Part I: Issues of 3D Shape Representation in Archaeology

The SHAPE Lab.
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The digitisation bottleneck

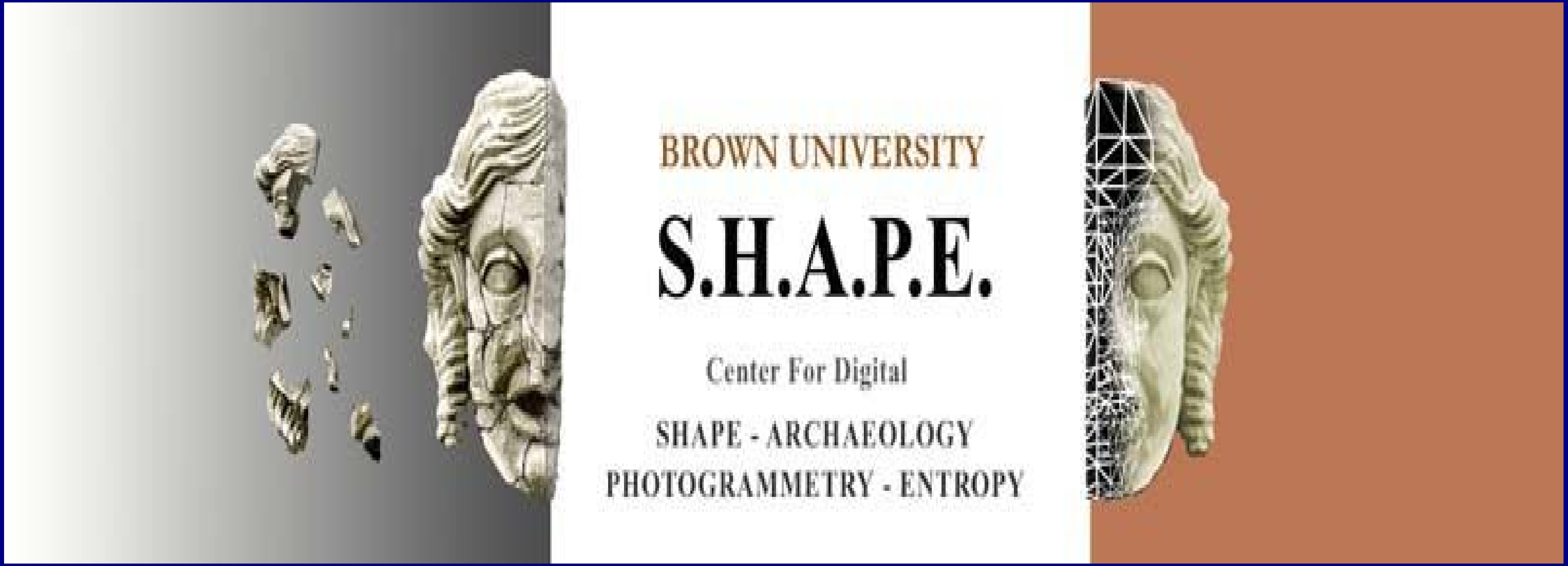
What can we really do with the technology?

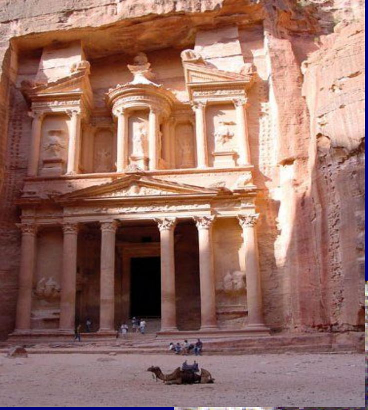
We may drown under too much data !!

One site can (easily) produce hundreds of thousands of registered artifacts.

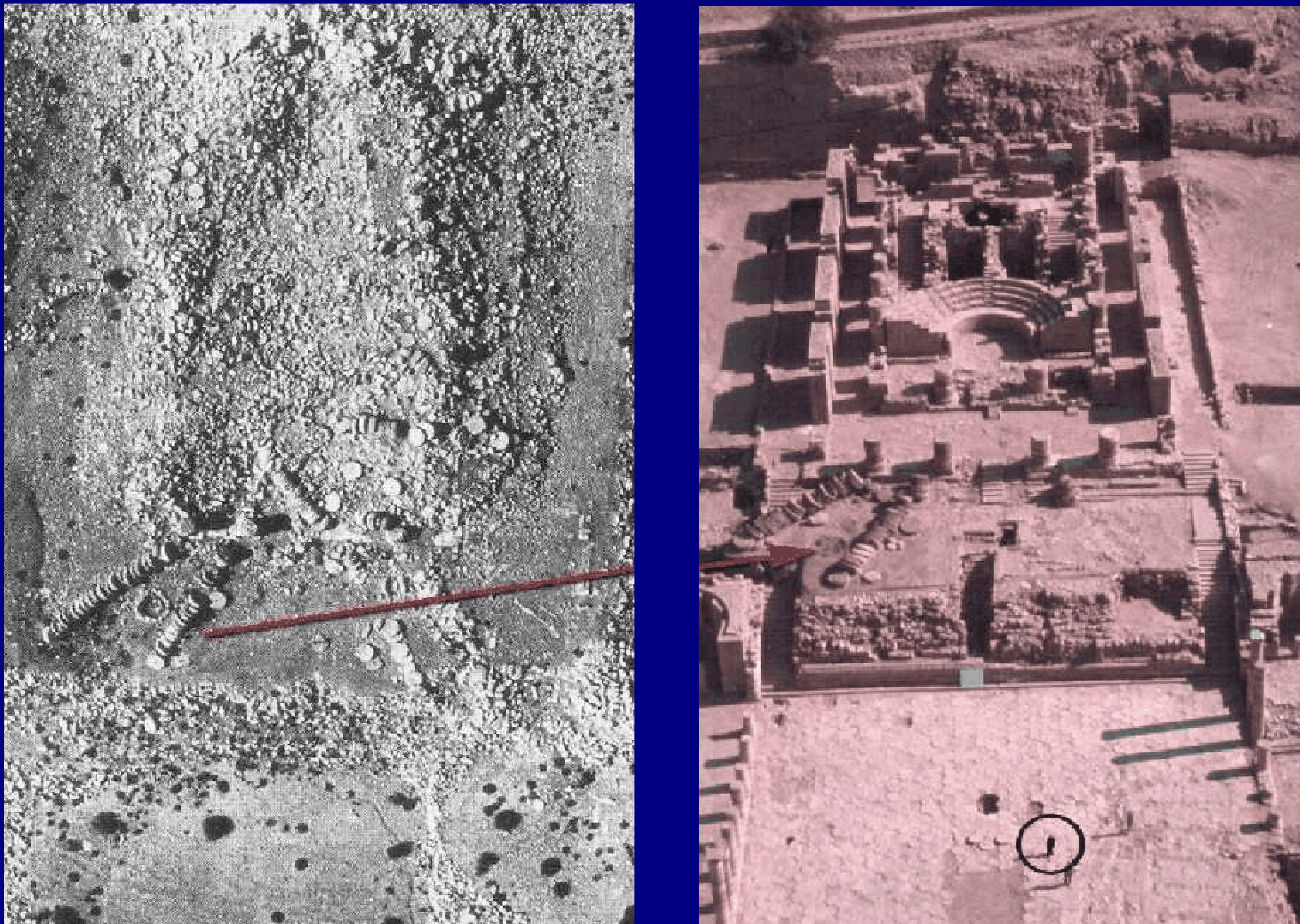
SHAPE Labs @ Brown University (since 1999) and Goldsmiths College (since 2004)

Combining 3D data and shape representation
to the benefit of archaeology

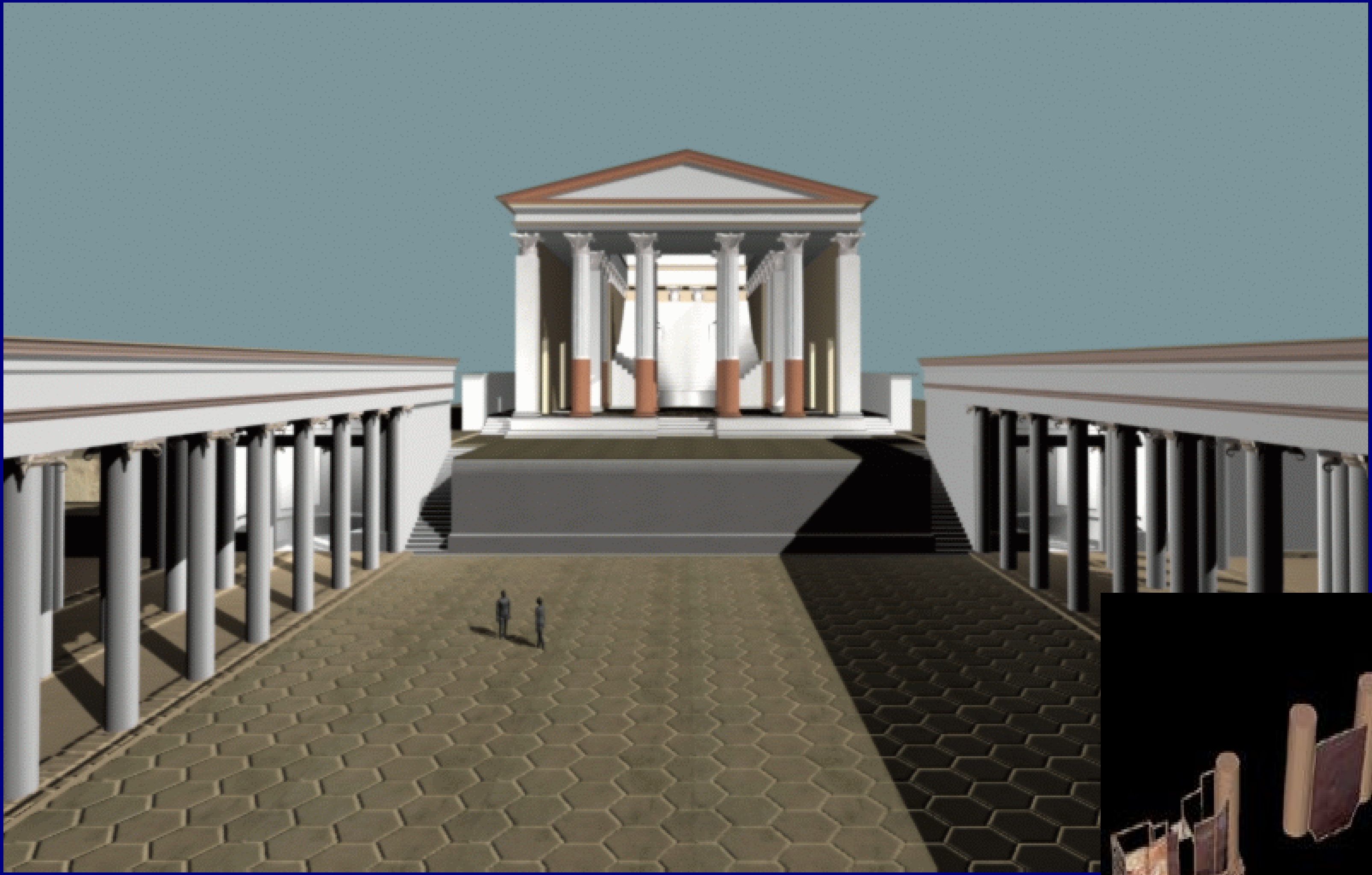




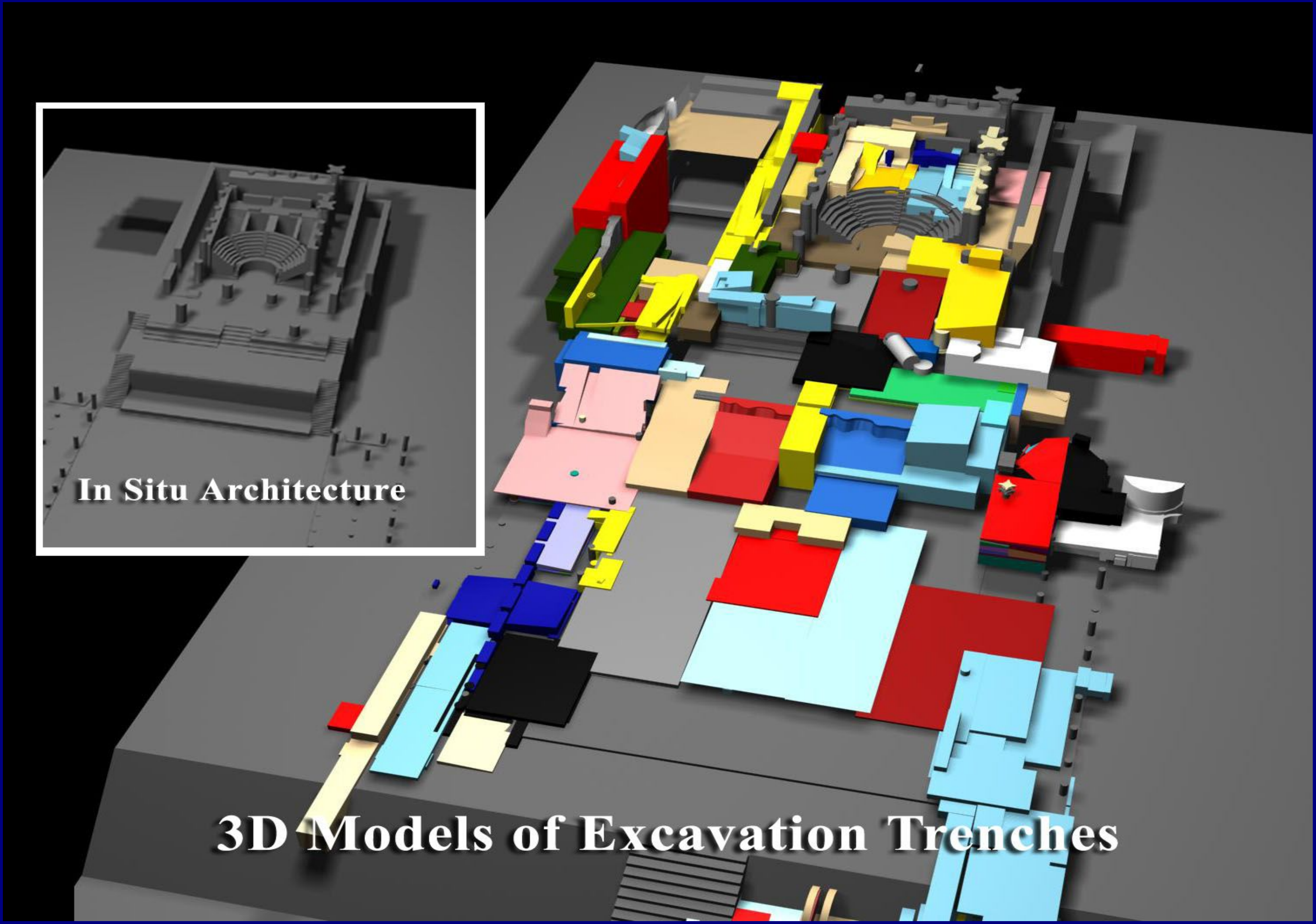
The Great Temple at Petra, Jordan.
Archaeological site: Martha Joukowsky and Brown University



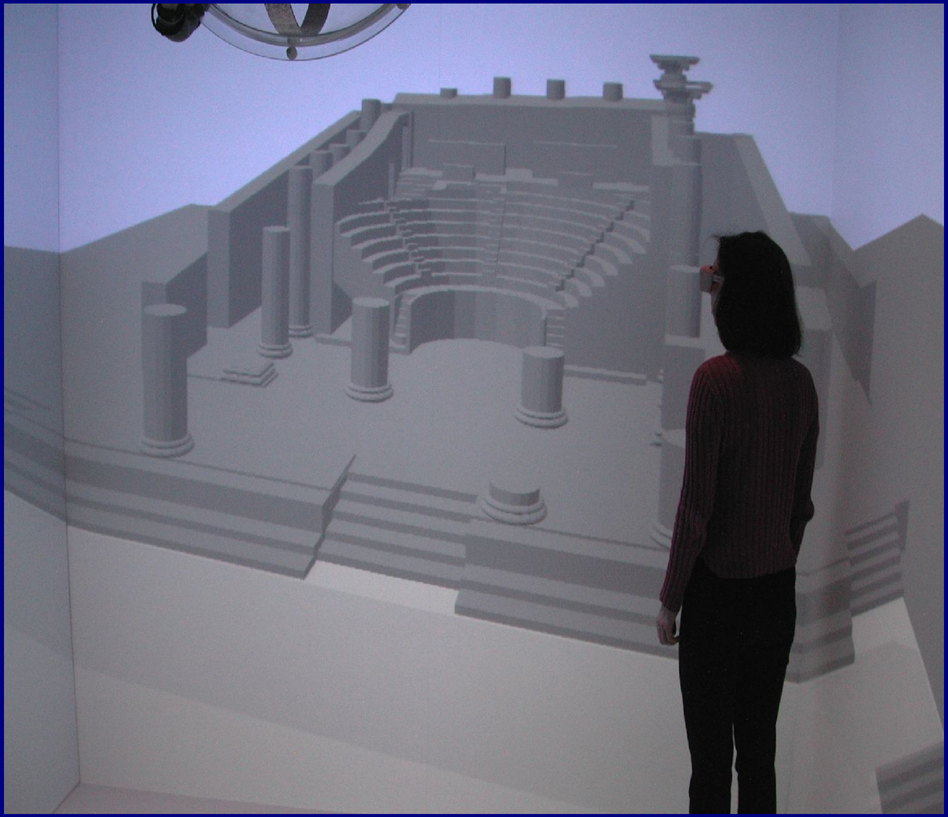
Basic (but not easy yet): 3D Reconstructions



More useful: 3D realistic context

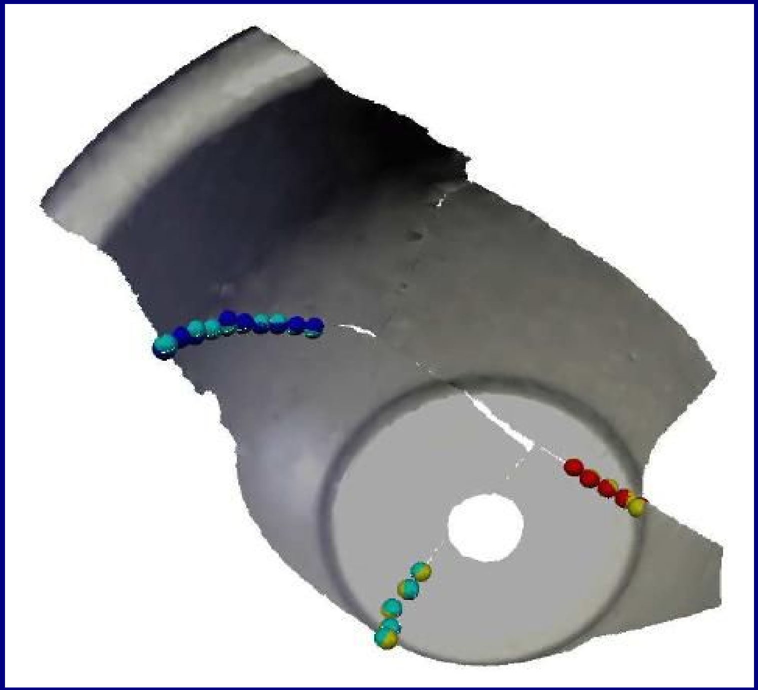


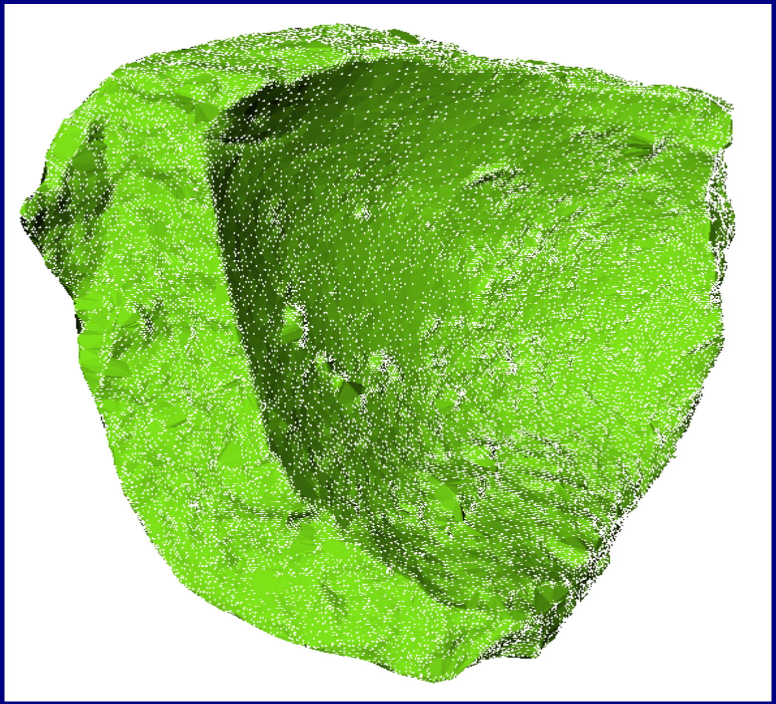
VR and large DB access @ Brown university: the ARCHAVE system for the Great Temple at Petra, Jordan.



Challenge: multi scale, multi user interplay with large multimedia DB via VR systems

Solving the 3D puzzle problem





Challenge: apply such ideas to free-form **sculptural** elements.

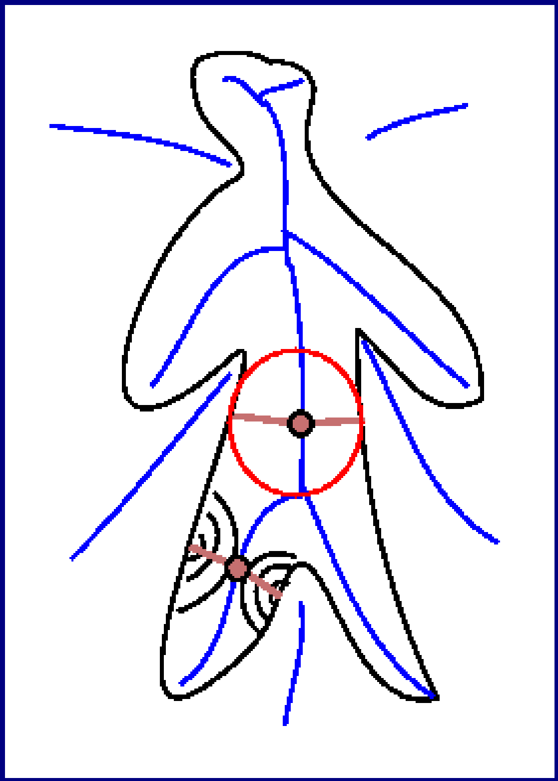
One, two, three, ...

- Digital Archaeology
- Context for the exploration of large DB
- 3D puzzle solvers
- ....
- Shape representation

Shape representation: From the Medial Axis to the Medial Scaffold

The Medial Axis Transform: Singular solutions of the Eikonal equation of geometric optics.

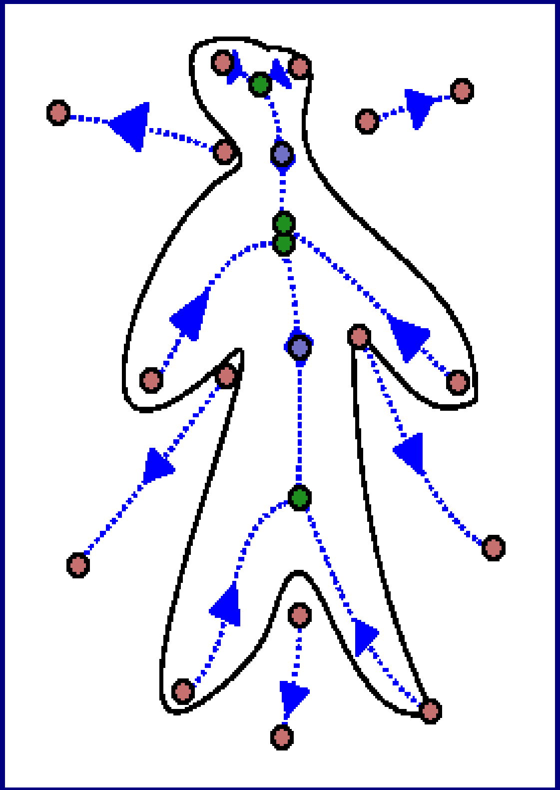
$$\sum_{i=1}^{n} \left(\frac{\partial u}{\partial x_i} \right)^2 = 1.$$



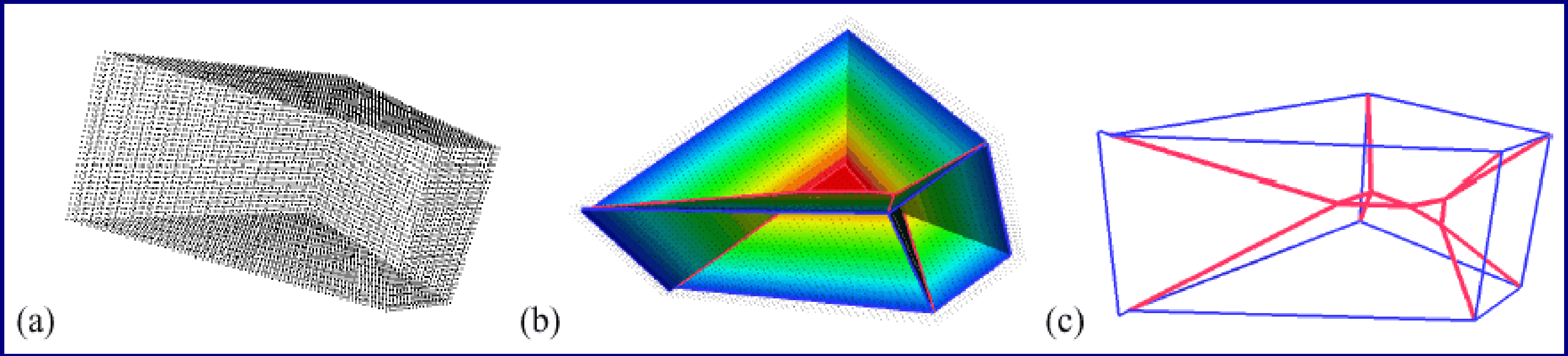
Blum (1960's & 70's):
Propagation vs.
Contact with disks

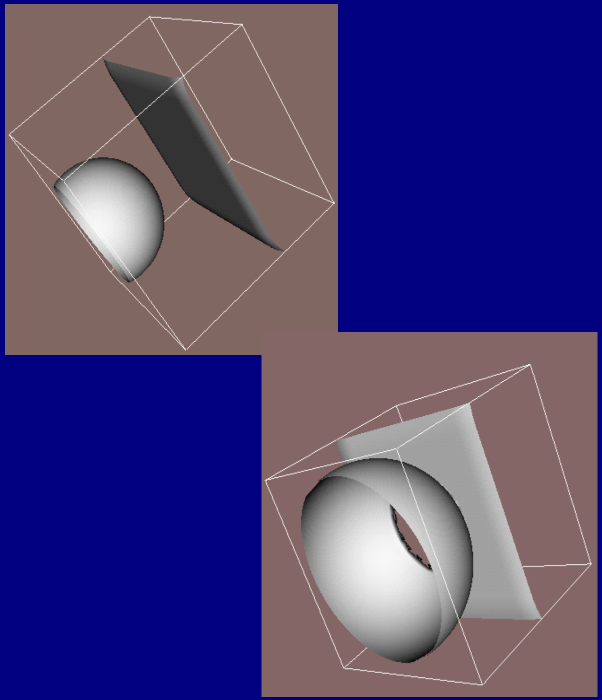
Wolter (1980+):
Geodesics, cut-locus

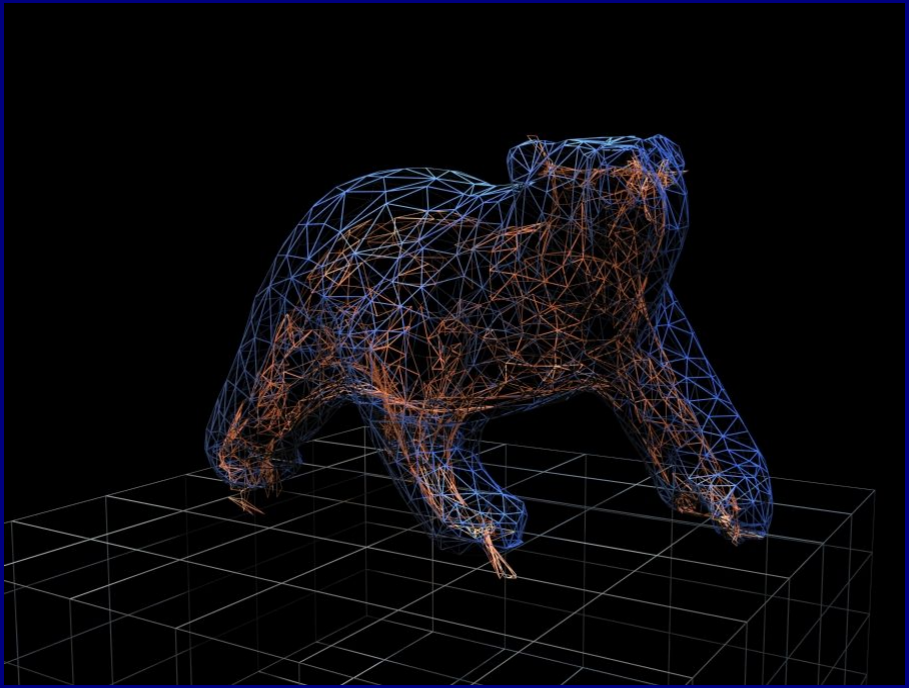
1995+:
Shock singularities
Contact typology



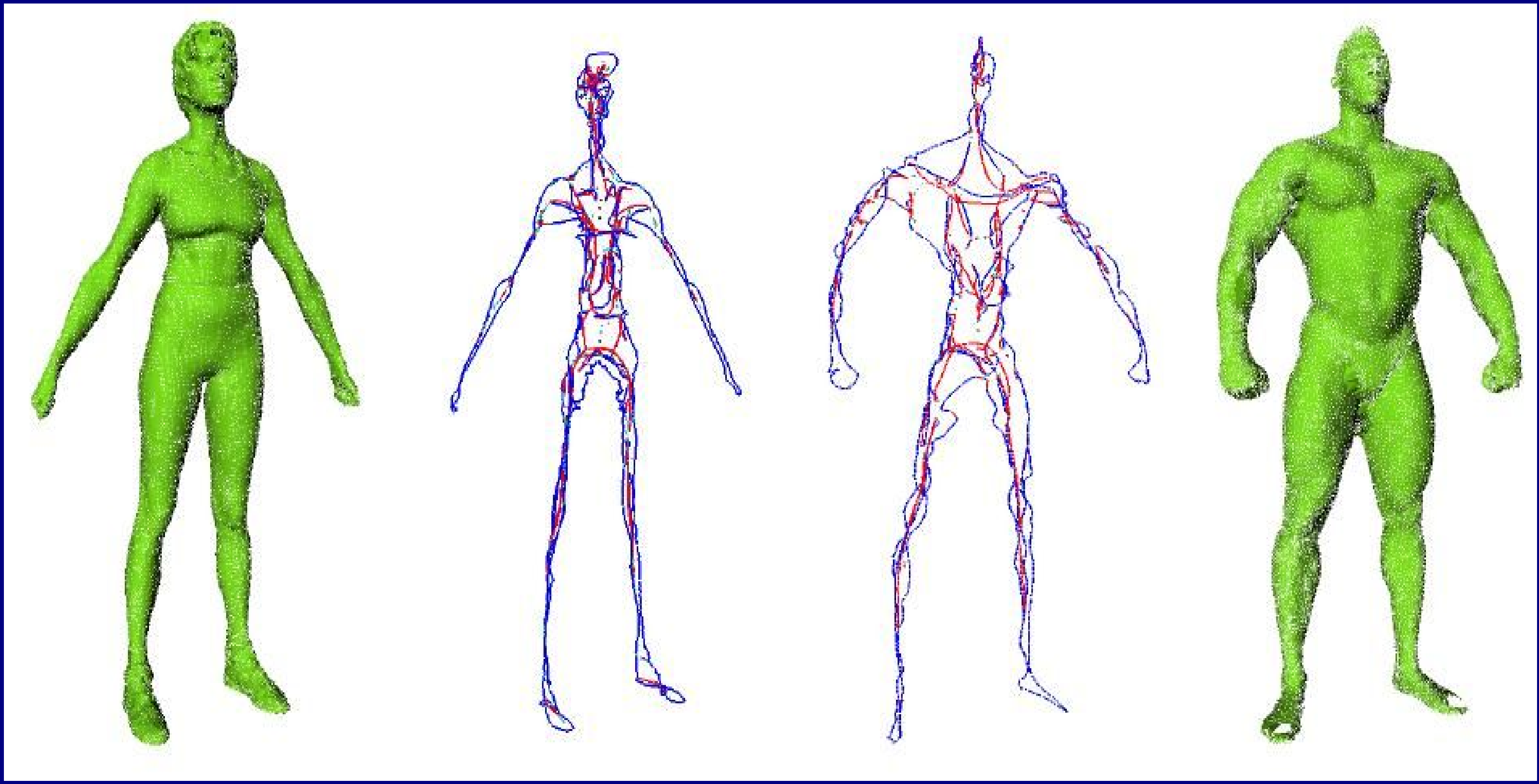
Shape representation: From the Medial Axis to the Medial Scaffold



