STRUCTURE OF GALAXIES

10. Formation of galaxies

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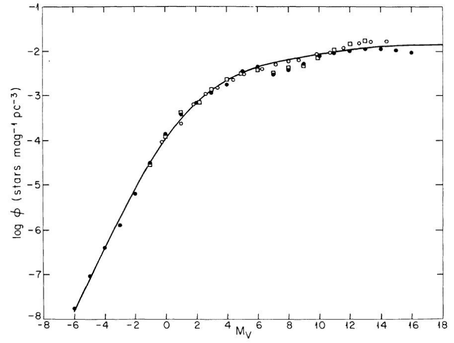
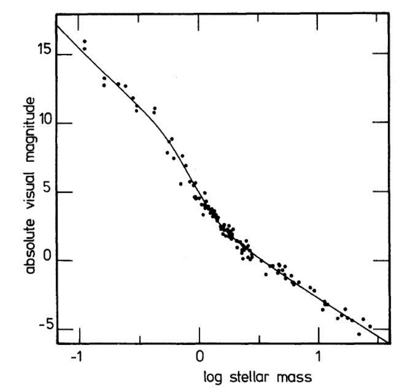
Outline

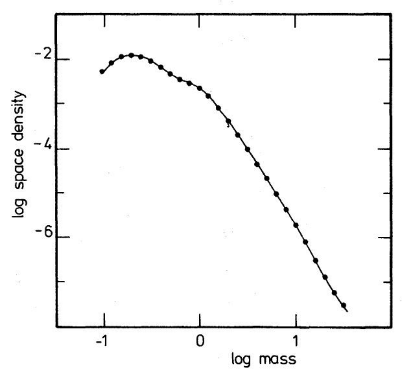
Evidence from our Galaxy

The disk population

The local Mass Function (density in $M_{\odot}$ pc^{-3} per mass interval of 0.1 log $M_{\odot}$ as a function of stellar mass) derives from the observed Luminosity Function (number of stars per magnitude interval per pc^3) using the Mass-Luminosity Relation.




15
10
5
0
-5
absolute visual magnitude
-1
0
1
log stellar mass


-2
-4
-6
log space density
-1
0
1
log mass

The stellar mass density in the solar neighborhood is then 0.038 $M_\odot$ pc^{-3} and the corresponding mass-to-light ratio is 1.2[1].

The (surface) density distributions can be derived from dynamical studies.

Take a tracer population of objects with density distribution ν and velocity dispersions $\sigma_{\rm xy}$

$$\sigma_{\rm xy} = \langle V_{\rm x} V_{\rm y} \rangle^{1/2}$$

[1] G. Gilmore & N. Reid, Mon.Not.R.A.S. 202, 1025 (1983)

The hydrodynamical equation then describes how the distribution and kinematics of this tracer population relates to the vertical gravitational force.

$$-K_z = \frac{1}{\nu}\frac{\partial}{\partial z}(\nu\sigma_{zz}^2) + \frac{1}{\nu R}\frac{\partial}{\partial R}(\nu R\sigma_{Rz}^2)$$

The second term can usually be neglected and if the tracer population is isothermal then

$$K_z = \sigma_{zz}^2 \frac{\partial}{\partial z} \ln \nu(z)$$

The Poisson equation relates the gravitational field to the total density distribution ρ.

At small distances z from the plane these equations can be combined to give

$$4\pi G\rho_\circ = \frac{\partial}{\partial z}\left[\frac{1}{\nu}\frac{\partial}{\partial z}(\nu\sigma^2_{\rm zz})\right]$$

One can use samples of for example K giants or (older) F dwarfs to this. This idea goes back to Kapteyn[2] and Oort[3].
Modern analyses of this kind have been done by Bahcall[4] and Kuijken & Gilmore[5].

Bahcall finds for the space density in the solar neighborhood $0.21 \pm 0.04\ \mathrm{M_\odot pc^{-3}}$.

[2] J.C. Kapteyn, Ap.J. 55, 302 (1922)
[3] J.H. Oort, Bull.Astron.Inst.Neth. 6, 249 (1932)
[4] J.N. Bahcall, Ap.J. 276, 156 and 169, Ap.J. 287, 926 (1984)
[5] K.H. Kuijken & G. Gilmore, Mon.Not.R.A.S. 239, 571, 605 and 651 (1989)

Observed are the following contributions.

Component	density
Main sequence stars	0.044
Subgiants and giants	0.002
White dwarfs	0.005
ISM (atomic & molec. gas, dust)	0.045
Population II	0.0001
Total	0.096

So in this case a total of about $0.1\ \mathrm{M_{\odot}pc^{-3}}$ is unaccounted for. This problem has been known for many years and is known as the "Oort limit".

Large numbers of brown dwarfs or stellar remnants cannot completely be ruled out.

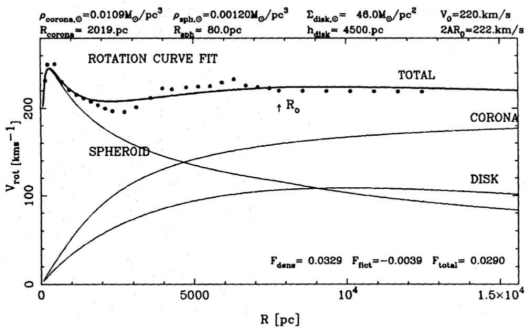
Kuijken & Gilmore on the other hand find that the local density is about 0.10 $\mathrm{M_\odot pc^{-3}}$ and that there is no convincing evidence for missing matter.

In terms of surface density of the Galactic disk, Bahcall finds a value of $66 \pm 8\ \mathrm{M_\odot pc^{-2}}$. This is distributed as follows:

Component	mass $\mathrm{M_\odot pc^{-2}}$	luminosity $\mathrm{L_\odot pc^{-2}}$
Main sequence stars	23.9	9.7
Subgiants and giants	1.0	13.3
White dwarfs	3.6	0.0
Interstellar medium	4.5	0.0
Unseen matter	33.0	0.0
(Population II)	(3.0)	(1.5)
Total	66.0	23.0

Kuijken & Gilmore find a total surface density of $46 \pm 9\ \mathrm{M_\odot pc^{-2}}$, of which $35 \pm 5\ \mathrm{M_\odot pc^{-2}}$ is in stars and $13 \pm 3\ \mathrm{M_\odot pc^{-2}}$ in gas and dust.

They also propose the following fit to the rotation curve of the Galaxy.



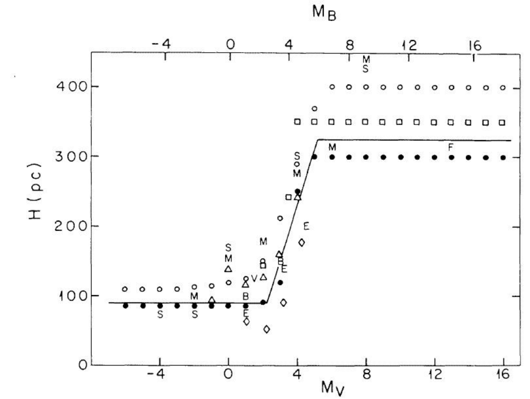
It follows that the Galaxy is NOT maximum disk.

With $\kappa \sim 31$ km sec^{-1} kpc^{-1} and $\sigma_{\rm RR} \sim 40$ km sec^{-1} the Toomre parameter can be determined as

$$Q \sim 2.1.$$

Disk stars have varying vertical distributions, according to the velocity dispersion – age relation.

This is also reflected in the (exponential) scaleheight derived from counts as a function of absolute magnitude.



The old disk scaleheight is about 325 pc.

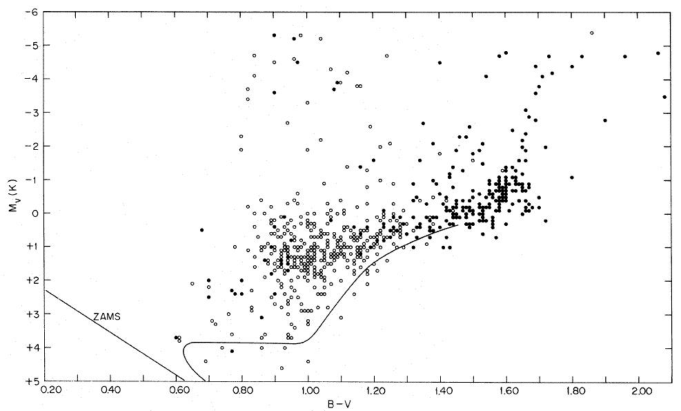
Subgiants and giants have emission ("chromospheric") components in the CaII K-line. The strength of this component gives the absolute magnitude and hence the distance.

