

STRUCTURE OF GALAXIES

Lecture 8. Abundance gradients, chemical evolution, absorption in disks.

a. Observations of abundance gradients.

Bulges have color gradients (become bluer with radius).

This is due to metallicity changes.

For a low $[\text{Fe}/\text{H}]$ in an old population:

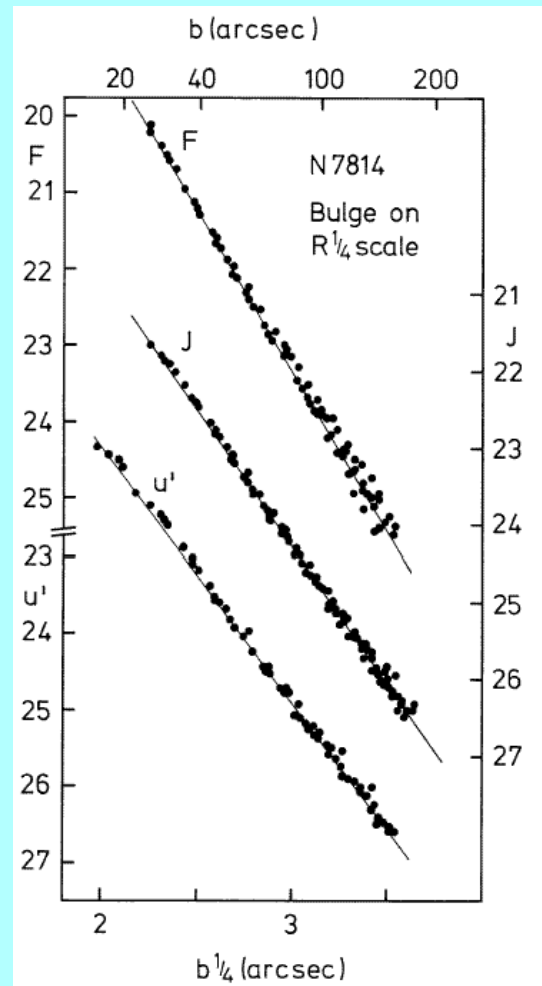
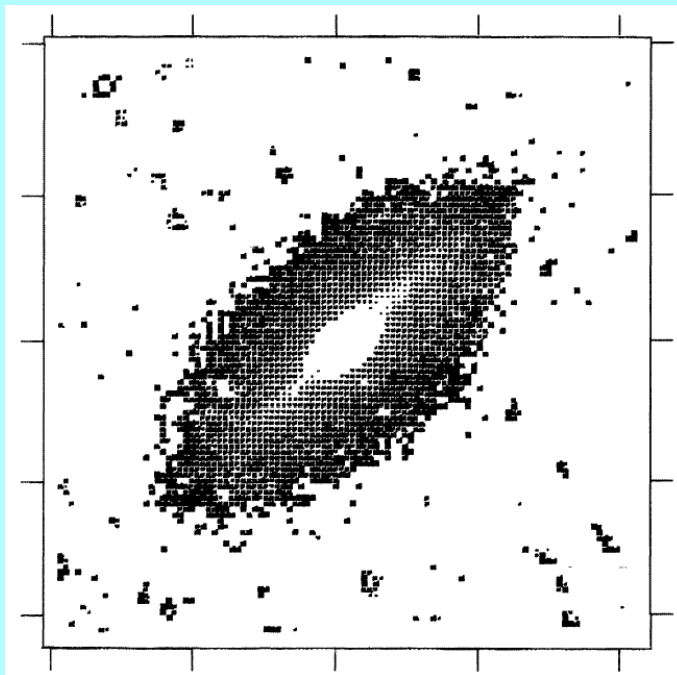
- The effective temperature of the **giant branch** is higher
- There is less **line-blanketing**
- The **horizontal branch** is more extended towards the blue.

The relation between color and metallicity can be calibrated using the integrated light of Galactic globular clusters.

The range in $(U-B), (B-V)$ in bulges is roughly that in globular clusters.

So the **range in metallicity** in bulges is **1 - 2 dex** in $[\text{Fe}/\text{H}]$.

NGC 7814*



$$\mu_{U'} = 14.87 + 3.32b^{1/4}$$

$$\mu_{J_{\text{opt}}} = 13.72 + 3.55b^{1/4}$$

$$\mu_V = 13.08 + 3.75b^{1/4}$$

$$\mu_F = 10.70 + 4.20b^{1/4}$$

$$\mu_J = 9.19 + 4.36b^{1/4}$$

$$\mu_K = 8.07 + 4.43b^{1/4}$$

*van der Kruit & Searle, A.&A. 110, 79 (1982)

Disks have gradients in emission line ratio's in HII regions.

Some prominent emission lines in spectra of HII-regions are the following:

Ion	Wavelength
[OII]	3726/3729
H δ	4101
H γ	4340
H β	4861
[OIII]	4959/5007
H α	6562
[NII]	6548/6583
[SII]	6716/6731

An often used parameter is the “**excitation**”, which is the ratio of the strengths of the **[OIII]** and **H β** lines.

These are at about the same wavelength, so this ratio is not sensitive to extinction corrections.

The excitation could change due to a number of effects:

- Changing **dust content** and therefore radiation field
- Changing **stellar temperatures**; increasing T_{eff} gives increasing excitation
- Changing **abundance** because of cooling through O- and N-ions:
A lower oxygen abundance gives an increased T_e and then we get stronger O-lines; thus $[\text{OIII}]/\text{H}\beta$ *increases with decreasing metallicity*.

Detailed studies* have shown that the effect of abundance gradients is probably the most important.

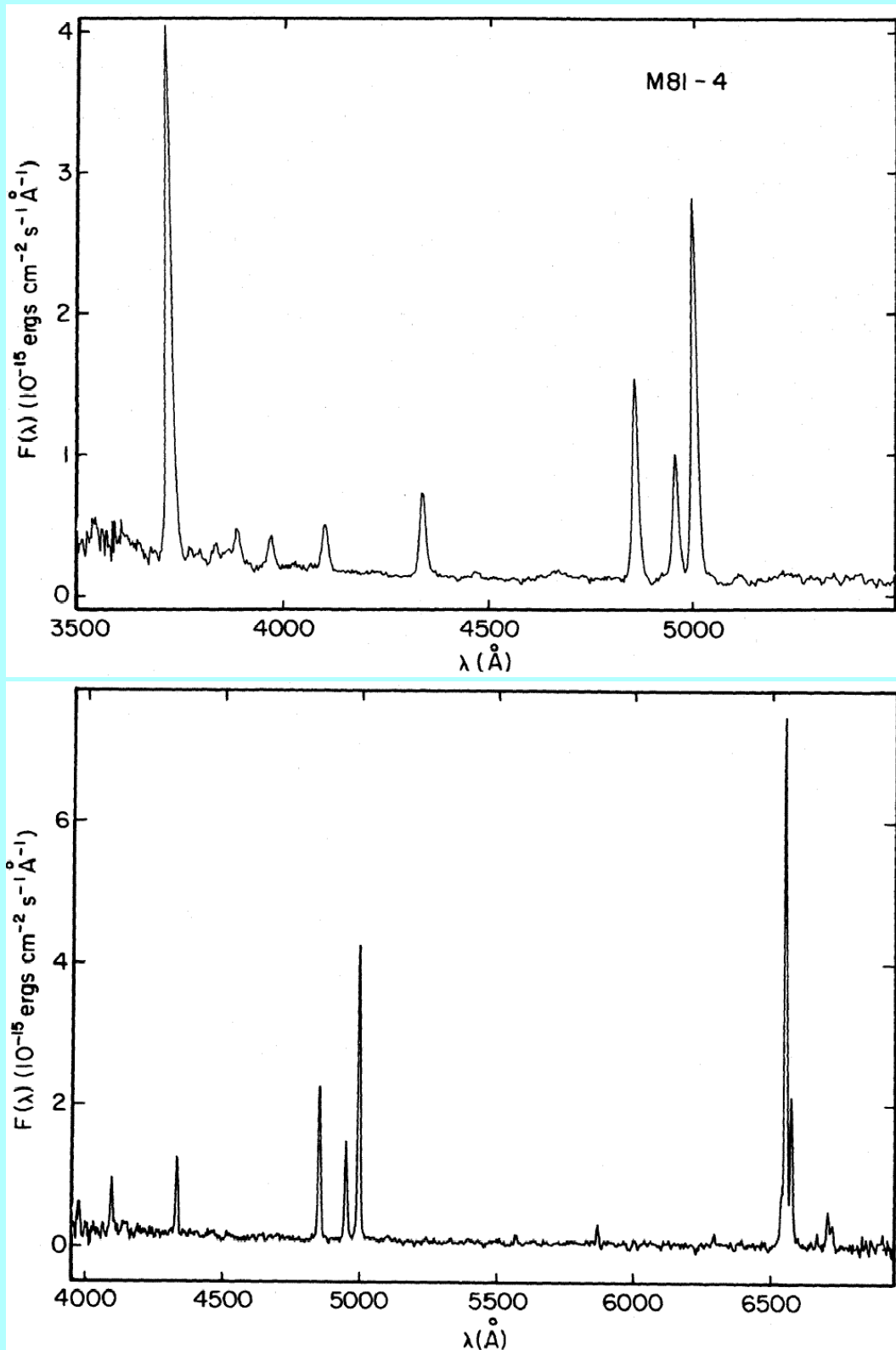
*Searle, Ap.J. 168, 327 (1973)

As an example we have a detailed look at measurement in M81* between 3 and 15 kpc.

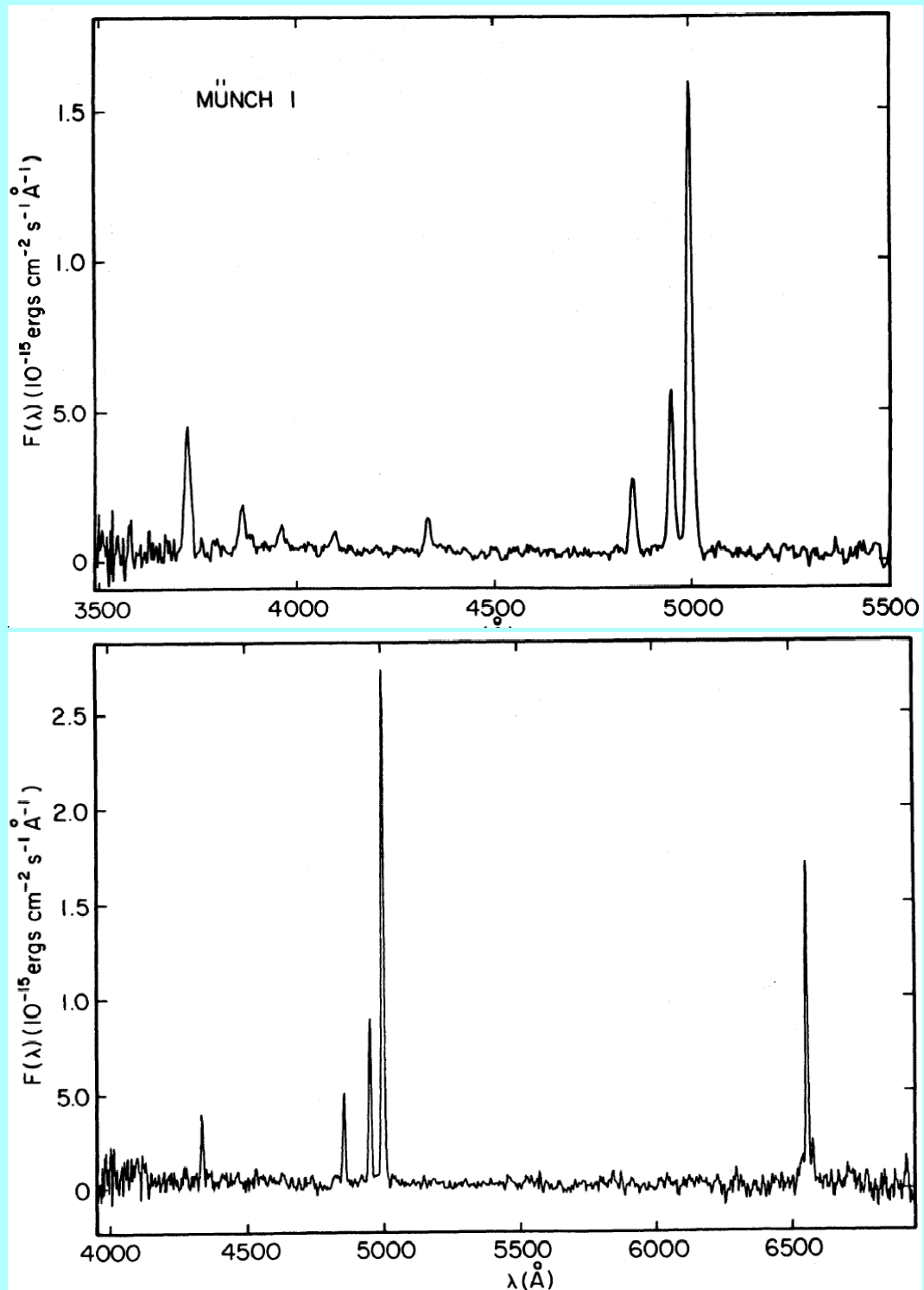


*Garnett & Shields, Ap.J. 317, 82 (1987)

