STRUCTURE OF GALAXIES

Lecture 5. Rotation curves, mass distributions, CO, warps.

Rotation curves and mass distributions.

The (infinitessimally thin) exponential disk, when self-gravitating, has a rotation curve of the following analytic form*:

$$V_{\mathrm{rot}}^2(R) = \pi G h \sigma_\circ \left(\frac{R}{h}\right)^2 [I_0 K_0 - I_1 K_1]$$

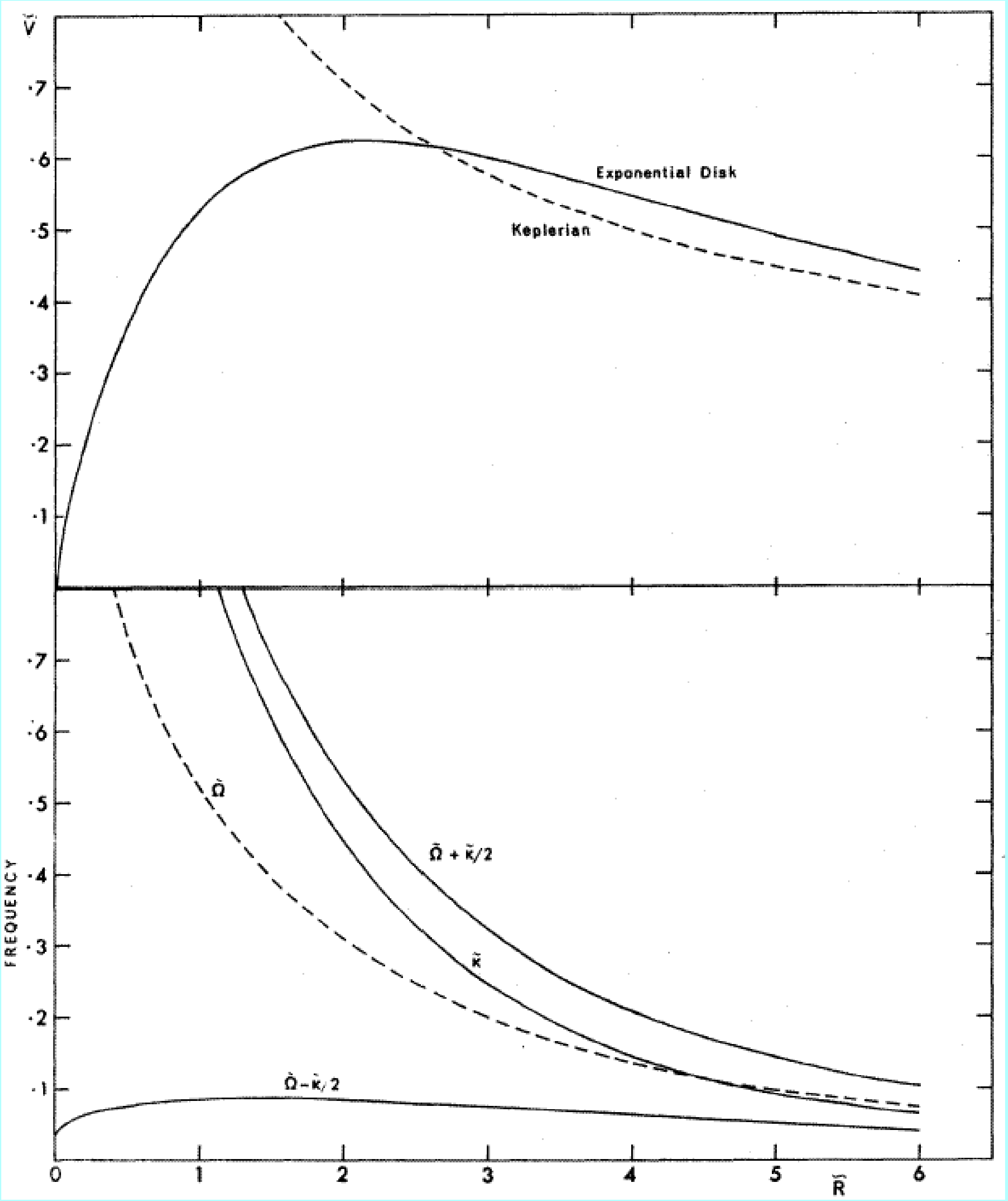
with $\sigma_\circ$ the central surface density and I and K modified Bessel functions *evaluated at* $R/2h$.

• This curve rizes from the center to a maximum at $R = 2.2h$ with

$$V_{\mathrm{max}} = 0.8796 (\pi G h \sigma_0)^{1/2}$$

• and it becomes Keplerian at large R.

*Freeman, Ap.J. 160, 811 (1970)

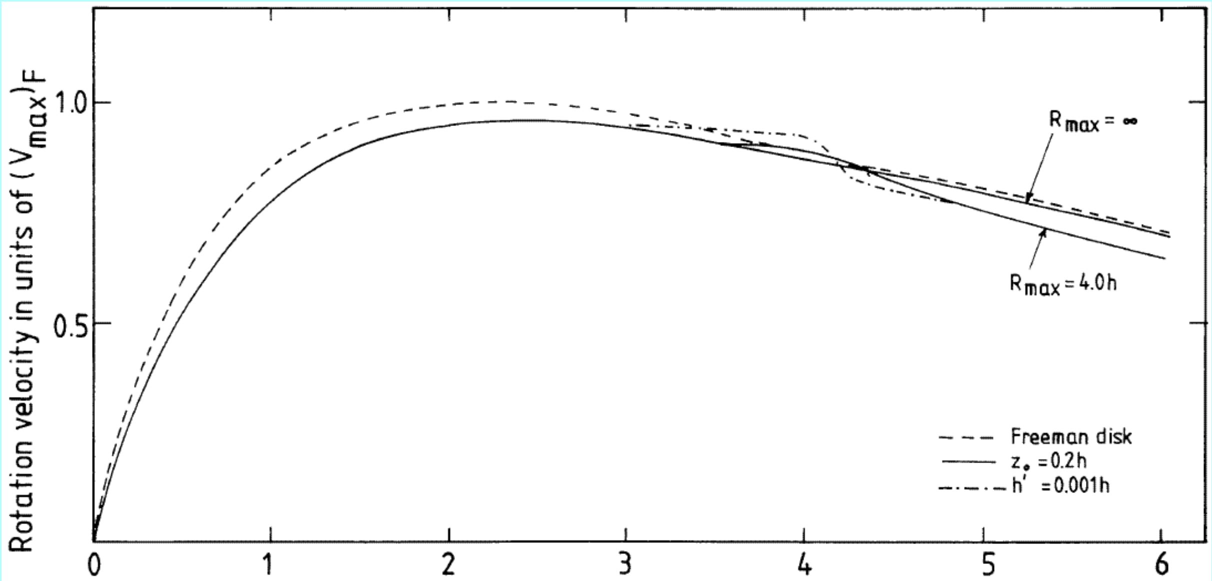


In the figure axes are dimensionless, so $\tilde{R} = R/h$ and $\tilde{V} = V\sqrt{h/GM}$ with $M = 2\pi\sigma_\circ h^2$.

The lower half of the figure has the angular frequency Ω, the epicyclic frequency κ and the Lindblad resonance frequencies $\Omega \pm \kappa/2$.

These frequencies is in units $\sqrt{GMh^3}$.

The rotation curve changes slightly when allowance is made for the finite thickness and the truncation*.



The dashed line has a infinitely thin disk, the full-drawn line has a finite thickness ($z_\circ = 0.2h$) without and with a shallow truncation (the scale-length changes by a factor 5 at $R_{\rm max}$). The dot-dashed curve has a very sharp edge.

*van der Kruit & Searle, A.&A. 110, 61 (1982)

Observations of spiral galaxies show flat rotation curves that do not show the decline, even beyond the optical edge.

So add a dark halo with $\rho \propto R^{-2}$ at large R.

This can be an isothermal sphere* or some other analytical function†.

In practice one may infer a predicted rotation curve from the disk by calculating from the observations.

In the general case that the disk density distribution is $\rho(R, z)$, the rotation curve from the corresponding self-gravitating disk is

$$V_c^2(R) = -8GR \int_0^\infty r \int_0^\infty \frac{\partial \rho(r,z)}{\partial r} \frac{K(p) - E(p)}{(Rrp)^{1/2}} \, dz \, dr$$

with

$$p = x - (x^2 - 1)^{1/2} \text{ and } x = \frac{R^2 + r^2 + z^2}{2Rr}$$

*e.g. Carignan & Freeman, Ap.J. 294, 494 (1985)

†Begeman, Ph.D. thesis (1987)

When the density distribution is separable in $\sigma(R)$ and $Z(z)$ this becomes

$$V_\mathrm{c}^2 = -8GR \int_0^\infty r\sigma(r) \int_0^\infty \frac{\partial Z(z)}{\partial z} \frac{K(p) - E(p)}{(Rrp)^{1/2}} \, dz \, dr$$

The vertical distribution can for example be assumed to be the isothermal sheet.

We may in addition have a bulge with observed surface density $\sigma(r)$; then for the self-gravitating case we have

$$V_\mathrm{c}^2(R) = \frac{2\pi G}{R} \int_0^R r\sigma(r) \, dr +$$

$$\frac{4G}{R} \int_R^\infty \left[\mathrm{arcsin} \ \left(\frac{R}{r}\right) - \frac{R}{(r^2 - R^2)^{1/2}} \right] r\sigma(r) \, dr$$

For the dark halo the assumed the density law

$$\rho(R) = \rho_\circ \left[1 + \left(\frac{R}{R_\mathrm{c}}\right)^2\right]^{-1}$$

results in

$$V_\mathrm{c}^2(R) = 4\pi G\rho_\circ R_\mathrm{c}^2 \left[1 - \frac{R_\mathrm{c}}{R} \,\mathrm{arctan}\ \left(\frac{R}{R_\mathrm{c}}\right)\right]$$

To get the total rotation curve for a system consisting of three components add these circular velocities in quadrature:

$$V_{\mathrm{circ}}(R) = \left[V^2_{\mathrm{disk}}(R) + V^2_{\mathrm{bulge}}(R) + V^2_{\mathrm{halo}}(R)\right]^{1/2}$$

