STRUCTURE OF GALAXIES

7. Structure of galaxy disks

Piet van der Kruit
Kapteyn Astronomical Institute
University of Groningen
the Netherlands

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Vertical dynamics

Isothermal case

First I recall some parameters of disk kinematics.

The epicyclic frequency κ describes the motion of objects with velocitites small compared to rotation and follows from the rotation curve:

$$\kappa = 2\{B(B-A)\}^{1/2}$$

A and B are the Oort constants, which follow from

$$A = \frac{1}{2}\left(\frac{V_{\rm rot}}{R} - \frac{dV_{\rm rot}}{dR}\right)$$

$$B = -\frac{1}{2}\left(\frac{V_{\rm rot}}{R} + \frac{dV_{\rm rot}}{dR}\right)$$

The Poisson equation for the case of axial symmetry is

$$\frac{\partial K_{\mathrm{R}}}{\partial R} + \frac{K_{\mathrm{R}}}{R} + \frac{\partial K_{\mathrm{z}}}{\partial z} = -4\pi G\rho(R,z)$$

At small z the first two terms on the right are equal to $2(A-B)(A+B)$ and this is zero for a flat rotation curve. So for practical purposes we may use the plane-parallel case:

$$\frac{dK_{\mathrm{z}}}{dz} = -4\pi G\rho(z)$$

The hydronamic equation for the axi-symmetric case is

$$\frac{d}{dz}\left[\rho(z)\langle V_{\mathrm{z}}^2\rangle\right] = \rho(z)K_{\mathrm{z}}$$

For an isothermal distribution this becomes

$$\frac{d\rho(z)}{dz} = \frac{\rho(z)K_{\mathrm{z}}}{\langle V_{\mathrm{z}}^2 \rangle}$$

The equations for the isothermal sheet are the solutions of this set of equations.

$$\rho(z) = \rho_\circ \,\mathrm{sech}^{\,2}\left(\frac{z}{z_\circ}\right)$$

$$\sigma = 2z_\circ\rho_\circ$$

$$K_{\mathrm{z}} = -4\pi G\rho_\circ z_\circ \tanh\left(\frac{z}{z_\circ}\right)$$

When a second isothermal component (II) with negligible mass moves in this force field we have

$$\frac{\partial \rho_{\rm II}}{\partial z} = \frac{\rho K_{\rm z}}{\langle V_{\rm z}^2 \rangle_{\rm II}}$$

$$\rho_{\rm II}(z) = \rho_{\rm II}(0)\ {\rm sech}^{\ 2p} \left(\frac{z}{z_\circ} \right)$$

$$p = \frac{\langle V_{\rm z}^2 \rangle}{\langle V_{\rm z}^2 \rangle_{\rm II}}$$

Extended case

Disks are not entirely isothermal, since velocity dispersions of the stellar generations increase with age. Therefore replace the solution by the set[1]

$$\rho(z) = 2^{-2/n} \rho_{\rm e} \,\mathrm{sech}^{\,2/n} \left(\frac{nz}{2z_{\rm e}} \right)$$

Consider the extremes $n = \infty$ (the exponential) and $n = 1$ (the isothermal) and one intermediate case $n = 2$.

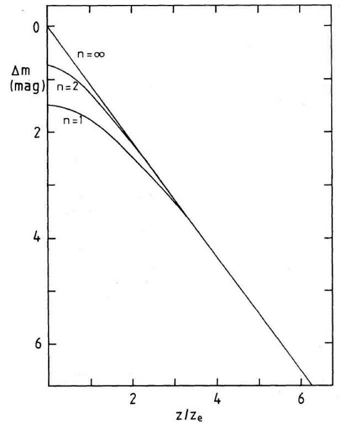
$$n = 1 \qquad \rho(z) = \frac{\rho_{\rm e}}{4} \,\mathrm{sech}^{\,2} \left(\frac{z}{2z_{\rm e}} \right)$$

$$n = 2 \qquad \rho(z) = \frac{\rho_{\rm e}}{2} \,\mathrm{sech} \left(\frac{z}{z_{\rm e}} \right)$$

We then calculate various properties. Eventually this can be used to evaluate the effects of assuming isothermal distributions.