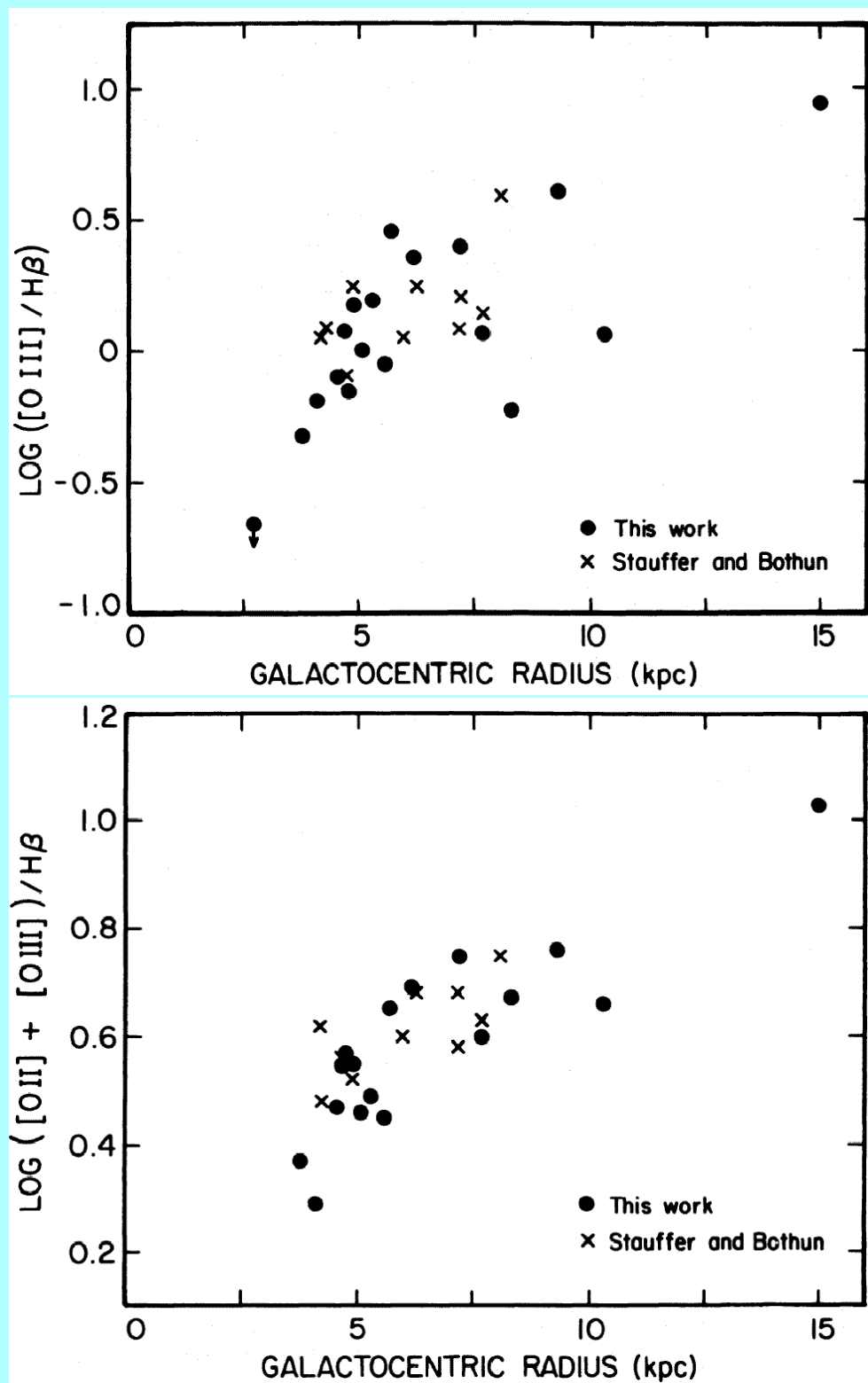
This is the spectrum of an HII-region at $R = 7$ kpc.



This is the spectrum of an HII-region at $R = 15$ kpc.



Here we see the gradients in $[\text{OIII}]/\text{H}\beta$ ratio and the $([\text{OIII}]+[\text{OII}])/\text{H}\beta$ ratio.

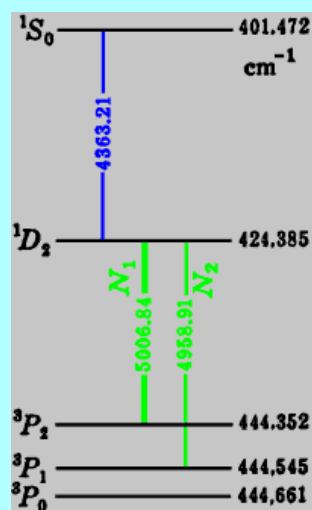


The use of [OIII] and [OII] has the advantage that two levels of ionisation of the oxygen are taken.

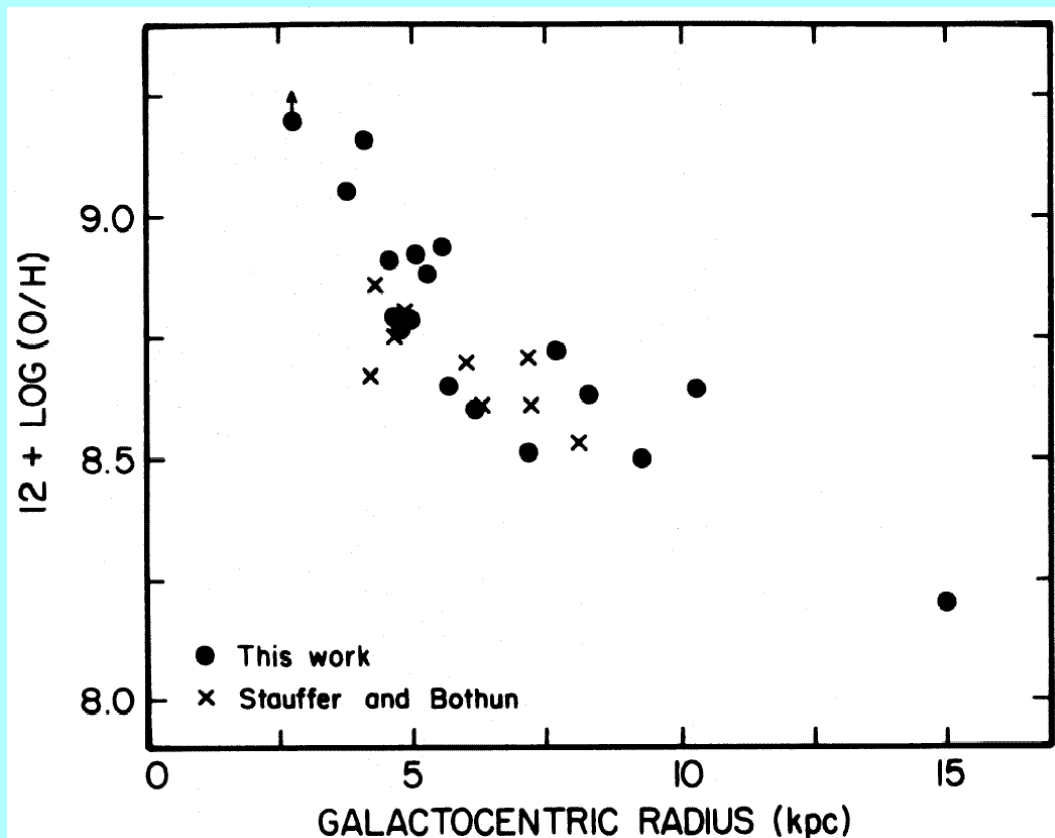
The disadvantage is that the extinction corrections are important.

The line ratio's must be transformed into abundances. The calibration of excitation into abundance can be done in two ways:

- Measure the weak [OIII] line at λ 4363 in addition to the lines at λ 4959 and 5007. Then lines are measured involving the same level and from this the electron temperature T_e can be calculated. This allows the determination of the oxygen-hydrogen ratio.



- The second possibility is to calculate full sets of photoionization models of HII regions.



The result is a gradient of $-0.08 \text{ dex kpc}^{-1}$ in $[\text{O}/\text{H}]$.

This is a typical value for spirals, including our own.

b. Chemical evolution.

Take a volume (either a whole galaxy or a part of it) and define within that volume:

M_g = Mass in gas

M_* = Mass in stars

M_Z = Mass in heavy elements

$Z(t) = M_Z(t)/M_g(t)$ = Abundance

$$y = \frac{\text{Mass injected in new metals}}{\text{Mass locked in long – lived stars}} = \text{Yield}^*$$

The **Instantaneous Recycling Approximation** says that star evolution of heavy stars is instantaneous and that the products are mixed instantaneously into the interstellar medium.

*Searle & Sargent, Ap.J. 173,25 (1972)

Assume the system is **closed** (no inflow or outflow of gas).

Then the fundamental equations are:

$$\frac{dM_Z}{dt} = y \frac{dM_*}{dt} - Z(t) \frac{dM_*}{dt}$$

$$\frac{dM_g}{dt} = - \frac{dM_*}{dt}$$

A. The Simple Model.

This assumes that $Z(t=0) = Z_o = 0$.

Define

$$x = \frac{M_g(t)}{M_{\text{tot}}}$$

The fundamental equations can then be solved to give

$$Z(t) = y \ln \left(\frac{1}{x} \right)$$

The metal abundance of the gas is an increasing function of the **gas fraction** x and **time**.

Stars have the abundance of the gas at the time of their birth.

The fraction of stars at time t with abundance $Z \leq Z_1 (\leq Z(t))$ is:

$$F(Z) = \frac{1 - x_1}{1 - x}$$

$$x_1 = \exp - \left(\frac{Z_1}{y} \right)$$

So

$$\langle Z \rangle = y \frac{1 - x(1 - \ln x)}{1 - x}$$

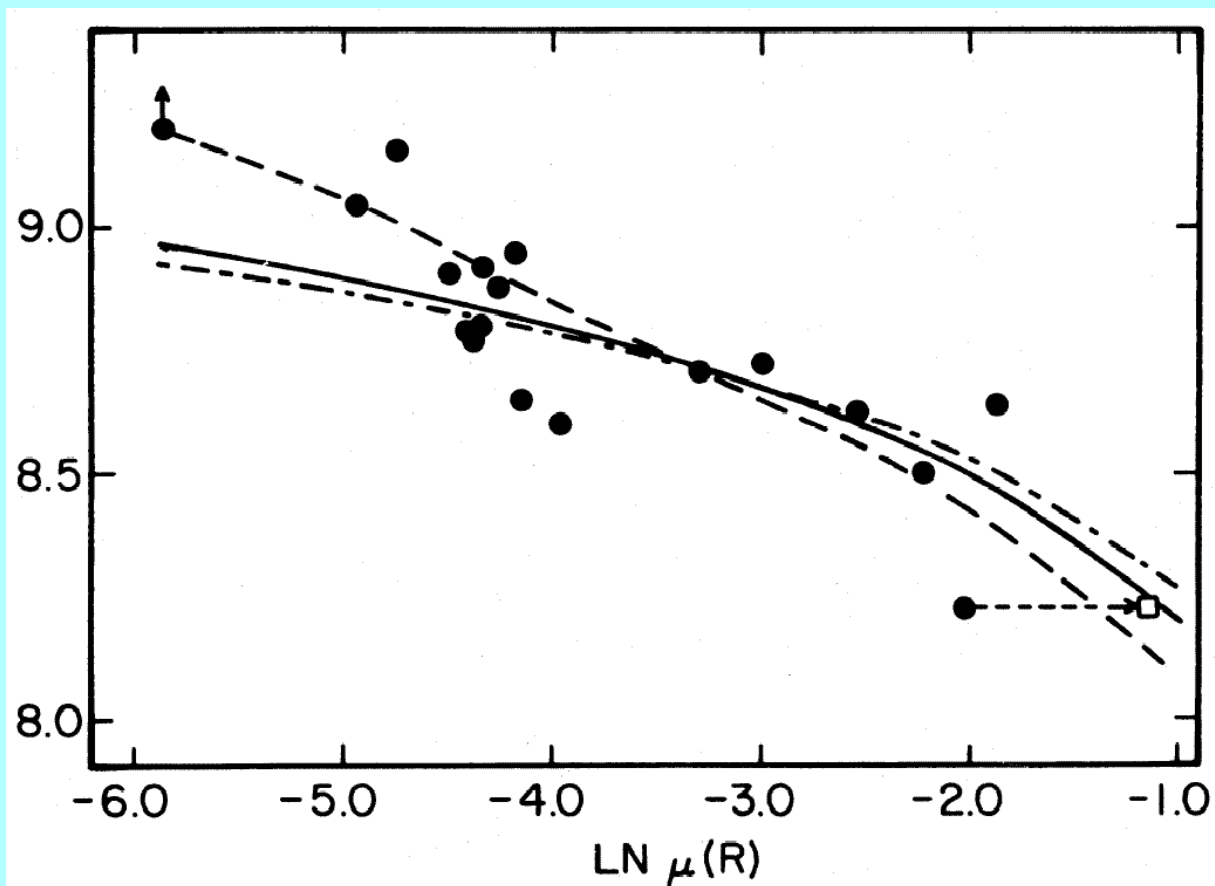
Use up all the gas ($x \rightarrow 0$), then $\langle Z \rangle \rightarrow y$.

So: **Abundance of gas** $\rightarrow \infty$.

The **mean** abundance of stars $\rightarrow y$.

The observations in M81 can be used to test this model.

For that purpose the radius has been replaced by the gas fraction (from the HI and the photometry) $\mu(R)$.



The Simple Model is the full-drawn line.

The Simple Model has the **G-dwarf problem**: it predicts far too many stars of low metallicity.

