Tessellations in Wireless Communication Networks: Voronoi and Beyond it

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The World a Jigsaw: Tessellations in the Sciences

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OUTLINE

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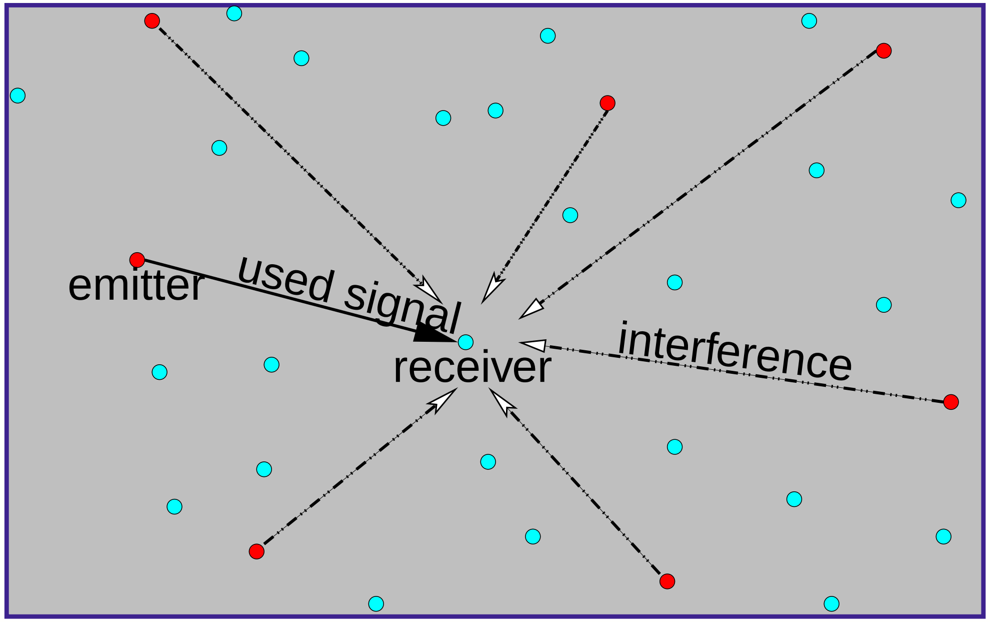
IV Power control in CDMA: Evaluating capacity of some Voronoi architecture.

I INTRODUCTION TO WIRELESS COMMUNICATION

- Physical layer,
- Multiple access,
- Network layer.

Physical layer

- Emitter sends information emitting power P_e.



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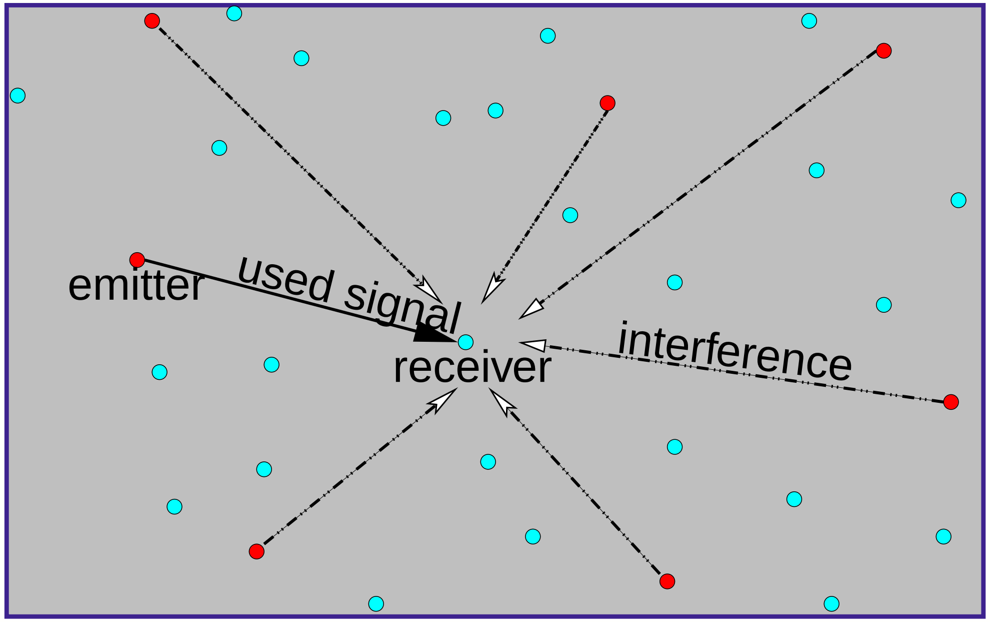

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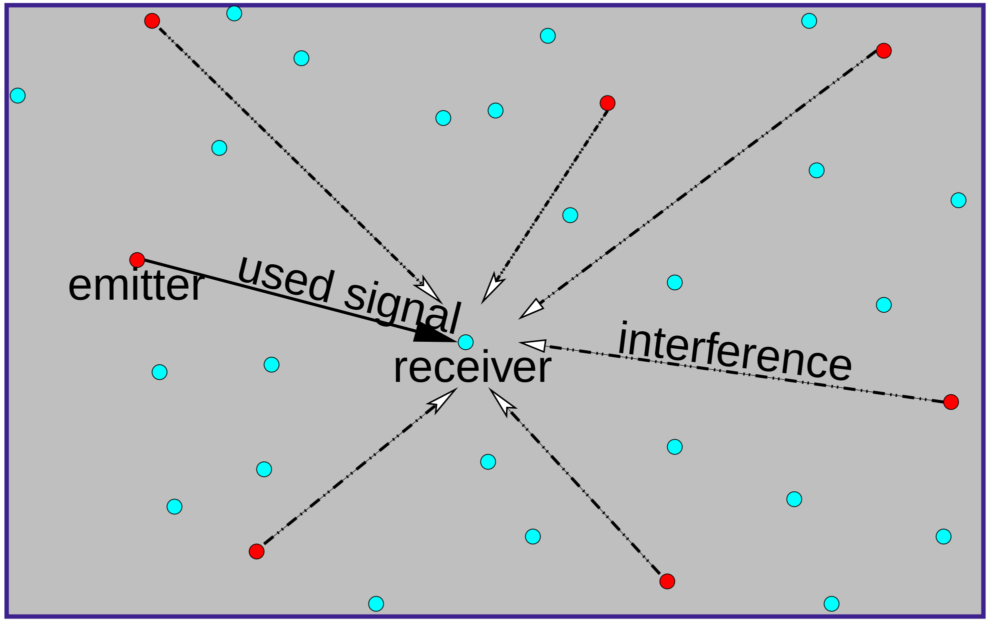

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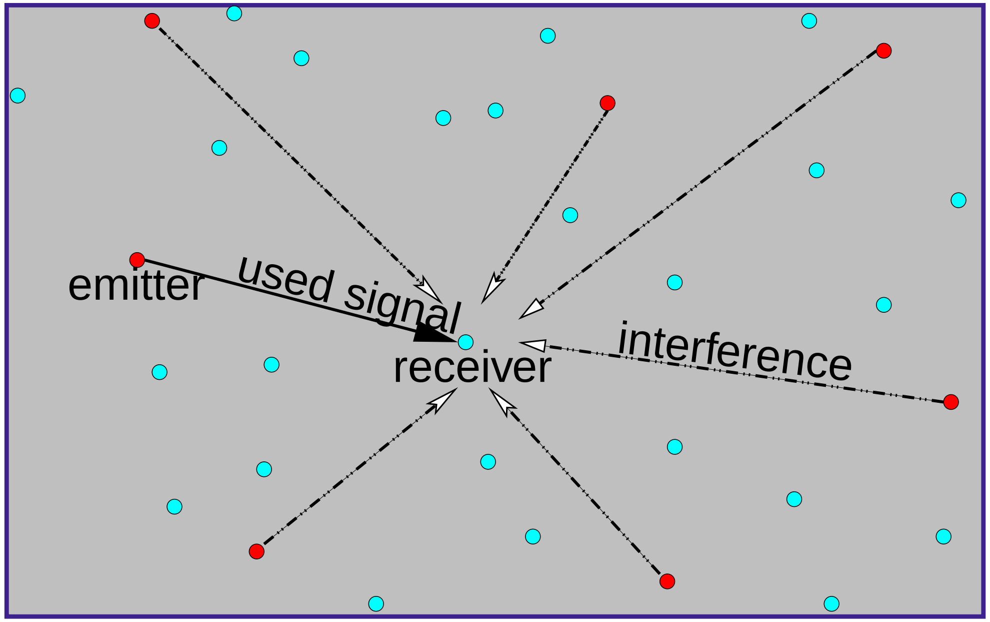
  $\text{SINR} = P_r/(N+I) > \text{threshold}\, T$.



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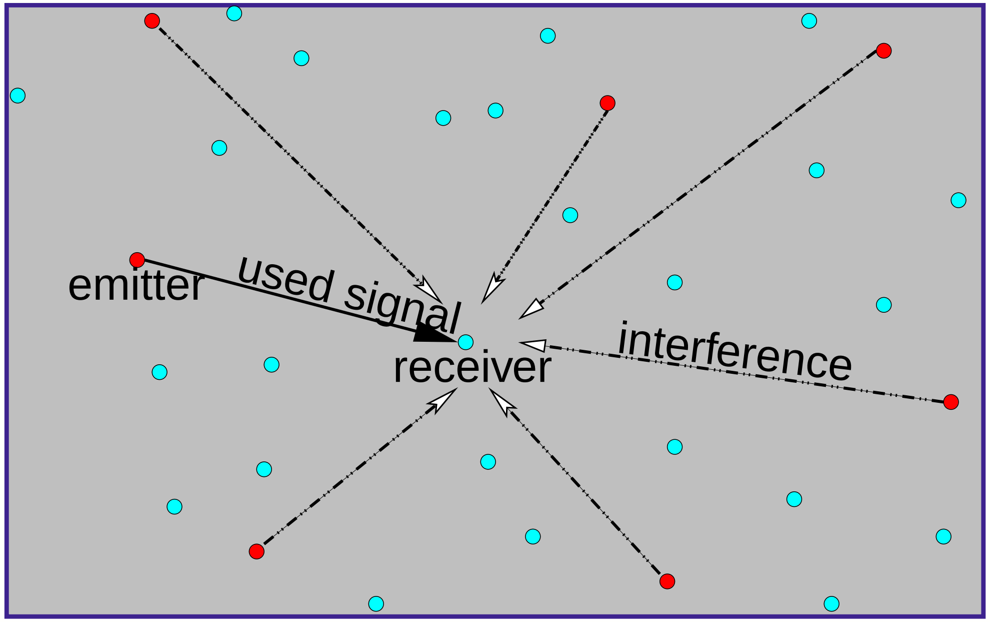

- T is related to the bitrate (amount of information transmitted per unit time); it depends on coding used, exist information-theoretic bounds (Shannon theorem).

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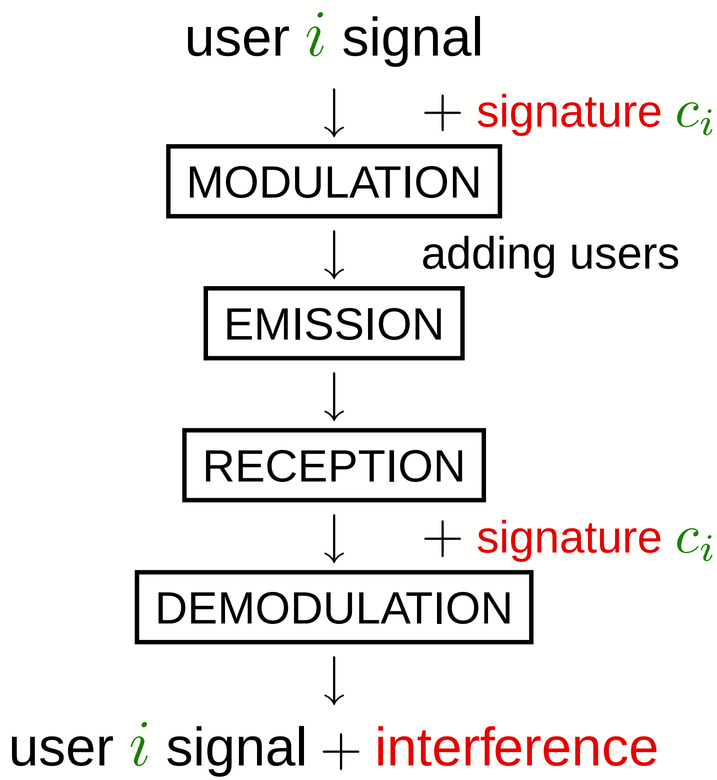
- TDMA: Time division multiple access; different channels get different time-slots of a given unit time interval (technology of the present GSM).
- CDMA: Code division multiple access; different channels get different (pseudo)-orthogonal codes to modulate their signals with (technology of the arriving UMTS).

CDMA basic principles

- Single channel with a bandwidth of 1.25 MHz; All users transmit simultaneously on this single channel.