This has been done for a sample of about 700 bright stars[6].

The line in the figure (next frame) is the (sub-)giant branch of NGC 188 (age $\approx 10 \times 10^9$ years).

This shows that the old disk population contains stars with ages at least up to that age.

[6] O.C. Wilson, Ap.J. 205, 823 (1976)


M_V (K)
-6
-5
-4
-3
-2
-1
0
+1
+2
+3
+4
+5
ZAMS
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
2.00
B−V

ELS and the collapse of the Galaxy

This classical paper ELS[7] contains a study of properties of samples of high- and low-velocity dwarfs.

These samples have determinations of parallax, proper motion, radial velocity and photometry and spectral type.

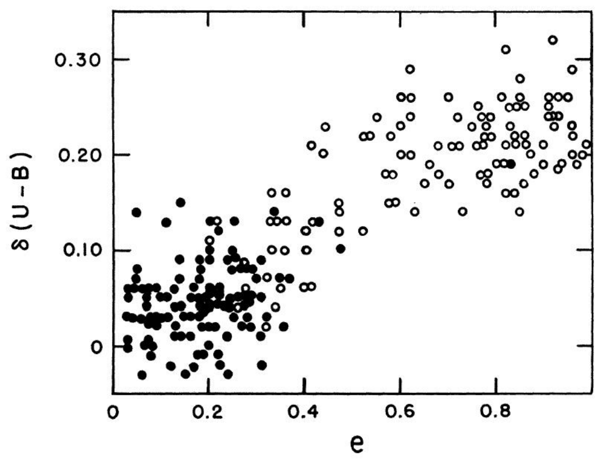
They determined the three components of the space velocity and computed from that the "excentricity" (from the radial excusion in the plane) and the angular momentum.

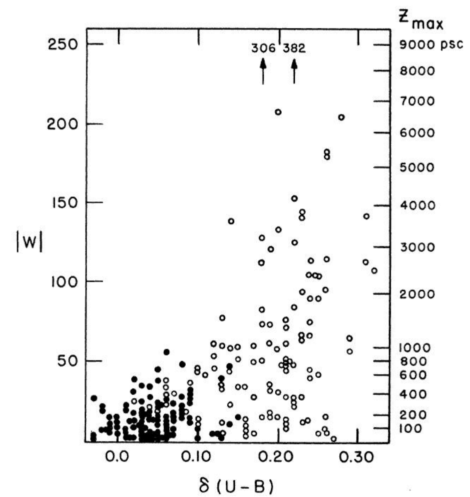
The ultraviolet excess $\delta(U-B)$[8] is an indication of the metallicity, since for these stars it results from line blanketing (more absorption lines in the UV than in the visual).

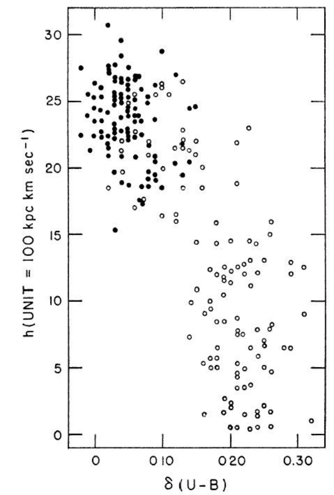
[7] O.J. Eggen, D. Lynden-Bell & A. Sandage, Ap.J. 136, 748 (1962)

[8] Difference in color observed from that expected from the spectral type

- The vertical velocity, orbital excentricity and angular momentum correlate with the UV-excess.



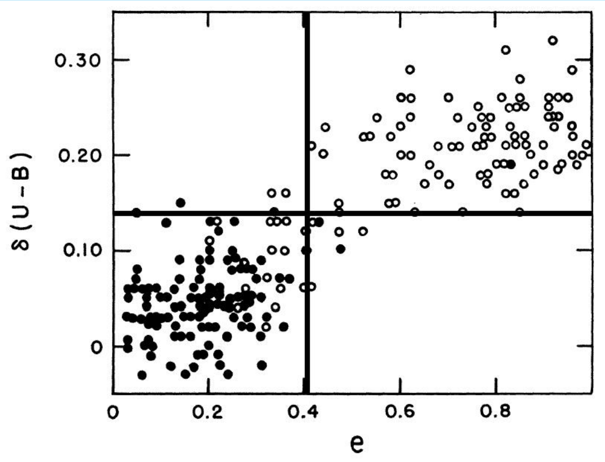

250
200
150
100
50
|W|
0.0
0.10
0.20
0.30
δ (U − B)
306
382
Z max
9000 psc
8000
7000
6000
5000
4000
3000
2000
1000
800
600
400
200
100

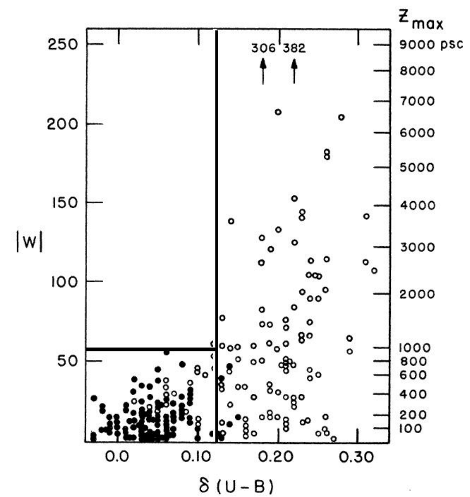

30
25
20
15
10
5
0
h (UNIT = 100 kpc km sec⁻¹)
0
0.10
0.20
0.30
δ (U − B)

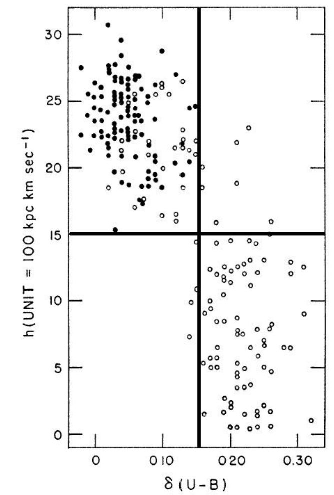
- The continuous progression of metal content from halo to disk stars provides evidence that the Galaxy collapsed.
- Metal-poor stars go up to $z \approx 10$ kpc while the old disk only goes up to $z \approx 400$ pc. The vertical collapse is thus about a factor 25.
- The occurence of very high excentricities among halo stars indicates rapid disk collapse. A strong increase in gravitation will elongate circular orbits when the collapse proceeds on timescale less than the orbital period ($\approx 10^8$ years).
- From the observed angular momentum the estimated radial collapse factor is about 10.

ELS described the process as a continuous one, but even their figures can be interpreted as showing two discrete components.

However, their graphs may be interpreted differently.

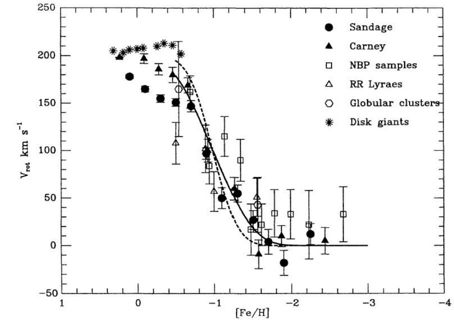



z_{max}
250
200
150
100
50
|W|
306 382
9000 psc
8000
7000
6000
5000
4000
3000
2000
1000
800
600
400
200
100
0.0
0.10
0.20
0.30
δ (U – B)


30
25
20
15
10
5
0
h(UNIT = 100 kpc km sec⁻¹)
0
0.10
0.20
0.30
δ (U – B)

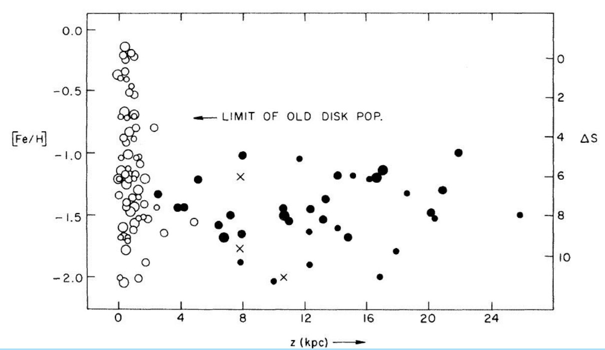
There is more evidence for the basic discreteness of Galactic structure.

One example is the asymmetric drift (the lagging behind in rotation of components with higher velocity dispersion) as a function of metallicity [Fe/H].



Here we see the rotation velocity with respect to an inertial frame.

Also the upper limit of the distribution of metallicity of disk and halo RR Lyrae stars[9] does not show a gradual decline with height above the plane.