The surface densities are

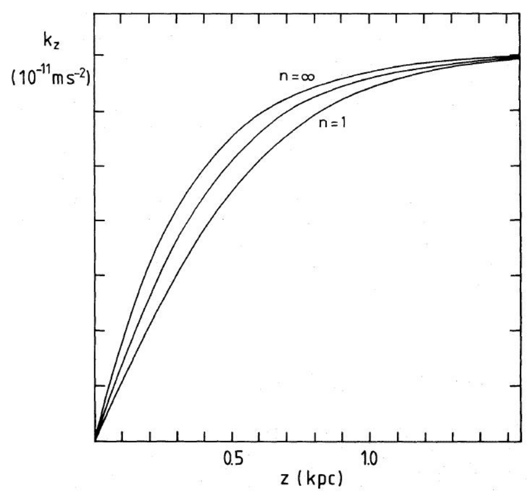
$$n = 1 \qquad \sigma = \rho_e z_e$$
$$n = 2 \qquad \sigma = \frac{\pi}{2}\rho_e z_e$$
$$n = \infty \qquad \sigma = 2\rho_e z_e$$



The vertical force is

$$
\begin{aligned}
n = 1 \qquad K_{\rm z} &= -2\pi G\sigma \tanh\left(\frac{z}{2z_{\rm e}}\right) \\
n = 2 \qquad K_{\rm z} &= -4G\sigma \arctan\left\{\sinh\left(\frac{z}{z_{\rm e}}\right)\right\} \\
n = \infty \qquad K_{\rm z} &= -2\pi G\sigma \left\{1 - \exp - \left(\frac{z}{z_{\rm e}}\right)\right\}
\end{aligned}
$$

The following graphs are for parameters approximately those of the solar neighborhood.


k_z
$(10^{-11} m s^{-2})$
$n=\infty$
$n=1$
0.5
1.0
z (kpc)

Velocity dispersion (squared) as a function of z

$$n = 1 \qquad \langle V_z^2 \rangle = 2\pi G \sigma z_e$$

$$n = 2 \qquad \langle V_z^2 \rangle = \left(\frac{\pi^2}{2}\right) G \sigma z_e \cosh\left(\frac{z}{z_e}\right) \left\{1 - \left(\frac{2}{\pi^2}\right) \arctan^2\left(\sinh\left(\frac{z}{z_e}\right)\right)\right\}$$

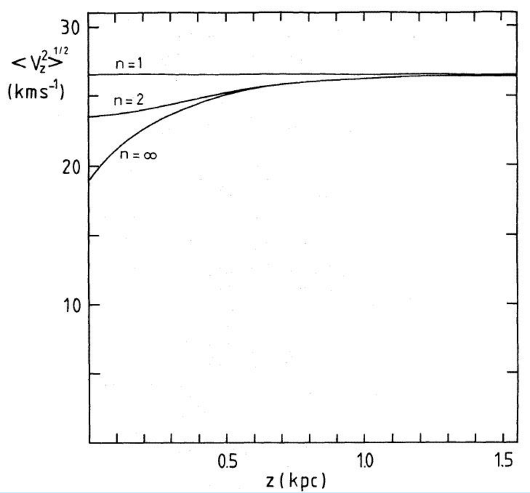
$$n = \infty \qquad \langle V_z^2 \rangle = \pi G \sigma z_e \left\{2 - \exp\left(\frac{z}{z_e}\right)\right\}$$

and integrated over all z

$$n = 1 \qquad \langle V_z^2 \rangle_{\rm FO} = 2\pi G \sigma z_e$$

$$n = 2 \qquad \langle V_z^2 \rangle_{\rm FO} = (1.705)\pi G \sigma z_e$$

$$n = \infty \qquad \langle V_z^2 \rangle_{\rm FO} = (3/2)\pi G \sigma z_e$$


30
20
10
$<V_z^2>^{1/2}$
(kms^{-1})
n = 1
n = 2
n = ∞
0.5
1.0
1.5
z (kpc)