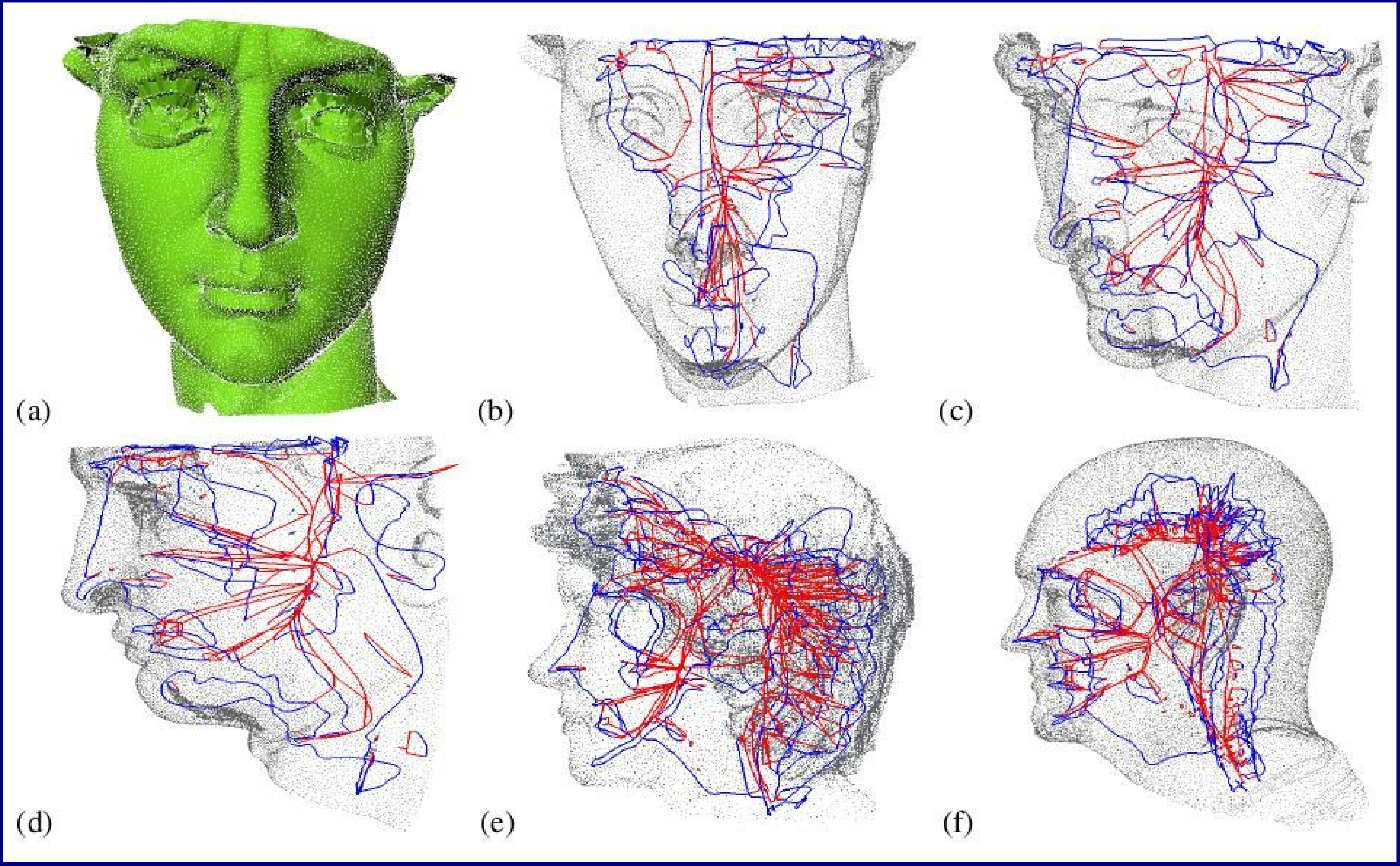




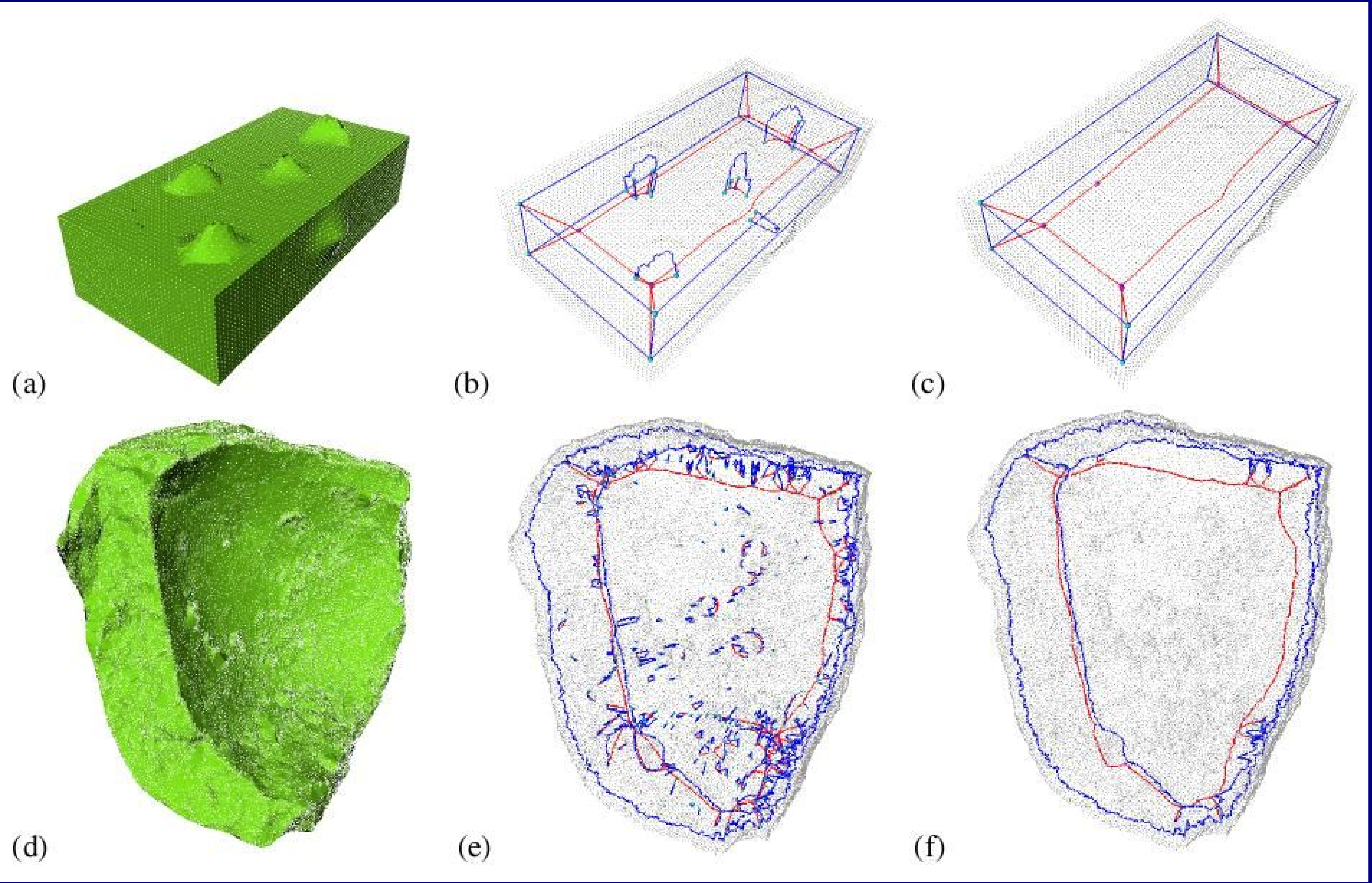
Shape representation: From the Medial Axis to the Medial Scaffold



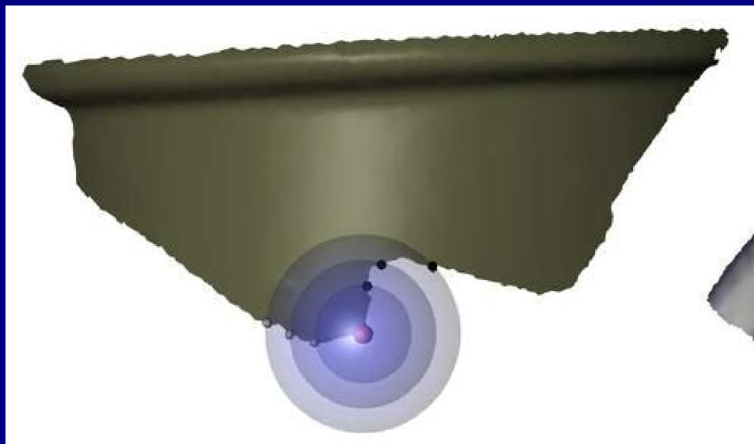
Shape representation: From the Medial Axis to the Medial Scaffold

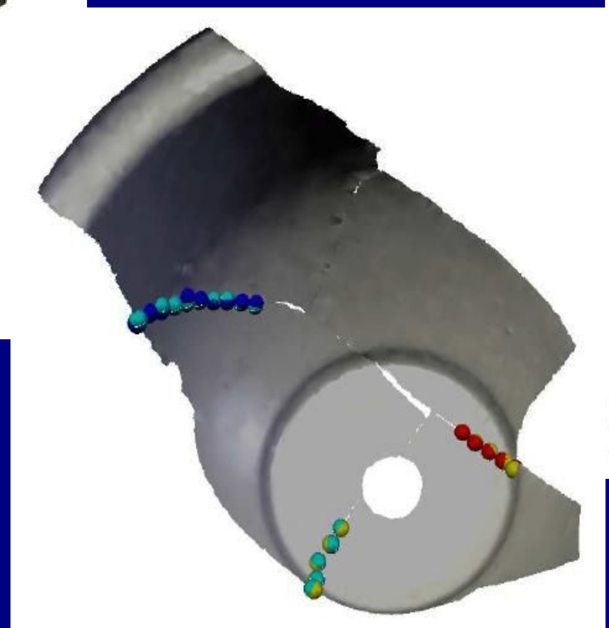


Digital Shape Understanding



Shape Understanding for Digital Archaeology





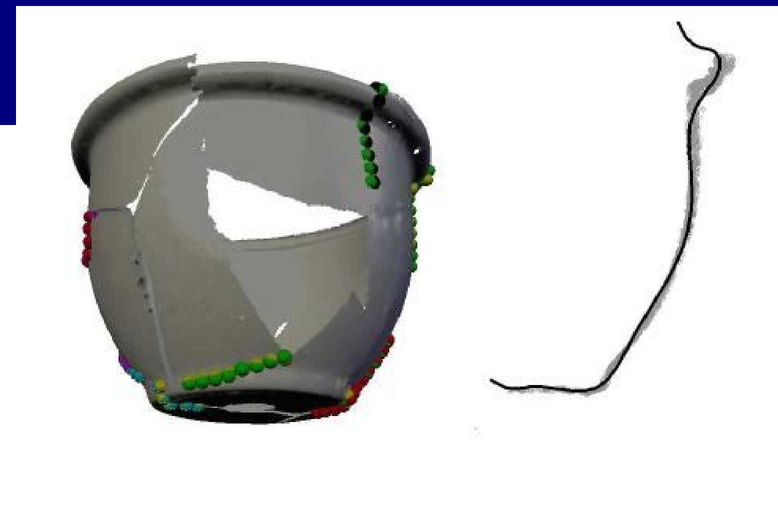


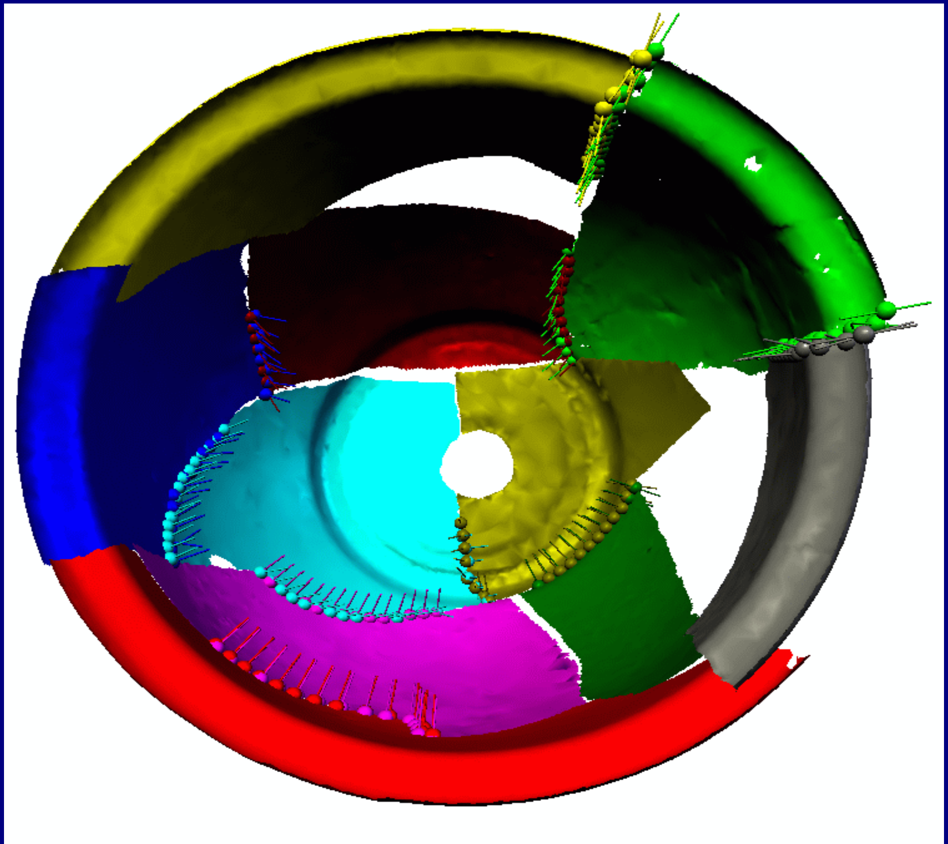
Figure 5: A correctly assembled pot of 13 sherds where only the 10 matched sherds shown were available.

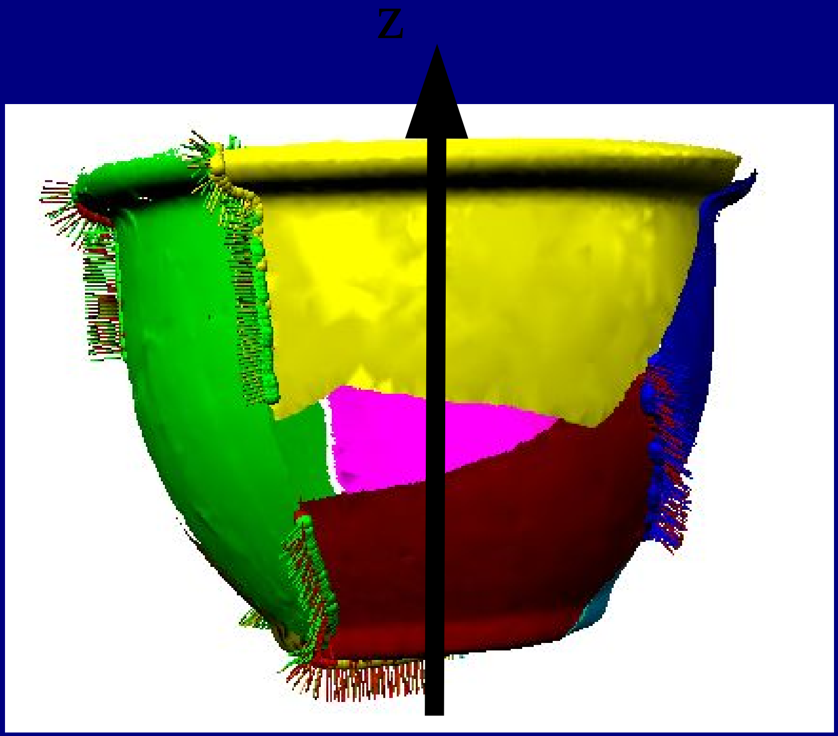
hendrix.lems.brown.edu/~arw

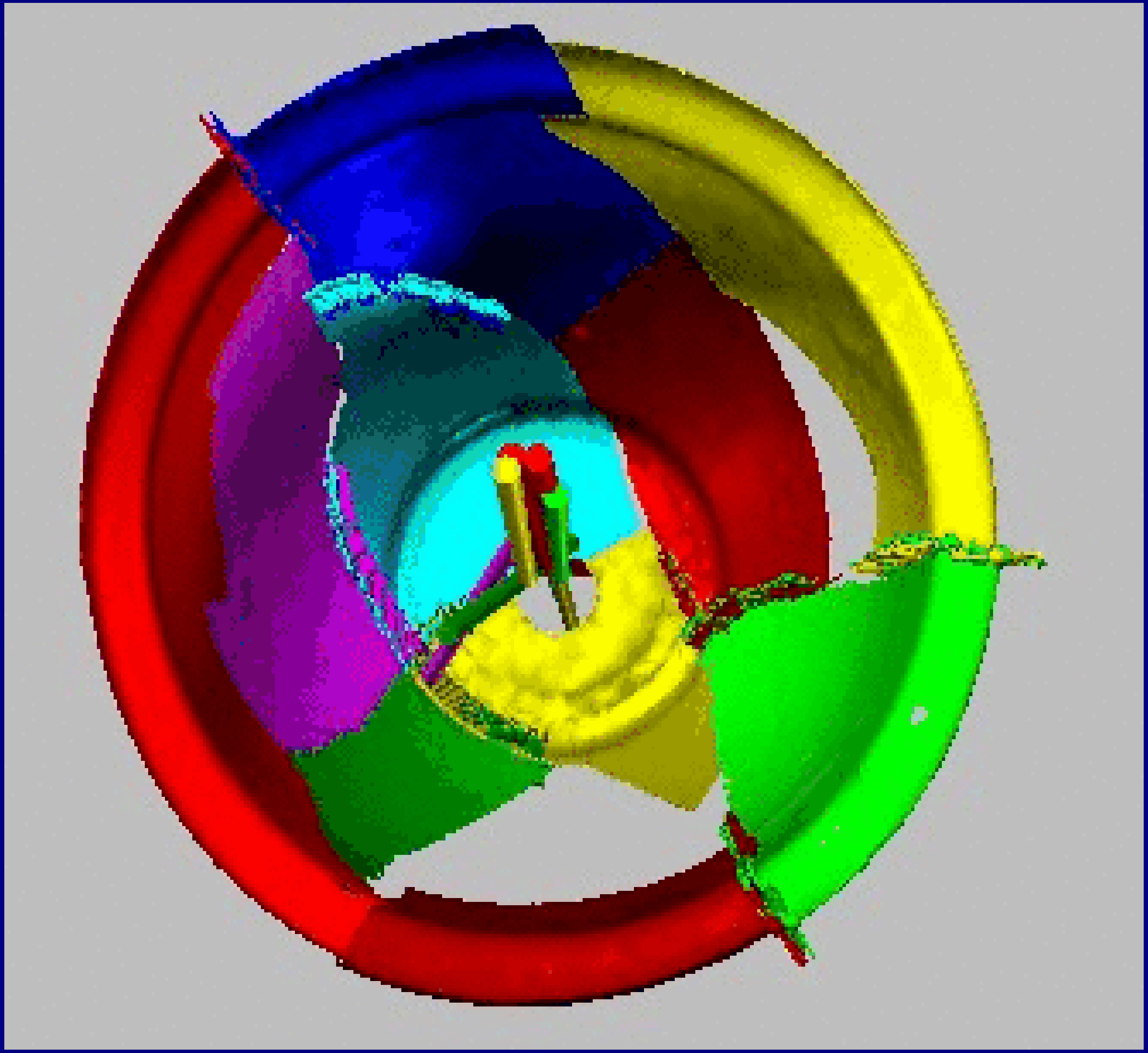
Bayesian assembly of 3D Axially Symmetric Shapes from Fragments

Andrew Willis and David Cooper, Brown University

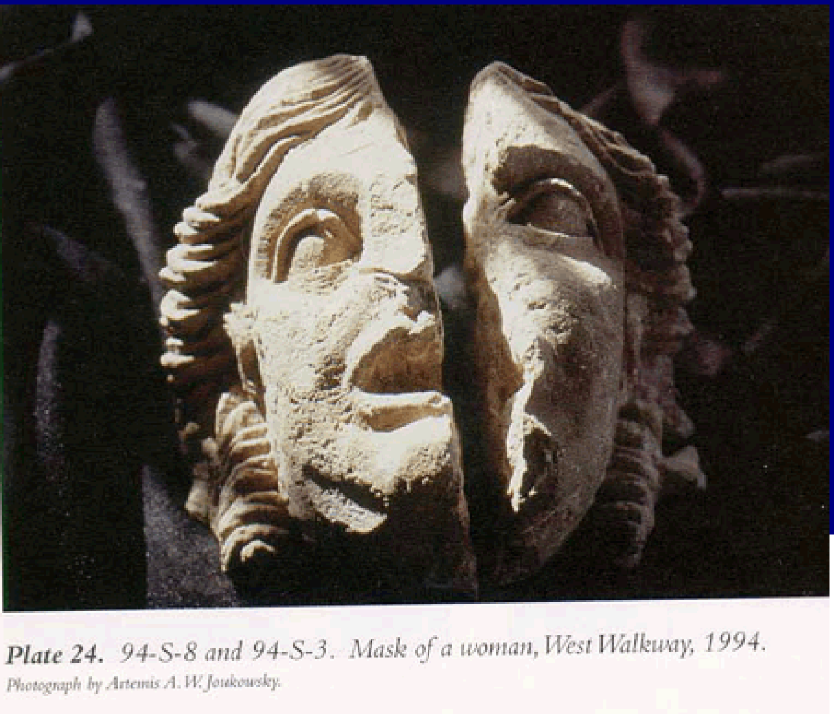
3D puzzle solver: correct 10 piece assembly







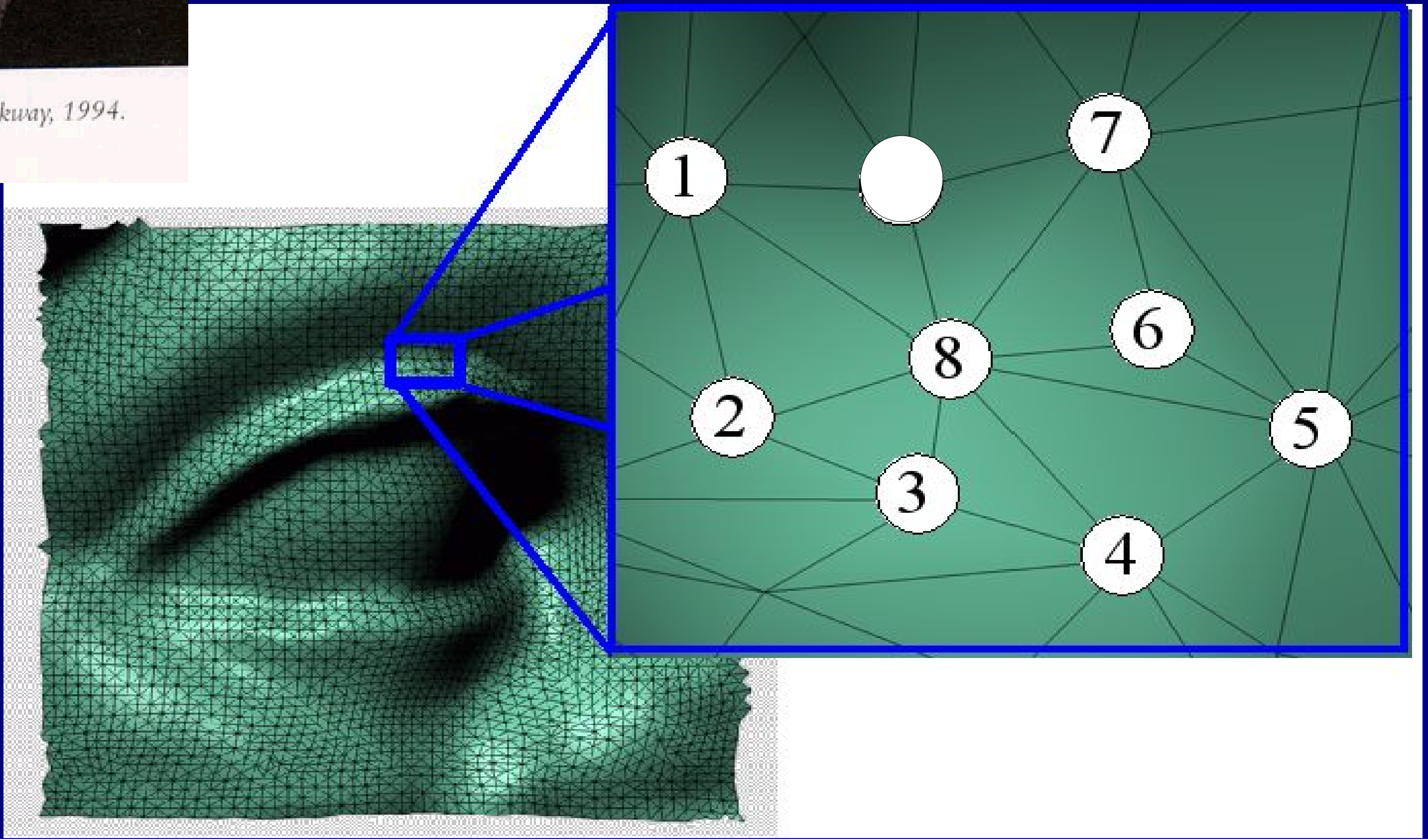
Deformable Models : Smart Materials



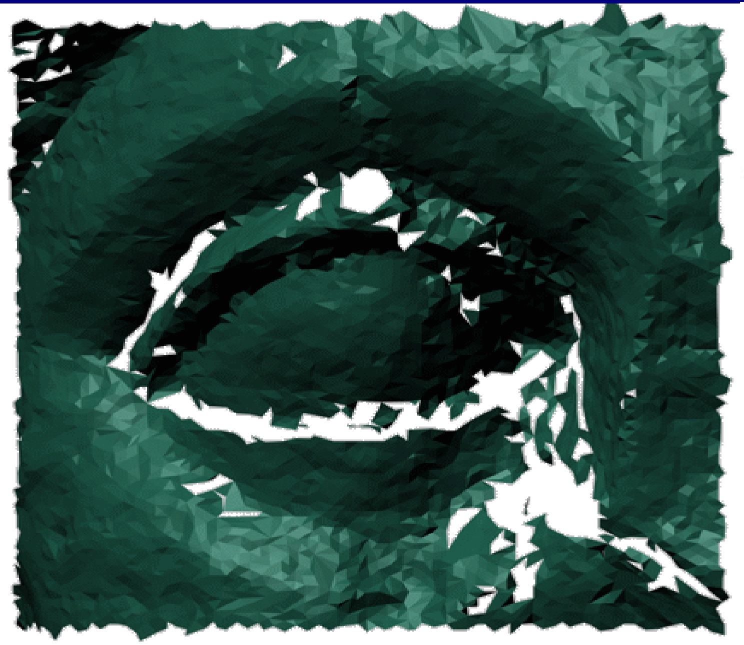
- Virtual archaeological reconstruction
- Virtual sculpting

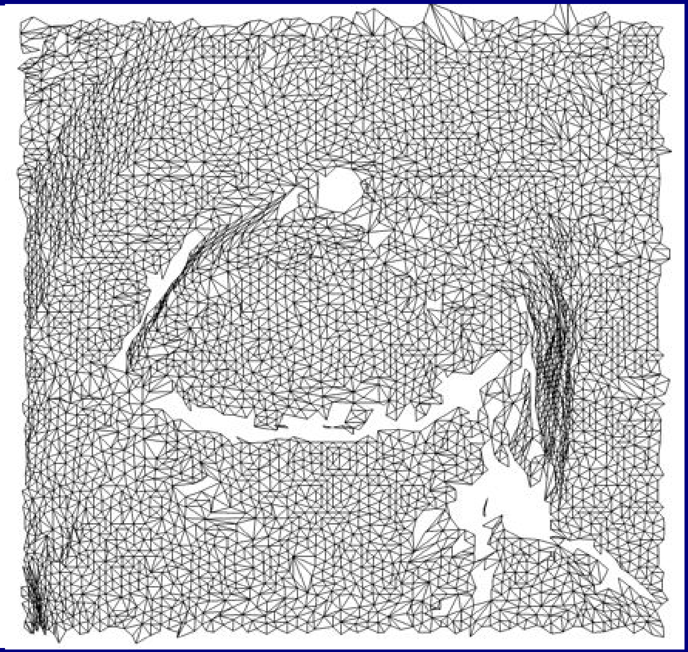
Markov Random Fields as Gibb's Distributions on Vertices of 3D Surface Meshes.

Cliques and Clique Potentials model smart materials.



Hole Filling and Surface Fairing



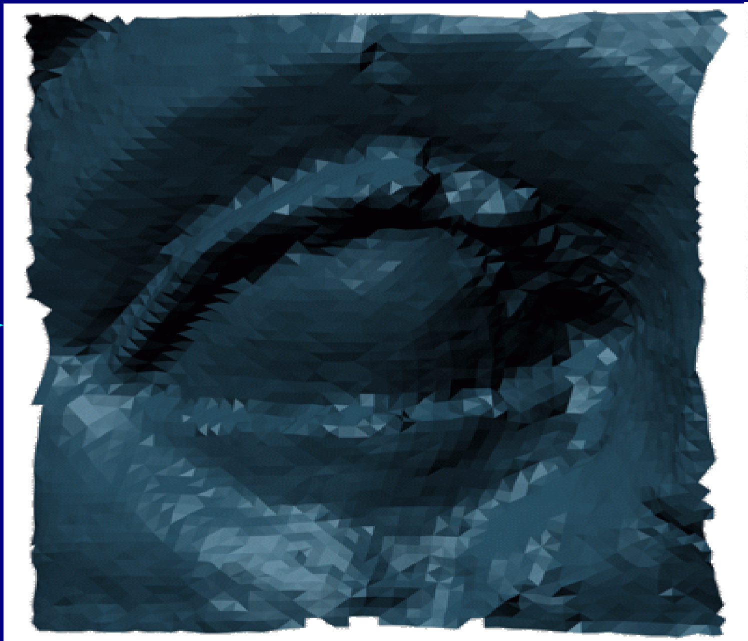


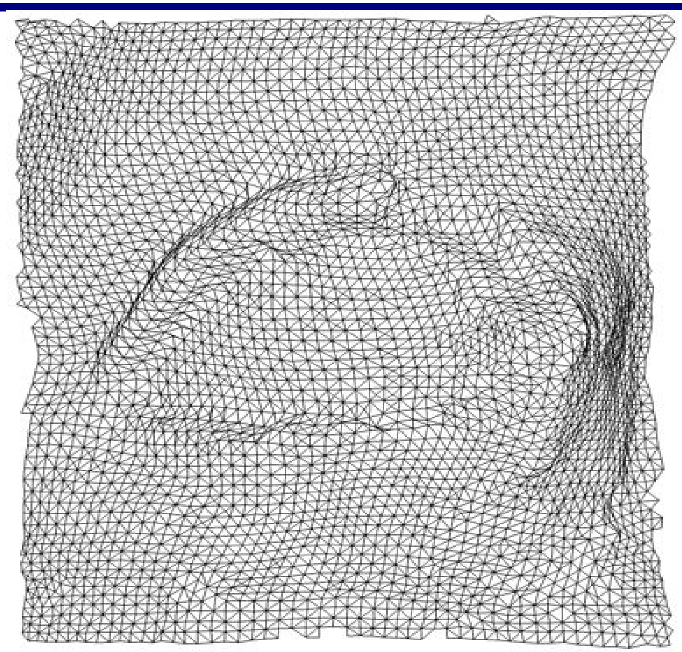
Irregular mesh with holes produced by 3D laser scanner.

$$V_1(\mathbf{p}_i) = \alpha_1 \|\mathbf{p}_i - \mathbf{p}_i^0\|^2$$

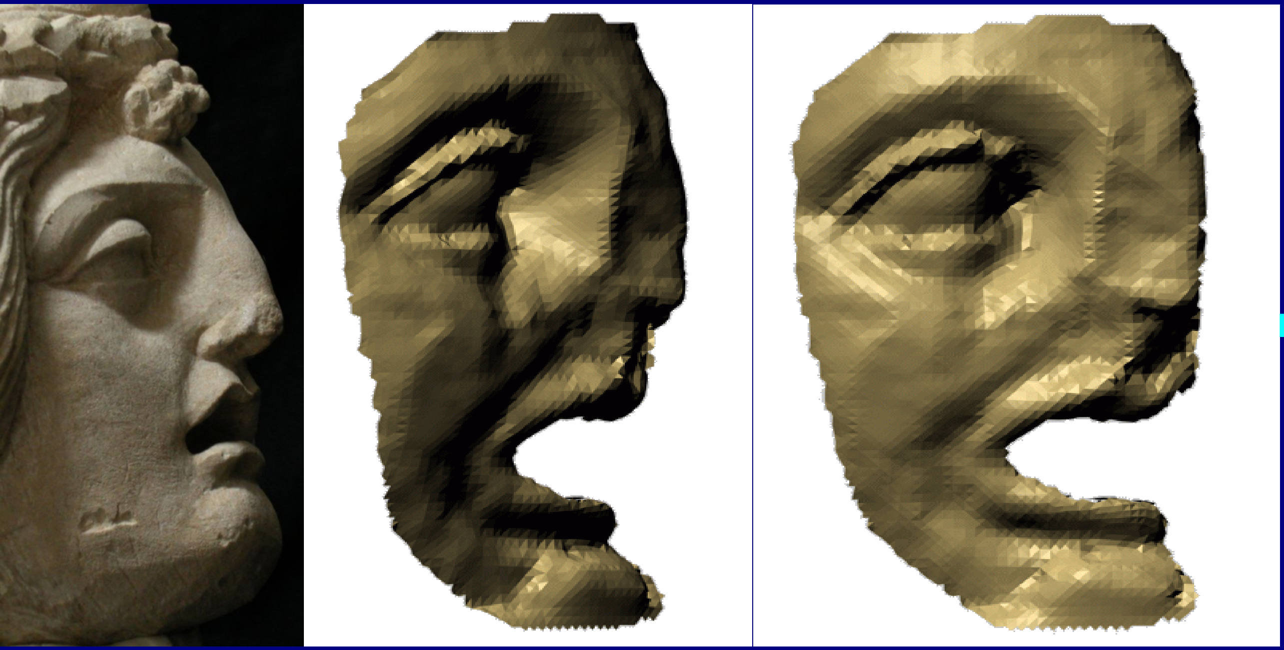
$$V_2(\mathbf{p}_i, \mathbf{p}_j) = \alpha_2 \|\mathbf{p}_i - \mathbf{p}_i^0 - \mathbf{p}_j + \mathbf{p}_j^0\|^2$$

MRF-produced surface using above as input data.



