

# STRUCTURE OF GALAXIES

**Lecture 10. Formation of galaxies, origin of angular momentum, why exponential disks and  $R^{1/4}$  bulges, spiral structure.**

This lecture will almost exclusively be concerned with **spiral galaxies**.

**Two paradigms:**

**ELS\*** studied the motions of stars in the solar neighborhood and found correlations between metal abundance and the kinematics.

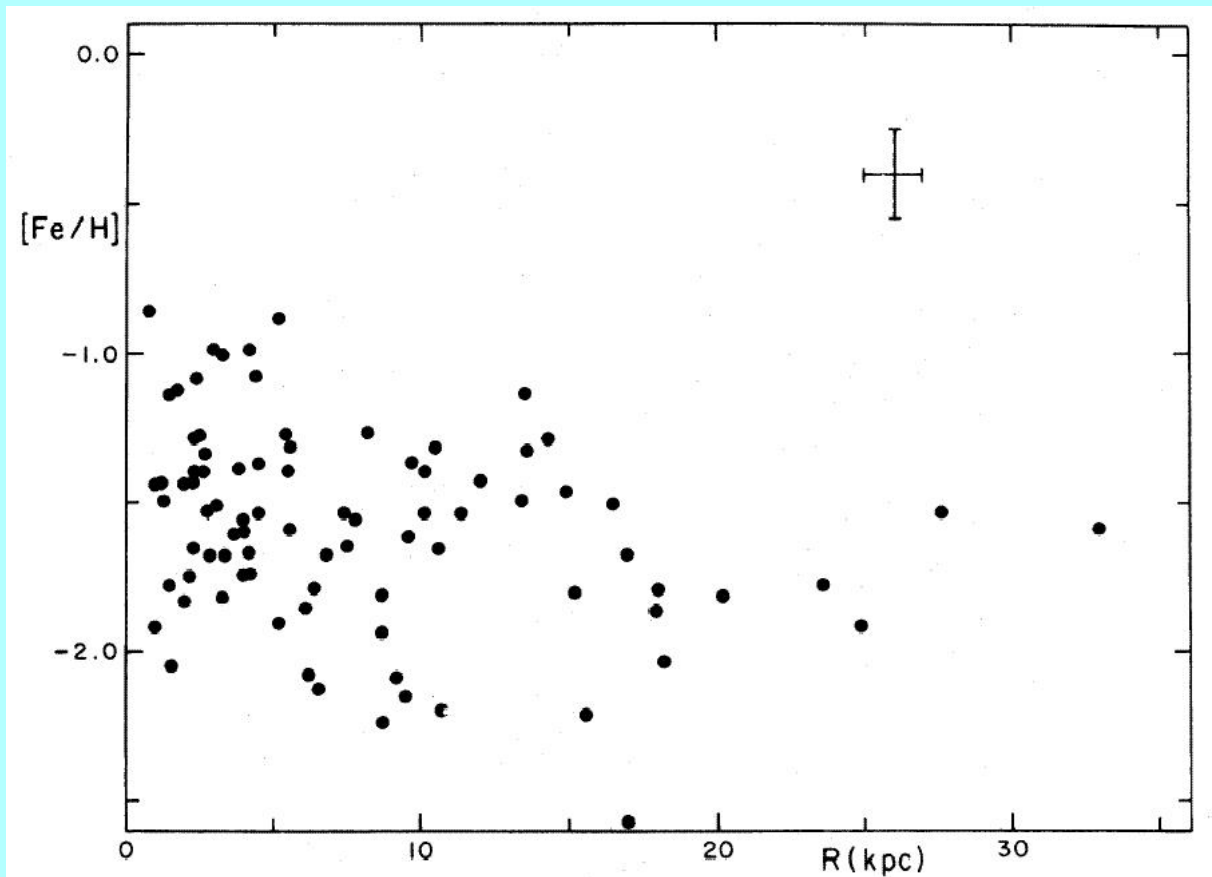
**They concluded that the Galaxy was formed during a relatively rapid collapse.**

**SZ†** studied the abundance distributions of **globular clusters**.

\*Eggen, Lynden-Bell & Sandage, Ap.J. 136, 748 (1962)

†Searle & Zinn, Ap.J. 225, 357 (1978)

Beyond 8 kpc from the center the distribution over abundance is fairly wide, but does not change with galactocentric distance.

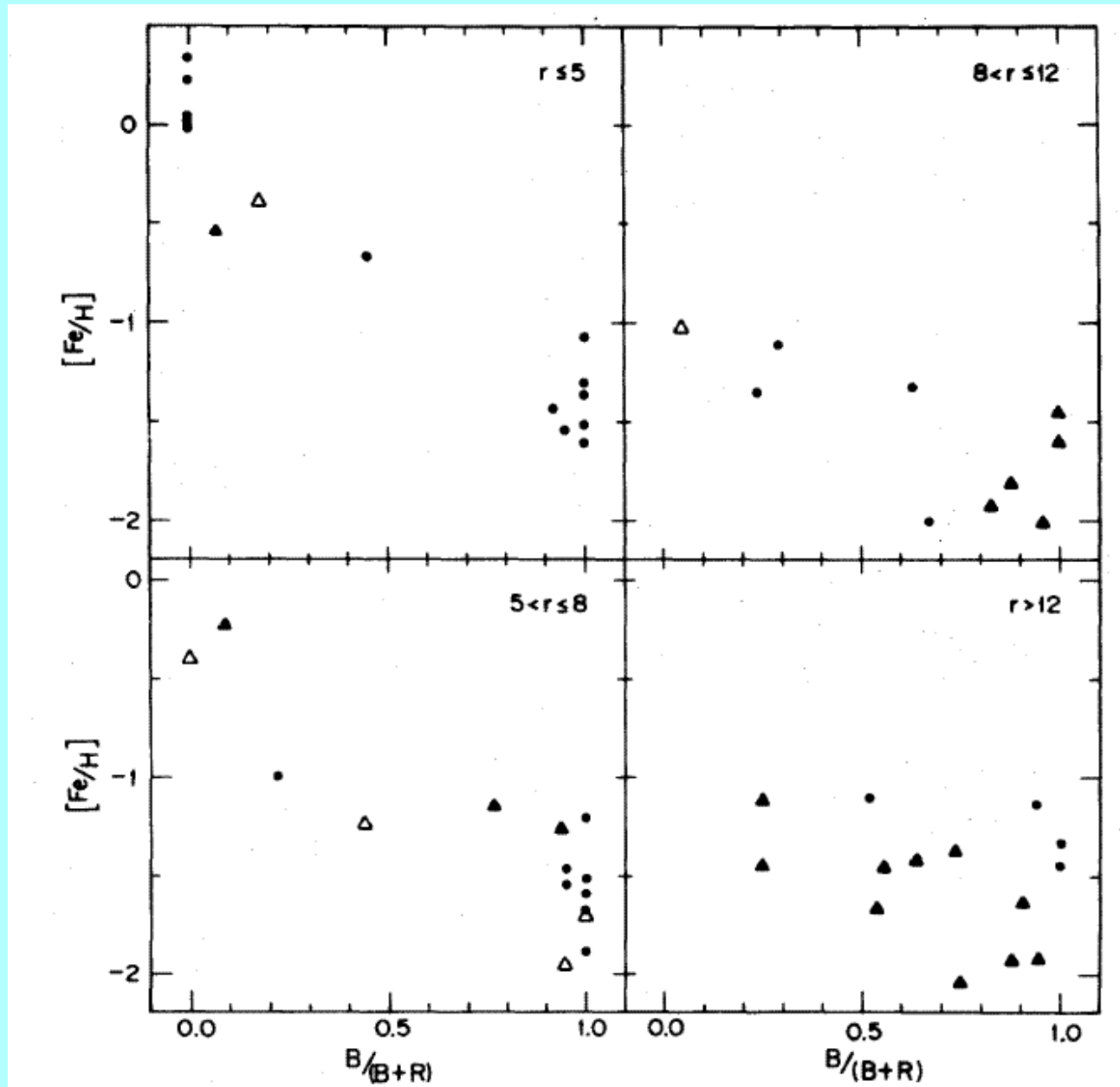


The extent of the Horizontal Branch depends in first instance on metallicity.

However, it has been known that the HB-morphology varies also among clusters of the same metallicity.

This is called the second parameter.

It is characterized by the parameter  $B/(B+R)$ .  $B$  is the number of HB-stars to the blue of the RR-Lyrae gap and  $R$  the number to the red.



SZ found that in the tightly bound inner regions  $B/(B + R)$  correlates well with abundance, but in the outer halo there is a great diversity of HB-morphology at a given abundance.

They suggested that the second parameter is age.

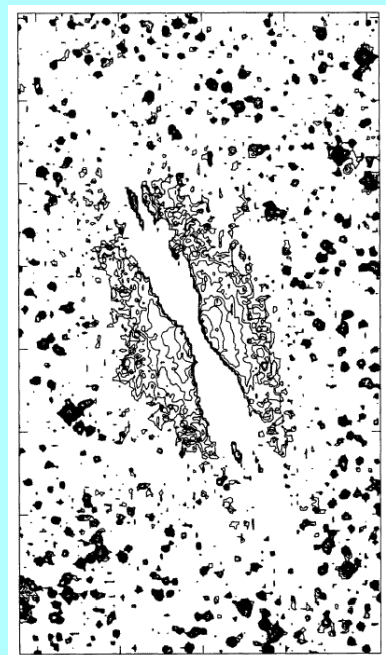
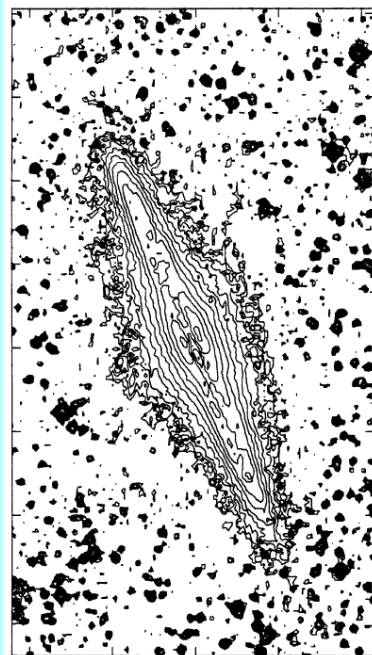
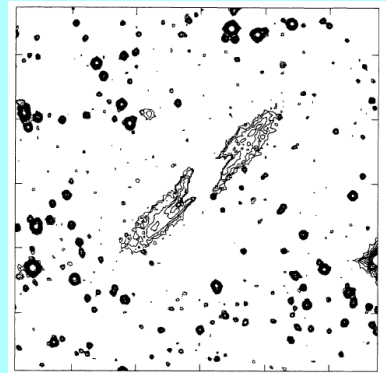
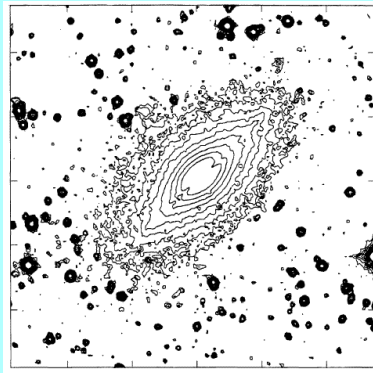
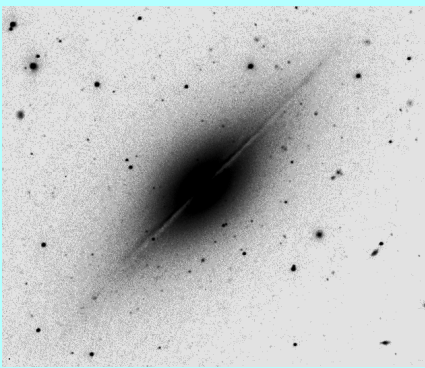
They concluded that all the above is consistent with a picture in which the build-up of the halo occurs over an extended period during which small fragments (of up to  $\sim 10^8 M_{\odot}$  or so) continue to fall in.