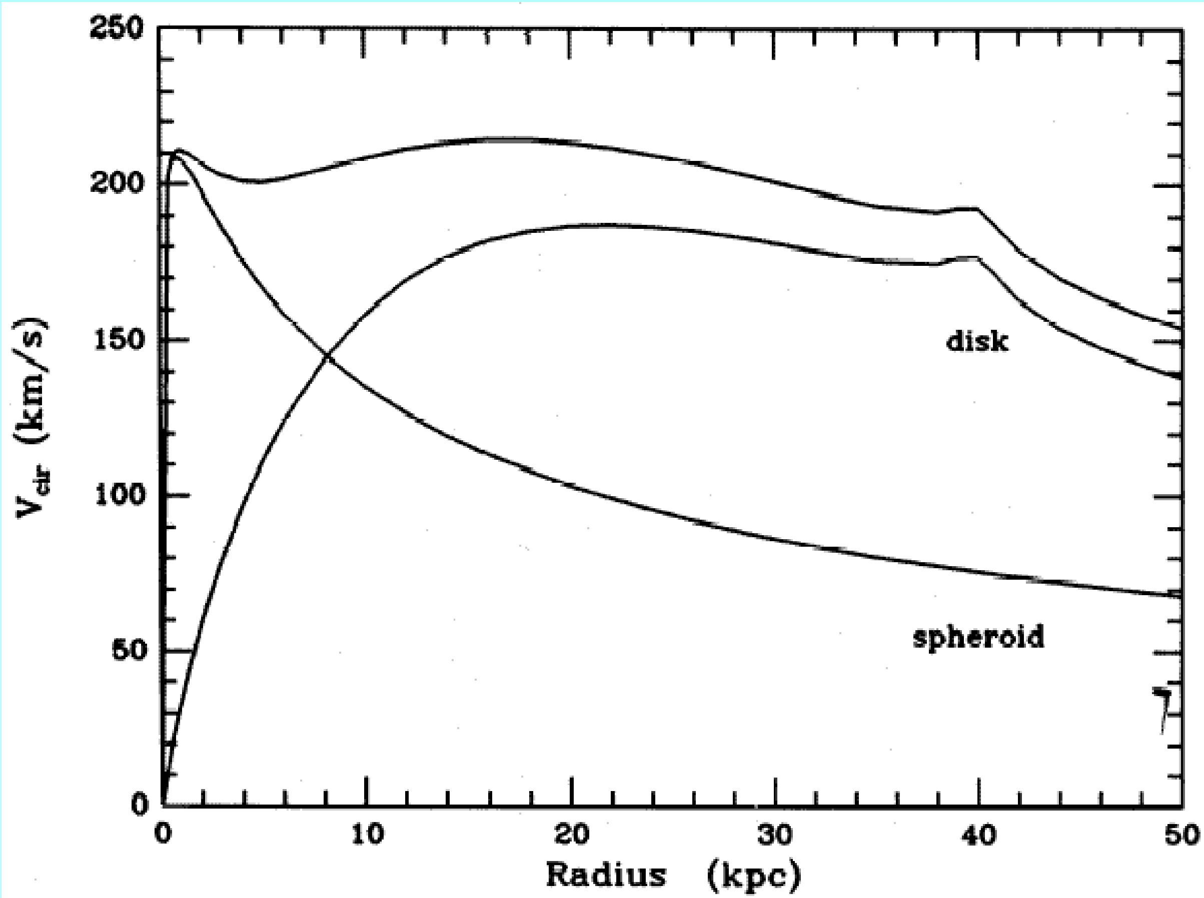
One can make things easier by fitting an exponential disk to the observations and use the analytic form of the corresponding rotation curve.

If in addition there is gas, this should be treated in the same way.

In practice we have for the stars only surface *brightness* distributions, so we need an undetermined mass-to-light ratio M/L in order to turn this into a surface *density* distribution.

From the solar neighborhood we can only find that M/L is of order a few in solar units.

In principle one can make an approximately flat rotation curve by a careful tuning of the disk and bulge contributions.

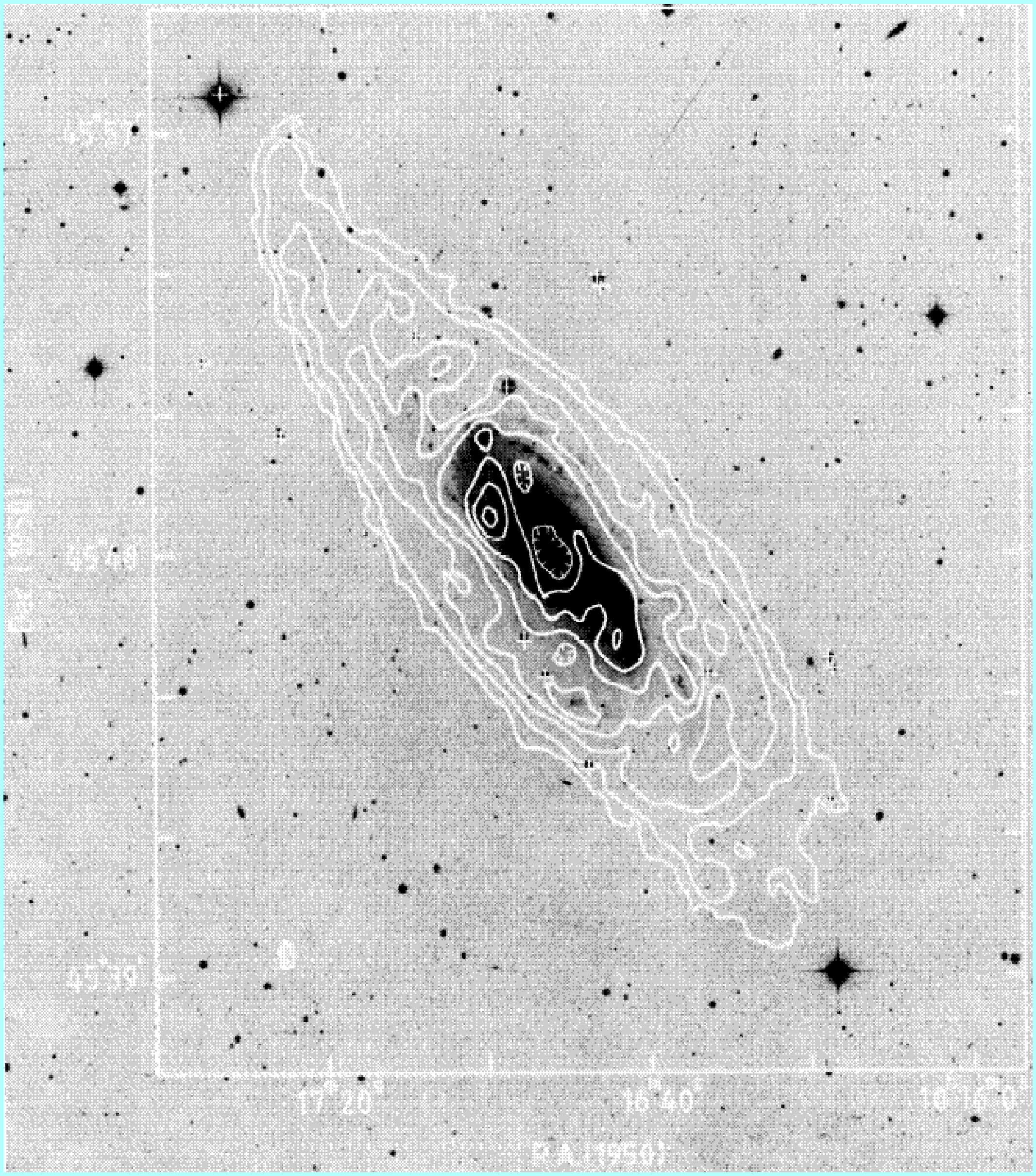


However, the rotation will drop beyond the edge, contrary to what is observed.

The following is from the analysis of the rotation curve of NGC 3198*, which has essentially no bulge.

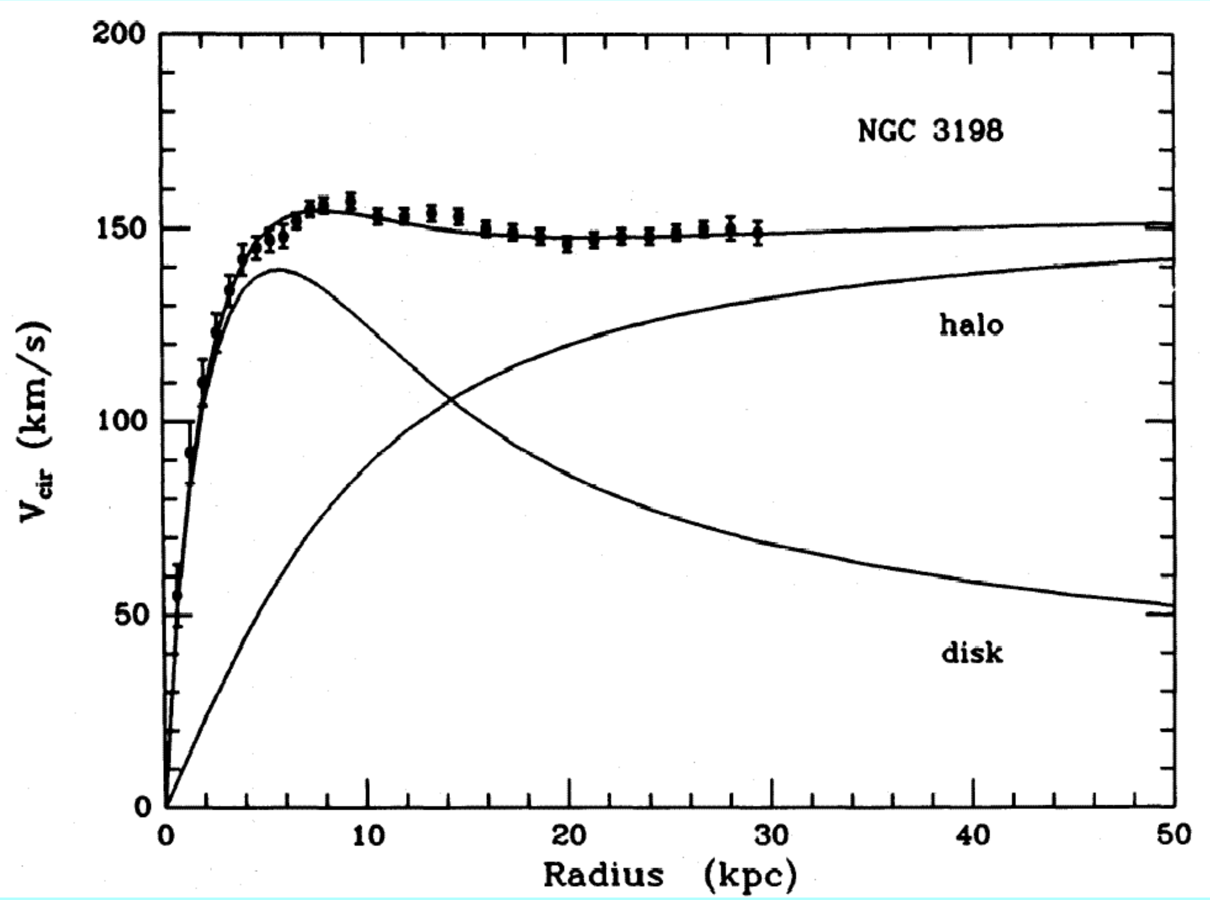
*van Albada, Bahcall, Begeman & Sancisi, Ap.J. 295, 305 (1985)

The HI extends out to 11 scalelengths.



The usual procedure is to choose an M/L of the disk that gives the maximum amplitude of the disk rotation curve that is allowed by the observations.

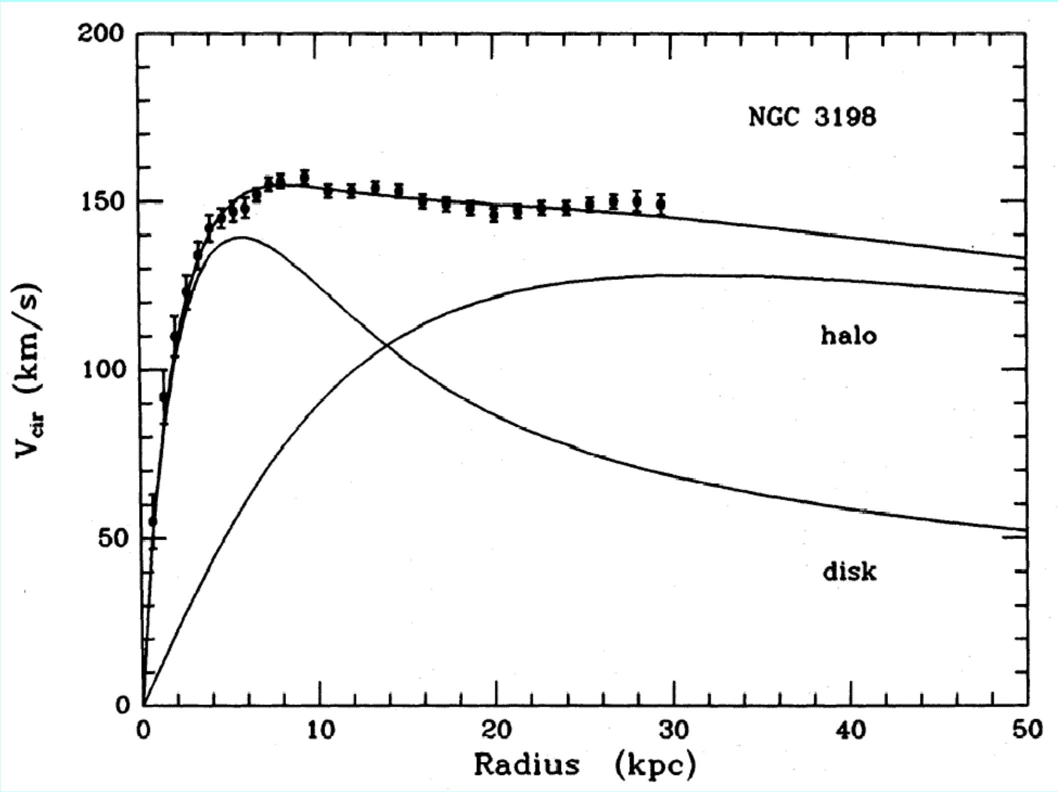
The two free parameters of the dark halo, core radius R_C and central density $\rho_\circ$ are then used to fit the rotation curve.