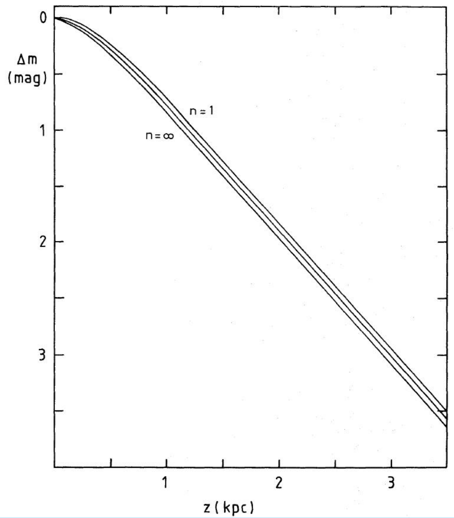
For a second isothermal component with

$$p = \frac{\langle V_{\mathrm{z}}^2 \rangle_{\mathrm{z}=0}}{\langle V_{\mathrm{z}}^2 \rangle_{\mathrm{II}}}$$

$$
\begin{aligned}
n = 1 \qquad \rho_{\mathrm{II}}(z) = &\quad \rho_{\mathrm{II}}(0)\, \mathrm{sech}^{\,2p} \left(\frac{z}{2z_{\mathrm{e}}} \right) \\
n = 2 \qquad \rho_{\mathrm{II}}(z) = &\quad \rho_{\mathrm{II}}(0) \exp \left[- \left(\frac{8}{\pi^2} \right) pI \left(\frac{z}{z_{\mathrm{e}}} \right) \right] \\
n = \infty \qquad \rho_{\mathrm{II}}(z) = &\quad \rho_{\mathrm{II}}(0) \exp \left[- \frac{2pz}{z_{\mathrm{e}}} + 2p \left\{ 1 - \exp \left(\frac{z}{z_{\mathrm{e}}} \right) \right\} \right]
\end{aligned}
$$

where the function I for $n = 2$ is

$$I(y) = \int_0^y \arctan(\sinh x)\, dx$$


0
Δm
(mag)
1
2
3
n = 1
n = ∞
1
2
3
z (kpc)

The thickness of an HI-layer can then be expressed in terms of

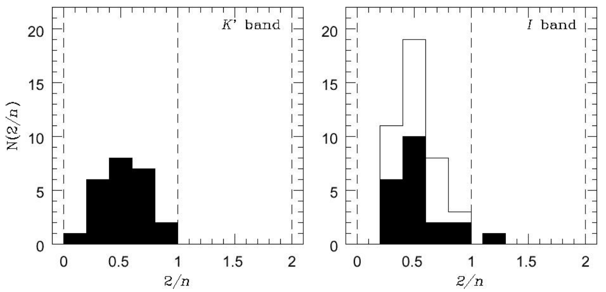
$$d_{\mathrm{HI}} = \left(\frac{\langle V_{\mathrm{z}}^2 \rangle_{\mathrm{HI}} z_{\mathrm{e}}}{G \sigma} \right)^{1/2}$$

and the full width at half maximum is

$$\begin{aligned} n = 1 \qquad W_{\mathrm{HI}} &= 1.33\, d_{\mathrm{HI}} \\ n = 2 \qquad W_{\mathrm{HI}} &= 1.18\, d_{\mathrm{HI}} \\ n = \infty \qquad W_{\mathrm{HI}} &= 0.94\, d_{\mathrm{HI}} \end{aligned}$$

From measurements of a sample of edge-on galaxies[2] the index n has been determined as

$$2/n = 0.54 \pm 0.20$$



[2] R. de Grijs, R.F. Peletier & P.C. van der Kruit, A.&A. 327, 996 (1997)

Toomre's stability criterion

Next we need to consider Toomre's[3] criterion for local stability:

$$Q = \frac{\langle V_{\rm R}^2 \rangle^{1/2} \kappa}{3.36 G \sigma}$$

$\langle V_{\rm R}^2 \rangle^{1/2}$ is the stellar velocity dispersion in the R-direction, σ is the local disk surface density and κ is the epicyclic frequency.

An approximate derivation of Toomre's criterion can be made for an infinitesimally thin disk.

1. At small scales the Jeans instability needs to be considered.

[3] A. Toome, Ap.J. 139, 1217 (1964)

Take an area with radius R and surface density σ. The equation of motion is

$$\frac{d^2R}{dt^2} = -\pi G\sigma$$

Solve this and apply for $R = 0$; this gives the free-fall time

$$t_{\mathrm{ff}} = \left(\frac{2R}{\pi G\sigma}\right)^{1/2}$$

A star moves out to radius R in a time

$$t = \frac{R}{\langle V^2\rangle^{1/2}}$$

and this must for marginal stability be equal to the free-fall time.