These fragments loose gas after a while (due to supernova explosions) and will have a mean metal abundance equal to the effective yield.

The effective yield will have a range and distribution that is stochastic and should show no correlation with galactocentric distance.

## Basic two-component structure.

In spite of the possible presence of a thick disk (which has of order 10% of the disk mass), spiral galaxies are basically consist of two distinct components with discrete flattening\*.



\*van der Kruit & Searle, A.&A. 110, 79 (1982)

This seems to point to two discrete epochs of star formation:

Before collapse – dissipationless – Population II

After collapse – dissipational – Thick and thin disk; Population I

We will look at these two separately.

### A. Bulge formation.

The observed properties of bulges are:

- $R^{1/4}$ -law.
- Generally color (=abundance) gradients
- Isochromes have the same shape as isophotes (in NGC 7814)

Color gradients are often interpreted as evidence for **dissipational** collapse.

However, then the more metal-rich parts should be more flattened than the metal-poor parts.

Further numerical experiments\* of dissipationless collapse with violent relaxation shows:

- From irregular initial conditions follows an  $R^{1/4}$  distribution
- Statistical conservation of binding energy and thus gradients.

The properties of bulges are consistent with them forming early on in a dissipationless collapse over a longer timescale with fragments falling in for a few Gyr.

\*van Albada, Mon.Not.R.A.S. 201, 939 (1982)

## B. Disk formation.

Disk formation is of course dissipational.

First we have to look into the question of the **origin of angular momentum**.

The angular momentum in disks is due to **tidal torques** between (proto-)galaxies in the early universe\*.

It can be described by a dimensionless parameter

$$\lambda = J|E|^{1/2}G^{-1}M^{-5/2} \approx 0.08$$

where  $J$  is the total angular momentum,  $E$  the total energy and  $M$  the total mass.

Numerical experiments give

$$\lambda = 0.07 \pm 0.03$$

This predicts **insufficient** angular momentum to explain rotation of disk galaxies in traditional models without dark matter.

\*Peebles, A.&A. 11, 377 (1969)



The canonical working model has the following characteristics \*:

- Disk and dark halo have the same distribution of **specific angular momentum** (= angular momentum per unit mass).
- Disks collapse with **detailed conservation** of angular momentum.

For tidal torques to work one needs  $\sim 10$  times as much mass in dark halo as in the disk.

Finally we need **Mestel's hypothesis**<sup>†</sup>.

Mestel noted that the rotation and mass distribution in the disk of the Galaxy gave a distribution of specific angular momentum similar to that of a **uniformly rotating, uniform sphere**.

The hypothesis then is that disks form from such a Mestel-sphere with detailed conservation of angular momentum.

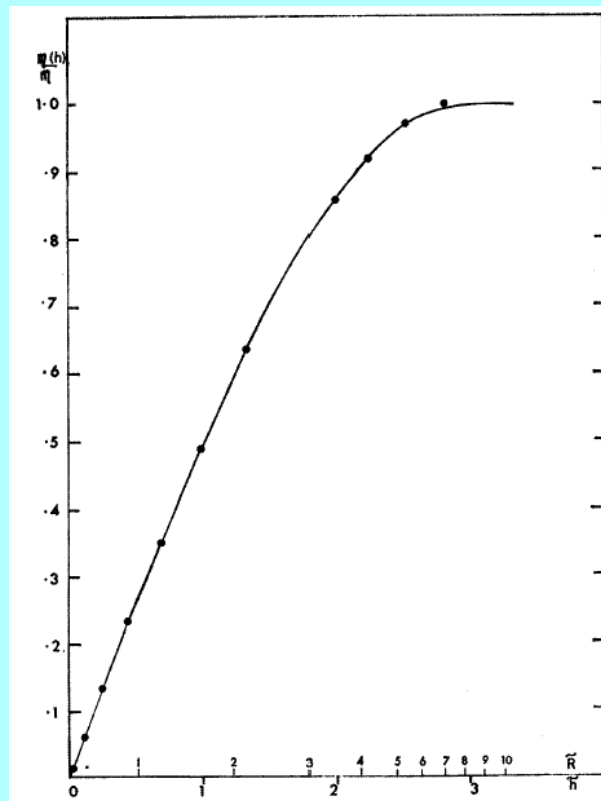
\*Fall & Efstathiou, Mon.Not.R.A.S. 193, 189 (1980)

<sup>†</sup>Mestel, Mon.Not.R.A.S. 126, 553 (1963)

The normalized distribution of specific angular momentum  $h_s$  in the Mestel sphere is

$$\frac{M(h_s)}{M} = 1 - \left(1 - \frac{h_s}{h_{\max}}\right)^{3/2}$$

Freeman\* has noted already that the self-gravitating exponential disk also has roughly this distribution.



The curve is for the exponential disk and the points for the Mestel sphere.

\*Ap.J. 160, 811 (1970)

Gunn\* noted that in a flat rotation curve the Mestel distribution would in centrifugal equilibrium give an approximately exponential radial surface density distribution.

On this basis we can consider the following **scenario for disk galaxy formation**<sup>†</sup>.

We make the following assumptions based on the discussion above:

- The protogalaxy is a Mestel sphere.
- The angular momentum results from tidal torques and  $\lambda \sim 0.07$ .
- There is a uniform mix of dark and luminous matter (so they have the same specific angular momentum distribution).

For the protogalaxy the total mass is  $M$ , and at maximum expansion the density is  $\rho_0$  and the radius  $R_m = (3M/4\pi\rho_0)^{1/3}$ .

\*in “Astrophysical Cosmology”, ed. Brück, Coyne & Longair, Pont. Acad. Scient, Vatican, p. 233 (1982)

†van der Kruit, A.&A. 173, 59 (1987)

At maximum expansion then the potential energy is

$$\Omega = -\frac{3GM^2}{5R_m}$$

and the total angular momentum

$$J = \frac{2}{5}Mh_{\max}$$

At maximum expansion the energy is essentially gravitational ( $|E| = |\Omega|$ ; in virial equilibrium it is a factor 2 smaller). Then

$$h_{\max} = \frac{5}{2} \left( \frac{5}{3} \right)^{1/2} G^{1/2} \lambda M^{1/2} R_m^{1/2}$$

Now first consider the **halo formation**.

There is some star formation in the inner regions to form the **Population II** stars. These settle dissipationlessly in the bulge.

So we get an  $R^{1/4}$ -bulge with an abundance gradient.

The **dark matter** settles dissipationlessly in something like an isothermal sphere.

Assume the amount of dark matter to be

$$M_H = (1 - \Gamma)M$$

Let this settle in an isothermal sphere with radius  $R_H$ . Then the potential energy can be calculated as

$$\Omega_H = -G(1 - \Gamma)^2 \frac{M^2}{R_H}$$

The viral theorem requires (after completion of the collapse of the dark halo) that

$$E_H = \frac{\Omega_H}{2} = -G(1 - \Gamma)^2 \frac{M^2}{2R_H}$$

But originally the energy was

$$E_H = -G(1 - \Gamma) \frac{3M^2}{5R_m}$$

Energy is conserved during dissipationless collapse, so

$$R_H = \frac{5}{6}(1 - \Gamma)R_m$$

$$V_m^2 = \frac{6}{5} \left( \frac{G}{1 - \Gamma} \right) \frac{M}{R_m}$$

Now look at the **disk formation**.

The remaining gas has a mass  $\Gamma M$  (minus bulge stars, but assume this to be small).

This then settles in a disk *with* dissipation, but conserves the specific angular momentum distribution.

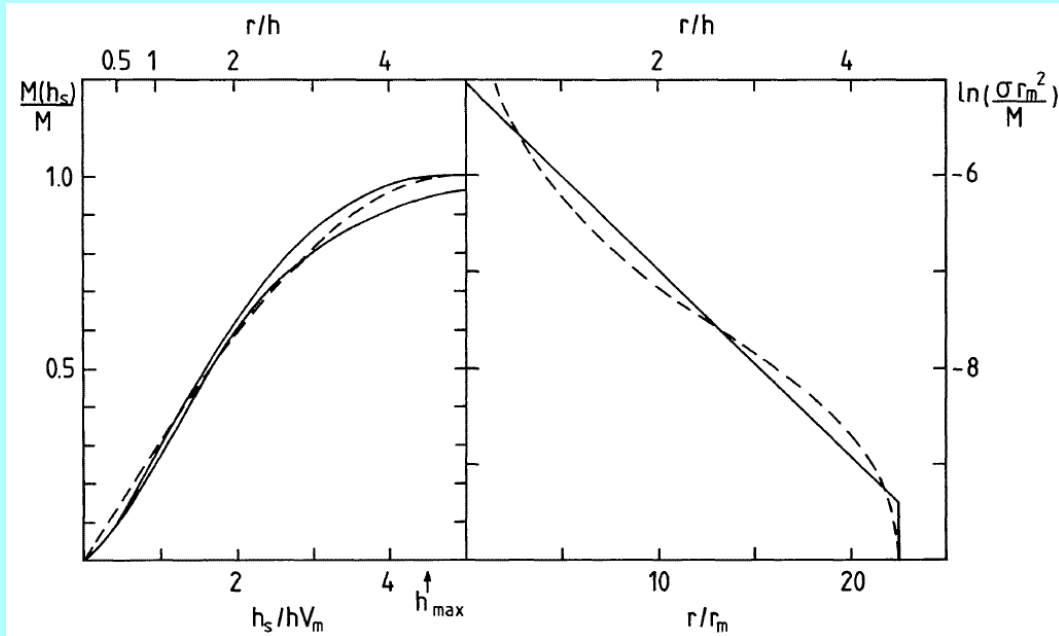
The force field in which this happens is that of the dark halo. Parametrize the final (flat) rotation curve is as\*

$$V_{\text{rot}}^2 = V_m^2 \frac{R^2}{R_m^2 + R^2} \left[ 1 - \gamma \ln \left( \frac{R^2}{R_m^2 + R^2} \right) \right]$$

From real galaxies we know that the precise value of  $\gamma$  is not important (but  $\gamma \approx 0.1$ ) and  $R_m \approx (0.1 - 0.5)h$ .

\*as also used by Fall & Efstathiou

Then calculate the surface density distribution of the disk that results; this is a roughly exponential disk with an edge at  $\approx 4.5(\equiv \beta)h$ .



On the left we have the specific angular momentum distribution (dashed line is the Mestel distribution; full drawn lines are exponential disks with a flat rotation curve for an edge at infinity and  $4.5h$ ).

On the right we see the surface density distribution from the Mestel distribution in the flat rotation curve (dashed) and a pure exponential truncated at  $4.5h$ .

Assume for simplicity that  $\Gamma \ll 1$ , so the disk does not seriously affect the force field.

The figure shows an inner excess; this may in reality be the bulge.

How does the thick disk originate? Is it a relic of the violent processes at the moment of disk collapse?

The outer HI beyond the optical truncation and the observed warps may be the result of later infall. Is that why warps start at the optical edge?

