2.4 Main Problems

Despite accounting for nearly all observations, there are some serious problems with ΛCDM. The first problem is the *Flatness Problem* the universe today is very close to being flat (as opposed to being spherical or hyperbolic) but the solution of the Friedmann equations with $k = 0$ (flat universe) is unstable and a small deviation (i.e. small amounts of curvature) would lead to either $\Omega \to 0$ or $\Omega \to \infty$. The current value of Ω (Ω_0) is close to unity. This would mean that at earlier times Ω must have been even closer to unity, but that would be an extraordinary coincidence.
This problem can be solved by introducing *inflation*, a period of rapid expansion between $\sim 10^{-36}$ and $\sim 10^{-33}$ s after the big bang. Analogously, a sphere appears to be flat (locally) after being inflated. What causes inflation is still a mystery, the (hypothetical) field that is responsible is called *inflaton*.
The curvature of the universe is usually expressed as

$$\Omega_k = -\frac{k}{a^2H^2} \tag{2.17}$$

This shows that large amounts of expansion (inflation) causes Ω_k to approach zero and therefore the universe to be (practically) flat. This explains why $\Omega \approx 1$. DE manifests itself in GR through the cosmological constant Λ. From the Friedmann equations (2.7) and (2.8) and the conservation equation (2.9) and assuming flat space ($k = 0$) the following relation for H can be derived:

$$H = \sqrt{\frac{8\pi}{3}} a^{-3(1+w)/2} \tag{2.18}$$

where $w = \frac{p}{\rho}$, which leads to

$$a \sim t^{\frac{2}{3(1+w)}} \tag{2.19}$$

A period of accelerated expansion requires $w < -\frac{1}{3}$. Inflation stabilizes the $\Omega = 1$ path. The second main problem is the *Horizon Problem*: the CMB is (almost) isotropic, even across distance between points that are causally disconnected. Inflation solves this problem as well. The regions that are currently causally disconnected were connected before the period of inflation, in which they separated faster than the speed of light.
Another big problem is the *Cosmological Constant Problem.* Assuming the expansion of the universe is due to DE, and DE can be attributed to vacuum energy, the value of the cosmological constant must be $\sim 10^{-47}$GeV. Quantum Field Theory estimates the energy of vacuum to be $\sim 10^{71}$ GeV [6]. Why is the measured value for the cosmological constant so very small compared to the theoretical predictions based on the zero point energy of vacuum in Quantum Field Theory? It is off by some 120(!) orders of magnitude. The current most accepted solution is the "multiverse": there are many different universes each with a different value for the CC, and we just happen to live in the one with a very small value.
The last big problem is the *Coincidence Problem.* The ratio of DE and matter today is close to unity:

$$\frac{\rho_{DE}}{\rho_m} = \frac{\Omega_\Lambda}{\Omega_m} \sim \mathcal{O}(1). \tag{2.20}$$

But it scales with the expansion factor:

$$\frac{\Omega_\Lambda}{\Omega_m} \propto a^3 \tag{2.21}$$

This makes it a remarkable coincidence, since measuring earlier or later in the life of the universe would lead to a completely different value. Moreover the period in which the ratio is this way is incredibly short. At this very moment the universe is undergoing a metamorphosis: from matter dominated to DE dominated. The coincidence problem can be tackled in many different ways, e.g. anthropic reasoning "solves" this problem by arguing that this is not a coincidence, since in order for life to exist the ratio must be close to unity. This solution is however still subject to debate, just like virtually all other solutions, including solutions to the other mentioned problems.

2.3 Observational Cosmology

One of the most important aspects of cosmology is testing hypotheses derived in the theoretical field. This is often a challenging task, as observations with telescopes are the only way the universe can be probed.
Distances to astronomical objects must be determined in order to do most of the relevant calculations. A commonly used method is to measure the lightcurve (brightness against time) of type Ia supernovae. When the mass of a white dwarf star in a binary system reaches a certain limit ("Chandrasekhar Mass"; $\sim 1,4M_{\odot}$), it explodes and the resulting absolute magnitude of such an event is known. Comparing the absolute magnitude to the apparent magnitude reveals a good estimate of the distance. The redshift can then also be calculated, which in turn is used to place constraints on Ω_Λ and Ω_m. The accelerated expansion of the universe was discovered using type Ia supernovae. Two other very useful features of the universe are the CMB and Baryonic Acoustic Oscillations (BAO). Measurements of the CMB constrain the total energy density ($\Omega_{tot} = \Omega_m + \Omega_\Lambda + \Omega_k$) as well as ($\Omega_m + \Omega_\Lambda$), the latter of which can be combined with the supernova observations. BAO are "pressure and density waves" in the galaxy power spectrum, caused by pressure waves in the primordial plasma of baryons and photons. When atoms formed the density fluctuations were "frozen in time" and are visible in the universe as variations in density on a scale of ~ 150 Mpc. BAO can also be used to constrain Ω_m and Ω_Λ (see figure 2.1) This method is based on ΛCDM. Using wCDM models instead of ΛCDM, with a perfect fluid (arbitrary w), gives constraints in the (Ω_m, w) plane, revealing that $w \approx -1$ (see figure 2.2) Since GR is very well established in the solar system, theories that include an extra force ("fifth force") to impact larger scales should be shielded from the solar system, and other high density regions in general. The strength of the fifth force depends on the local density, becoming weaker as the density rises. Usually new theories can therefore not be tested close to earth. But voids (low density regions in large scale structure) are an excellent alternative, as the extra force is strongest there.

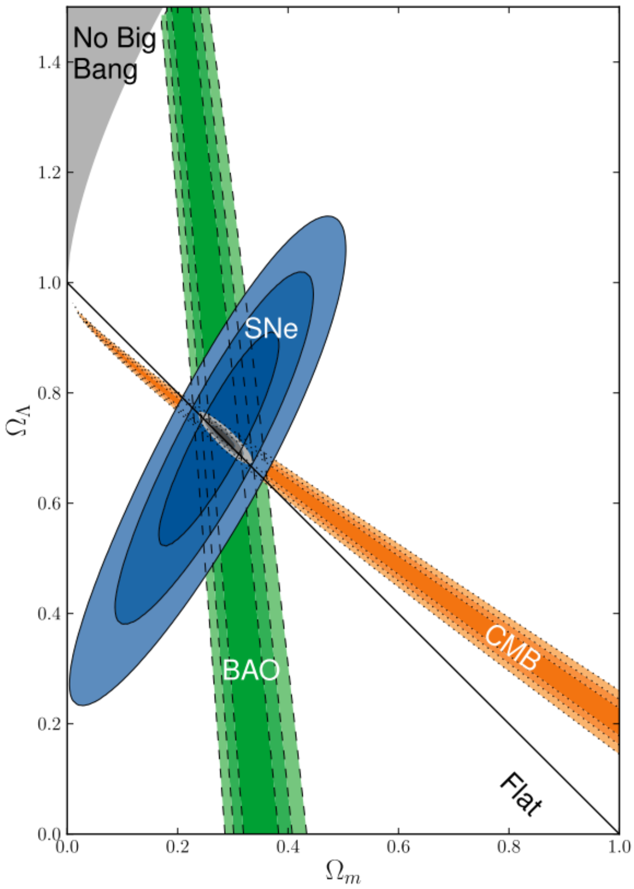

FIGURE 2.1: Constraints placed on Ω_m and Ω_Λ (68.3%, 95.4 % and 99.7% confidence regions). Taken from the Supernova Cosmology Project [7]

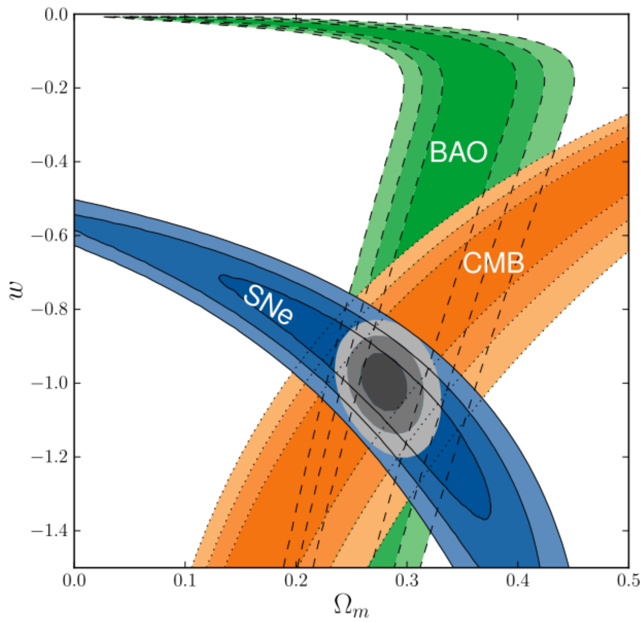

FIGURE 2.2: Constraints placed on Ω_m and w (68.3%, 95.4 % and 99.7% confidence regions). Taken from the Supernova Cosmology Project [7]

Solar system tests are used to constrain extra degrees of freedom. They force the necessity for screening mechanisms on several theories of modified gravity. Examples include measurements of the deflection of light rays by the sun, and the anomalous perihelion precession of the planets (Mercury is often used).
Some alternatives for GR, unlike GR itself, do not obey the Weak Equivalence Principle (WEP). It states that the trajectory of a freely falling body is independent of its internal structure and composition. This is easily tested by monitoring two different objects in a gravitational field to check whether they undergo the same gravitational acceleration [8][9]. Many modified gravity theories do obey the WEP. Some of them alternatively predict a deviation from the inverse square law ($1/r^2$) of gravity. These deviations can be tested in the laboratory by analyzing the forces between objects under certain conditions [10].