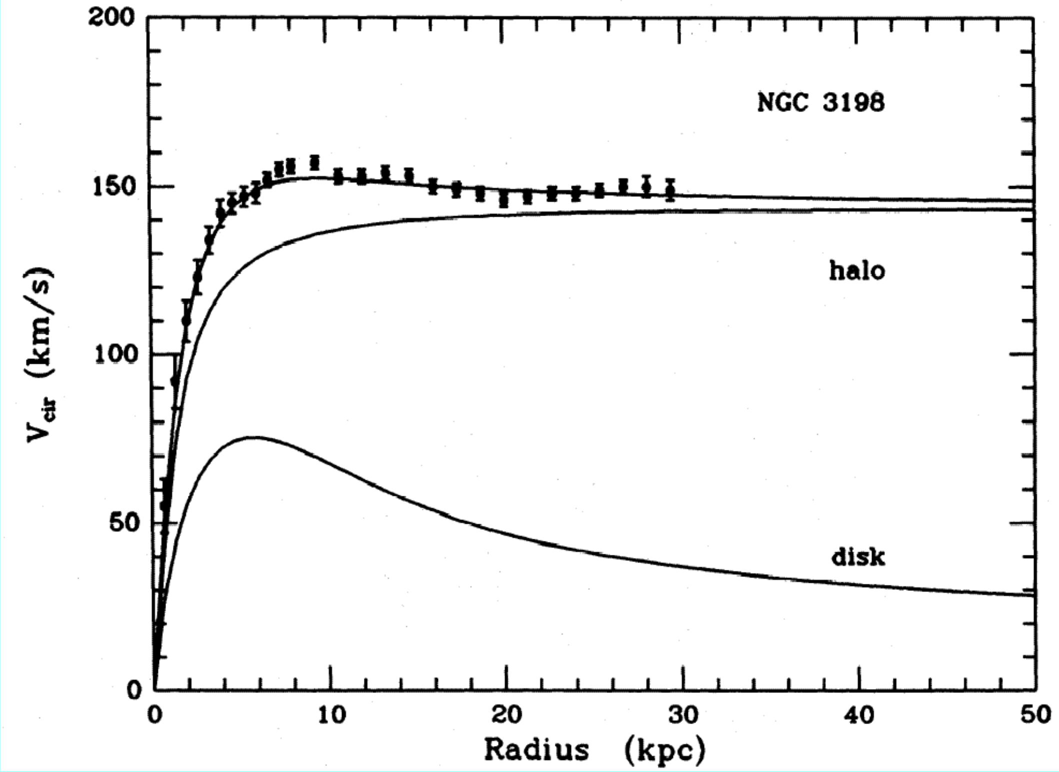
This is called the "maximum disk hypothesis".

This particular model for NGC 3198 has a total mass of 15×10^{10} $M_\odot$ within 30 kpc. Within this radius the ratio of dark to visible matter is 3.9. At the optical edge this ratio is 1.5.

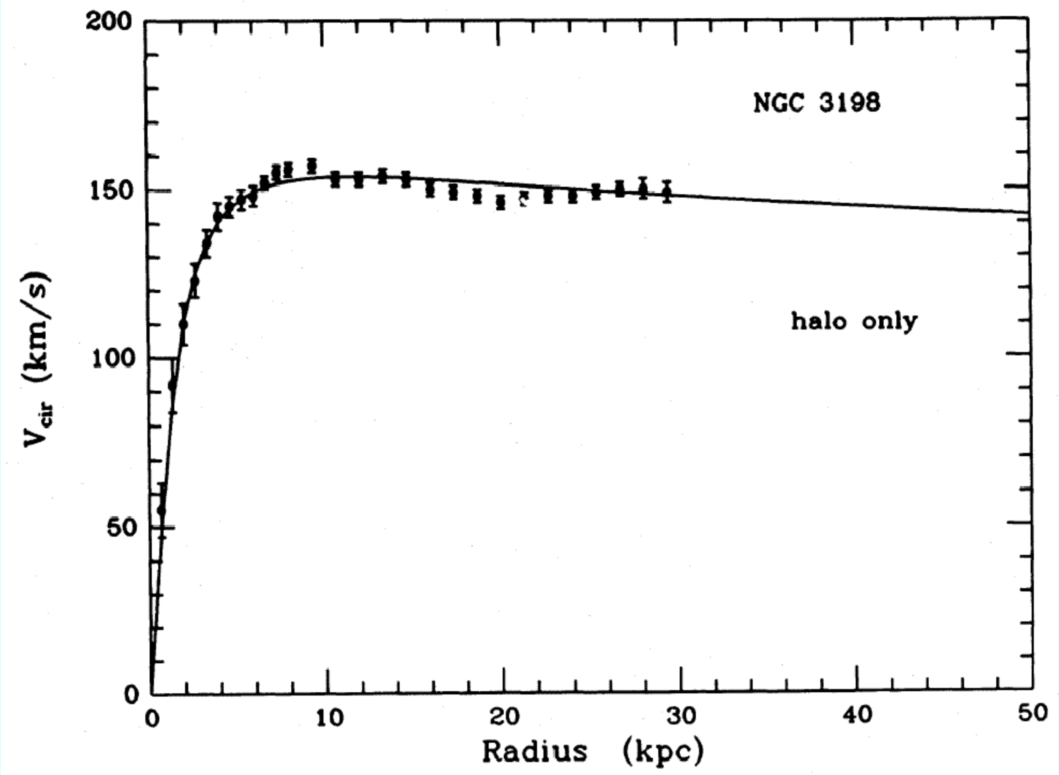
By adjusting the halo parameters one can minimize the dark halo mass by assuming that the rotation curve falls beyond the last measured point.



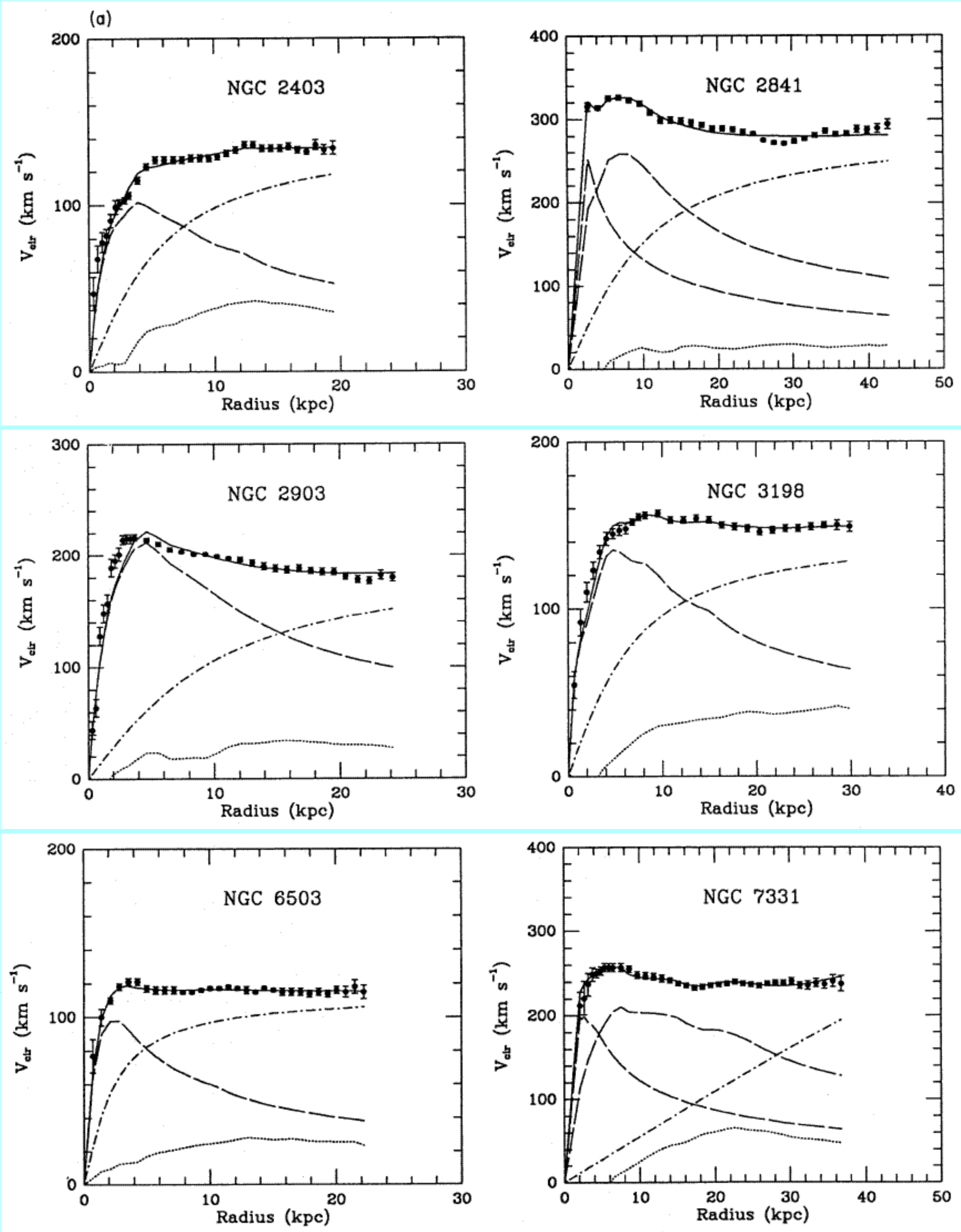
The difficulty with the maximum disk hypothesis is that it is possible to make similar good fits with lower disk masses ...



... and even no disk mass at all.



Begeman* observed 8 spirals, of which HI in NGC 2841 goes out to 17.8 h (43 kpc).



*Ph.D. thesis (1987); Begeman, Broeils & Sanders, Mon.Not.R.A.S. 249, 523 (1991)

Begeman's maximum disk fits have

$$(M/L)_{\text{disk}} = 3.1 \pm 1.2 \text{ (9.4 for NGC 2841)}$$

$$(M_{\text{halo}})_{\text{R}_{\text{opt}}} = 44 \pm 9\% \text{ (34 \% for NGC 2841)}$$

Broeils* made maximum disk fits to a sample of 23 galaxies with extended HI, accurate rotation curves and photometry. He studied the distribution of the parameters from the fits.

The global mass-to-light ratio M/L out to the maximum radius observed is in the range 10 to 20 (in B).

The ratio of the dark to luminous matter at some fiducial radius (either R_{25} or $R = 7h$) correlates well with the maximum rotation velocity and reasonably well with integrated magnitude and morphological type.

The question is whether these conclusions are influenced by the assumption of a maximum disk.