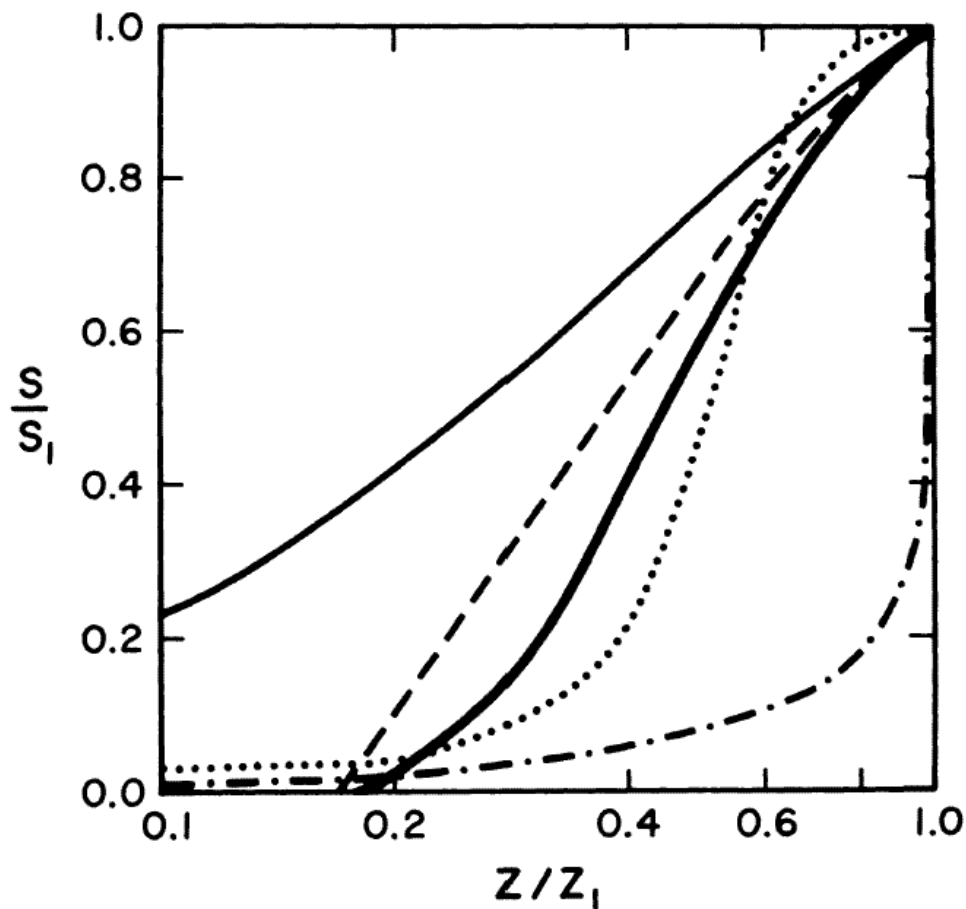


Figure 5 Metallicity distributions. S/S_1 is the fraction of G-K dwarfs in the solar neighborhood with metal abundance less than Z , where Z_1 is the present interstellar abundance (except as noted below). *Heavy line*: schematic representation of the data after removing an estimated dispersion due to observational errors (after Pagel & Patchett 1975). *Light solid line*: the “simple model” [Equation (3)]. *Dashed line*: effect of a finite initial abundance, $Z_0 = 0.17 Z_1$. *Dash-dotted line*: an infall model [Equation (4)]. *Dotted line*: the infall model with a log gaussian distribution of Z at all times, with $\sigma(\log Z) = 0.2$. In this case, Z_1 is the value at which $S/S_1 \approx 1$ (cf Tinsley 1975a).

The simple model predicts that of the **G-dwarfs** in the solar neighborhood more than 40% should have a metallicity less than 0.2 of solar.

B. The Extended Simple Model.

The assumptions are the same as in the simple model, except that $Z_0 \neq 0$.

This is also known as **Prompt Initial Enrichment**.

Then everywhere replace y with $y + Z_0$ and the equations look the same.

The solution then is

$$Z(t) = Z_0 + y \ln \left(\frac{1}{x} \right)$$

So, now when we use up all gas, we get

Abundance of gas $\rightarrow \infty$.

Mean abundance of stars $\rightarrow y + Z_0$.

Because the metallicity of the gas is initially finite, there are (much) fewer metal-poor stars.

C. Inflow Model.

Assume an inflow $f(t)$ of unprocessed material.

This means that there is less gas in the beginning compared to the simple model and the enrichment then proceeds much faster and therefore decreases the predicted number of G-dwarfs.

The second fundamental equation becomes

$$\frac{dM_g}{dt} = -\frac{dM_*}{dt} + f(t)$$

This model cannot be solved analytically in the general case, but it can be done for the extreme inflow model, where $M_g = \text{constant}$.

Define

$$\mu = \frac{M_*(t)}{M_g}$$

Then

$$Z(t) = y \{1 - \exp(-\mu)\}$$

It can then be found that

$$F(Z) = \frac{\mu}{\mu_1}$$

$$\langle Z \rangle = y - \frac{y}{\mu} + \frac{y}{\mu} \exp(-\mu)$$

If we now use up all gas, we get

$$\mu \rightarrow \infty \text{ and } \langle Z \rangle \rightarrow y.$$

Abundance of gas $\rightarrow y$.

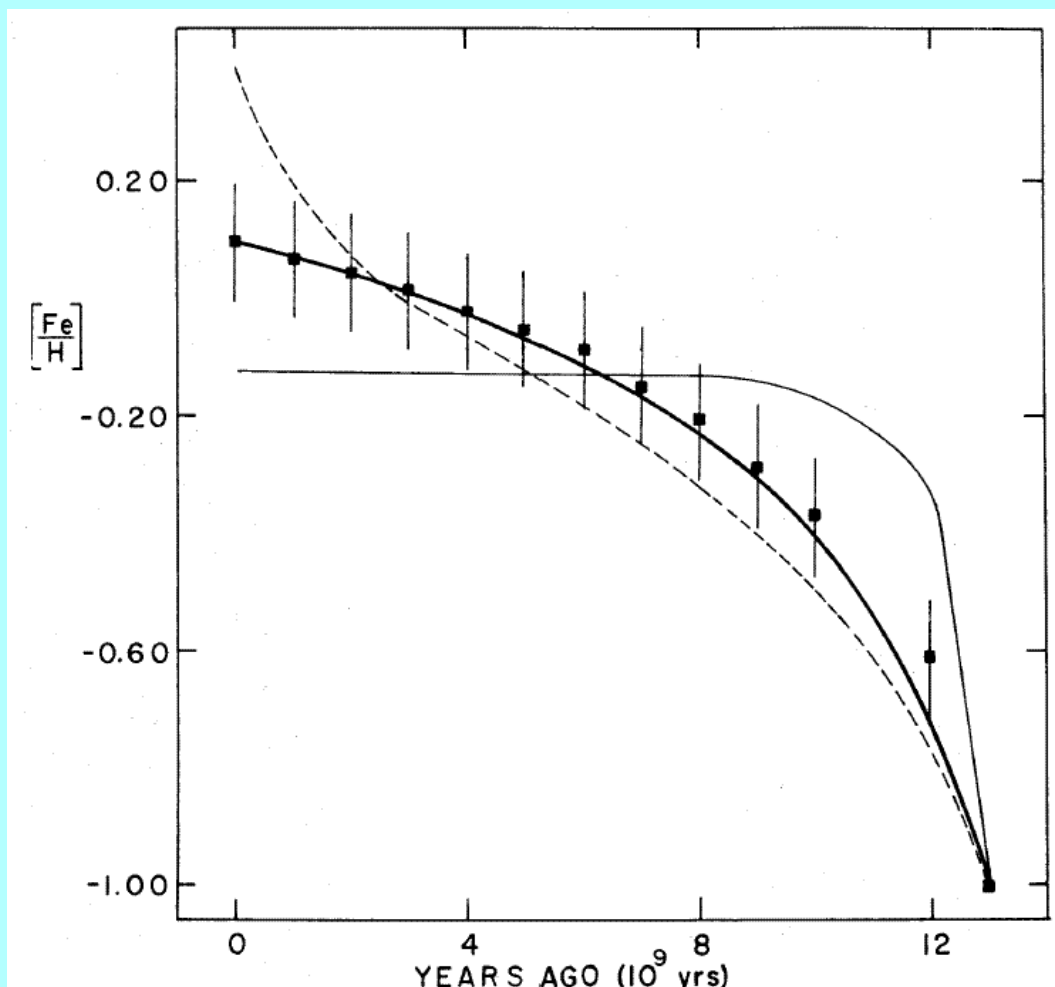
Mean abundance of stars $\rightarrow y$.

The extreme inflow model is much too extreme in that it now predicts too few metal-poor stars. So, M_{gas} must have decreased with time.

The inflow is possibly seen in the high-velocity clouds.

The best fit can be found for the solar neighborhood with a combination of prompt initial enrichment and inflow.

When the time is made explicit (e.g. by assuming that the SFR is constant) this model can reproduce the **metallicity - age relation***.



*Twarog, Ap.J. 242, 242 (1980)

D. Simple Model with Bells and Whistles.*

This term is now used for any model that relaxes the assumptions of the simple model, but was used originally for models with outflow of processed material.

Let there be an outflow of processed material $g(t)$.

Then the fundamental equations become

$$\frac{dM_Z}{dt} = y \frac{dM_*}{dt} - Z(t) \frac{dM_*}{dt} - Z(t)g(t)$$

$$\frac{dM_g}{dt} = - \frac{dM_*}{dt} - g(t)$$

For an illustrative case that can be solved analytically, take

$$g(t) = \alpha \frac{dM_*}{dt}$$

*Mould, P.A.S.P. 96, 773 (1984)

Then we have the fundamental equations back with y replaced with an *effective yield*

$$y' = \frac{y}{1 + \alpha}$$

The solution is then

$$Z(t) = \frac{y}{1 + \alpha} \ln \left(\frac{1}{x} \right)$$

Use up all gas, then:

Abundance of gas $\rightarrow \infty$.

Mean abundance of stars $\rightarrow y' = y/(1 + \alpha)$.

For elliptical galaxies there is a *mass - metallicity relation**. This can be explained if elliptical galaxies have (had) outflow of processed material, which must have been more pronounced in smaller systems.

*Mould, P.A.S.P. 96, 773 (1984)

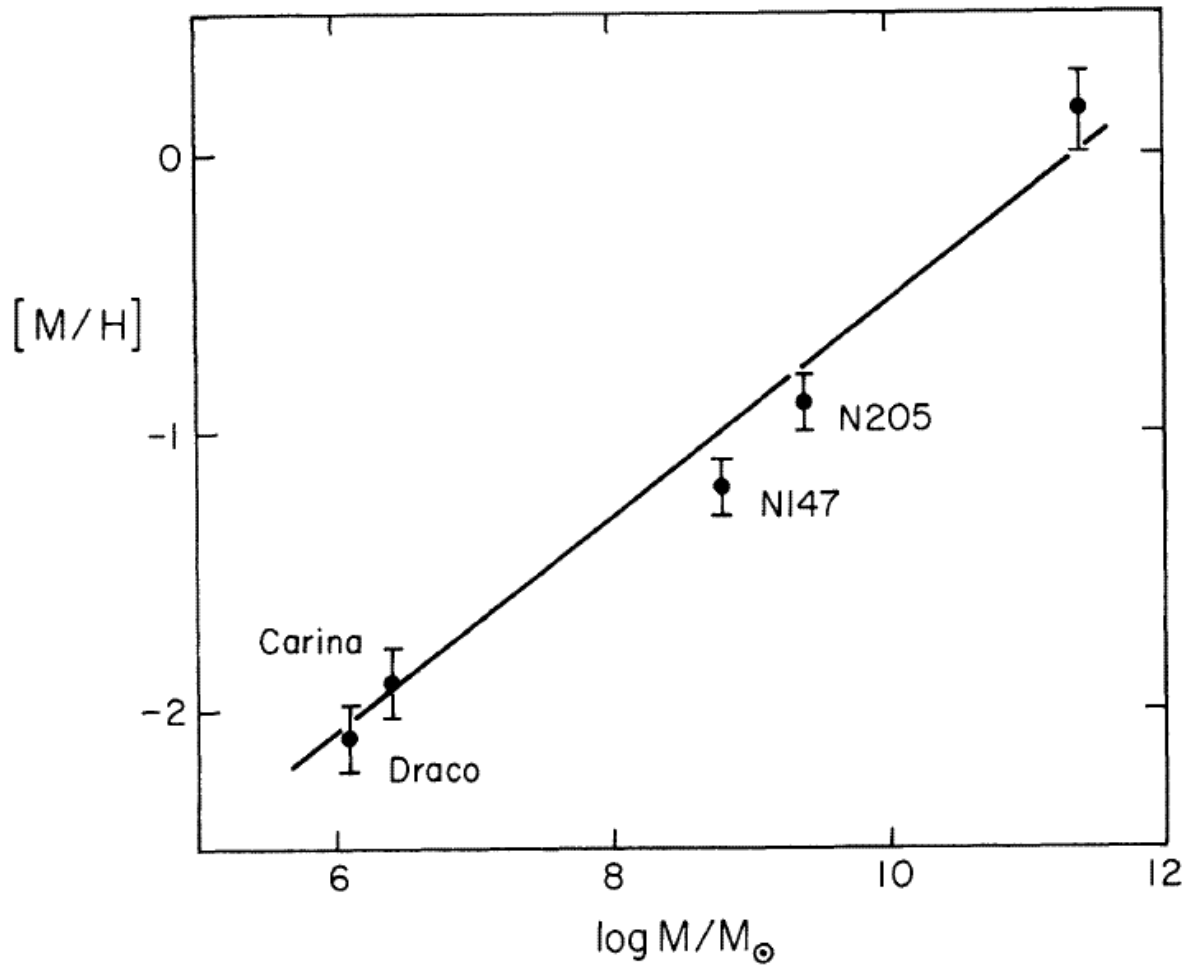
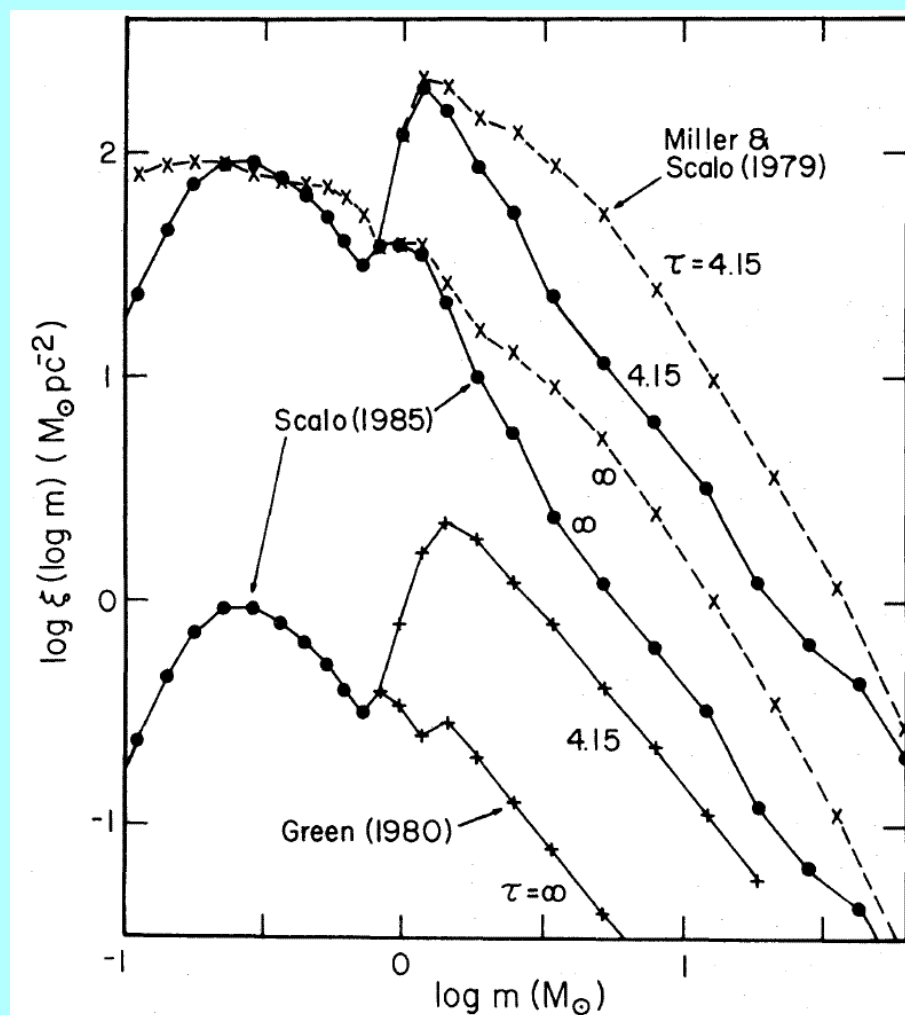