An interesting way to distinguish DE models from modified gravity is to analyze the evolution of the linear growth of matter perturbations ($\delta_m \equiv \frac{\delta\rho_m}{\rho_m}$). It is determined by the following differential equation:

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{eff}\rho_m\delta_m \tag{2.22}$$

where $G_{eff} = Q(a)G$ and $Q(a) = \frac{2+4\Omega_m^2}{3+3\Omega_m^2}$.
A solution of this equation is $\delta_m \propto D(t)$, where $D(t)$ is the linear growing mode, normalized to unity at the present time: $D(t) = \frac{\delta_m(t)}{\delta_m(t_{\rm now})}$.

Substituting and changing variables ($t \to a$) then gives

$$\frac{a^2}{D}\frac{d^2D}{da^2} + \left(3 + a\frac{d\ln E}{da}\right)\frac{a}{D}\frac{dD}{da} = \frac{3}{2}\Omega_m(a)Q(a) \tag{2.23}$$

Looking at the growth rate of clustering $f(a) = \frac{d\ln(D)}{d\ln(a)} \simeq \Omega_m^\gamma$, where γ ("growth index") depends on $w(z)$ and is a useful tool for testing different gravity theories. For GR with DE $\gamma \simeq \frac{3(w-1)}{6w-5}$, which gives $\gamma \simeq 6/11 \approx 0.55$ for the ΛCDM model, as $w \simeq -1$. [11]

Chapter 3

Overview of Modified Gravity

3.1 Theories of Modified Gravity

Modified gravity theories must obey certain restrictions in order to be a reasonable alternative to General Relativity.

There are constraints from both the theoretical and the observational field. Theoretical considerations force any proper theory to obey the following rules. For the full list see [4] and references therein.

- *Action principle*
  The theory must be derivable from an action principle. This is to guarantee that the conservation of energy, linear momentum and angular momentum are taken care of automatically.
- *Relativistic Invariance*
  The action must be a relativistic scalar to ensure that all equations derived from it are relativistically invariant.
- *Equivalence Principle*
  To make the theory obey the Equivalence Principle it must be a metric theory, i.e., all other laws of physics (i.e. not gravitational laws), must be expressed with the metric $g_{\mu\nu}$ replacing the Lorentz metric.
- *Causality*
  The theory should obviously be consistent with the rules of causality, no superluminal propagation is allowed.

- *Positive Energy*
  Fields cannot have negative energy; any bounded system must have positive energy. This is to provide vacuum stability.

Observations have also provided a great deal of conditions to be met. The MG theory must be compatible with the following list of observational phenomena (taken from [4]) without assuming DM and/or DE (depending on the theory and its application):

- *Extragalactic Phenomena*
  The theory should correctly predict extragalactic phenomena, such as the dynamics of galaxies in clusters and of clusters in superclusters.

- *Gravitational Lensing*
  Gravitational lensing should be identical to GR with DM to match observations.

- *Solar System Tests*
  Solar system test, mainly the perihelion precession of planets, the deflection of light and the delay in radar signals, must be well predicted. Since the gravitational field varies relatively slowly in the solar system, tests performed there can only address the weak field limit of theories.

- *Binary Pulsars*
  The theory must correctly predict the observed pulse times of arrival from binary pulsars. This allows for detection of relativistic time delay, periastron precession and the orbit's decay due to gravitational radiation. Pulsars are essentially very stable clocks. In a binary pulsar the clock moves and through the Doppler effect the orbital velocity can be determined, together with the parameters characterizing the orbit, such as the period and the eccentricity. Observations can be compared to the theory in question. If the theory can correctly predict values for the two masses orbiting each other with the correct orbit parameters the theory has passed the test [12]. Binary pulsar allow, in contrast to solar system test, the strong field regime of theories to be tested. For (mathematical) details regarding binary pulsars as test see, for example, [13].

- *Cosmology*
  The theory should account for cosmological phenomena, such as the Hubble expansion, the structure of the CMB, the element abundances in the universe etc.

Obviously it is possible to create a very general modified gravity theory that adds many new degrees of freedom and depends on several free parameters in order to provide a good fit to observational data. Consequently these kind of models require a great deal of fine tuning. Requiring serious fine tuning is not a good sign. Theories are judged in three different but not completely independent ways: Apart from the aforementioned theoretical and experimental ones, there are also esthetical ones ("Occam's Razor"), whether the proposed model is natural enough to be considered as predictive, or needs so much fine tuning that it becomes an uninteresting ad hoc fit to experimental data. These criteria are related, as fine tuning can be necessary to explain observations or hide theoretical inconsistencies.

Following is a non exhaustive list of several different proposals to modify gravity. The theoretical basis is discussed along with constraining observations and experiments.

3.1.1 MOND

Physics

Milgrom proposed to modify Newton's Universal Law of Gravitation in the limit of very small accelerations. He came up with a relation between the acceleration and the corresponding gravitational force (and thus gravitational potential):

$$F_g = m_g \mu \frac{a}{a_0} a \tag{3.1}$$

Where

$$\mu(x \gg 1) \approx 1, \qquad \mu(x \ll 1) \approx x \tag{3.2}$$

This form ensures that the gravitational force reduces to the standard Newtonian $\mathbf{F} = m\mathbf{a}$ when $a \gg a_0$. The theory does not provide information about the form of $\mu(a/a_0)$. A few example of used functions are $\mu(x) = \frac{1}{\sqrt{1+x^2}}$ (to fit rotation curves) and the simple interpolating function $\mu(x) = \frac{x}{1+x}$. The value for a_0 was found empirically to be $\sim 2 \cdot 10^{-8}$ cm/s^2 [2]

Note that none of the other assumptions for gravitation in the non-relativistic regime have been changed. An example of the effects this theory has on a simple system such as the harmonic oscillator is that the well known equation of motion

$$\ddot{x} = \frac{k}{m} \tag{3.3}$$

transforms into

$$\ddot{x}\mu\left(\frac{\ddot{x}}{a_0}\right) = \frac{k}{m}x \tag{3.4}$$

when $x \lesssim \frac{10^{-10}}{\omega^2}$. The small size of the scale at which MOND dominates the dynamics makes it very difficult to perform tests on earth [14].
The modified formula for gravitational force however doesn't constitute an entire theory by itself. It is just an effective formula [15].

Astronomy

The formula has been extensively tested and seems to be consistent with e.g. the flat rotation curve, the Faber-Jackson and Tully-Fisher relations, and gravitational lensing [15]. During earlier epochs, MOND should have dominated when the deceleration of the Hubble expansion was smaller than a_0. The size of a causally connected region during these epochs was larger than the scale where MOND starts to take over. This means that the "early MOND Universe" is the same as the early universe described by the standard FLRW metric (2.5) and the Friedmann equations (2.7) and (2.8): MOND does not seem to have a significant effect on the cosmology of the early universe. Certain regions however (with $M \sim 10^{11} M_\odot$) recollapse at a redshift of ~ 26 "...which implies that large galaxies should be in place as virialized objects by redshift of 5 to 10. This is earlier than the epoch of galaxy formation in the standard CDM paradigm" [16]. Cluster sized objects reach maximum expansion at $z \sim 3$ (hierarchical bottom-up structure formation; larger mass collapses after smaller mass), so one should be able observe slightly more massive, more clustered galaxies.
Big bang nucleosynthesis in the MOND model is unchanged, but structure formation is marginally more rapid [17]. Unfortunately for MOND there are observations that would still require the presence of DM even in MOND, although less than in the ΛCDM model. Even after adding the maximum allowed number of 2 eV neutrinos the dynamical mass would still fall short [18]. This is obviously an embarrassment for a model that was primarily developed to replace DM. Especially observations of the Bullet Cluster (1E 0657-558) provide strong evidence for DM, even in MOND the DM would comprise at least half of the total mass in this cluster.
But MOND isn't necessarily ruled out completely, but rather requires less DM, it managed to reduce the matter discrepancy by a factor of ~ 5 [19].

3.1.1.1 TeVeS Theory

Physics

To upgrade MOND to a full-fledged theory, it must be generalized to include the relativistic regime. Tensor Vector Scalar Theory has been developed to do just that. It introduces another scalar field ϕ, and a vector field A_μ next to the already present $g_{\mu\nu}$ from GR, hence the name 'Tensor Vector Scalar Theory'.

