Using the Delaunay tessellation to compute the phase-space structure of dark-matter halos

Itai Arad[1], Avishai Dekel[1] and Anatoly Klypin[2]

[1]Racah Institute of Physics,
The Hebrew University,
Jerusalem, Israel

[2]Astronomy Department,
New Mexico State University,
New Mexico, USA

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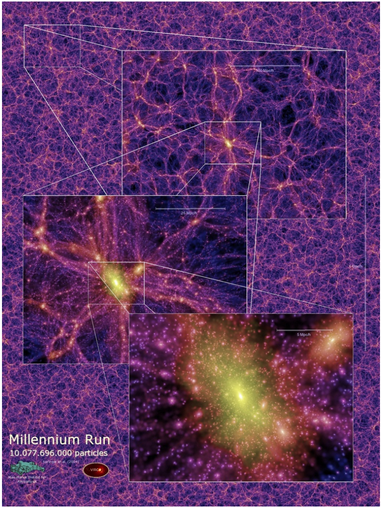
Outline

Cosmological N-body simulations



Provide many insights to the dynamics of Dark Matter (DM).

Yet also pose many open questions...

Open Questions

Universality: all DM halos seem to have (up to scaling) the same density profile (NFW)

$$\rho(r) \propto \frac{1}{r(1+r)^2}$$

- What is the exact inner slope (is there a cusp?)
- Physical mechanism? - Accretion? Violent Relaxation?
- What is the dependence on the cosmological model?
- Aditionally, Taylor & Navarro (2001) demonstrated

$$f_{poor\ man}(r) \stackrel{\text{def}}{=} \frac{\rho(r)}{\sigma^3(r)} \propto r^{-1.875}$$

So phase-space density is intersting! - but how does it come about?

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The Vlasov-Poisson Equation (Collisionless Boltzmann equation)

Gravity is an attractive long-range force. Therefore every DM particles only feel the mean-field gravitational potential and binary collisions are negligible.

$$\dot{\mathbf{x}}(t) = \mathbf{v}(t)\ , \quad \dot{\mathbf{v}}(t) = -\nabla \cdot \Phi(\mathbf{x})\ .$$

So if we define the *phase-space density function* $f(\mathbf{x}, \mathbf{v})$

$$f(\mathbf{x}, \mathbf{v}) d\mathbf{x} d\mathbf{v} = \text{how much mass in } d\mathbf{x} d\mathbf{v}$$

then it changes smoothly over time and we get a continuity equation - the Vlasov-Poisson equation:

$$\frac{d}{dt} f = \partial_t f + \nabla_x f - \nabla_x \Phi \cdot \nabla_v f = 0\ , \quad \nabla^2 \Phi(\mathbf{x}) = 4\pi^2 G \int d\mathbf{v}\, f(\mathbf{x}, \mathbf{v})$$

Phase-Space density is therefore the fundamental field in the problem!

Estimating $f(\mathbf{x}, \mathbf{v})$ numerically

In a hi-res simulation there are usually $10^4 \rightarrow 10^6$ particles in a halo. Phase-space density varies over 9 orders of magnitude!

Simple Box counting and other naive approaches do not work!

Our solution: use DTFE (Schaap & van de Weygaert, 2000)

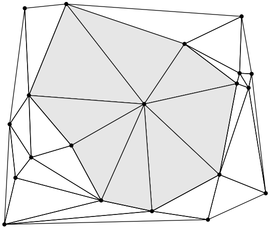
DTFE - Delaunay Tessellation Field Estimator

1. Estimate f at the location of every atom i:

$$f_i = \frac{7m_i}{\sum_\alpha |D_\alpha|}$$

2. Estimate the average f for each Delaunay cell:

$$f_\alpha = \frac{1}{7}\sum_i f_i$$



Global quantities

$$\int \Psi(\mathbf{x},\mathbf{v}) d\mathbf{x} d\mathbf{v} \rightarrow \sum_\alpha \Psi(\mathbf{x}_\alpha,\mathbf{v}_\alpha)|D_\alpha|$$

Technical Difficulties

Consider a typical sample with 10^6 particles in 6D...