This then gives the Jeans length

$$R_{\mathrm{Jeans}} = \frac{2\langle V^2 \rangle}{\pi G \sigma}$$

2. At large scale we need to consider stability resulting from differential rotation.

Take an area with radius $R_\circ$; the angular velocity from differential rotation is

$$\Omega = B$$

The centrifugal force is then

$$F_{\mathrm{cf}} = R_\circ \Omega^2$$

Let it contract to radius R, then the angular velocity becomes

$$\Omega = \frac{R_\circ^2 B}{R^2}$$

and the centrifugal force

$$F_{\mathrm{cf}} = R\Omega^2 = \frac{R_\circ^4 B^2}{R^3}$$

If the contraction is dR then

$$\frac{dF_{\mathrm{cf}}}{dR} = -\frac{3R_\circ^4 B^2}{R^4}$$

Now look at the gravitational force

$$F_{\rm grav} = -\frac{G\pi R_\circ^2 \sigma}{R^2}$$

This is correct to within a factor 2 for a flat distribution. Then

$$\frac{dF_{\rm grav}}{dR} = \frac{2\pi G R_\circ^2 \sigma}{R^3}$$

At $R = R_\circ$ these two must compensate each other, so

$$R_{\rm crit} = \frac{2\pi G \sigma}{3B^2}$$

and the disk is stable for all $R > R_{\rm crit}$.

3. Toomre's stability criterion then follows by considering that the disk is stable at all scales if the minimum radius for stability by differential rotation is equal to or smaller than the maximum radius for stability by random motions (the Jeans radius).

Thus

$$\langle V^2 \rangle^{1/2}_{\rm crit} = \frac{\pi}{\sqrt{3}} \frac{G\sigma}{B}$$

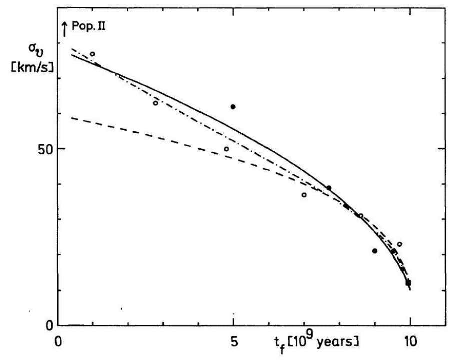
In practice $B \approx -A$ (for flat rotation curves), so we can write

$$\langle V^2 \rangle^{1/2}_{\rm crit} \sim 2\pi \left(\frac{2}{3} \right)^{1/2} \frac{G\sigma}{\kappa} = 5.13 \frac{G\sigma}{\kappa}$$

Toomre in his precise treatment found a constant of 3.36.

Stellar velocity dispersions in disks

Stars increase their velocity dispersion with age[4]; this is referred to as the velocity dispersion – age relation.

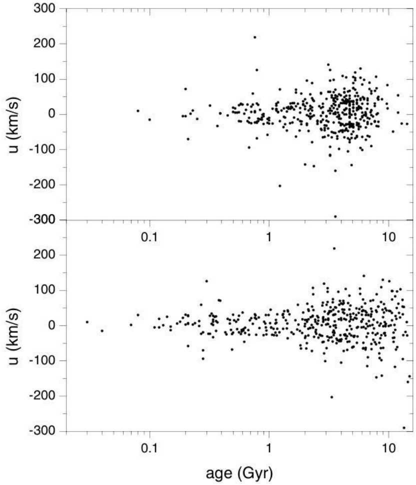


[4] R. Wielen, A.&A. 60, 262 (1977)

With the HIPPARCOS astrometric satellite better data are possible.

Here is a more recent version of the relation.[a]

[a] H. J. Rocha-Pinto et al. A.&A. 423, 517 (2004)



Increase of the u peculiar velocity with age, for uncorrected and corrected chromospheric ages.

Stars are formed with the velocity dispersion of the interstellar medium (about 10 km/s). Then the velocity dispersion increases roughly as $\sigma \propto t^{\alpha}$ with $\alpha \sim 0.5$.

There are three general mechanisms proposed for this:

- Stars are in their orbits scattered by concentrations in the ISM[5], now identified as Large Molecular Clouds.
- Spiral structure systematically increases the random motions of the stars[6].
- Infall of small companion galaxies has the same effect on disks[7].

[5] L. Spitzer & M. Schwarzschild, Ap.J. 114, 385 (1951)

[6] B. Barbanis & L. Woltjer, Ap.J. 150, 461 (1967); Carlberg & Sellwood, Ap.J. 292, 79 (1985)

[7] H. Velázquez & S.D.M. White, Mon.Not.R.A.S. 304, 254 (1999)

The scattering becomes less as the stars move outside the gas layer.