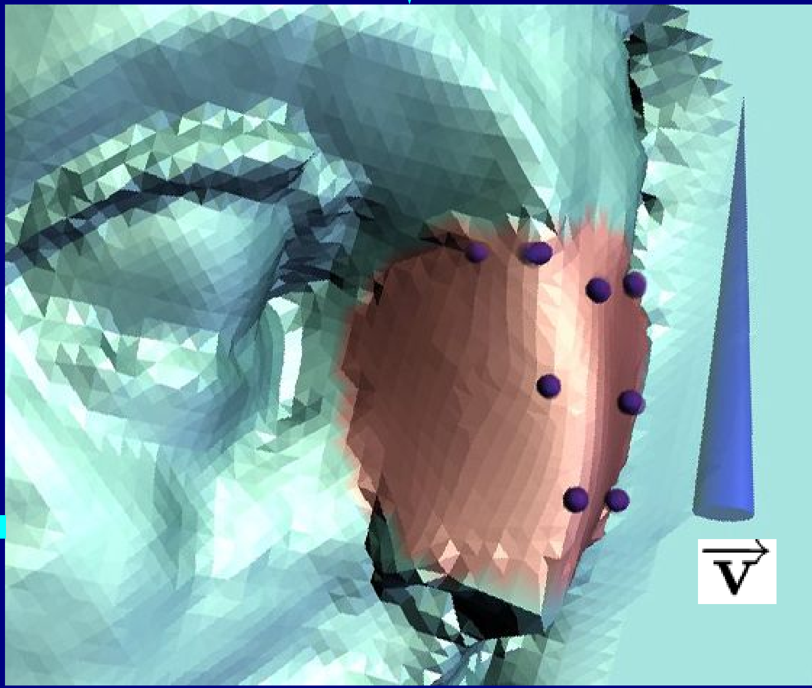


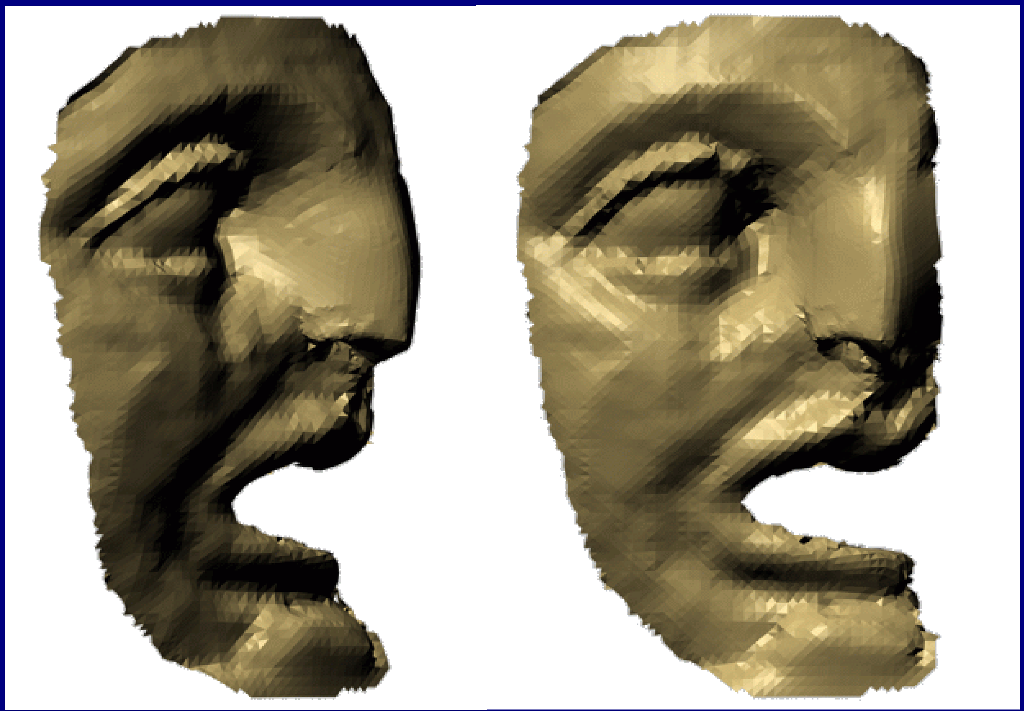
Virtual Nose Reconstruction



$$\|\kappa_{\overrightarrow{\mathbf{v}}}\|^2$$

$$(\kappa_{\overrightarrow{\mathbf{v}_\perp}} - \frac{1}{K}\Sigma_{k=1}^{K}\kappa_{\overrightarrow{\mathbf{v}_\perp}}(j_k))^2$$



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Part II: The Medial Scaffold for 3D Shape Representation

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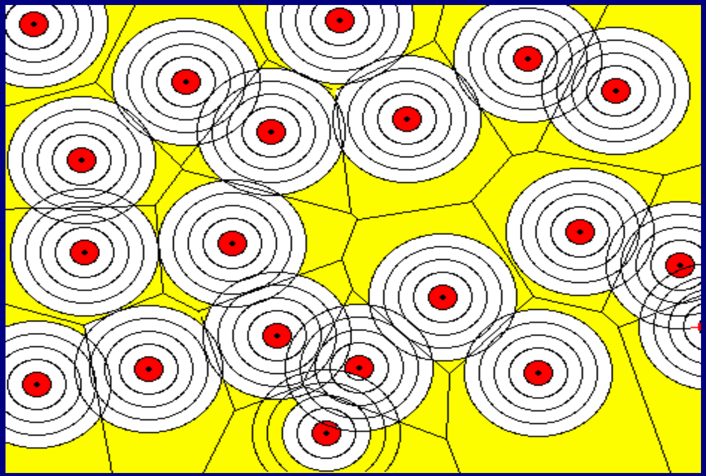
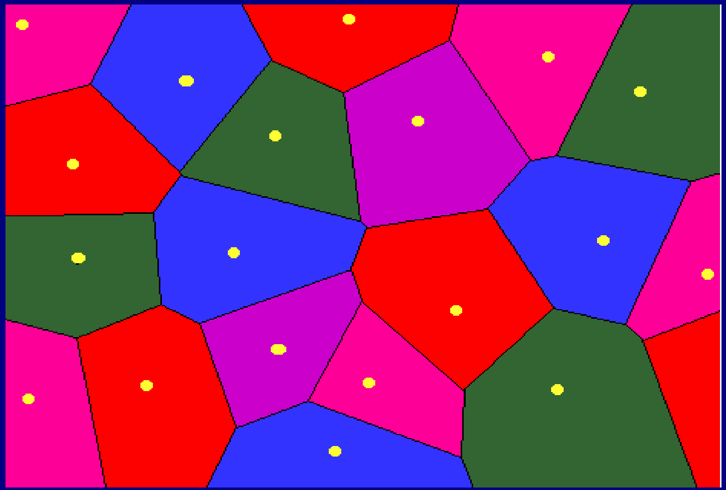
Outline

- Medial scaffold for 3D shape representation
- Transitions of the medial axis

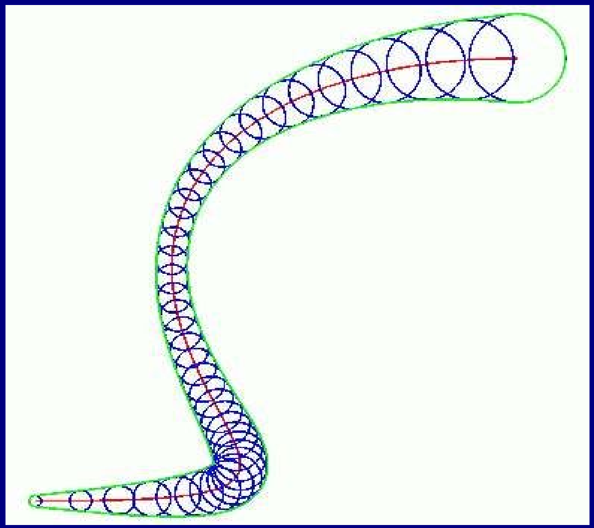
Shape representation: From the Medial Axis to the Medial Scaffold

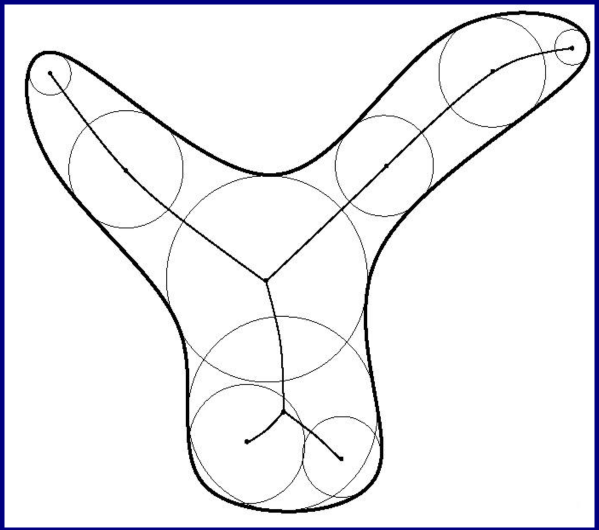
Wave
propagation
Oberlin,
Teichman *et al.*



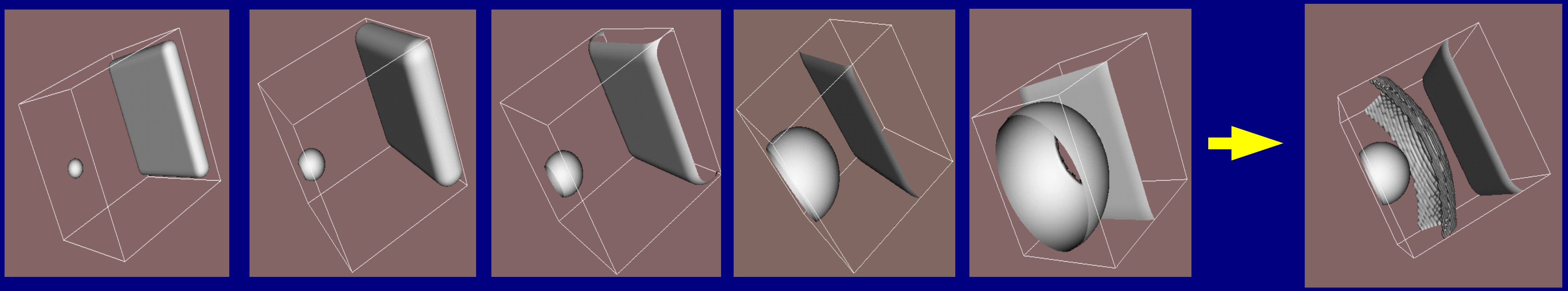


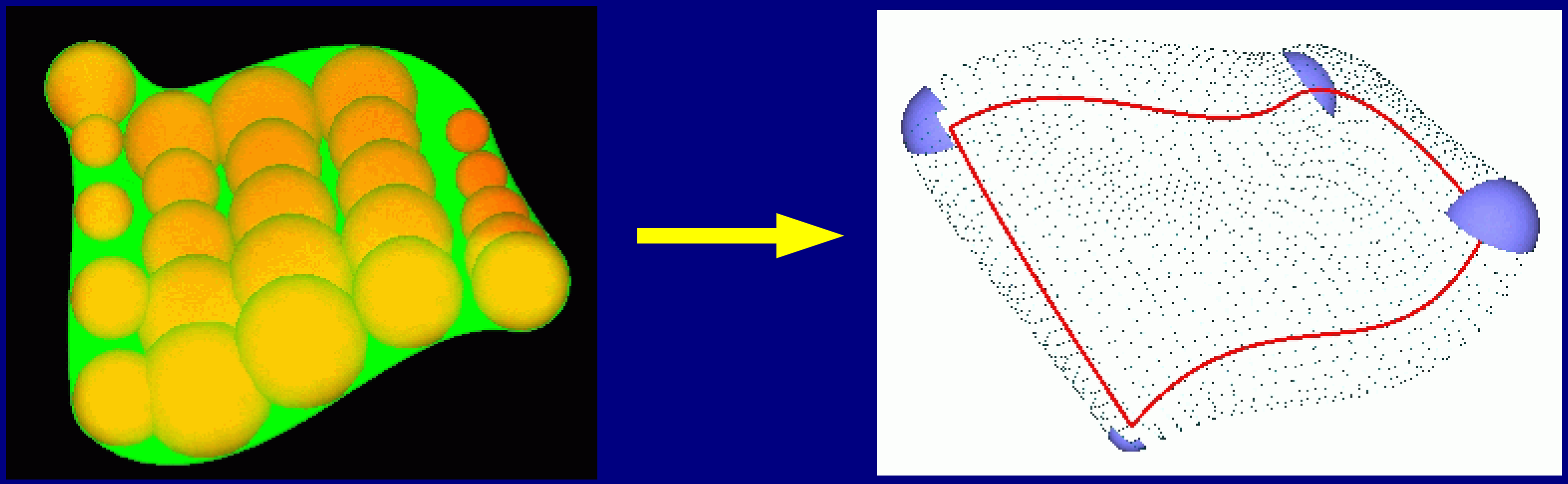
Maximal
disks
Wolter *et al.*



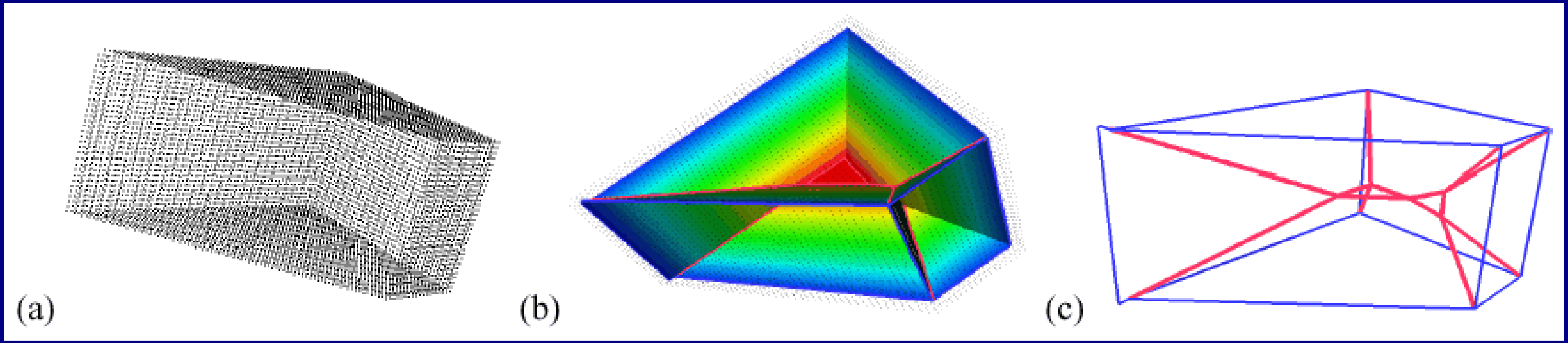


Shape representation: From the Medial Axis to the Medial Scaffold





Shape representation: From the Medial Axis to the Medial Scaffold



In 3D: the MA is a set of connected surfaces with boundary properties.

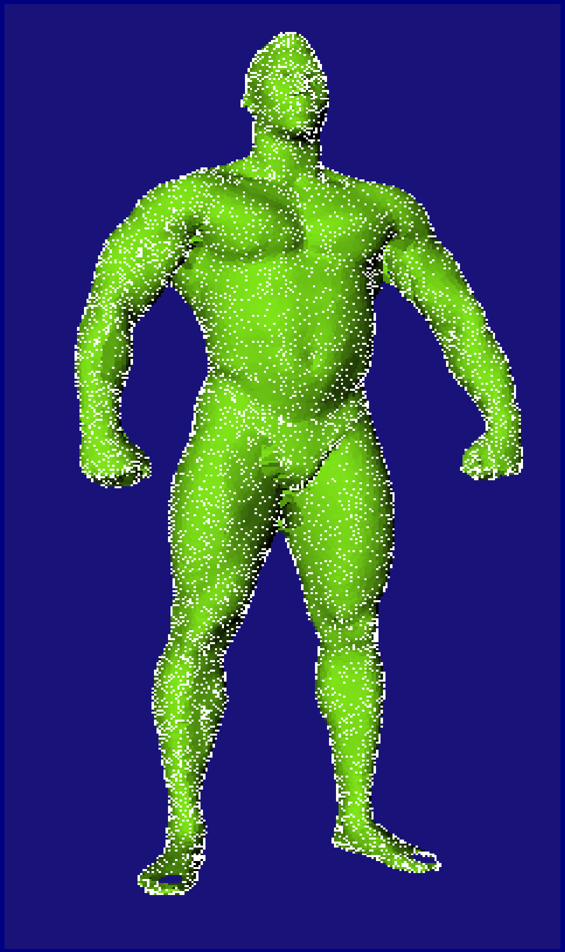
In 2D: the MA is directly a set directed graphs.

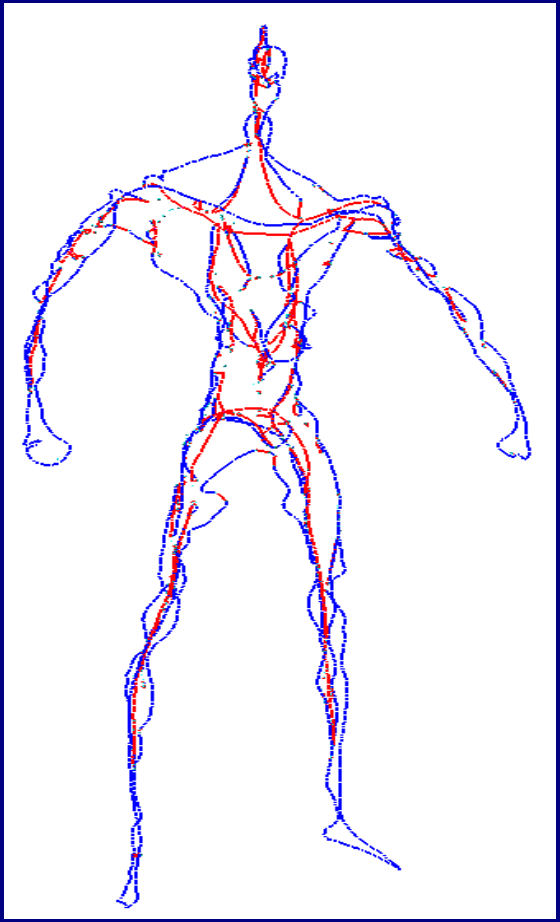
Map the 3D MA to an equivalent 3D graph structure:

- by studying the radius flow along the MA, *i.e.* as a **vector field** .

Shape representation: From the Medial Axis to the Medial Scaffold

Leymarie-Kimia:
Keep only singular points of the flow (radius) to build a **graph** (2001).

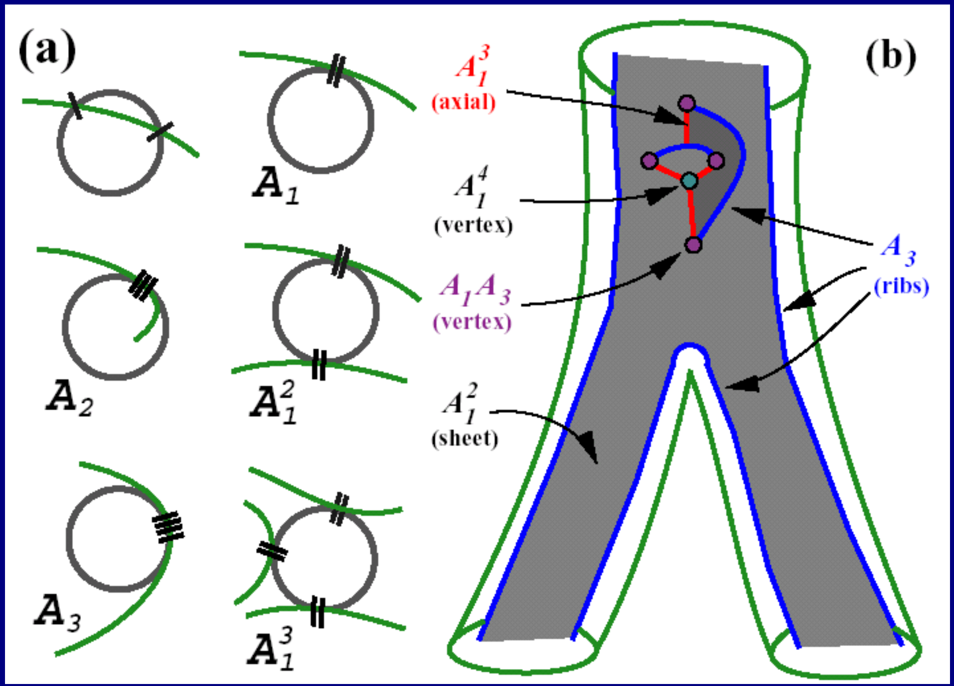




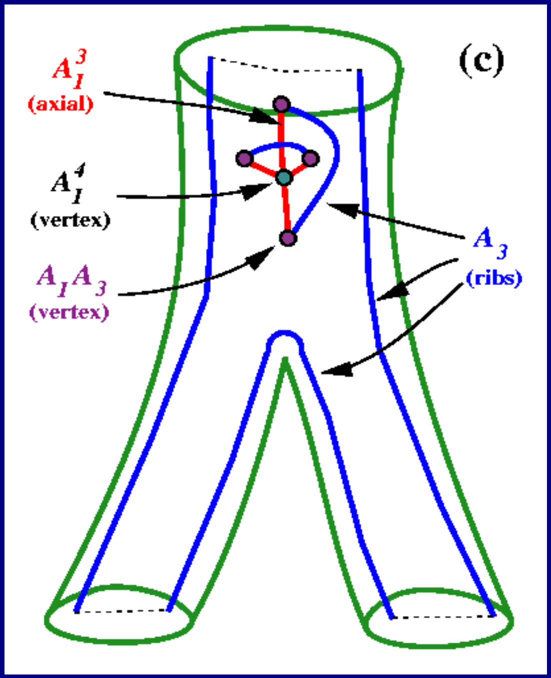
Shape representation: From the Medial Axis to the Medial Scaffold

- 3D: Five types of points from contact theory [Giblin-Kimia PAMI04]:
  - Sheet: A_1^2
  - Links: A_1^3 (Axial), A_3 (Rib)
  - Nodes: A_1^4 (Voronoi vertices) , A_1A_3

$\mathbf{A}_k^n$: contact at n distinct points, each with $k+1$ degree of contact



Leymarie-Kimia '01:
Keep only singular points of the flow (radius) to build a graph.



Shape representation: From the Medial Axis to the Medial Scaffold

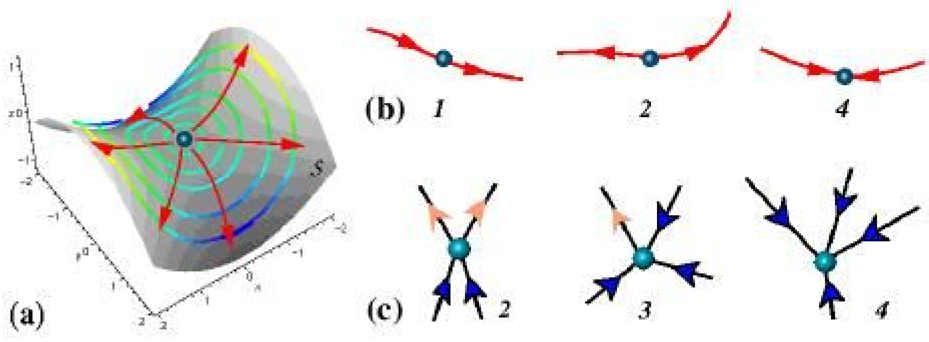
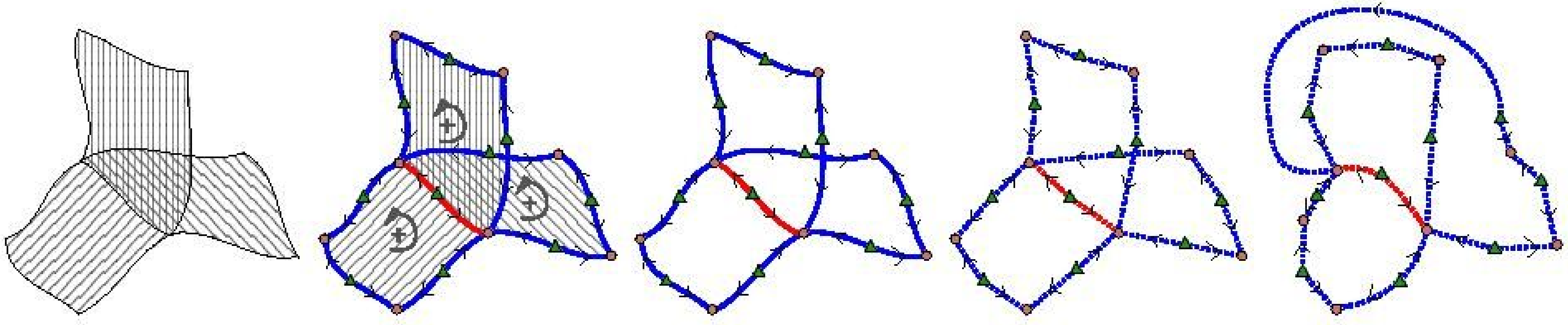


Figure 4: (a) Generic situation for the initial flow of an A_1^2 shock sheet: from the initial shock source, an A_1^2–2 point, a 3D surface S radially grows-out to form the sheet; arrows indicate some directions of increasing radius values along the $\mathcal{MA}$ sheet. (b) Flows at A_1^3 and A_3 shock curves. (c) Flows at A_1^4 shock points, where the number of *inward* flows is indicated.

Shocks	Regular	Source	Relay	Sink
Sheet	A_1^2–1	A_1^2–2	A_1^2–3*	A_1^2–4
Rib	A_3–1	A_3–2	A_3–3*	A_3–4
Axis	A_1^3–1	A_1^3–2	A_1^3–3*	A_1^3–4
Rib end	-	A_1A_3–2	A_1A_3–3	A_1A_3–4
Axis end	-	-	A_1^4–2, A_1^4–3	A_1^4–4

Table 2: Final classification of 18 possible shock points based on contact with spheres, A_k^n, and flow type. There are 3 regular shock types, and 15 singular ones which are the sources, relays, or sinks for the flow. * We consider degeneracies, *i.e.*, part of A_1^2–3, A_3–3 and A_1^3–3, as a special case of relay where shocks flow simultaneously in and out.

Shape representation: From the Medial Axis to the Medial Scaffold



(a) $\mathcal{MA}$ (b) $\mathcal{MS}^+$ (c) $\mathcal{MS}$ (d) $\mathcal{MS}^-$ (e) $\mathcal{TS}$

Figure 6: From the "classical" $\mathcal{MA}$ static representation to the medial scaffolds. (a) Typical situation in 3D, where three medial sheets intersect into a medial curve. (b) Representation by the *augmented medial scaffold*, where medial nodes (red dots) along curves are connected by links; hyperlinks' cyclic order is indicated by a counterclockwise arrow. (c) Representation by the *medial scaffold* ($\mathcal{MS}$), where the interior of sheets is implicit. (d) The *reduced medial scaffold*, is obtained from the $\mathcal{MS}$ by discarding the geometry of shock curves. (e) The *topological medial scaffold* is obtained from the $\mathcal{MS}$ when only the topology of the graph structure is preserved. **Red** dots correspond to $\mathcal{MA}$ vertices, *i.e.*, A_1^4 or A_1A_3. **Green** triangles correspond to shock sources of curves, *e.g.*, A_1^3−2 points, which are needed for capturing the $\mathcal{MS}$. A_1^3 and A_3 curves are shown in **red** and **blue**, respectively.

Medial Scaffold of Point Generators

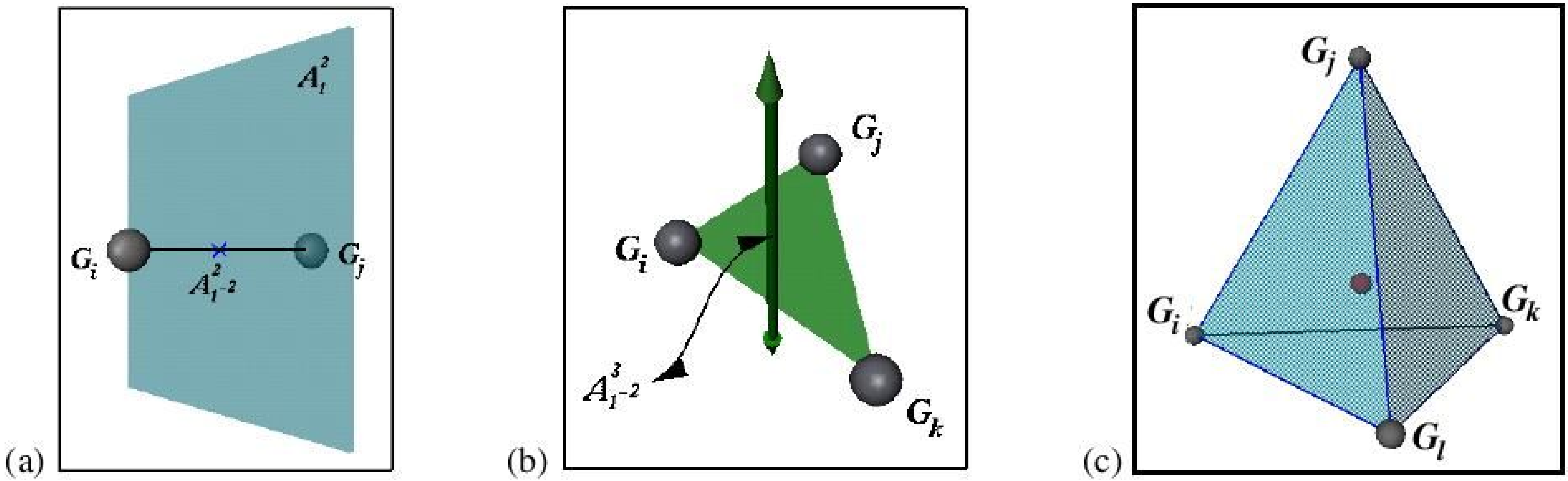
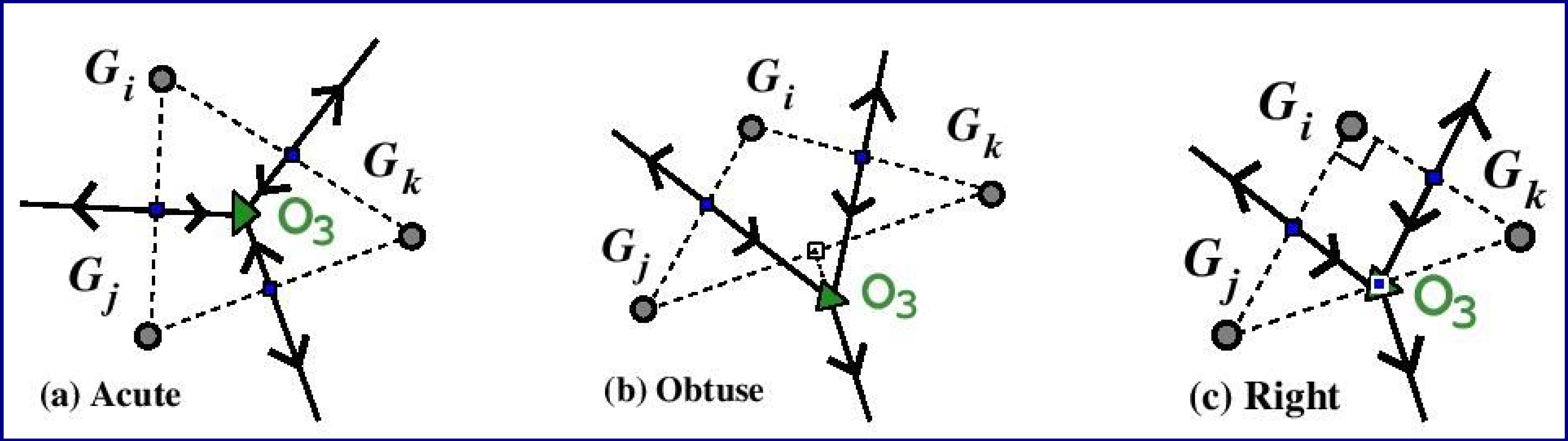


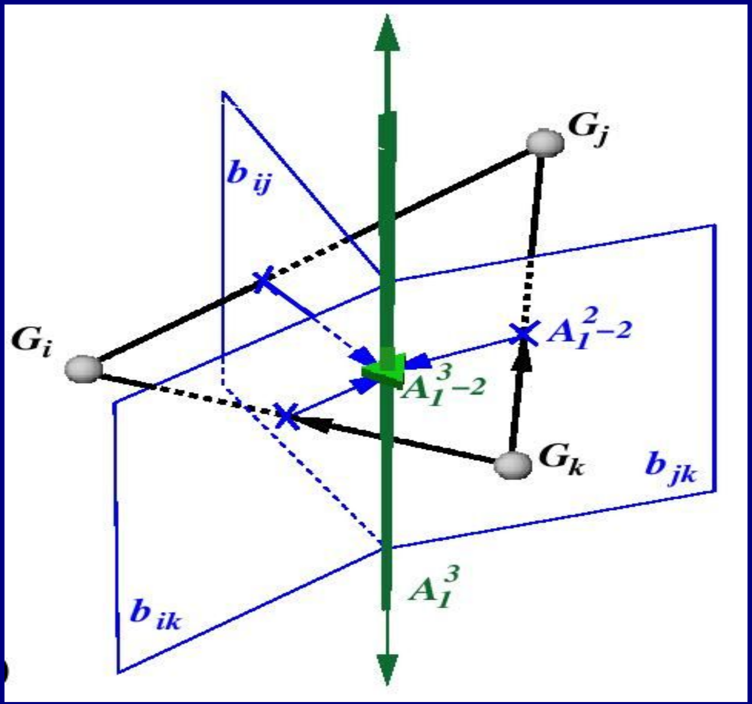
Figure 7: (a) Two point generators generate an A_1^2 shock sheet initiated at an $A_1^2{-}2$ shock source at mid-distance between them. (b) Three point generators generate an A_1^3 shock curve flowing in two opposite directions initiated at the $A_1^3{-}2$ shock positioned at the circumcenter of the triangle defined by the triplet of generators. (c) Four point generators generate an A_1^4 shock vertex located at the circumcenter of the corresponding tetrahedron.