From an examination of the figure we deduce the resulting scalelength

$$h = \frac{h_{\max}}{\beta V_m} = \frac{25\lambda}{6\sqrt{2}\beta} \frac{R_m}{(1 - \Gamma)^{1/2}}$$

and the central surface density

$$\sigma_o = \frac{36}{625} \left( \frac{4}{3\sqrt{\pi}} \right)^{2/3} \left( \frac{\beta}{\lambda} \right)^2 \frac{\Gamma}{1 - \Gamma} \rho_o^{2/3} M^{1/3}$$



In models of hierarchical clustering, galaxies form at about the same time and

$$\frac{\delta\rho}{\rho} \propto M^{-(3+n)/6}$$

with  $n = -1.5$  to  $0$ .

So  $\rho_o$  is about constant and has only a small dependence on  $M$ . Then we get  $\sigma_o$  about constant for  $\Gamma$  constant.

For  $\lambda = 0.07$  and  $\beta = 4.5$  we get ( $V$  in  $\text{km s}^{-1}$ ,  $M$  in  $M_\odot$ ,  $R$  in kpc,  $\rho$  in  $M_\odot \text{ pc}^{-3}$ , etc.):

$$\frac{\Gamma}{(1 - \Gamma)^{1/2}} = 1.5 \frac{\sigma_o h}{V_m^2}$$

$$R_m = 22 \frac{h}{(1 - \Gamma)^{1/2}}$$

$$R_H = 18(1 - \Gamma)h$$

$$M = 4.2 \times 10^6 (1 - \Gamma)^{1/2} V_m^2 h$$

$$\rho_o = 9.7 \times 10^8 (1 - \Gamma)^2 \frac{V_m^2}{h^2}$$

Now apply this to our Galaxy, which has  $h = 5$  kpc,  $V_m = 220 \text{ km s}^{-1}$ ,  $\sigma_o = 400 \text{ M}_\odot \text{ pc}^{-2}$ .

$$\Gamma = 0.06$$

$$R_m = 115 \text{ kpc}$$

$$R_H = 90 \text{ kpc} = 18h$$

$$M = 1.0 \times 10^{12} \text{ M}_\odot$$

$$\rho_o = 2 \times 10^{-4} \text{ M}_\odot \text{ pc}^{-3}$$

For other galaxies we find  $\Gamma = 0.04 - 0.11$  and  $\rho_o \approx 10^{-4} \text{ M}_\odot \text{ pc}^{-3}$ .

For  $\Omega = 1$  it has been deduced that

$$\frac{\delta\rho}{\rho} = \frac{9\pi^2}{16}$$

For  $H = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$  this then implies a redshift of galaxy formation of about 3.5.

Finally calculate the disk luminosity

$$L_{\text{disk}} = \frac{L}{M} \Gamma^2 (1 - \Gamma) V_{\text{m}}^4 \mu_{\text{o}}^{-1}$$

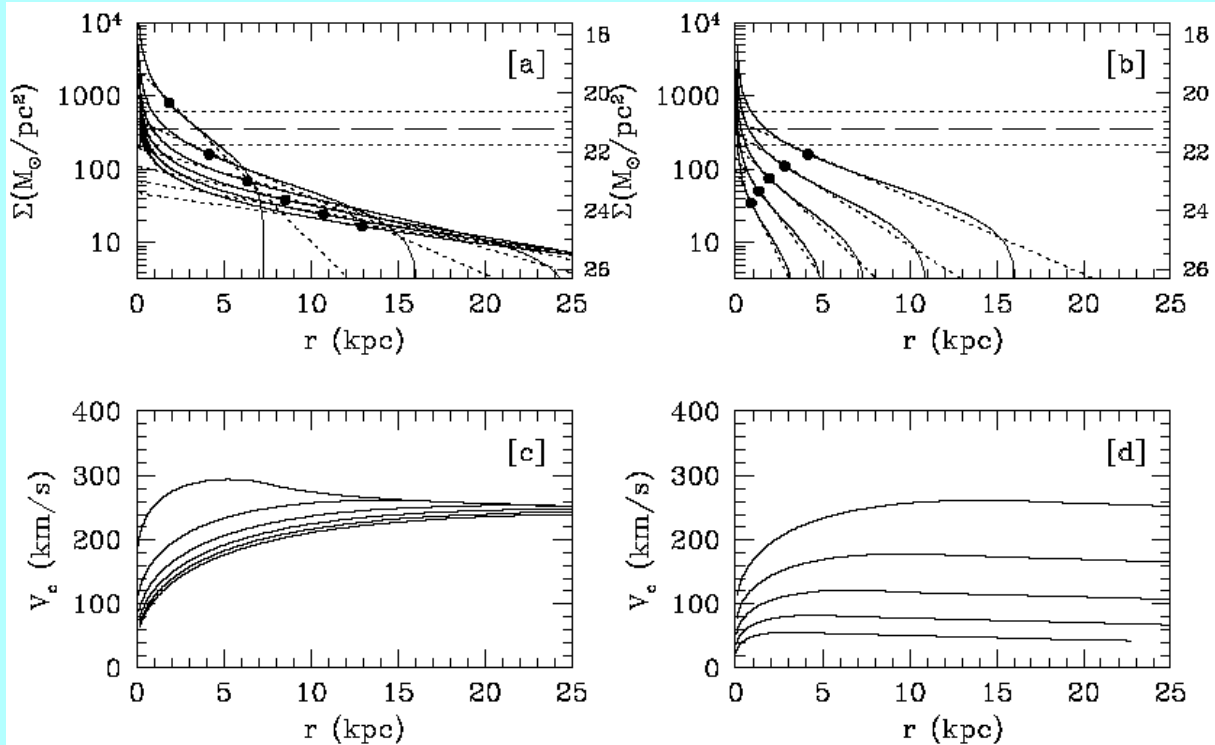
So with Freeman's law, constant  $(M/L)$  and  $\Gamma$  we get the Tully-Fisher relation.

This schematic model has been greatly improved by Dalcanton et al.\*.

They do more realistic calculations, taking all gravitation into account, take a range in  $\lambda$ , etc.

The resulting disk density profiles and rotation curves are in the following figure.

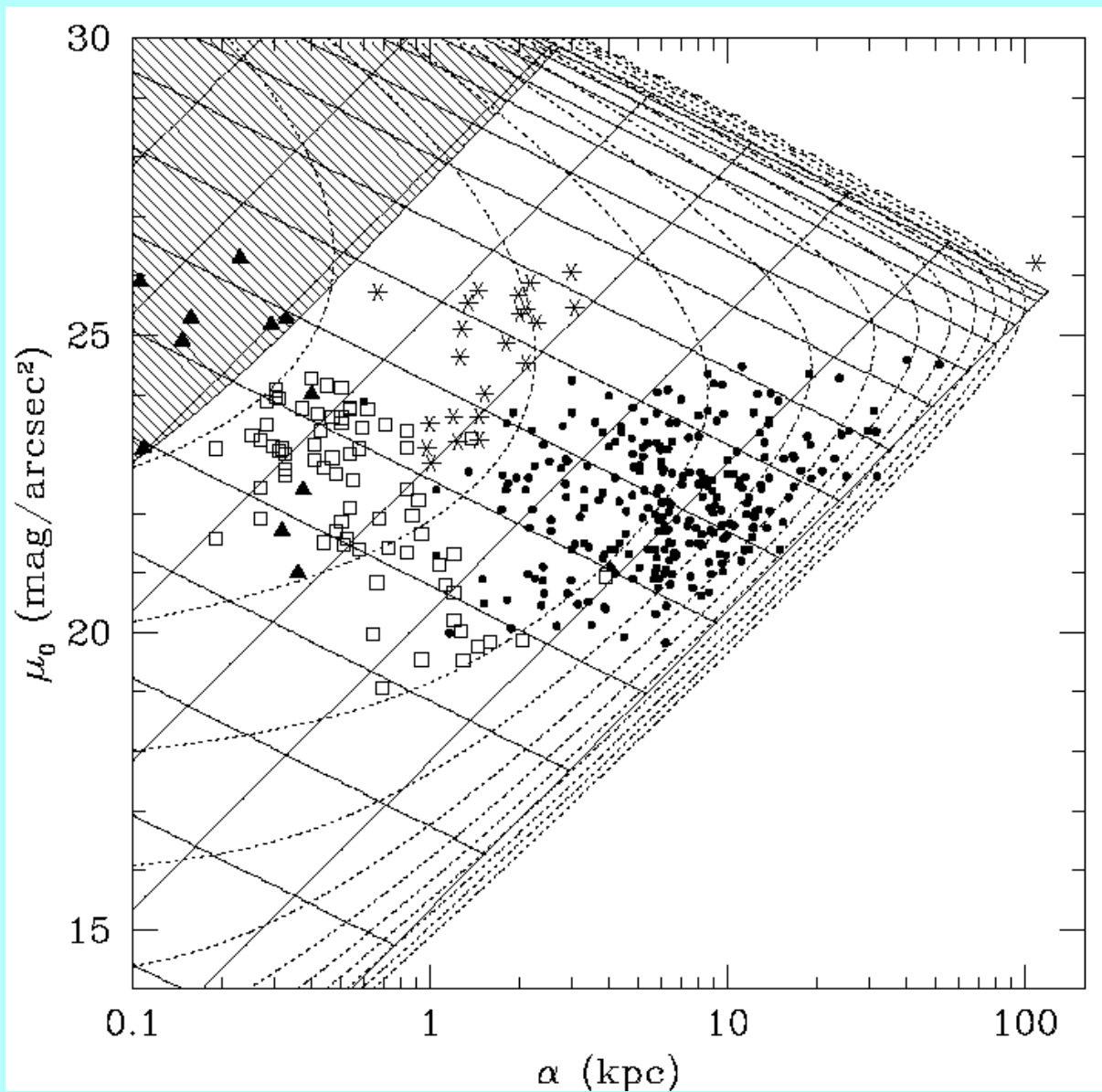
\*Dalcanton, Spergel & Summers, Ap.J. 482, 659 (1997)



On the left we have models for  $\lambda = 0.03 - 0.18$ ;  $M = 10^{12} M_{\odot}$ . On the right we have  $\lambda = 0.06$ ;  $M = 10^{10} - 10^{12} M_{\odot}$ .

The assumption of a range in  $\lambda$  now translates in a range of predicted central surface densities.

The models project in the (surface brightness - scalelength) plane as follows.