FIG. 2—A mass-metallicity relation for elliptical galaxies. The unlabeled point shows the metallicity inferred for the brightest ellipticals from integrated light models.

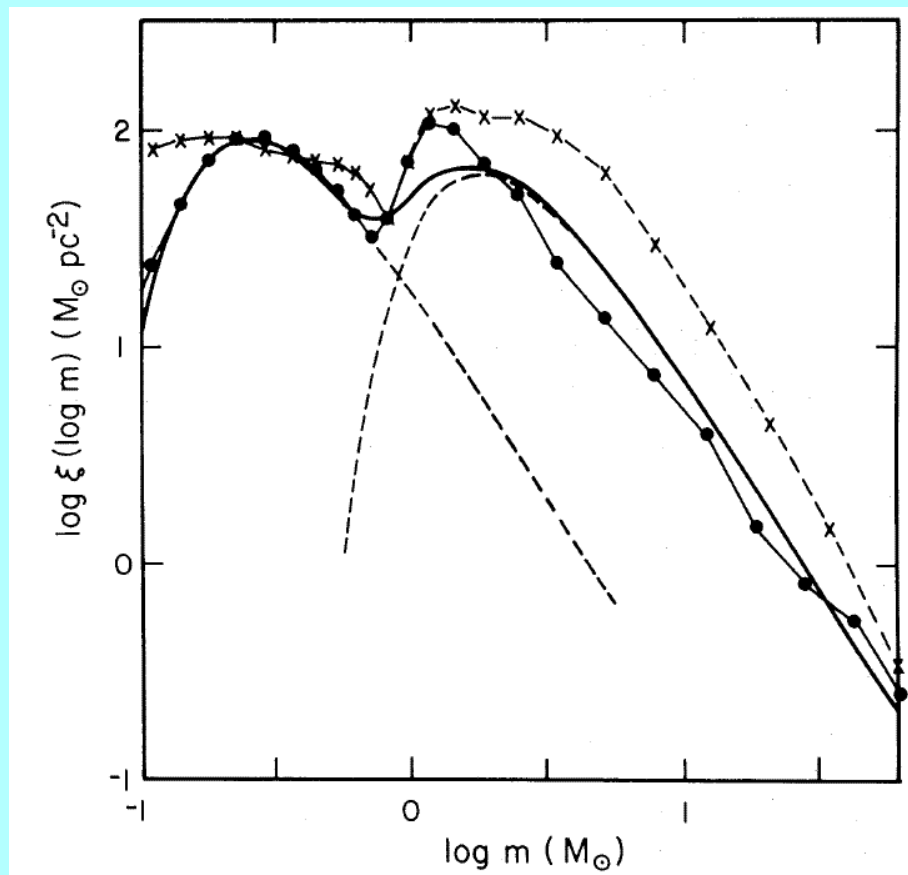
Bi-modal star formation.

It is possible to relax the **continuity constraint** in the determination of the IMF and assume bi-modal star formation*.

This is based on the idea of two modes of star formation, that are independent.



*Larson, Mon.Not.R.A.S. 218, 409 (1986)



If C is the number of stars formed $(\log M)^{-1} \text{ pc}^{-2} \text{ Gyr}^{-1}$:

$$C(\log M, t) = SFR_1(t) \cdot IMF_1(\log M) + SFR_2(t) \cdot IMF_2(\log M)$$

$$IMF_k(\log M) = 2.55 M_k M^{-2} \exp \left[- \left(\frac{M_k}{M} \right)^{3/2} \right]$$

$$SFR_k(t) = A_k \exp \left(- \frac{t}{\tau_k} \right)$$

- Low masses: $\tau_1 = \infty$, $M_1 = 0.30M_{\odot}$,
 $A_1 = 1.85M_{\odot}\text{pc}^{-2}\text{Gyr}^{-1}$
- High masses: $\tau_2 = 3.4 \text{ Gyr}$, $M_2 = 2.2M_{\odot}$,
 $A_2 = 41M_{\odot}\text{pc}^{-2}\text{Gyr}^{-1}$

Effects of bi-modal star formation:

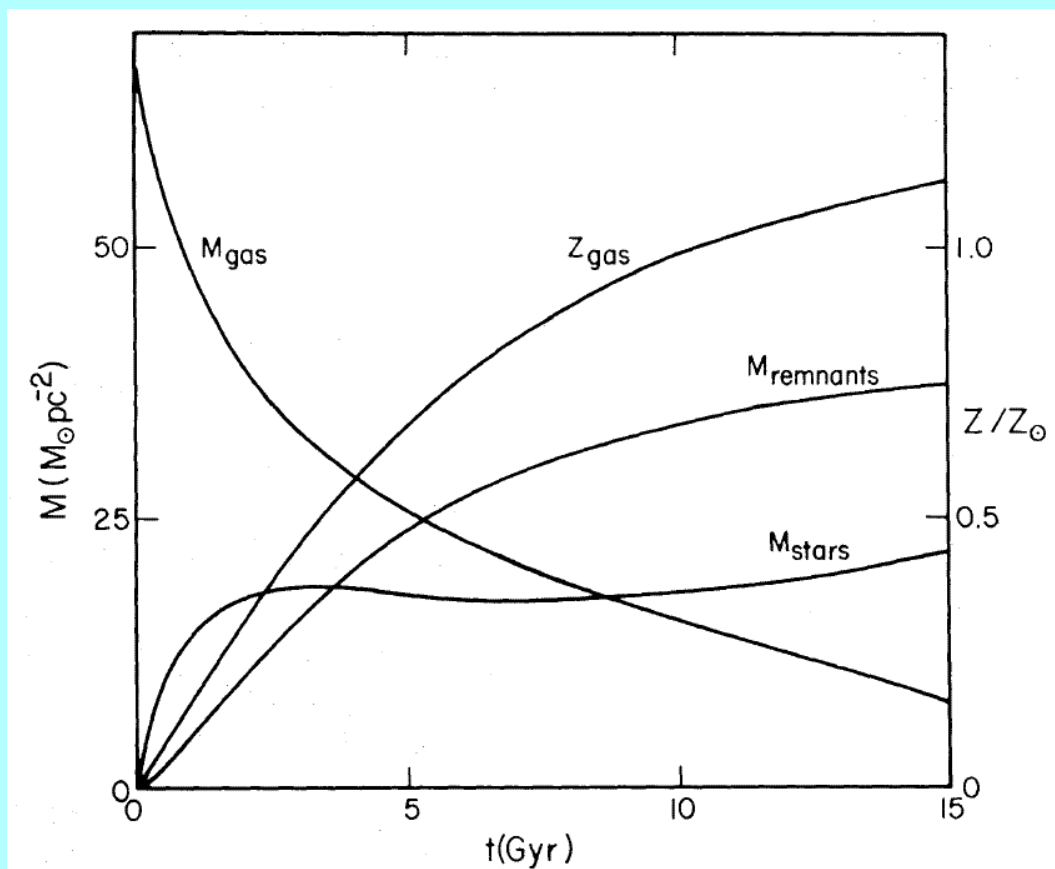
- This explains in a natural way the occurrence of **two types of associations**; the O-associations having OB-stars and the T-associations having only T Tauri stars.
- A smaller amount of mass has gone into long-lived stars per unit luminosity of newly formed stars during the whole history.
This solves the problem of the **gas consumption time-scale** (why do all galaxies use their gas in another Hubble time or less?).

- More mass is in invisible remnants of massive stars (white dwarfs, etc.).

For $M_{\text{remnant}} = 0.38 + 0.15M_*$ this adds up to about 3/4 of the mass density.

This solves the **local missing mass problem** (Oort limit), but is only compatible with observations if the fading time is less than 10 Gyr.

- Rapid early increase in $[\text{Fe}/\text{H}]$ combined with low relative SR in low-mass stars. This solves the **G-dwarf problem** of the simple model for chemical evolution.



Comparison to observations.

- Abundance gradients in bulges:

This results from a change in the effective yield with radius due to changing escape rates of processed gas.

- Overall abundances of ellipticals:

There is a correlation of $[\text{Fe}/\text{H}]$ with M_V , which follows if for more massive systems the gas has more difficulty to escape.

- Disk abundance differences between galaxies:

Earlier types have higher metallicities, because more gas has been used in star formation.

- Gas abundance gradients in disks:

This results from radial gradient in relative gas consumption and content.

- Stellar abundance gradients in disks:

No gradients should result if most of the gas is used up (the mean stellar abundance is then equal to the yield); at least it should be smaller than in the gas.

c. Internal absorption.

The earliest study is by Holmberg*.

He defined an apparent face-on surface brightness from the apparent magnitude m and the angular major-axis diameter a

$$\mu'_{\text{obs}} = m + 5 \log a$$

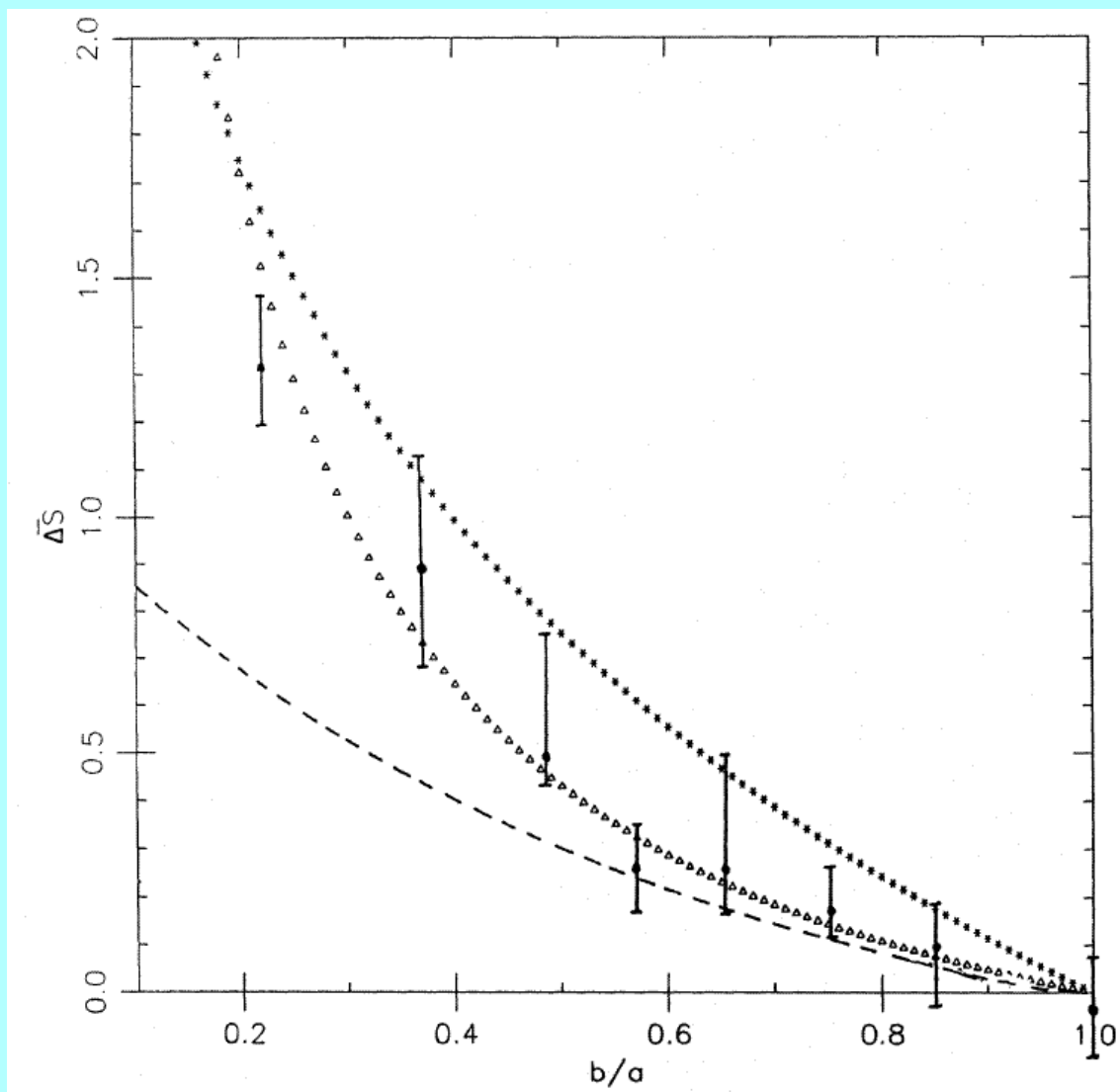
He then plotted this against the axis ratio b/a on the sky.