*Ph.D. thesis (1992)

The maximum disk hypothesis could lead to the following spurious results:

- Large V_{max} results in disk surface density and therefore large $(M/L)_{\text{disk}}$.
- Large $(M/L)_{\text{disk}}$ results in less dark matter.

Indeed:

$$\left(\frac{M}{L}\right)_{\text{disk}} = (0.014 \pm 0.003)V_{\text{max}} + (0.72 \pm 0.60)$$

with $r = 0.67$.

$$\left(\frac{M_{\text{dark}}}{M_{\text{lum}}}\right)_{\text{R}_{25}} = (2.37{\pm}0.39){-}(0.42{\pm}0.12)\left(\frac{M}{L}\right)_{\text{disk}}$$

with $r = 0.62$.

There are independent ways in which the maximum disk hypothesis can be checked.

The truncation feature in the rotation curve:

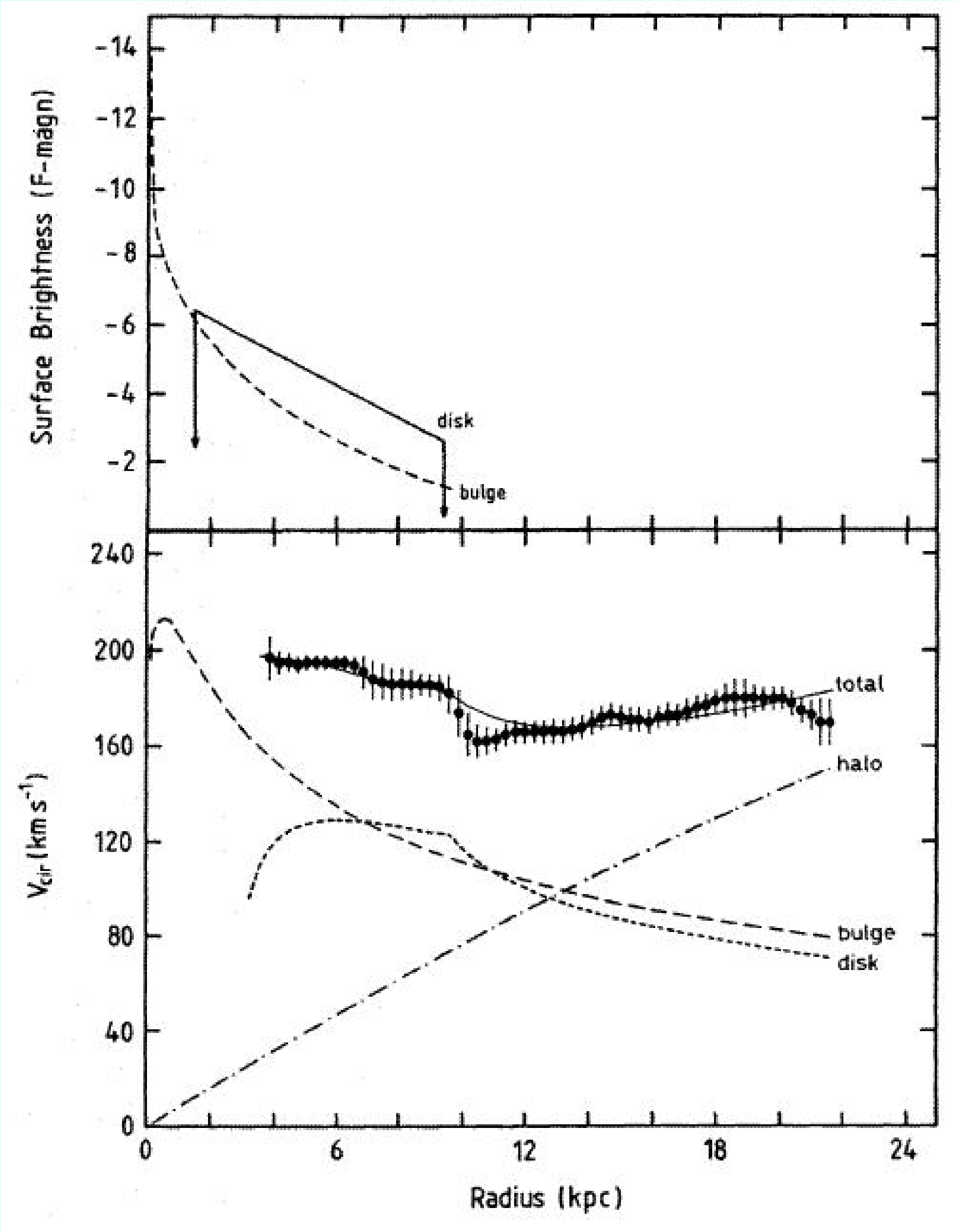
NGC 5907*: $(M_{\text{halo}})_{\text{R}_{\text{opt}}} \approx 60\%$ (so not maximum disk)

NGC 4013†: $(M_{\text{halo}})_{\text{R}_{\text{opt}}} \approx 25\%$

*Casertano, Mon.Not.R.A.S. 203, 735 (1983)

†Bottema, A.&A. 306, 345 (1996)

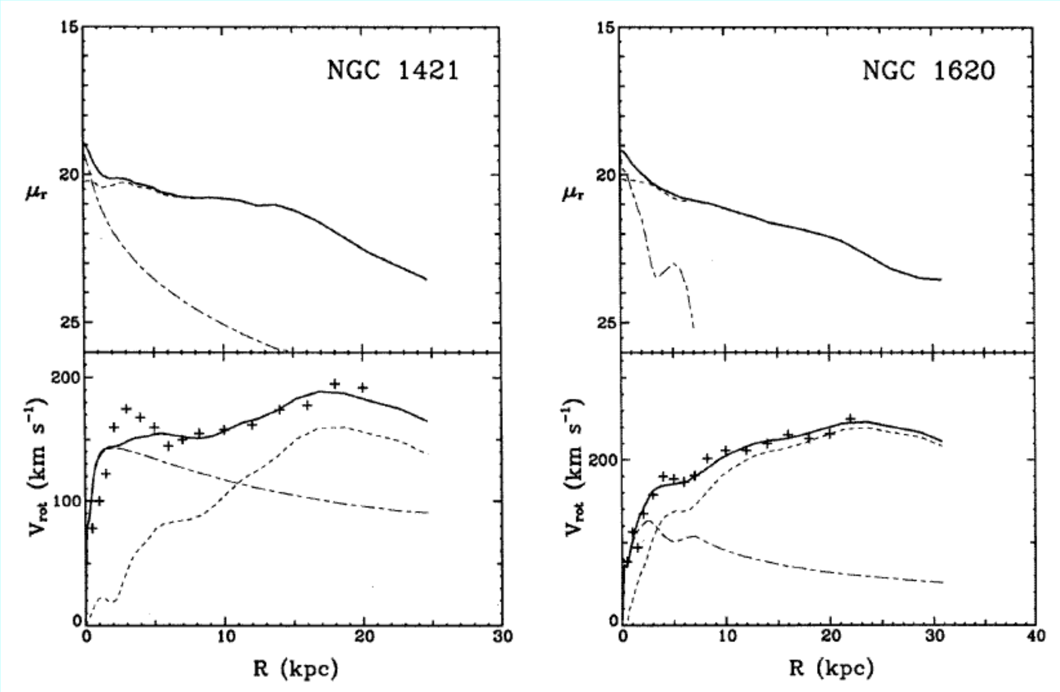
In NGC 4013 the disk and bulge must dominate dynamically in the inner regions.



This is Bottema's analysis of the rotation curve of NGC 4013.

“Wiggles” in rotation curves

The inner parts op rotation curves can often be fit wihtout a dark halo and features in luminosity profiles seem to correspond features in rotation curves*.



This suggests maximum disks, but even if disks are not dynamically dominant in the inner parts the wiggles can still be reproduced.†

*E.g. Kent, A.J. 91, 1301 (1986)

†van der Kruit, IAU Symp. 164, 227 (1995)

Maximum rotation versus scalelength

Recently* the following argument has been used.

For a pure exponential disk the maximum in the rotation curve occurs at $R = 2.2h$ with an amplitude of

$$V_{\text{max}} \propto \sqrt{h\sigma_\circ} \propto \sqrt{\frac{M_{\text{disk}}}{h}}$$

For fixed disk-mass M_{disk} this gives

$$\frac{\partial \log V_{\text{max}}}{\partial \log h} = -0.5$$

So at a given absolute magnitude (or mass) lower scalelength disks should have higher rotation.

If galaxies are maximum disk (in practice $V_{\text{disk}} \sim 0.85 V_{\text{total}}$) this should be seen in scatter of the Tully-Fisher relations.

This is not observed and the estimate is that on average $V_{\text{disk}} \sim 0.6 V_{\text{total}}$.

*Courteau & Rix, Ap.J. 513, 561 (1999)

Thickness of the HI-layer.

The thickness of the gas layer can be used to measure the surface density of the disk independent of the rotation curve.

The density distribution of the exponential, locally isothermal disk was:

$$\rho_*(R,z) = \rho_*(0,0)\ \exp\left(-R/h\right)\ \mathrm{sech}^{\,2}(z/z_\circ)$$

If the HI has a velocity dispersion $\langle V_\mathrm{z}^2 \rangle_\mathrm{HI}^{1/2}$, and if the stars dominate the gravitational field

$$\rho_\mathrm{HI}(R,z) = \rho_\mathrm{HI}(R,0)\ \mathrm{sech}^{\,2p}(z/z_\circ)$$

with

$$p = \frac{\langle V_\mathrm{z}^2 \rangle_*}{\langle V_\mathrm{z}^2 \rangle_\mathrm{HI}}$$

The full width at half maximum of this distribution is:

$$W_\mathrm{HI} = 1.663 p^{-1/2} z_\circ \ \text{ for } p \gg 1$$

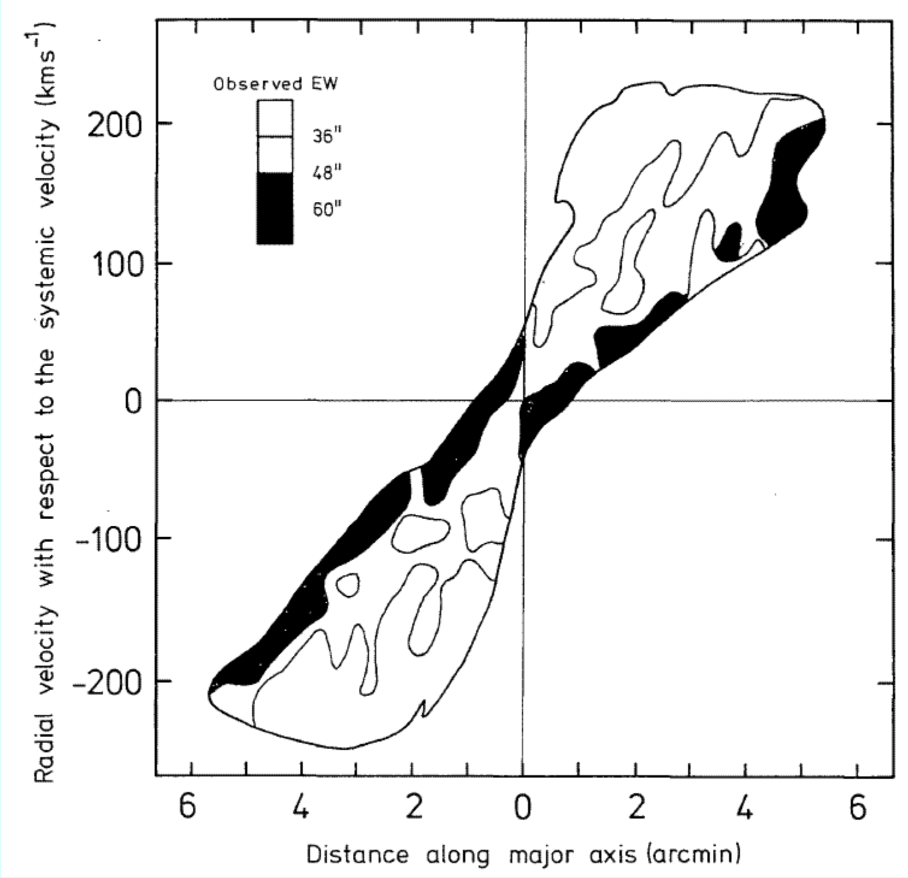
$$W_\mathrm{HI} = 1.763 p^{-1/2} z_\circ \ \text{ for } p = 1$$

Then to within 3 %

$$W_{\text{HI}} = 1.7 \langle V_{\text{z}}^2 \rangle_{\text{HI}}^{1/2} \left[\frac{\pi G (M/L) \mu_\circ}{z_\circ} \right]^{-1/2} \exp{(R/2h)}$$

So the gas layer increases exponentially in thickness with an e-folding of $2h$.