The Spitzer-Schwarzschild mechanism appears to be incapable of explaining the ratio of the R- and z-velocity dispersions (the axis ratio of the velocity ellipsoid), scattering too little in z.

It is most likely that both processes contribute[8].

[8] A. Jenkins & J. Binney, Mon.Not.R.A.S. 257, 305 (1990)

Observations of stellar velocity dispersions

1. Z-velocity dispersion

If disks have constant mass-to-light ratios M/L, the density can be described by

$$\rho(R,z) = \rho(0,0)\,\exp\left(-R/h\right)\,\mathrm{sech}^{\,2}(z/z_\circ)$$

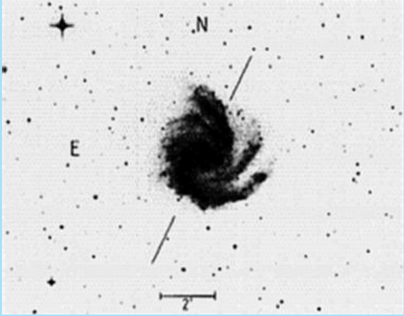
The vertical velocity dispersion then is

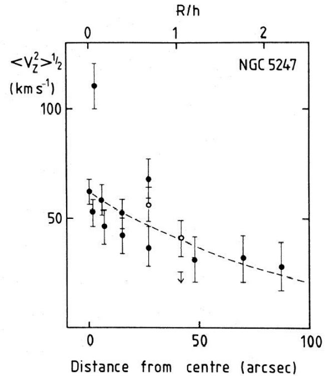
$$\langle V_{\mathrm{z}}^2\rangle^{1/2} = \sqrt{2\pi G\rho(R,0)}z_\circ$$

and it is expected that

$$\langle V_{\mathrm{z}}^2\rangle^{1/2} \propto \exp\left(-R/2h\right)$$

This can be tested by observations in face-on systems, e.g. NGC 5247[9].





[9]P.C. van der Kruit & K.C. Freeman, Ap.J. 303, 556 (1986)

The fit is

$$\langle V_z^2 \rangle^{1/2} = (62 \pm 7) \exp\left[-(0.42 \pm 0.10)\ R/h\right] \mathrm{km\ s}^{-1}$$

This is consistent with M/L about constant.

There are two recent, new developments:

- Use of Integral Field Unit (IFU) spectroscopy makes it possible to sample larger areas.
- Use of tracers as Planetary Nebulae (PN) makes it possible to go to fainter parts of disk.

The Disk Mass Project[10] uses IFU spectroscopy on a sample of relatively face-on galaxies.

The vertical velocity dispersions generally "follow the light", providing evidence for constant M/L.

Mass-to-light ratios are generally lower than expected and disks are in general less than maximum.

[10] M.A.W. Verheijen et al., in: Island Universes ed. R.S. de Jong, p. 95 (2007), see also www.astro.rug.nl/~islands/Verheijen.pdf

The use of Planetary Nebulae[11] makes it possible to move out far in the disks.

These authors find that most disks have a constant mass-to-light ratio out to at least ~ 3 scalelengths and the disks are submaximal (in late-type galaxies).

At large radii, the vertical velocity dispersions become independent of radius, suggesting flaring as a result of instabilities.

[11] K.A. Hermann & R. Ciardullo, Ap.J. 705, 1686 (2009); see also www.astro.queensu.ca/GalaxyMasses09/data/Ciardullo_GMasses09.pdf

R- and θ -velocity dispersions

These are more difficult to measure, since we will need to use edge-on galaxies.

From fundamental kinematics we have

$$\frac{\langle (V_\theta - V_{\mathrm{t}})^2 \rangle}{\langle V_{\mathrm{R}}^2 \rangle} = \frac{B}{B - A}$$

So, if we know the rotation curve we know the ratio of the radial and tangential velocity dispersion.

The other property to consider is the asymmetric drift.