[9] T.D. Butler, T.D. Kinman & R.P. Kraft, A.J. 84, 993 (1979)

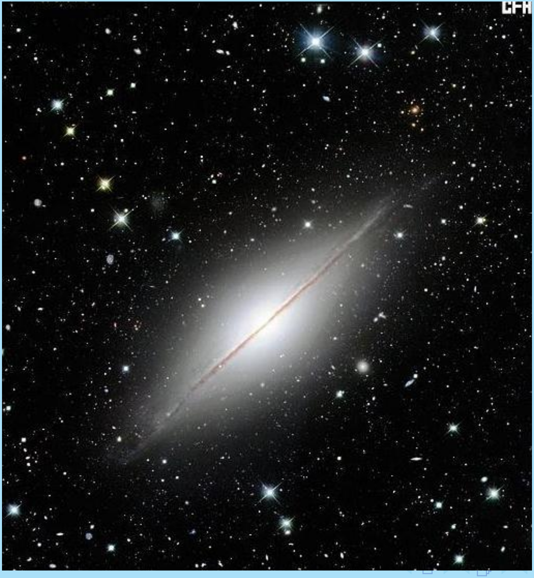
The surface photometry of NGC 7814[10] reveals some important information.

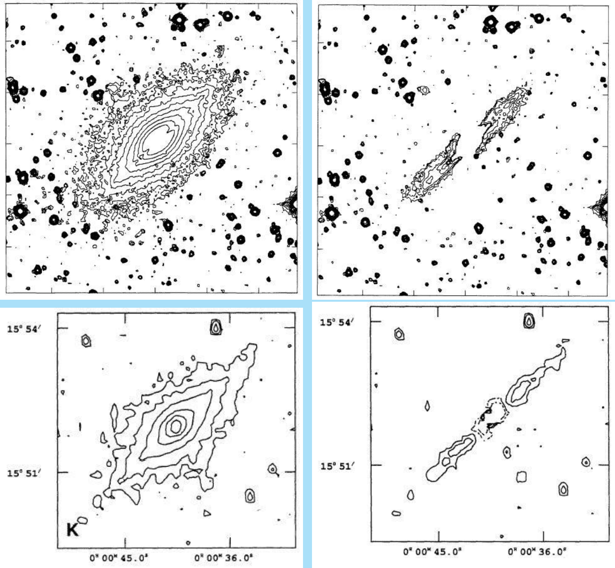
This galaxy is bulge-dominated, but the photometry showed bulge isophotes with all identical axis ratios.

Analysis of the data then showed that it is possible to separate the surface brightness distribution into two distinct components (spheroid and disk) with discretely different flattenings.

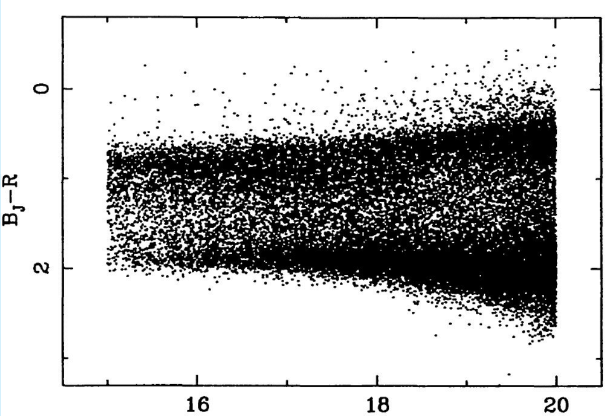
This seemed to indicate that star formation occured in two discrete epochs, one before and one after disk collapse.

[10] P.C. van der Kruit & L. Searle, A.&A. 110, 79 (1982)




15° 54′
15° 51′
K
0ʰ 00ᵐ 45.0ˢ
0ʰ 00ᵐ 36.0ˢ
15° 54′
15° 51′
0ʰ 00ᵐ 45.0ˢ
0ʰ 00ᵐ 36.0ˢ

We see also the basic two-component structure in the colors at faint star counts.



- Few stars bluer than $(B-V)\sim 0.4$. This corresponds to the MS turn-off of the extremely metal-poor halo population.
- The peak at $(B-V)\sim 0.6$. This is the MS turn-off of the halo population.
- The peak at $(B-V)\sim 1.5$. This is the cool MS of the disk population.
- The absence of stars redder than $(B-V)\sim 2.0$. This indicates the absence of large amounts of M-dwarfs to provide the missing local mass.

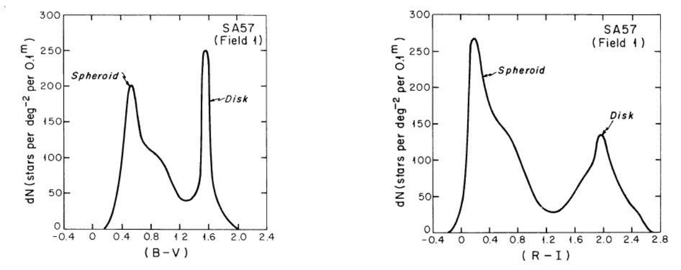
The thick disk

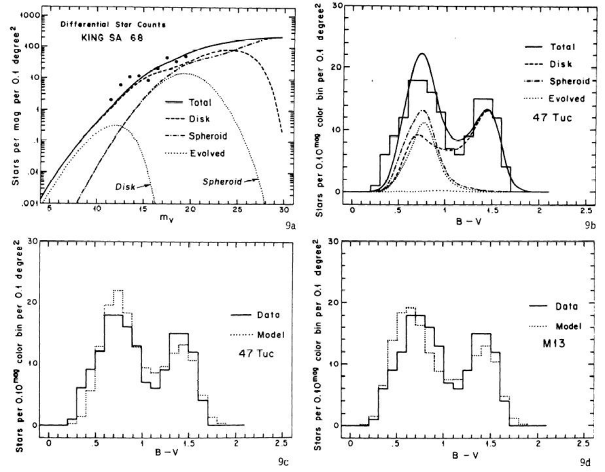
The old situation (before about 1980) was that there was no clear evidence for a substabtial Intermediate Population II and there were basically two discrete components (halo and disk).

Bahcall & Soneira[11] built a Galaxy model with distinct disk and halo components. This was later improved as the Standard Galaxy Model[12].

[11] Ap.J. Suppl. 44, 73 (1980)
[12] Bahcall & Soneira, Ap.J.Suppl. 55.67 (1984)

They showed that it could very well reproduce faint star counts and color distributions in two "Selected Areas", for which deep data were available, namely SA 57 $(l, b) = (65{,}86)$ and SA 68 (111,-46).




Differential Star Counts
KING SA 68
Total
Disk
Spheroid
Evolved
Disk
Spheroid
Stars per mag per 0.1 degree²
m_V
9a
47 Tuc
Stars per 0.10^{mag} color bin per 0.1 degree²
B − V
9b
Data
Model
47 Tuc
9c
Data
Model
M13
9d

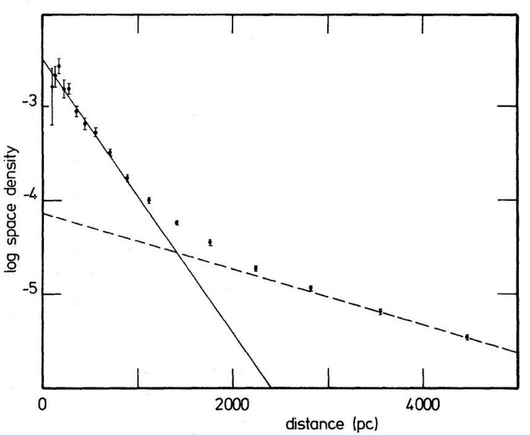
Gilmore & Reid[13] did deep star counts in South Galactic Pole.

They selected only those stars near the MS turn-off on the basis of their colors. For these turn-off and subgiants they determined photometric parallaxes.

They found two components in the disk:

- The "thin disk" (really the old disk) with exponential scaleheight $h_z \approx 300$ pc
- A new component that they called the "thick disk" with $h_z \approx 1350$ pc.
- The local normalisation was such that the thick disk has in the plane $\approx 2\%$ of the stars and this corresponds to $\approx 9\%$ of the face-on surface brightness.

[13] G. Gilmore & N. Reid, Mon.Not.R.A.S. 202, 1025 (1983)


-3
-4
-5
log space density
0
2000
4000
distance (pc)

Bahcall and collaborators[14] conclude that the Standard Galaxy Model BS84 is consistent with counts in all fields available and inconsistent with a model including a thick disk and inconclusive when a metal-rich Luminosity Function (LF) is used for the thick disk.