Shocks	Regular	Source	Relay	Sink
Sheet	$A_1^2{-}1$	$A_1^2{-}2$	-	-
Axis	$A_1^3{-}1$	$A_1^3{-}2$	$A_1^3{-}3$	-
Axis end	-	-	$A_1^4{-}2$, $A_1^4{-}3$	$A_1^4{-}4$

Table 4: Only eight types of shock points arise from point generators.

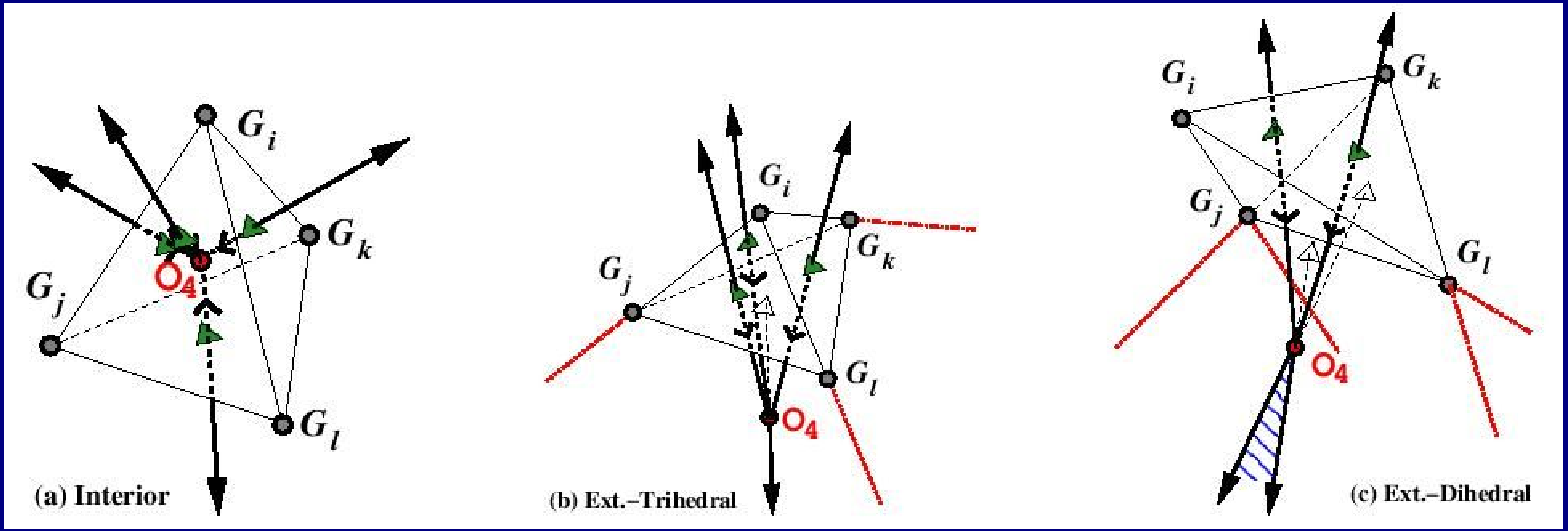
Medial Scaffold of **Point** Generators

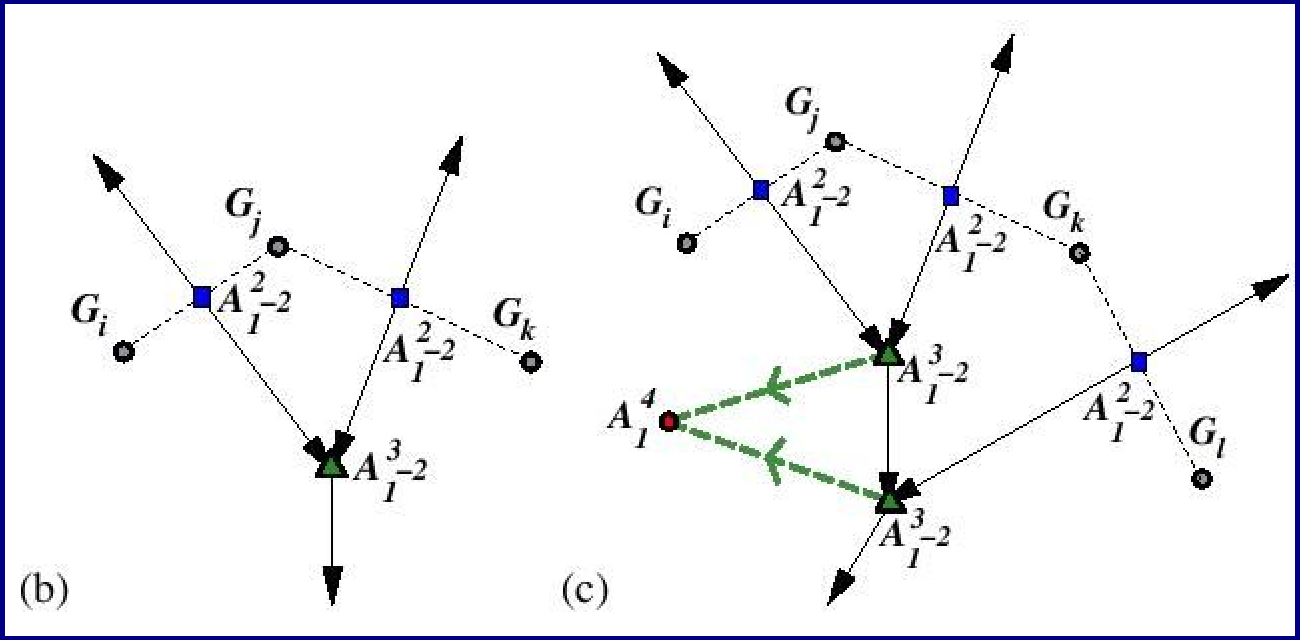




NB: Relationship with Morse theory (Siersma, Hart, Edelsbrunner, *et al.*)

Medial Scaffold of **Point** Generators





Medial Scaffold of Point Generators
Computation: Visibility property

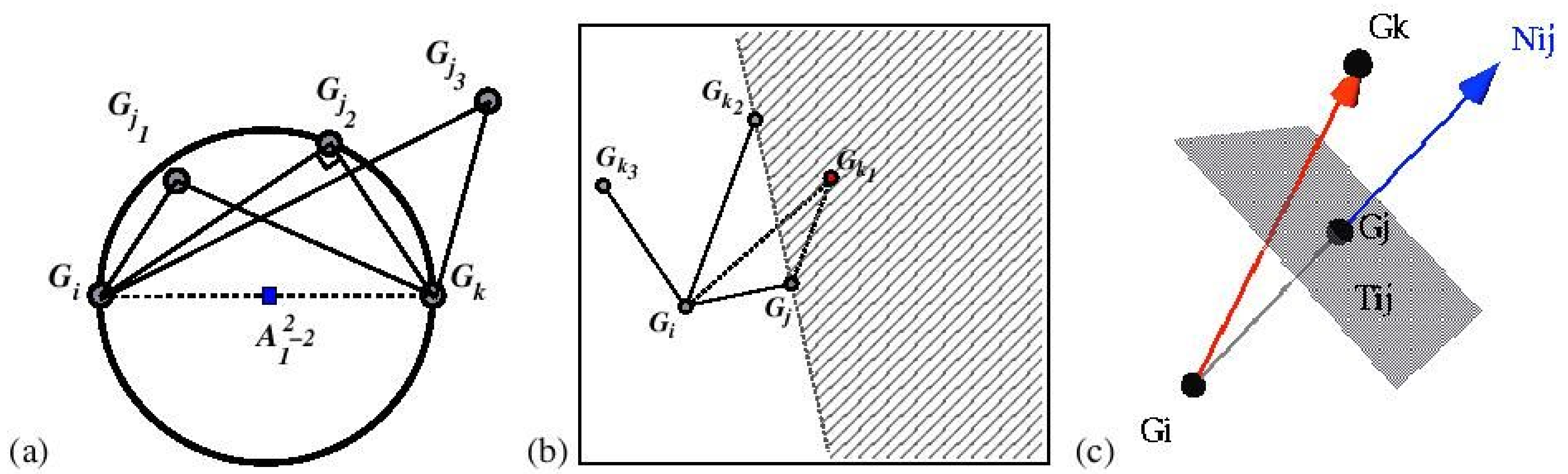
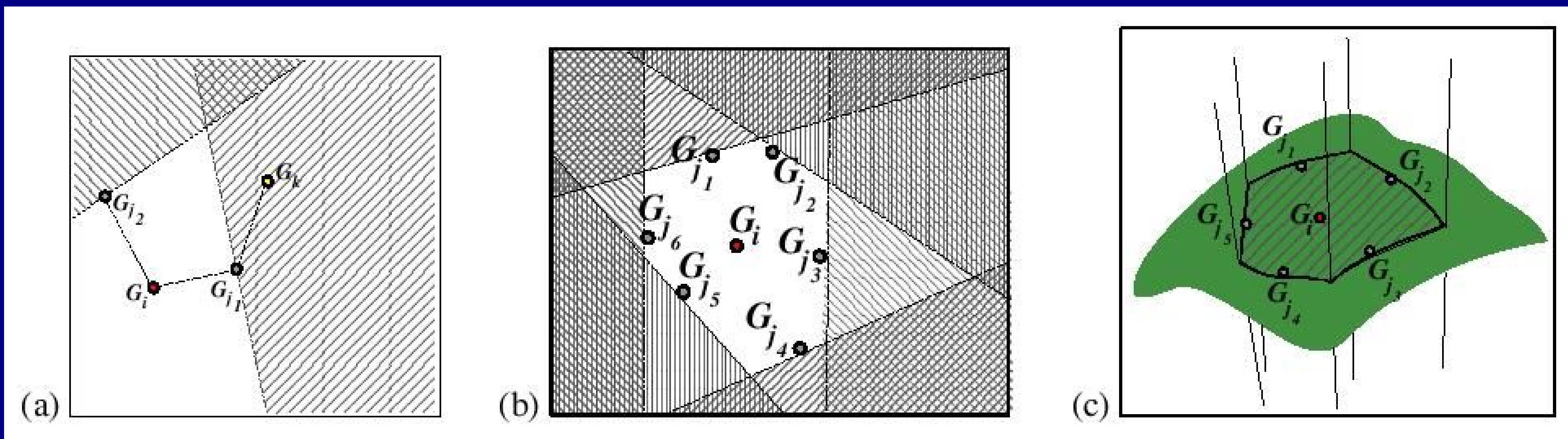
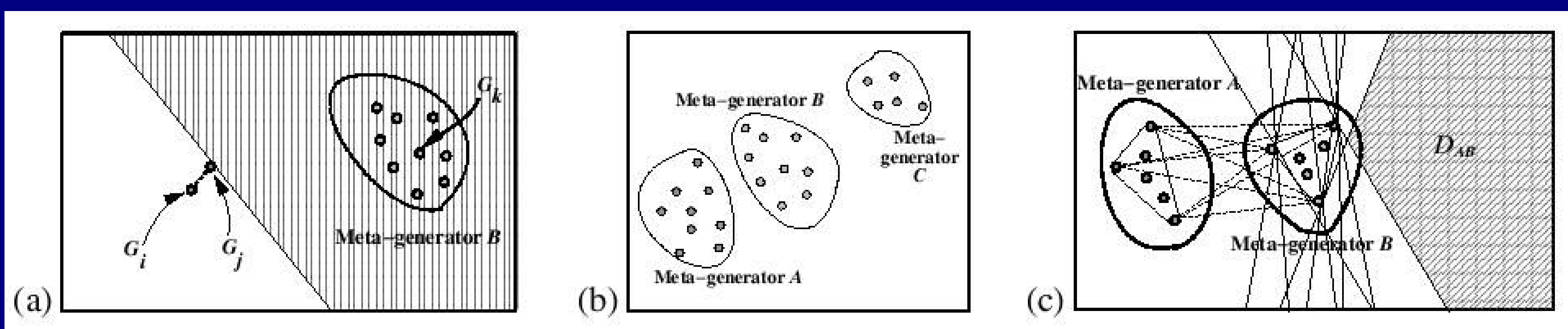


Figure 11: (a) The $A_1^2{-}2$ shock between $\mathbf{G}_i$ and $\mathbf{G}_k$ forms iff there exists no generator $\mathbf{G}_j$ inside the circle (sphere) with diameter $\mathbf{G}_i\mathbf{G}_k$. Thus, the angle $\angle\mathbf{G}_i\mathbf{G}_j\mathbf{G}_k$ must either be acute or at most a right angle. (b) Thus, when considering potential pairings of generators with $\mathbf{G}_i$, the presence of $\mathbf{G}_j$ implies that generators in the shaded area cannot possibly form a pairing with $\mathbf{G}_i$ as $\angle\mathbf{G}_i\mathbf{G}_j\mathbf{G}_k$ is then obtuse for those points; (c) in 3D, this region is a half-space (delimited by the plane $\mathbf{T}_{ij}$).

Medial Scaffold of Point Generators

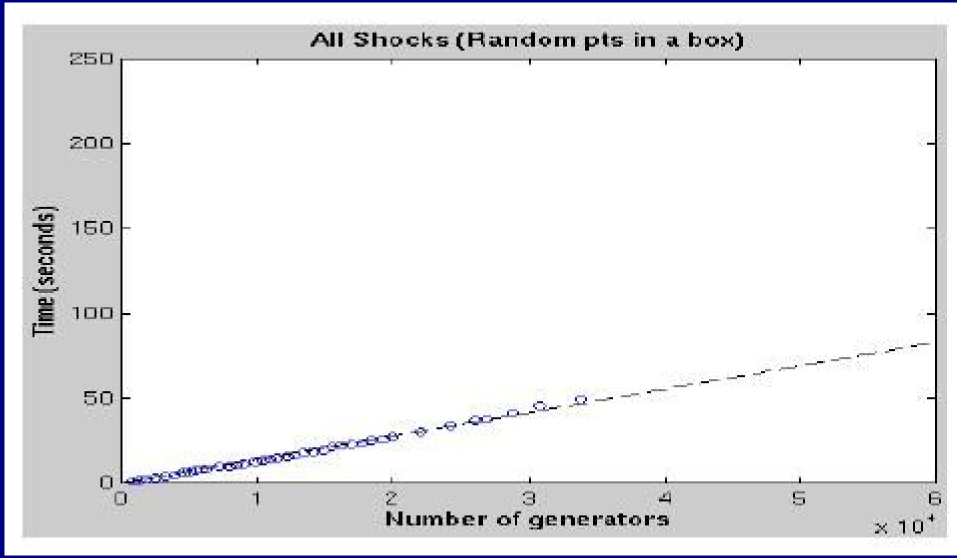
Computation: Visibility property





Medial Scaffold of Point Generators

Computation: complexity



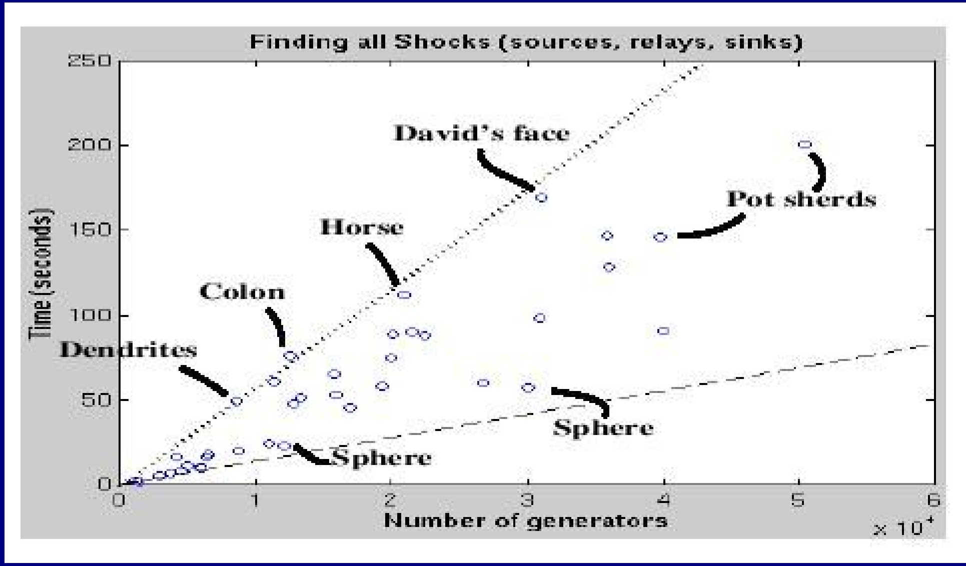
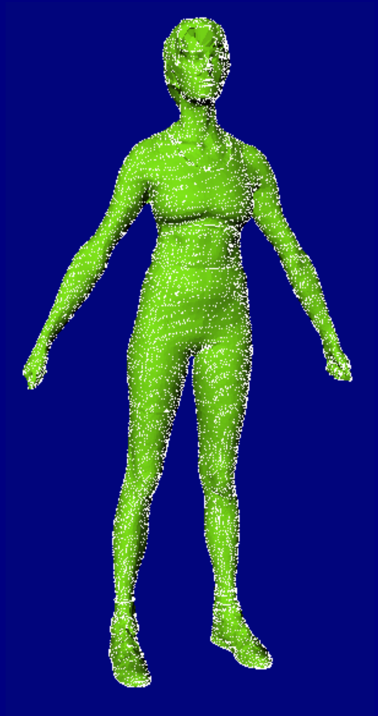
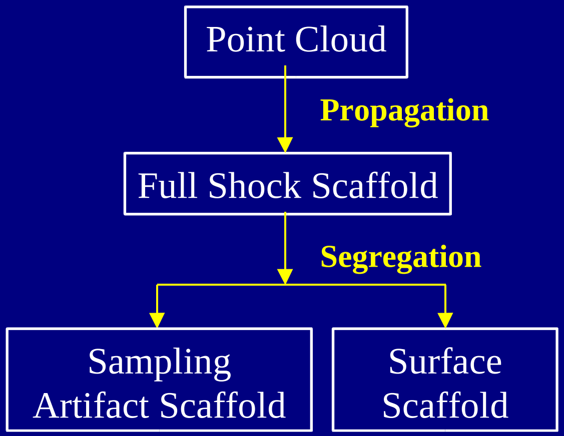
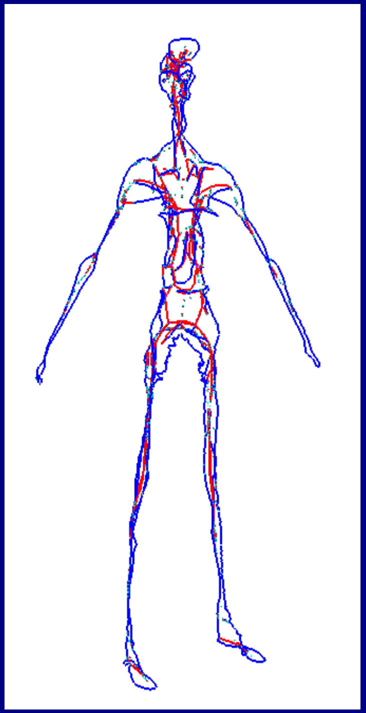


Figure 18: Timing results for the construction of the shock scaffold for a set of artificially generated random samples from a 3D block (Left), and for a set of 40 shapes including pot sherds, body scans, blood vessels, mechanical parts, *etc.* (Right). All experiments were performed on an SGI Octane 2 (IRIX 6.5)® with a C language implementation, but using non-optimized development code. Horizontal scaling: $\times 10^4$.

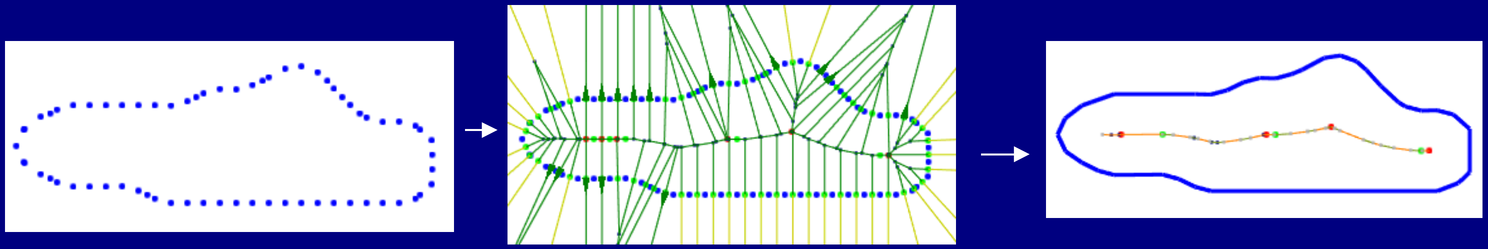
Medial Scaffold of Point Generators
Computation: Surface samplings