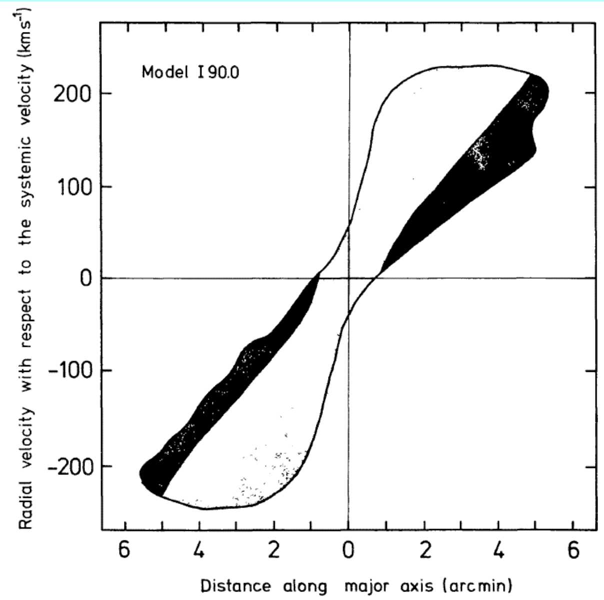
We look at the measurements of the HI-layer in NGC 891*.

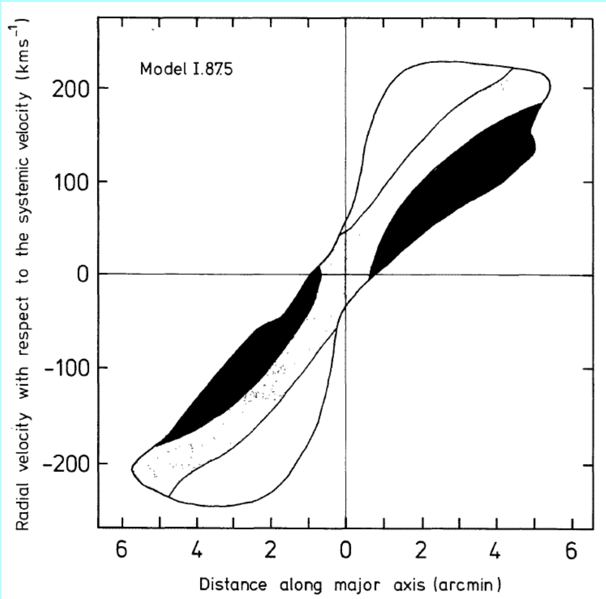


*van der Kruit, A.&A. 99, 298 (1981)

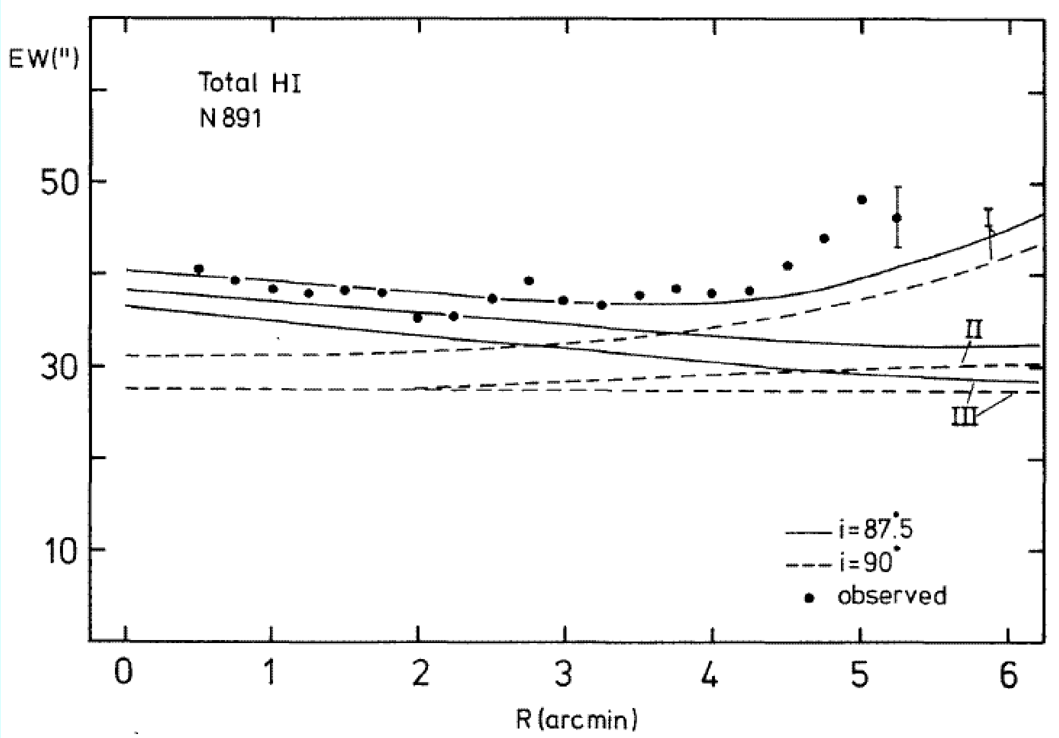
Three particular models were then calculated:
Model I (40% of the mass within the optical radius in the disk),
Model II with all the mass (including the dark mass) in the disk, and
Model III with a constant thickness of the HI-layer.

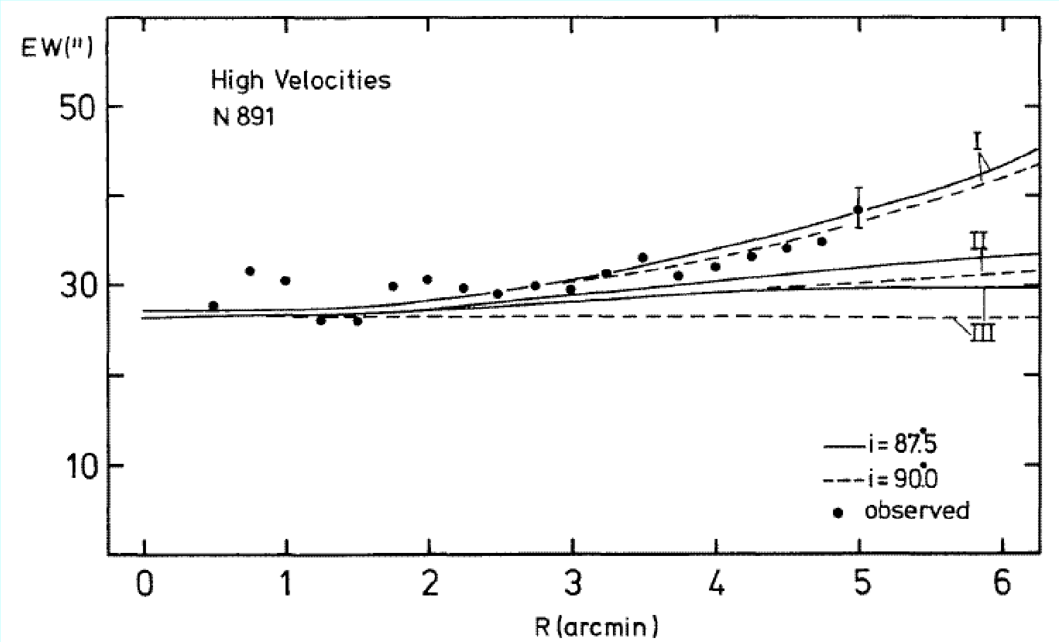
The $W_{\rm HI}$ in the observations were then calculated for disks with inclinations of 87.5 and 90°.





Also the thickness over all velocities and the "high" velocities (190 to 230 km/s) can be compared to observations.





NGC 891 is not maximum disk. Also this analysis shows that the dark matter cannot be in the disk.

Stellar kinematics and stability.

Later on we will look at stellar velocity dispersions and stability of stellar disks (Lecture 6).

The vertical motions of the stars can be combined with the thickness of stellar disks to provide estimates of the disk surface densities.

For the isothermal sheet with space density $\rho(z)$

$$\rho(z) = \rho(0)\ \mathrm{sech}^{2}(z/z_\circ)$$

we had for the velocity dispersion

$$\langle V_z^2 \rangle^{1/2} = \sqrt{2\pi G \rho(0)}\, z_\circ = \sqrt{\pi G \sigma}\, z_\circ$$

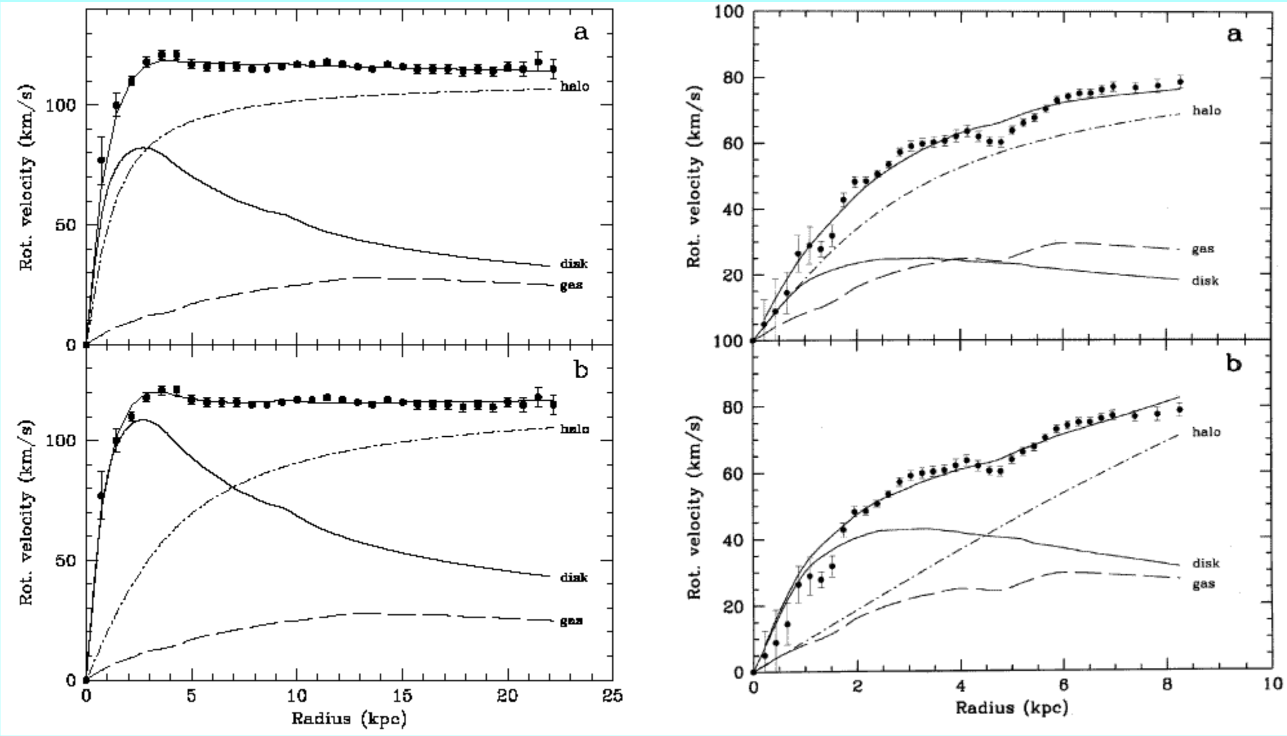
with σ the surface density.