The most used method to specify TeVeS theory is to write the action in two mixed frames; write the action in the 'Bekenstein frame' for the gravitational fields, and in the physical frame for the matter fields. This way the Einstein equivalence principle is obeyed. The three gravitational fields are the metric $\tilde{g}_{\mu\nu}$ ('Bekenstein Metric'), the vector field A_μ ('Sanders Vector Field') and the scalar field ϕ. The scalar field ϕ plays the role of DM, as the Newtonian potential Φ_N is replaced by total potential $\Phi = \alpha\Phi_N + \phi$ with $\alpha \approx 1$. Φ_N and ϕ are related through

$$\nabla \cdot [\mu_s \nabla\phi] = \nabla^2\Phi_N = 4\pi G\rho \tag{3.5}$$

Where μ_s depends on the strength of the scalar field ($g_s = |\nabla\phi|$), and Φ_N is the (Newtonian) potential generated by the baryonic density ρ [20]. The vector field A_μ is constrained to be unit-timelike:

$$\tilde{g}^{\mu\nu} A_\mu A_\nu = -1 \tag{3.6}$$

This constraint is a phenomenological requirement to give the right bending of light around massive objects. To preserve Einstein's equivalence principle it is useful to define a single 'universally coupled metric'

$$g_{\mu\nu} \equiv e^{-2\phi}\tilde{g}_{\mu\nu} - 2\sinh(2\phi)A_\mu A_\nu \tag{3.7}$$

to be able to use only the 'Bekenstein Metric' $\tilde{g}_{\mu\nu}$ in further analysis. The action in TeVeS theory is split into 4 parts

$$S = S_{\tilde{g}} + S_A + S_\phi + S_m. \tag{3.8}$$

Where $S_{\tilde{g}}$, S_A, S_ϕ and S_m are the actions for $\tilde{g}_{\mu\nu}$, A_μ, ϕ and the matter respectively. $S_{\tilde{g}}$ is given by

$$S_{\tilde{g}} = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \tilde{R} \tag{3.9}$$

notable is that it resembles the standard Einstein-Hilbert Action of GR. S_A is given by

$$S_A = -\frac{1}{4\kappa} \int d^4x \sqrt{-\tilde{g}} \left[K F^{\mu\nu} F_{\mu\nu} - 2\lambda (A_\mu A^\mu + 1) \right], \tag{3.10}$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ and λ is a Lagrange multiplier to guarantee (3.6) holds. K is a (dimensionless) constant. The action for the scalar field ϕ is

$$S_\phi = \frac{1}{2\kappa} \int d^4x \sqrt{-\tilde{g}} \left[\mu \breve{g}^{\mu\nu} \tilde{\nabla}_\mu \phi \tilde{\nabla}_\nu \phi + V(\mu) \right], \tag{3.11}$$

where μ is some constant dimensionless scalar field, $\tilde{\nabla}_\mu$ is the connection corresponding to $g_{\mu\nu}$ and $\breve{g}^{\mu\nu}$ is defined as

$$\breve{g}^{\mu\nu} \equiv \tilde{g}^{\mu\nu} - A^\mu A^\nu \tag{3.12}$$

to preserve causality. Finally the action for the matter part is

$$S_m = \int d^4x \sqrt{-g} L[g, \chi^A, \nabla \chi^A] \tag{3.13}$$

for some matter field χ^A. Note that S_m depends on $g_{\mu\nu}$ rather than $\breve{g}^{\mu\nu}$ or $\tilde{g}^{\mu\nu}$.
Using the variational principle gives the field equations as well as two constraints. The first constraint (for μ) is

$$\breve{g}^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = -\frac{dV}{d\mu}. \tag{3.14}$$

The other obtained constraint is (3.6). The field equations are

$$\begin{aligned}\tilde{G}_{\mu\nu} = \kappa & \left[T_{\mu\nu} + 2\left(1 - e^{-4\phi}\right) A^{\rho} T_{\rho(a} A_{\nu)}\right] \\ & + \mu \left[\tilde{\nabla}_{\mu}\phi\tilde{\nabla}_{\nu}\phi - 2A^{\rho}\tilde{\nabla}_{\rho}\phi A_{(a}\tilde{\nabla}_{b)}\phi\right] \\ & + \frac{1}{2}\left(\mu V,_{\mu} - V\right)\tilde{g}_{\mu\nu} \\ & + K\left[F^{\rho}{}_{\mu}F_{\rho\nu} - \frac{1}{4}F^{\rho\sigma}F_{\rho\sigma}\tilde{g}_{\mu\nu}\right] \\ & - \lambda A_{\mu}A_{\nu}\end{aligned} \tag{3.15}$$

Where $\tilde{G}_{\mu\nu}$ is the Einstein tensor of $\tilde{g}_{\mu\nu}$. The field equations for A_μ and ϕ are

$$K\tilde{\nabla}_{\rho}F_{\rho_{\mu}} = \lambda A_{\mu} - \mu A^{\nu}\tilde{\nabla}_{\nu}\phi\tilde{\nabla}_{\mu}\phi + \kappa(1 - e^{-4\phi})A_{\nu}T_{\nu\mu} \tag{3.16}$$

$$\tilde{\nabla}_{\mu}\left[\mu\breve{g}^{\mu\nu}\tilde{\nabla}_{\nu}\phi\right] = \kappa e^{-2\phi}\left[g^{\mu\nu} + 2e^{-2\phi}A^{\mu}A^{\nu}\right]T_{\mu\nu} \tag{3.17}$$

TeVeS is free from ghosts (negative kinetic terms), guaranteeing its stability and making it an even more viable alternative to GR [21]. Details and additional information can be found in [22][23].

Astronomy:

In TeVeS for a homogeneous and isotropic (flat) universe the metrics $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are

$$ds^2 = a^2(-d\eta^2 + dr^2) \tag{3.18}$$

$$d\tilde{s}^2 = b^2(-e^{-4\phi}d\tilde{\eta}^2 + d\tilde{r}^2) \tag{3.19}$$

where a and b are the scale factors, related through $a = be^{-\phi}$. The Friedmann equation (in the $\tilde{g}_{\mu\nu}$ frame) is given by

$$3\frac{\dot{b}^2}{b^2} = a^2\left[\frac{1}{2}e^{-2\phi}(\mu V,_{\mu} + V) + 8\pi e^{-4\phi}\rho\right] \tag{3.20}$$

where ρ is the matter-energy density field (not including the scalar field). The vector field is constant and points in the time direction. $V(\mu)$ is a free function. In order to restrain TeVeS to reduce to both MOND and Newtonian dynamics in the right limits the derivative of $V(\mu)$, $V,_\mu$, must be of the form

$$V,_{\mu} = -\frac{1}{16\pi\ell_B}\frac{\mu^2}{(1 - \mu/\mu_0)^m}f(\mu) \tag{3.21}$$

where ℓ_B is some scale, μ_0 is a (dimensionless) constant and $f(\mu)$ is an arbitrary function with the only constraint that $f(0) \neq 0$. The μ^2 term is needed to include the MOND limit ($\mu \to 0$) and the $(1 - \mu/\mu_0)$ part is used to reach the Newtonian limit ($\mu \to \mu_0$) [24]. To describe the background dynamics the equation of motion for ϕ is needed:

$$\ddot{\phi} = \dot{\phi}\left(\frac{\dot{a}}{a} - \dot{\phi}\right) - \frac{1}{\mu + 2V_{,\mu}/V_{,\mu\mu}}\left[3\mu\frac{\dot{b}}{b}\dot{\phi} + 4\pi a^2 e^{-4\phi}(\rho + 3p)\right] \tag{3.22}$$

where p denotes the pressure (again not including the scalar field).
In the other frame the (effective) Friedmann equation is given by

$$3\frac{\dot{a}^2}{a^4} = 8\pi\frac{e^{-4\phi}}{(1 + \frac{d\phi}{d\ln a})^2} \tag{3.23}$$

and the (effective) density of the ϕ is

$$\rho_\phi = \frac{1}{16\pi}e^{2\phi}(\mu V_{,\mu} + V) \tag{3.24}$$

[25] [26]

In [4] and [27] it is shown that all these conditions are mostly satisfied: in a spherically symmetric situation the theory is consistent with disk galaxies that have low surface brightness, dwarf (spheroidal galaxies), and the outer regions of spiral galaxies: Gravitational lensing in TeVeS is the same as in GR with DM; The three standard solar system tests (mentioned in the list above) produce the same results in both TeVeS and GR with DM up to at least the precision of the used equipment. The observations of binary pulsars however force a large significant fine tuning upon TeVeS.