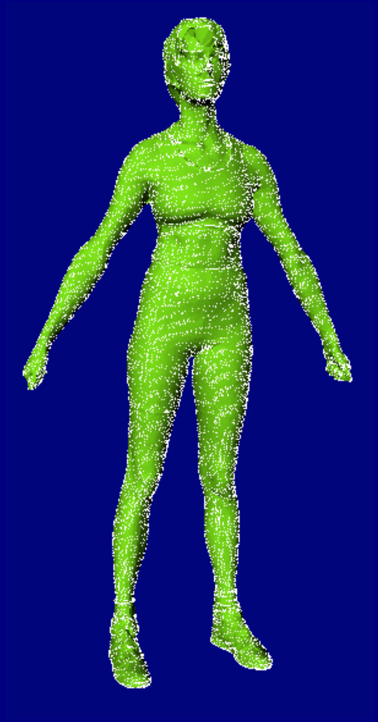
2D is easy for smooth outlines

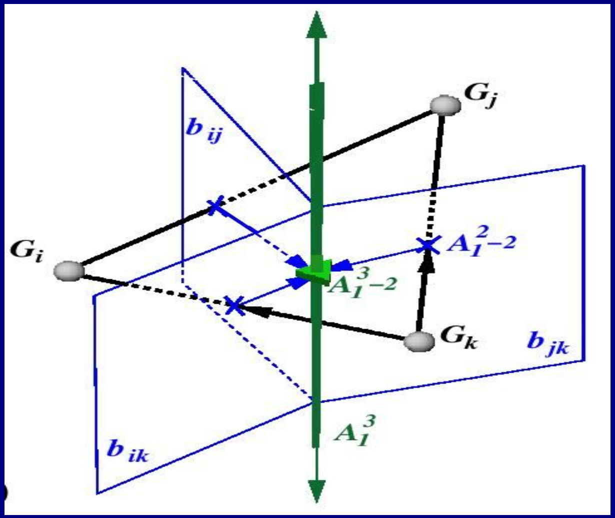


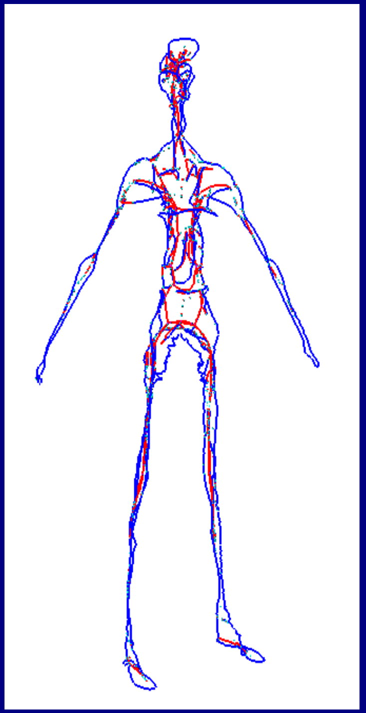
NB: Work of Amenta *et al.*

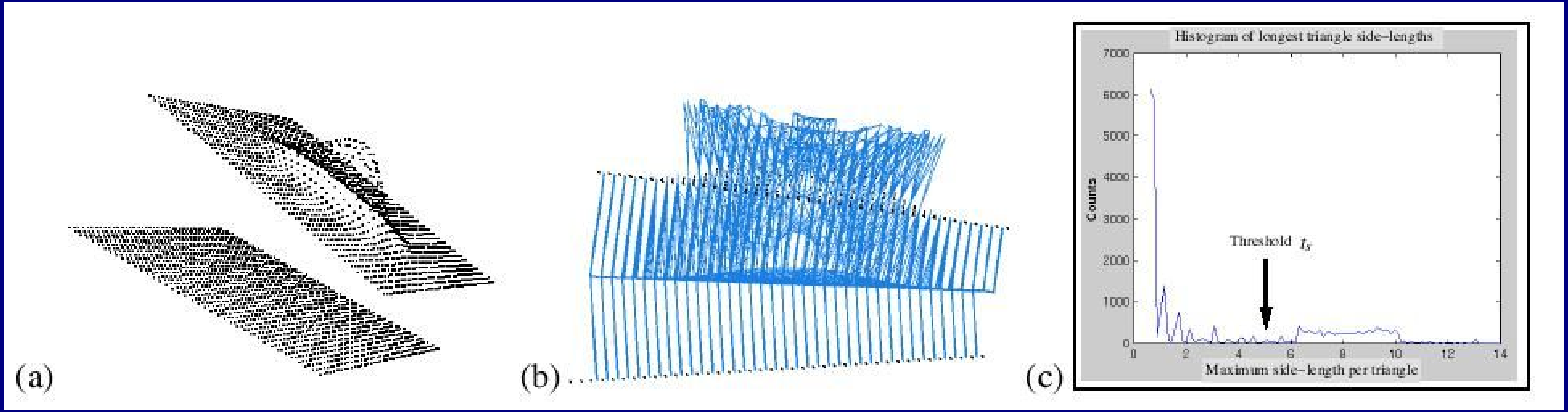
Medial Scaffold of Point Generators

Computation: Surface samplings

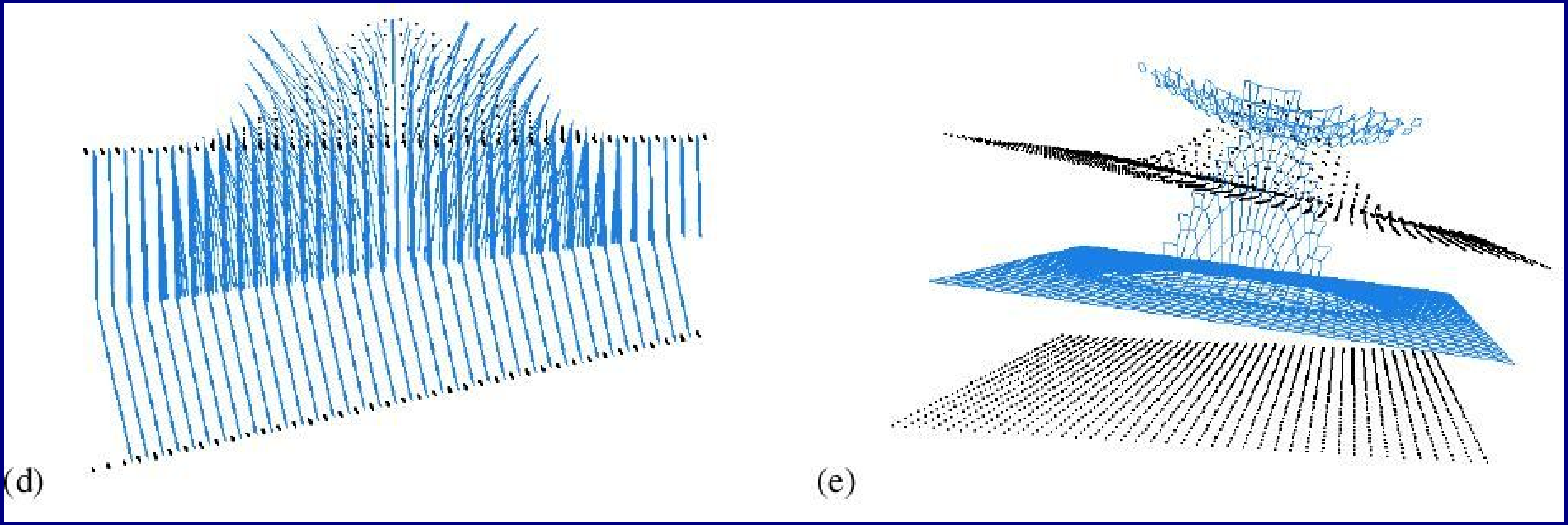


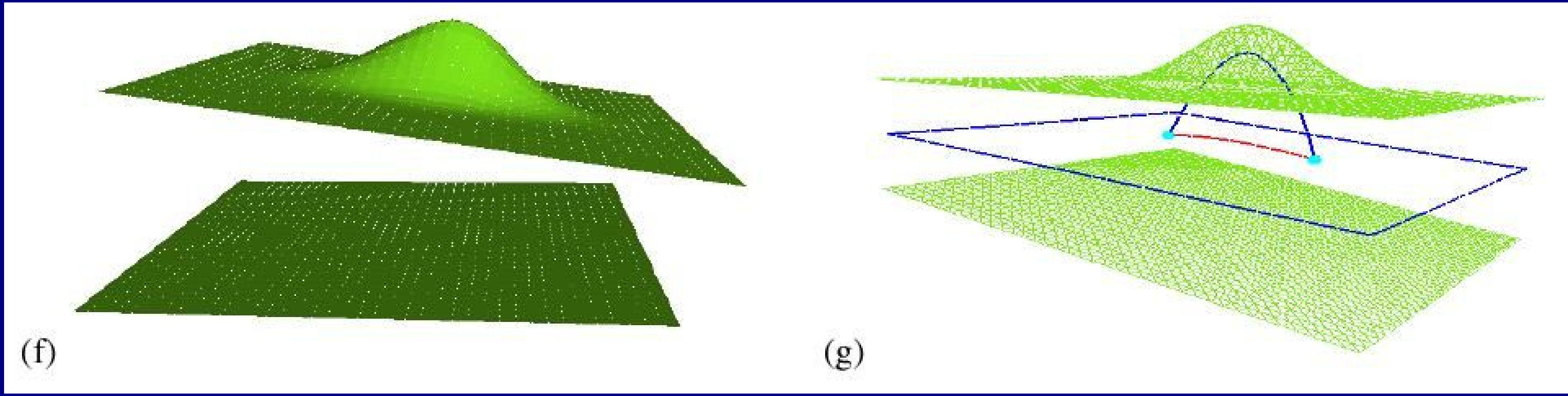




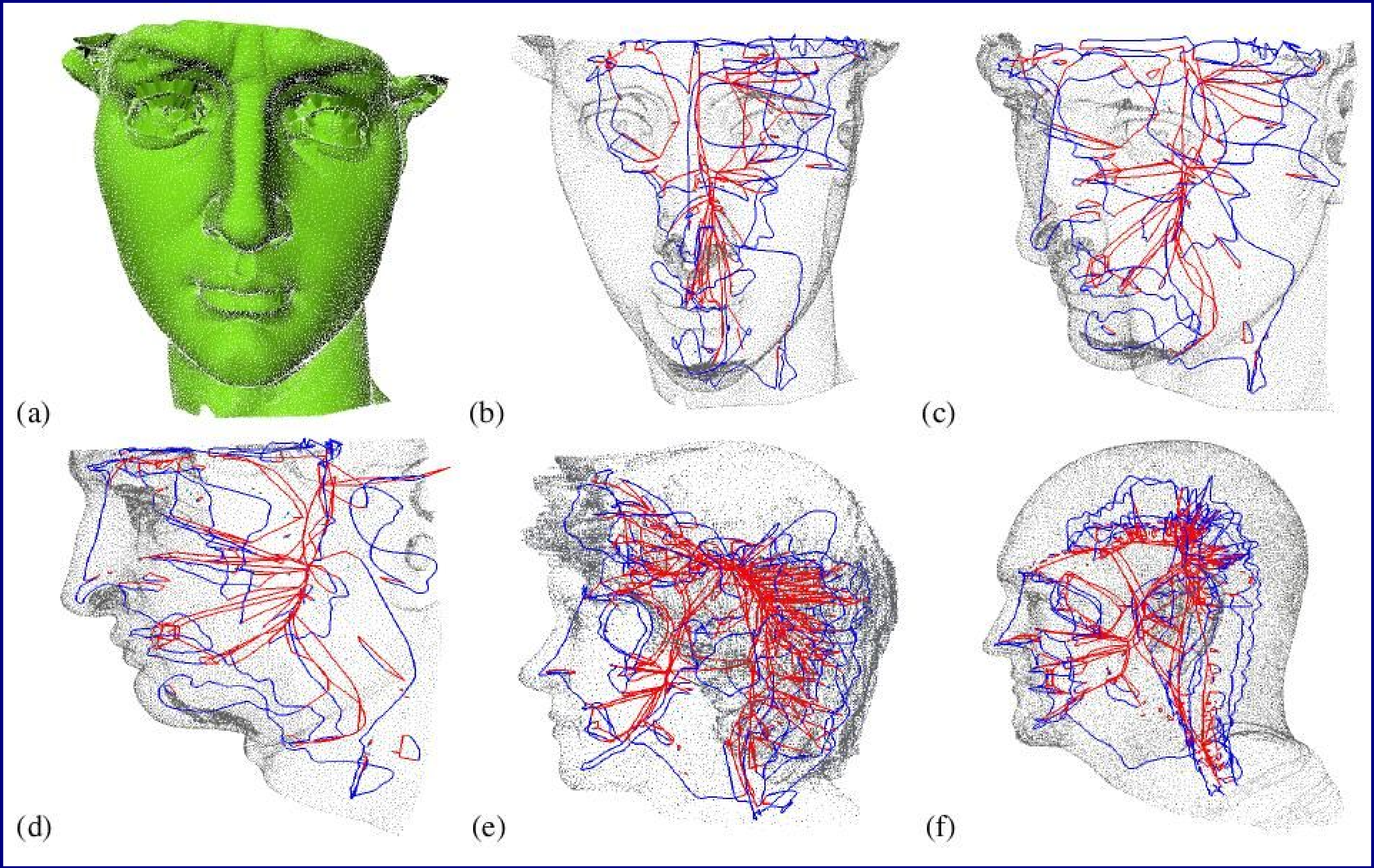


Medial Scaffold of Point Generators
Computation: Surface samplings





Medial Scaffold of Point Generators
Computation: Surface samplings



Outline

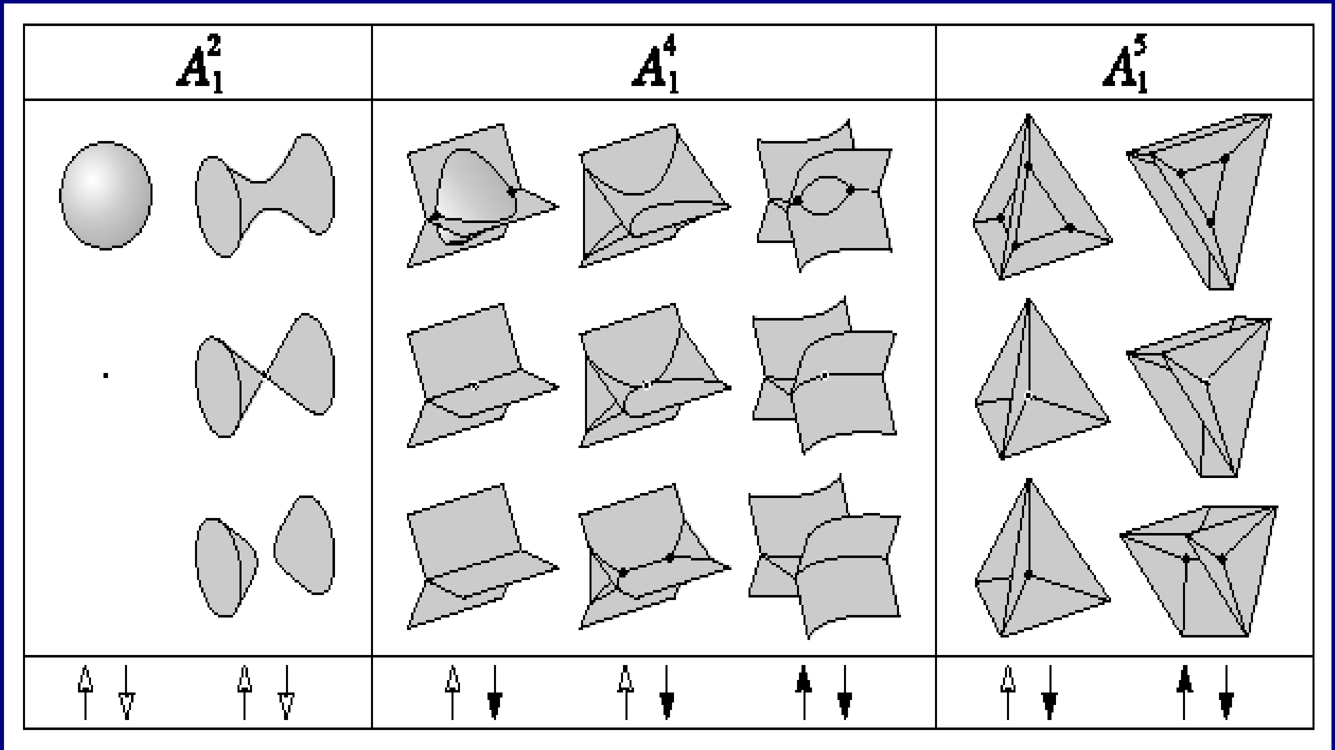
- Medial scaffold for 3D shape representation
- Transitions of the medial axis

Transitions of the 3D graph structure

Study the topological events of the graph structure under perturbations and shape deformations.

Singularity theory (Arnold *et al.*, since the 1990's):

- In 3D, 26 topologically different perestroikas of linear shock waves.



"Perestroikas of shocks and singularities of minimum functions"
I. Bogaevsky, 2002.

Transitions of the 3D graph structure

Study the topological events of the graph structure of the MA under perturbations and shape deformations.

Transitions of the MA (Giblin & Kimia, ECCV 2002):

- Under a 1-parameter family of deformations, only **seven transitions** are relevant.

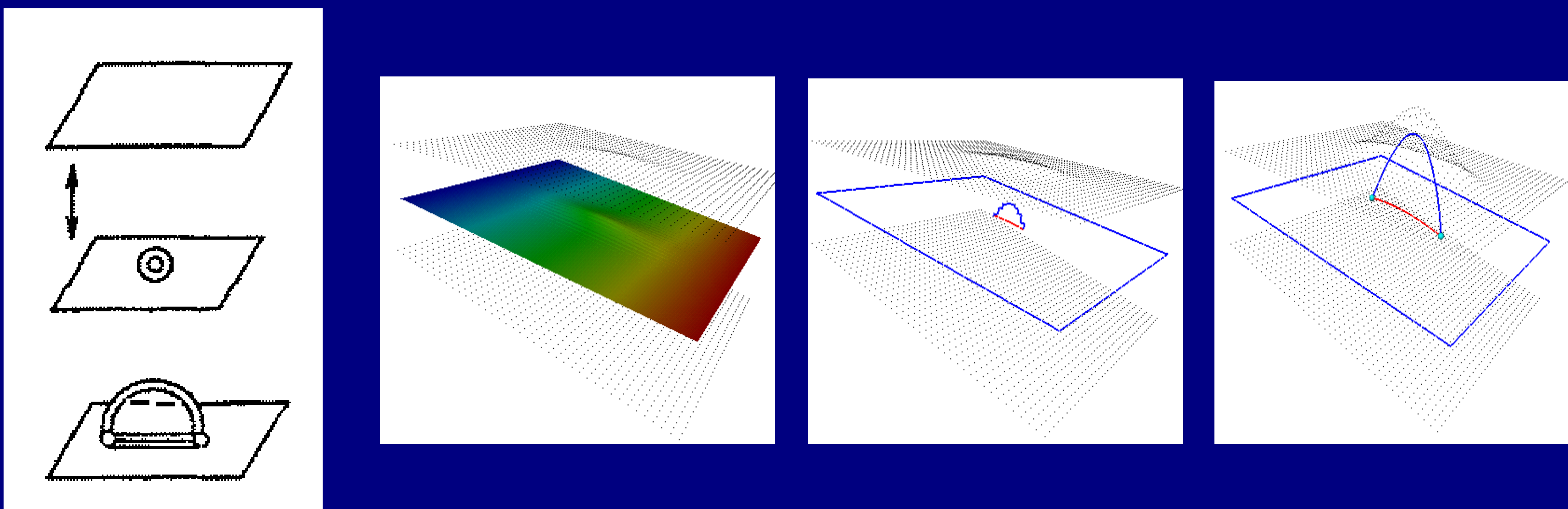
Transition	Collision of Types
A_1^4	A_1^3 - A_1^3
A_1^5	A_1^4 - A_1^4, A_1^4 - A_1^3
A_5	A_1A_3 - A_1A_3, A_3 - A_3
$A_1A_3 - I$	A_1A_3 - A_1A_3
$A_1A_3 - II$	A_1A_3 - A_1A_3, A_1^3 - A_3
$A_1^2A_3 - I$	A_1^4 - A_1A_3
$A_1^2A_3 - II$	A_1^3 - A_1A_3

Transitions of the 3D graph structure

Study the topological events of the graph structure of the MA under perturbations and shape deformations.

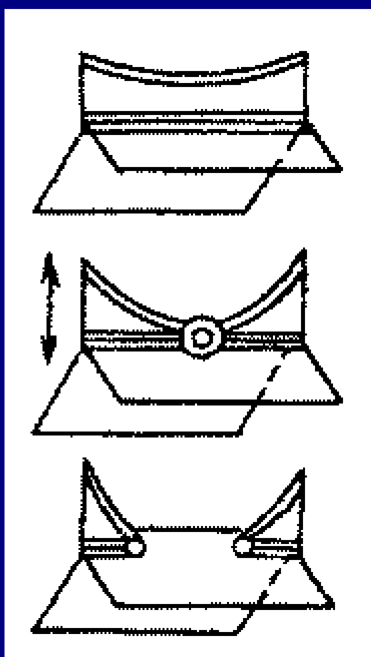
Transitions of the MA (Giblin & Kimia, ECCV 2002):

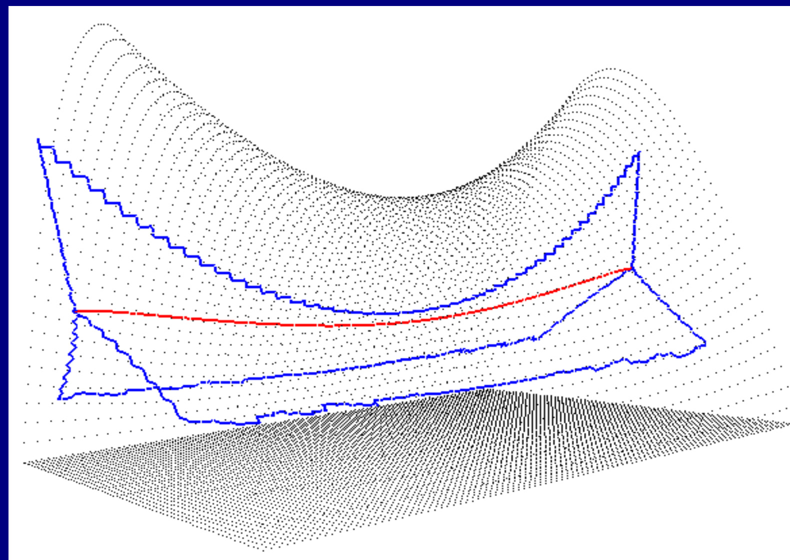
- Under a 1-parameter family of deformations, only **seven transitions** are relevant.



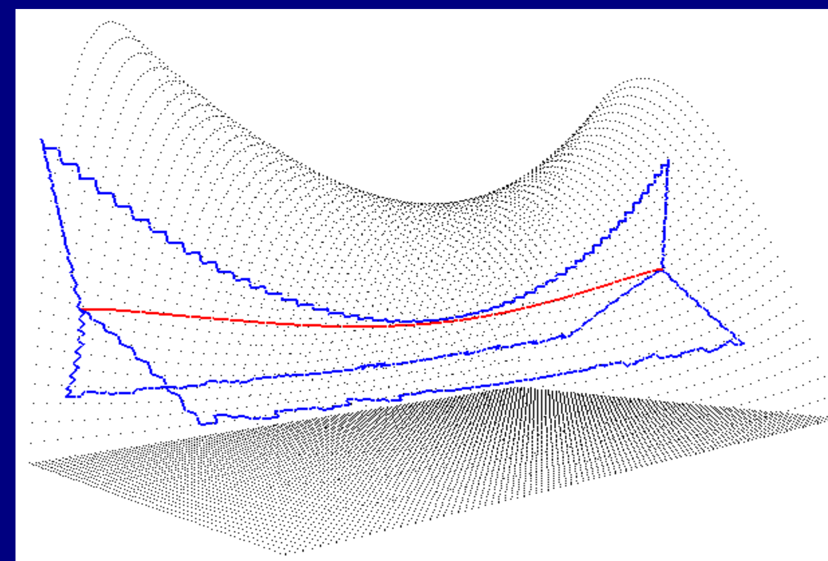
A_1A_3-I (protrusion-like, Leymarie, PhD, 2002)

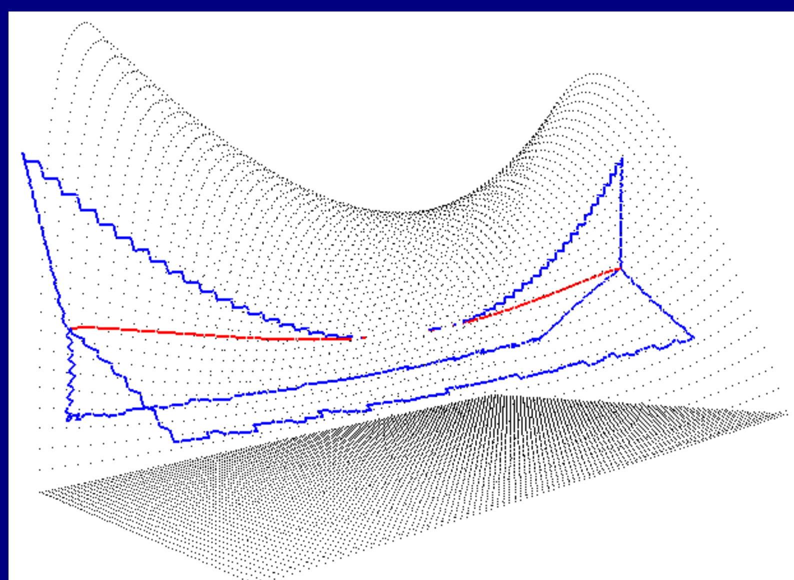
Transitions of the 3D graph structure



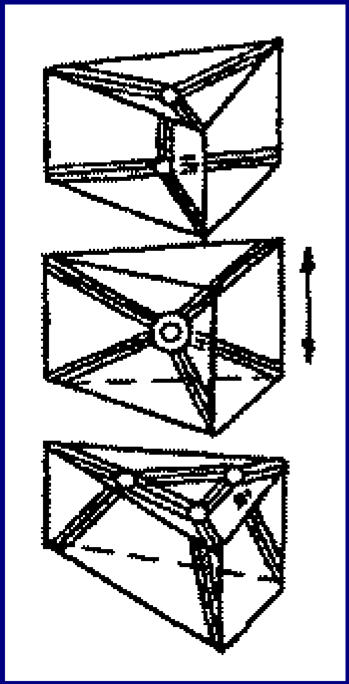


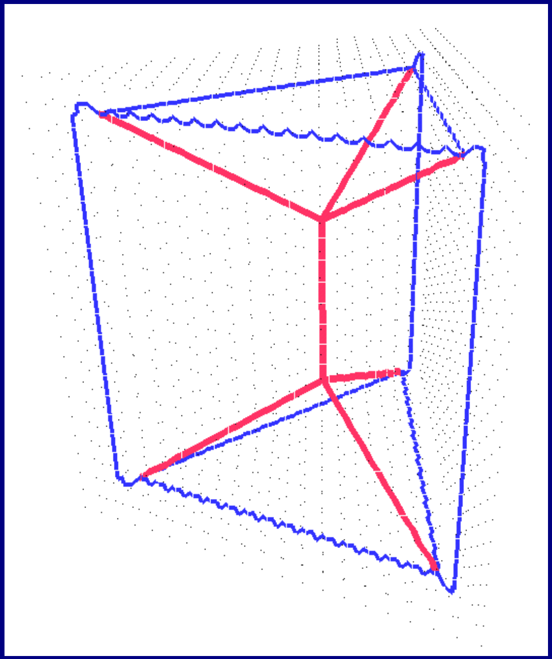
A_1A_3-II (pulling apart-like)



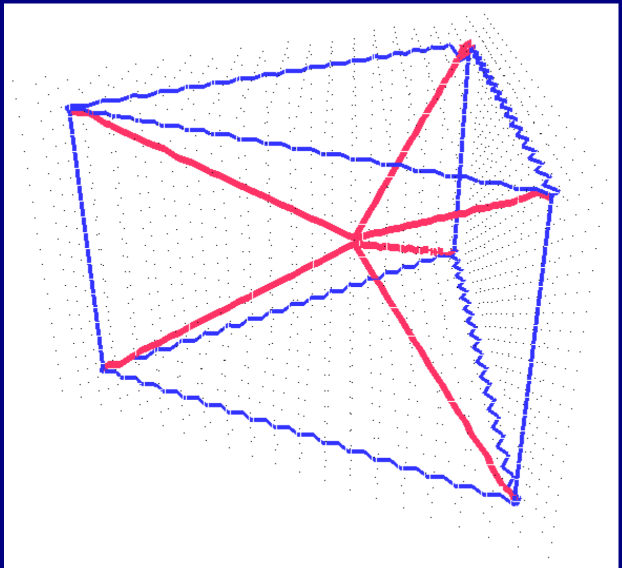


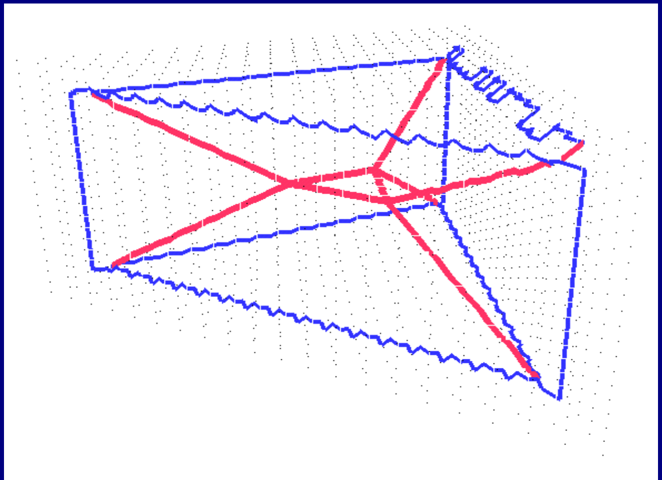
Transitions of the 3D graph structure



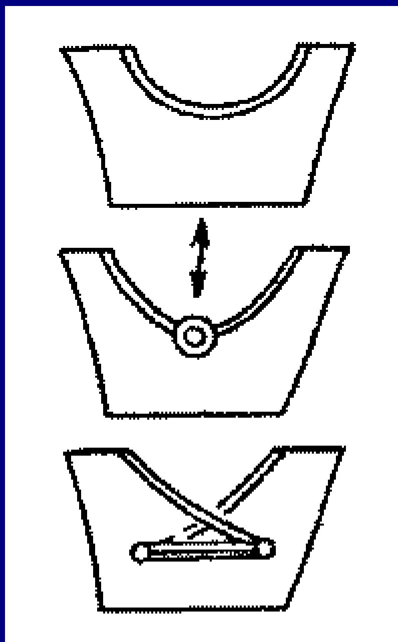


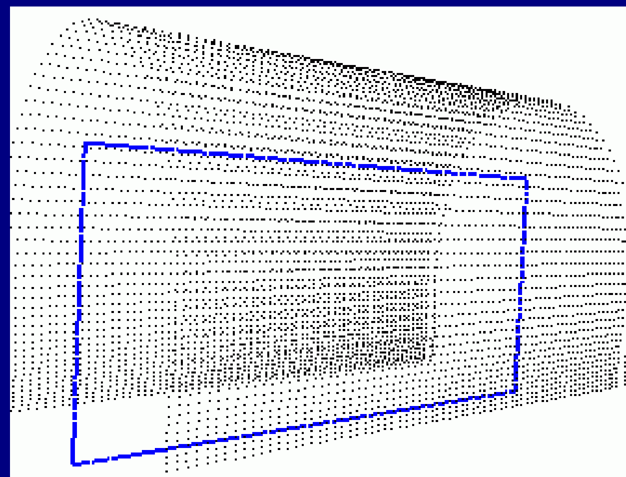
A_1^5 (compression-like)

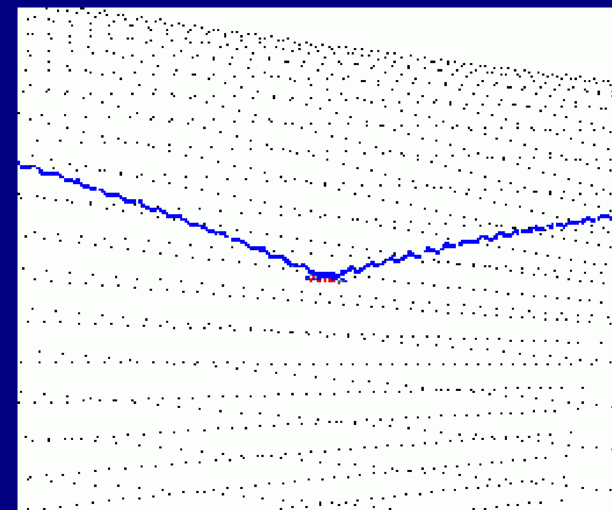


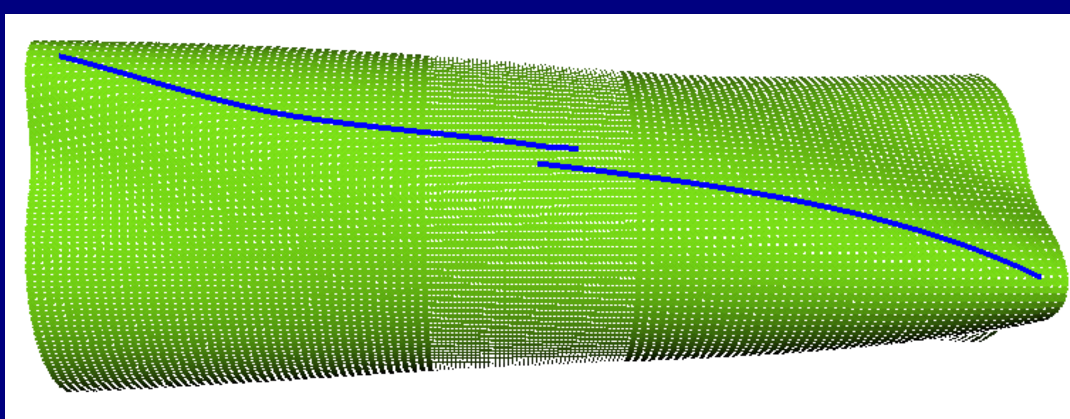


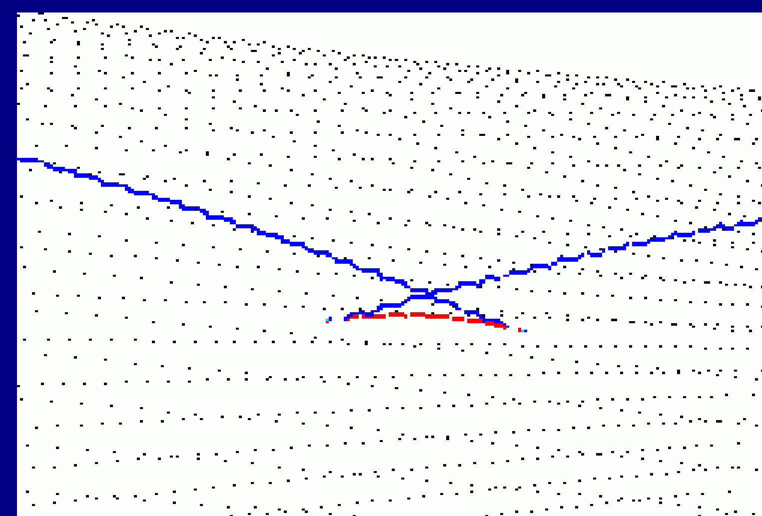
Transitions of the 3D graph structure







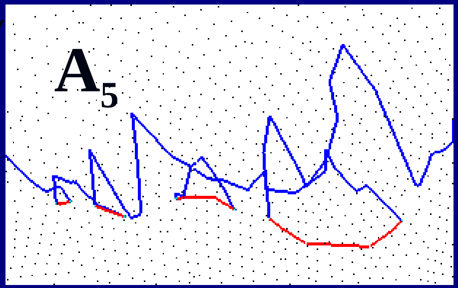


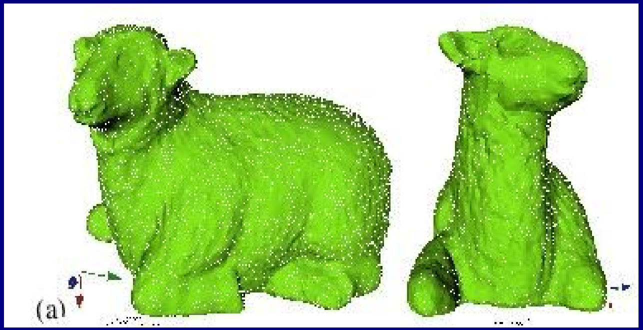


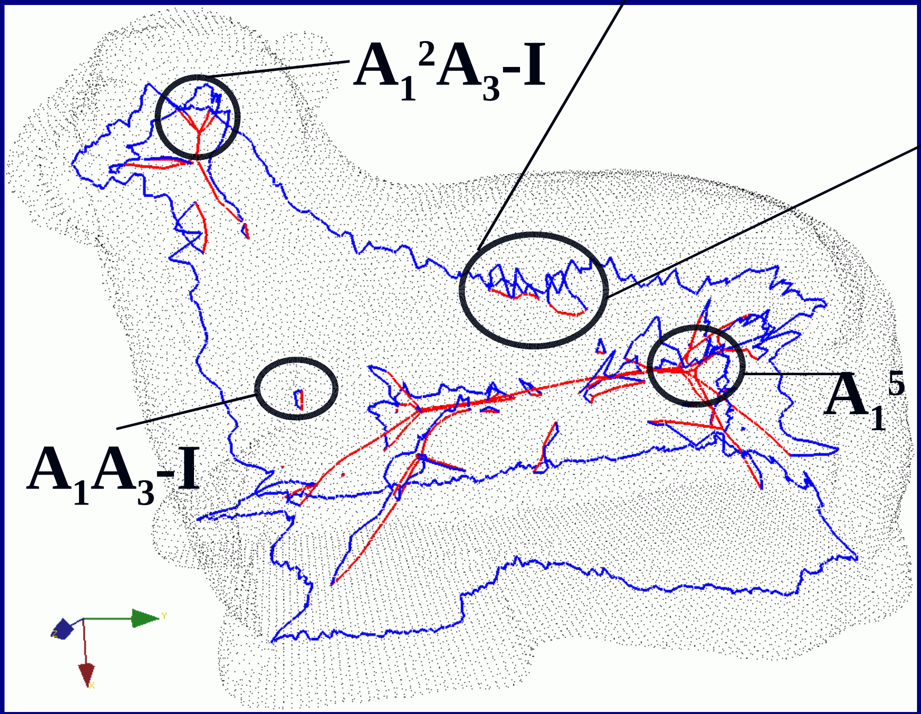
$\mathbf{A}_5$ (ridge merging-like)

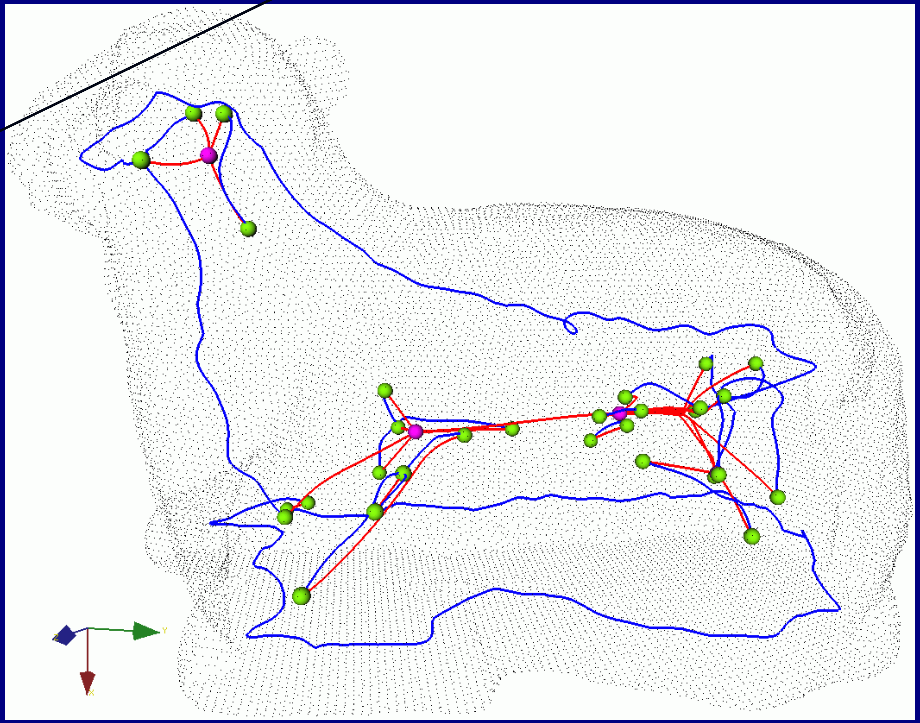
Scaffold Regularization

- Transition removal, *i.e.*, remove topological instability
- Smoothing





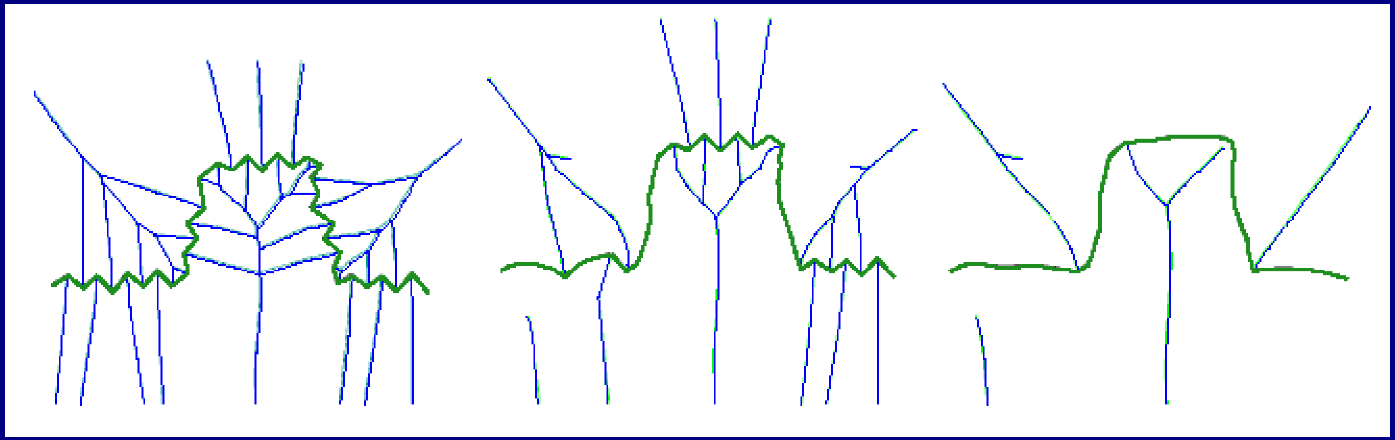




Blue: A_3 links, **Red**: A_1^3 links Green: A_1A_3 nodes, Pink: A_1^4 nodes

2D Scaffold Regularization

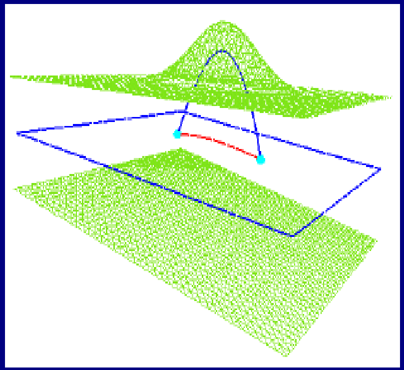
- Transition removal, *i.e.*, remove topological instability
- 2D Boundary Smoothing (via symmetry transforms) ordered by "scale" (Kimia, Tek, 2001)

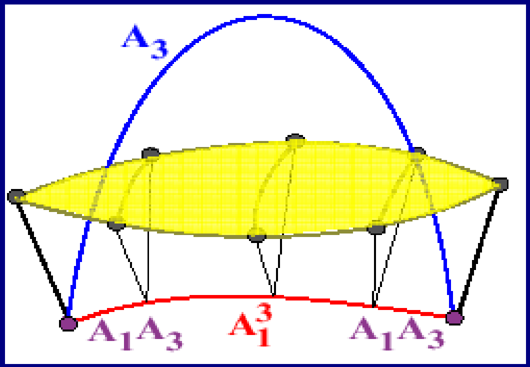


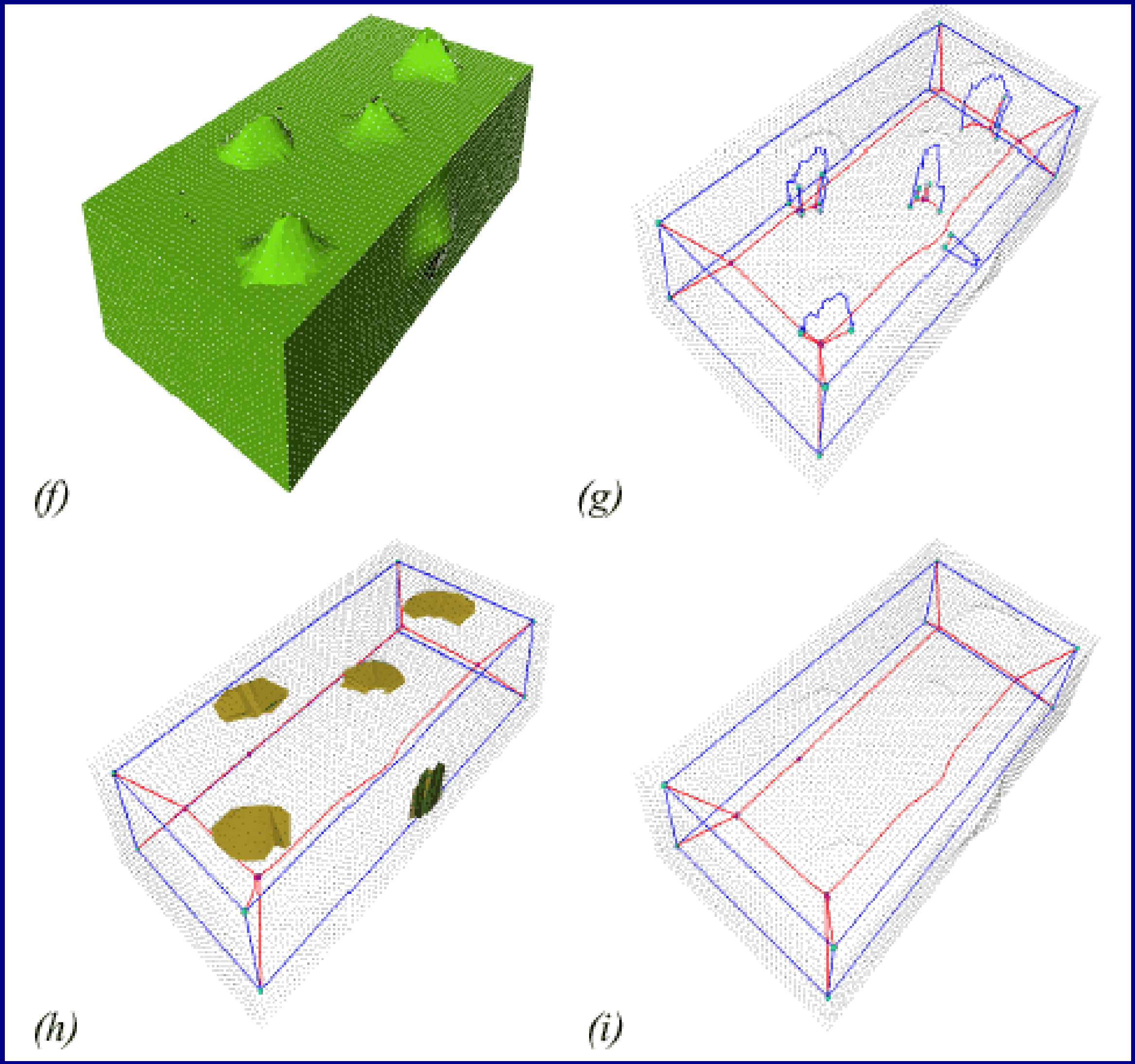
Iterative removal of MA branches, ordered by boundary support (*i.e.*, how much of the contour is represented), coupled with local boundary model adjustment, results in corner enhancement and small perturbations' smoothing.