Bottema* found that the stellar velocity dispersion at a fiducial radius correlates total rotation velocity.

Using this relation we can estimate the disk surface density if we know $z_\circ$ and the rotation curve.

*A.&A. 275, 16 (1993)

Analysis* on a high surface brightness and a low-surface brightness galaxy gives a model according to the stellar velocity dispersion as at the top and the maximum disk hypothesis as at the bottom.



Statistical analysis of samples of galaxies gives† then is

$$\frac{V_{\text{rot,disk}}}{V_{\text{rot,obs}}} = 0.57 \pm 0.22.$$

*Bottema, A.&A. 328, 517 (1997)

†Bottema, A.&A. 275, 16 (1993); Kregel, van der Kruit & de Grijs, Mon.Not.R.A.S. 334, 646 (2002)

Our Galaxy

The measured surface density of the stellar disk in the solar neighbourhood is 50* to 80† $M_\odot$ pc^{-2} and the scalelength of the disk 4 to 5 kpc‡.

With this it can be estimated that the luminous matter provides a maximum rotation velocity of 155 ± 30 km/s, while the observed value is 225 ± 10 km/s.

The Galaxy is then not maximum disk.

However, one can change the parameters within uncertainties to get any answer§.

*Kuijken and Gilmore, Mon.Not.R.A.S. 239, 605 (1989): $\sigma_\odot = 46 \pm 9 M_\odot pc^{-2}$

†Bahcall, Ap.J. 287, 926 (1984): $\sigma_\odot = 80 \pm 20 M_\odot pc^{-2}$

‡van der Kruit, A&A. 157,230 (1986)

§Sellwood & Sanders, Mon.Not.R.A.S. 233, 611 (1988); Sackett, Ap.J. 483, 103 (1997); Bottema, A.&A. 328, 517 (1997)

Modified dynamics

Flat rotation curves may show that classic Newtonian gravity does not work at large distances*.

For this purpose Modified Newtonian Dynamics (MOND)† was developed.

This has an accelaration $\vec{g}$, which is related to Newtonian acceleration $\vec{g}_{\mathrm{N}}$ as

$$\vec{g}\left(\frac{g}{a_\circ}\right)\left[1+\left(\frac{g}{a_\circ}\right)^2\right]^{-1/2}=\vec{g}_{\mathrm{N}}$$

with $a_\circ \sim 1.2 \times 10^{-8}$ cm sec^{-2}.

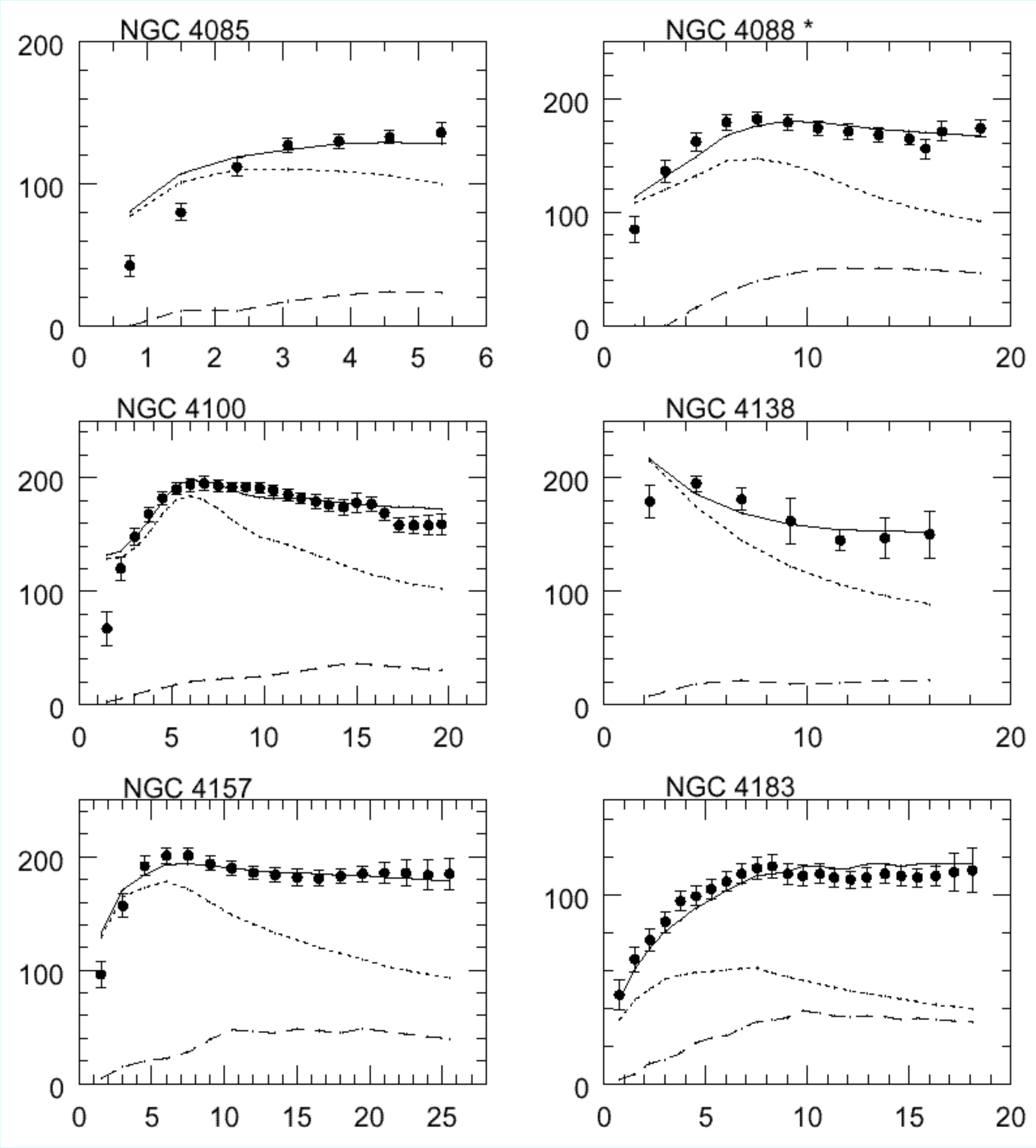
For large $g/a_\circ$ this reduces to Newtonian gravity and at low accelerations is becomes $g = (g_{\mathrm{N}} a_\circ)^{1/2}$.

The result is that flat rotation curves can be produced without introducing a dark halo .

*e.g. Sanders, Mon.Not.R.A.S. 223, 539 (1986); Begeman, Broeils & Sanders, Mon.Not.R.A.S. 249,523 (1991)

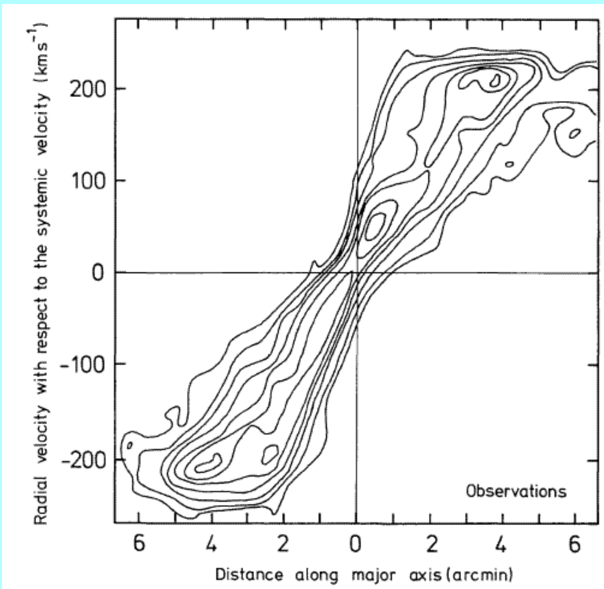
†e.g. Milgrom, Ap.J. 270, 365 (1983)

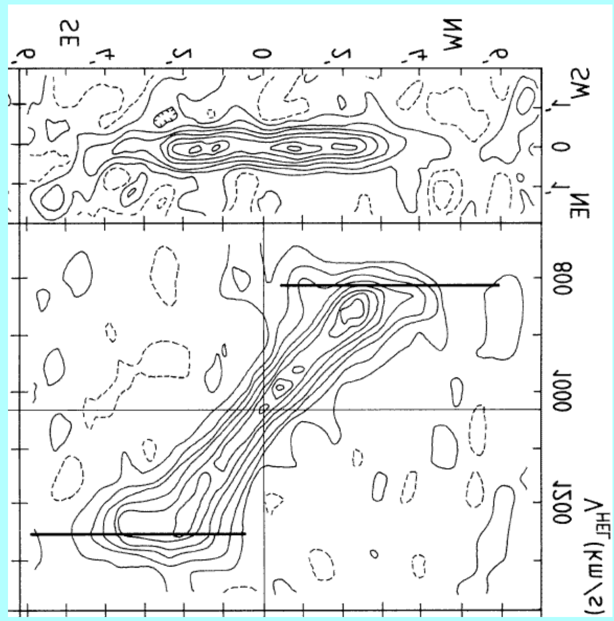
Here are some fits to actual rotation curves*. The full lines are the MOND-fits and the other lines show Newtonian curves for the stars and gas.

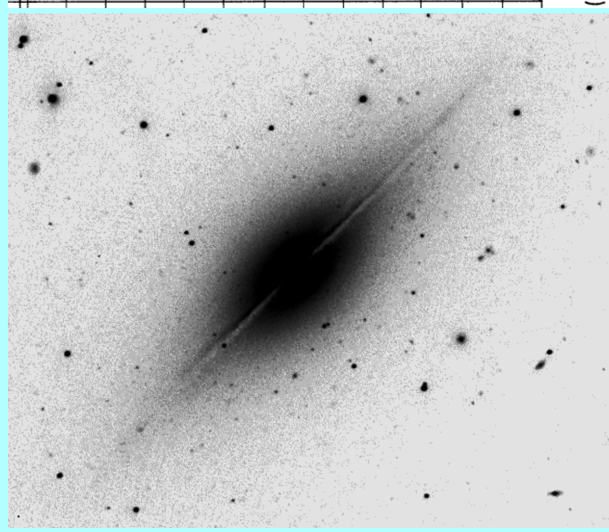


*Sanders & Verheijen, Ap.J. 503, 97 (1998)

NGC 891 and 7814 have the same rotation curves but completely different light distributions.







This is inconsistent with MOND.