3.1.2 Quintessence

A very simple extension to the ΛCDM Models is the quintessence. Quintessence models are strictly speaking not theories of modified gravity, but rather theories of dynamical DE. Quintessence replaces the cosmological constant with a minimally coupled scalar field Q (i.e. only feeling gravity) rolling down a corresponding potential $V(Q)$. Q is not constant across space-time but depends on the local density. The kinetic contribution to the Lagrangian is linear in the kinetic energy (canonical). The Lagrangian is then given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu Q \partial^\mu Q - V(Q) \tag{3.25}$$

$V(Q)$ is an arbitrary function and any given possible history of the universe can be modeled using the right form. The Lagrangian leads to the equations of motion:

$$\ddot{Q} + 3H\dot{Q} = -dV/dQ \tag{3.26}$$

The density and pressure can be shown to be

$$\begin{aligned} \rho_Q &= \frac{1}{2}\dot{Q}^2 + V(Q) + \frac{1}{2}(\nabla Q)^2 \\ p_Q &= \frac{1}{2}\dot{Q}^2 - V(Q) - \frac{1}{6}(\nabla Q)^2 \end{aligned} \tag{3.27}$$

Combining the above relations and using $w_Q = p_Q/\rho_Q$ gives:

$$w_Q = \frac{K - V}{K + V} \tag{3.28}$$

Where K and V are given by

$$\begin{aligned} K &= \frac{1}{2}\rho_Q(1 - w) \\ V &= \frac{1}{2}\rho_Q(1 + w) \end{aligned} \tag{3.29}$$

4 different regimes can be distinguished in the behavior of scalar fields.

- Fast Roll
  - Fast rolling fields have kinetic energy greater than potential energy, or, in terms of w, $w > 0$. For certain forms of the potential there are many different possible initial conditions that lead to a reasonable energy density. An example of a fast rolling model is a tracker models. These models follow attractor trajectories such that at certain epochs their equation of state is determined by the dominant energy density component. These models however have problems reaching $w \lesssim -0.7$, while observations show that $w < 0.7$. These models are thus considered unreliable[28].

- Slow Roll
  - Slow rolling fields have a large potential energy and relatively small kinetic energy $w \approx -1$. This requires domination of DE in the energy density to explain accelerated expansion. Good models require a combination of fast and slow rollers because quintessence models with only slow rolling fields have the same fine tuning and coincidence problems as the cosmological constant.
- Steady Roll
  - Steady rolling fields have a linear potential and have different epochs of fast and slow rolling. The potential does not have a minimum, letting the field roll even past zero, where it becomes negative. Such a universe would eventually collapse, and end in a "Big Crunch", the opposite of the big bang [29].
- Oscillation
  - Oscillating fields have a potential of the form $V(Q) \sim Q^n$. For even n the fields will have at least one minimum. The fields will eventually reach that minimum and oscillate around it. For $n = 2$ the fields behave like nonrelativistic matter and for $n = 4$ the fields act like radiation. An example of an oscillating fields is the axion, a DM candidate.

For more details on dynamical DE, quintessence models and the above summary see [28][30][31][32]

Astronomy

The main goal of quintessence (in Latin: "Quinta Essentia", means "fifth element") is to solve the coincidence problem and the cosmological constant problem. In this the cosmological constant is not constant but varies with time, rolling down a potential from being large in the early universe and small today. The Matter and DE ratio in quintessence is fixed (solving the coincidence problem). Tracker models solve the coincidence problem by letting many different initial condition converge into a single solution. The equation of motion for tracker fields (in a flat background) is

$$\ddot{Q} + 3H\dot{Q} + V_{,Q} = 0 \tag{3.30}$$

with $H^2 = \frac{8\pi}{3}(\rho_m + \rho_r + \frac{1}{2}\dot{Q}^2 + V)$. ρ_m and ρ_r are the matter and radiation density respectively. One can define Ω_Q (similarly to Ω_m, Ω_k etc.):

$$\Omega_Q \equiv \frac{\rho_Q}{\rho_c} \tag{3.31}$$

Tracking solutions have the property $-1 < w_Q < w_B$ (where w is the equation of state parameter for the background). [33] claims that for tracking to occur the potential must obey two constraints, the first one is

$$\frac{V_{,QQ}\,V}{V_{,Q}^2} > 1 \tag{3.32}$$

and the second constraint is that $\frac{V_{,QQ}V}{V_{,Q}^2}$ is (nearly) constant over the range of all Q; from $V(Q) = \rho_B$ all the way to $V(Q) = \rho_B$ at the matter-radiation equality epoch (spanning over 100+ orders of magnitude). There are many different potentials satisfying these conditions. For example all inverse power-law potentials ($V(Q) \propto 1/Q^a, \;\; a > 0$) obey them. Fields like the above mentioned power-law ($V(Q) \propto Q^n$) do not exhibit tracking behavior and thus do require fine tuning, making them less desirable. [33][34] Supernova observations support quintessence models (in a flat universe) as being viable extensions to GR [35].

3.1.3 f(R) Theories

Physics

$f(R)$ theories modify the Einstein equations by replacing R (Ricci scalar) by an arbitrary function of R. They try to generalize the Lagrangian of the Einstein-Hilbert action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R \tag{3.33}$$

by replacing R with $f(R)$:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) \tag{3.34}$$

Where $g = |g_{\mu\nu}|$, and $f(R)$ is some function of the Ricci Scalar. The Ricci Scalar is defined as $R = g^{\mu\nu}R_{\mu\nu}$. $R_{\mu\nu}$ is the Ricci tensor, which represents the deviation from euclidean space.

Three different classes of $f(R)$ theories can be distinguished based on the variational principle (formalism) used on the Einstein-Hilbert action to derive Einstein's equations; 'Metric $f(R)$ Gravity', 'Palatini $f(R)$ Gravity' and 'Metric-affine $f(R)$ Gravity'. In the first formalism one varies only with respect to the metric, while in the second formalism one varies with respect to the metric and connection, assuming they are independent.

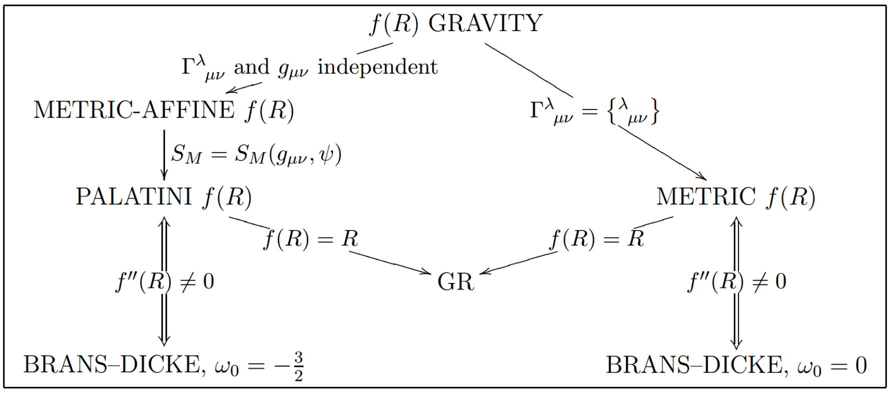

FIGURE 3.1: Schematic Diagram of $f(R)$ Theories. This diagram shows the connection between the different $f(R)$ theories. Taken from [38]

In the last, and most general formalism, one very varies with respect to the metric and connection, without the assumption of independence. This means that the last formalism is the most general, reducing to the other two under certain restrictions, see figure 3.1 and [36] [37].

Figure 3.1 shows how the different $f(R)$ theories are connected, showing the different restrictions placed on the various relevant parameters to switch between theories. It is worth noting that apart from $f(R)$ theories there is also a somewhat similar class of theories called 'Modified Gauss-Bonnet Gravity', $f(R, \mathcal{G})$, in which the action not only depends on R (more generally $f(R)$) but also on $\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}$ (called the Gauss-Bonnet Invariant) [39][40].
An obvious way to create an $f(R)$ theory is to replace R by an infinite power series:

$$f(R) = \dots \frac{\alpha_2}{R^2} + \frac{\alpha_1}{R} - 2\Lambda + R + \frac{R^2}{\beta_2} + \frac{R^3}{\beta_3} \dots \tag{3.35}$$

where α_i and β_i are the coefficients of the terms in the expansion with the right dimension. But nearly all of the possible functions based on (3.35) are merely toy models devoid of any practical use since most of them do not reduce to the correct Newtonian limit[41][42], or are unable to pass solar system test[43].