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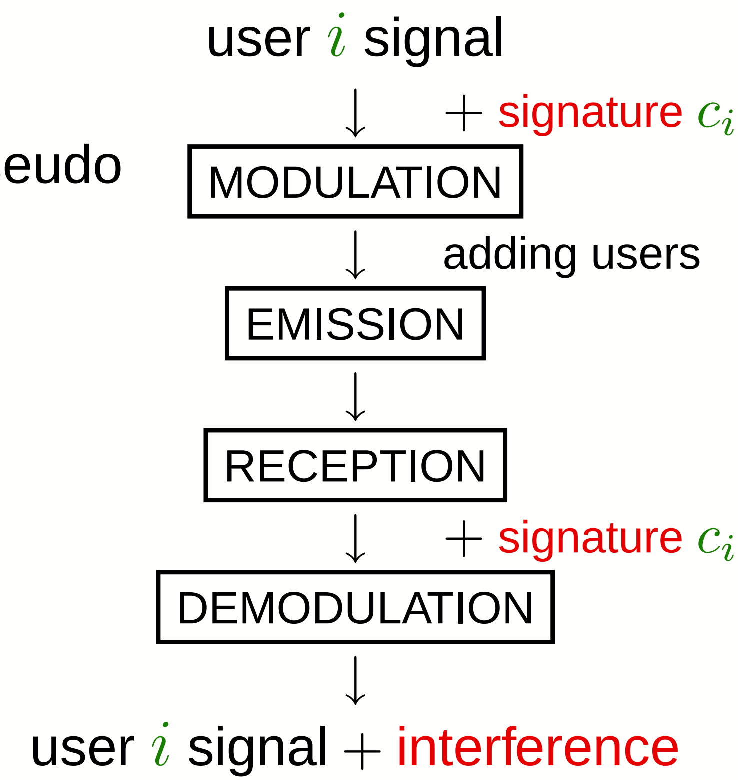

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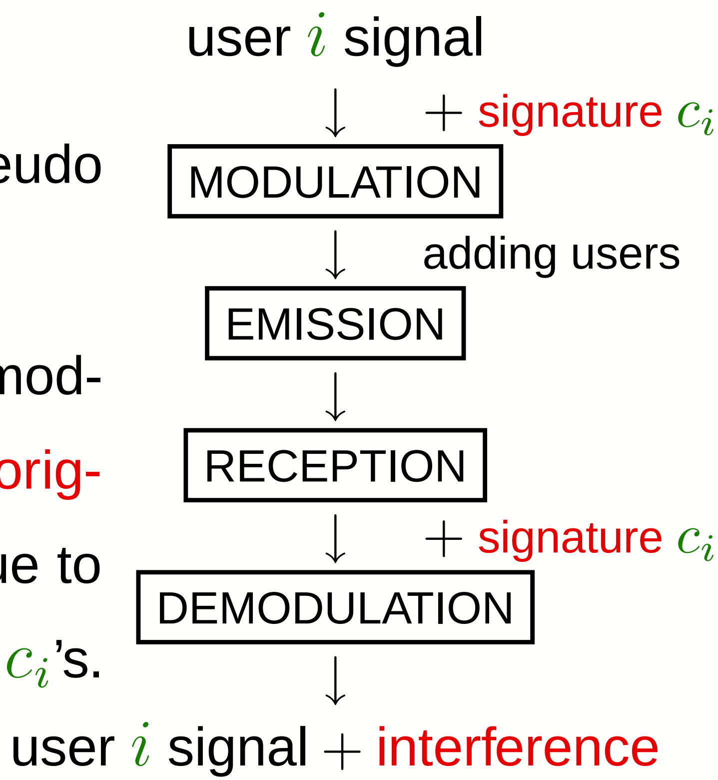
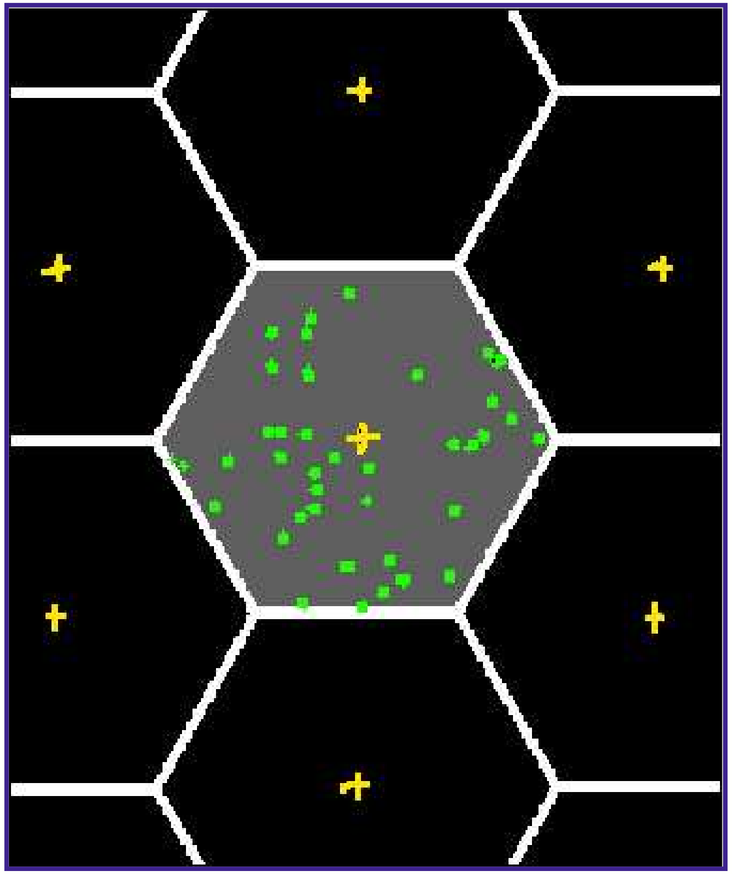
- The signature process c_i is used by the receiver to modulate the total received signal. This gives back the original signal of user i plus some (Gaussian) noise due to the lack of perfect orthogonality between signatures c_i's. This is the interferences.



Network layer

Cellular networks: Infrastructure of base stations or access points provided by an operator. Individual users talk to these stations and listen to them.

regular



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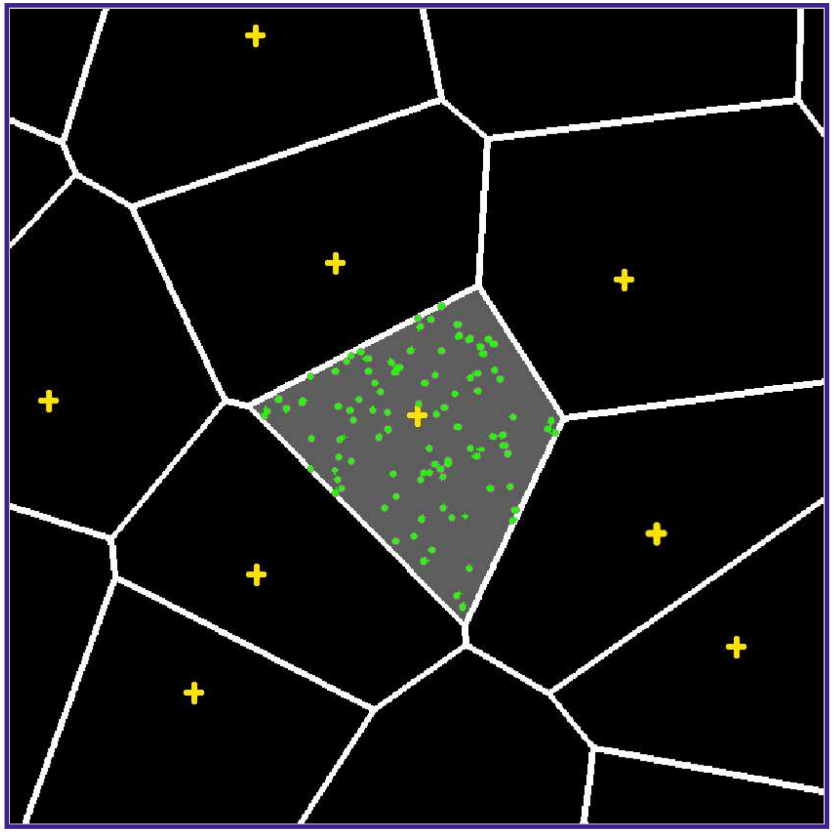

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- How do the cells really look like?

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- Evaluate Quality-of-Service characteristics of a "typical user" (e.g. call blocking probability).

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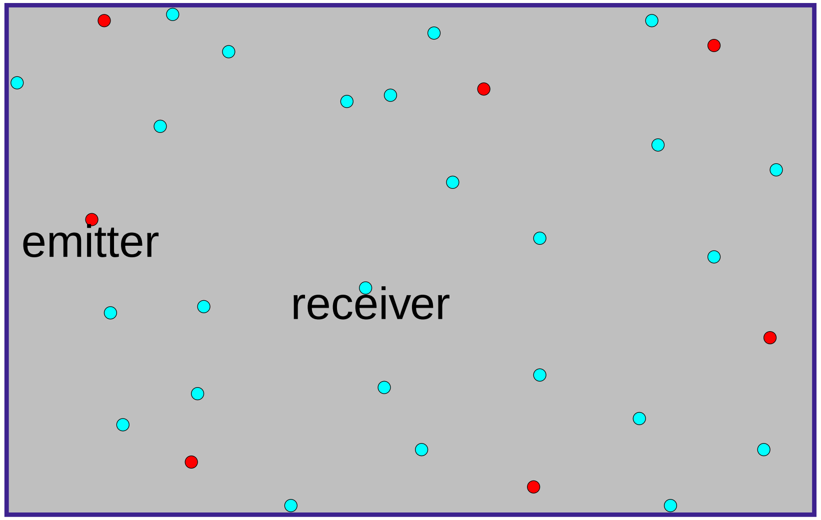

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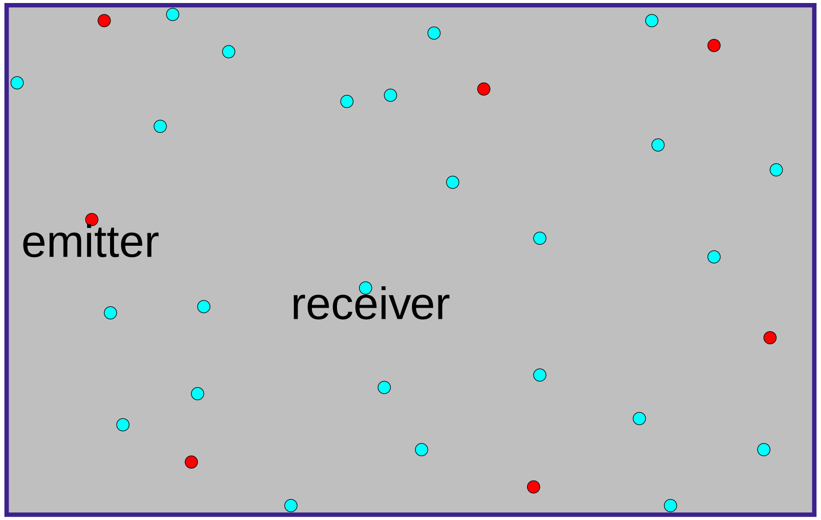

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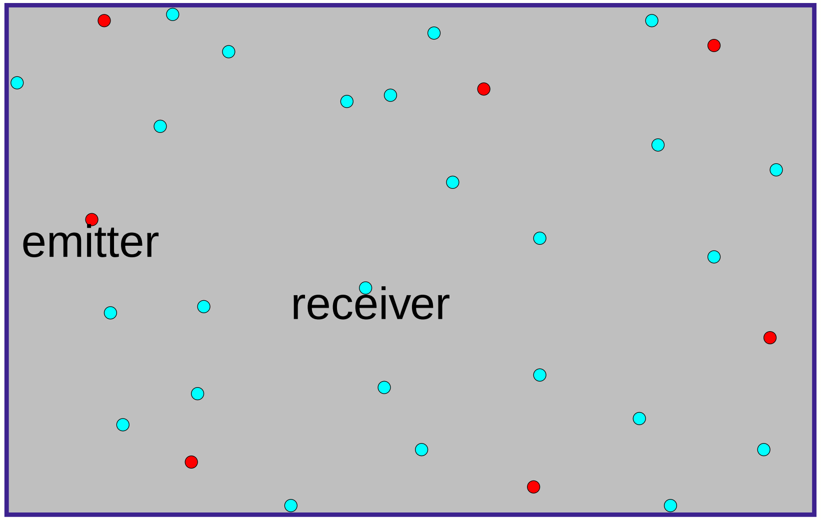

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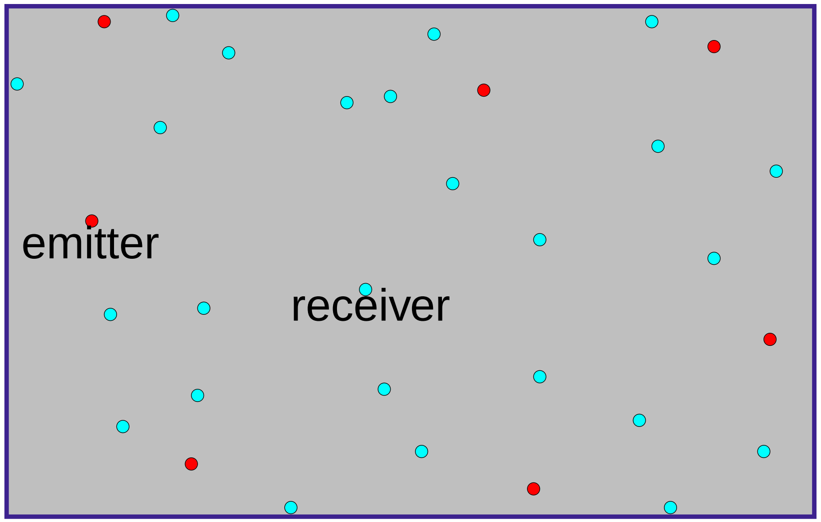
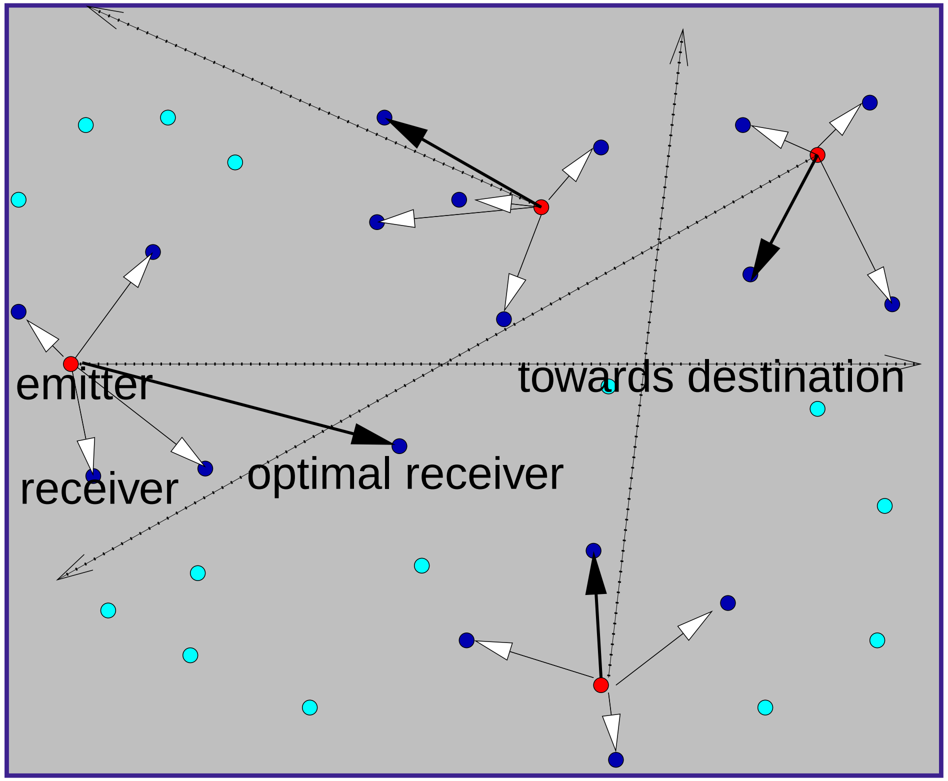
- Users switch between emitter and receiver modes.