Gilmore and collaborators[15] present a model with a thick disk (G84) and claims consistency with the count: for the thick disk they use the LF of the globular cluster 47 Tuc ([Fe/H] ≈ -0.7).

[14] J.N. Bahcall & R.M Soneira, Ap.J.Suppl. 55, 67 (1984) (BS84); J.N. Bahcall *et al.*, 299, 616 (1985)

[15] G. Gilmore, Mon.Not.R.A.S. 207, 223 (1984) (G84); G. Gilmore *et al.*, Mon.Not.R.A.S. 213, 257 (1085)

So the earlier disagreement due to choice of the LF of the intermediate component; BS84 and G84 reproduce star counts only if the LF of metal-rich globular cluster is used for it.

The conclusion then is that star counts by themselves are not conclusive evidence for a thick disk or Intermediate Population II.

We first look at external galaxies.

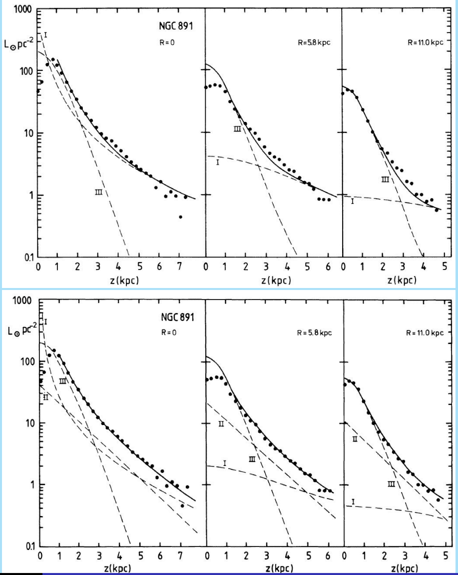
Thick disks in external galaxies

We can look at edge-on external galaxies, such as NGC 891, which is very similar to our Galaxy[16] and construct equivalent "BS84" and "G84" models.

	Galaxy	NGC 891	Galaxy	NGC 891
	"BS84" old disk		"G84" old disk	
h (kpc)	4.5 - 5	4.9	4.5 - 5	4.9
$z_\circ$ (kpc)	0.6 - 0.7	0.99	0.6 - 0.7	0.99
$R_{\rm max}$ (kpc)	22	21	22	21
$L_{\rm tot}$ ($L_\odot$)	$\sim 1.1 \times 10^{10}$	6.7×10^{10}	$\sim 1.1 \times 10^{10}$	6.7×10^{10}
	"BS84" thick disk		"G84" thick disk	
$h_{\rm R}$ (kpc)	no thick disk	no thick disk	~ 4.5	5
$h_{\rm z}$ (kpc)	no thick disk	no thick disk	~ 1.3	1.5
$L_{\rm tot}$ ($L_\odot$)	no thick disk	no thick disk	$\sim 2 \times 10^8$	2×10^8
	"BS84" spheroid		"G84" spheroid	
$R_{\rm e}$ (kpc)	~ 2.7	2.3	~ 2.7	2.3
$(1-e^2)^{1/2}$	~ 0.7	~ 0.6	~ 0.7	~ 0.6
$L_{\rm tot}$ ($L_\odot$)	$\sim 1.5 \times 10^9$	1.2×10^9	$\sim 1.0 \times 10^9$	4.9×10^8

[16] P.C. van der Kruit, A.&A. 140, 470 (1984)

Both these models fit the surface photometry well.



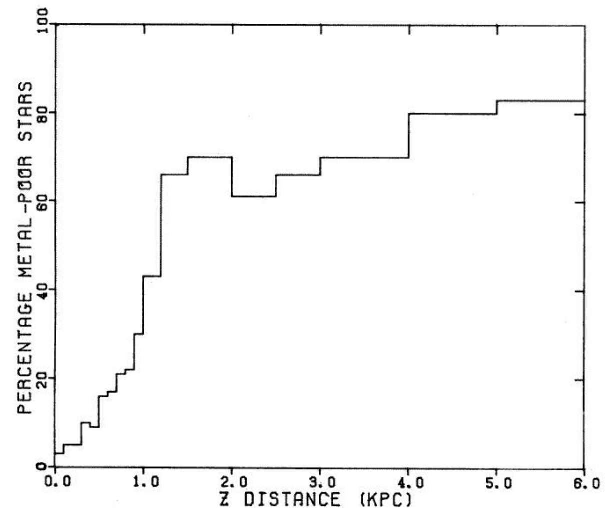
Kinematical evidence for a thick disk

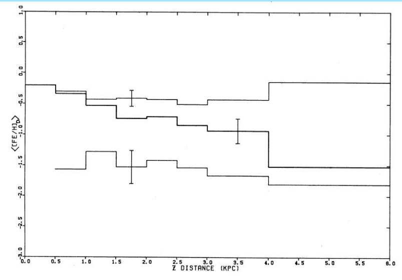
Hartkopf & Yoss[17] compiled DDO photometry and vertical velocities of G & K giants.

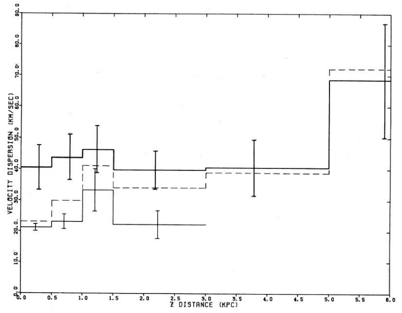
The distribution is separable into two components, each about isothermal:

- $\langle[\mathrm{Fe/H}]\rangle \approx -0.4$; $\langle W^2\rangle^{1/2} \approx 20$ km s^{-1}
- $\langle[\mathrm{Fe/H}]\rangle \approx -1.5$; $\langle W^2\rangle^{1/2} \approx 40$ km s^{-1}

[17] W.I. Hartkopf & K.M. Yoss, 87, 1679 (1982)


PERCENTAGE METAL-POOR STARS
0
20
40
60
80
100
0.0
1.0
2.0
3.0
4.0
5.0
6.0
Z DISTANCE (KPC)





Rose[18] found Red Horizontal Branch stars in the North Galactic Pole field similar to the ones in globular cluster M71 ([Fe/H] ~ -0.6).

These are too metal poor to be old disk stars.

[18] J.A. Rose, A.J. 90, 787 (1985)

They constitute 5% of all non-halo giants in the field and have $h_z \lesssim 0.5$ kpc and $\langle W^2 \rangle^{1/2} \sim 40$ km s^{-1}.

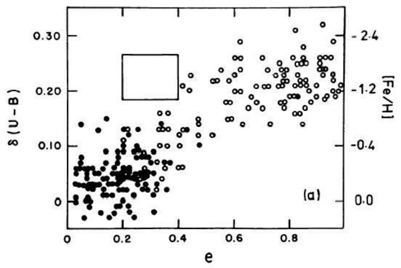
This is fully consistent with the thick disk of Gilmore & Reid.

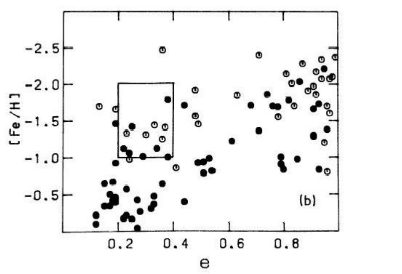
Norris *et al.*[19] found stars with [Fe/H] ≤ -1.0 ; $e \leq 0.4$.

This area is empty in the ELS study and would correspond to positions of stars in an Intermediate Population II.

These stars have $\langle W^2 \rangle^{1/2} = 61 \pm 9$ km s^{-1}.

[19] J. Norris, M.S. Bessel & A.J. Pickles, Ap.J.Suppl. 58, 463 (1985)





At the left the ELS diagram. The rectangle gives corresponding areas in both diagrams.

The thick disk is real and could be an Intermediate Population II.

It is probably discrete from the Old Disk Population and possibly also from the Halo Population II.

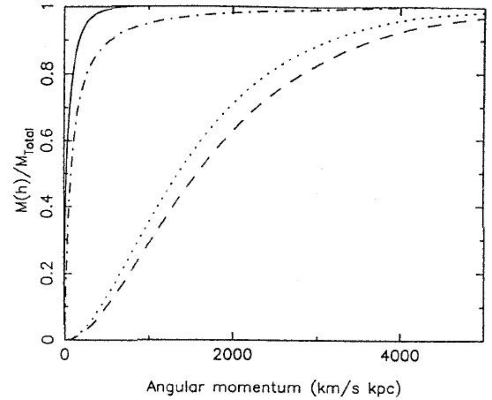
In face-on surface brightness is only of the order of 10% compared to the disk in the solar neighborhood.