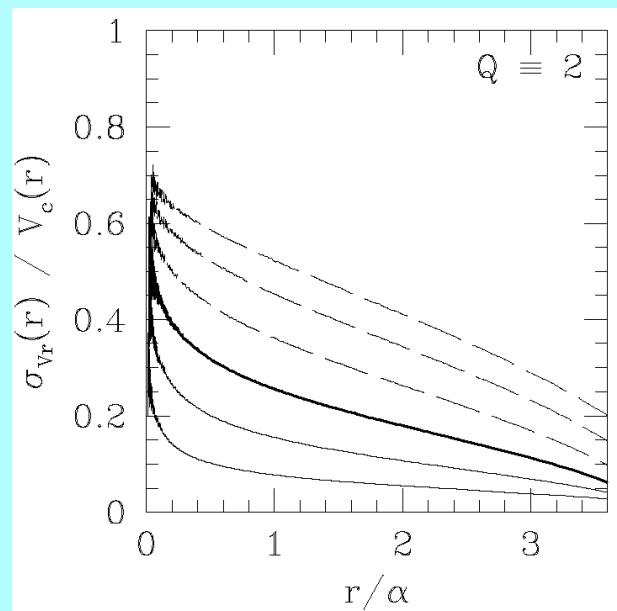
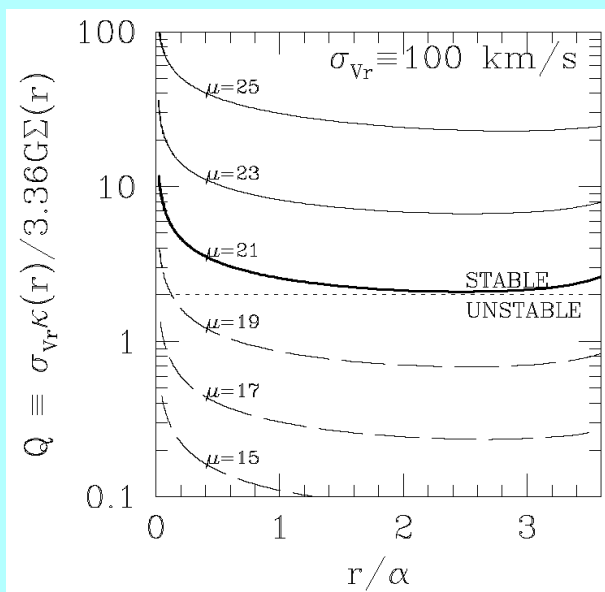
The dashed line are lines of equal expected density in this plane for  $M/L = 3$  in B,  $\Gamma = 0.05$  and  $H = 50 \text{ km s}^{-1} \text{ kpc}^{-1}$ . This is based on an assumed mass distribution as a **Schechter function**.

The solid lines with positive slope are of equal mass and those of negative slope of constant angular momentum.

In the hatched region gas pressure is expected to prevent the galaxies from collapsing.

The data are various not statistically complete samples (the filled triangles are Local Group spheroidals).

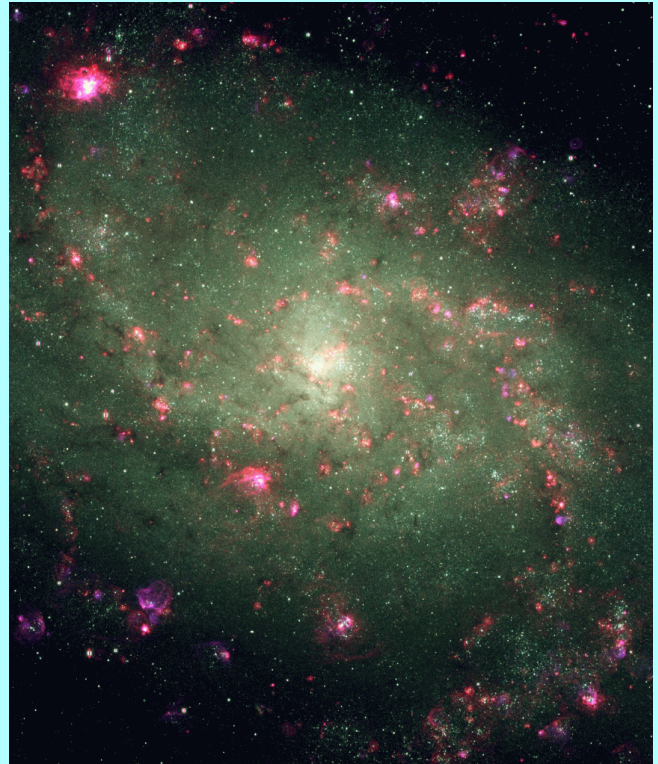
The **disk stability** and **stellar velocity dispersion** as a function of radius gives the following results.



Although in broad terms probably still applicable, this model will have to be augmented to incorporate the effects of **infall of companions**, such as the Sagittarius Dwarf into our own Galaxy.

## Spiral structure.

In general terms we can distinguish two types of spiral structure, namely **grand design** and **flocculent**.





A comparative study of these two classes\* suggests that in grand-design spiral structure there seems to be a strong **underlying spiral wave in the stellar disk**, while not in flocculent ones.

The **density wave theory**<sup>†</sup> was a response to the “**winding dilemma**”, where material arms would wind up in a matter of  $10^8$  years or less.

The density wave is a spiral pattern, whose shape does not change with time, and which moves through the stellar and interstellar disk.

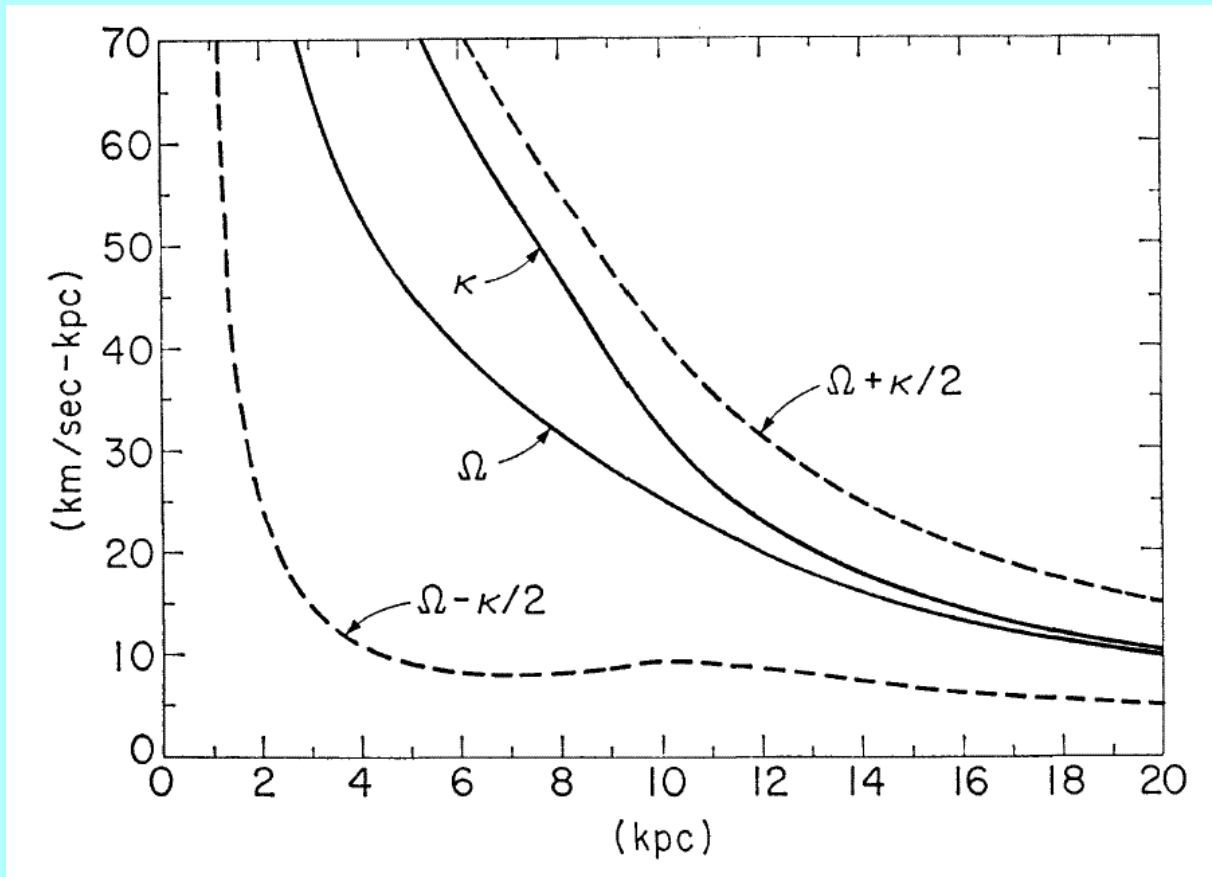
At the basis of a good description we can take the deduction that in the disk of our Galaxy (and in many others) the **inner Lindblad resonance**  $\Omega - \kappa/2$  is fairly constant.

In this resonance a star goes through two epicycles during one revolution around the center. That means it describes a closed oval orbit in coordinate system moving with  $\Omega - \kappa/2$ .

\*Elmegreen & Elmegreen, Ap.J.Suppl. 54, 127 (1984)

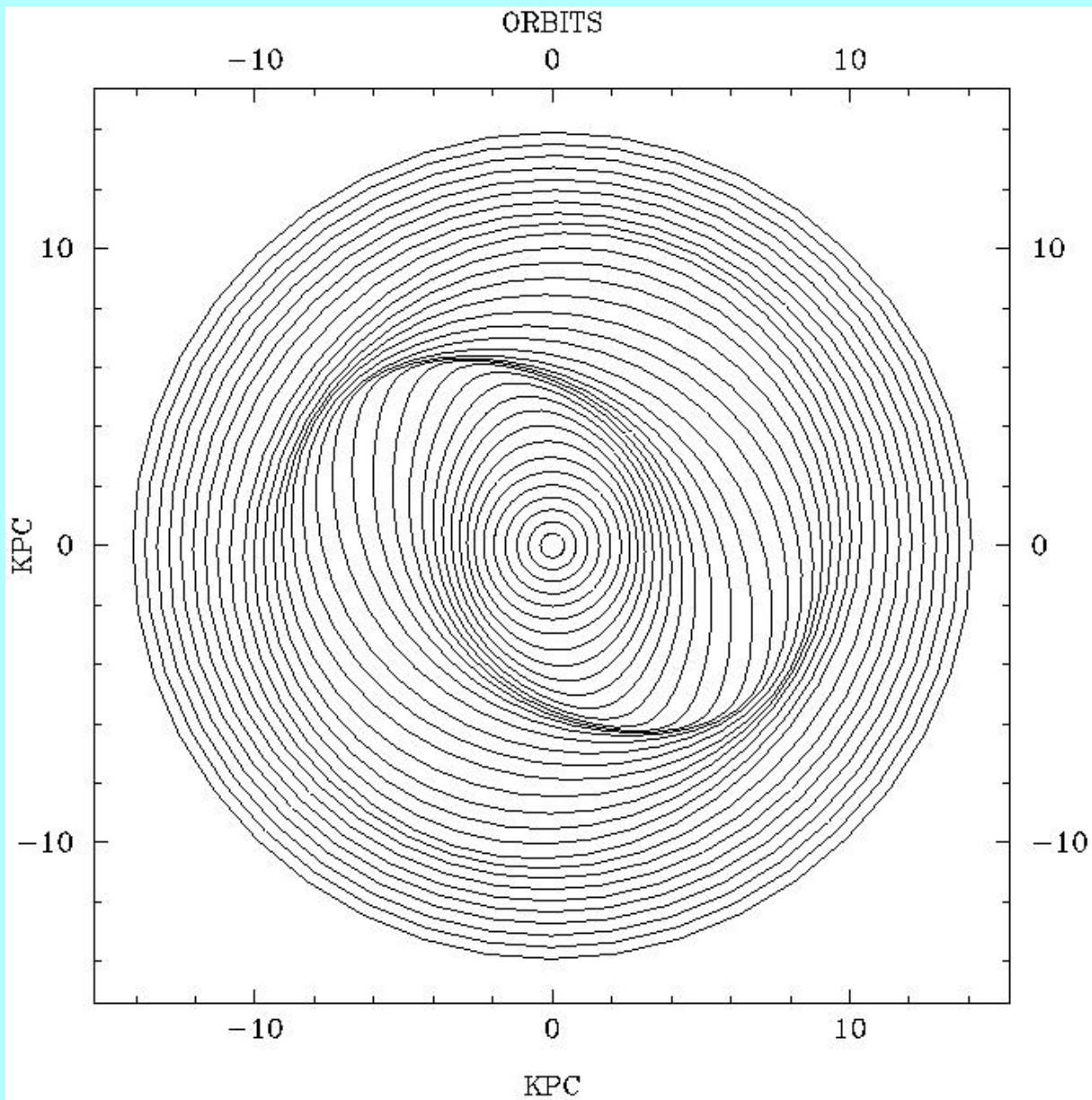
<sup>†</sup>Lin & Shu, Ap.J. 140,646 (1964), Lin, Yuan & Shu, Ap.J. 155, 721 (1969)





In a disk where this property is constant over most radii we can get the following situation, where the stars are forced in orbits that line up as a spiral pattern.