- Emitter sends a packet in some given direction far away via several hops.



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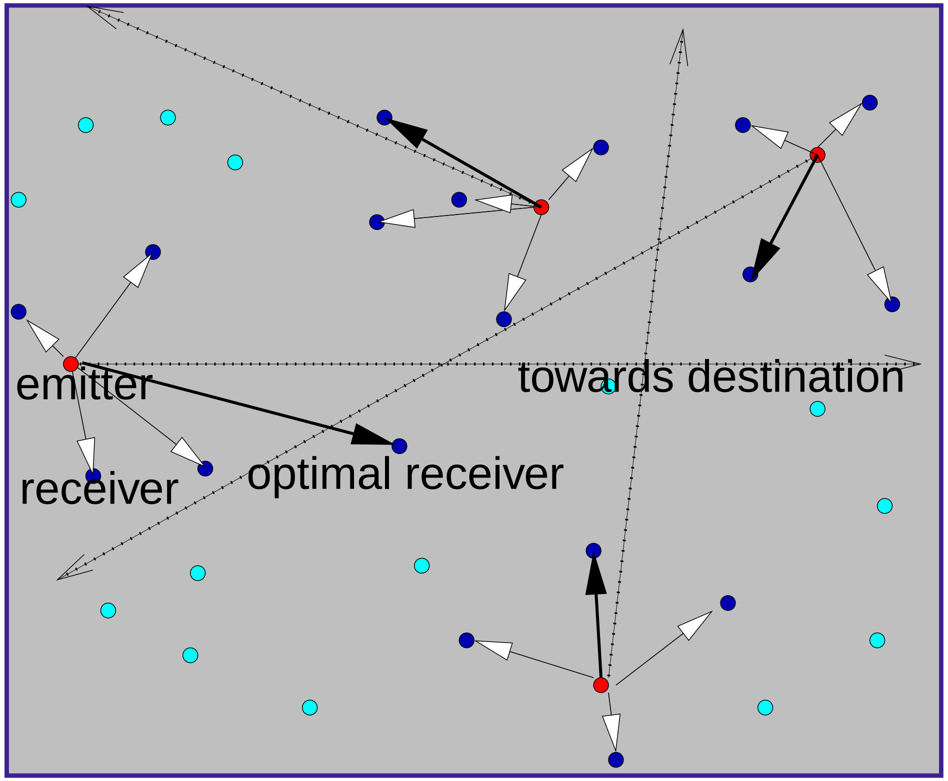

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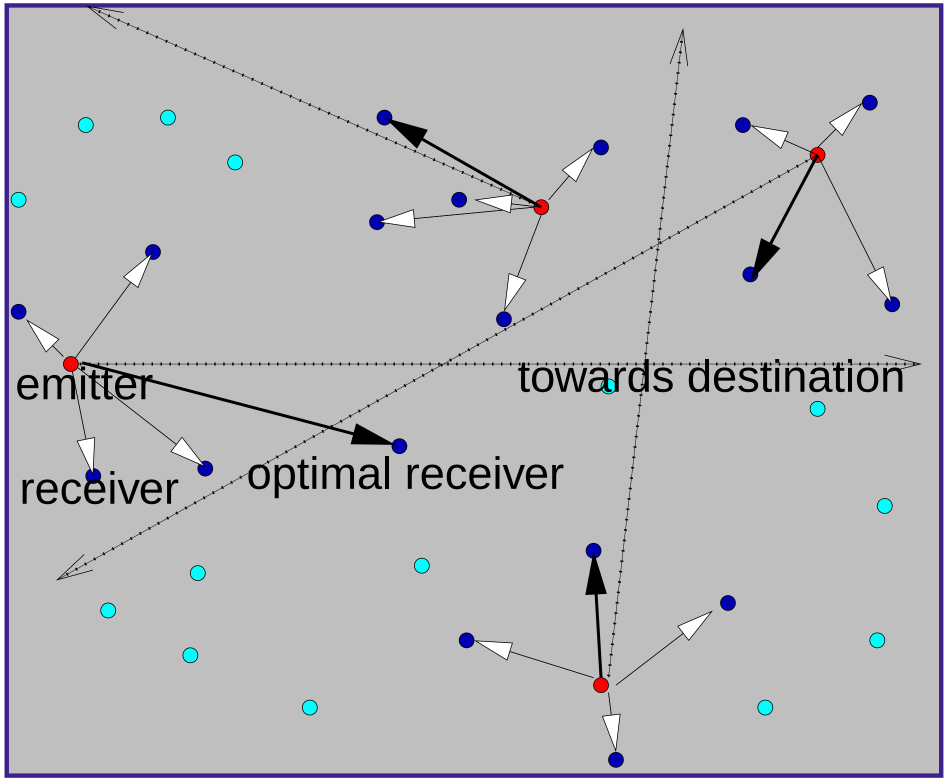

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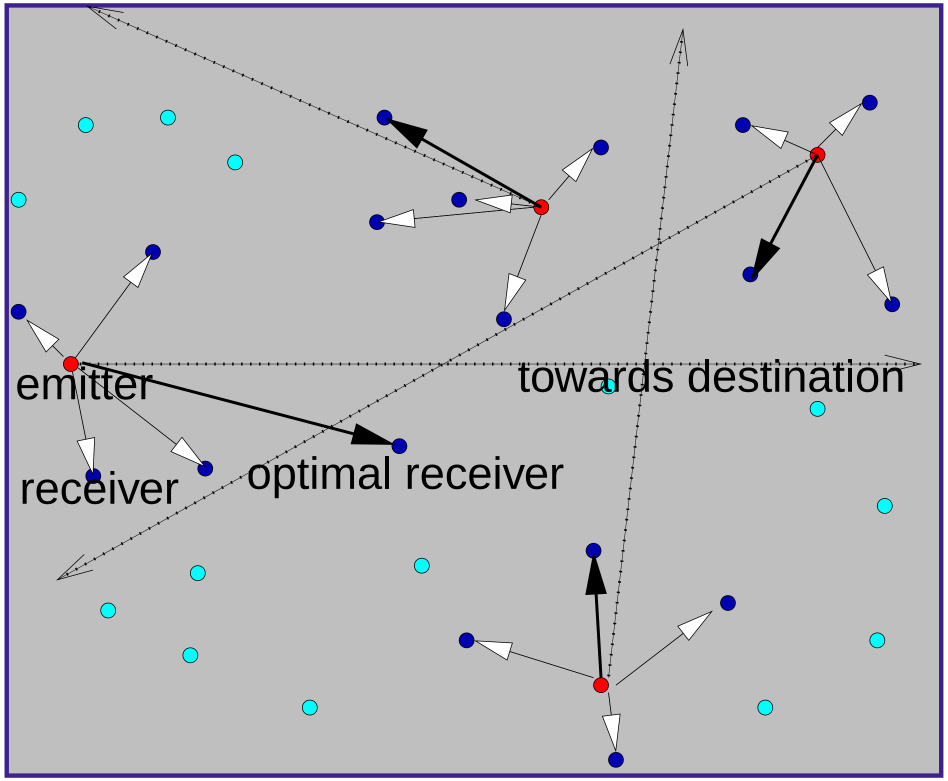
- In the case of no reception, emitter re-emits the packet next authorized time.



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- Capacity: How much own traffic every node can send, given it has to relay traffic of other nodes?

Sensor networks: Variants of ad-hoc networks.

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- They send collected information in an ad-hoc manner to some “sink” locations.

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- Issues: Coverage, connectivity, energy (batery) saving.

II BASIC GEOMETRIC MODELS

- Poisson point process,
- Voronoi tessellation and Delaunay graph,
- Boolean model,
- Shot-Noise model.

Poisson Point Process

Planar Poisson point process (p.p.) Φ of intensity λ:

- Number of Points $\Phi(B)$ of Φ in subset B of the plane is Poisson random variable with parameter $\lambda|B|$, where $|\cdot|$ is the Lebesgue measure on the plane; i.e.,

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Laplace transform of the Poisson p.p.

$$\mathcal{L}_\Phi(h) = \mathbf{E}\big[e^{\int h(x)\,\Phi(\mathrm{d}x)}\big] = e^{-\lambda\int(1-e^{h(x)}),\mathrm{d}x}\,,$$

where $h(\cdot)$ is a real function on the plane and $\int h(x)\,\Phi(\mathrm{d}x) = \sum_{X_i\in\Phi} h(X_i)$.

Poisson p.p. is the basis of the stochastic-geometry modeling of communication networks.

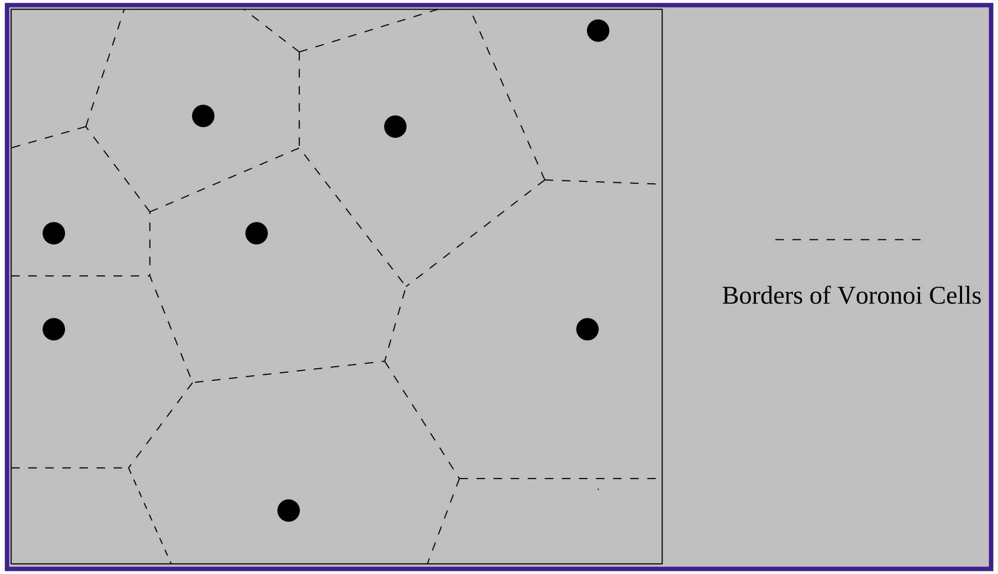
This modeling consist in treating the given architecture of the network as a snapshot of a (homogeneous) random model, and analyzing it in a statistical way. In this approach the physical meaning of the network elements is preserved and reflected in the model, but their geographical locations are no longer fixed but modeled by random points of, typically, homogeneous planar Poisson point processes.

Consequently, any particular detailed pattern of locations is no longer of interest. Instead, the method allows for catching the essential spatial characteristics of the network performance basically through the densities of these point processes (i.e., the densities of the network devices).

Voronoi Tessellation (VT) and Delaunay graph

Given a collection of points $\Phi = \{X_i\}$ on the plane and a given point x, we define the Voronoi cell of this point $\mathcal{C}_x = \mathcal{C}_x(\Phi)$ as the subset of the plane of all locations that are closer to x than to any point of Φ; i.e.,

$$\mathcal{C}_x(\Phi) = \{y \in \mathbb{R}^2 : |y - x| \le |y - X_i| \; \forall X_i \in \Phi\} \,.$$



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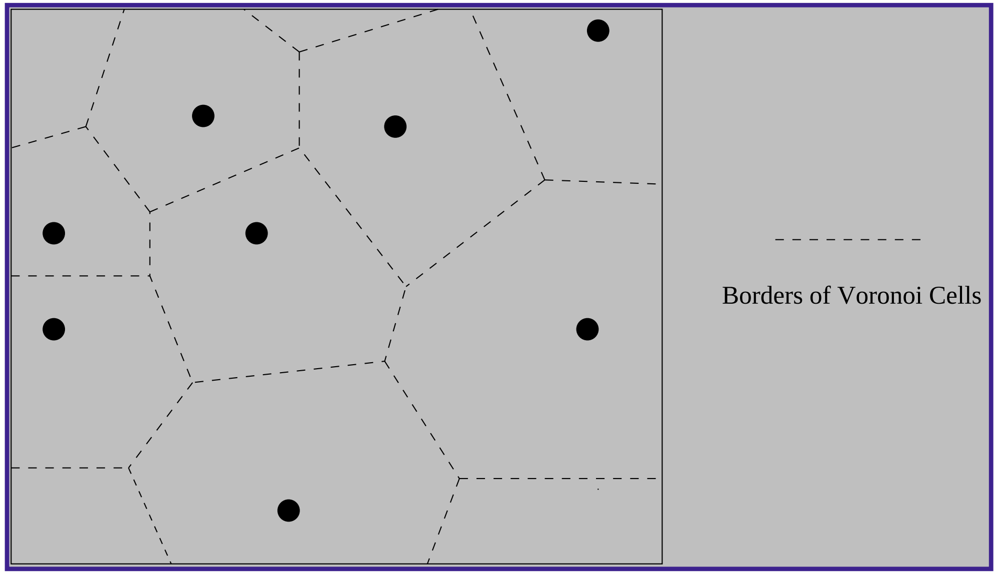
When $\Phi = \{X_i\}$ is a Poisson p.p. we call the (random) collection of cells $\{\mathcal{C}_{X_i}(\Phi)\}$ the Poisson-Voronoi tessellation (PVT).



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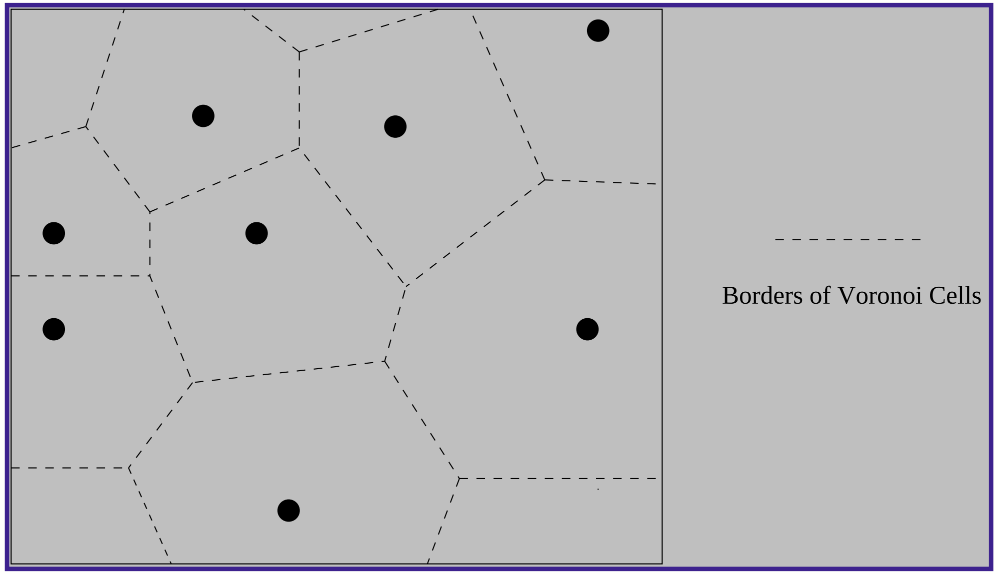


VT is a frequently used generic model of tessellation of the plane.