3D Scaffold Regularization

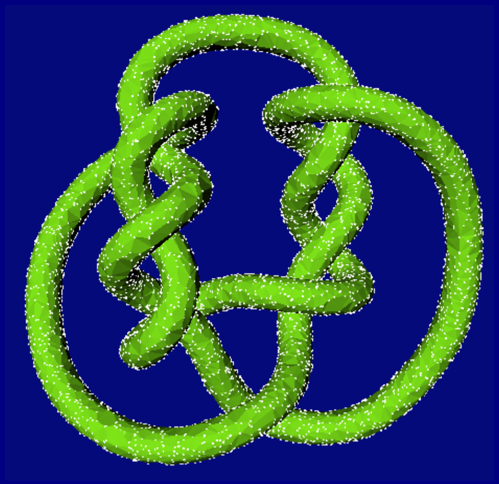
(Leymarie, Kimia, Giblin, 2003-4)





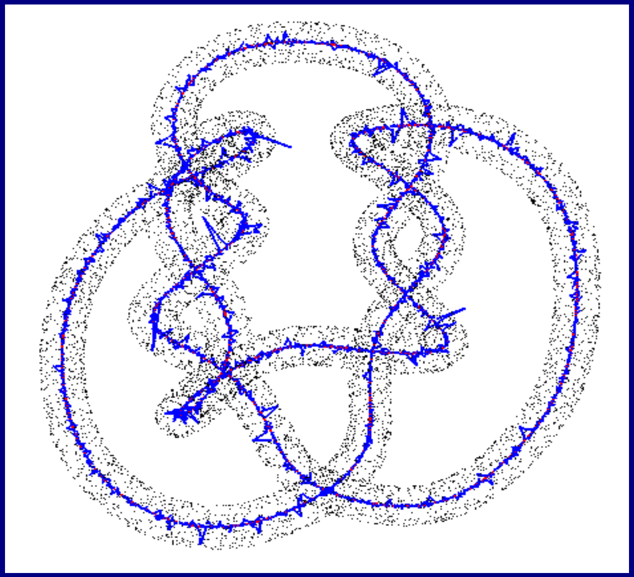


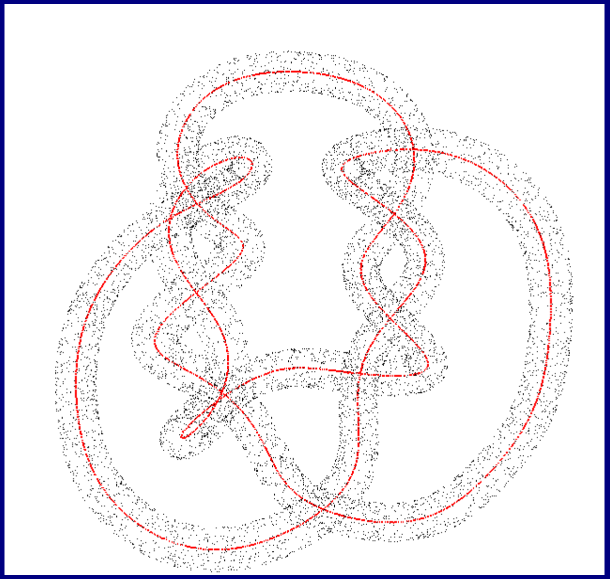
3D Scaffold Regularization

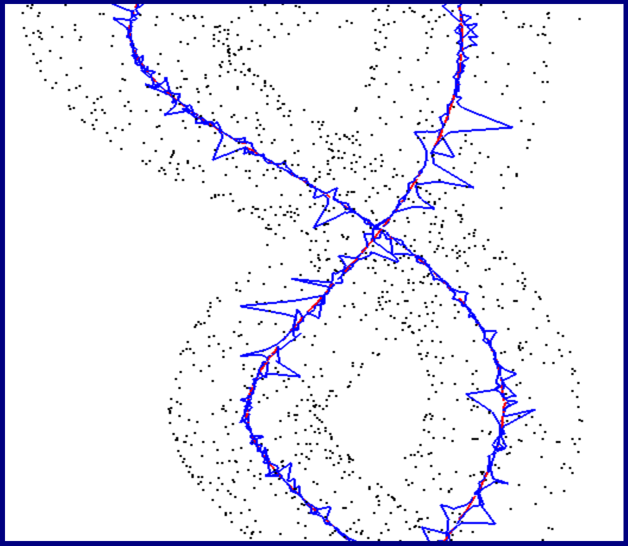


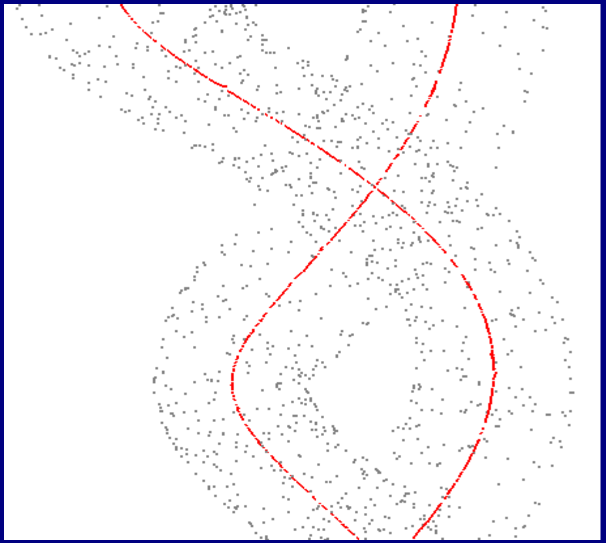
Knot data: 10K random samples

Towards
generalized cylinders



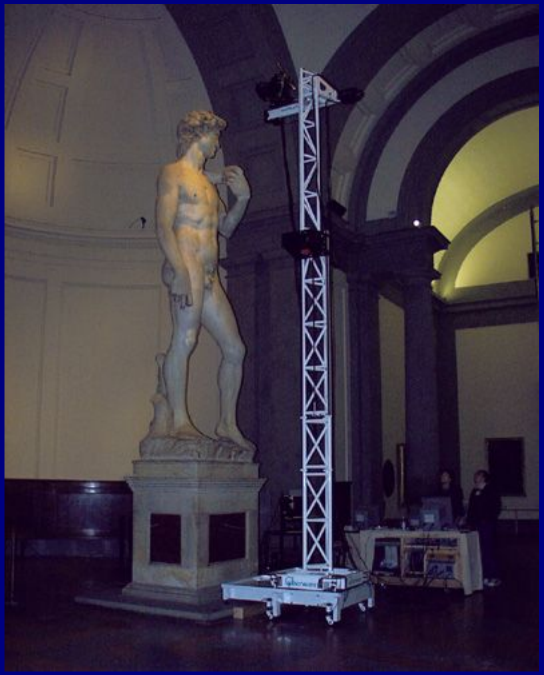




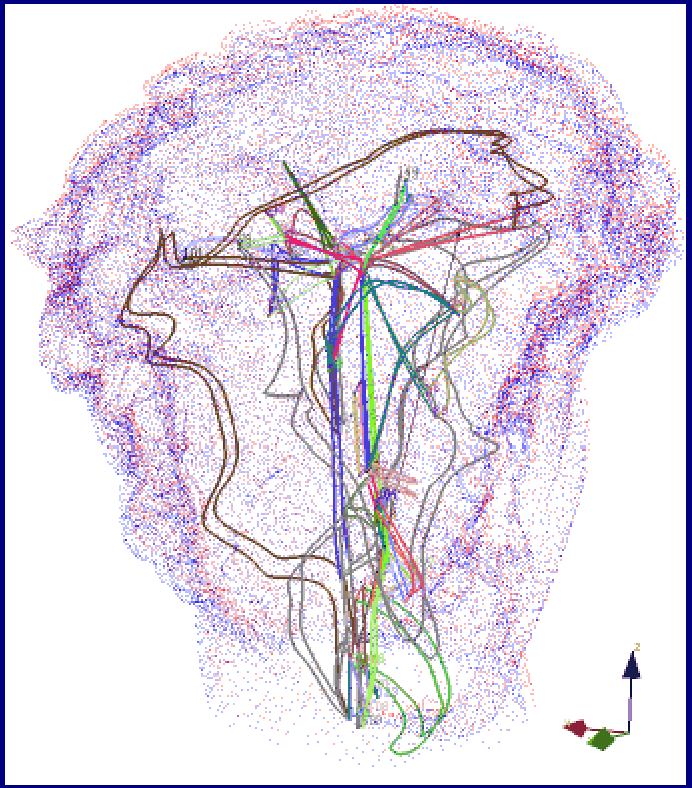


Visual Search in large 3D DB

Graph matching via graduated assignement
(presented at 3DPVT, Greece, Sept. 2004, Chang, Leymarie & Kimia)
a solution to the **Global Registration** problem.



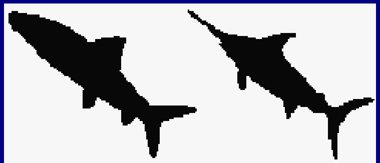
Digital Michelangelo
Stanford, Firenze, NRC

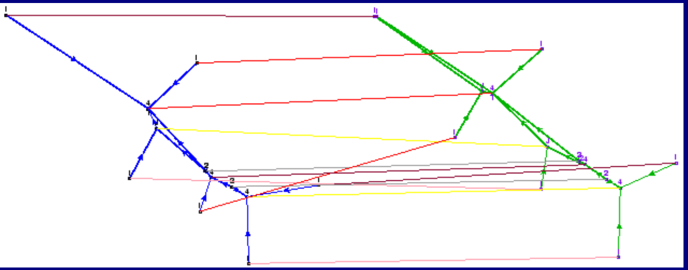


Challenge: robust automatic extraction of graphs,
dealing with topological instabilities/events.

Compare Medial Scaffolds by Graph Matching

- Intractability
  - Weighted graph matching: **NP**-hard
  - One special case: Largest common subgraph: **NP**-complete
  - Only “good” sub-optimal solutions can be found
- Graduated Assignment [Gold & Rangarajan PAMI’96]
  - [Sharvit *et. al*. JVCIR’98] index 25-shape database by matching 2D shock graphs

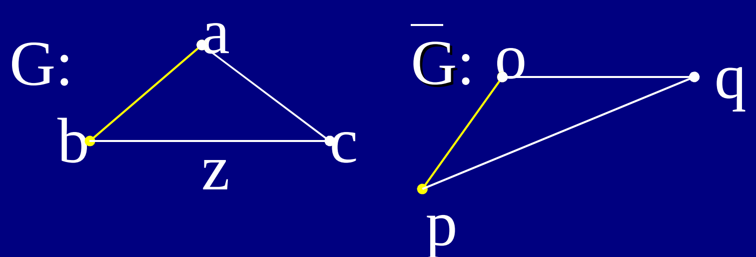




  - 3D hypergraph matching:
    - Additional dimension
    - Generally not a tree, might have isolated loops
    - No inside/outside: non-closed surfaces or surface patches

Graduated Assignment

Quadratic weighted graph matching



G, $\bar{G}$: 2 undirected graphs
I: # of nodes in G, $\bar{I}$: # of nodes in $\bar{G}$
$\{G_i\}$, $\{\bar{G}_{\bar{i}}\}$ nodes
$\{G_{ij}\}$, $\{\bar{G}_{\bar{i}\bar{j}}\}$ edges: adjacency matrices of graphs

The match matrix
$\mathbf{M}_{i\bar{i}}$ = 1 if node i in G corresponds to node $\bar{i}$ in $\bar{G}$,
= 0 otherwise

Then objective function to maximize over the space of **M** is:

$$E(\mathbf{M}) = \sum_{i=1}^{I}\sum_{\bar{i}=1}^{\bar{I}}\sum_{j=1}^{I}\sum_{\bar{j}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}}\mathbf{M}_{j\bar{j}}L_{i\bar{i}j\bar{j}} + \alpha \sum_{i=1}^{I}\sum_{\bar{i}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}}N_{i\bar{i}}$$

Cost of matching G_{ij} to $\bar{G}_{\bar{i}\bar{j}}$. If the nodes match, how similar the links are.

Cost of matching G_i to $\bar{G}_{\bar{i}}$

$\mathbf{L}_{i\bar{i}j\bar{j}}$: link similarity between G_{ij} and $\bar{G}_{\bar{i}\bar{j}}$
$\mathbf{N}_{i\bar{i}}$: node similarity between G_i and $\bar{G}_{\bar{i}}$

Modified Graduated Assignment for 3D Medial Scaffold Matching

$$E(\mathbf{M}) = \begin{array}{l} \alpha \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} N_{i\bar{i}} + \beta \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} \mathbf{M}_{j\bar{j}} L_{i\bar{i}j\bar{j}} \\ + \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} \sum_{k=1}^{I} \sum_{\bar{k}=1}^{\bar{I}} \mathbf{M}_{i\bar{i}} \mathbf{M}_{j\bar{j}} \mathbf{M}_{k\bar{k}} H_{i\bar{i}j\bar{j}k\bar{k}} \,, \end{array}$$

Node cost: (radius)

$$N_{i\bar{i}}(G_i, \overline{G}_{\bar{i}}) = \begin{cases} 0, & \text{if } G_i \text{ and } \overline{G}_{\bar{i}} \text{ have different types,} \\ 1 - \left|\frac{r_i - r_{\bar{i}}}{max(R, \overline{R})}\right|, & \text{otherwise,} \end{cases}$$

Link cost: (length)

$$L_{i\bar{i}j\bar{j}} = \begin{cases} 0, & \text{if any of links } ij \text{ and } \bar{i}\bar{j} \text{ are missing,} \\ 1 - \left|\frac{l_{ij} - l_{\bar{i}\bar{j}}}{max(L, \overline{L})}\right|, & \text{otherwise,} \end{cases}$$

Sheet (hyperlink) cost: (corner angle)

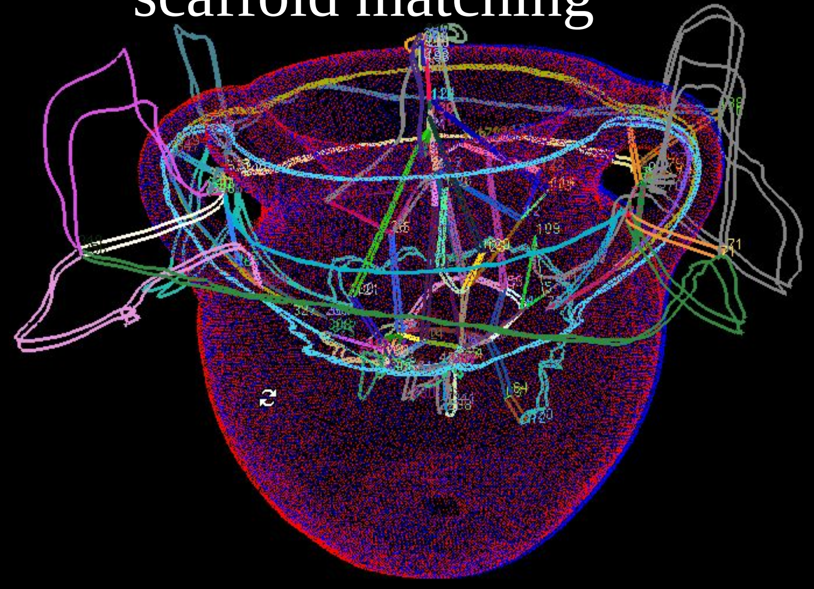
$$H_{i\bar{i}j\bar{j}k\bar{k}} = \begin{cases} 0, & \text{if any links } ij,\ jk,\ \bar{i}\bar{j},\ \bar{j}\bar{k} \text{ are missing,} \\ 1 - |\angle ijk - \angle \bar{i}\bar{j}\bar{k}|, & \text{otherwise,} \end{cases}$$

Medial scaffold matching example

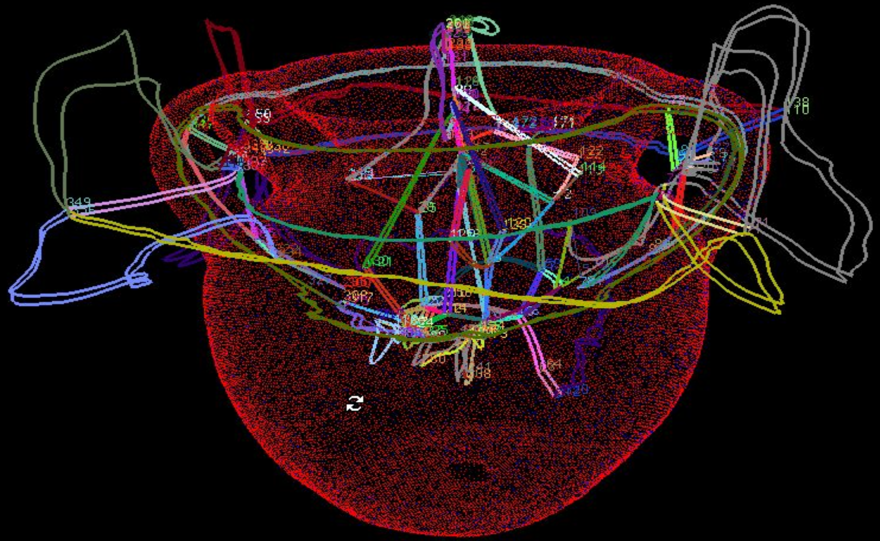
Archaeological pot

Two scans of the outside surface of a pot (50K and 40K). The inner surface of the pot is missing.

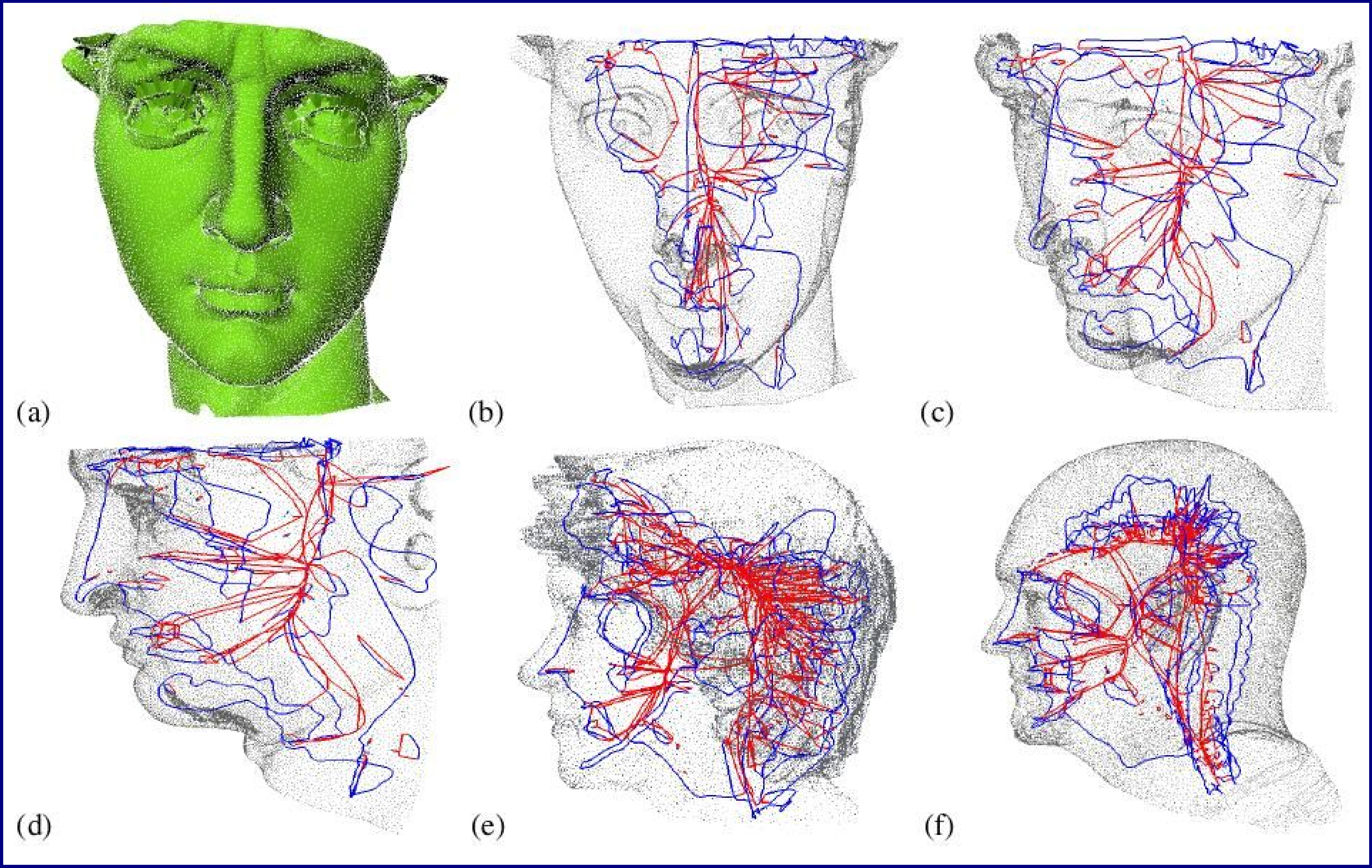
Initial alignment via scaffold matching



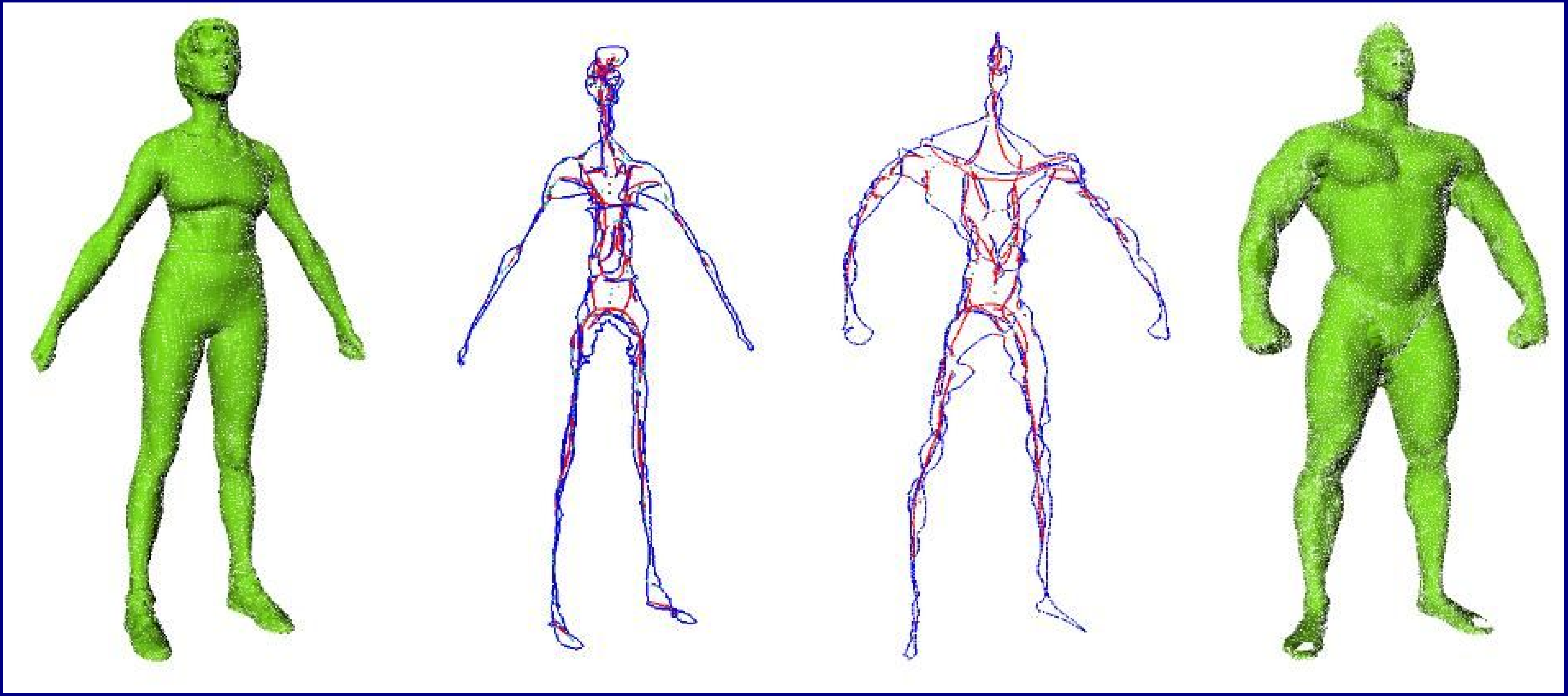
Final registration after ICP



Towards 3D Object Recognition

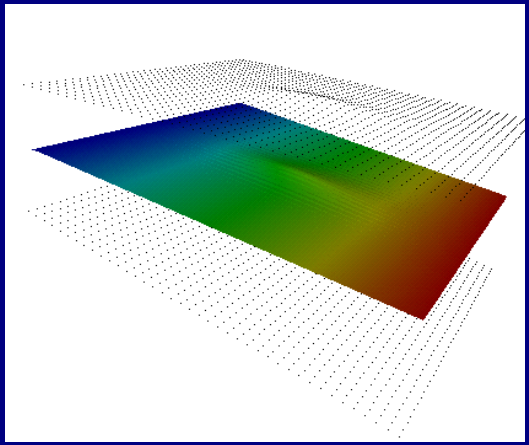


Towards 3D Object Recognition

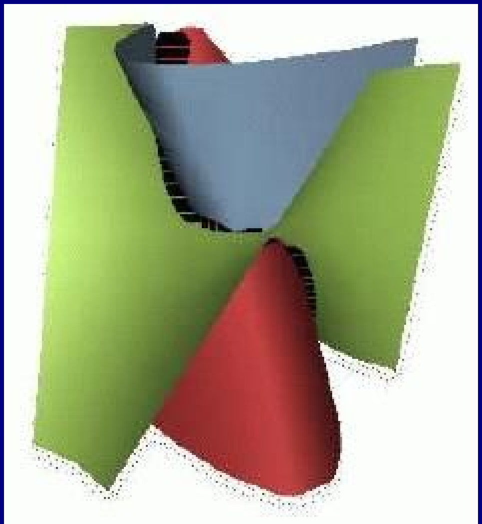


Conclusions – Limitations + Future

- Perturbations smaller than contact sphere radius are not “visible” from the graph structure alone.

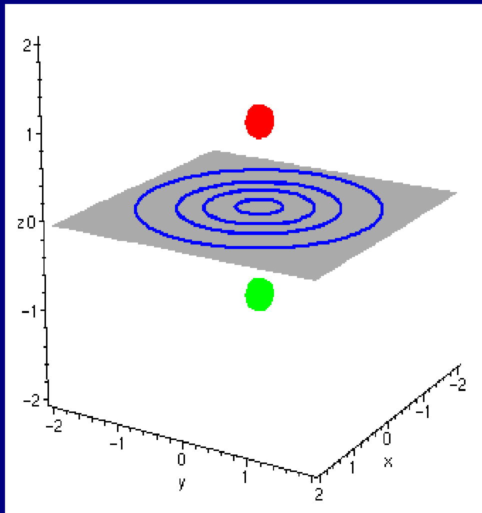


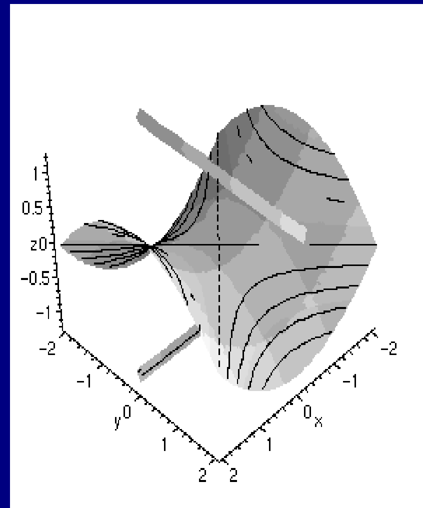
- Complementary to recent anisotropic smoothing methods (explicit ridges).
- Cost measures for 3D shape matching are at an early stage of development.

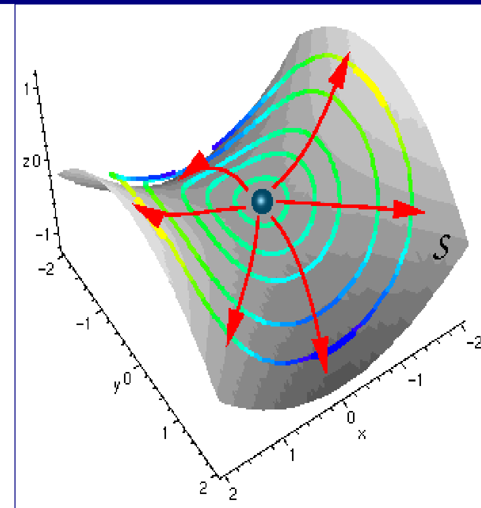


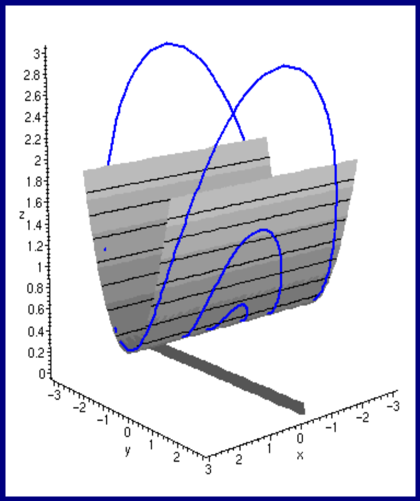
- Extension to other generators (inputs) is underway (polygons, curved patches).