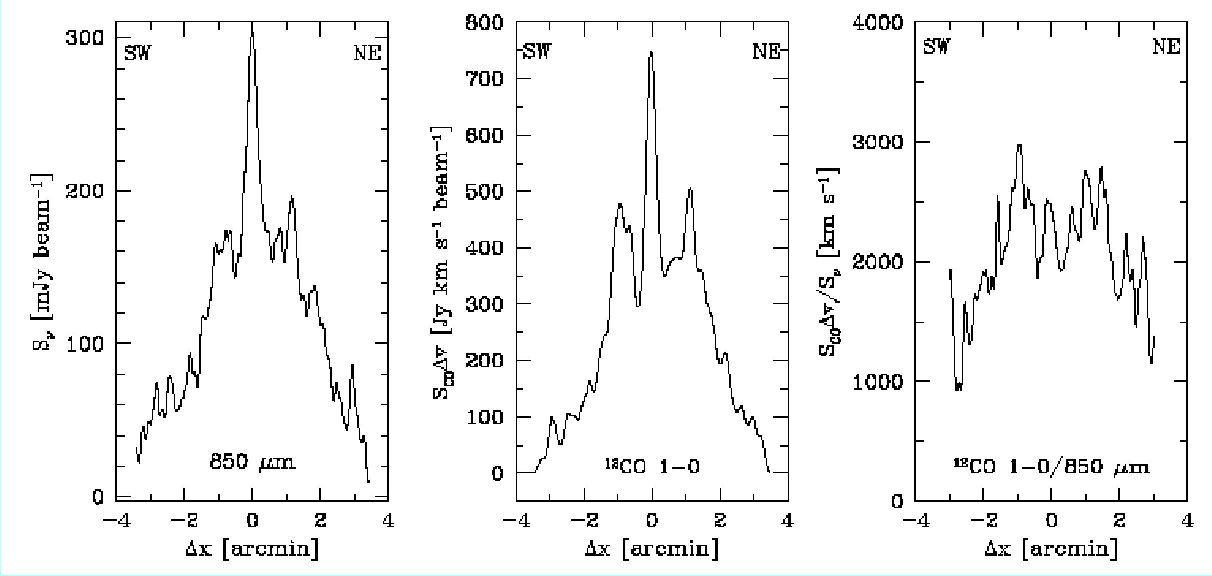
CO and H_2.

The distribution of molecular hydrogen is often inferred from observations of CO at (sub-)millimeter wavelengths.

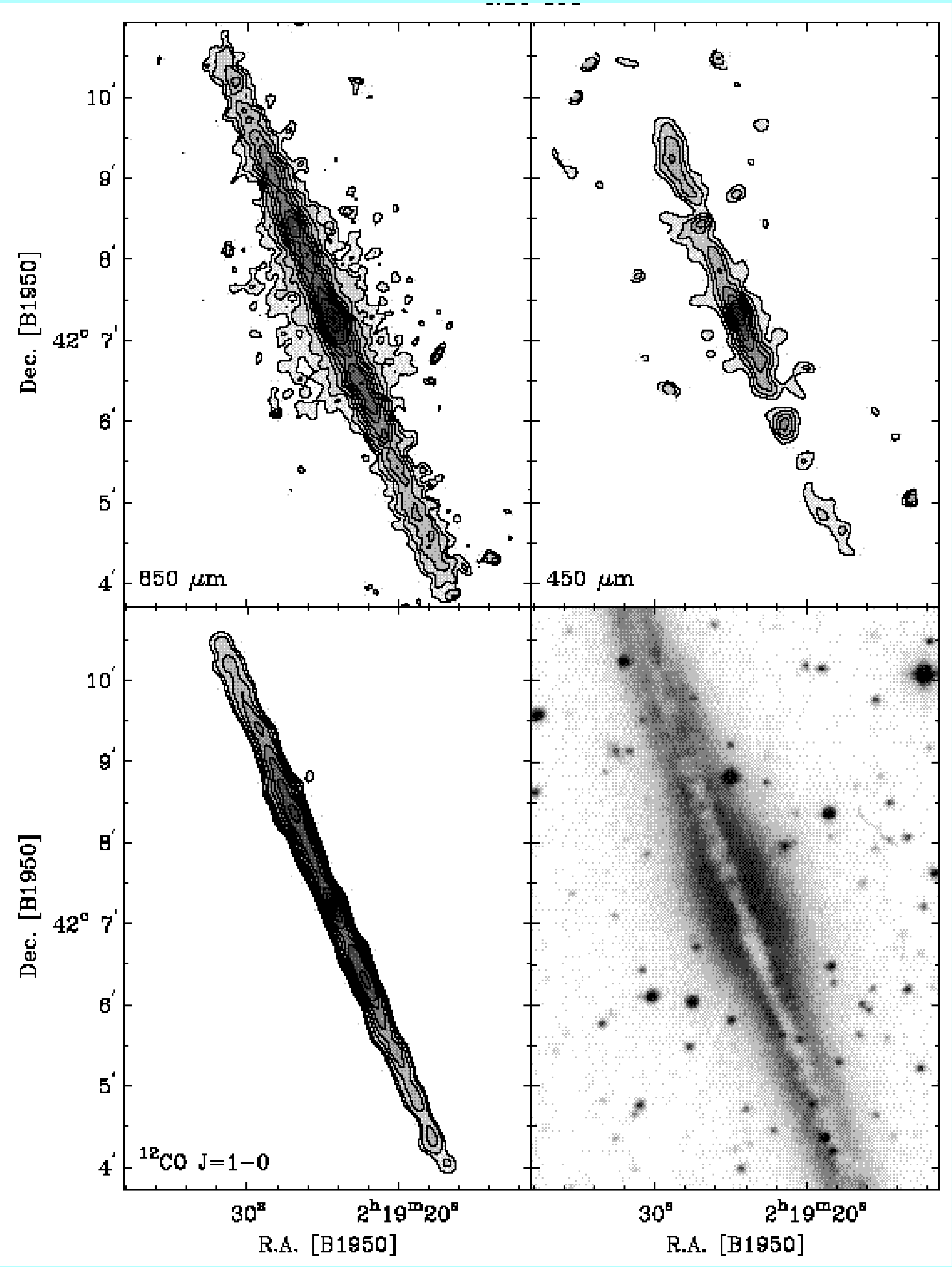
The assumption is that everywhere the ratio between these two molecules is the same.

Here are some observations of NGC 891*.

Also near-infrared observations are shown that show the distribution of the dust.



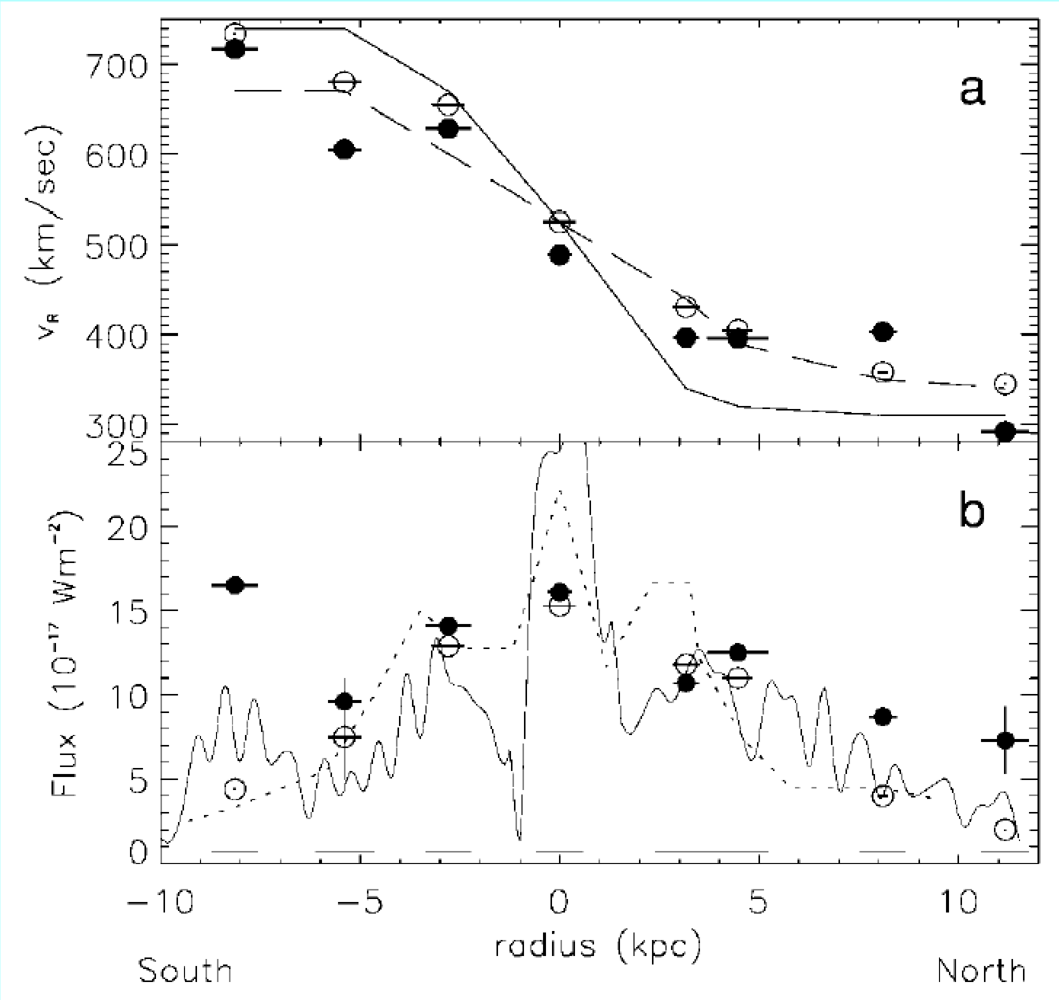
*Israel, van der Werf & Tilanus, A.&A. 334, L83 (1999)



10′
9′
8′
42° 7′
6′
5′
4′
Dec. [B1950]
850 μm
450 μm
12CO J=1−0
30s
2h19m20s
R.A. [B1950]

Only recently has it been possible to directly measure lines of H_2 with the Infrared Space Observatory (ISO)*.

We see here observations of the S(0) (28.2 μ) (filled) and S(1) (17.0 μ) (open) lines, compared with CO-observations.

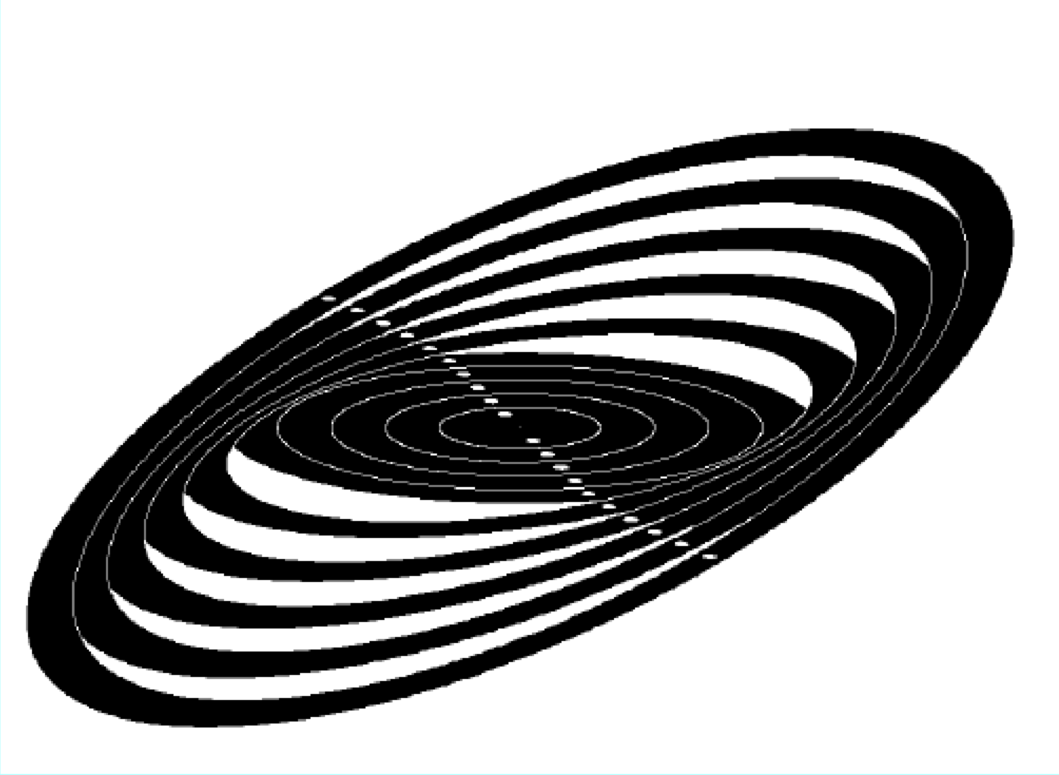


*Valentijn & van der Werf, Ap.J. 522, L29 (1999)

Warps.