Points denote locations of various structural elements (devices) of the network (base station antennas and/or network controllers in cellular networks, concentrators in fixed telephony, access nodes in ad hoc networks, etc.).

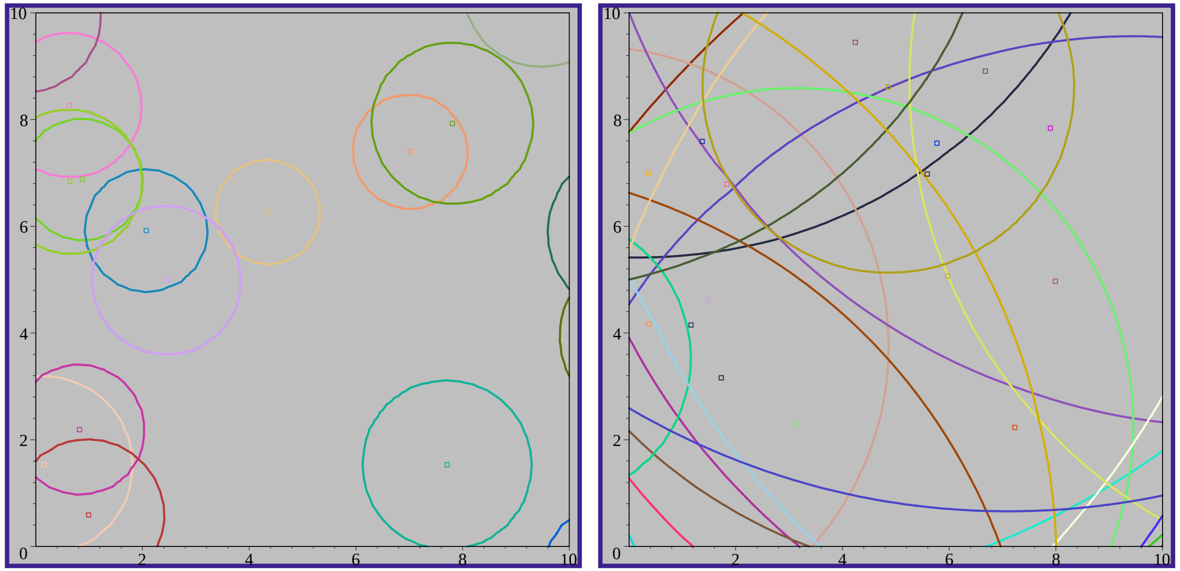
Cells denote mutually disjoint regions of the plane served in some sense by these devices.

Delaunay graph is a “protocol” model of the neighbourhood.

Boolean Model (BM)

Let $\tilde{\Phi} = \{(X_i, G_i)\}$ be a marked Poisson p.p., where $\{X_i\}$ are points and $\{G_i\}$ are iid random closed stets (grains). We define the Boolean Model (BM) as the union

$$\Xi = \bigcup_i X_i \oplus G_i \qquad \text{where } x \oplus G = \{x + y : y \in G\}.$$



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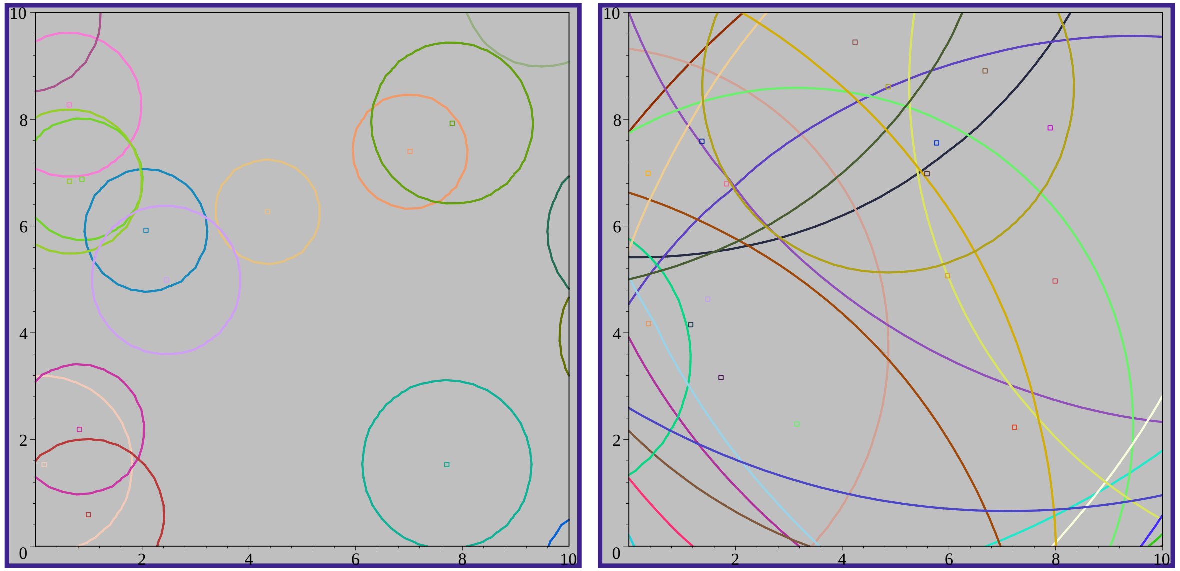

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Known:

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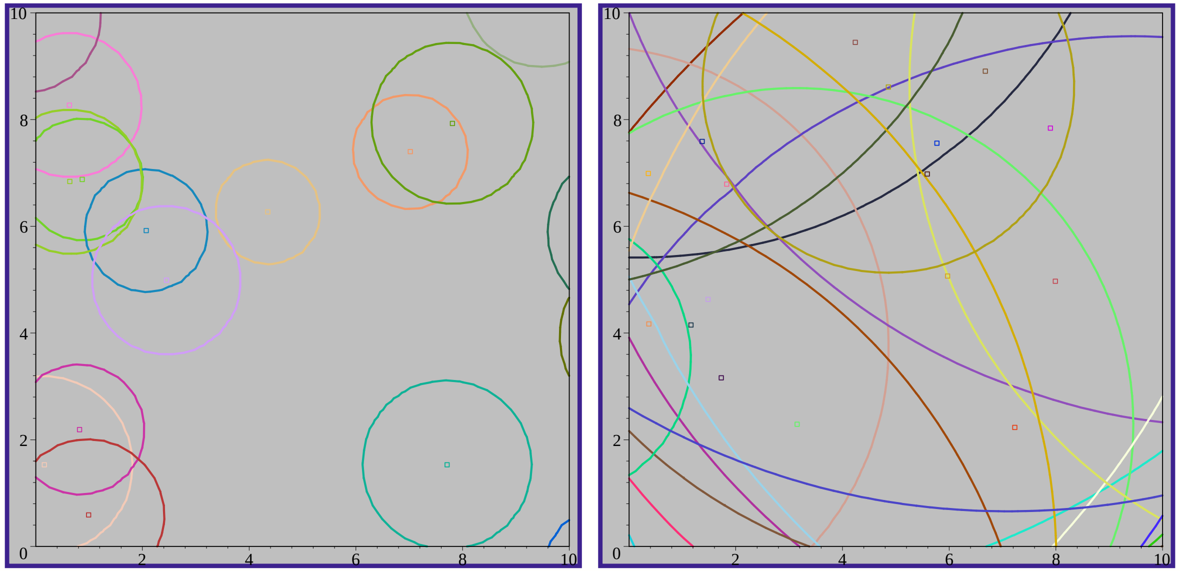
- Asymptotic results ($\lambda \to \infty$) for the probability of complete covering of a given set.



BM with spherical grains of random radii

BM is a generic coverage model.

Points denote locations of various structural elements (devices) of the network.

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In wireless networks it is a simplified model for the study of coverage and connectivity. It takes into account transmission regions, but it ignores the interference effect.

Shot-Noise (SN) model

Let $\tilde{\Phi} = \{(X_i, S_i)\}$ be a marked p.p., where $\{X_i\}$ are points and $\{S_i\}$ are iid random variables. Given a real response function $L(\cdot)$ of the distance on the plane we define the Shot-Noise field

$$I_{\tilde{\Phi}}(y) = \sum_i S_i L(y - X_i)\,.$$

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When $\tilde{\Phi}$ is a marked Poisson p.p. then we call $I_{\tilde{\Phi}}$ the Poisson SN.

For the Poisson SN, the Laplace transform of the vector $(I_\Phi(y_1), \ldots, I_\Phi(y_n))$ is known for any $y_1, \ldots, y_n \in \mathbb{R}^2$ (via Laplace transform of the Poisson p.p.).

SN is a good model for interference in wireless networks.

Marks S_i correspond to emitted powers.

Response function correspond to attenuation function.

III SINR COVERAGE MODEL

In between Voronoi and Boolean

$\Phi = \{X_i, (S_i, T_i)\}$ marked point process (Poisson)

$\{X_i\}$ points of the p.p. on $\mathbb{R}^2$ — antenna locations,

$(S_i, T_i) \in (\mathbb{R}^+)^2$ possibly random mark of point X_i — (power,threshold)

cell attached to point X_i: $C_i(\Phi, W) = \left\{ y : \frac{S_i l(y - X_i)}{W + \kappa I_\Phi(y)} \geq T_i \right\}$

where $I_\phi(y) = \sum_{i \neq 0} S_i l(y - X_i)$ shot noise process, κ interference factor, $W \geq 0$ external noise, $l(\cdot)$ attenuation (response) function.

C_i is the region where the SINR from X_i is bigger than the threshold T_i.

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Coverage PROCESS: $$\Xi(\Phi; W) = \bigcup_{i \in \mathbb{N}} C_i(\Phi, W).$$

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Motivation I: CDMA handoff cells

x_0 — a point in $\mathbb{R}^2$ (location of an antenna),

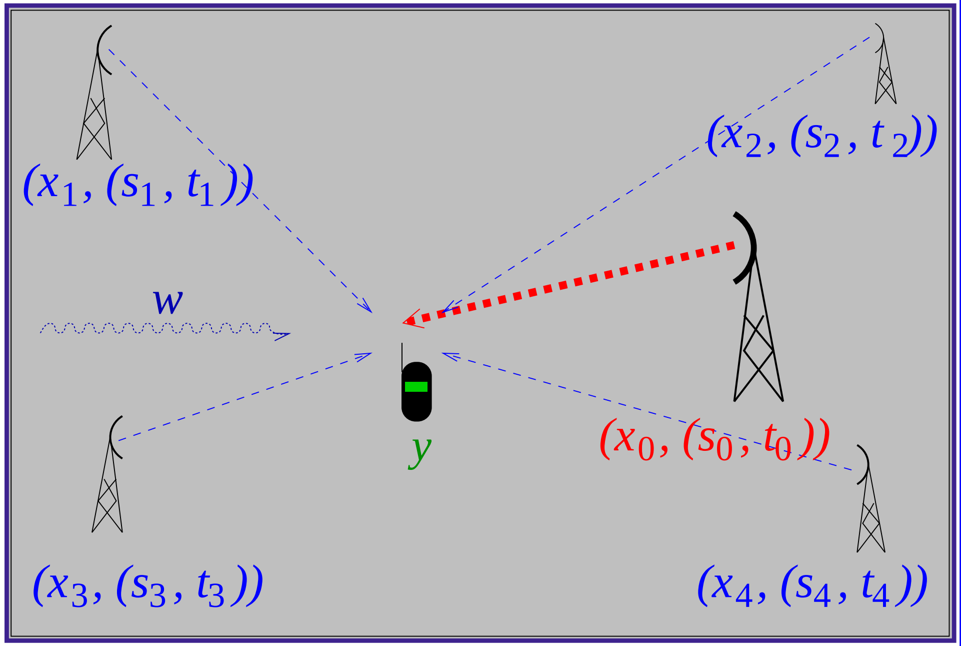
$s_0 \geq 0$ and $t_0 \geq 0$ — (pilot signal power of the antenna and SINR threshold (bit energy-to-noise spectral power density $E_b/\mathcal{N}_O$) for the pilot signal),

$\phi = \{x_i, (s_i, t_i)\}$ — pattern of antennas,

$w \geq 0$ — external noise,

$0 \leq \kappa \leq 1$ — orthogonality factor,

$l(\cdot)$ — attenuation function



$$\frac{s_0 l(y - x_0)}{w + I_\phi(y)} \geq t_0$$

Parameter values

Intensity of Poisson process of base stations $\lambda_{BS} \sim 0.2$ BS/km^2.

Pilot signal power $s_0 \sim 30$ mW

SINR threshold (bit energy-to-noise spectral power density $E_b/\mathcal{N}_O$) for the pilot $t_0 \sim -14$ dB

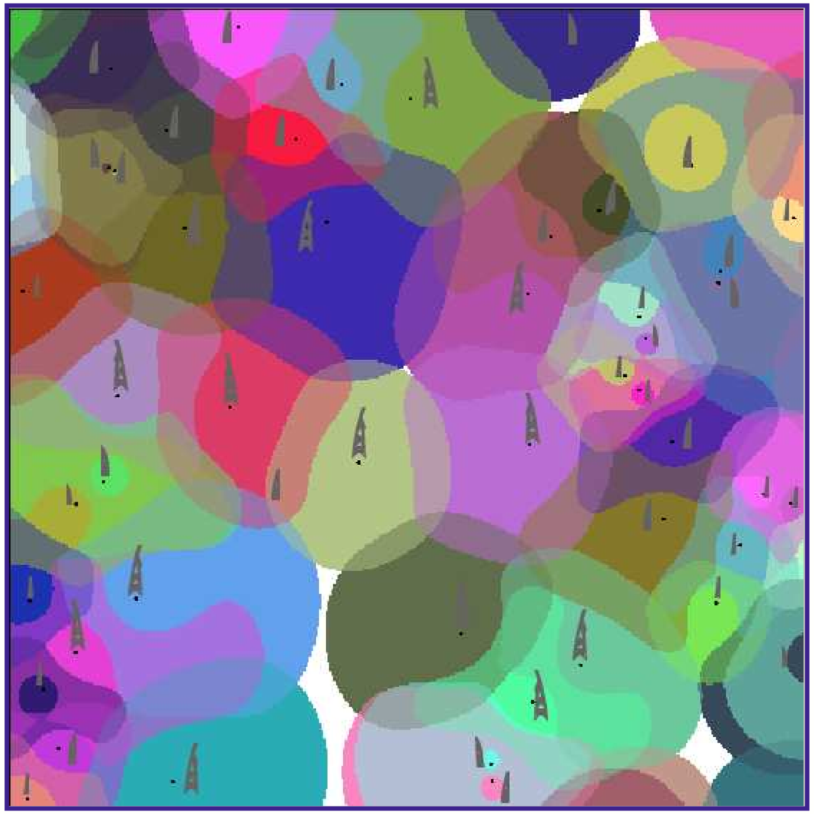
External noise $w \sim -105$ dB

Interference factor for pilots from different BS's $\kappa = 1$

Attenuation function

$l(x) = A \max(|x|, r_0)^{-\alpha}$ or

$l(x) = (1 + A|x|)^{-\alpha}$ with $\alpha \sim 3 - 6$.