In a coordinate frame, rotating with the **pattern speed**  $\Omega_p = \Omega - \kappa/2$ , the spiral pattern remains unchanged.

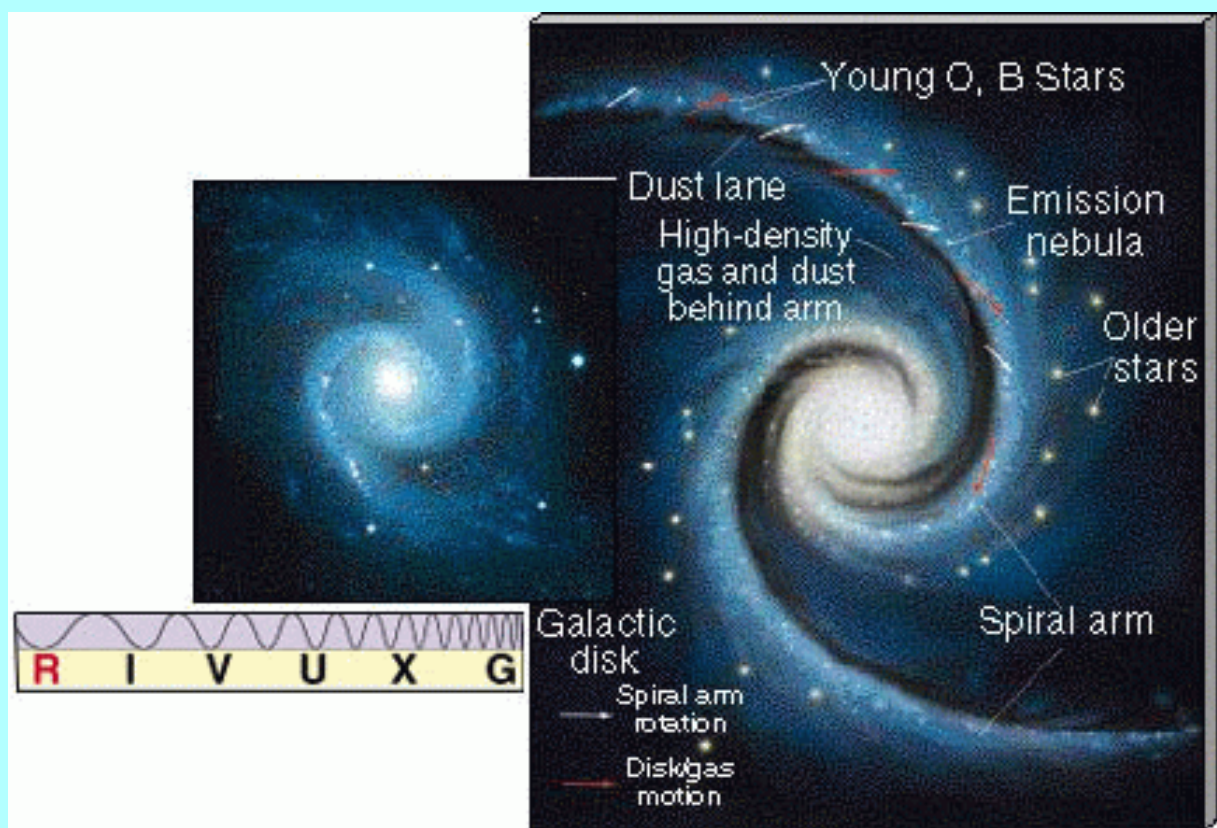


In the original density wave theory the density perturbations maintain themselves. The response of the stars to the perturbed gravitational field by the density concentrations in the arms results in a continuing pattern of density perturbations.

It was realized later by Toomre and others that the **dissipation of energy** in the waves is quick enough ( $\sim 10^8$  years) that rejuvenation is required regularly.

It took until the first part of the seventies, before the **underlying wave in the stellar disk** was discovered in surface photometry\*.

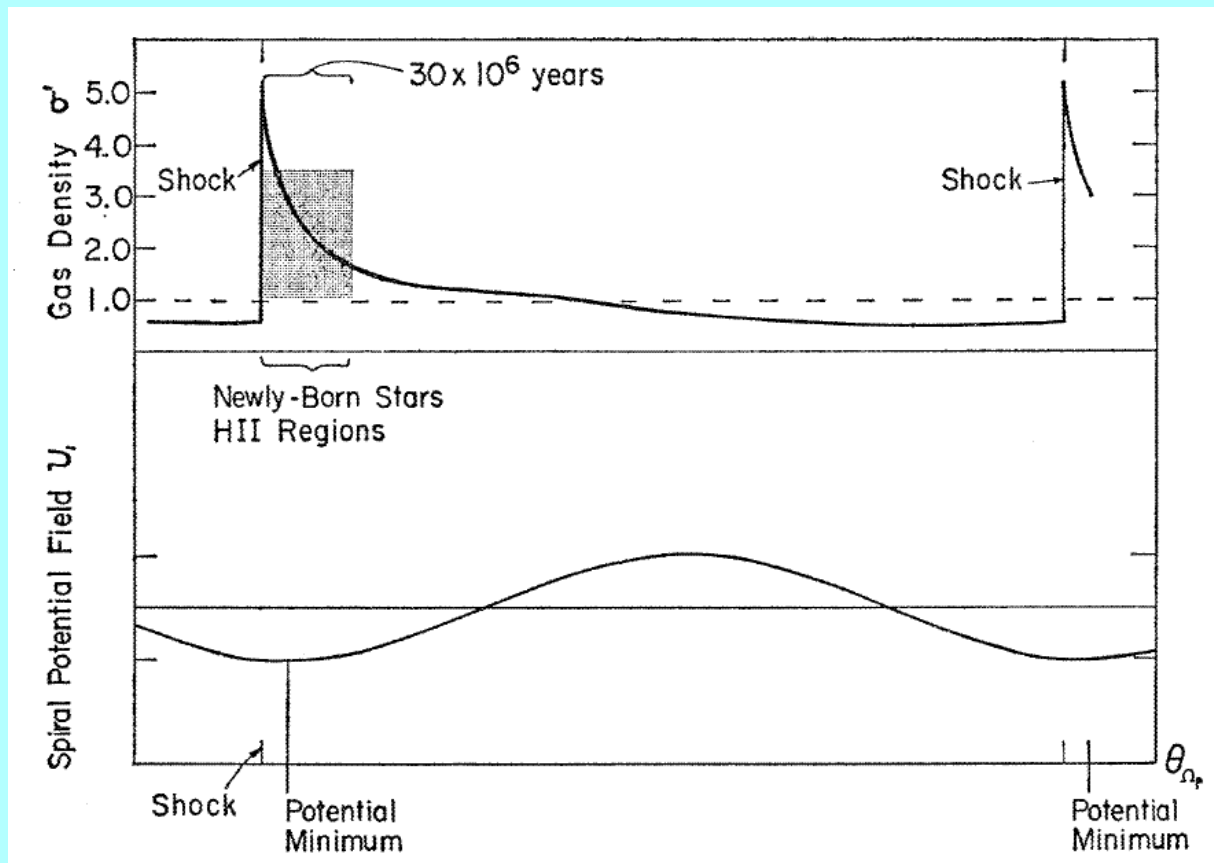
The strongest confirmation came from studies of the interstellar medium.



\*Schweizer, Ap.J.Suppl. 31, 313 (1976)

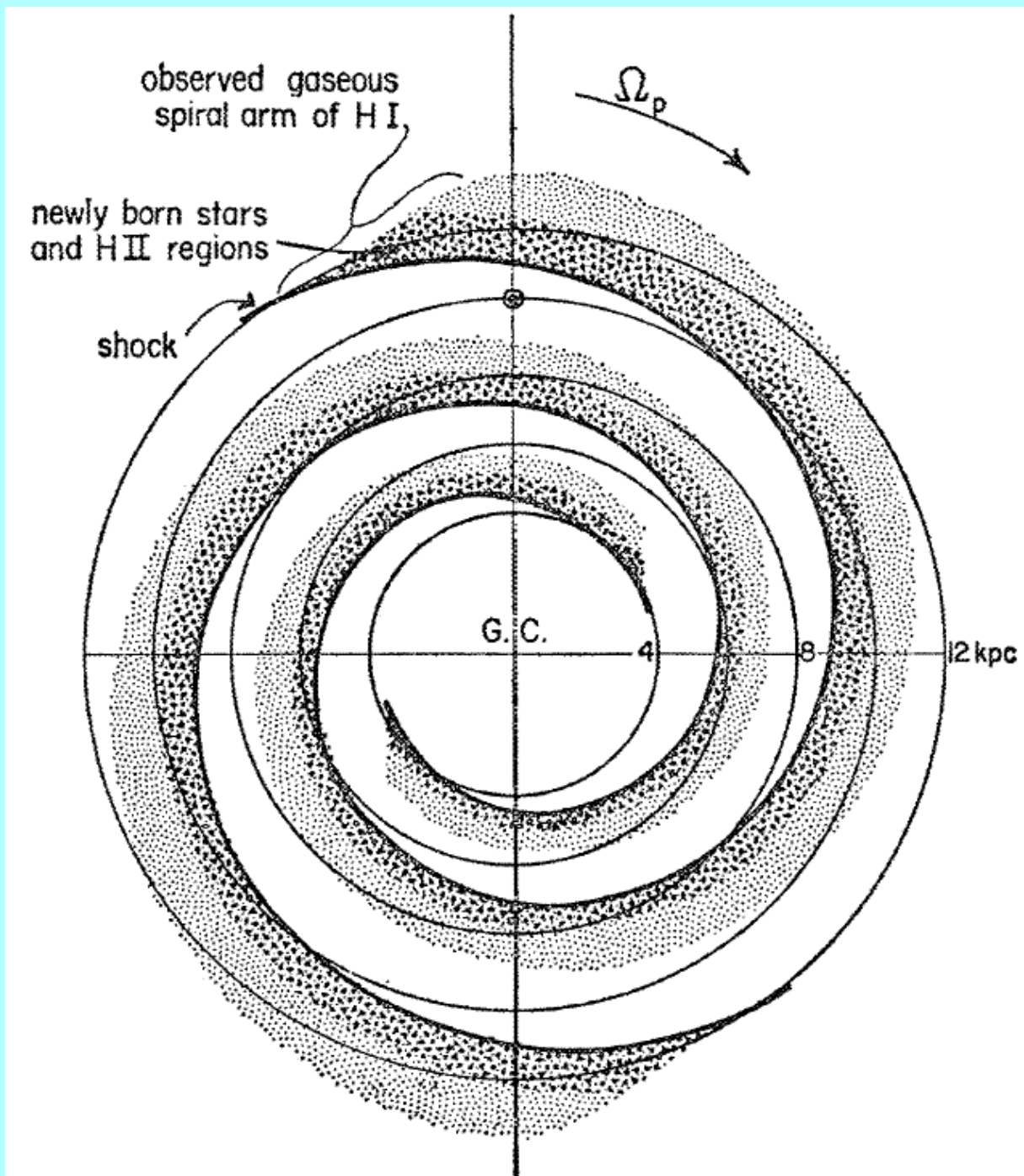
The response of the gas and dust is a-linear, since the relative velocities involved are **supersonic**.