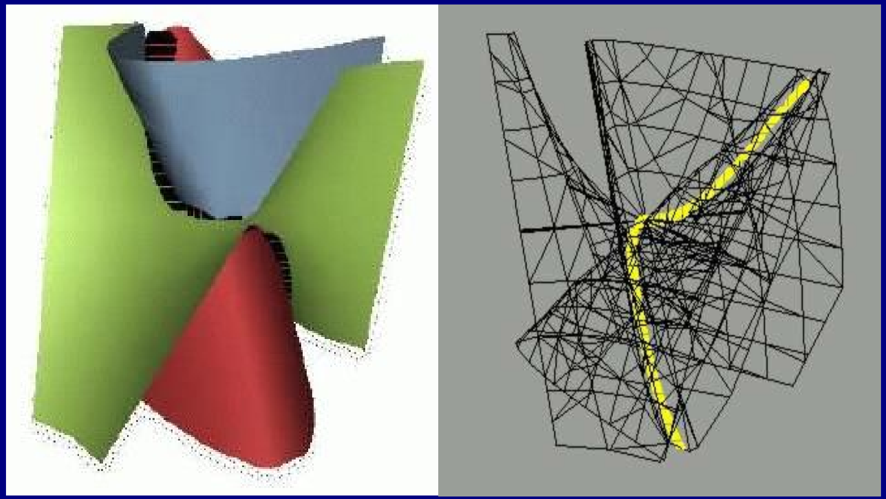
Conclusions – Extended generators

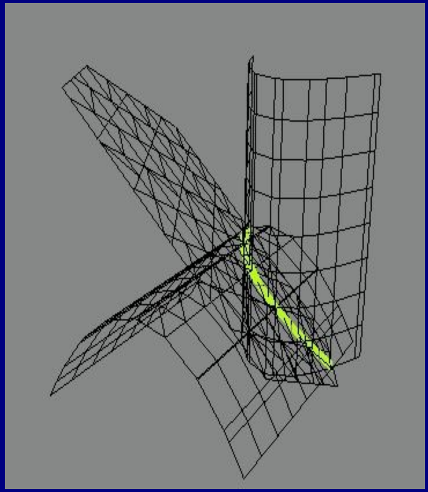












Requires hybrid methods: algebraic (Groebner bases) and numerical (intervals)

Part III:
Science, Art & Shape

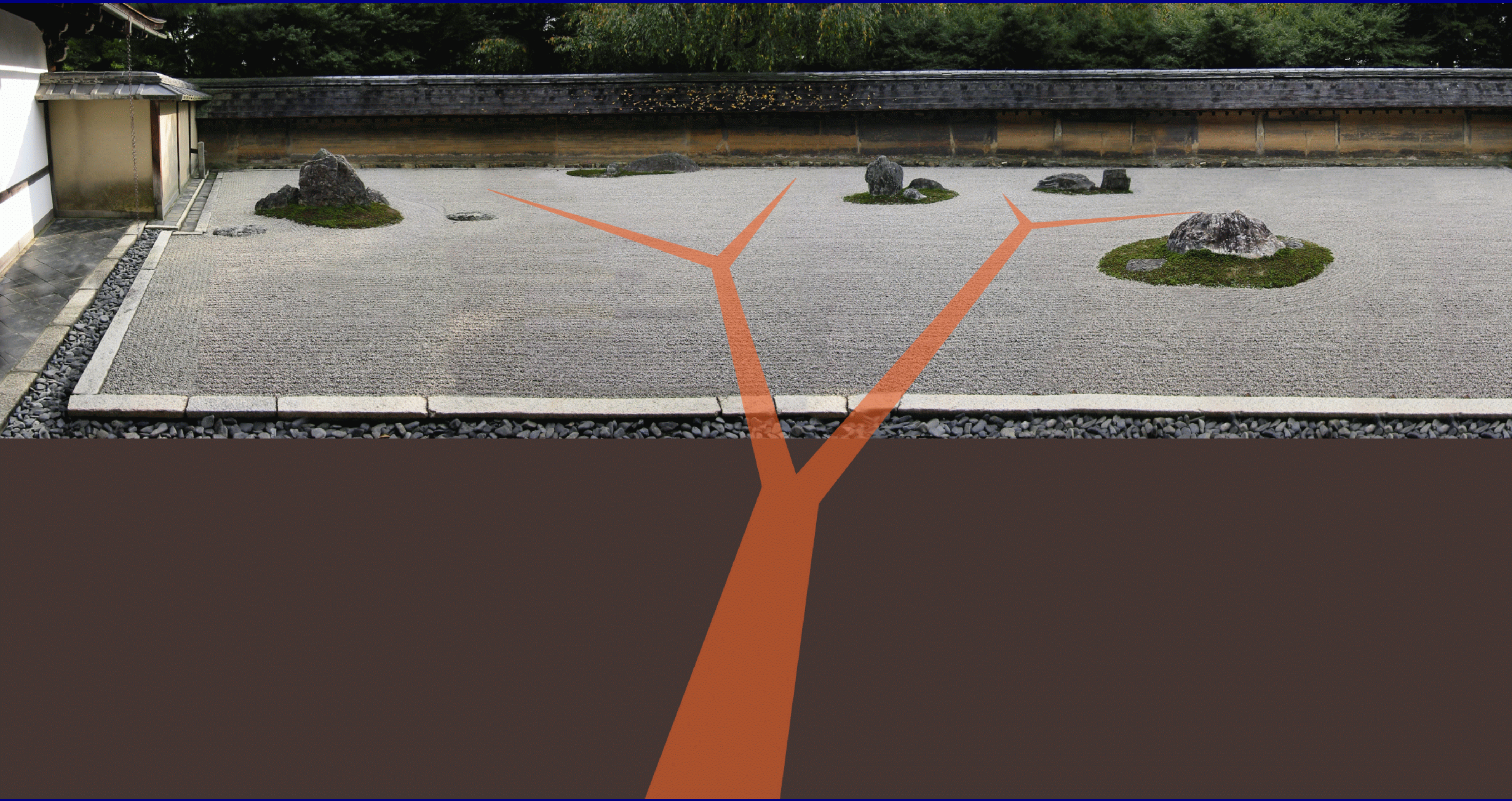
The SHAPE Lab.
Goldsmiths College, University of London

Frederic Fol Leymarie
< ffl@gold.ac.uk >
www.gold.ac.uk/~ffl

The perception/design of gardens

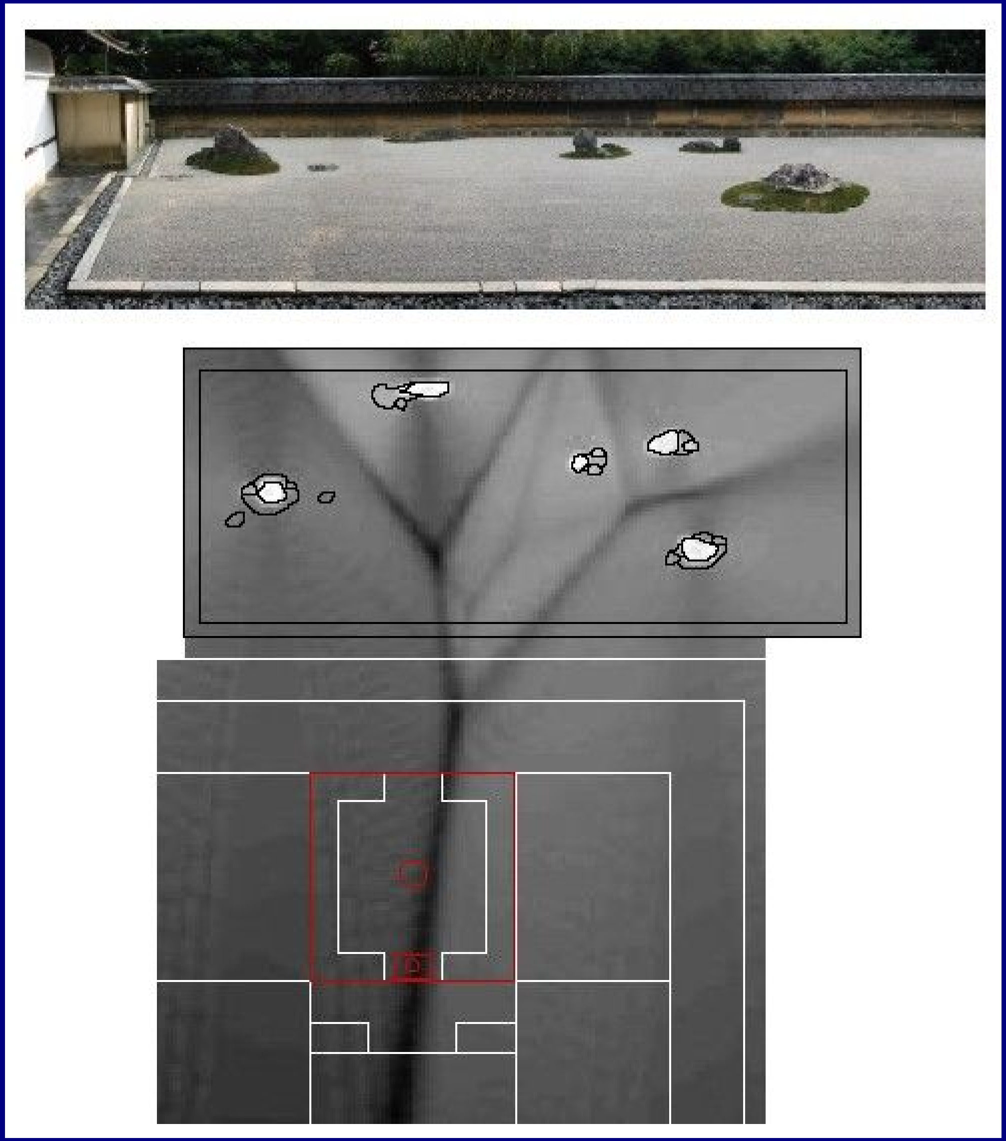
Ryonji garden, Japan, 15th century

The perception/design of gardens



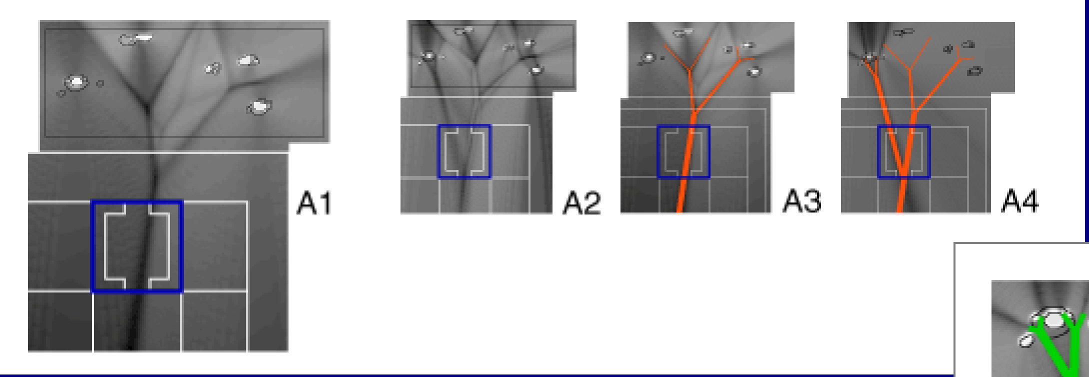
Gert van Tonder et al. --- Nature, 2002.

The perception/design of gardens

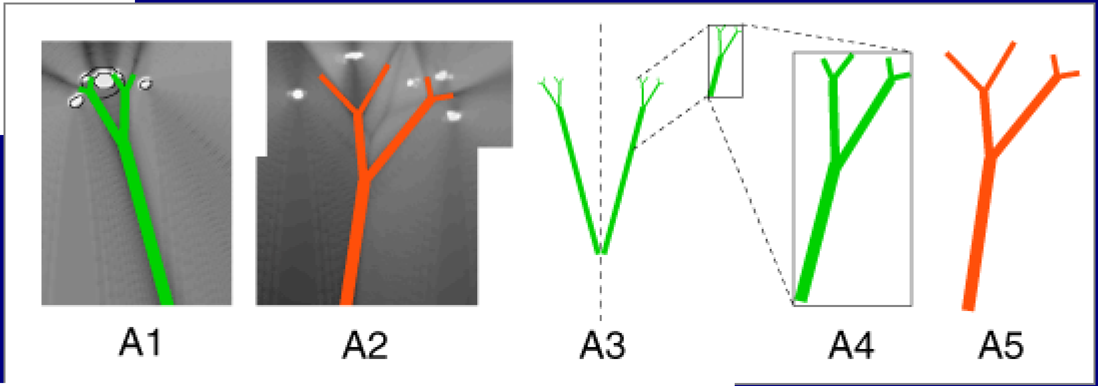


Gert van Tonder et al. --- Nature, 2002.

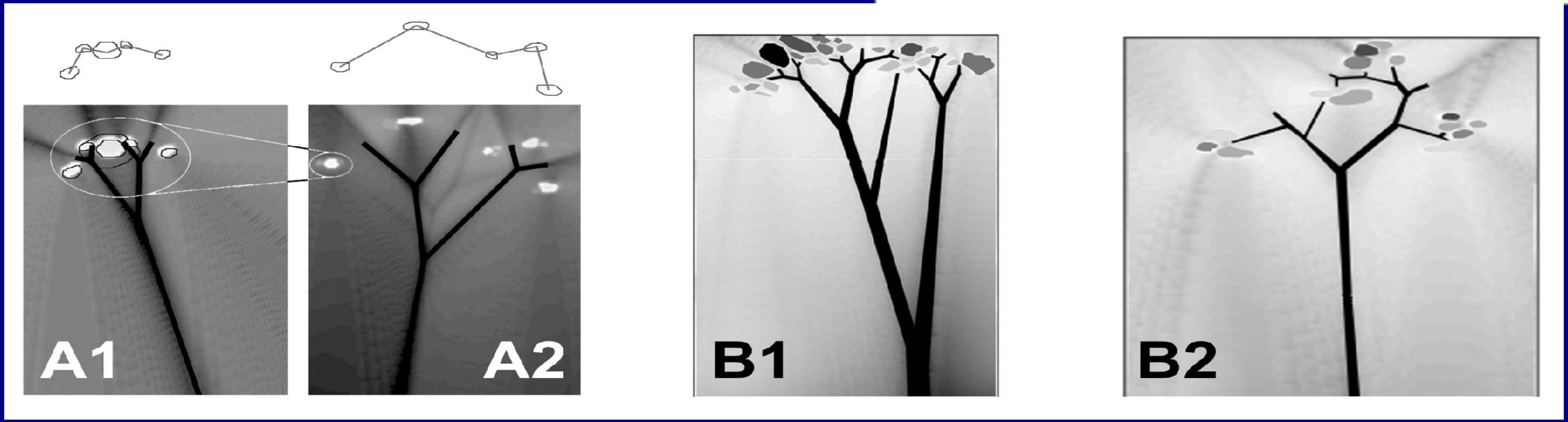
The perception/design of gardens



Gert van Tonder et al. --- **Stylistic signature of creators.**



Fractal-like designs



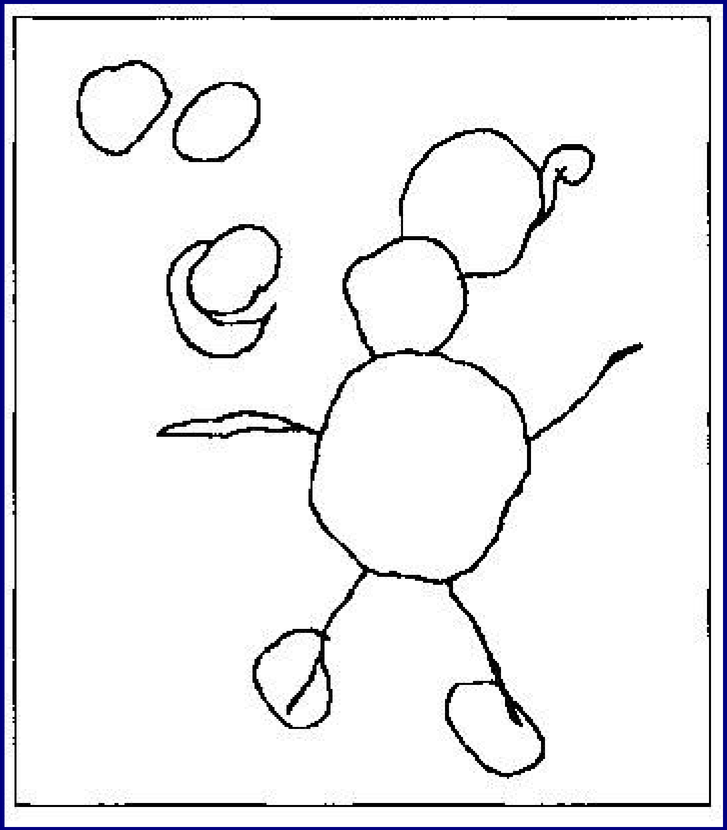
Local (A1) vs. global (A2) MA's in Ryonji

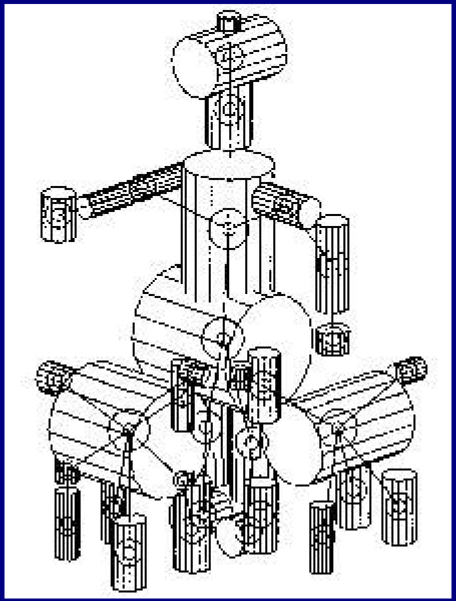
B1: Zakkein (no longer exists) – B2: Akisato Ritoh (1799)

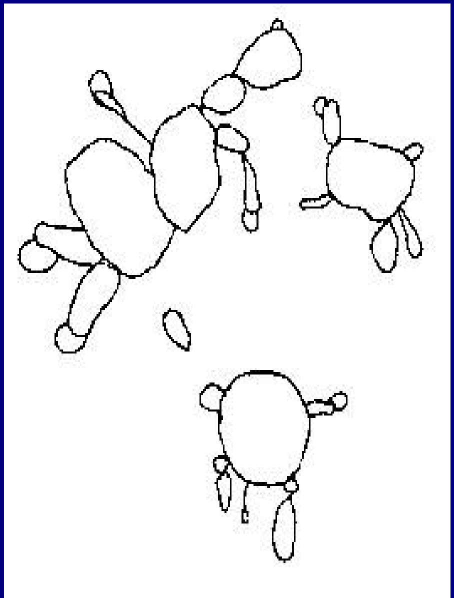
Drawing

Lascaux paintings

Drawing

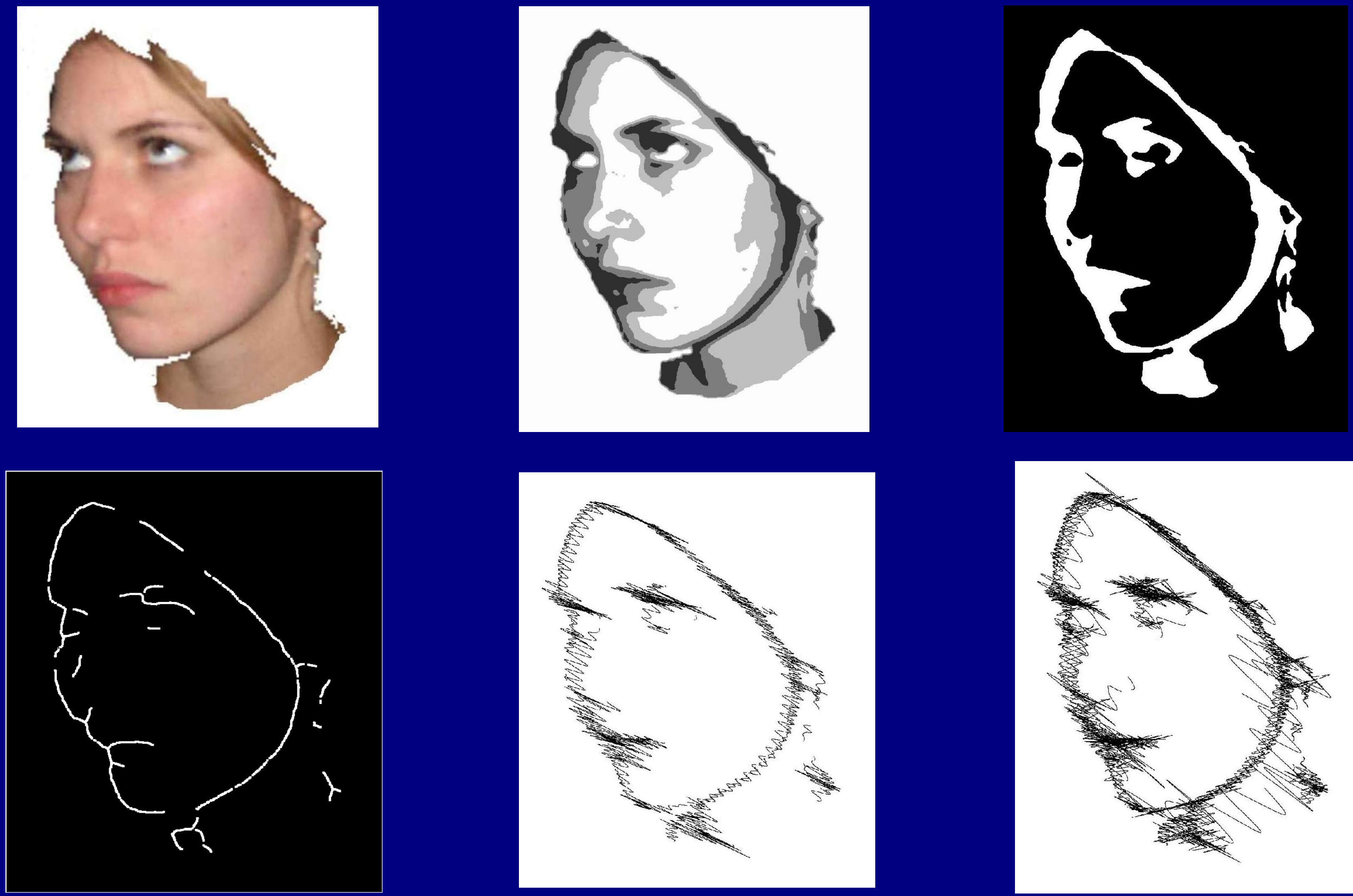






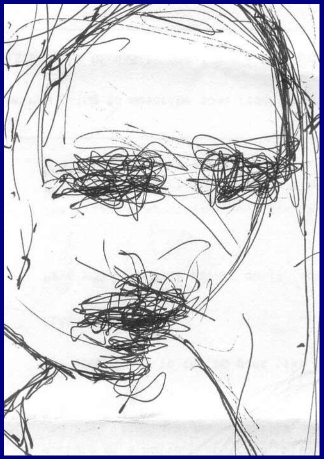
ROSE: Representation Of Spatial Experience --- Ed Burton

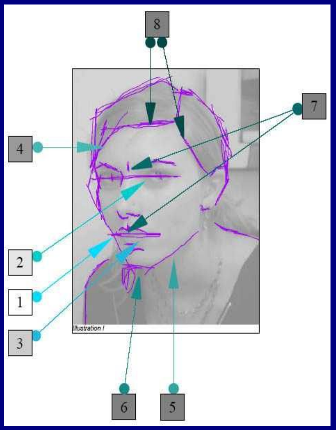
Drawing

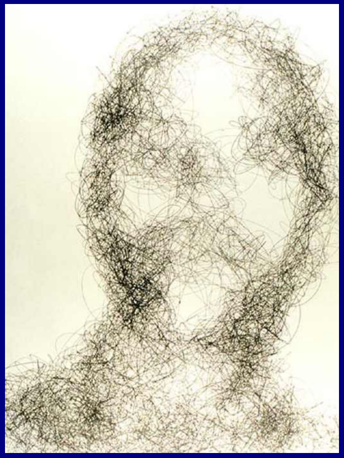


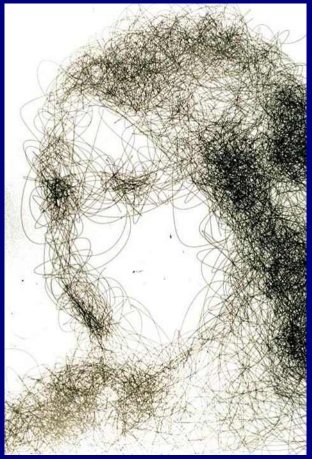
AIKON: Automated/Artistic IKONograph ---
P. Tresset & F. F. Leymarie @ Goldsmiths

Drawing



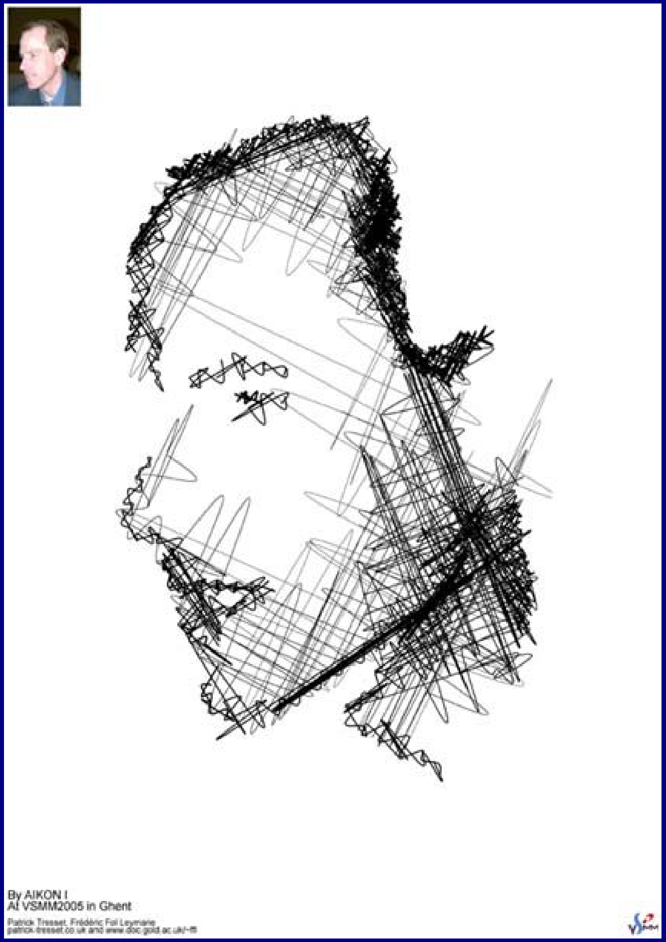


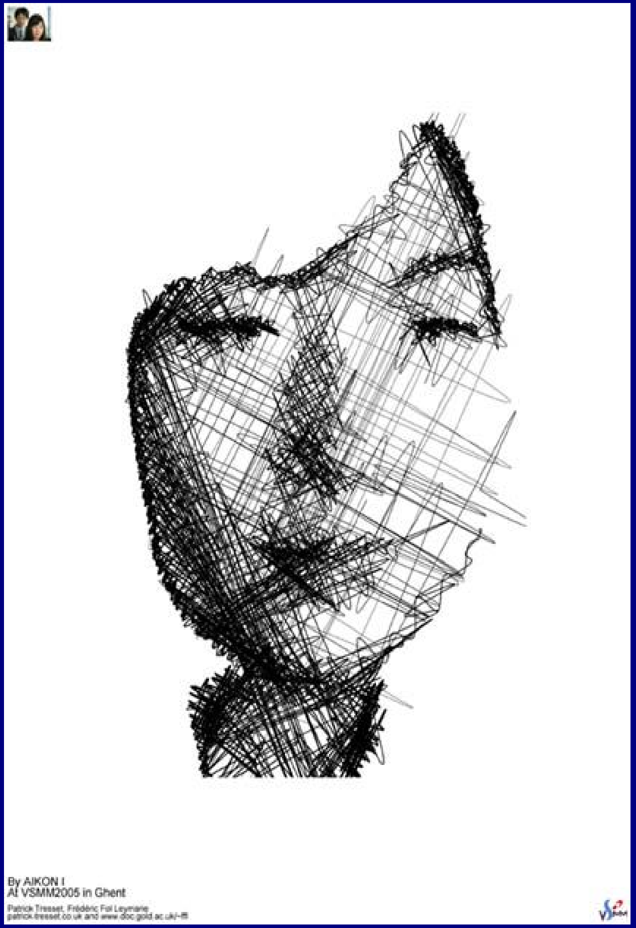




AIKON: Automated/Artistic IKONograph ---
P. Tresset & F. F. Leymarie @ Goldsmiths

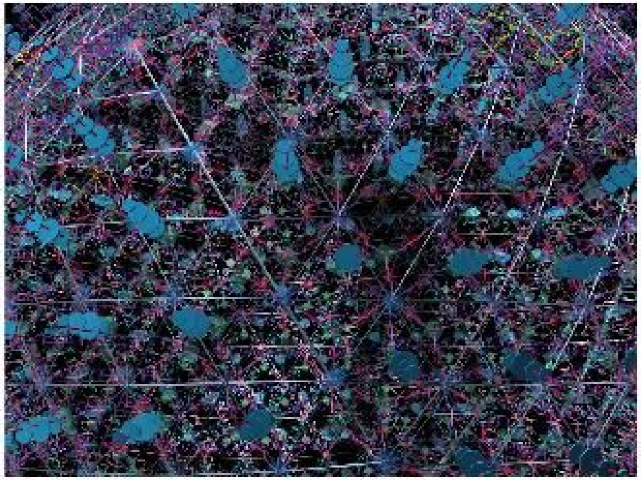
Drawing



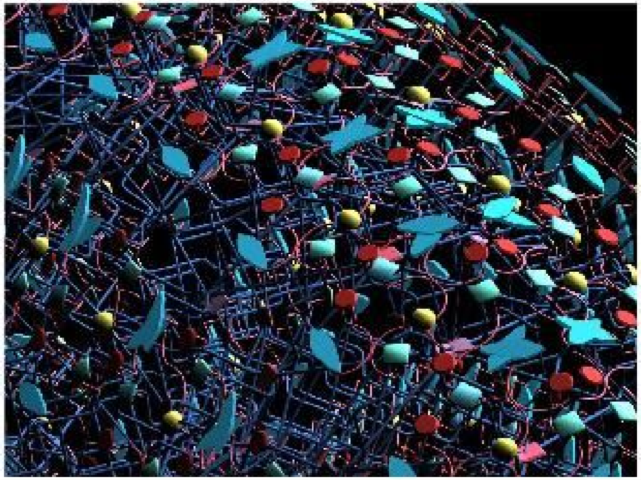


AIKON: Automated/Artistic IKONograph ---
P. Tresset & F. F. Leymarie @ Goldsmiths

(a)



(b)



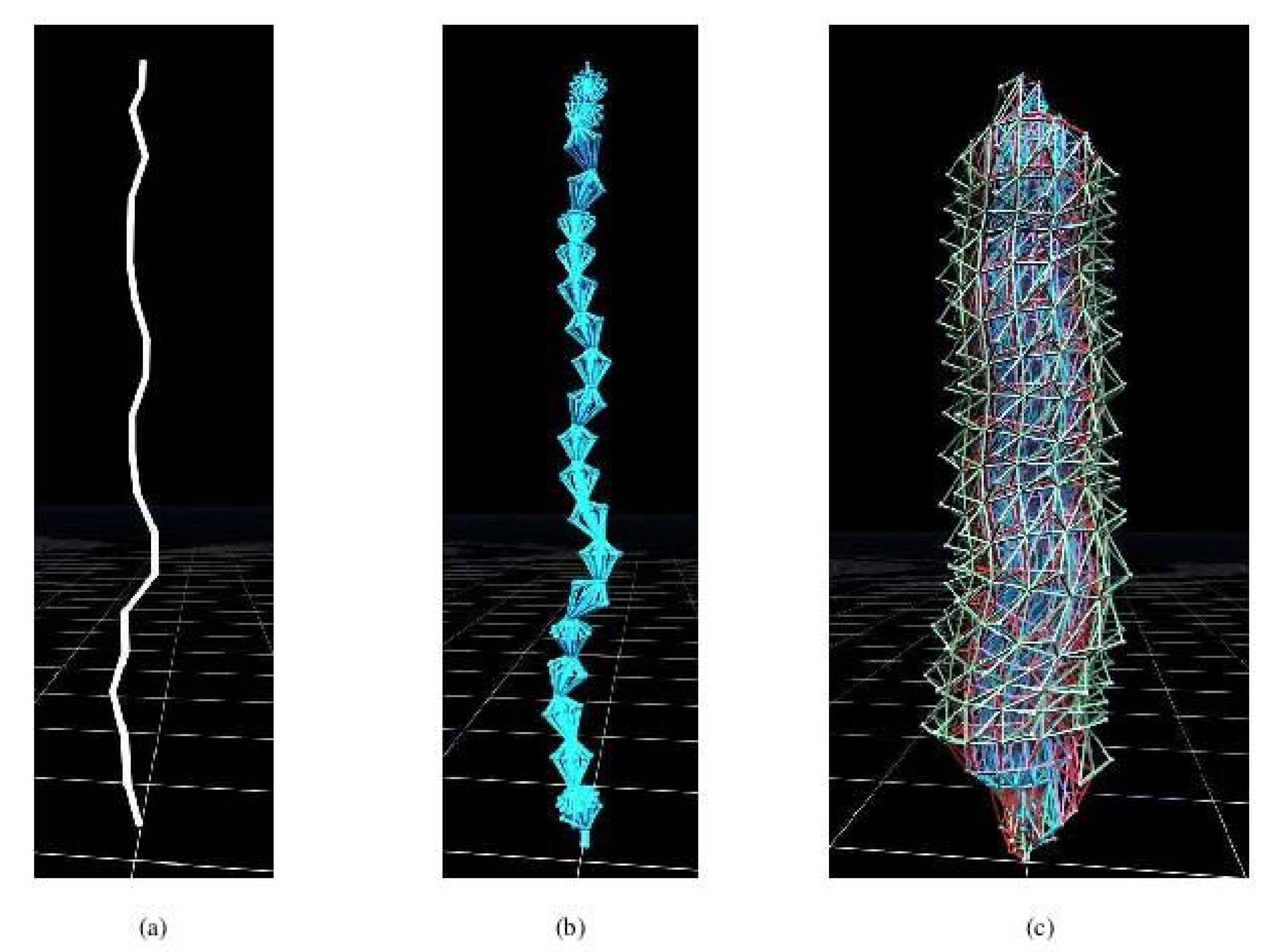
Biomimetics & Sculpting

Collaboration with Mid-Ocean Studio

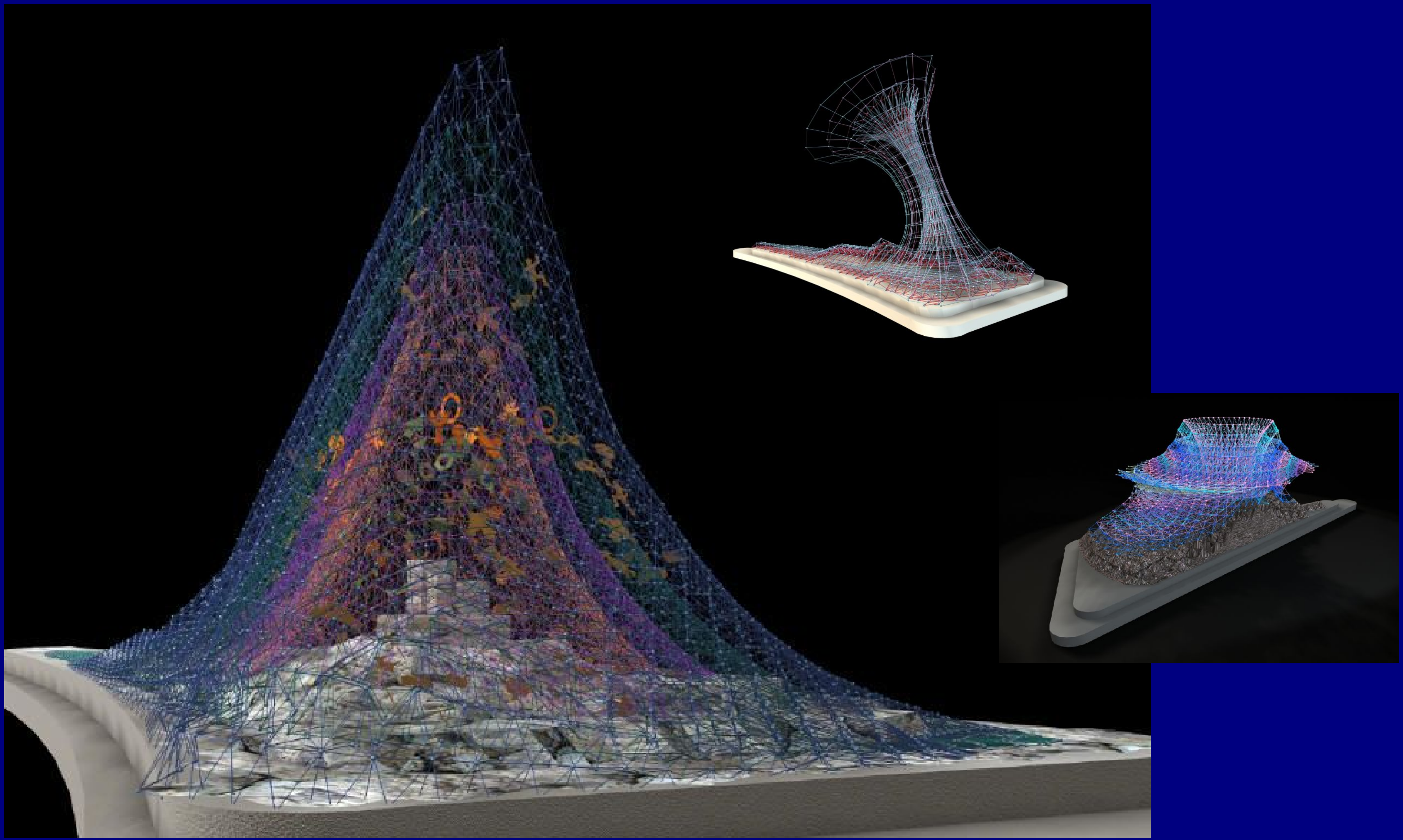
Brower Hatcher's manifesto:

Paradigm for sculpting
where a deformable, layered,
approximately regular
scaffold structure is used as
a framework upon which
other sculptural elements
can be associated.