So we may distinguish the following components[20] (lengths in kpc and velocities in km/s).

Component	Pop I	Old Disk	Thick disk	Halo
h_z	0.1	0.3	$\sim$ 1.5	$\sim$ 4
$\langle[\mathrm{Fe/H}]\rangle$	$\sim$ 0.0	-0.3	-0.6	-1.5
$\sigma_{[\mathrm{Fe/H}]}$	$\sim$ 0.15	$\sim$ 0.2	$\sim$ 0.3	$\sim$ 0.5
Asym. Drift	small	$\sim$ 10	$\sim$ 40	$\sim$ 150
$\langle W^2\rangle^{1/2}$	$\sim$ 10	25	45	100

[20] See also G. Gilmore, R.F.G. Wyse & K.H. Kuijken, Ann.Rev.A.&A. 27, 555 (1989)

It is possible to make an estimate of the cumulative distribution $M(h)/M_{\rm total}$ of specific angular momentum[21] h in each of these components.



The solid line is the bulge, the dashed-dotted line the halo, the dotted curve the thick disk and the dashed curve the old (thin) disk.

The bulge is related to the halo, but the thick disk to the disk.

[21] Angular momentum per unit mass

Globular clusters

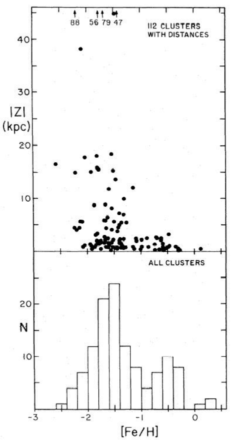
Globular clusters have long been known to be made up of two sub-systems, one following the traditional halo and with metal-poor clusters and one flattened and with less metal-poor systems.

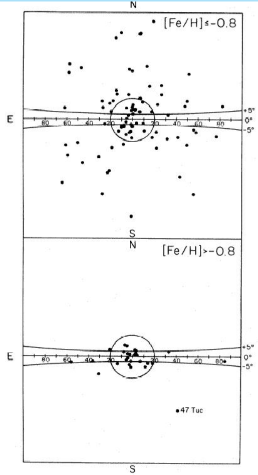
These have been called G- and F-clusters or disk- and halo-clusters.

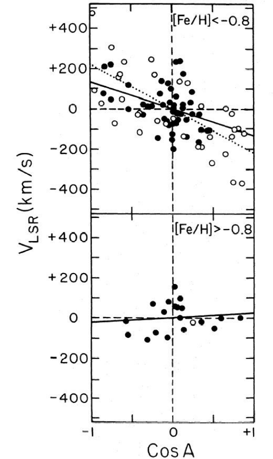
They also display a bi-modal metallicity distribution with a division at $[\mathrm{Fe/H}] \approx -0.8$.

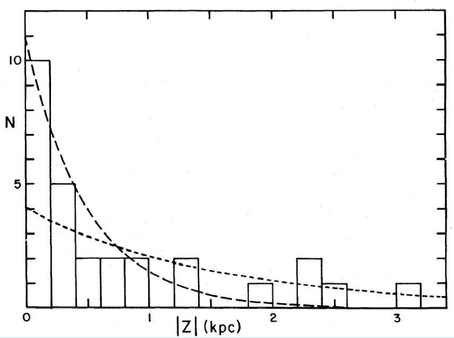
Also there is a clear difference in asymmetric drift (or rotation velocity of the group as a whole) and velocity dispersion.

This is seen in the radial velocity with respect to the Local Standard of Rest (LSR) as a function of the angle A with the apex of the LSR.


88 56 79 47
112 CLUSTERS
WITH DISTANCES
|Z|
(kpc)
ALL CLUSTERS
N
[Fe/H]


N
[Fe/H]≤-0.8
E
S
N
[Fe/H]>-0.8
47 Tuc
S


[Fe/H]<-0.8
[Fe/H]>-0.8
V_LSR (km/s)
Cos A

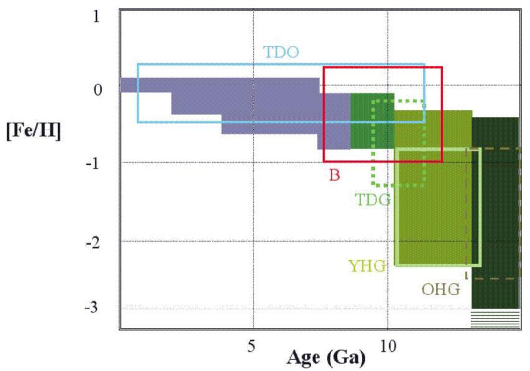


The metal-rich clusters form a disk-system with properties much like the thick disk[22].

[22] R. Zinn, Ap.J. 293, 424 (1985)

	disk-clusters	halo-clusters
[Fe/H]	> -0.8	< -0.8
h_z (kpc)	0.5-1.5	–
$V_{\rm rot}$ (km/s)	152 ± 29	50 ± 23
$\sigma_{\rm los}$ (km/s)	72 ± 11	116 ± 9

A summary picture of the structure of the Galaxy is given in this age-metallicity relation[23].

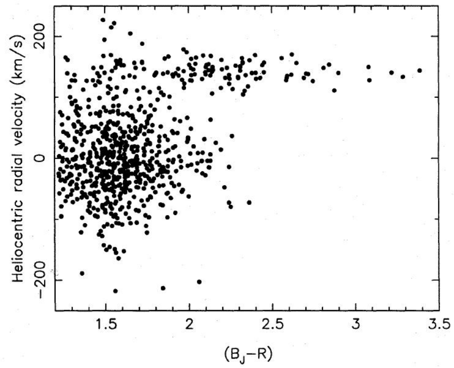


TDO = thin disk open clusters
TDG = thick disk globular clusters
B = bulge
YHG = young halo globular clusters
OHG = old halo globular clusters

[23] K.C. Freeman & J. Bland-Hawthorn, Ann.Rev.A.&A. 40, 487 (2002)

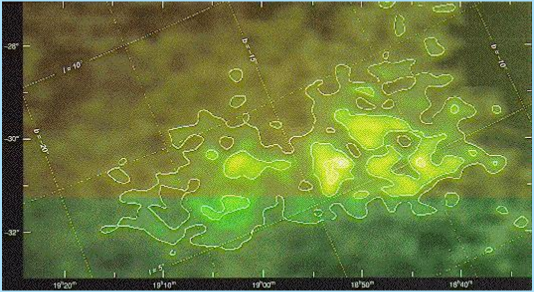
The Sagittarius dwarf

In the course of a study of the kinematics of a sample of stars in the Galactic bulge[24] a curious feature in the distribution was found.



[24] R.O. Ibata, G. Gilmore & M.J. Irwin, Mon.Not.R.A.S. 277, 781 (1995)

Tracing it accross the sky mapped out the Sagittarius Dwarf.

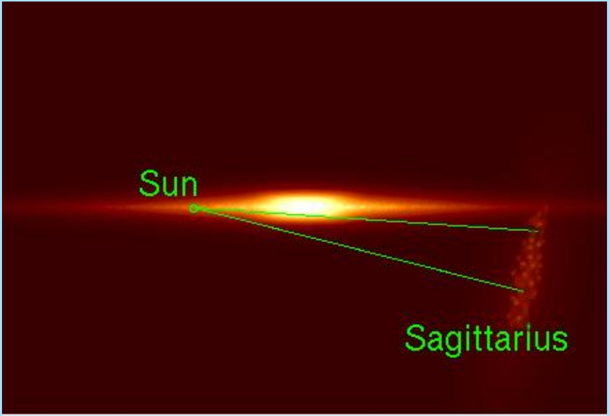


The distance is about 24 kpc and it is comparable in size and luminosity to a large dwarf spheroidal galaxy.

It apparently is approaching the disk of the Galaxy.

Detailed follow-up studies[25] indicate that it is on an orbit with a period of about 1 Gyr and it must have gone through the disk a few times before.

[25] R.A. Ibata, R.F.G. Wyse, G. Gilmore, M.J. Irwin & N.B. Suntzeff, A.J. 113, 634 (1997)


Sun
Sagittarius

Evidence from galaxies at high redshift

First we look at some results of the Sloan Digital Sky Survey (SDSS)[26], that surveyed a large part of the northern sky outside the Galactic Plane in five optical wavelenght bands.