- Each particles has about $7,000$ Delaunay cells around it
- With 200 neighboring particles
- Totally, there are about 10^9 Delaunay cells in the system.
- If each cell is represented by the indices of its 7 particles then we need a total of

$$7 \times 4 \times 10^9 \text{ bytes} = 28\text{GB}$$

just to store the tessellation.

conclusion

We need a program the caches some of the information to the disk, and possibly makes some of the analysis while finding the Delaunay cells.

The implementation - SHESHDEL

- Is a free software (GNU GPL).
- Written in C. Sequential algorithm. Based on a sequntial algorithm of Tanemura, Ogawa & Ogita (1983) + improvements due to van de Weygaert (1994) (mainly use of $k-d$ trees).
- The program goes particle by particle, finding all its Delaunay cells. After every K paritcles, data is cached to/from the disk.
- Once all calculations of a given cell have been done - it is no longer kept.
- Suitable for running on an ordinary PC. For example, 10^6 particles will take about 3 days on a recent laptop (Intel Pentium M 1.7 GHz with 1GB RAM), and will take 7GB disk-space.
- Fairly modular and extendiable.

Results - the $v(f)$ function

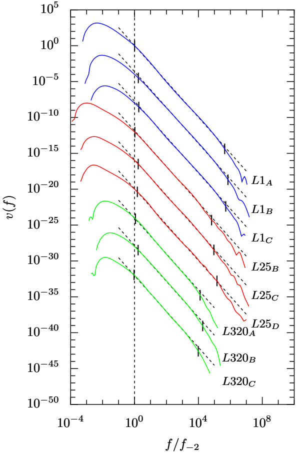
We first looked at the $v(f)$ function:

$$v(f_0) \stackrel{\text{def}}{=} \int d\mathbf{x} d\mathbf{v}\, \delta[f(\mathbf{x}, \mathbf{v}) - f_0]$$

$v(f_0)df_0$ - The volume occupied by $f_0 < f(\mathbf{x}, \mathbf{v}) < f_0 + df_0$

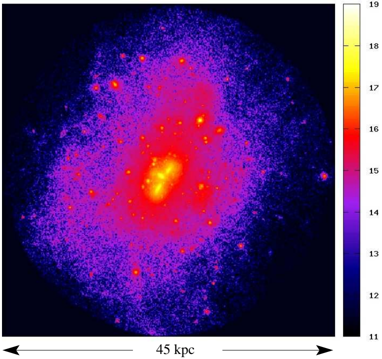
Very well described by a power-law $f^{-2.5}$

However, a smooth halo with $v(f) \propto f^{-2.5}$ implies $\rho(r) \propto r^{-2}$!



Results - substructure

Looking at a typical halo



Substructure is much more pronounced in phasespace - due to its small velocity dispersion

Results - substructure

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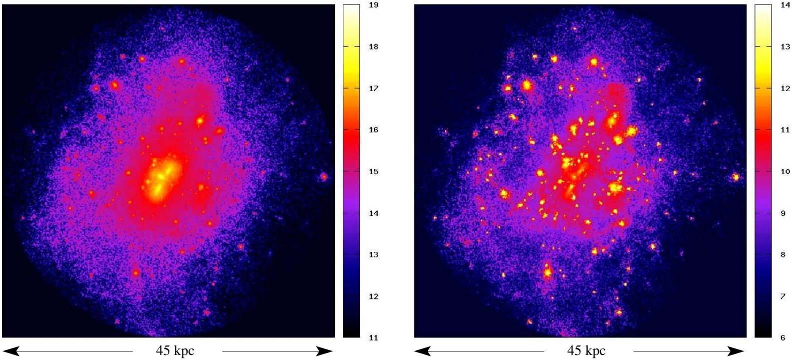


Substructure is much more pronounced in phasespace - due to its small velocity dispersion