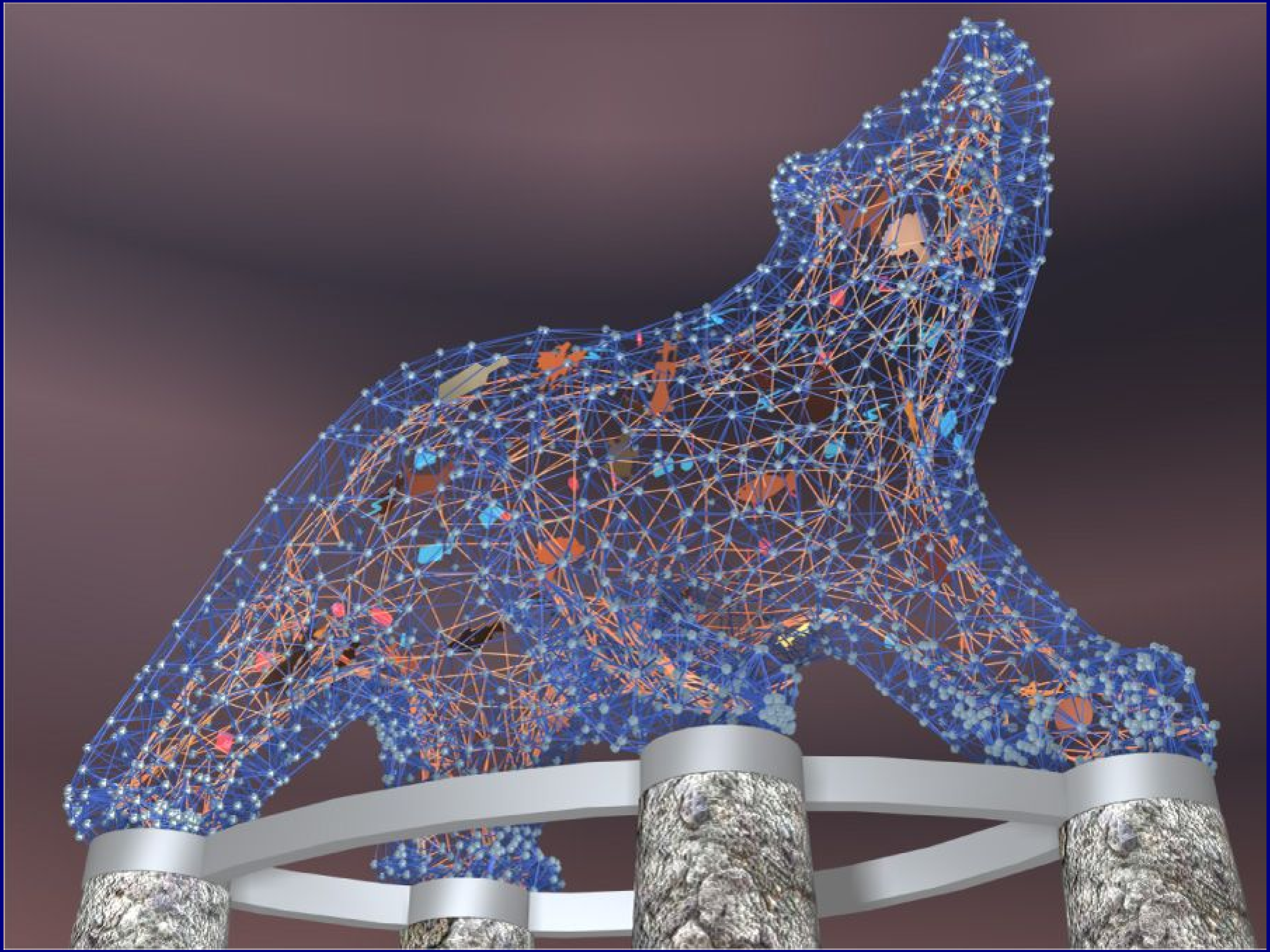
Biomimetics & Sculpting



Biomimetics & Sculpting

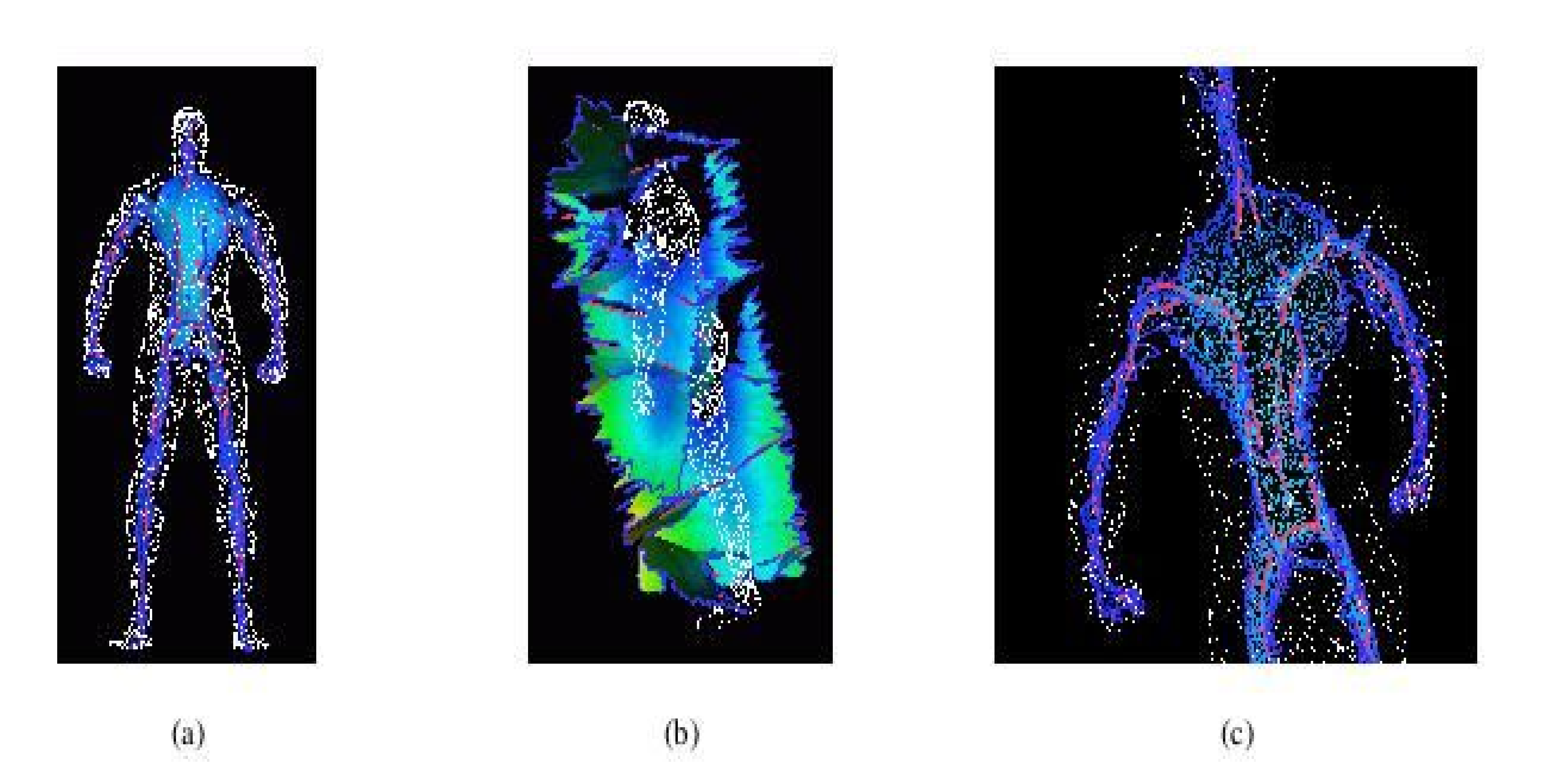


Collaboration between Mid-Ocean Studio,
Brown University & Goldsmiths College



Kelowna, British Columbia, Canada

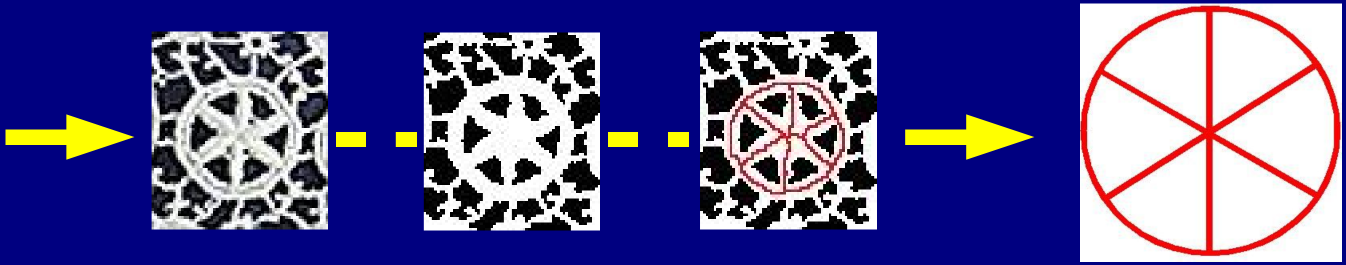
Biomimetics & Sculpting



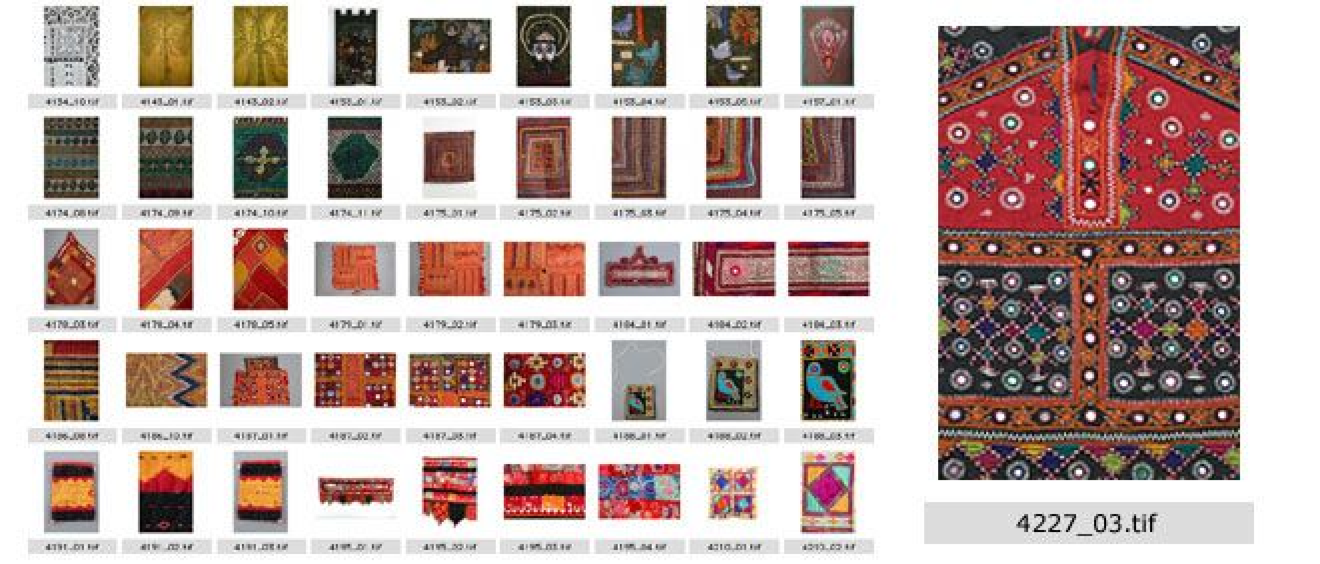
Using the Medial Scaffold to initiate biomimetism.

Visual Search in large image DB

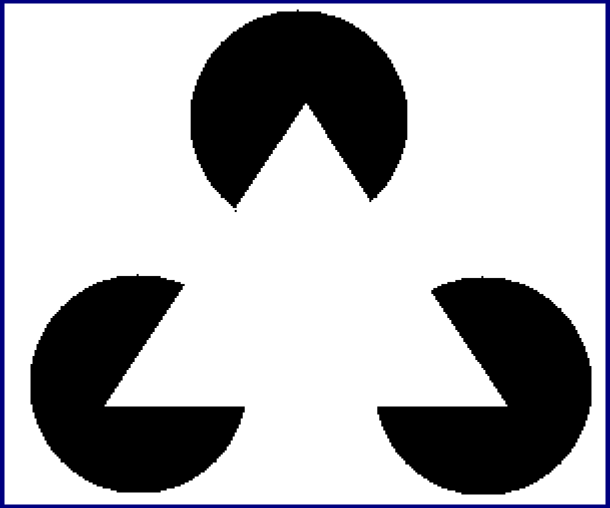
Expertise: Search via patterns represented as shock **graphs**.



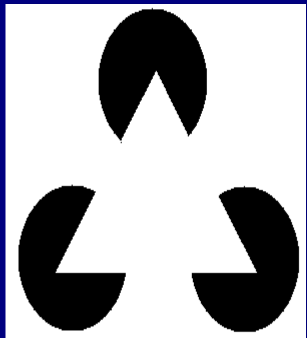
Challenge: robust automatic extraction of graphs, dealing with topological instabilities/events.



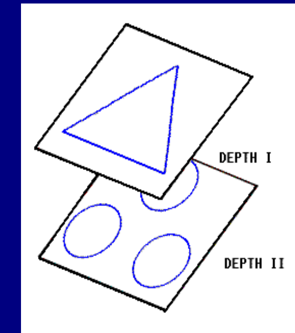
Perception
Illusory contours & Gestalt

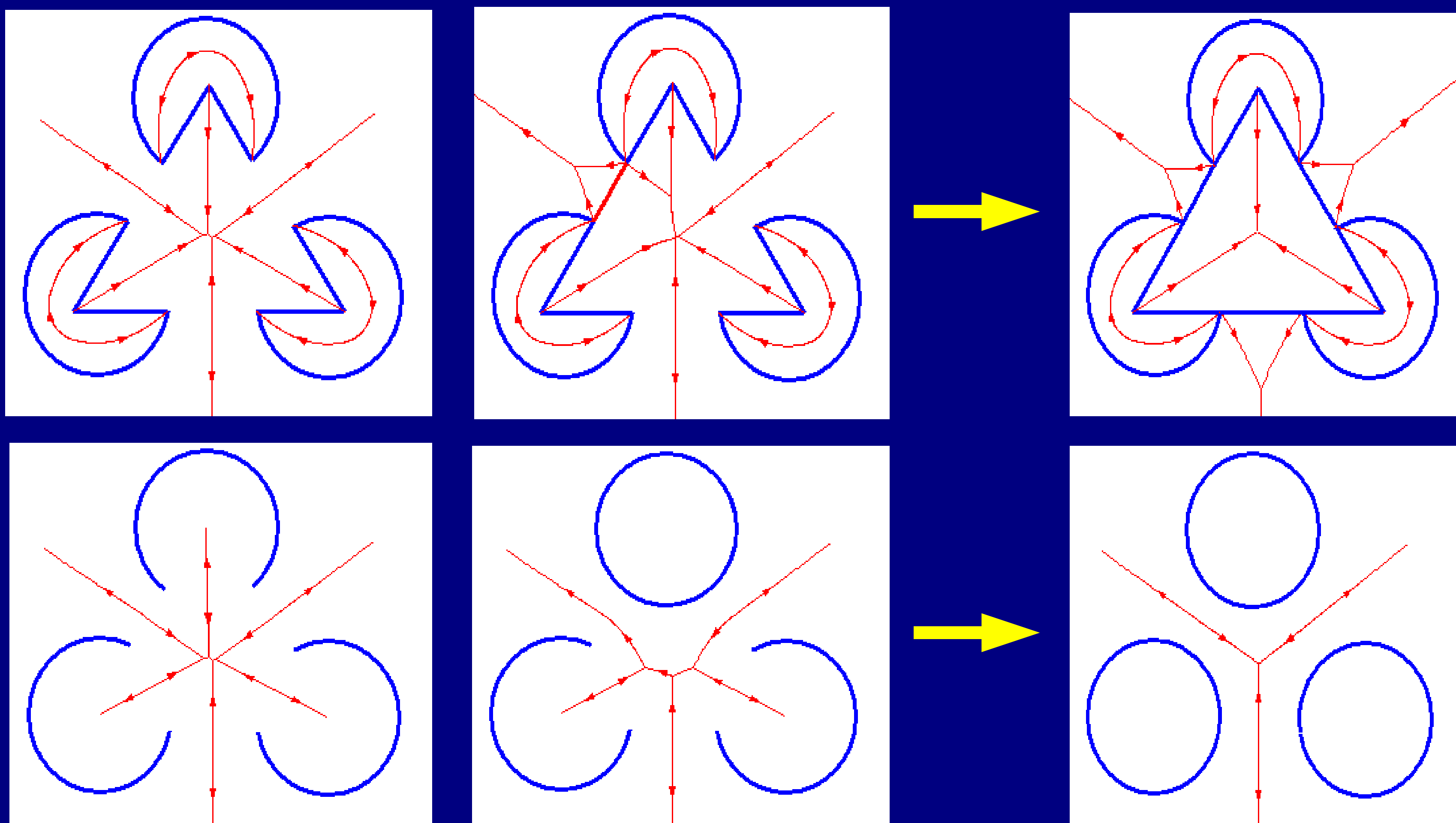


Kanisza triangle: 2D --> 3D



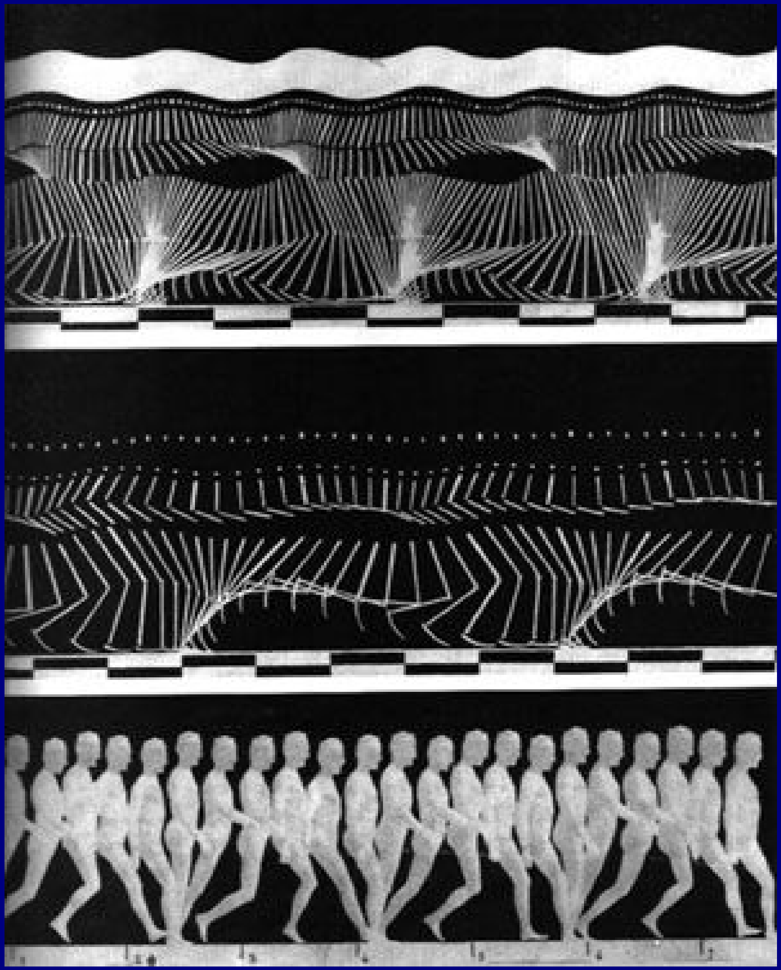
Perception





Kanisza triangle: Symmetry transforms --- Ben Kimia *et al.*

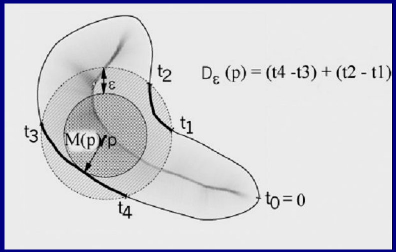
Perception --- Motion

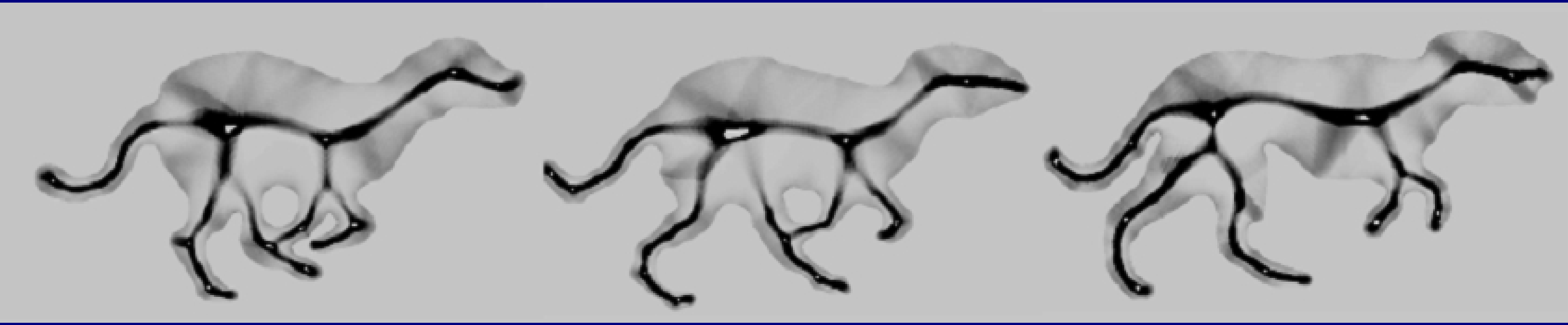


Etienne-Jules Marey --- Motion studies (1886)

M. Duchamp
Nude descending
a staircase
(1912)

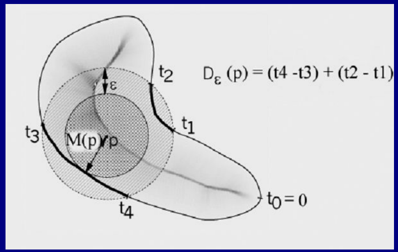
Perception --- Motion

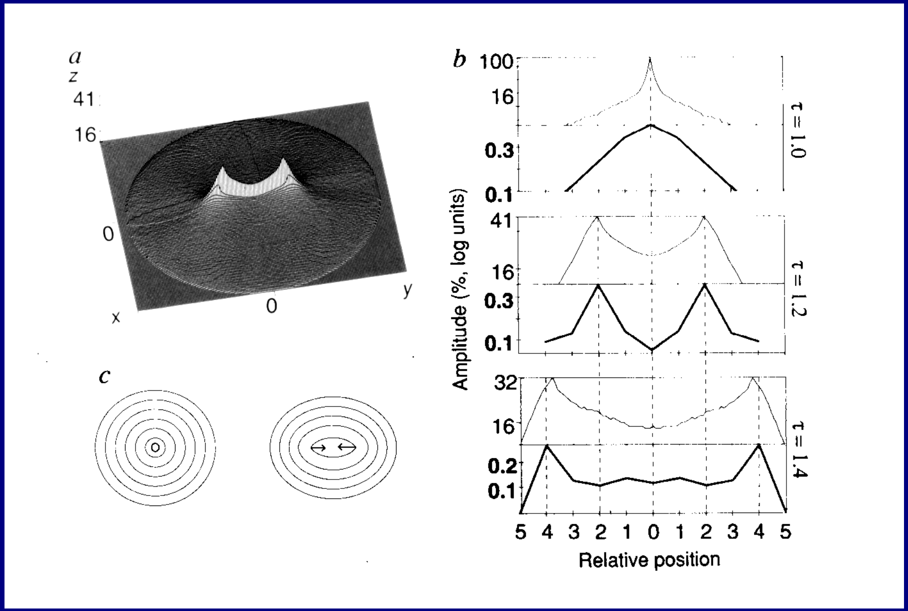


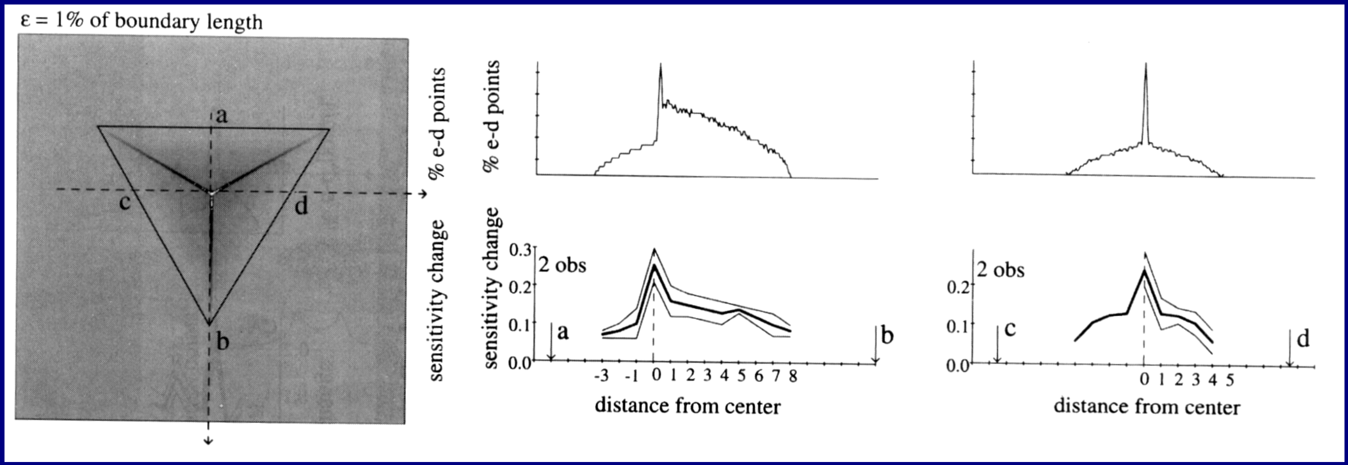


Sensitivity maps: I. Kovacs, B. Julesz *et al.*

Perception – Brain activity

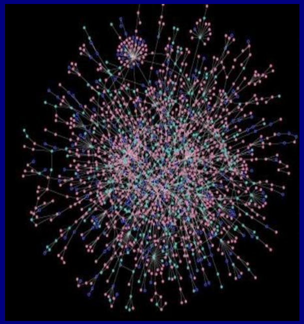
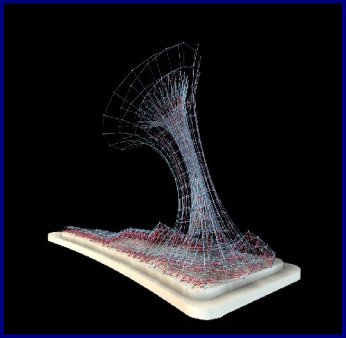
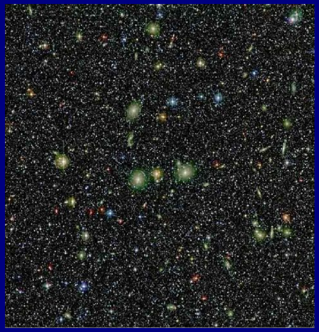






Sensitivity maps: I. Kovacs, B. Julesz *et al.*

Conclusions

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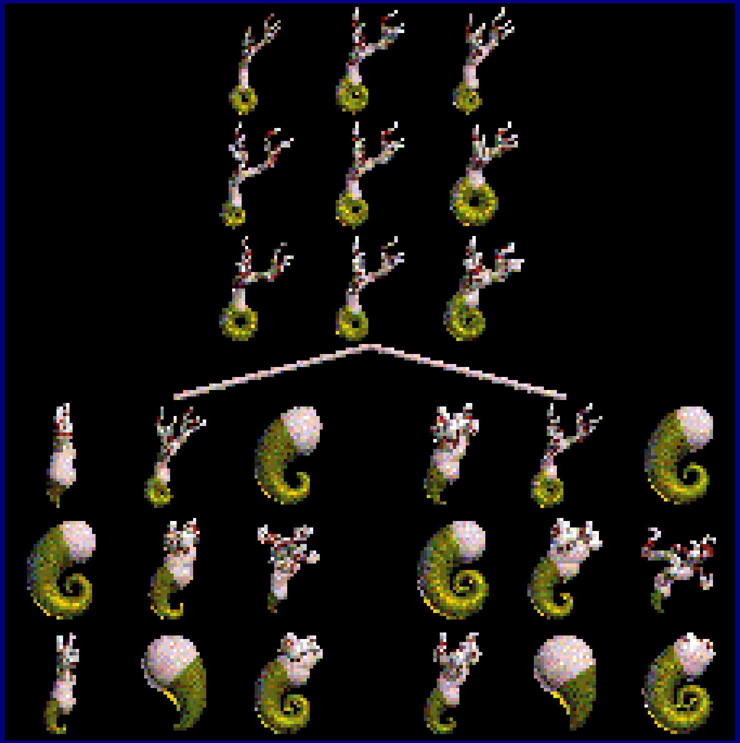
Special thanks to Rien v. d. Weygaert & Vincent Icke

Project *Mutations*



FormGrow: S. Todd & W. Latham (early 1990's)

Project *Mutations*

William Latham's organic art

Special thanks:

Michael Leyton (Rutgers), Ben Kimia (Brown)

The “US team” : Engineering, Applied Maths, Archaeology at Brown, the Mid Ocean studio

The “UK team” : Computing Dept. at Goldsmiths, Digital Studios (Robert Zimmer and Janis Jefferies), artists Patrick Tresset and William Latham, mathematician Peter Giblin.

Other collaborators: Franz-Erich Wolter (Hannover), Gert van Tonder (Kyoto), Liliana Albertazzi (Bolzano), ...

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