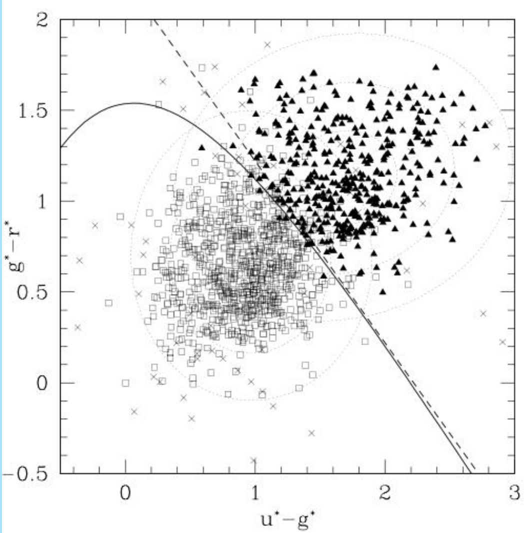
In the SDSS there is a routine to identify galaxies and do photometry and of many objects low-resolution spectra are taken.

The following is from a study[27] of the colors a sample of almost 150,000 galaxies at high Galactic latitude, of which 287 have been studied for morphology and 500 have spectra.

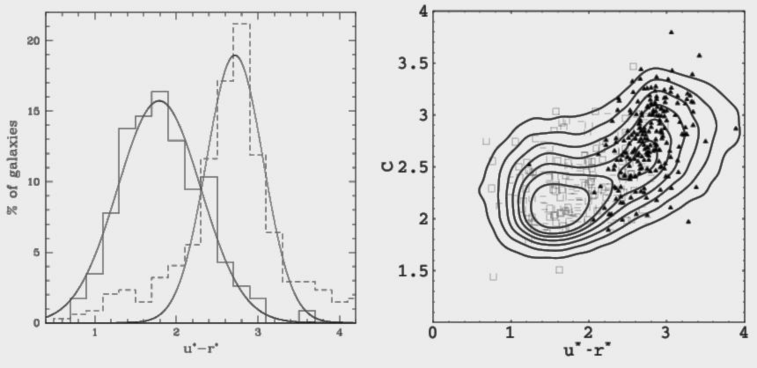
[26] D.G. York et al.. A.J. 120, 1579 (2000)
[27] Strateva et al., Ap.J. 122, 1861 (2001)

Fron statistical studies it turns out that the distribution of the colors is bi-modal.

The separator is at $(u^* - r^*) = 2.22$ (dashed line).



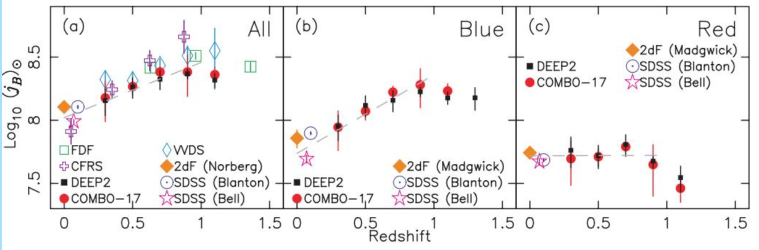
The important result is that the red peaks correspond to early types (E, S0, Sa) and the blue peak to late types (Sb, Sc, Irr).



c is a concentration index; triangles early, squares late types.

It is possible to study the time evolution of both groups from samples at different redshifts.

An extensive study[28] shows that the luminosity density of blue galaxies has decreased by 0.6 dex since $z \sim 1$, while that for the red galaxies has remained constant.



[28]S.M. Faber et al., Ap.J.665, 265 (2007)

When one looks in detail[29] there is little change in the mass function of massive galaxies with redshift out to $z \sim 1$.

However, the morphological mix changes, with more early-types at later times.

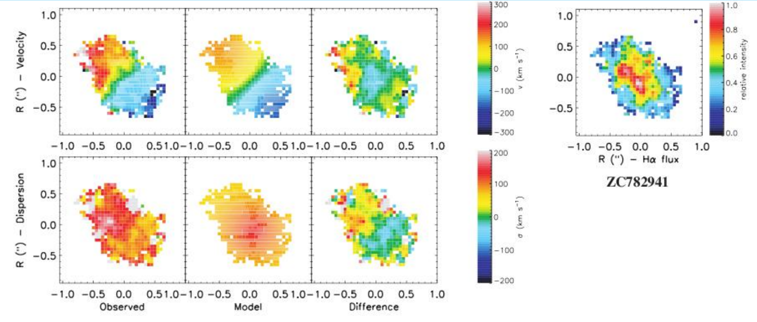
E/S0 galaxies dominate the higher mass population, spirals that at lower masses. The transition changes from $(1-2) \times 10^{11}$ M$_\odot$ at $z \sim 1$ to 3×10^{10} M$_\odot$ at $z = 0$

This "downsizing" phenomenon means that the most massive galaxies stop forming stars first and lower mass galaxies later.

[29] K. Bundy, R.S. Ellis & C.J. Conselice, Ap.J. 625, 621 (2005)

More massive galaxies then evolve into spheroidal systems at earlier times, and this morphological transformation may be completed 1-2 Gyr after star formation ceases.

It is possible now to derive velocity fields of star-forming galaxies at large redshift through emission lines.[30]

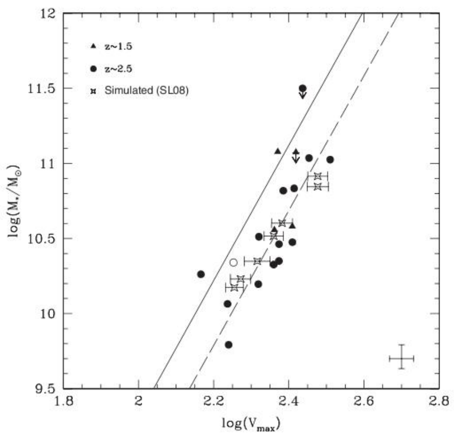


[30] Cresci et al., Ap.J. 697, 115 (2009)

A large fraction of star-forming galaxies at $z \sim 2$ have large, rotating disks.

The dynamical mass correlates with the stellar mass from stellar population models

They show a (stellar mass) Tully-Fisher relation with the same slope as at present, but with different zero-point.



The disks at high redshift have regular velocity fields and result most likely not from mergers, but rather from smooth, but rapid gas inflow.

- In most massive galaxies star-formation ceased early and the result was elliptical galaxies.
- Many of the current disks in large galaxies are basically already in place at redshifts of about 2, when the Universe was only $1 - 2$ Gyr old.
- Bulges probably formed later and are still forming from merging and capturing of satellites.

Galaxy formation

Background

a. Two paradigms

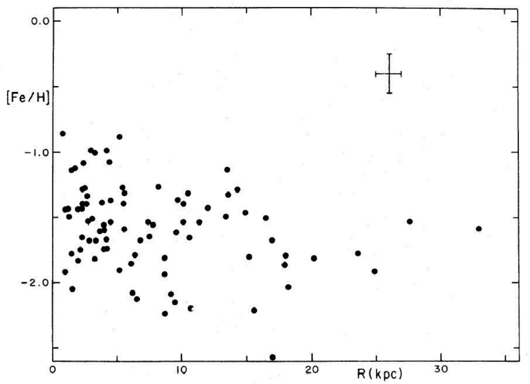
ELS[31] studied the motions of stars in the solar neighborhood and found correlations between metal abundance and the kinematics.

They concluded that the Galaxy was formed during a relatively rapid collapse.

SZ[32] studied the abundance distributions of globular clusters.

[31] O.J. Eggen, D. Lynden-Bell & A.R. Sandage, Ap.J. 136, 748 (1962)
[32] L. Searle & R. Zinn, Ap.J. 225, 357 (1978)

Beyond 8 kpc from the center the distribution over abundance is fairly wide, but does not change with galactocentric distance.

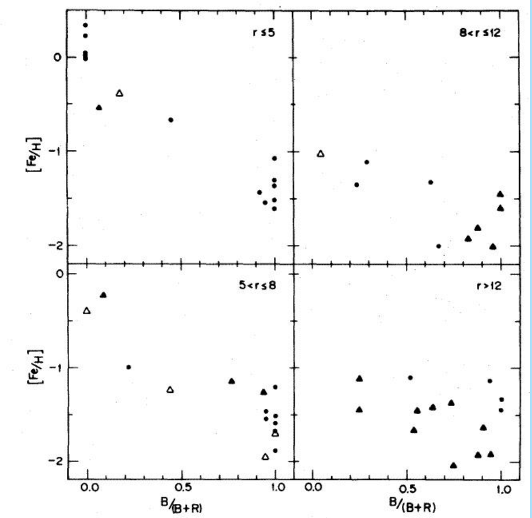


The extent of the Horizontal Branch depends in first instance on metallicity.

However, it has been known that the HB-morphology varies also among clusters of the same metallicity.

This is called the second parameter.