This gives **shocks** at the inner sides of the spiral arms and associated **dustlanes** and **starformation**.\*.



The “delay” between dustlanes and HII-regions concerns the time between onset of gravitational instability and birth of MS-stars.

\*Roberts, Ap.J. 158, 123 (1969)

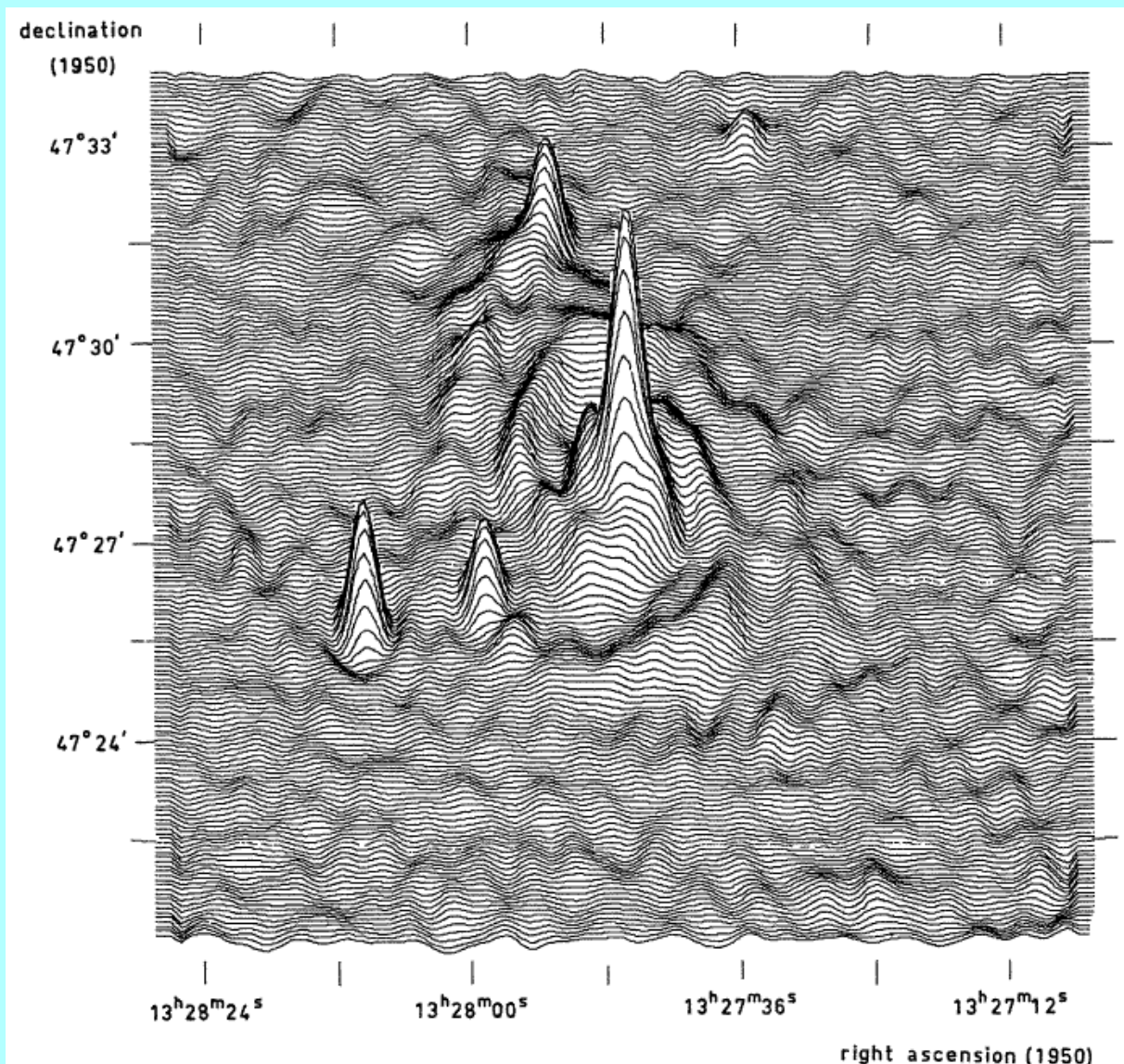


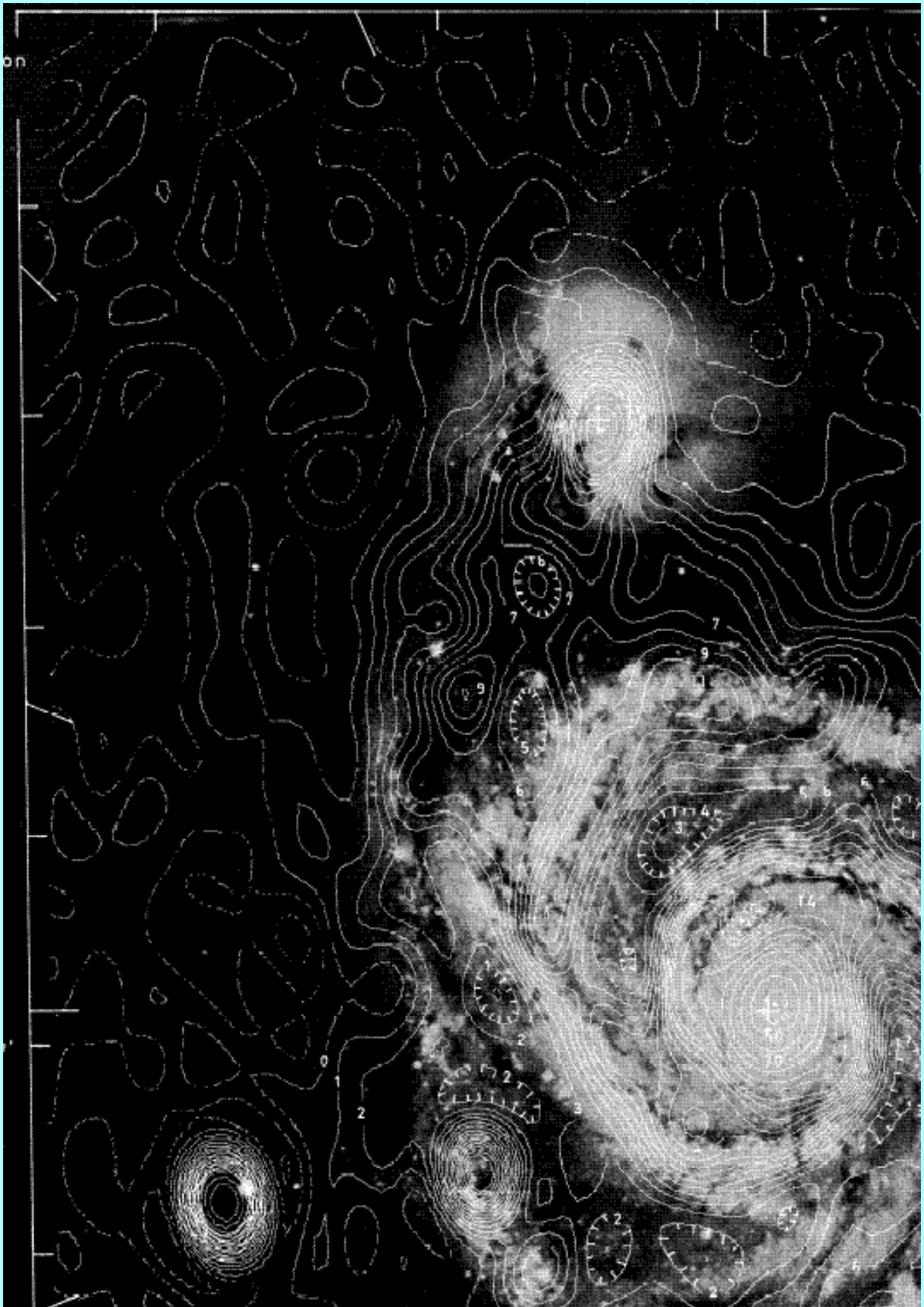
It was also confirmed by **radio continuum** studies with the new WSRT\* in **M51**.

\*Matthewson, van der Kruit & Brouw, A.&A. 17, 468 (1972)

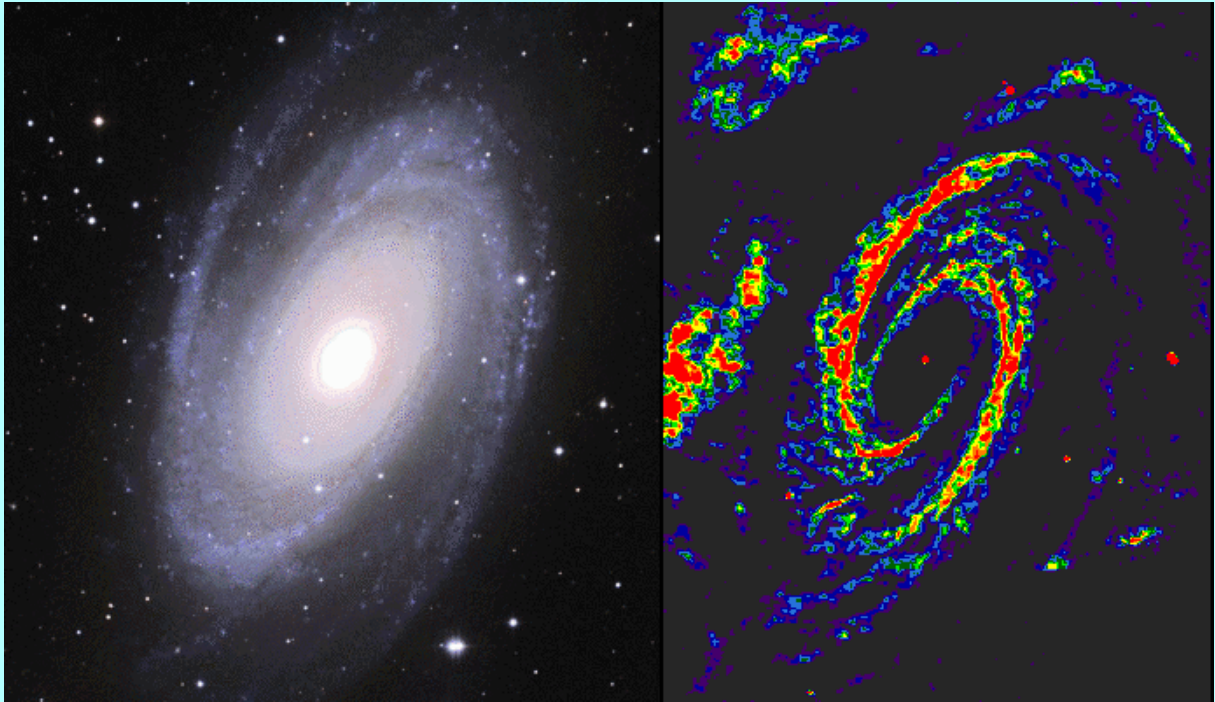


The compression holds at least for the **magnetic field** and possibly the **relativistic electrons**, so the **synchrotron radiation** will be enhanced at the inside of the arms and at the dustlanes.





The next thing was to try and measure the **streaming motions** due to the density wave. This was tried in particular in M81.