Astronomy

To create useful $f(R)$ theories a few boundary conditions must be set. These boundary conditions are needed to unify inflation and late time acceleration (dark energy). To ensure the inflation in the early universe

$$\lim_{R\to\infty} f(R) = -\Lambda_i \tag{3.36}$$

where Λ_i is an effective cosmological constant for the early universe, with values of the order of $\Lambda_i \sim 10^{20\sim 38}\text{eV}^2$. Or one could chose

$$\lim_{R\to\infty} f(R) = \alpha R^m \tag{3.37}$$

with m a positive integer and constant α. $f,_R > -1$ and $\alpha > 0$ or anti-gravity will occur. This leads to $f(R) > 0$ at the early universe, but to account for the current acceleration one should impose

$$f(R_0) = -2\tilde{R}_0, \quad |f,_R(R_0)| \ll (10^{-33}eV)^4 \tag{3.38}$$

where $\tilde{R}_0 = R_0 + \kappa T_{matter}$, T_{matter} is trace of the matter energy-momentum tensor. The last boundary condition is

$$\lim_{R\to 0} f(R) = 0, \tag{3.39}$$

meaning there is a flat space-time solution[44].

[45] analyzed several $f(R)$ models, based on their growth index. They can be written in the following form:

$$f(R) = R - \lambda R_c f_1(R/R_c) \tag{3.40}$$

where $R_c(> 0)$ defines a characteristic value for R; when $\lambda \approx 1$ R_c roughly corresponds to the value for R in the present time. $\lambda(> 0)$ is a free parameter. The investigated models differed in the form of $f_1(R/R_c)$:

$$f_{1a} = \frac{(R/R_c)^{2n}}{(R/R_c)^{2n}+1}, \quad n > 0 \tag{3.41}$$

$$f_{1b} = 1 - (1 + (R/R_c)^2)^{-n}, \quad n > 0 \tag{3.42}$$

$$f_{1c} = 1 - e^{-(R/R_c)} \tag{3.43}$$

$$f_{1d} = \tanh(R/R_c) \tag{3.44}$$

For the right choice of λ and n these models can all pass the observational constraints. The first and second model, (3.41) and (3.42), are viable models when $\lambda \approx 1.55$ and $n = 1$. This leads to a growth index $\gamma_0 \equiv \gamma(z = 0) \simeq 0.41$, dropping to 0 with increasing

z. This is notably different from the ΛCDM model which has $\gamma_0 \approx 0.55$.
For the third model (3.43) γ_0 is dispersed between 0.40 and 0.55 for $2 \leq \lambda \leq 8$, but for $1 \leq \lambda \leq 2$ the value for γ_0 lies between 0.40 and 0.43. The last model (3.44) has $0.40 \leq \gamma_0 \leq 0.55$ only now $1.5 \leq \lambda \leq 4$. Again the growth index converges to $0.40 \leq \gamma_0 \leq 0.43$ but now for $0.905 \leq \lambda \leq 1.5$.

The three different types of $f(R)$ theories (Metric, Palatini and Metric-affine $f(R)$ Gravity) will be discussed in the following sections.

3.1.3.1 Metric $f(R)$ Theories

Physics

In the metric formalism the action is varied with respect to the metric. The action in $f(R)$ gravity is given by:

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + \int d^4x\mathcal{L}_M(g_{\mu\nu}, \Psi_M) \tag{3.45}$$

Where $\mathcal{L}_M$ is the matter Lagrangian, dependent on the metric and the matter fields Ψ_M. After varying (3.45) with respect to the metric the field equations are obtained.

$$f,_R R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu}[\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]f,_R R = T_\mu\nu \tag{3.46}$$

where ∇_μ is the covariant derivative associated with the Levi-Civita connection of the metric and $\Box \equiv \nabla^\mu\nabla_\mu$. Some algebraic manipulation results in:

$$f,_R(R)R - 2f(R) = 0 \tag{3.47}$$

at a de Sitter point corresponding to a vacuum solution (energy-momentum tensor vanishes), where R is constant. For the standard FLRW metric (2.6) with $k = 0$ (i.e. flat space time) the field equations are

$$3f,_R H^2 = \frac{1}{2}(f,_R R - f) - 3H\dot{f},_R + \kappa\rho_M \tag{3.48}$$

$$-2f,_R \dot{H} = \ddot{f},_R - H\dot{f},_R + \kappa(\rho_M + p_M) \tag{3.49}$$

An example of a solution to (3.47) is $f(R) = \alpha R^2$. In another model where $f(R) = R+\alpha R^2$ (Starobinsky model) the inflationary expansion ends when αR^2 becomes smaller than R. This is followed by a reheating stage in which the oscillation of R leads to the gravitational particle production. For more details and the derivation of (3.47), (3.48) and (3.49) see for example [36], [37] or [44] and references therein.

The metric $f(R)$ gravity corresponds to generalized Brans-Dicke Theory with $\omega_{BD} = 0$ (with a potential) [37][46], Brans-Dicke theory will be discussed in Section 4.1.4.

Astronomy

(Metric) $f(R)$ gravity is only useful if it predicts accelerated expansion without DE (or inflation). In [36] the authors accomplish this by defining the effective energy density and effective pressure:

$$\rho_{eff} = \frac{Rf_{,R} - f(R)}{2f_{,R}} - \frac{3H\dot{R}f_{,RR}(R)}{f_{,R}} \tag{3.50}$$

$$P_{eff} = \frac{\dot{R}^2 f_{,RRR} + 2H\dot{R}f_{,R} + \ddot{R}f_{,RR} + \frac{1}{2}(f - Rf_{,R})}{f_{,R}} \tag{3.51}$$

by plugging

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + Pg^{\mu\nu} \tag{3.52}$$

into the field equations (3.46) where u^μ is the four-velocity of an observer. Dividing (3.50) and (3.51) leads to the effective equation of state parameter w_{eff}

$$w_{eff} = \frac{\dot{R}^2 f_{,RRR} + 2H\dot{R}f_{,RR} + \ddot{R}f_{,RR} + \frac{1}{2}(f - Rf_{,R})}{\frac{Rf_{,R} - f}{2} 3H\dot{R}f_{,RR}} \tag{3.53}$$

3.1.3.2 Palatini $f(R)$ Theory

Physics

In the Palatini Formalism the action (3.45) is not only varied with respect to the metric, but also with respect to the connection, which leads to:

$$\delta S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}[\left(f_{,R}\, R_{(\mu\nu)} - \frac{1}{2}f(R)g_{\mu\nu}\right)\delta g_{\mu\nu} + f_{,R}\, g^{\mu\nu}\delta R_{\mu\nu}] + \delta S_m \tag{3.54}$$

where $R_{(\mu\nu)}$ is the symmetric part of $R_{\mu\nu}$
This equation leads to the following field equations:

$$f_{,R}\, R_{\mu\nu}(\Gamma) - \frac{1}{2} f(R) g_{\mu\nu} = \kappa T^{(M)}_{\mu\nu} \tag{3.55}$$

$$- \nabla_\lambda \left(\sqrt{-g} f_{,R}\, g^{\mu\nu}\right) + \delta^\nu_\lambda \nabla_\rho \left(\sqrt{-g} f_{,R}\, g^{\mu\rho}\right) + \\ 2\sqrt{-g} f_{,R} \left(g^{\mu\nu} S^\sigma_{\sigma\lambda} - \delta^\nu_\lambda g^{\mu\rho} S^\sigma_{\sigma\rho} + g^{\mu\sigma} S^\nu_{\lambda\sigma}\right) = H^{\nu\mu}_\lambda \tag{3.56}$$

Milgrom, the inventor of MOND[2], has also used the Palatini formalism on his theory using the Lagrangian:

$$L_N = \frac{1}{\kappa}(\vec{g}^2 - 2\phi \vec{\nabla} \vec{g}) + \rho \left(\frac{1}{2}\vec{v}^2 - \phi\right) \tag{3.57}$$

where $\rho = \sum_i m_i \delta(\vec{x} - \vec{x}_i)$; variations over ϕ and $\vec{g}$. This leads to the following set of equations:

$$\ddot{\vec{x}}_i = -\vec{\nabla}\phi(\vec{x}_i \tag{3.58}$$

$$\vec{\nabla}\vec{g} = -\frac{\kappa}{2}\rho \tag{3.59}$$

$$\vec{\nabla}\phi = \mu \left(\left| \frac{\vec{\nabla}\vec{g}}{a_0} \right| \right) \vec{g} \tag{3.60}$$

More details can be found in [47] and [48]. In the Palatini version of $f(R)$ theories the independent connection does not introduce new dynamical degrees of freedom in contrast to the metric formalism. Rather, it changes the way matter generates the space-time curvature corresponding to the metric by generating new terms for matter on the right side of the field equations [47]. The Palatini $f(R)$ gravity corresponds to generalized Brans-Dicke theory with $\omega_{BD} = -3/2$ and a potential [47][37], for Brans-Dicke Theory see section 3.1.4.