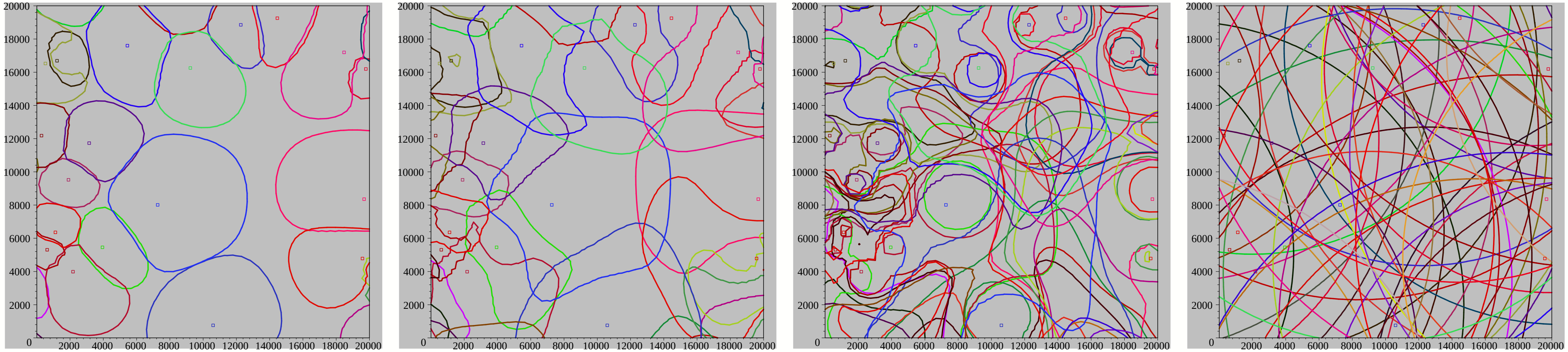
This is a relatively simple model, which takes into account only locations of the Base Stations, their pilot signal powers and SIR's for the pilots.

In particular there is no any pattern of mobiles assumed yet and it does not take into account power control issues.

Aplications to ad-hoc networks

- Gupta & Kummar (2000) studied the capacity of ad-hoc networks under similar model.
- Percolation in a variant of this model was studied by Douse et al. (2003, 2006) to address connectivity issues of large ad-hoc networks.
  (BTW, percolation of the classical Boolean model was proposed as a connectivity model for wireless communication networks by Gilbert back in 1961!)

Snapshots and qualitative results

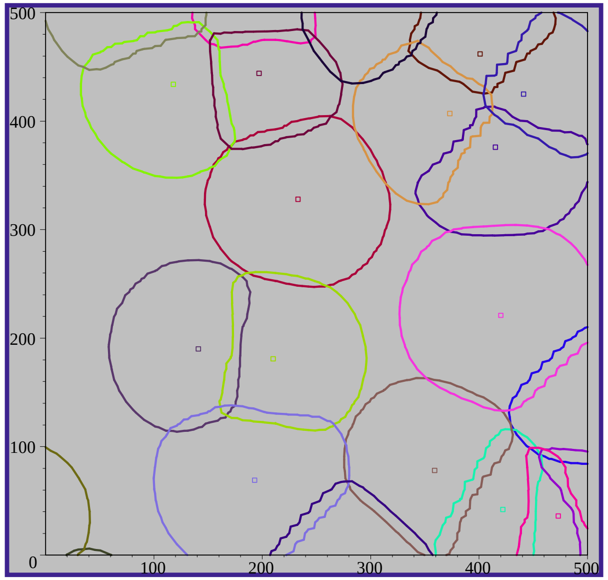


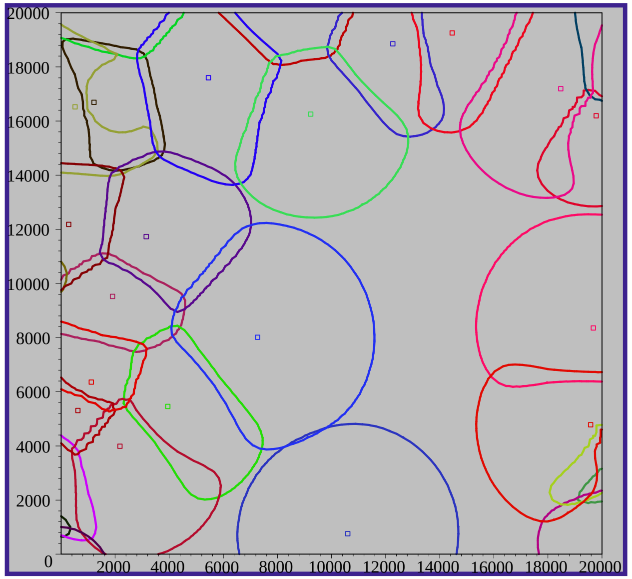
$\kappa = 0.5$ $\kappa = 0.1$ $\kappa = 0.01$ $\kappa = 0$

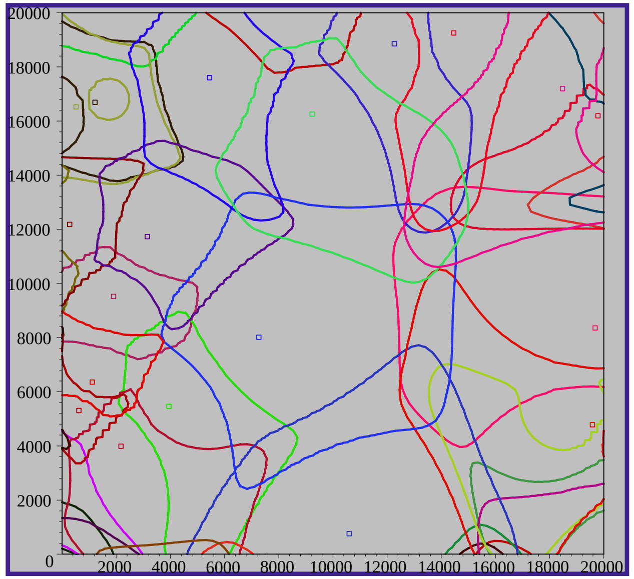
Constant emitted powers S_i, $e_i \equiv 1$, $T = 0.4$ and

interference factor $\kappa \to 0$.

Small interference factor allows one to approximate SINR cells by a Boolean model (quantitative results via perturbation methods).

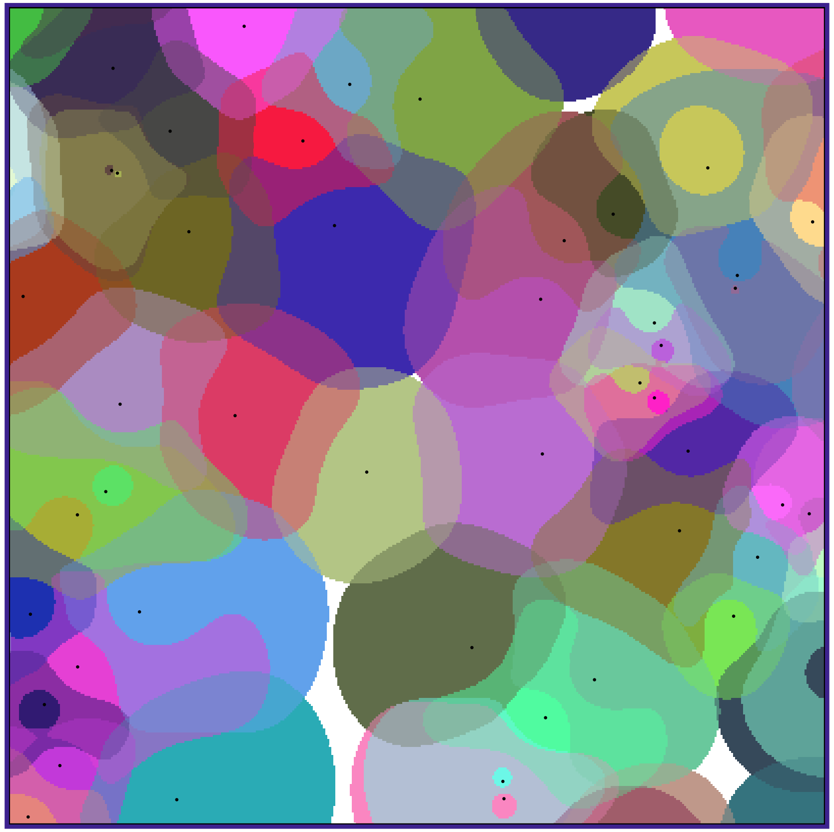


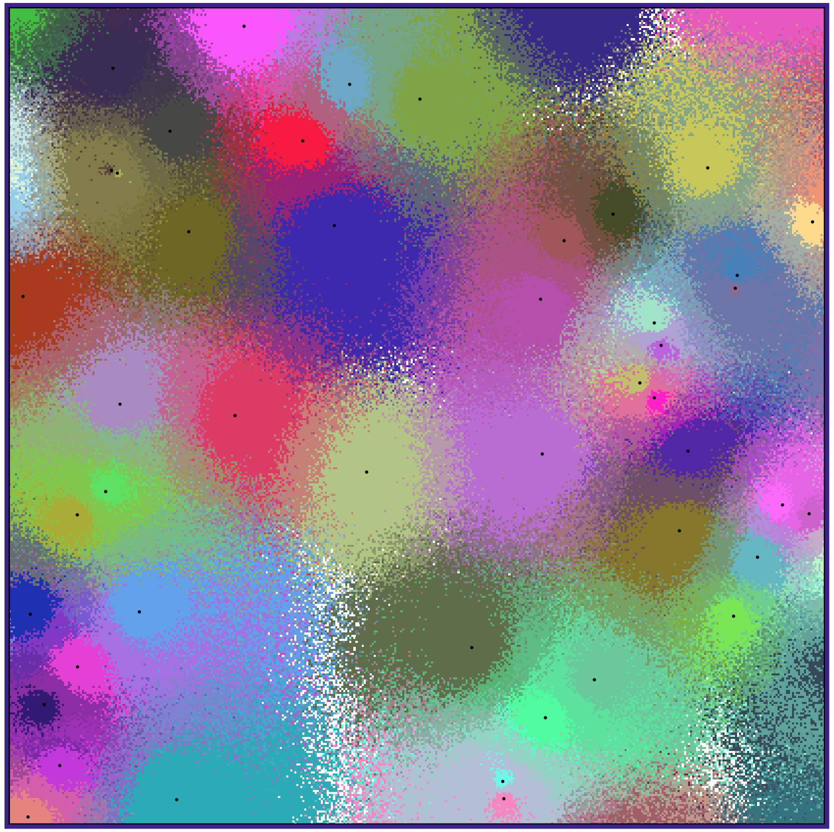




Constant emitted powers S_i, $e_i \equiv 1$, $T = 0.4$, $W = 0$, $l(r) = (Ar)^{-\beta}$ and attenuation exponent $\beta \to \infty$.

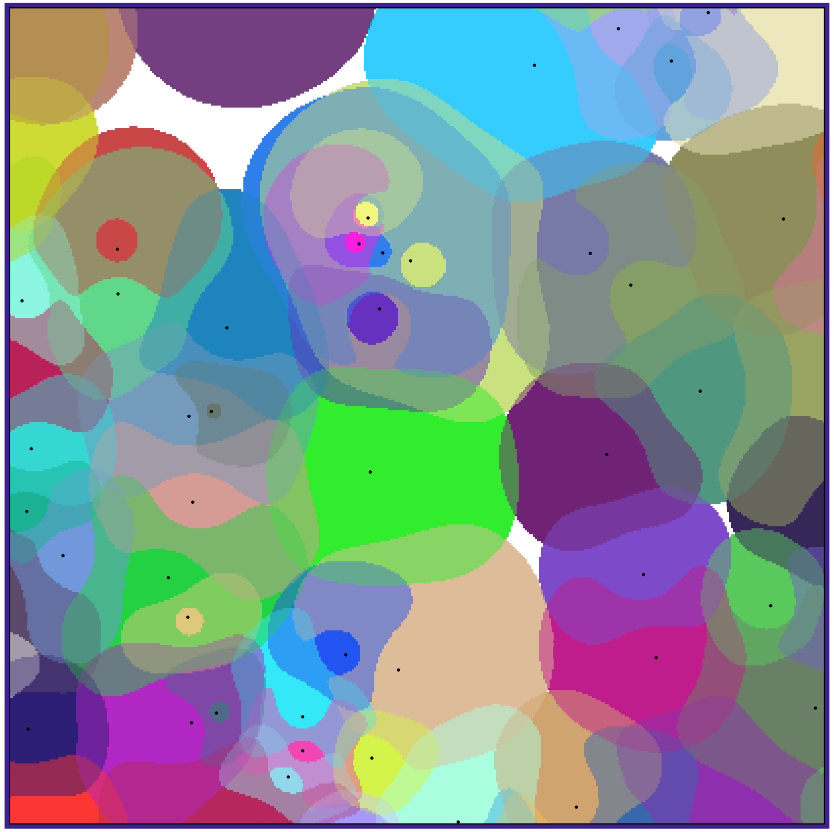
SIR cells tend to Voronoi cells whenever attenuation is stronger, e.g. in urban areas.