It is characterized by the parameter $B/(B+R)$. B is the number of HB-stars to the blue of the RR-Lyrae gap and R the number to the red.


r ≤ 5
8 < r ≤ 12
5 < r ≤ 8
r > 12
[Fe/H]
0
−1
−2
0.0
0.5
1.0
B/(B+R)

SZ found that in the tightly bound inner regions $B/(B+R)$ correlates well with abundance, but in the outer halo there is a great diversity of HB-morphology at a given abundance.

They suggested that the second parameter is age.

They concluded that all the above is consistent with a picture in which the build-up of the halo occurs over an extended period during which small fragments (of up to $\sim 10^8$ $M_\odot$ or so) continue to fall in.

These fragments loose gas after a while (due to supernova explosions) and will have a mean metal abundance equal to the effective yield.

The effective yield will have a range and distribution that is stochastic and should show no correlation with galactocentric distance.

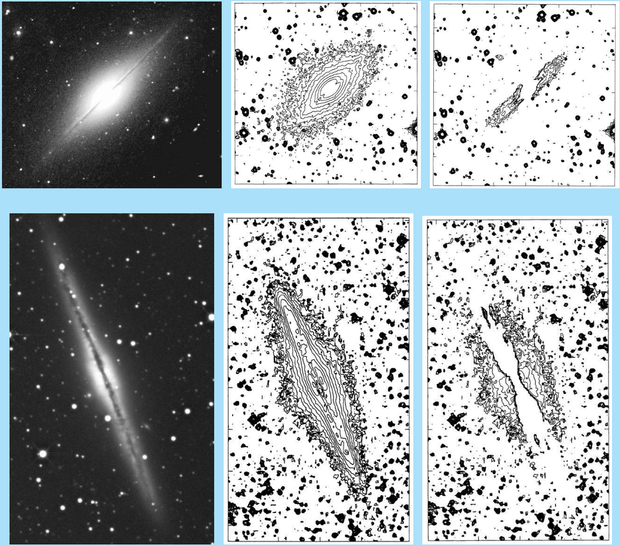
b. Basic two-component structure.

In spite of the possible presence of a thick disk (which has of order 10% of the disk mass), spiral galaxies are basically consist of two distinct components with discrete flattening[33].
This seems to point to two discrete epochs of star formation:

Before collapse – dissipationless – Population II

After collapse – dissipational – Thick and thin disk; Population I

[33] P.C. van der Kruit & L. Searle, A.&A. 110, 79 (1982)



Bulge formation

The observed properties of bulges are:

- $R^{1/4}$-law.
- Generally color (=abundance) gradients
- Isochromes have the same shape as isophotes (in NGC 7814)

Color gradients are often interpreted as evidence for dissipational collapse.

However, then the more metal-rich parts should be more flattened than the metal-poor parts.

Further numerical experiments[34] of dissipationless collapse with violent relaxation shows:

- From irregular initial conditions follows an $R^{1/4}$ distribution
- Statistical conservation of binding energy and thus gradients.

The properties of bulges are consistent with them forming early on in a dissipationless collapse over a longer timescale with fragments falling in for a few Gyr.

[34] T.S. van Albada, Mon.Not.R.A.S. 201, 939 (1982)

Disk formation

Disk formation is of course dissipational.

First we have to look into the question of the origin of angular momentum.

The angular momentum in disks is due to tidal torques between (proto-)galaxies in the early universe[35].

It can be described by a dimensionless parameter

$$\lambda = J|E|^{1/2}G^{-1}M^{-5/2} \approx 0.08$$

where J is the total angular momentum, E the total energy and M the total mass.

[35] P.J.E. Peebles, A.&A. 11, 377 (1969)

Numerical experiments give

$$\lambda = 0.07 \pm 0.03$$

This predicts insufficient angular momentum to explain rotation of disk galaxies in traditional models without dark matter.

The canonical working model has the following characteristics [36]:

- Disk and dark halo have the same distribution of specific angular momentum (= angular momentum per unit mass).
- Disks collapse with detailed conservation of angular momentum.

For tidal torques to work one needs $\sim$ 10 times as much mass in dark halo as in the disk.

[36] M. Fall & G. Efstathiou, Mon.Not.R.A.S. 193, 189 (1980)

Finally we need Mestel's hypothesis[37].

Mestel noted that the rotation and mass distribution in the disk of the Galaxy gave a distribution of specific angular momentum similar to that of a uniformly rotating, uniform sphere.

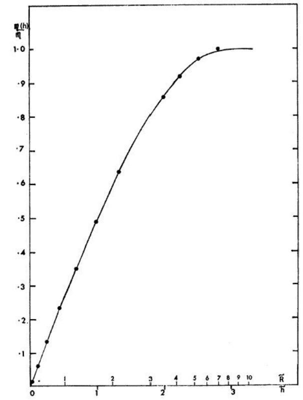
The hypothesis then is that disks form from such a Mestel-sphere with detailed conservation of angular momentum.

The normalized distribution of specific angular momentum $h_{\rm s}$ in the Mestel sphere is

$$\frac{M(h_{\rm s})}{M} = 1 - \left(1 - \frac{h_{\rm s}}{h_{\rm max}}\right)^{3/2}$$

[37] L. Mestel, Mon.Not.R.A.S. 126, 553 (1963)

Freeman[38] has noted already that the self-gravitating exponential disk also has roughly this distribution.



The curve is for the exponential disk and the points for the Mestel sphere.

[38] K.C. Freeman, Ap.J. 160, 811 (1970)

Gunn[39] noted that in a flat rotation curve the Mestel distribution would in centrifugal equilibrium give an approximately exponential radial surface density distribution.

On this basis we can consider the following scenario for disk galaxy formation[40].

We make the following assumptions based on the discussion above:

- The protogalaxy is a Mestel sphere.
- The angular momentum results from tidal torques and $\lambda \sim 0.07$.
- There is a uniform mix of dark and luminous matter (so they have the same specific angular momentum distribution).

[39] J.E. Gunn, in "Astrophysical Cosmology", ed. Brück, Coyne & Longair, Pont. Acad. Scient, Vatican, p. 233 (1982)
[40] P.C. van der Kruit, A.&A. 173, 59 (1987)

For the protogalaxy the total mass is M, and at maximum expansion the density is $\rho_\circ$ and the radius $R_{\rm m} = (3M/4\pi\rho_\circ)^{1/3}$.

At maximum expansion then the potential energy is

$$\Omega = -\frac{3GM^2}{5R_{\rm m}}$$

and the total angular momentum

$$J = \frac{2}{5} M h_{\rm max}$$

At maximum expansion the energy is essentially gravitational ($|E| = |\Omega|$; in virial equilibrium it is a factor 2 smaller). Then

$$h_{\rm max} = \frac{5}{2}\left(\frac{5}{3}\right)^{1/2} G^{1/2}\lambda M^{1/2} R_{\rm m}^{1/2}$$

Now first consider the halo formation.

There is some star formation in the inner regions to form the Population II stars. These settle dissipationlessly in the bulge.

So we get an $R^{1/4}$-bulge with an abundance gradient.

The dark matter settles dissipationlessly in something like an isothermal sphere.

Assume the amount of dark matter to be

$$M_{\rm H} = (1 - \Gamma)M$$

Let this settle in an isothermal sphere with radius $R_{\rm H}$. Then the potential energy can be calculated as

$$\Omega_{\rm H} = -G(1 - \Gamma)^2 \frac{M^2}{R_{\rm H}}$$

The viral theorem requires (after completion of the collapse of the dark halo) that

$$E_{\rm H} = \frac{\Omega_{\rm H}}{2} = -G(1-\Gamma)^2 \frac{M^2}{2R_{\rm H}}$$

But originally the energy was

$$E_{\rm H} = -G(1-\Gamma)\frac{3M^2}{5R_{\rm m}}$$

Energy is conserved during dissipationless collapse, so

$$R_{\rm H} = \frac{5}{6}(1-\Gamma)R_{\rm m}$$

$$V_{\rm m}^2 = \frac{6}{5}\left(\frac{G}{1-\Gamma}\right)\frac{M}{R_{\rm m}}$$

Now look at the disk formation.

The remaining gas has a mass ΓM (minus bulge stars, but assume this to be small).

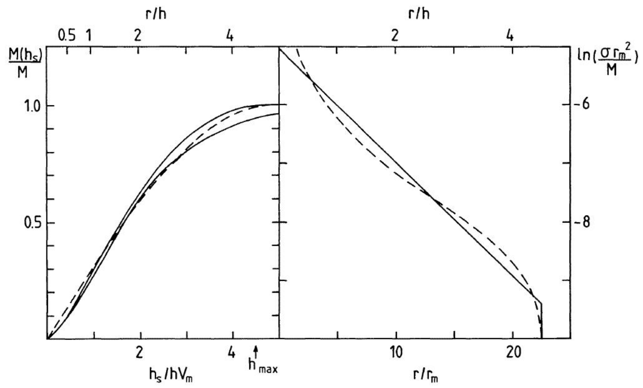
This then settles in a disk *with* dissipation, but conserves the specific angular momentum distribution.