The inclination i is related to the axis ratio as

$$\sec i = a/b$$

for a not too edge-on disk ($a/b < 3$).

*Medd. Lund Obs. Ser. 2, No. 136 (1958)



Holmberg's fit to the data (triangles) then is

$$\mu'_{\text{obs}}(i) = \mu'(0) + A_B \{\sec i - 1\}$$

$A_B = 0.40 \text{ mag}$ for Sa-Sb

$A_B = 0.28 \text{ mag}$ for Sc

So his conclusion was that disk of galaxies are
not optically thick.

However, it should be realised that Holmberg's fit is not physical, since it is actually that of a dust sheet in front of a stellar disk.

Later with the IRAS satellite it was found that often for galaxies $L_{\text{FIR}}/L_{\text{opt}} \sim 1$ or more.

Also realise that for a thin, opaque dust layer in the central plane of stellar disk we expect:

- $A_{\text{B}} = 0.75$ mag.
- No change in color index
- $L_{\text{FIR}}/L_{\text{opt}} \sim 1$

In the Galaxy we are *not* in an optically thick part of the disk.

Extinctions towards the poles are estimated between 0 and 0.2 mag in B.

But there may be denser parts and towards the center absorption may in general increase in galaxies.

Models and observations of edge-on galaxies.

Kylafis & Bahcall* model the surface brightness distribution of NGC 891.

They assume for the space density of stars

$$L(R, z) = L_o \exp \left(-\frac{R}{h_s} \right) \operatorname{sech}^2 \left(\frac{z}{z_s} \right)$$

and for the dust (absorption coefficient)

$$\kappa(R, z) = \kappa_o \exp \left(-\frac{R}{h_d} \right) \operatorname{sech}^2 \left(-\frac{z}{z_d} \right)$$

This produces very good fits of z-profiles with:

$$z_s \sim 0.93 \text{ kpc}$$

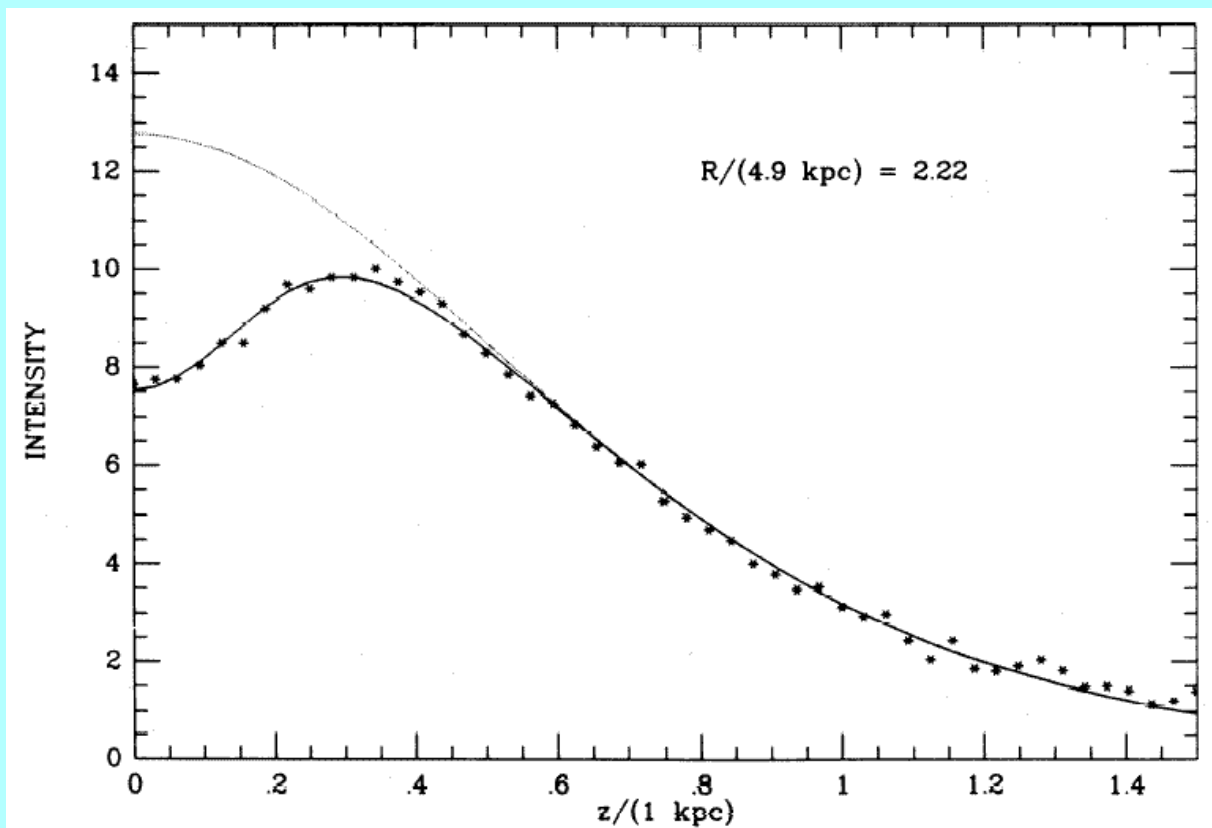
$$z_d \sim 0.22 \text{ kpc}$$

$$h_s \sim 4.9 \text{ kpc}$$

$$h_d \sim (0.8 - 1.5) h_s$$

$$\tau_o = 2\kappa_o h_d \sim 10.3$$

*Ap.J. 317, 637 (1987)



Translated to face-on this is:

$$\tau(R) = 2\kappa_0 z_d \exp\left(-\frac{R}{h_d}\right) = 0.46 \exp\left(-\frac{R}{h}\right)$$

A **optical depth** τ of 0.46 corresponds to an **extinction** of 0.50 mag.

For a central thin dust-layer this means, that we miss **58%** of the light from the **backside** and the **total disk surface brightness** is decreased by **18%**.

However, the sech^2 may not be a good approximation at low z due to young stars, in which case there is more extinction than implied here.

Wainscoat *et al.** make a similar model of IC 2531 on the basis of UBVRIJHK photometry.

They use exponentials in the z -direction:

Old disk:

$$L_{\text{old}}(R, z) = L_{\text{old}}(0, 0) \exp \left(-\frac{R}{h} - \frac{|z|}{h_z} \right)$$

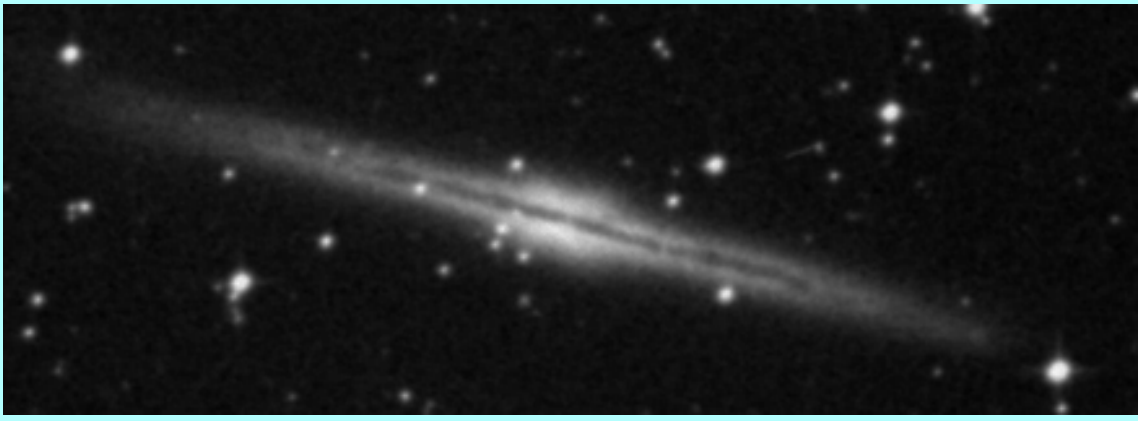
Young disk:

$$L_{\text{young}}(R, z) = L_{0,y}(0, 0) \exp \left(-\frac{R}{h} - \frac{|z|}{h_{z,y}} \right)$$

Dust absorption in path δd along line-of-sight d :

$$\delta A_{\lambda}(R, z) = A_{\lambda,0} \exp \left(-\frac{R}{h_d} - \frac{|z|}{h_{z,d}} \right) \delta d$$

*Wainscoat, Freeman & Hyland, Ap.J. 337, 190 (1989)



Adopted and resulting parameters:

color	old disk	young disk	λ	A_λ/A_V
U-V	1.09	-0.77	U	1.531
B-V	0.78	-0.04	B	1.324
V-R	0.50	-0.01	R	0.816
V-I	1.14	0.00	I	0.597
V-J	1.80	0.01	J	0.282
V-H	2.51	0.02	H	0.175
V-K	2.76	0.03	K	0.112