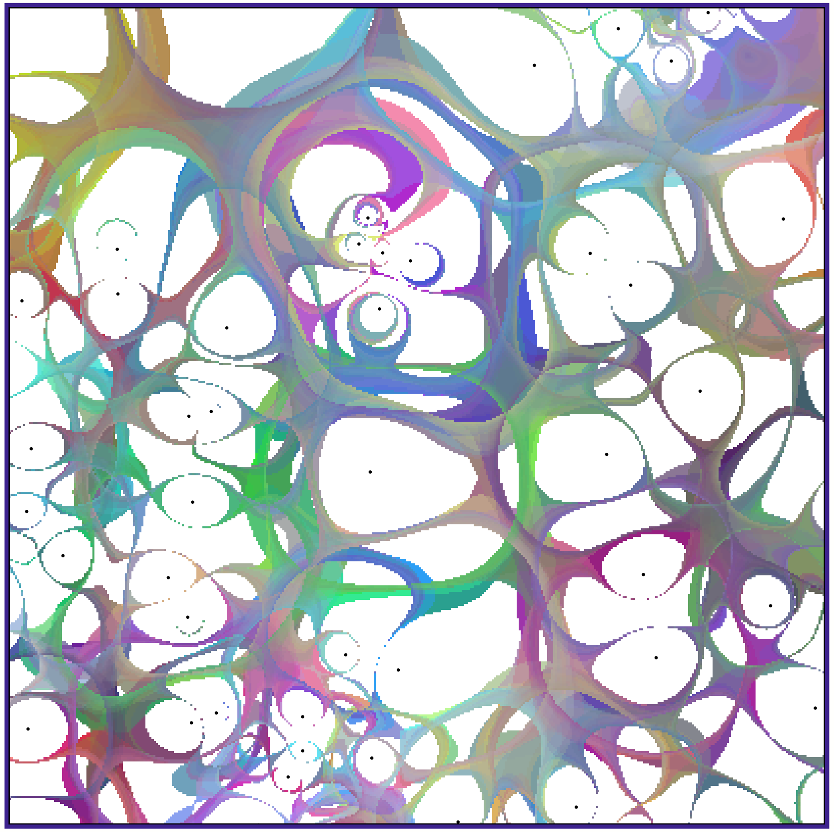




Cells without and with point dependent fading.

Fading reflects variations in time and space of the channel quality about its average state.





Cells with macrodiversity $K = 1$ and the gain of the macrodiversity $K = 2$.

Macrodiversity K: possibility of being connected simultaneously to K stations and to combine signals from them.

Probability for a typical cell to cover a point

Given: Φ — marked Poisson point process representing antennas in $\mathbb{R}^2$, $(0,(S,T))$ — additional antenna located at fixed point 0 with random (S,T) distributed as any mark of Φ, independent of it (thus $\Phi \cup \{(0,(S,T)\}$ has Poisson Palm distribution), y — location (of a mobile) in $\mathbb{R}^2$.

Probability for C_0 to cover a given point y located at the distance R to the origin:

$$
\begin{aligned}
p_R &= \mathsf{P}\Big(y \in C_0 \Big) \\
&= \mathsf{P}\Big(S(1/T - 1)l(R) - W - I_\Phi(y) > 0 \Big).
\end{aligned}
$$

Res. For M/G case (general distribution of (S,T)) the coverage probability p_R can be given via Laplace transforms of $S(1/T-1)$, W and the Laplace transform of $I_\Phi(y)$ that is

$$\mathsf{E}[\exp(-\xi I_\Phi(y))] = \exp\Big[-\int_{\mathbb{R}^d}\Big(1-\mathcal{L}_S(\xi l(y-z))\Big)\,\mu(\mathrm{d}z)\Big],$$

where $\mathcal{L}_S(\xi)=\mathsf{E}[e^{-\xi S}]$ is the Laplace transform of S.

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Cor. Fourier transform of the Poisson shot-noise variable $I_\phi(y) \rightarrow$

Rieman Boundary Problem $\rightarrow$ probability of coverage by the typical cell.

Example

Fourier transform $\mathcal{F}_{I_\Phi}(\xi)$ of the homogeneous Poisson (intensity λ) shot noise with exponential S (parameter m) and attenuation $l(x) = A\max(|x|, r_0)^{-4}$

$$\mathcal{F}_{I_\Phi}(\xi) = \mathsf{E}\left[e^{-i\xi I_\Phi}\right]$$
$$= \exp\left[\lambda\pi\sqrt{\frac{iA\xi}{m}}\arctan\left(r_0^2\sqrt{\frac{m}{iA\xi}}\right) - \frac{\lambda}{2}\pi^2\sqrt{\frac{iA\xi}{m}} + \lambda\pi r_0^2\frac{r_0^4 - iA\xi - r_0^4 m}{iA\xi + r_0^4 m}\right],$$

for $\xi \in \mathbb{R}$, where the branch of the complex square root function is chosen with positive real part.

Special M/M case

Res. [Baccelli&BB&Muhlethaler (2004)] Assume that $\{S_i\}$ are exponential r.vs. with par. μ, $T_i = T$ are constant and denote $\mathcal{L}_W$ the Laplace transform of W. Then the probability for C_0 to cover a given point located at the distance R is equal to

$$p_R = \exp\left\{ -2\pi\lambda \int_0^\infty \frac{u}{1 + l(R)/(Tl(u))} \,\mathrm{d}u \right\} \mathcal{L}_W(\mu T/l(R))\,.$$

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proof: Say the emitter is at the origin and consider the corresp. Palm distribution $\mathbf{P}$;

$$\begin{aligned} p_R &= \mathbf{P}(S \geq T(W + I_{\Phi^1}/l(R))) \\ &= \int_0^\infty e^{-\mu s T/l(R)}\,\mathrm{d}\mathbf{P}(W + I_\Phi \leq s) \\ &= \mathcal{L}_{I_\Phi}(\mu T/l(R))\mathcal{L}_W(\mu T/l(R))\,, \end{aligned}$$

where $\mathcal{L}_{I_\Phi}(\cdot)$ is the Laplace transform of the value of the hom. Poisson SN I_Φ.

Cor. For the attenuation function $l(u) = (Au)^{-\beta}$ and $W = 0$

$$p_R(\lambda) = e^{-\lambda R^2 T^{2/\beta} C},$$

where $C = C(\beta) = \Big(2\pi\Gamma(2/\beta)\Gamma(1-2/\beta)\Big)/\beta$.

Some optimizations

One can study the following optimization problems for the expected effective transmission range $r \times p_r$:

- given the density of stations λ find the targeted range r that optimizes the expected effective transmission range

$$\rho = \rho(p) = \max_{r \geq 0}\{rp_r(p)\} = \frac{1}{T^{1/\beta}\sqrt{2\lambda pC}}$$

$$r_{\max} = r_{\max}(p) = \text{argmax}_{r\geq 0}\{rp_r(\lambda)\} = \frac{1}{T^{1/\beta}\sqrt{2\lambda C}}$$

- given the targeted range R find the density of emitters λ that optimize the spatial density of successful transmission $\lambda \times p_R$:

$$\lambda_{\max} = \lambda_{\max}(R) = \text{argmax}_{\lambda \geq 0}\{\lambda p_R(\lambda)\} = \frac{1}{R^2 T^{2/\beta} C}$$

$$\max_{\lambda \geq 0}\{\lambda p_R(\lambda)\} = \frac{1}{R^2 T^{2/\beta} e C}$$

Probability for a typical cell to cover two points

(y_1, y_2) — two point to be covered by a given cell $C_0(\Phi, W)$ under Palm distribution of $\Phi \cup \{(0, (S, T)\}$

We need the joint Laplace transform of

$(I_\Phi(y_1), I_\Phi(y_2))$ that is given by

$$\mathsf{E}\Big[\exp\Big(-\xi_1 I_\Phi(y_1) - \xi_2 I_\Phi(y_2)\Big)\Big]$$
$$= \exp\Big[-\int_{\mathbb{R}^d}\Big(1 - L_S(\xi_1 l(y_1 - z) + \xi_2 l(y_2 - z))\Big)\mu(\mathrm{d}z)\Big].$$

Coverage probability via perturbation of Boolean model

valid for small interference factor κ

Denote $p_R^{(\kappa)} = \mathsf{P}(x \in C_0^{(\kappa)})$, where $|x| = R$ and
$C_0^{(\kappa)} = \Big\{ y \in \mathbb{R}^2 : Sl(y) \geq \kappa I_\Phi(y) + W \Big\}$.
Assume $F_*(u) = \mathsf{P}((Sl(x) - W) \leq u)$ admits Taylor approximation at 0:

$$F_*(u) = F_*(0) + \sum_{k=1}^{h} \frac{F_*^{(k)}(0)}{k!} u^k + \mathcal{R}_*(u)$$

and $\mathcal{R}_*(u) = o(u^h) \quad u \searrow 0$.

Res.

$$p_R^{(\kappa)} =$$

$$\overbrace{\mathsf{P}\Big(Sl(x) \geq W\Big)}^{\text{value for the Boolean model}} \quad - \overbrace{\sum_{k=1}^{h} \kappa^k \frac{F_*^{(k)}(0)}{k!} \mathsf{E}\big[(I_\Phi(y))^k\big]}^{\text{correcting terms}} + \overbrace{o(\kappa^h)}^{\text{error}},$$

provided $\mathsf{E}\big[(I_\Phi(x))^{2h}\big] < \infty$.

Mean cell area formula

Denote the mean area of the cell of the BS located at 0 by $v_0 = \mathsf{E}[|C_0|]$.

Recall that p_R is the coverage probability for location at distance R.

Res. We have

$$v_0 = \int_{\mathbb{R}^2} p_{|y|} \, \mathrm{d}y.$$

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Recall that p_R is the coverage probability for location at distance R.

Res. We have

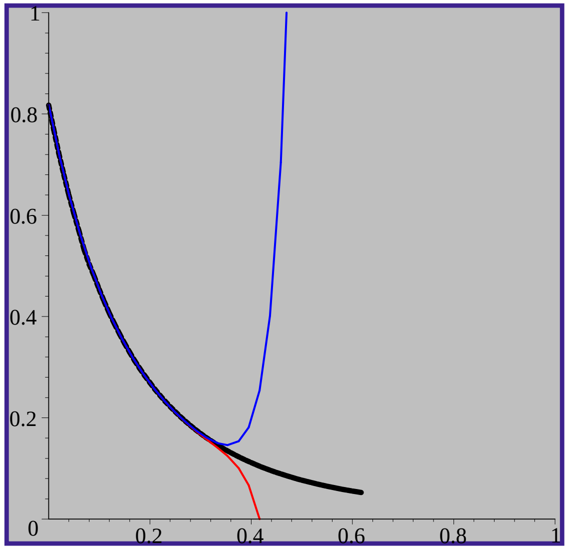
$$v_0 = \int_{\mathbb{R}^2} p_{|y|} \, \mathrm{d}y.$$

proof:

$$v_0 = \mathsf{E}\left[\int_{\mathbb{R}^2} 1(y \in C_0) \, \mathrm{d}y\right] = \int_{\mathbb{R}^2} p_{|y|} \, \mathrm{d}y.$$

Numerical examples

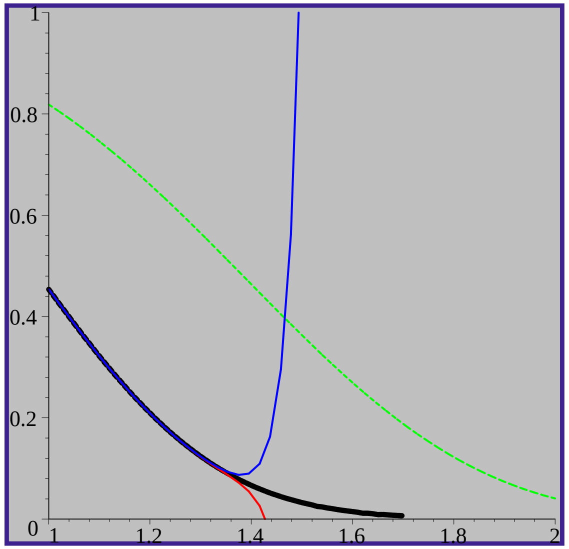
$p^{(\kappa)}(1)$



κ

Probability of coverage as a function of the interference coeffi cient

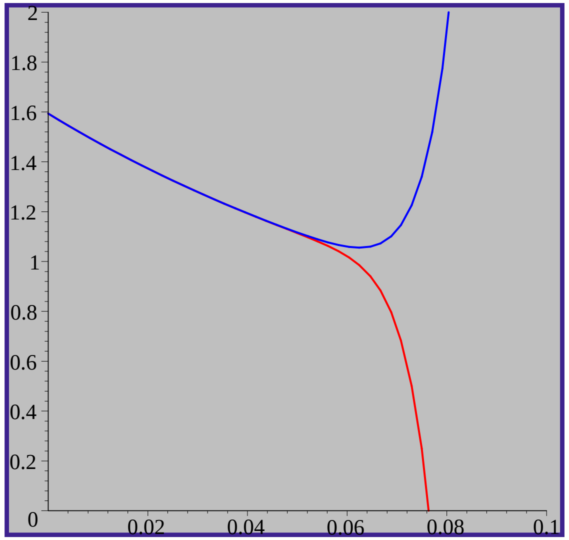
$p^{(0.1)}(y)$



y

Probability of coverage as a function of the distance.

$v^{(\kappa)}(1)$



κ

Mean cell area as a function of the interference coeffi cient.

Overlapping of cells

Deterministic scenario: given n cells $C(x_i, s_i, t_i; \phi, w)$, $i = 1, \ldots, n$

Res. The inequality $\sum_{i=1}^{n} t_i/(1+t_i) < 1$ is a necessary condition for $\bigcap_{i=1}^{n} C(x_i, s_i, t_i; \phi, w) \neq \emptyset$

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Random scenario:

Cor. If the distribution of the ratio T is such that $T \geq \tau$ for some $\tau > 0$, then the number K_y of cells of the coverage process Ξ covering any given point y is a.s. bounded

$$K_y < \frac{1+\tau}{\tau}.$$

(Given point cannot be covered by $(1 + \tau)/\tau$ or more cells, no matter how close they are located and how their signal is strong — "pole handoff number".)

Example: For the maximal pilot's bit energy-to-noise spectral power density $\tau = E_b/\mathcal{N}_O = -14$ dB the pole handoff number (theoretical maximal handoff number) $K \leq 26$.