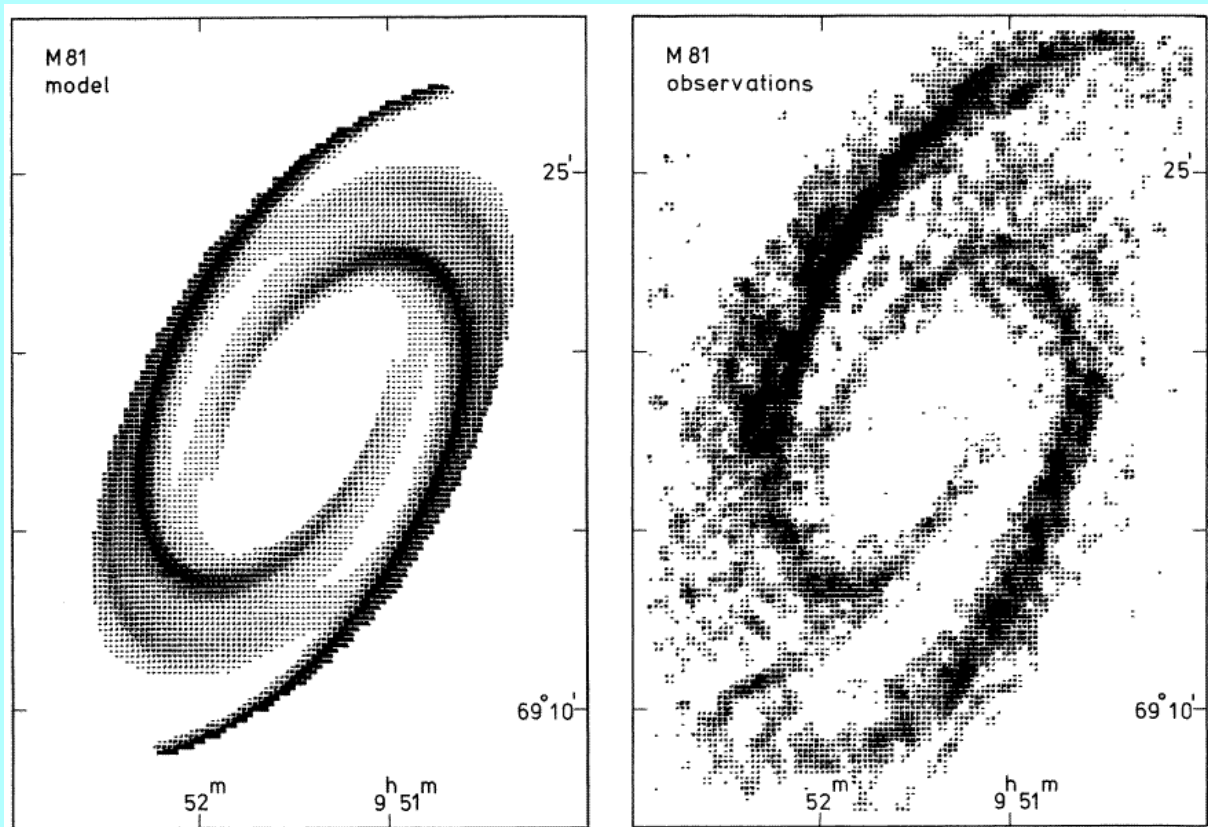
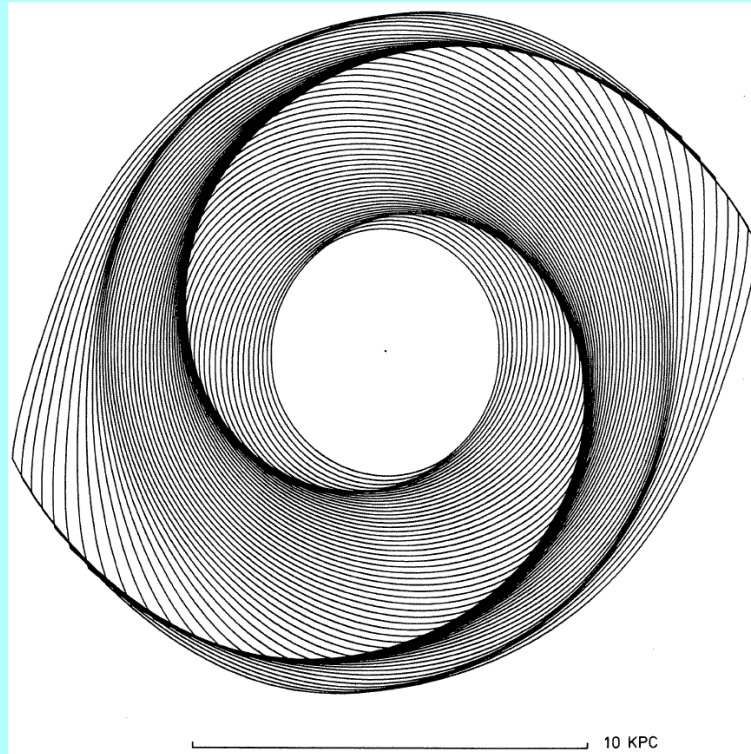
The Ph.D. thesis of **Visser**\* analysed this in detail.

Using the photometry of Scheizer and HI-measurements at Westerbork, he was able to find an internally consistent representation of the observations.

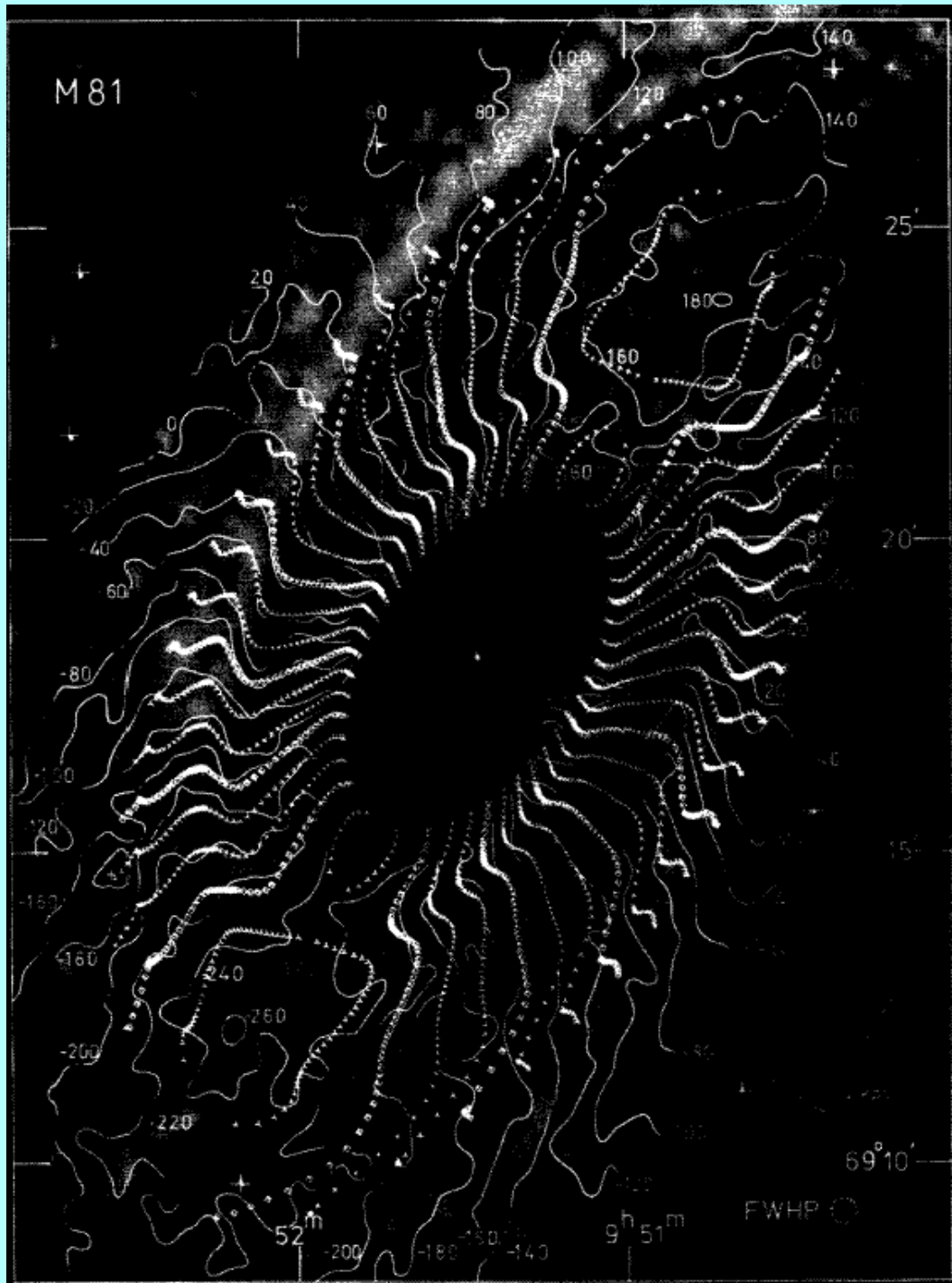
\*1978; see also A.&A. 88, 159 (1980)



Here are the (non-linear) streamlines of the gas.

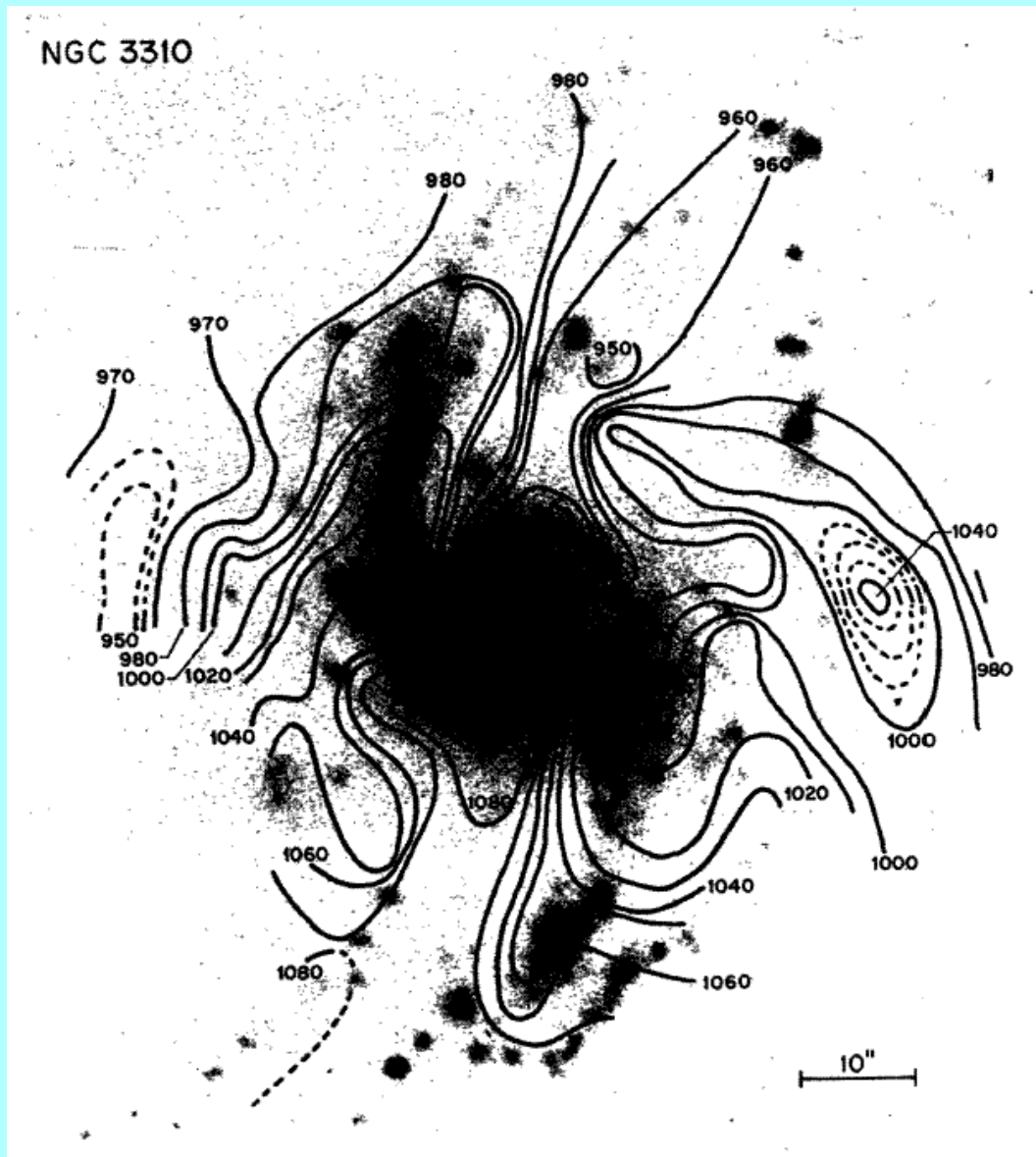


The streaming motions are of the order of  $10 \text{ km s}^{-1}$ .