3.1.3.3 Metric-Affine $f(R)$ Theories

Varying with respect to both the metric and the connection without assuming independence the following field equations are acquired [49]:

$$f_{,R}\, R_{(\mu\nu)} - \frac{1}{2} f(R) g_{\mu\nu} = \kappa T_{\mu\nu} \tag{3.61}$$

$$\frac{1}{\sqrt{-g}}\left[-\nabla_\lambda(\sqrt{-g} f_{,R}\, g^{\mu\nu}) + \nabla_\sigma(\sqrt{-g} f_{,R}\,(R) g^{\mu\sigma}\delta^\nu_\lambda)\right] + 2f_{,R}\,(R) g^{\mu\sigma} S_{\sigma\lambda}{}^{\nu} = \kappa(\Delta^{\mu\nu}_\lambda - \frac{2}{3}\Delta_\sigma{}^{\sigma[\nu}\delta^{\mu]}{}_\lambda) \tag{3.62}$$

$$S^\sigma_{\mu\sigma} = 0 \tag{3.63}$$

Where $\Delta^{\mu\nu}_\lambda \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\Gamma^\lambda{}_{\mu\nu}}$, the so called *hypermomentum*, indicating the variation of the matter action S_M with respect to the connection. The hypermomentum is somewhat analogous to the stress-energy tensor [36], and "it encapsulates all the information related to the spin angular momentum of matter, the intrinsic part of dilation current and the shear current"[50]. When the matter action depends at most linearly on the connection (which is true for e.g. scalar and gauge fields, where there is no dependence, and for fermion fields, where the matter action depends linearly on the connection) and depends only on first order derivatives of the matter fields, $\Delta^{\mu\nu}_\lambda$ will only depend algebraically on the connection. This leads to the following equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa \mathcal{T}_{\mu\nu} \tag{3.64}$$

Where $R_{\mu\nu}$ and R are the Ricci tensor and the Ricci scalar of the metric $g_\mu\nu$ respectively, and $\mathcal{T}_{\mu\nu}$ will be a second rank tensor which depends on the metric, $\Delta^{\mu\nu}_\lambda$ and $T_{\mu\nu}$. $\mathcal{T}_{\mu\nu}$ reduces to $T_{\mu\nu}$ when $\Delta^{\mu\nu}_\lambda = 0$. Eq. (3.64) describes GR with modified matter interactions [50].

For more details on Metric-Affine Gravity see for example [23],[49],[51],[50].

Simulations

Apart from solar system tests numerical simulations can be used to provide (often stronger) constraints on modified gravity theories. A particularly interesting $f(R)$ model that benefits from simulations is the model proposed by [52]:

$$f(R) = -m^2 \frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1} \quad n > 0 \tag{3.65}$$

m is the mass scale, the authors used $m = \frac{\kappa^2 \bar{\rho}_0}{3}$ where $\bar{\rho}_0$ is the average density in the present epoch. This model is equivalent to (3.41). It passes solar system and cosmological tests by using the chameleon screening mechanism and predicts the accelerated expansion of the universe without a cosmological constant. When $R/m^2 \ll 1$, (3.65) can be approximated by the expansion

$$f(R) \approx -\frac{c_1}{c_2} m^2 + \frac{c_1}{c_2^2} m^2 \left(\frac{m^2}{R}\right)^n \tag{3.66}$$

$f,_R$ is then

$$f,_R = -\frac{nc_1}{c_2^2} \left(\frac{m^2}{R}\right)^{n+1} \tag{3.67}$$

The ratio c_1/c_2 is constraint by the fact that the model should predict the correct expansion history, corresponding to ΛCDM: $m^2 c_1/c_2 = 16\pi\bar{\rho}_\Lambda$. Filling this in and rewriting in terms of $f,_{R_0}$, the current background value for the field gives

$$f(R) = -16\pi\bar{\rho}_\Lambda - \frac{f,_{R_0}}{n} \frac{\bar{R}_0^{n+1}}{R^n} \tag{3.68}$$

$$f,_R = f,_{R_0} \left(\frac{\bar{R}_0}{R}\right)^{n+1} \tag{3.69}$$

Looking into models with $n = 1$, the only free parameter to be constraint is $f,_{R_0}$. A lot of effort has been put into numerical simulations of this model with
$|f,_{R_0}| = 10^{-4}, 10^{-5}, 10^{-6}$. The code used in these simulations is ECOSMOG. It is based on the RAMSES code (for N-body hydrodynamical simulations, structure formation) [53] and performs N-body simulations for modified gravity theories [54]. For example [55] looked into clustering of galaxies in redshift space. Figure 3.2 shows that for higher k the model significantly deviates from GR, implying observations should be able to distinguish the two.

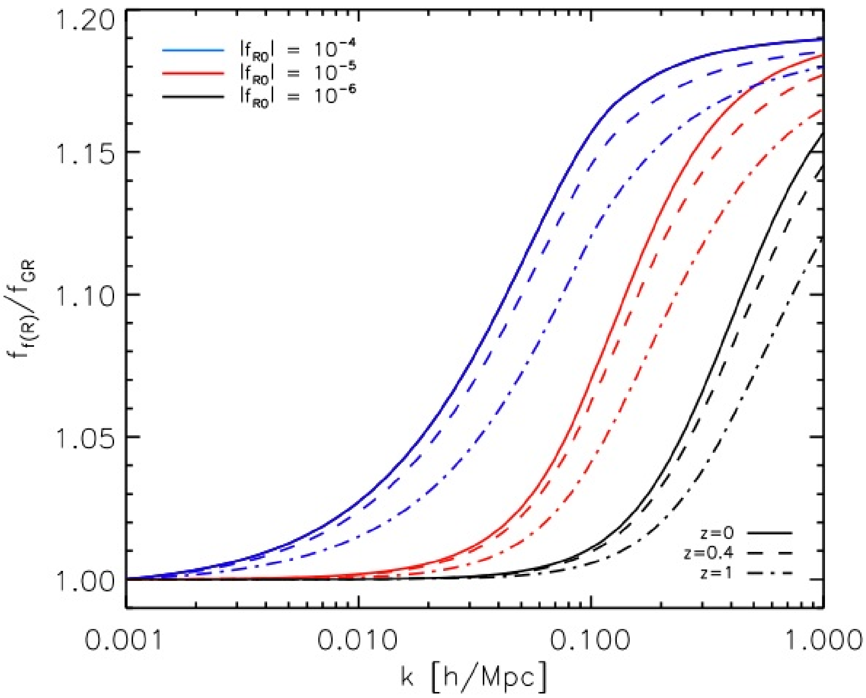

FIGURE 3.2: Ratio of linear growth rate against scale compared to GR for different parameters. Taken from [55]

Similar work was carried out in [56]. The authors investigated the same model with the same code to examine the properties of a halo (similar to the Virgo Cluster) at different redshifts.

In figure 3.3 the density along the line of sight is shown for the model and ΛCDM. F4, F5 and F6 correspond to $|f_{,R_0}| = 10^{-4}, 10^{-5}$ and 10^{-6} respectively which have $\frac{c_1}{c_2^2} = 0.168$, 0.0168 and 0.00168 respectively. Simulations of the model show small but noticeable deviation from ΛCDM.