Moment expansion of the number of cells K_y covering y

Res. The factorial moment of K_y is given by

$$\mathsf{E}[K_y^{(n)}] = \mathsf{E}[K_y(K_y-1)\ldots(K_y-n+1)_+]$$

$$= \int_{(\mathbb{R}^d)^n} \mathsf{P}\Bigg(y \in \bigcap_{k=1}^{n} C\Big(x_k, S_k, T_k; \Phi + \sum_{\substack{i=1 \\ i\neq k}}^{n} \varepsilon_{(x_i,(S_i,T_i))}, W\Big)\Bigg)$$

$$\times \mu(\mathrm{d}x_1)\ldots\mu(\mathrm{d}x_n).$$

Little law

In particular, for a homogeneous Poisson point process with intensity λ

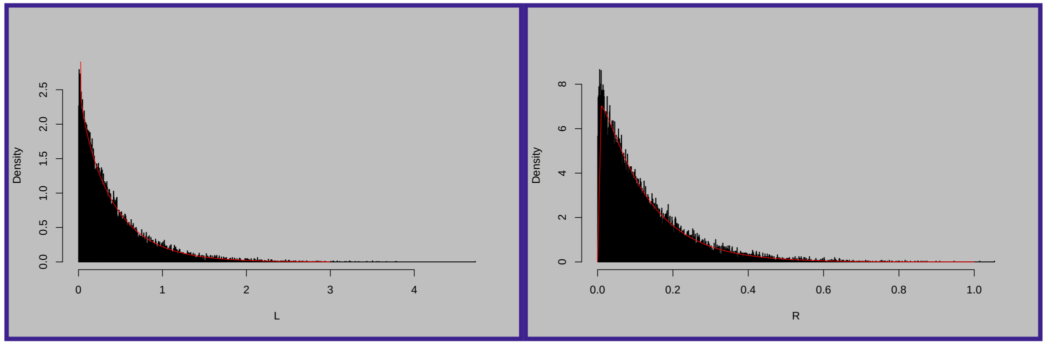
$$\mathsf{E}[K_0] = \lambda \mathsf{E}[|C_0|]\,,$$

where $|C_0|$ is the area of the typical cell. Moreover, in this case the volume fraction p (fraction of the space covered by Ξ) is given by

$$p = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} \mathsf{E}\big[(K_0)^{(k)}\big]\,.$$

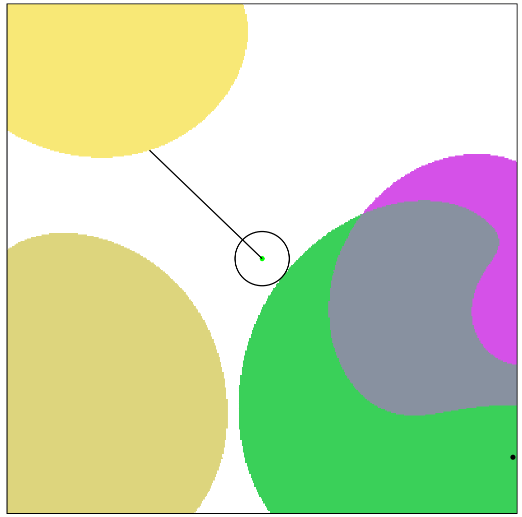
Contact distribution functions

Example of contact d.f.'s estimation



Histograms of linear L and spherical R

contact d.f. given the point is not covered.



$\mathsf{E}L$	$var L$	$\mathsf{E}R$	$var R$
0.423 km	0.191 km^2	0.121 km	0.013 km^2

Conditional distribution of the model

Two finite sets of points: $z_1, \ldots, z_n$ and $z'_1, \ldots, z'_p$.

Condition:

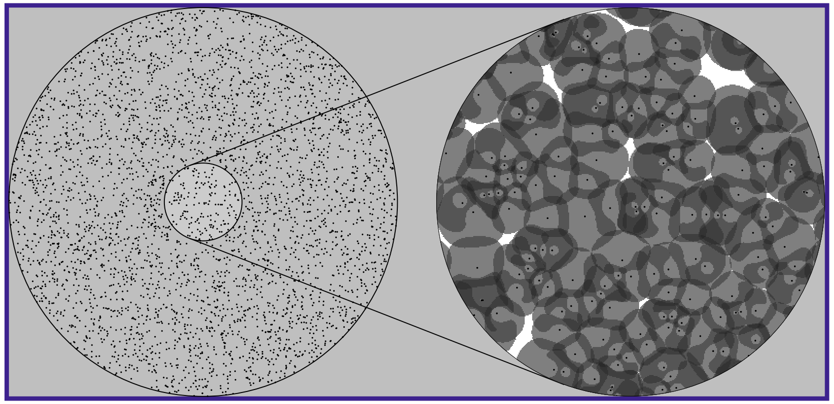
points z_i are covered by at least n_i cells
and
points z'_i are covered by at most n'_i cells,

for some given numbers $n_1, \ldots, n_n$ and $n'_1, \ldots, n'_p$.

This type of conditions allows one to consider cases where the exact number of cells covering a point is specified.

Almost exact simulation of the shot-noise

For a given size of observation window (radius R) one selects a larger influence window (radius R') in order to get good estimate of the shot-noise term I_ϕ in the smaller observation window.



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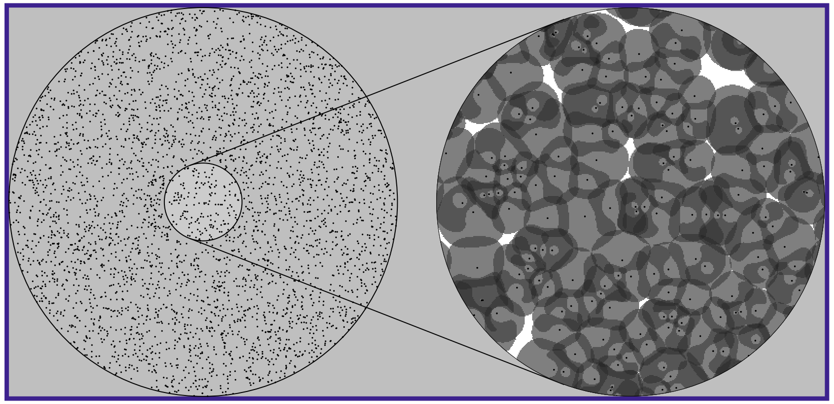

Th. If the attenuation functions is of the form $l(x,y) < C/|x-y|^\beta$ for some constants $C > 0, \beta > 0$ and if the distribution of S has finite moment $E[S^{1/(\beta/2-\delta)}] < \infty$ for some $\delta \in [1, \beta/2]$, then one can show that for any $R, \varepsilon, \alpha > 0$, there exists $R' > 0$ such that

$$P\left(\sup_{|y|<R} \sum_{|X_i|>R'} S_i l(y, X_i) < \varepsilon\right) > 1 - \alpha\,.$$

Perfect simulation in the observation window

One constructs a Markov process $(\tilde{Z}_t)$ of patterns of points that has for its stationary distribution the conditional distribution.

> Points are generated at exponential periods and located in the window but only if their presence does not violate conditions of maximal coverage of the points z_i'. Points located in the window stay there for exponential times and are removed, but only if their absence does not violate the conditions of maximal coverage of the points z_i'. If a particular removal would lead to the violation, then the point are exponentially perpetuated.

The exact stationary distribution of the Markov process $(\tilde{Z}_t)$ is obtained using backward simulation (coupling from the past) similar to that proposed by Kendall.

Macroeconomic optimization example: densification / magnification

Increase the mean power m of existing antennas or

increase the density λ of antennas?

C total budget of an operator per km^2,

C_λ cost of one antenna,

C_m cost of increasing the power of one antenna by 1W.

constraint:

$$\lambda C_\lambda + C_m \lambda m = C.$$

<u>Macroeconomic optimization example:</u> **densification / magnification**

Increase the mean power m of existing antennas or

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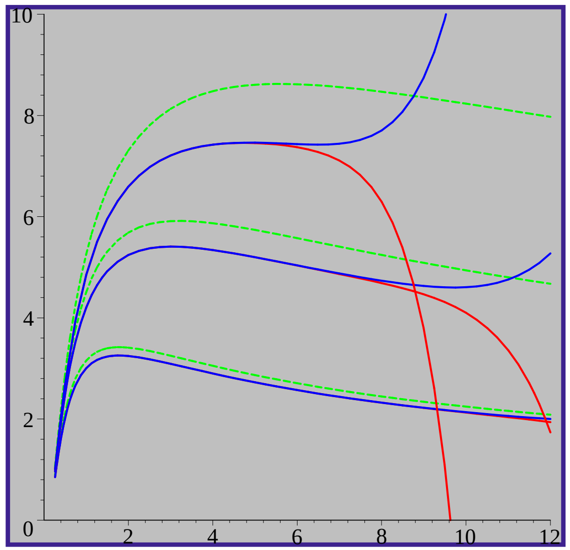
C_m cost of increasing the power of one antenna by 1W.

constraint:

$$\lambda C_\lambda + C_m \lambda m = C.$$

mean handoff level $\mathsf{E} N_0$



mean antenna power m

Plots of mean handoff as a functions of mean antenna power m under budget constraint with $C = 1000, C_\lambda = 500$ and from the top: $C_m = 1, 2, 5$.

Solution: Plot maximum $=$ Optimal confi guration.

References

1. Baccelli & Bł aszczyszyn (2001), On a coverage process ranging from the Boolean model to the Poisson Voronoi tessellation, with applications to wireless communications *Adv. Appl. Probab.* **33**

2. Tournois (2002), Perfect Simulation of a Stochastic Model for CDMA coverage *INRIA report* **4348**,

3. Baccelli, Bł aszczyszyn & Tournois (2002), Spatial averages of downlink coverage characteristics in CDMA networks (*INFOCOM*).

IV POWER CONTROL IN CDMA

Evaluating capacity of the Voronoi architecture

- Voronoi network architecture,

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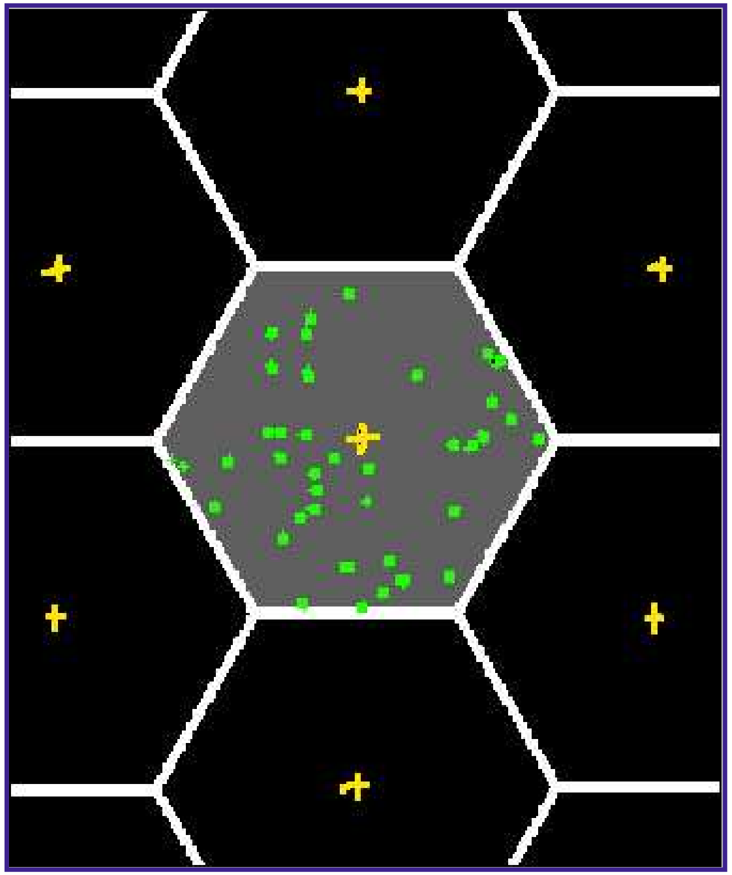
- References.

Network architecture models

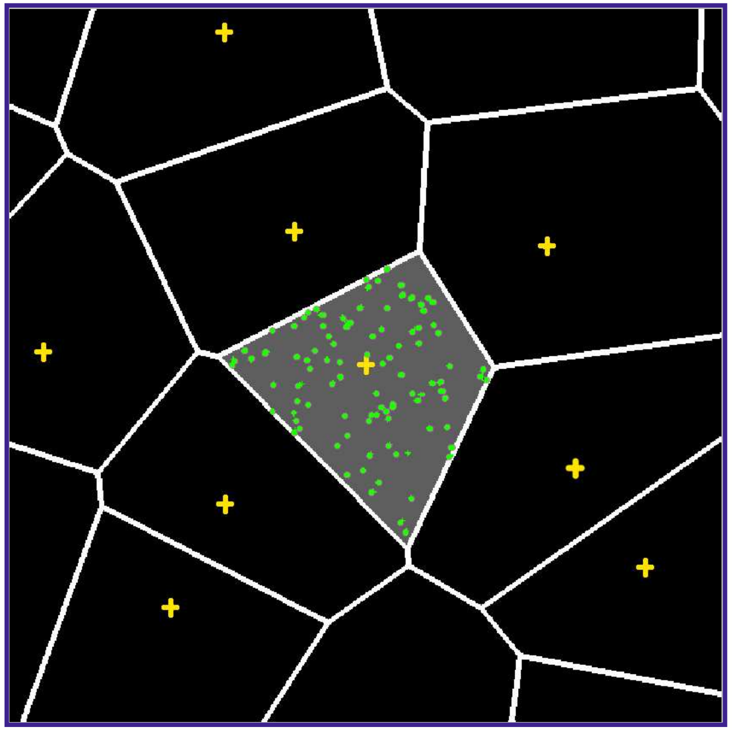
Hexagonal (Hex) model ("too regular")

- BS's $\{Y_j\}$ located according to hexagonal grid, p.p, with spatial density λ_{BS}.
- All antenna parameters are i.i.d. marks.
- All mobiles form independent Poisson p.p. $\mathcal{N}_M$ with spatial density λ_M.
- Each mobile is served by the nearest BS.