Warps are now known to be fairly common.

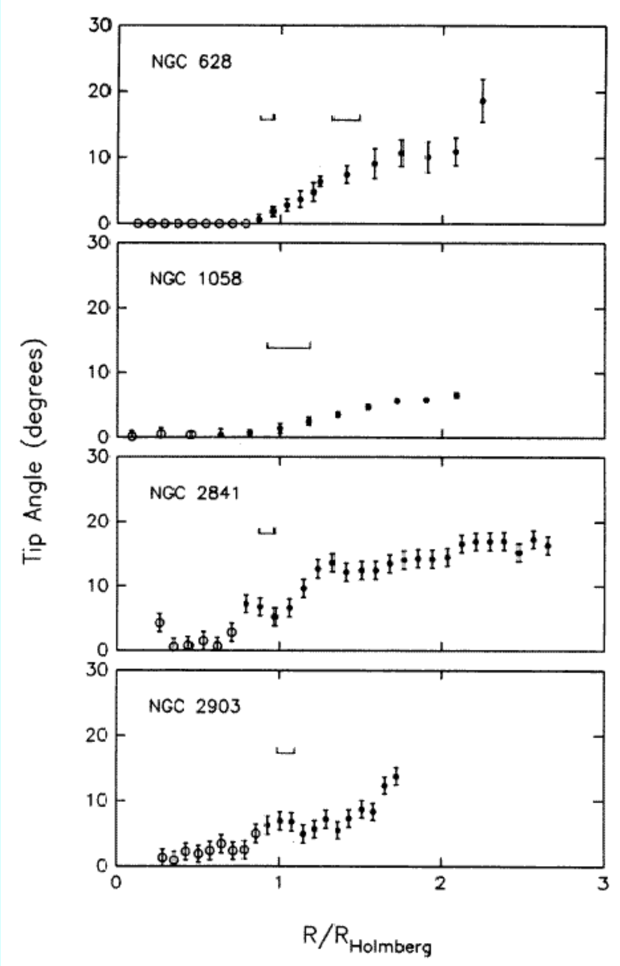
In inclined galaxies they are studied by fitting a "tilted-ring model" to the distribution and velocity field of the HI.

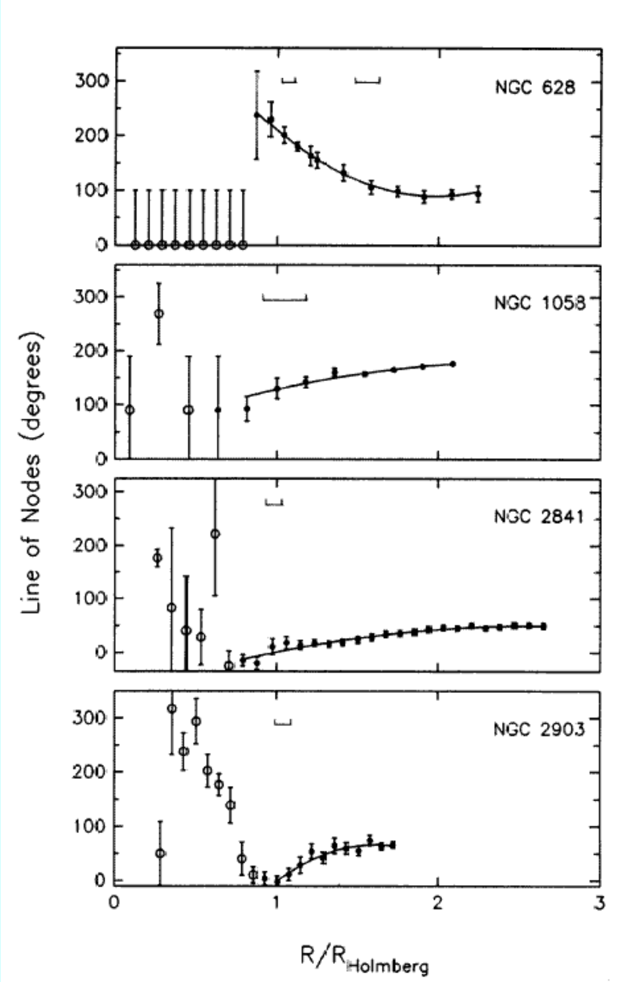


Each ring has a line of nodes where it crosses the plane of the inner disk and a tip angle.

Briggs* collected the available data on 12 well-studied galaxies.

*Ap.J. 352, 15 (1990)





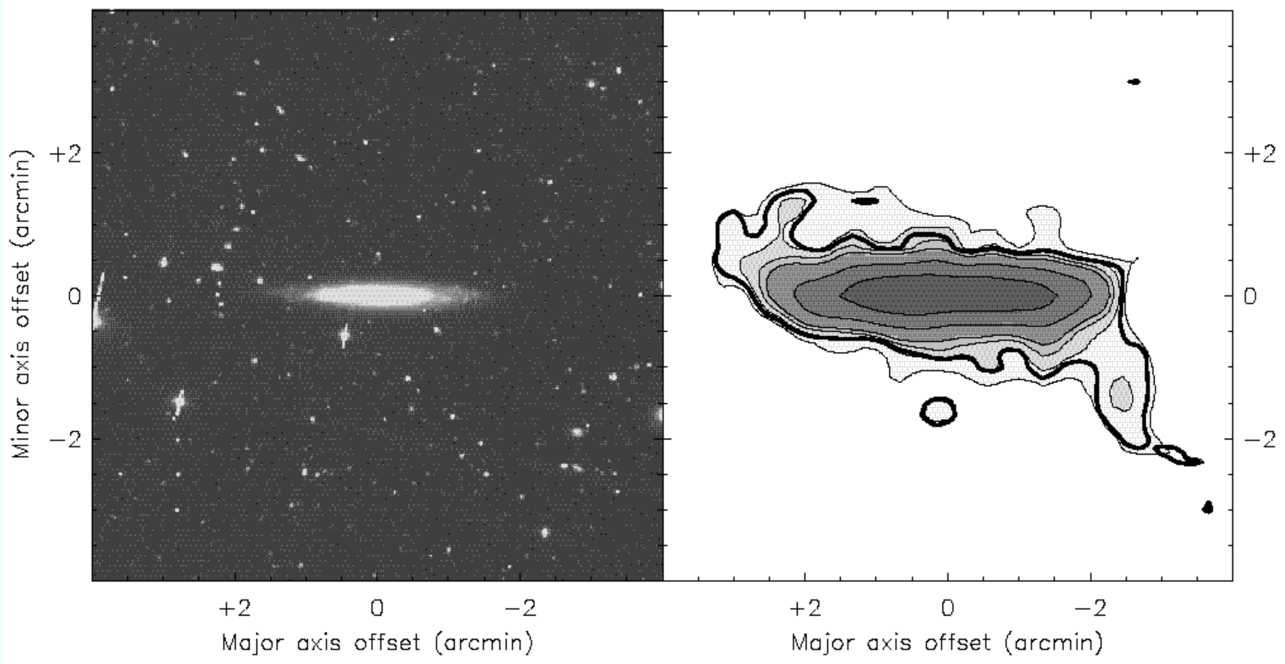
Outside the optical radius the line of nodes advances in the plane of the galaxy in the direction of rotation (forms a leading spiral arm).

This is consistent with the expectation that the outer rings precess more slowly.

Studies* of the time evolution of warps can in principle set limits on parameters of the dark halo, in particular its oblateness.

*Casertano & Sparke, Mon.Not.R.A.S. 234, 873 (1988); Hofner & Sparke, Ap.J. 428, 466 (1994)

García Ruiz* has done a survey of edge-on galaxies.



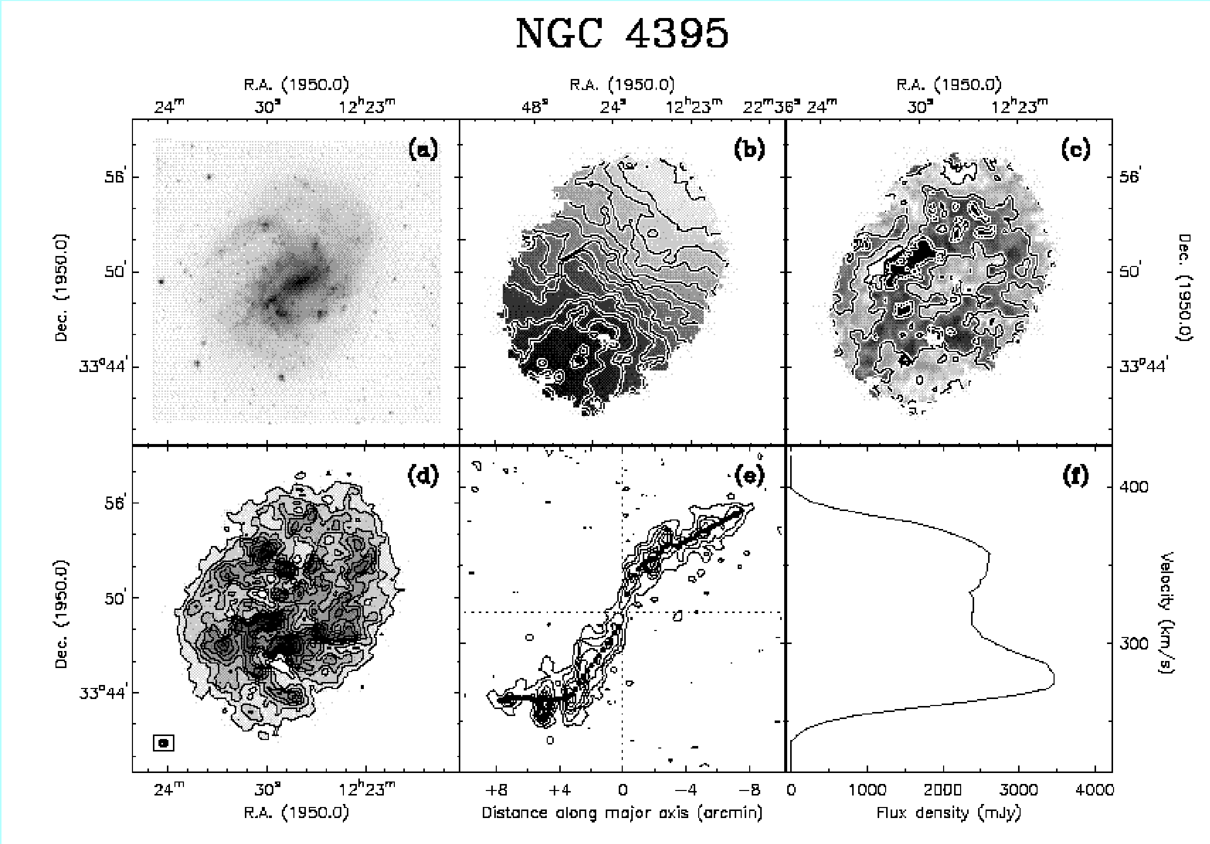
• All galaxies, in which the HI is more extended than the stellar disk have warps.

• The warp usually starts near the edge of the stellar disk.

• Galaxies in rich environments tend to have larger and more asymmetric warps.

*Ph.D. thesis (2001)

Often spiral galaxies are "lob-sided"* in their outer HI, such as NGC4395.



(panel c has residual velocities)

This has been explained as disks that are lying off-center in a dark halo†.

*Schoenmakers, Ph.D. thesis (1999), Swaters, Schoenmakers, Sancisi & van Albada, Mon.Not.R.A.S. 304, 330 (1999)

†Levine & Sparke, Ap.J. 496, L13 (1998); Noodermeer, Sparke & Levine, Mon.Not.R.A.S. 328, 1064 (2001)