$$h \sim 6.4 \text{ kpc}$$

$$h_z \sim 0.53 \text{ kpc}$$

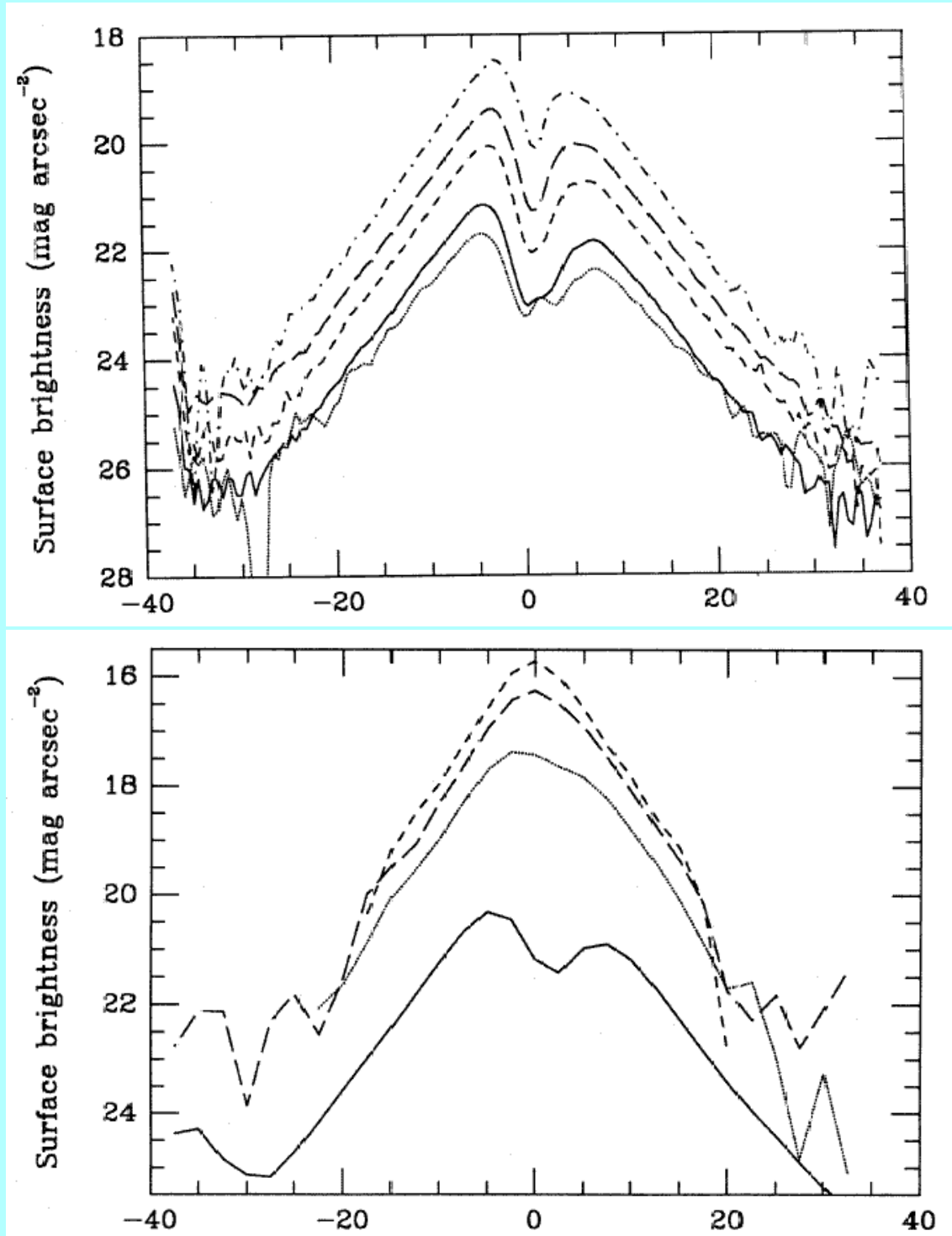
$$h_{z,d} = h_z/4$$

$$h_{z,y} = h_z/8$$

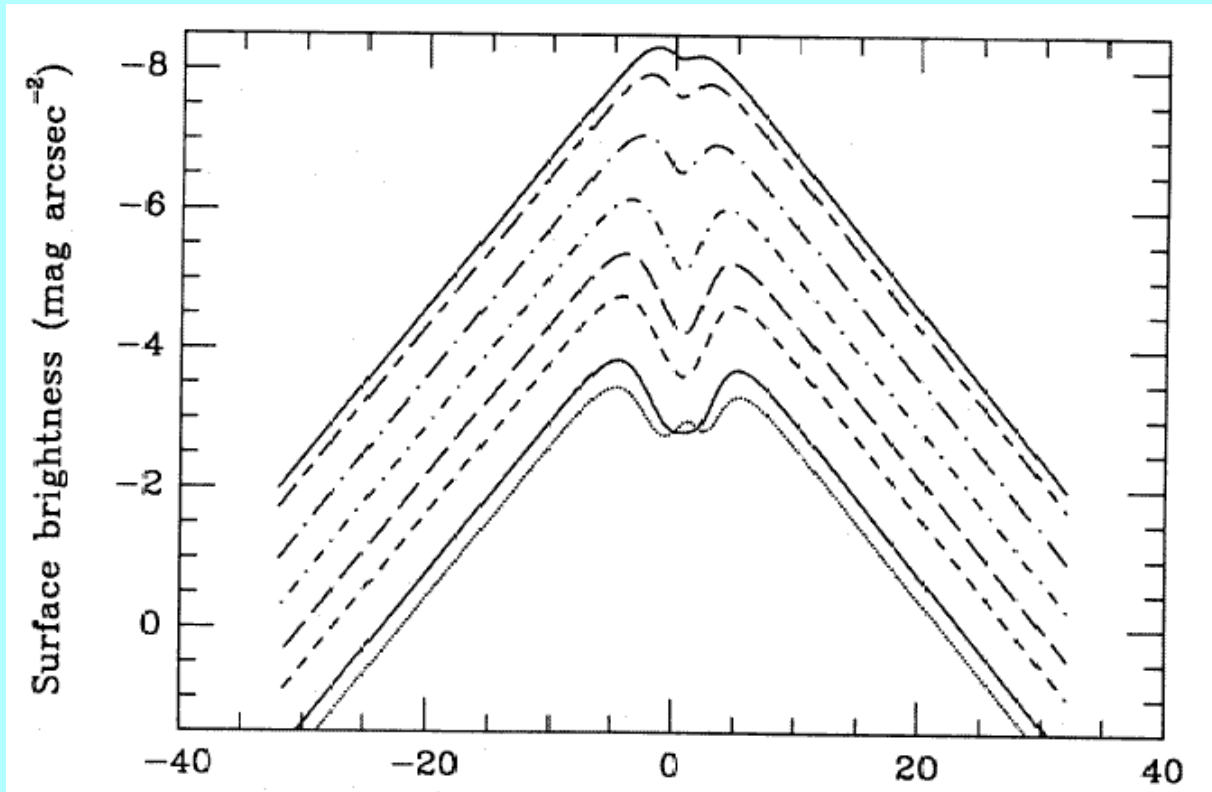
$$A_V(0,0) = 1.6 \text{ mag kpc}^{-1}$$

$$L_{\text{young}}(0,0) = L_{\text{old}}(0,0) \text{ in V - band}$$

Here are **z-profiles** in U, B, V, R, and I (top to bottom) in the top-panel and in V, J, H and K in the bottom-panel.

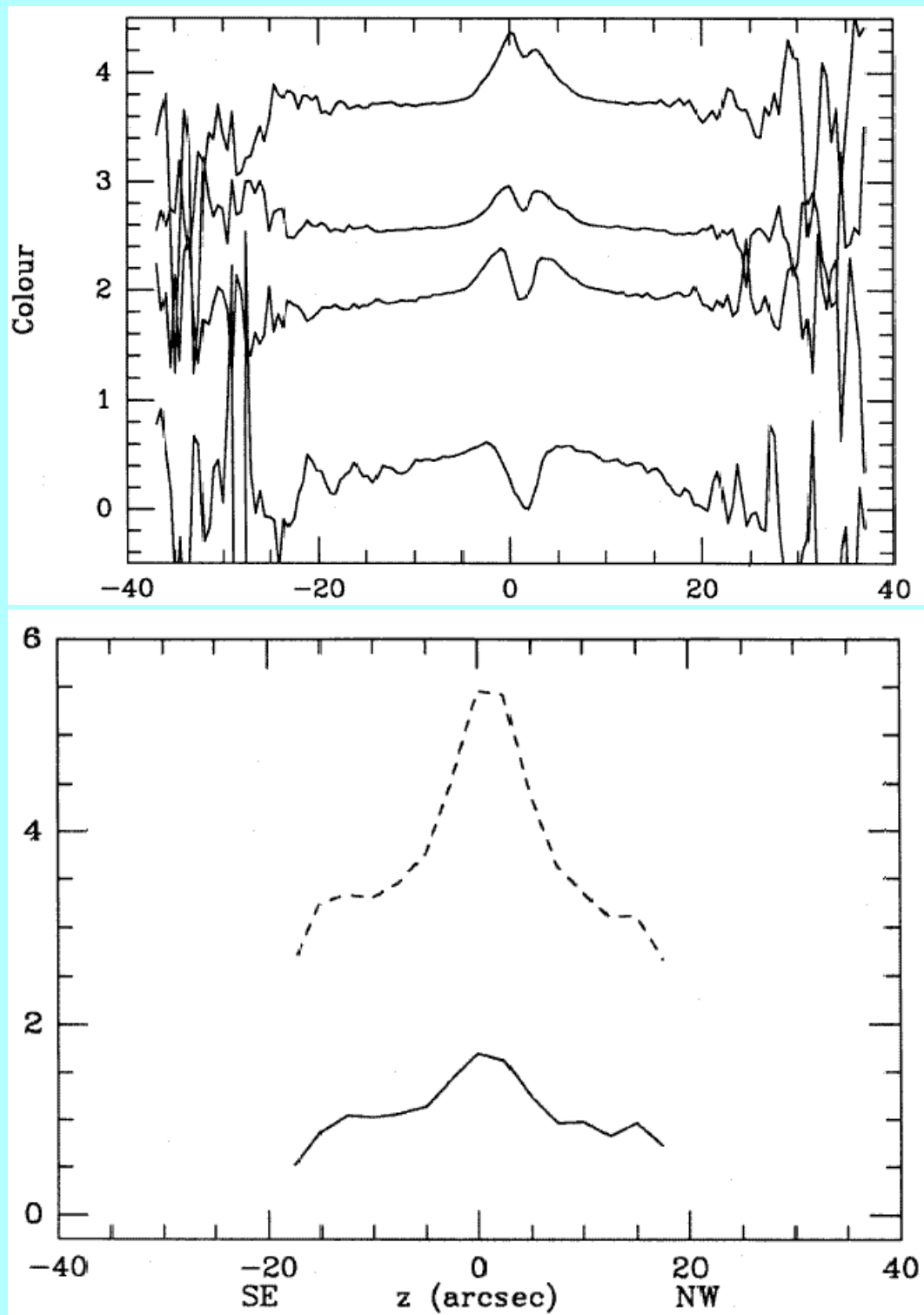


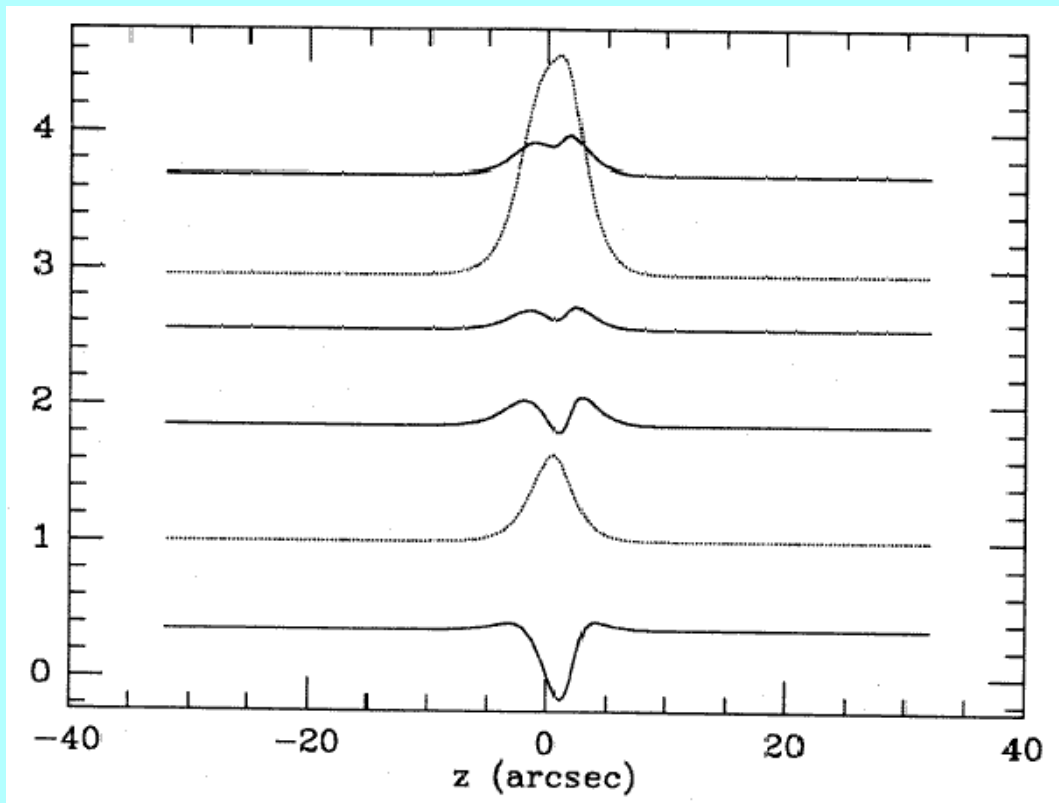
The models then give these profiles (U at the bottom, K at the top).



Also the **color profiles** can be calculated and compared to observations.

We see first the observed profiles: from bottom to top (U - B), (B - V), (V - R) and (V - I) in the top-panel and bottom to top (J - K) and (J - I) in the bottom panel.





Here the color profiles from the model. Solid lines from bottom to top: (U - B), (B - V), V - R) and (V - I); dotted lines from bottom to top: (J - K) and (H - K).

Thus $A_B(0,0) = 1.9 \text{ mag kpc}^{-1}$ or $\kappa_o = 1.75 \text{ kpc}^{-1}$.

So we get face-on in the B-band:

$$\tau(R) = \kappa_o h_{z,d} \exp\left(\frac{R}{h}\right) = 2.80 \exp\left(-\frac{R}{h}\right)$$

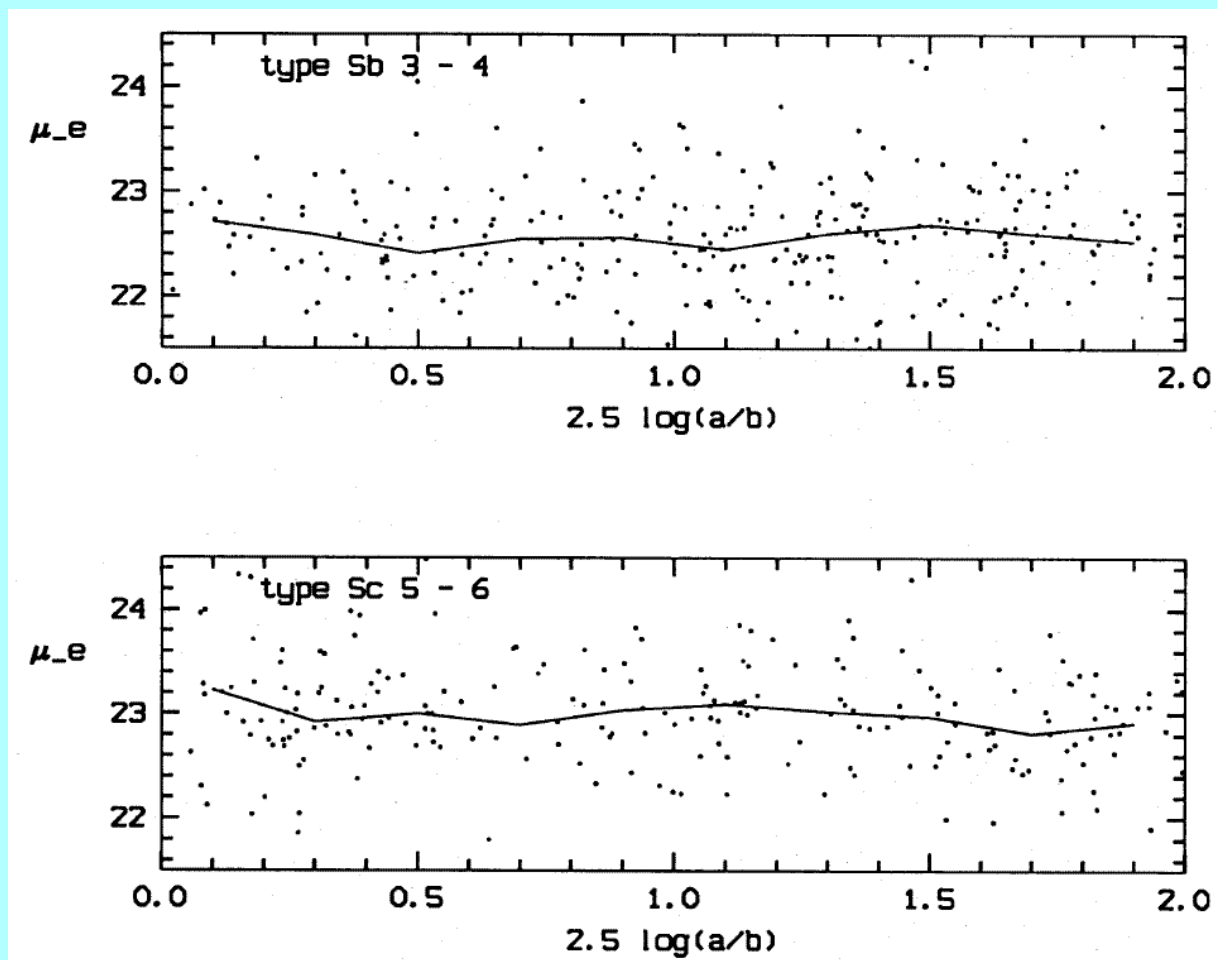
A τ of 2.80 corresponds to an extinction of **3.0 mag**. For a very thin dust-layer the total light from the disk is decreased by **47%**.