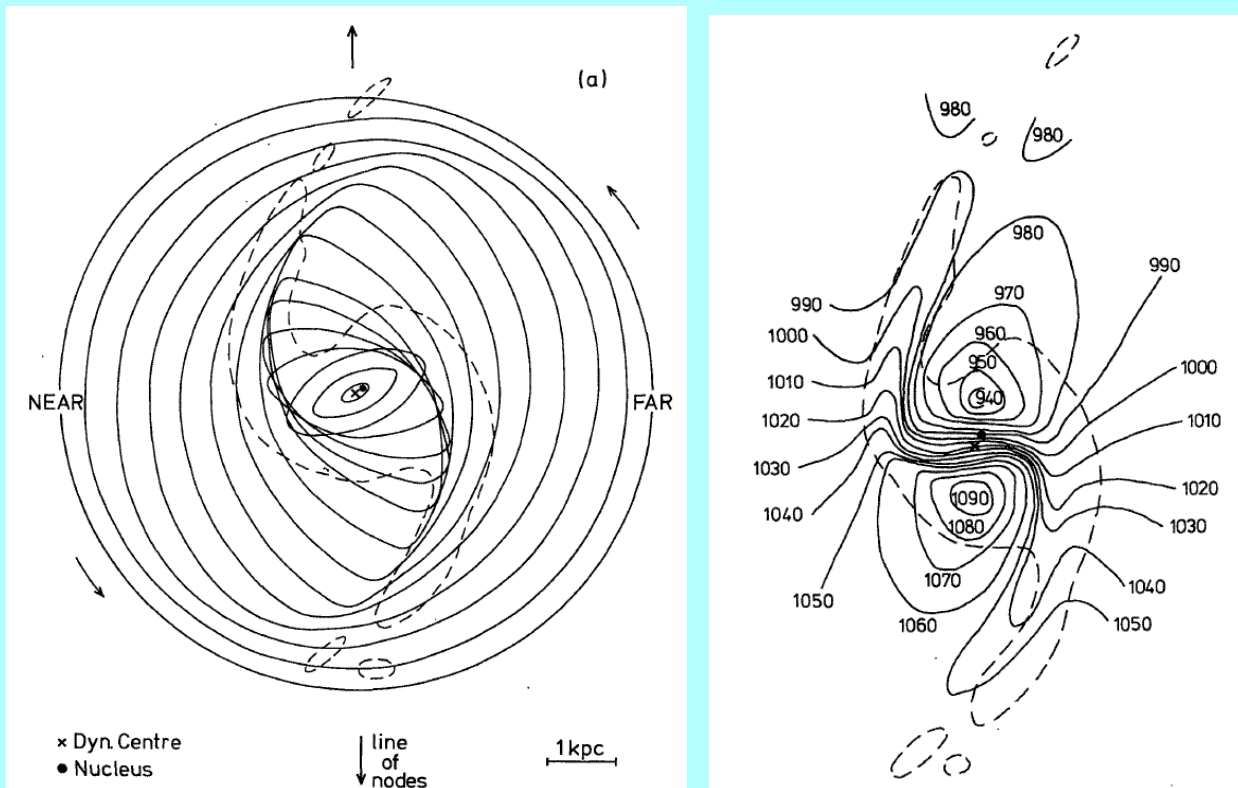
A very exceptional case is the **disturbed, star burst galaxy NGC 3310**, which is probably an example of a recent encounter\*.

The velocity field looks as follows.



\*van der Kruit & de Bruyn, A.&A. 48, 373 (1976); van der Kruit, A.&A. 49, 161 (1976)

The streaming motions are here up to a third or so of the rotation velocity.

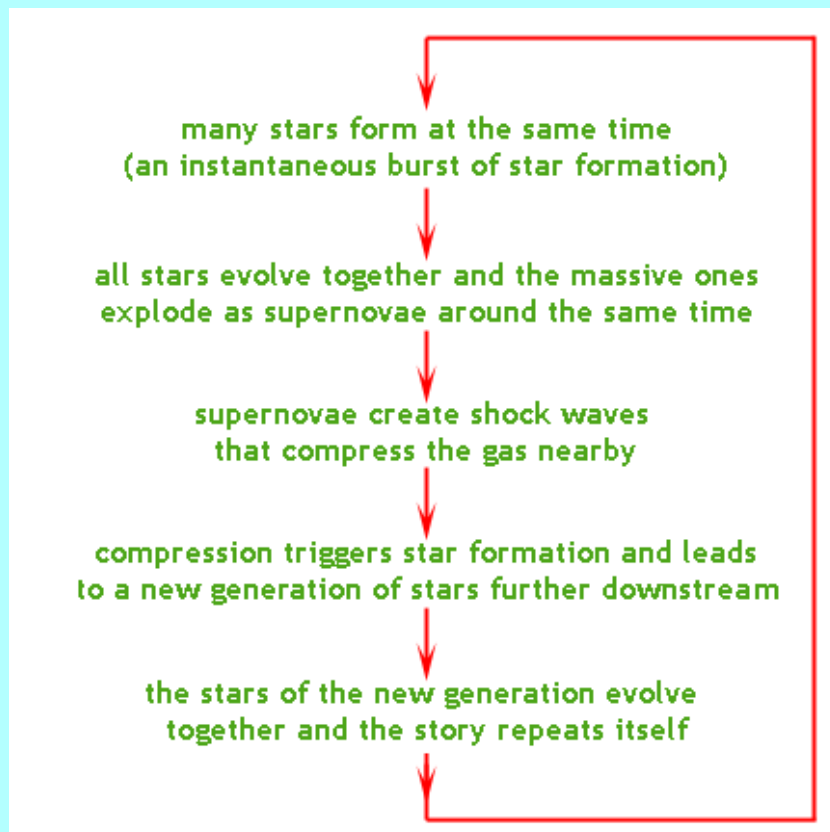


Density waves may be generated by **tidal interactions**, such as in M51 or in NGC 3310, or through Toomre's **swing amplification**.

The **flocculent** spiral structure is probably the result of **stochastic self-propagating star formation**\*.

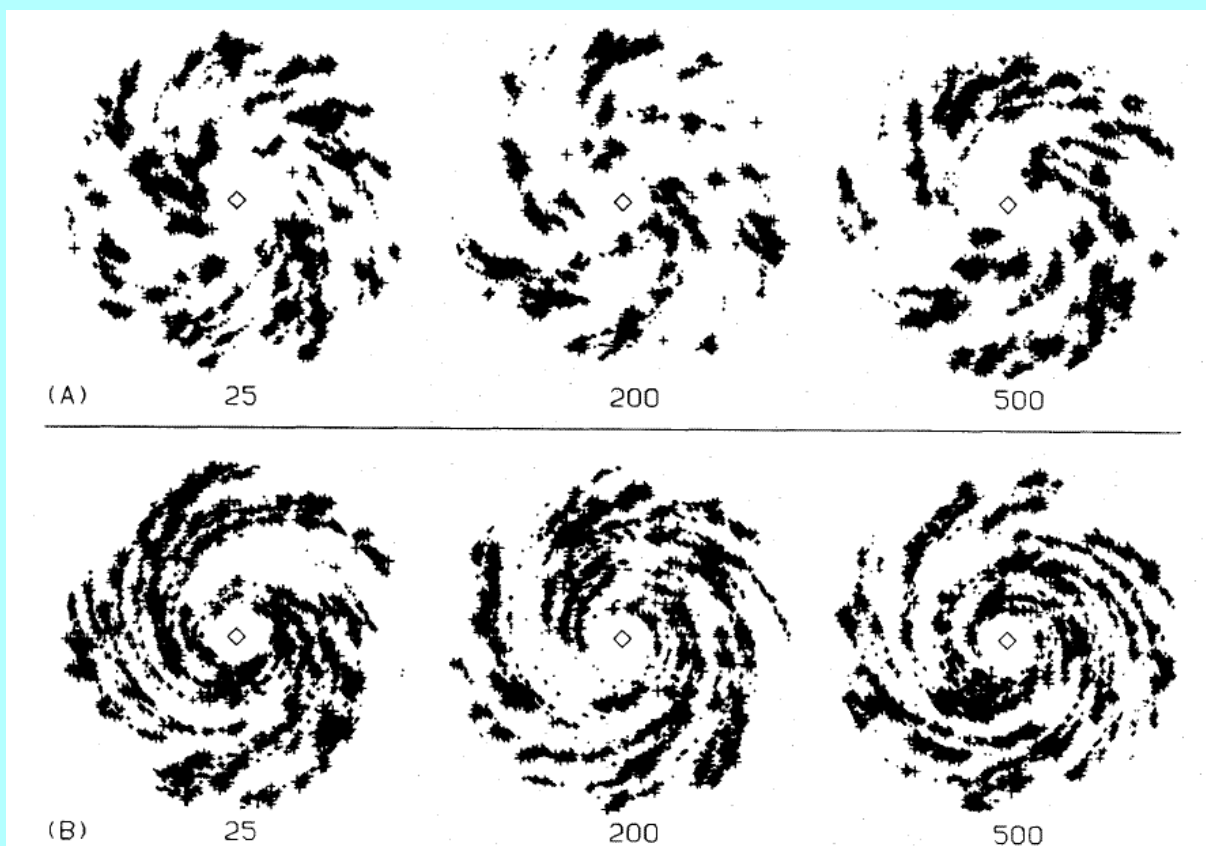
In this model star formation through supernova explosions is postulated to stimulate star formation in the neighborhood.

Such structures are then drawn out by differential rotation into arm-like features.



\*Gerola & Seiden, Ap.J. 223, 129 (1978)

Here are some simulations.



Since the propagation and induced star formation is never 100%, also this will die out unless there is also spontaneous star formation.

It has been suggested\* that grand-design spiral structure is produced by bars or tidal encounters, while flocculent spiral structure results if the disk is left by itself.

\*Kormendy & Norman, Ap.J. 233, 539 (1979)