Phase-space density

I. Arad, A. Dekel, A Klypin

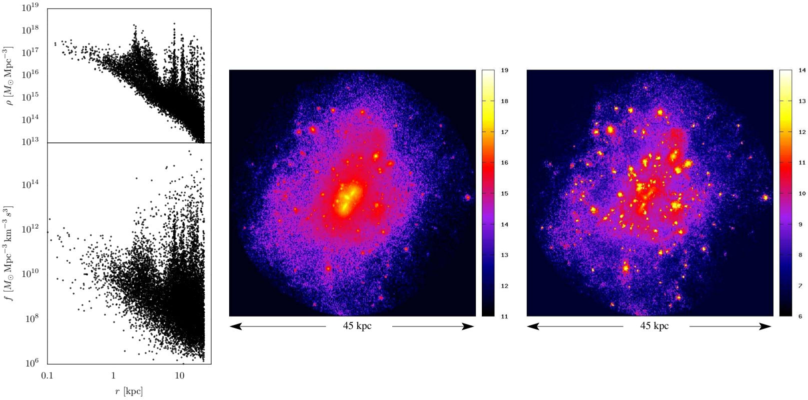
Motivation
- N-body simulations
- The Vlasov-Poisson Equation

Our Work
- Estimating $f(\mathbf{x},\mathbf{v})$ from an N-body simulation
- Results

Problems and Prospects

Results - substructure

Looking at a typical halo



So the $v(f) \propto f^{-2.5}$ power-law does not reflect the smooth background structure - but instead the distribution of substurcture.

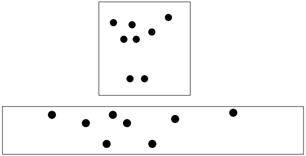
Problems and Prospects

- Velocity vs. Position scaling
- Fluctuations and smoothing
- 6D structure finding

Velocity Vs. Position

The problem

There does not seem to be any natural dimensional scaling between velocity and position. Using different units leads to different tessellations

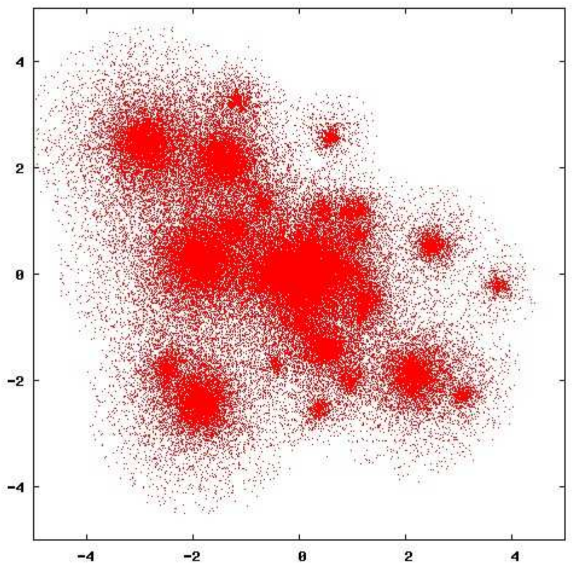


Possible Solutions

- Scale v, x by the mean dispersion in the halo (not optimal).
- Use local metric, by either local dispersion, or some local timescale $T = \mathbf{v}/\nabla\Phi(\mathbf{x})$ - computationally very hard. Is really needed?
- Use a global canonical transformation ($\mathbf{x}, \mathbf{v}$ are actually conjugate)

$$X = \frac{\partial F(V,x)}{\partial V} \;, \quad v = -\frac{\partial F(V,x)}{\partial x} \;.$$

Fluctuations and Smoothing
Comparison with FiEstAS (Ascasibar Y., Binney J., 2005)



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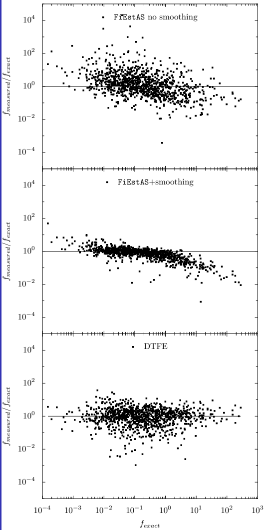
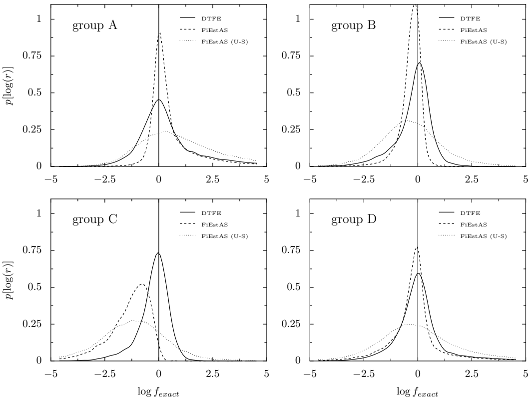