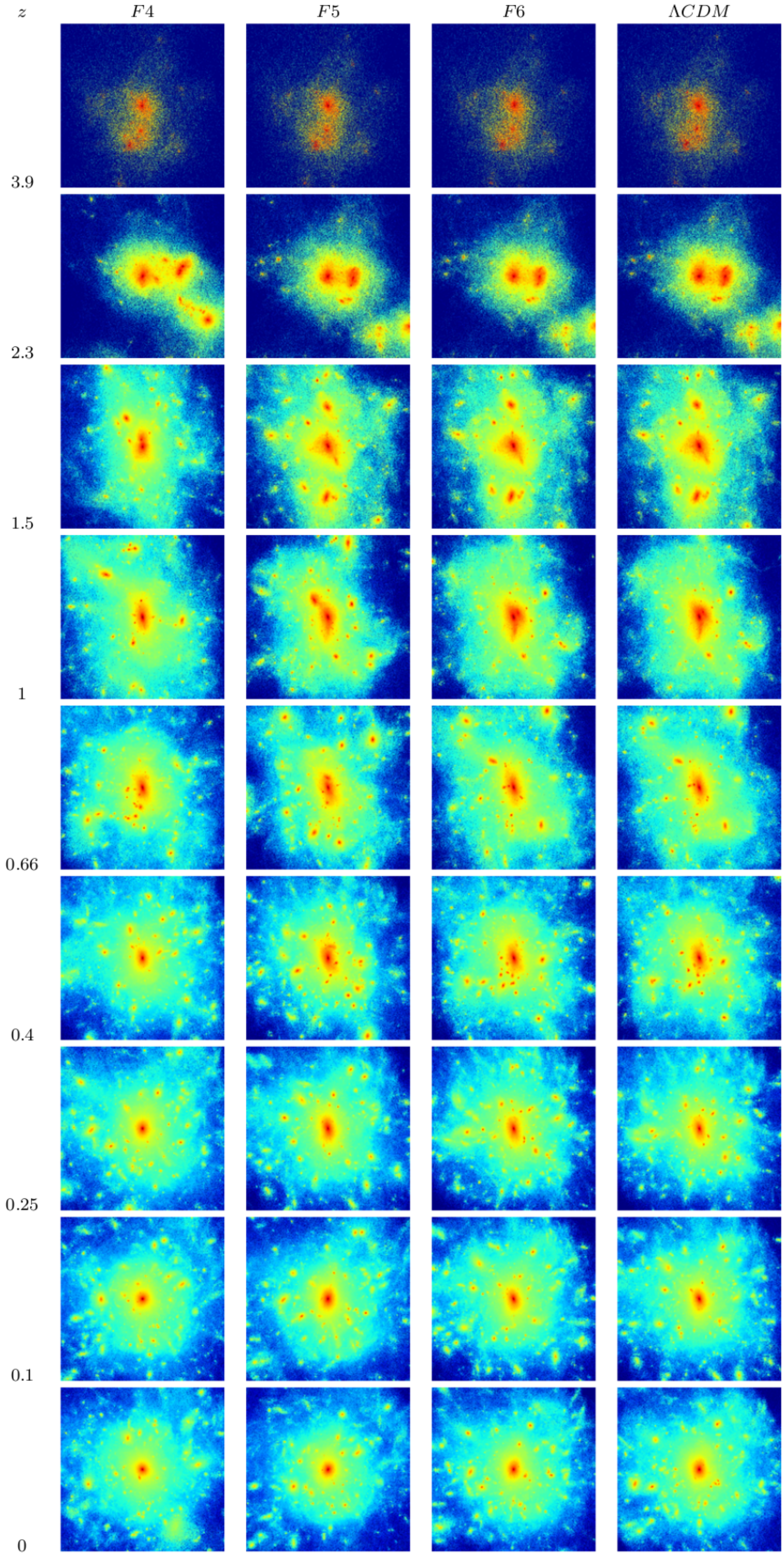

FIGURE 3.3: Density along line of sight for the model and ΛCDM at various redshifts. Taken form [56]

The same model was investigated using the same code (ECOSMOG) in the context of voids in [57]. Two different classes of voids were modeled; with radii ranging from 15 to 25 Mpc/h and 35 to 55 Mpc/h. The void density profiles are shown for F4, F5 and F6 and ΛCDM in 3.4. The left figures correspond to a small void and the right figures to a larger void. The bottom two graphs show the deviation of the model from GR. Smaller voids generally have a sharper edge, as is visible in the graph. The fact that the over-dense void edge ($r \sim r_{void}$) is bigger in the model is most likely due to the fact that the voids in the model have different radii compared to their GR counterparts. The paper concludes that in this particular $f(R)$ model voids grow marginally larger and faster. It will be difficult to distinguish the model from GR observationally [57]. Voids are emptier and steeper in this particular $f(R)$ model [58].

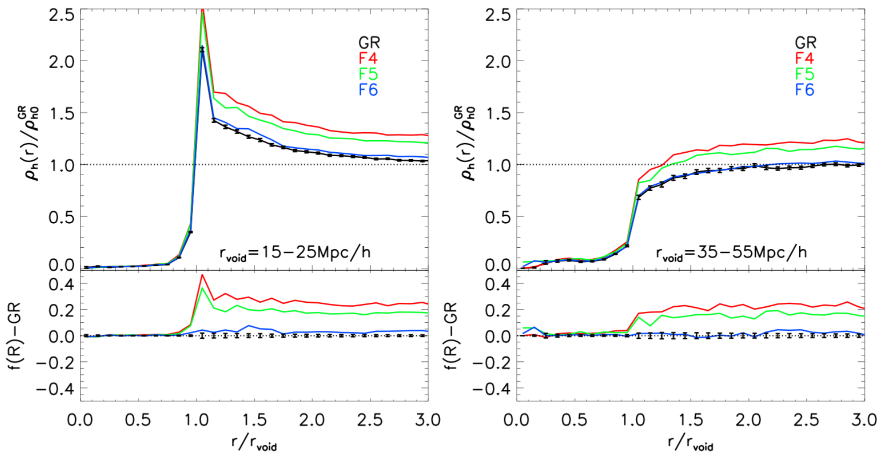

FIGURE 3.4: Void profiles for the model and ΛCDM (GR).Taken form [57]

For a list of constraints on $f_{,R_0}$, see [59] and references therein.

3.1.4 Brans-Dicke

Physics

Brans-Dicke Theory (BD), sometimes referred to as Jordan-Brans-Dicke Theory, is a Scalar-Tensor gravity theory. This theory adds an extra scalar field ϕ to the tensor equations to account for the measurable gravitational constant. The value of this scalar field depends on the point in space-time. The theory is based on two principles, Mach's principle and Dirac's Large Number Hypothesis (LNH). The first principle says that "motion of any individual body is to be defined with respect to the entire universe" [60], but there are many other interpretations [61]. LNH states that large dimensionless numbers in physics are related (e.g. the gravitational constant and the radius and mass of the universe are related: $GM_{universe} \sim R_{universe}$) [62]. The premise of BD theory is that the gravitational constant G is replaced by a scalar field ϕ, depending on the matter distribution in the universe, with $G \sim \phi^{-1}$. The Lagrangian (without a potential) on which the action is based in BD is

$$\mathcal{L}_{BD} = \phi R + 16\pi \mathcal{L}_m + \mathcal{L}_\phi(\phi, \partial_\mu \phi) \tag{3.70}$$

Notice that this is just the normal Lagrangian from GR multiplied by $G^{-1} = \phi$, except for the last term which is there to ensure the Lagrangian leads to a second order equation. The action is then

$$S_{BD} = \int d^4x \sqrt{-g} \mathcal{L}_{BD} \tag{3.71}$$

On basis of unit consistency (and simplicity) the chosen $\mathcal{L}_\phi$ is

$$\mathcal{L}_\phi = -\omega \frac{\partial_\mu \phi \partial^\mu \phi}{\phi} \tag{3.72}$$

Combining the above equations results in

$$\Box \phi - \frac{\partial_\mu \phi \partial^\mu \phi}{2\phi} \frac{\phi}{2\omega} R \tag{3.73}$$

Which relates ϕ to the Ricci curvature R using ω [63]. The value for ω is not given by the theory and can only be determined by observation. ω represents a measure of the fraction of the gravitational force caused by the scalar field. The strength of the coupling of the scalar field compared to that of gravity can be written as

$$\alpha = \left(\omega + \frac{3}{2}\right)^{-1} \tag{3.74}$$

In the limit $\omega \to \infty$ the coupling of the scalar field becomes negligible and BD theory reduces to GR [64][65], but it is important to note that there are problems arise when taking this limit, especially when the trace of the energy-momentum tensor vanishes [66].

Physics/Astronomy

Solar system test however indicate that $\omega > 250$, which means that the scalar force is smaller than 0.2% of the gravitational force; the predictions based on the theory are essentially the same as those based on GR, rendering Brans-Dicke theory (without potential) unappealing [67]. Many other tests resulted in large values as well, even using slightly different models. Some calculations neglected the potential of the scalar field while others used the Newtonian approximation and spherical symmetry. The most stringent condition was found in a solar system test done by the Cassini Spacecraft: $\omega > 40000$ [68].

When the scalar field gradient is large compared to some constant a_0 ($[a_0] = [L][T]^{-2}$) $\omega \to \infty$ and theory reduces to GR. When $\omega \to -\frac{3}{2}$ the scalar force becomes arbitrarily large compared to gravity with increasing distance from a massive object [65].
In the weak field limit the theory reduces to a theory with field equations

$$\nabla^2 \phi_1 = 4\phi(1-\lambda)\rho \tag{3.75}$$

$$\nabla \cdot \left[\mu\left(\frac{\lambda \nabla \phi_2}{a_0} \nabla \phi_2\right)\right] = 4\pi\lambda\rho \tag{3.76}$$

where

$$\lambda = \frac{1}{2\omega_0 + 4} \tag{3.77}$$

$$\mu(x) = \frac{dF}{dx} \tag{3.78}$$

that are equivalent to the standard Newtonian theory and reduce to MOND (in cases of high symmetry)[67].

According to the theory gravitation decreases in time, at a rate depending on ω. The age of the universe t_u can be expressed as a function of ω and the Hubble time t_H

$$t_u = \frac{2 + 2\omega}{4 + 3\omega} t_H \tag{3.79}$$

For $\omega = 6$ the value of t_u/t_H is $7/11 \approx 0.64$, only slightly smaller than the ratio $2/3$ based on the Einstein-de Sitter model. Applying the theory observations (e.g. the anomalous precession of Mercury's perihelion) leads to $\omega \gtrsim 6$ [64].