Other indications.

- Valentijn* claimed that in complete samples the observed mean surface brightness correlates little with axis ratio.

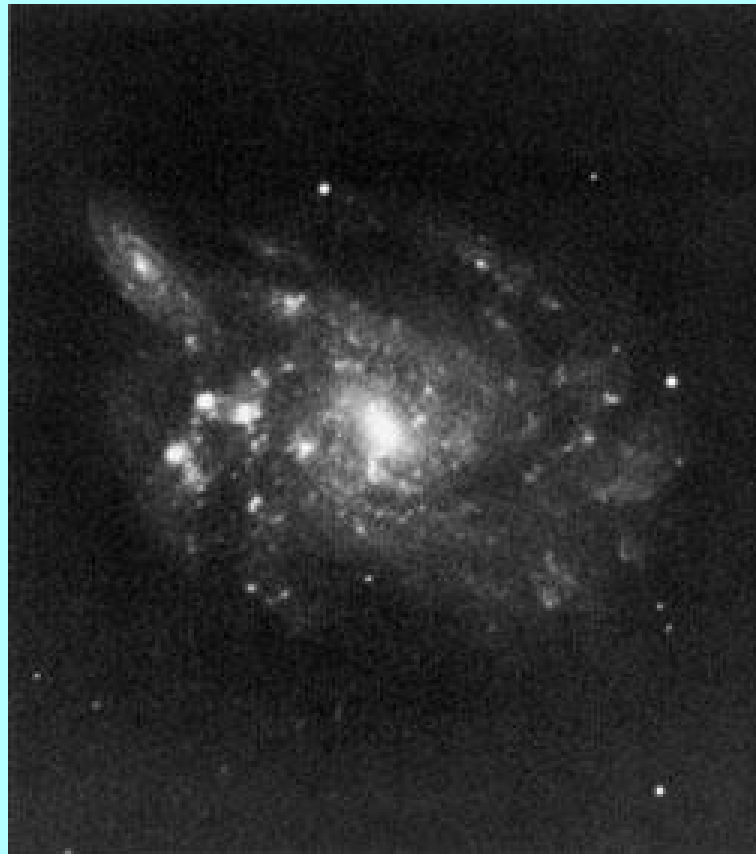
One expects more inclined galaxies to be systematically brighter in surface brightness.

So maybe disks are optically thick.



*Mon.Not.R.A.S. 266, 614 (1994); also Burstein, 1980, unpublished

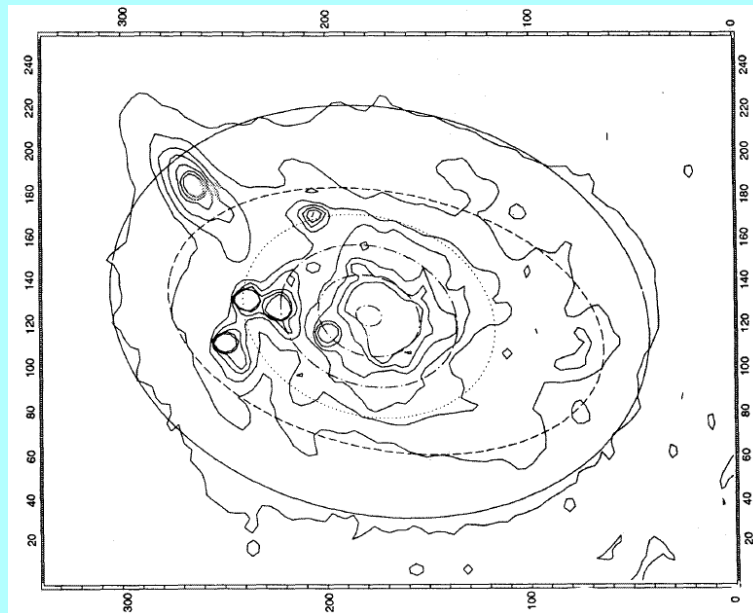
- Jura* proposed that Freeman's law results from absorption. He also noted the similar surface brightness in the (obscured!) Milky Way.
- A very effective test in principle is to look for galaxies seen through disks as in the pair NGC450/UGC807†.



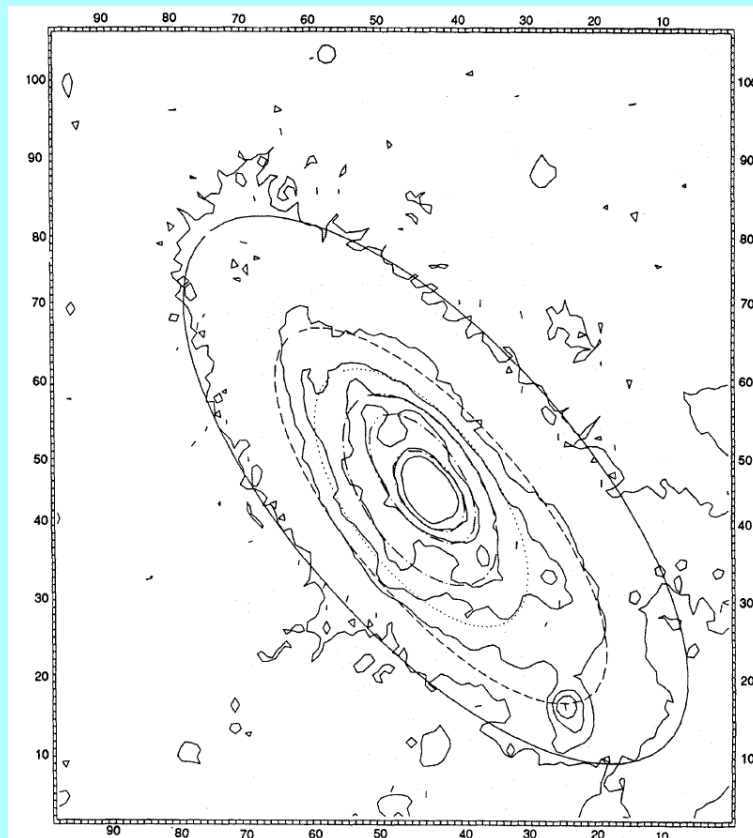
In the photometry we can deduce the surface brightness distribution of NGC 450 in the area of overlap.

*Ap.J. 238, 499 (1980)

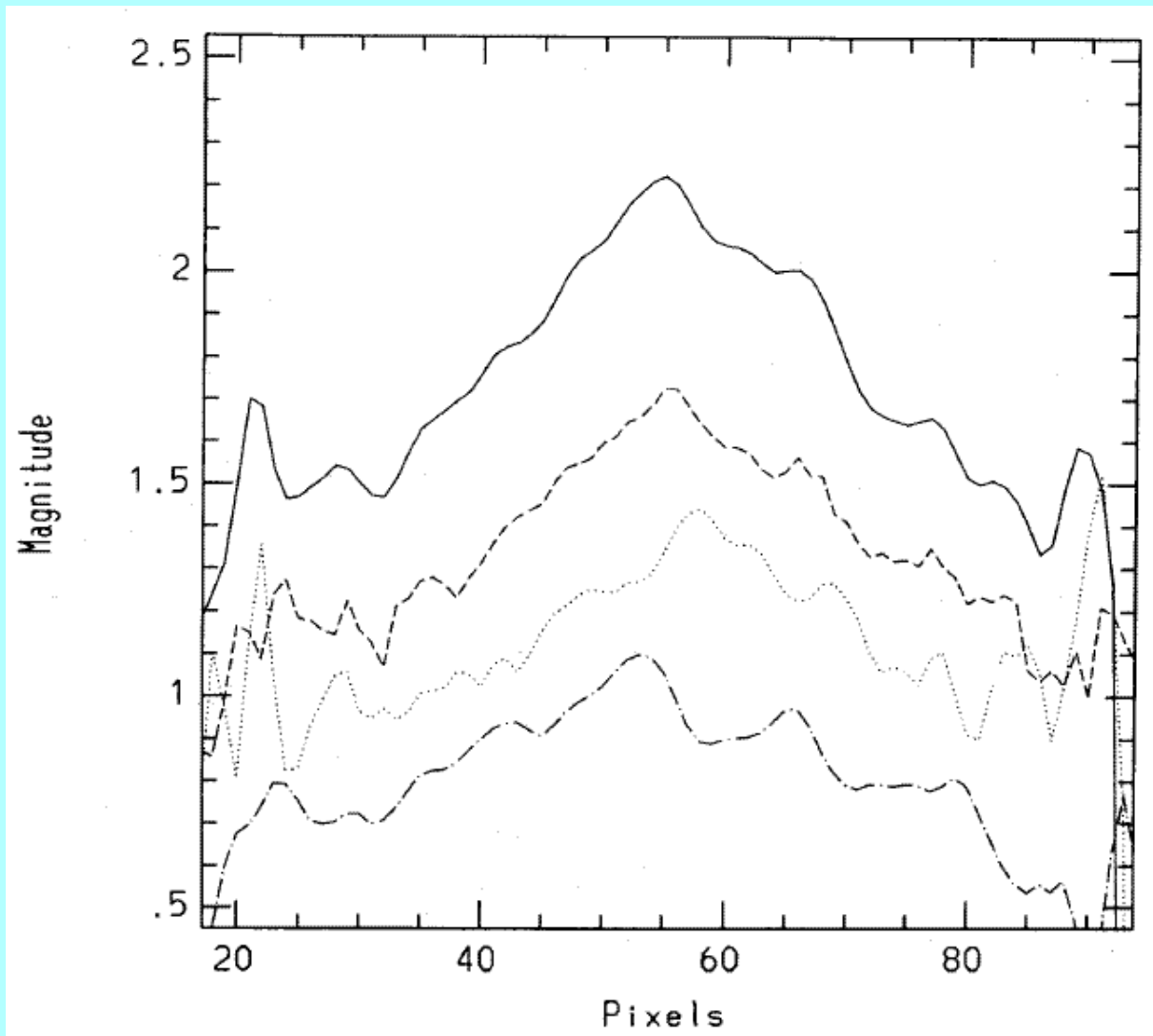
†Andredakis & van der Kruit, A.&A. 265, 396 (1992)



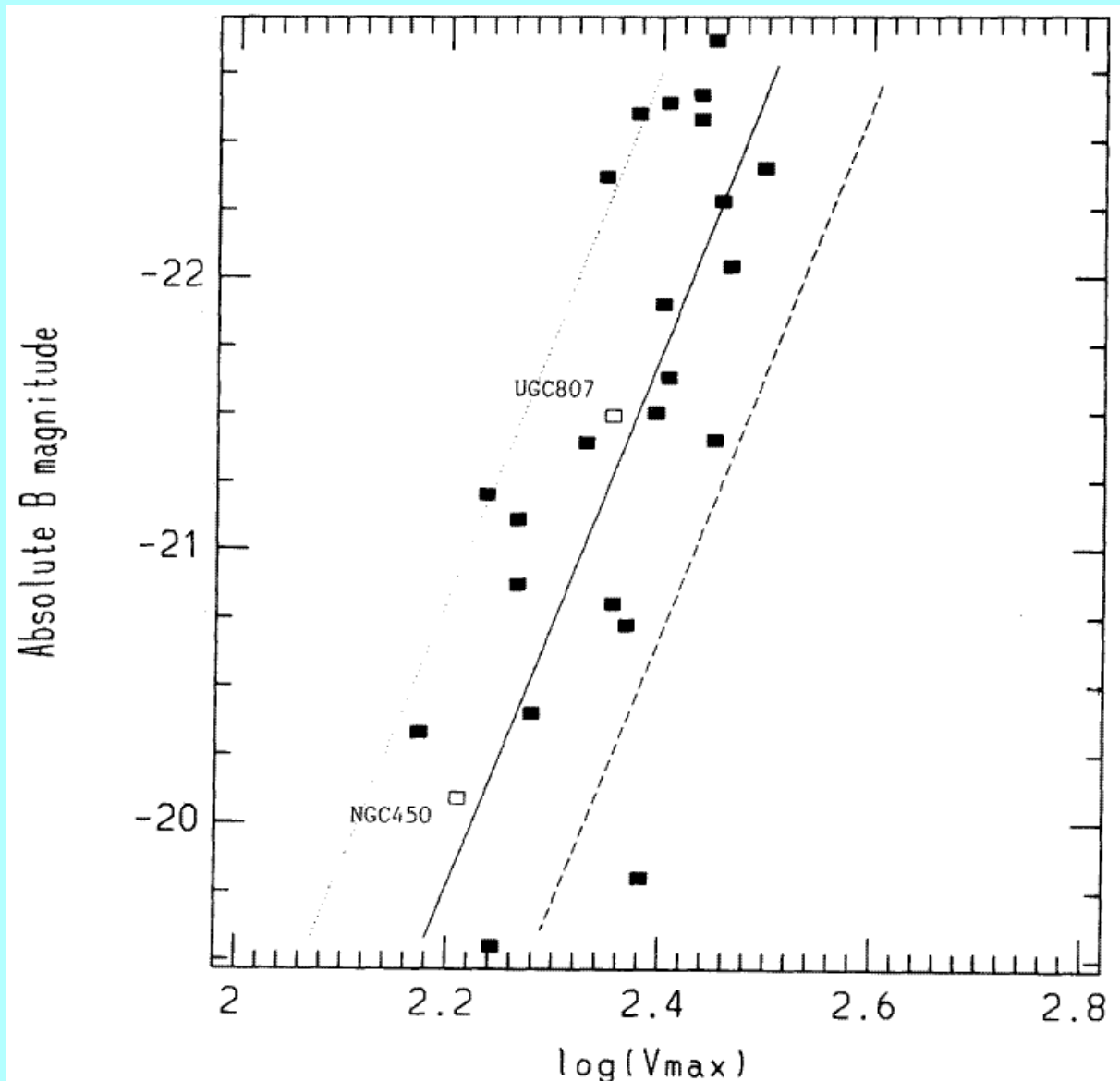
Subtraction then gives the “unaffected” image of UGC807.