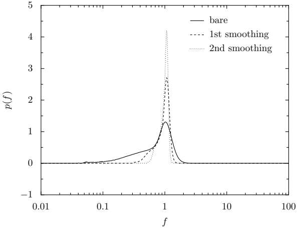
Fluctuations and Smoothing

Comparison with FiEstAS (Ascasibar Y., Binney J., 2005)

- DTFE is much slower than FiEstAS, but gives better results at high densities.
- To be more attractive it needs a smoothing scheme.
- One possibility - hierarchy of smoothing:

$$f_i^{(0)} = \frac{7m_i}{\sum_\alpha |D_\alpha|} \qquad f_\alpha^{(0)} = \frac{1}{7}\sum_i f_i^{(0)} ,$$

$$f_i^{(1)} = \frac{\sum_\alpha |D_\alpha| f_\alpha^{(0)}}{\sum_\alpha |D_\alpha|} \qquad f_\alpha^{(1)} = \frac{1}{7}\sum_i f_i^{(0)} ,$$



- Conserves mass, but produces large biases - Perhaps because of boundary effects.

Structure finder in phase-space

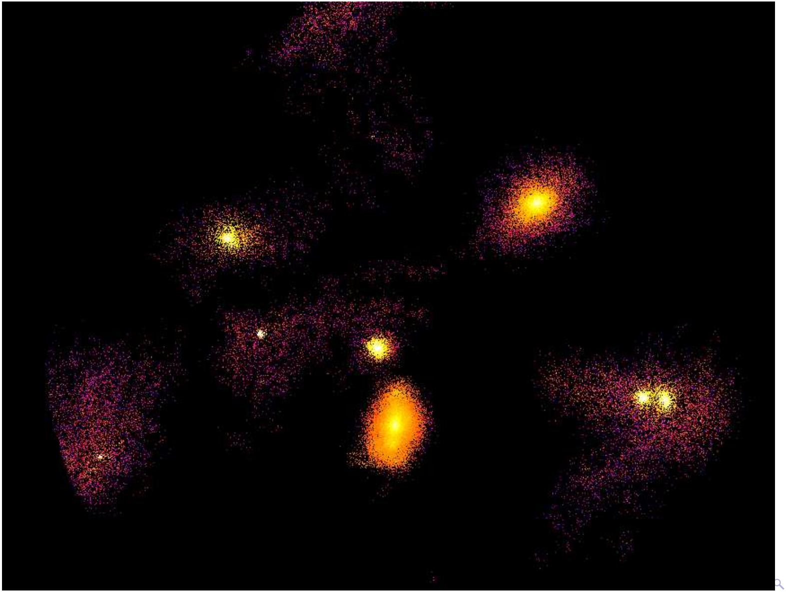
We can use the Delaunay mesh to find substructure in haloes (work in progress by Michael Maciejewski, IAP)

- Find a local maxima of $f(\mathbf{x},\mathbf{v})$ (all neighboring particles have lower density)
- Look recursively at all its neighbors.
- Process stops below a given density threshold, or some more sophisticated condition.

Advantages

- Fewer free parameters.
- Greater contrast of substructure density in phase-space.
- Ability to detect streams - remnants of merger events.

Structure finder in phase-space



Conclusions

- Understanding the phase-space structure of dark-matter halos in crucial to understanding the physics behind their structure. Much work remains in this area.
- The DTFE can be used to estimate the phase-space density. However, it is a very expansive process which also involves conceptual difficulties (e.g. position vs. velocity) as well as technical difficulties (e.g. smoothing the fluctuations).
  It is still not clear that this is the preferred method.
- Substructure in DM halos is much more pronounced in phase-space than in real-space. This gives rise to interesting applications in structure finding.
- The scale invariance in $v(f)$ is due to the distribution of substructure - and not due to the smooth background. The relation $f(r) = \frac{\rho(r)}{\sigma^3(r)} \propto r^{-1.875}$ is still not understood.