An interesting example of an action (in the $f(R)$ gravity theory class) is treated in [69]. It shows the connection between general Scalar-Tensor theories, quintessence, $f(R)$ theories and BD. Starting with

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}f(\varphi, R) - \frac{1}{2}\zeta(\varphi)(\nabla\varphi)^2\right] + S_m \tag{3.80}$$

where φ is a scalar field and ζ is some function of φ, the authors considered theories of the type

$$f(\varphi, R) = F(\varphi)R - 2V(\varphi) \tag{3.81}$$

where $F \equiv \frac{\partial f}{\partial R}$ and $V(\varphi)$ is a potential. They then introduced a new scalar field

$$\phi = \frac{\sqrt{6}}{2}\ln F \tag{3.82}$$

so that the action (in the Einstein Frame) becomes

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}\frac{1}{2}(\tilde{\nabla}\phi)^2 - \frac{RF - f}{2F^2}\right] + S_m \tag{3.83}$$

The tilde denotes a quantity in the Einstein Frame. This means that $f(R)$ gravity is equivalent to a more general scalar-tensor theory with action (3.80), of the type described by (3.81) with potential V (in Jordan Frame) given by $V = \frac{RF-f}{2}$. The coupling strength between DE and (non-relativistic) matter can be expressed by defining

$$Q = -\frac{F_{,\phi}}{2F} \tag{3.84}$$

Comparing Q with (3.82) shows that $f(R)$ gravity corresponds with $Q = -\frac{1}{\sqrt{6}}$.
If one instead defines the scalar field

$$\phi = \int\left[\sqrt{\frac{3}{2}\left(\frac{F_{,\varphi}}{F}\right)^2 + \frac{\zeta}{F}}\right]d\varphi \tag{3.85}$$

the action changes slightly:

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - \frac{V}{F^2}\right] + S_m \tag{3.86}$$

This leads to

$$F = e^{-2Q\phi}, \ \ \zeta = (1 - 6Q^2)F\left(\frac{d\phi}{d\varphi}\right)^2 \tag{3.87}$$

The action (3.80) is now given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}FR - \frac{1}{2}(1-6Q^2)F(\nabla\phi)^2 - V\right] + S_m \tag{3.88}$$

Comparing this to the general action in BD (with a potential V):

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\chi R - \frac{1}{2}\frac{\omega_{BD}}{\chi}(\nabla\chi)^2 - V\right] + S_m \tag{3.89}$$

clearly shows that the two are equivalent when $\chi = F$ and $3 + 2\omega_{BD} = \frac{1}{2Q^2}$. When $Q \to 0$ the model reduces to a quintessence model.

The observational constraints on ω_{BD} carry over to Q when no potential is added: $\omega_{BD} > 40000$ means that $Q < 0.0025$. It is difficult to distinguish this from the $Q = 0$ case. But with a potential, w is allowed to be smaller, and Q will then be much larger. [69]

3.1.5 Horndeski Theory

Horndeski theory is the most general scalar-tensor model that has second-order equations of motion and obeys the weak equivalence principle. Horndeski Theory encompasses all scalar-tensor theories with second order field equations in curved 4-dimensional spacetime[70]. The reason for avoiding higher order field equations is to stay away from Ostrogradski instabilities. A requirement for modified gravity theories is the absence of ghosts. According to the Ostrogradski Theorem, these types of instabilities appear in theories with a non-degenerate Lagrangian (i.e. with higher time derivatives). A Lagrangian $\mathcal{L}(q, \dot{q}, \ddot{q})$ is non degenerate when $\frac{\partial^2\mathcal{L}}{\partial\ddot{q}^2} \neq 0$. Even though Horndeski was considered to be the most general scalar tensor models devoid of Ostrogradski instabilities, more general theories have been developed [71], e.g. "Generalized Generalized Galileons" (G^3)[72].
In Horndeski the action is given by

$$S_H = \int d^4x\,\sqrt{-g}\left[\sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_m(g_{\mu\nu})\right], \tag{3.90}$$

where $g_{\mu\nu}$ is the metric, $\mathcal{L}_m$ is the matter Lagrangian and $\mathcal{L}_i$ are given by

$$\mathcal{L}_2 = G_2(\phi, X) \tag{3.91}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi \tag{3.92}$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right] \tag{3.93}$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right] \tag{3.94}$$

G_2, G_3, G_4 and G_5 are arbitrary functions of the scalar field ϕ and $X = -\phi^{;\mu}\phi_{;\mu}/2$ (the canonical kinetic term). S_H is the most general action for a single scalar field that has second-order equations of motion and satisfies the weak equivalence principle[73]. The $\mathcal{L}_i$ are very general due to the dependence on the arbitrary G_i.
Horndeski theory is a generalization of many other, more specific theories of gravity, for example Brans-Dicke (BD) theory (with potential $V(\phi)$), is corresponds to functions $G_2 = -M_{pl}\omega_{BD}X/(2\phi) - V(\phi)$, $G_3 = 0$, $G_4 = M_{pl}\phi/2$, $G_5 = 0$, where M_{pl} is the reduced Planck mass and ω_{BD} is the Brans-Dicke parameter; The covariant Galileon corresponds to the functions $G_2 = \beta_2 X$, $G_3 = \beta_3 X$, $G_4 = M_{pl}^2/2 + \beta_4 X^2$ and $G_5 = \beta X^2$, where β_i are constants [74].

Astronomy

Horndeski theory is very general and in need of constraints in order to be useful in practice. Solar system tests demand a screening mechanism to screen the extra degree of freedom (”fifth force”) from high density regions. This is done by enforcing ”self-tuning”, i.e. the theory contains Minkowski space as a solution, in the presence of a cosmological constant. This results in the following set of Lagrangians, dubbed ”Fab Four” [75]:

$$\begin{aligned}
\mathcal{L}_{John} &= V_{John}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi & (3.95)\\
\mathcal{L}_{Paul} &= -\frac{1}{4}V_{Paul}(\phi)\epsilon^{\mu\nu\lambda\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\lambda\sigma\gamma\delta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\phi\nabla_\beta\phi & (3.96)\\
\mathcal{L}_{George} &= V_{George}(\phi)R & (3.97)\\
\mathcal{L}_{Ringo} &= V_{Ringo}(\phi)\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) & (3.98)
\end{aligned}$$

3.2 Screening

Most modified gravity theories introduce new degrees of freedom. GR however has been tested very well in the solar system meaning that the fifth force must be shielded from high density regions. This can be achieved using a so called screening mechanism. Theories adding an extra scalar degree of freedom almost always need a screening mechanism. A typical Lagrangian for such a scalar-tensor theory where the scalar field is conformally coupled to matter is given by

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, ...)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu{}_\mu \tag{3.99}$$

Where $Z^{\mu\nu}$ represents derivative self-interactions of the field, and $T^\mu{}_\mu$ is the trace of the matter stress-energy tensor. Close to a point source $T^\mu{}_\mu \to -\rho$, $\rho = M\delta^3(\vec{x})$. The resulting potential is

$$V(r) = -\frac{1}{4\pi r}\frac{g^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})} e^{-\frac{m(\bar{\phi})}{c_s(\bar{\phi})\sqrt{Z(\bar{\phi})}}r} M \tag{3.100}$$

Where a bar over a quantity denotes background values for that quantity. This however means that a light scalar field will produce a force $F \propto 1/r^2$. This problem can be fixed by realizing that g, m, Z or c_s depends on the background: [10]

- *Weak Coupling:* When g depends on the environment, the coupling is very small in regions of high density such that all test are satisfied. Examples of theories based on this are the symmetron [76] or varying dilaton [77] theories. In symmetron screening the coupling of the scalar field to matter is proportional to the vacuum expectation value of the field. In low density regions the field has a nonzero vacuum expectation value, but in high density regions it approaches zero. Dilaton screening is similar but has a different potential and coupling to matter.

- *High Mass:* An alternative is to make $m(\bar{\phi})$ depend on the local matter density. In regions of high density $m(\bar{\phi})$ acquires a very high mass and its interaction range becomes very short, making it unobservable, whereas in low density regions it mediates a force. Chameleon screening is based mostly on this assumption [78].

- *Kinetic Screening:* The third option is to make Z (the kinetic function) large environmentally. This leads to kinetic screening, where first or second derivatives become relevant. The mechanism where the second derivative becomes important is called Vainshtein screening [79]. Kinetic screening is used in for example *K-Inflation* models [80], while Vainshtein screening is seen in models based on brane constructions in higher dimensions, in limits of massive gravity theories [81] [82].

- *Sound Speed:* The last possibility is to make c_s very large. This however gives rise to superluminality.

Chapter 4

Conclusion

4.1 ΛCDM or Modified Gravity?

Unfortunately neither of the two paradigms seems to have enough proof backing it up to completely rule out the other. MG is still a relatively young field of study and will undoubtedly undergo much development in the coming years. As for now the ΛCDM model still stands proud as the cosmological standard model, despite craving a more fundamental understanding of both dark matter and dark energy. It would also be a shame to replace or modify GR, as it is an incredibly elegant theory capable of predicting almost all observations with great accuracy.

Both sides will benefit from better observational data, as is often the case in astrophysics. Tighter observational constraints will help weed out the bad MG theories and improve the good ones, as well as hopefully help to uncover the true nature of gravity.

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