The force field in which this happens is that of the dark halo. Parametrize the final (flat) rotation curve is as

$$V_{\rm rot}^2 = V_{\rm m}^2 \frac{R^2}{R_{\rm m}^2 + R^2} \left[1 - \gamma \ln \left(\frac{R^2}{R_{\rm m}^2 + R^2} \right) \right]$$

From real galaxies we know that the precise value of γ is not important (but $\gamma \approx 0.1$) and $R_{\rm m} \approx (0.1 - 0.5)h$.

Then calculate the surface density distribution of the disk that results; this is a roughly exponential disk with an edge at $\approx 4.5(\equiv \beta)h$.



On the left we have the specific angular momentum distribution (dashed line is the Mestel distribution; fulldrawn lines are exponential disks with a flat rotation curve for an edge at infinity and $4.5h$).

On the right we see the surface density distribution from the Mestel distribution in the flat rotation curve (dashed) and a pure exponential truncated at $4.5h$.

Assume for simplicity that $\Gamma \ll 1$, so the disk does not seriously affect the force field.

The figure shows an inner excess; this may in reality be the bulge.

How does the thick disk originate? Is at a relic of the violent processes at the moment of disk collapse?

The outer HI beyond the optical truncation and the observed warps may be the result of later infall. Is that why warps start at the optical edge?

From an examination of the figure we deduce the resulting scalelength

$$h = \frac{h_{\max}}{\beta V_{\rm m}} = \frac{25\lambda}{6\sqrt{2}\beta} \frac{R_{\rm m}}{(1-\Gamma)^{1/2}}$$

and the central surface density

$$\sigma_\circ = \frac{36}{625} \left(\frac{4}{3\sqrt{\pi}} \right)^{2/3} \left(\frac{\beta}{\lambda} \right)^2 \frac{\Gamma}{1-\Gamma} \rho_\circ^{2/3} M^{1/3}$$

In models of hierarchical clustering, galaxies form at about the same time and

$$\frac{\delta\rho}{\rho} \propto M^{-(3+n)/6}$$

with $n =$ -1.5 to 0.

So $\rho_\circ$ is about constant and has only a small dependence on M. Then we get $\sigma_\circ$ about constant for Γ constant.

For $\lambda = 0.07$ and $\beta = 4.5$ we get (V in km s^{-1}, M in M$_\odot$, R in kpc, ρ in M$_\odot$ pc^{-3}, etc.):

$$\frac{\Gamma}{(1-\Gamma)^{1/2}} = 1.5\frac{\sigma_\circ h}{V_{\rm m}^2} \qquad R_{\rm m} = 22\frac{h}{(1-\Gamma)^{1/2}} \qquad R_{\rm H} = 18(1-\Gamma)h$$

$$M = 4.2\times10^6(1-\Gamma)^{1/2}V_{\rm m}^2 h \qquad \rho_\circ = 9.7\times10^8(1-\Gamma)^2\frac{V_{\rm m}^2}{h^2}$$

Now apply this to our Galaxy, which has $h = 5$ kpc, $V_{\rm m} = 220$ km s^{-1}, $\sigma_\circ = 400\ M_\odot$ pc^{-2}.

$$\Gamma = 0.06 \qquad R_{\rm m} = 115\,{\rm kpc} \qquad R_{\rm H} = 90\,{\rm kpc} = 18h$$

$$M = 1.0 \times 10^{12}\,{\rm M_\odot} \qquad \rho_\circ = 2 \times 10^{-4}\,{\rm M_\odot\,pc^{-3}}$$

For other galaxies we find $\Gamma = 0.04$ - 0.11 and $\rho_\circ \approx 10^{-4}$ M$_\odot$ pc^{-3}.

For $\Omega = 1$ it has been deduced that

$$\frac{\delta\rho}{\rho} = \frac{9\pi^2}{16}$$

For $H = 75$ km s^{-1} Mpc^{-1} this then implies a redshift of galaxy formation of about 3.5.

Finally calculate the disk luminosity

$$L_{\rm disk} = \frac{L}{M}\Gamma^2(1-\Gamma)V_{\rm m}^4\mu_\circ^{-1}$$

So with Freeman's law, constant (M/L) and Γ we get the Tully-Fisher relation.

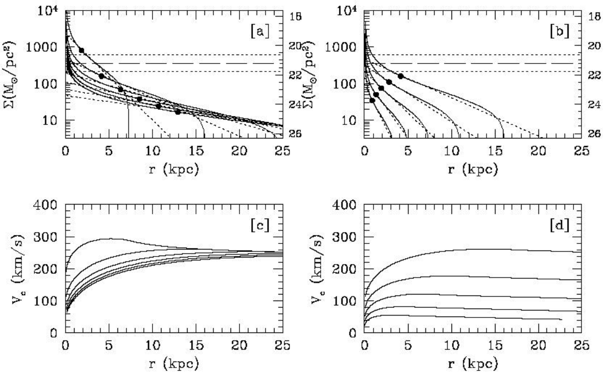
This schematic model has been greatly improved by Dalcanton et al.[41].

They do more realistic calculations, taking all gravitation into account, take a range in λ, etc.

The assumption of a range in λ now translates in a range of predicted central surface densities.

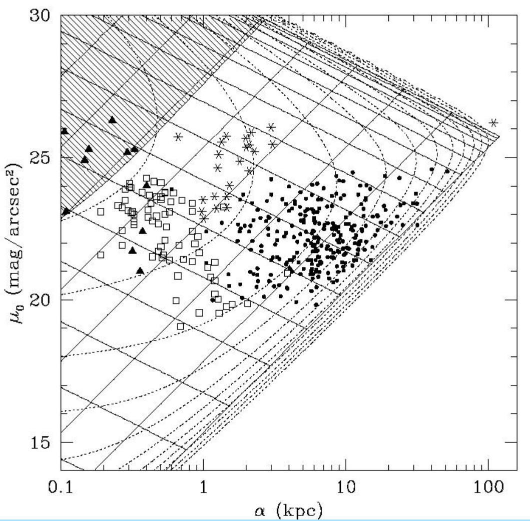
The resulting disk density profiles and rotation curves are in the following figure.

[41] J.J. Dalcanton, D.N. Spergel & F.J. Summers, Ap.J. 482, 659 (1997)



On the left we have models for $\lambda = 0.03$ - 0.18; $M = 10^{12}$ $M_\odot$. On the right we have $\lambda = 0.06$; $M = 10^{10} - 10^{12}$ $M_\odot$.

The models project in the (surface brightness - scalelength) plane as follows.



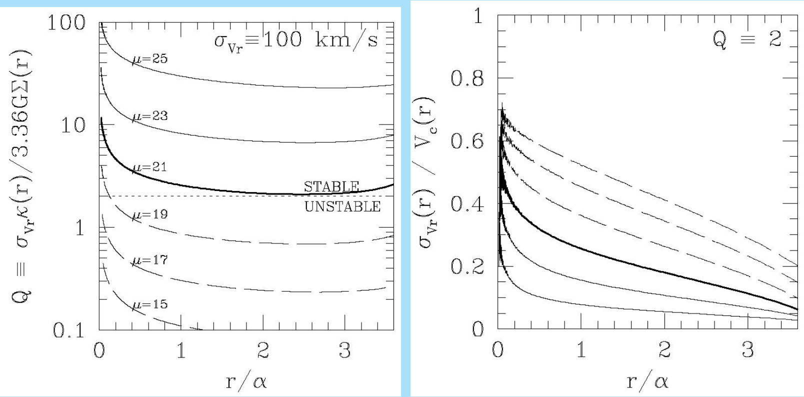
The dashed line are lines of equal expected density in this plane for $M/L = 3$ in B, $\Gamma = 0.05$ and $H = 50$ km s^{-1} kpc^{-1}. This is based on an assumed mass distribution as a Schechter function.

The solid lines with positive slope are of equal mass and those of negative slope of constant angular momentum.

In the hatched region gas pressure is expected to prevend the galaxies from collapsing.

The data are various not statistically complete samples (the filled triangles are Local Group spheroidals).

The disk stability and stellar velocity dispersion as a function of radius gives the following results.



Although in broad terms probably still applicable, this model will have to be augmented to incorporate the effects of infall of companions, such as the Sagittarius Dwarf into our own Galaxy.