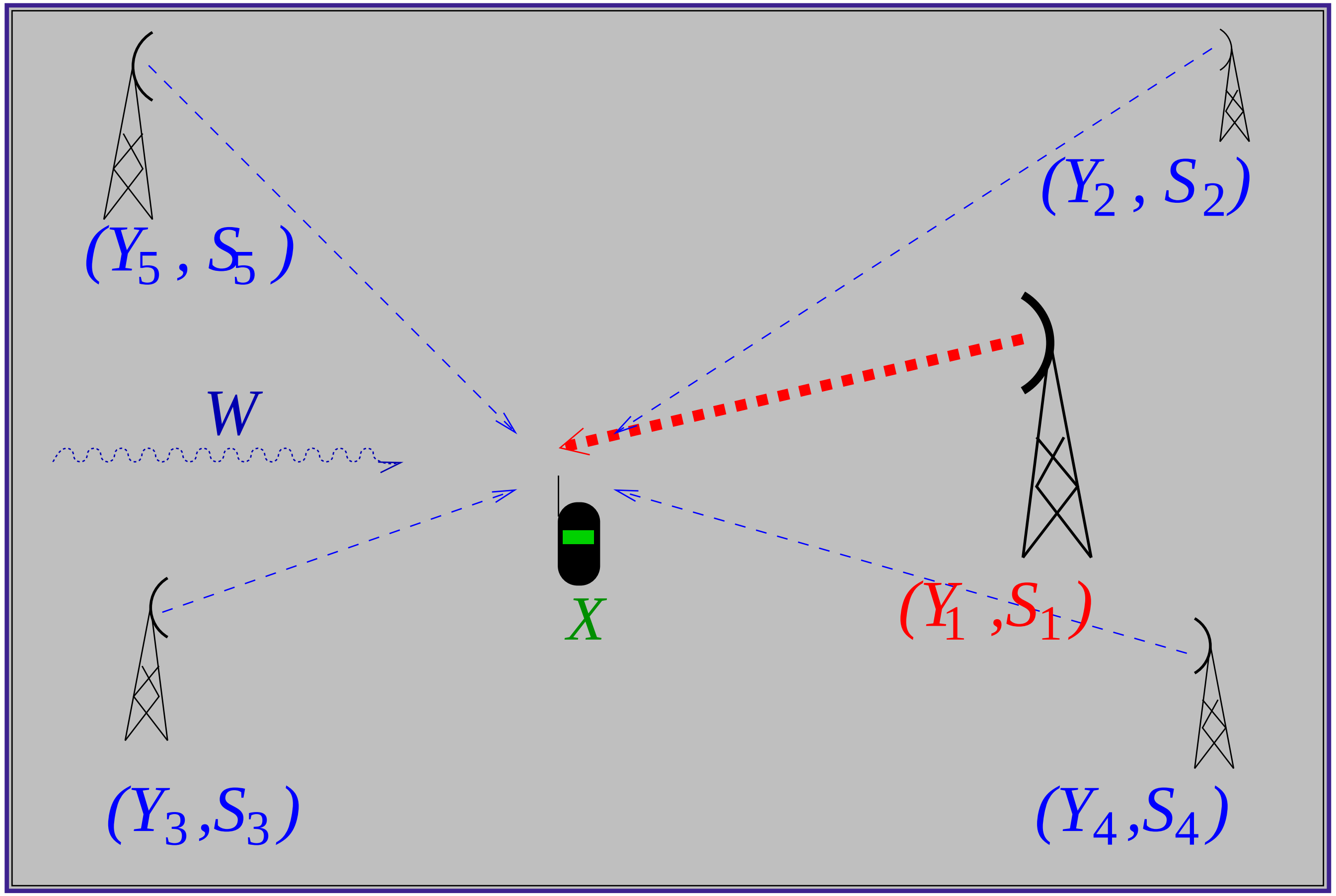


Poisson-Voronoi (P-V) model ("too random")

- BS's $\{Y_j\}$ located according to Poisson p.p, with intensity λ_{BS}.
- All antenna parameters are i.i.d. marks.
- All mobiles form independent Poisson p.p. $\mathcal{N}_M$ with intensity λ_M.
- Each mobile is served by the nearest BS. (Equivalently: Each BS j serves mobiles $\mathcal{N}_M^j$ in its Voronoi cell.)



CDMA: Interference limited radio channel



$$\frac{S_1 l(X - Y_1)}{W + I(X)} \geq C$$

CDMA Power allocation algebra

$\mathcal{N}_{BS} = \{Y_j\}_j$: locations of base-stations (BS's) in $\mathbb{R}^2$,

$\mathcal{N}_M = \{X_i^j\}_i$: locations of mobiles served by BS No. j,

C_i^j: SINR required for mobile X_i^j,

W_i^j: total non-traffic noise at X_i^j (from common overhead channels, thermal noise),

κ_j, γ: orthogonality factors,

$l(x, y)$: path-loss from y to x.

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κ_j, γ: orthogonality factors,

$l(x, y)$: path-loss from y to x.

Power allocation feasible if exist antenna powers $0 \leq S_i^j < \infty$ such that

$$\frac{S_i^j l(Y_j - X_i^j)}{W_i^j + \underbrace{\kappa_j l(Y_j, X_i^j) \sum_{i' \neq i} S_{i'}^j}_{\text{own-cell interference}} + \underbrace{\gamma \sum_{k \neq j} l(Y_k, X_i^j) \sum_{i'} S_{i'}^k}_{\text{other-cell interference}}} \geq C_i^j \quad \text{all } i, j.$$

Local and global problem

Power allocation feasible

$\Updownarrow$

Local power allocation feasible for each BS and Global power allocation feasible

Local problem

Fix BS j.

Fix total powers emitted on traffic channels by other BS's: $S_k = \sum_{i'} S^k_{i'}$ ($k \neq j$).

Power allocation is locally feasible in cell j if exist powers $0 \le S^j_i < \infty$ s.t.

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$$\kappa_j \sum_i \frac{C^j_i}{1 + \kappa_j C^j_i} < 1.$$

(local) pole capacity condition

Global problem

Suppose for each BS local power allocation is feasible.

Define:

$$a_{jk} = \gamma \sum_i \frac{H_i^j l(Y_k, X_i^j)}{l(Y_j, X_i^j)} \text{ for } j \neq k \quad \text{and} \quad a_{jj} = \sum_i \kappa_j H_i^j,$$

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Global power allocation is feasible if exist antenna powers $0 \leq S_j < \infty$ (total powers emitted on traffic channels) such that

$$\boxed{\mathbf{S} \geq \mathbf{b} + \mathbf{A}\,\mathbf{S}}$$

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- The minimal solution can be obtained as the limit of the iteration $\mathbf{A}\mathbf{b}, \mathbf{A}^2\mathbf{b}, \ldots$
- A sufficient condition for the spectral radius to be less than one is that $\mathbf{A}$ is substochastic (has row-sums less then 1). $\Rightarrow$ Decentralized Power Allocation Principle

Decentralized Power Allocation Principle (DPAP)

Each BS j verifies for the pattern $\mathcal{N}_M^j$ of the mobiles it controls if

$$\sum_{X_i^j \in \mathcal{N}_M^j} \underbrace{H_i^j \frac{\text{total path loss of user } i}{\text{own-BS path loss of user } i}}_{\text{user's } i \text{ weight}} < 1 \,.$$

Maximal load estimations of P-V and Hex model

(Wrong) Idea: Given density of BS's λ_{BS} find maximal density of mobiles λ_M, such that power allocation is feasible with probability 1.

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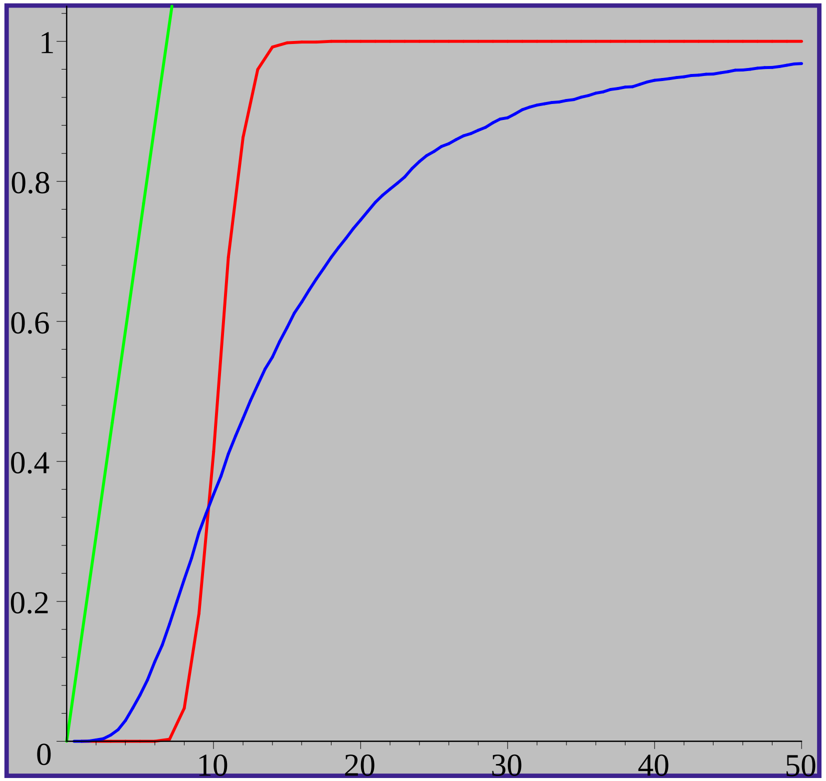
Conclusion: A reduction of mobiles (admission control) is necessary for any $\lambda_M > 0$. Calculate blocking probabilities.

Feasibility probabilities for DPAP

$$\mathbf{P}\Bigg(\underbrace{\sum_{X_i^j \in \mathcal{N}_M^j} H_i^j \frac{\text{total path loss of user } i}{\text{own-BS path loss of user } i} < 1}_{\text{DPAP satisfied}}\Bigg).$$

It says how often an non-constrained Poisson configuration of users in a given cell cannot be entirely accepted by the admission scheme DPAP.

Pr. of DPAP



λ_M

λ_{BS} =0.18 BS/km^2

$C = 0.011797$

$\gamma = 1$

$\kappa = 0.2$

$\alpha = 3$

Simulated DPAP failure probability for P-V model (more flat curve) and Hex model (more steep) curve.

Blocking rates under DPAP — spatial dymamic modeling

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  - for a given subset $A \subset C_0$, call inter-arrival times to A are independent exponential random variables with mean $1/\lambda(A)$, where $\lambda(\cdot)$ is some given intensity measure of arrivals to C_0 unit of time,
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  - call holding times are independent exponential random variables with mean τ.
- Call acceptance/rejection: given some configuration of calls in progress $\{X_m \in C_0\}$, accept a new call at x if $f(x) + \sum_m f(X_m) < 1$, where $f(\cdot)$ is the call weight function defined on C_0, and reject otherwise.

Define blocking rate associated with a given location in the cell as the fraction of users arriving according to the SBD process at this location that are rejected.

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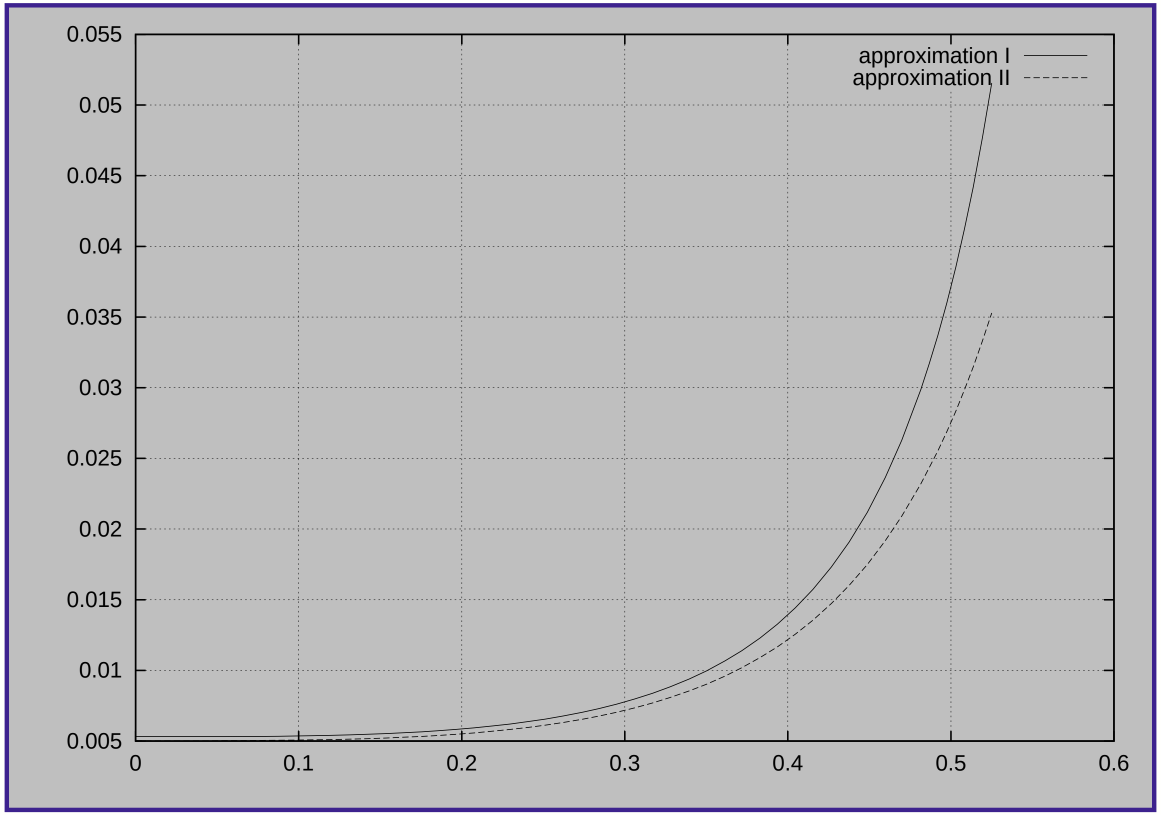
Rem. Note that $\sum_m f(X_m)$ is a compound Poisson r.v., whose distribution can be effectively approximated by Gaussian distribution.

Numerical results (assumptions correspond to UMTS)

Blocking rate



normalized distance

Approximations of the blocking probability as functions of the distance to BS for the mean number $\bar{M} = 27$ of users per cell.

References

- Baccelli & Blaszczyszyn & Tournois (2003) Downlink admission/congestion control and maximal load in CDMA networks, *IEEE INFOCOM*
- Baccelli & Blaszczyszyn & Karray (2004) Up and Downlink Admission/Congestion Control and Maximal Load in Large Homogeneous CDMA Networks, *Mobile Networks* **9(6)**,
- Baccelli & Blaszczyszyn & Karray (2005) Blocking Rates in Large CDMA Networks via a Spatial Erlang Formula, *IEEE INFOCOM*.

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- One particular "physical" model of coverage regions in cellular network was proposed and analyzed. Depending on the choice of parameters it can resemble the Voronoi or the Boolean model.
- Going more deep into the engineering details of the performance of the CDMA cellular network (power control aspect) we evaluated capacity of a large such network under simplified (Voronoi) model of its architecture.

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THANK YOU