This shows no color changes, so there is no significant **gradient** in absorption.



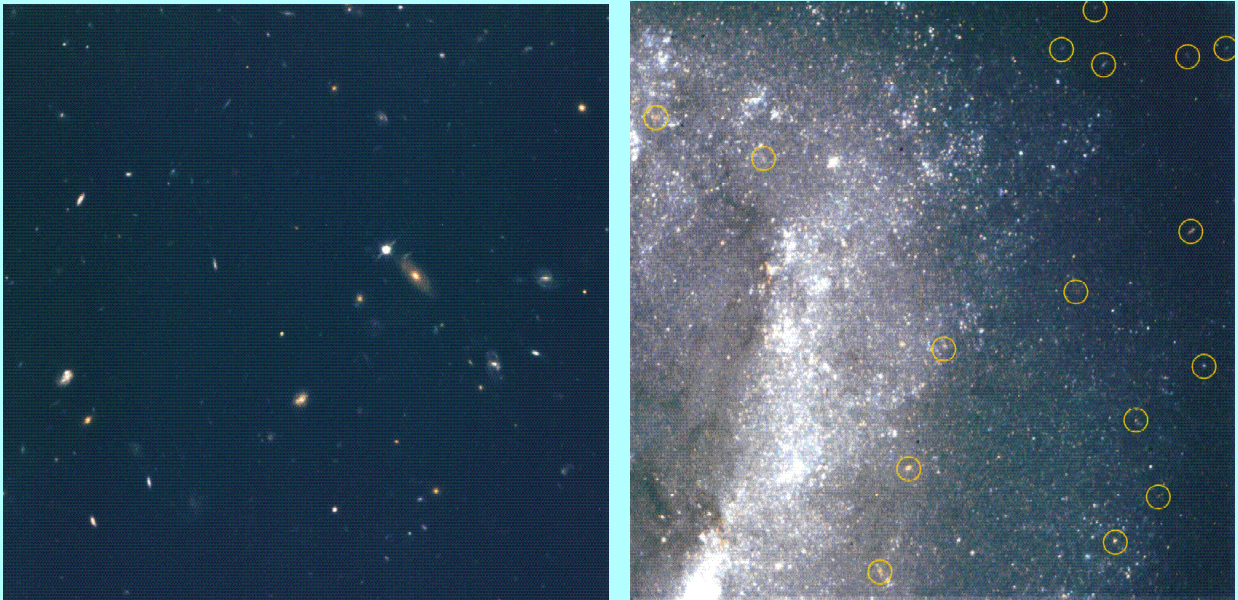
Both galaxies conform to the Tully-Fisher relation.



The maximum absorption allowed is 0.3 magnitudes in the V-band.

More sophisticated is to study images of galaxies with the **Hubble Space Telescope** and identify background galaxies.

Then the test can be done to add the **Hubble Deep Field** with the appropriate noise and background level and see what fraction of these galaxies are recovered.



With this **synthetic field method*** evidence for some absorption has been found.

*González, Allen, Dirch, Ferguson, Calzetta & Panagia, Ap.J. 506, 152 (1998)

- Disney *et al.** collected information from various sources, parametrizing it as

$$\mu_{\text{obs}}(i) = \mu_{\text{o}} - 2.5n_{\text{eff}} \log(a/b)$$

In a completely optically thin disk one expects $n_{\text{eff}} = 1$ and in an optically thick disk $n_{\text{eff}} \leq 1$.

Then for samples in the [Second Reference Catalogue \(RC2\)](#) and the [Revised Shapley-Ames Catalogue \(RSA\)](#) the following values are found for n_{eff} :

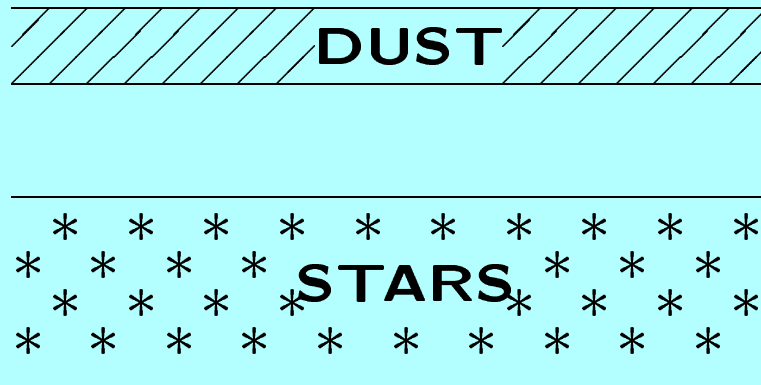
Type	Holmberg	RC2	RSA
Sa-Sb	0.46	0.72	0.46
Sbc	0.46	0.68	0.65
Sc	0.65	0.68	0.65
Sd	-	0.68	0.65
Sdm-Im	-	0.96	0.82

So there is certainly evidence for some absorption.

Now look at some simple models of Disney *et al.*

*Disney, Davies & Phillipps, Mon.Not.R.A.S. 239, 939 (1989)

♠ SCREEN MODEL



The dust layer has optical thickness τ , the stellar disk emissivity E^* and thickness T .

The observed surface brightness then becomes

$$L(i) = E^* T \sec i \exp \{-\tau \sec i\}$$

Note that Holmberg's μ' is $L'(i) = L(i) \cos i$, so

$$\mu'(i) = \mu'_o + A_B^0 \sec i = \mu'(0) + A_B^0 (\sec i - 1)$$

The total face-on absorption becomes

$$A_B^0 = 1.086\tau$$

For $\tau \ll 1$

$$L(i) = E^*T \sec i$$

The observed surface brightness is $L(\tau, i)$ and the bolometric surface brightness is $L(0, 0) \sec i$.

Consider a circular area πa^2 , then total luminosity is

$$L_{\text{bol}} = \pi a^2 L(0, 0) = \pi a^2 E^*T$$

The observed face-on luminosity is

$$L_{\text{opt}} = \pi a^2 L(\tau, 0)$$

If the dust re-radiates isotropically

$$L_{\text{FIR}} = L_{\text{bol}} - L_{\text{opt}} = \pi a^2 \{L(0, 0) - L(\tau, 0)\}$$

The FIR surface brightness at inclination i then is

$$L_{\text{FIR}}(i) = \sec i \{L(0, 0) - L(\tau, 0)\}$$

and we can calculate (drop the τ 's)

$$\frac{L_{\text{FIR}}}{L(i)} = \sec i \frac{E^*T - L(0)}{L(i)}$$

So we get for the Screen Model

$$\frac{L_{\text{FIR}}}{L(i)} = \exp \{ \tau \sec i \} - 1$$

For the optically thin case $\tau \ll 1$ this reduces to

$$\frac{L_{\text{FIR}}}{L(i)} = \tau \sec i$$

♠ SLAB MODEL

No make the model more realistic.



The results then become:

$$L(i) = \frac{E^* T}{\tau} [1 - \exp \{ -\tau \sec i \}]$$

$$A_{\text{B}}^0 = -2.5 \log \left\{ \frac{1 - \exp (-\tau)}{\tau} \right\}$$

$$\frac{L_{\text{FIR}}}{L(i)} = \sec i \frac{\tau - 1 + \exp \{-\tau\}}{1 - \exp \{-\tau \sec i\}}$$

For the **optically thick case** $\tau \gg 1$

$$L(i) = \frac{E^*T}{\tau} = \text{constant}$$

$$A_{\text{B}} = 2.5 \log \tau$$

$$\frac{L_{\text{FIR}}}{L(i)} = (\tau - 1) \sec i$$

and for the **optically thin case** $\tau \ll 1$

$$L(i) = E^*T \sec i$$

So L' is independent of i .

$$A_{\text{B}} = -2.5 \log \left(1 - \frac{\tau}{2} \right)$$

$$\frac{L_{\text{FIR}}}{L(i)} = \frac{\tau}{2}$$

♠ SANDWICH MODEL

In real galaxies the dust layer is thinner than the stellar disk.



Let the thickness of dust layer be pT . Then

$$L(i) = E^*T \sec i \left[\frac{1-p}{2} \{1 + \exp(-\tau \sec i)\} + \frac{p}{\tau \sec i} \{1 - \exp(-\tau \sec i)\} \right]$$

$$A_B^0 = -2.5 \log \left[\frac{1-p}{2} \{1 + \exp(-\tau)\} + \frac{p}{\tau} \{1 - \exp(-\tau)\} \right]$$

The optically thick case $\tau \gg 1$ now becomes

$$L(i) = E^*T \sec i \frac{1-p}{2}$$

$$A_B = -2.5 \log \left\{ \frac{1-p}{2} \right\}$$

$$\frac{l_{\text{FIR}}}{L(i)} = \sec i \frac{(1+p)\tau - 2p}{(1-p)\tau + 2p}$$

If $p \ll 1$

$$L(i) = \frac{E^*T}{2} \sec i$$

$$A_B = 0.753$$

$$\frac{L_{\text{FIR}}}{L(i)} = \sec i$$

The optically thin case $\tau \ll 1$ gives

$$L(i) = E^*T \sec i \left\{ 1 - \frac{1-p}{2} \tau \sec i \right\}$$

$$A_B = -2.5 \log \left\{ 1 - \frac{1-p}{2} \tau \right\}$$

$$\frac{L_{\text{FIR}}}{L(i)} = \frac{\tau}{2}$$

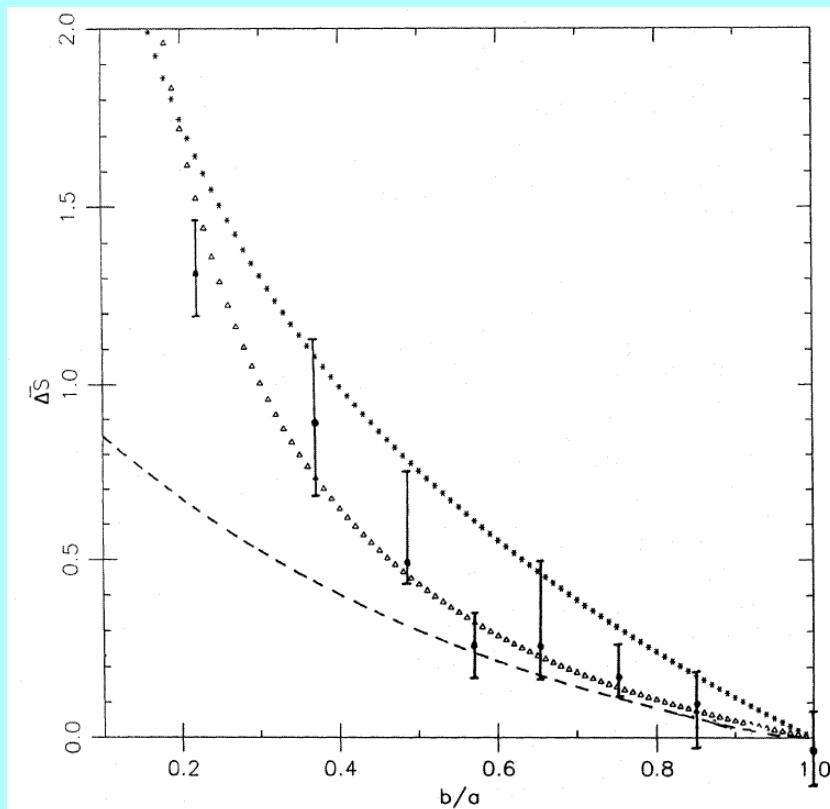
If $p \ll 1$

$$L(i) = E^* T \sec i \left(1 - \frac{\tau}{2} \sec i \right)$$

$$A_B = -2.5 \log \left(1 - \frac{\tau}{2} \right)$$

$$\frac{L_{\text{FIR}}}{L(i)} = \frac{\tau}{2}$$

- triangles: $\tau < 1$ Screen Model
- stars: $\tau \gg 1$ Slab Model
- dashes: $\tau \gg 1$ Sandwich Model ($p = 0.5$).



The optically **thin** Slab and Sandwich Models predict no dependence of Holmberg surface brightness on inclination.

So **observations are consistent with optically thick models, but the results are very geometry dependent and therefore not yet conclusive.**

The near-IR data are also not entirely conclusive. L_{FIR} can be very large compared to L_{opt} if star-formation occurs extensively in very thick, obscured, but localized area's (GMC's)

Disney *et al.* also calculate triplex models as above, which give similar results as these simple models.

We can still extend the analysis by looking at the **colors**.

In all models we had:

$$L(i) = E^* T F(p, \tau, i) \sec i$$

Take

$$\tau_V = 0.75\tau_B$$

$$\frac{L_B(i)}{L_V(i)} = \frac{E^*(B) F(p, \tau_B, i)}{E^*(V) F(p, \tau_V, i)}$$

The color change between inclination 0° and 70° then is:

$$\Delta(B - V) = -2.5 \log \left\{ \frac{F(p, \tau_B, 70) F(p, \tau_V, 0)}{F(p, \tau_V, 70) F(p, \tau_B, 0)} \right\}$$

For the Sandwich Model we have:

- Optically thin ($\tau \ll 1$):

$$F(p, \tau, i) = 0.5 \Rightarrow \Delta(B - V) = 0$$

- Optically thick ($\tau \gg 1$):

$$F(p, \tau, i) = \frac{1-p}{2} \Rightarrow \Delta(B - V) = 0$$

Here are some values for $\Delta(B-V)$ as a function of optical thickness.

τ	Screen	$p = 1$	$p = 0.5$	$p = 0.1$
0.1	0.05	0.02	0.02	0.02
0.5	0.26	0.09	0.06	0.04
1.0	0.52	0.13	0.04	-0.01
2.0	1.04	0.11	-0.04	-0.07
5.0	2.61	0.02	-0.04	-0.01
10.	5.22	0.02	0.02	0.02

- For small τ B is always more affected than V, so **redder with inclination**.
- For large τ at high inclination we see only up to the dust, so we have **unreddened colors**. However at face-on there is still reddening and disks become **bluer with inclination**.