The hydronamic equation can be written as

$$
\begin{aligned}
-K_{\rm R} \quad = \quad & V_{\rm t}^2 - \langle V_{\rm R}^2 \rangle \frac{\partial}{\partial R} \ln(\nu \langle V_{\rm R}^2 \rangle) + \\
& \frac{1}{R} \Big\{ \langle V_{\rm R}^2 \rangle - \langle (V_\theta - V_{\rm t})^2 \rangle + \\
& \langle V_{\rm z} V_{\rm R} \rangle \frac{\partial}{\partial z} (\ln \nu \langle V_{\rm z} V_{\rm R} \rangle) \Big\}
\end{aligned}
$$

Poisson's equation is

$$
\frac{\partial K_{\rm R}}{\partial R} + \frac{K_{\rm R}}{R} + \frac{\partial K_{\rm z}}{\partial z} = -4\pi G \rho
$$

For small z it can be shown that

$$\frac{\partial K_{\rm R}}{\partial R} + \frac{K_{\rm R}}{R} = 2(A-B)(A+B)$$

and for a flat rotation curve $A=-B$, so that

$$\frac{\partial K_{\rm z}}{\partial z} = -4\pi G\rho$$

Then

$$\langle V_{\rm z} V_{\rm R}\rangle = 0$$

Obviously we have

$$K_{\rm R} = V_{\rm rot}^2/R$$

For an exponential disk with constant M/L

$$\frac{\partial}{\partial R}\ln\nu = -\frac{1}{h}$$

The asymmetric drift equation then becomes

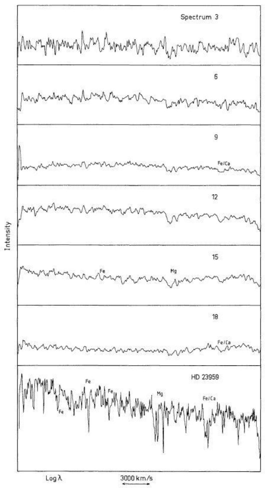
$$V_{\rm rot}^2 - V_{\rm t}^2 = \langle V_{\rm R}^2 \rangle \left[\frac{R}{h} - R \frac{\partial}{\partial R} \ln \langle V_{\rm R}^2 \rangle - \left\{ 1 - \frac{B}{B-A} \right\} \right]$$

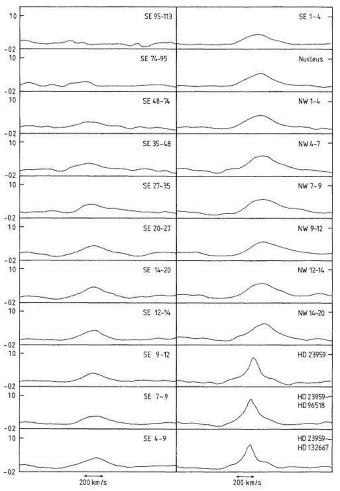
There are now two possibilities for observing. The first is to measure $\langle V_{\rm R}^2 \rangle^{1/2}$ directly from spectra.

The difficulty is the line-of-sight integration. This has to be treated by modeling as was done in the edge-on galaxy NGC 5170[12].

The profiles now have become asymmetric. In the next figure we see here the spectra and the cross-correlation peaks between galaxy and template spectra.

[12] R. Bottema, P.C. van der Kruit & K.C. Freeman, Ap.J. 178. 77 (1987)

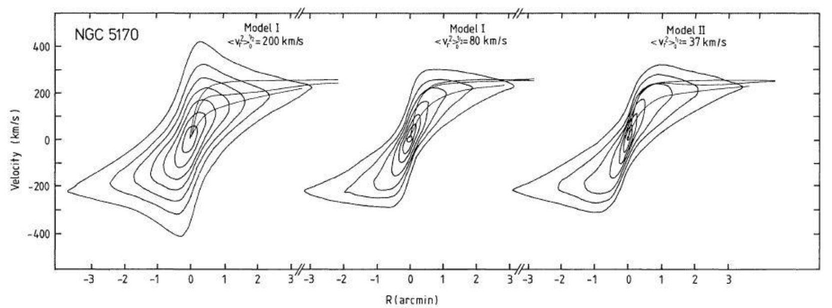

Spectrum 3
6
9
12
15
18
HD 23959
Intensity
Log λ
3000 km/s


SE 95-113
SE 1-4
SE 74-95
Nucleus
SE 48-74
NW 1-4
SE 35-48
NW 4-7
SE 27-35
NW 7-9
SE 20-27
NW 9-12
SE 14-20
NW 12-14
SE 12-14
NW 14-20
SE 9-12
HD 23959
SE 7-9
HD 23959-HD 96518
SE 4-9
HD 23959-HD 132667
200 km/s
200 km/s

Using an estimate of the circular motion from the HI-rotation curve one can calculate the profiles in a stellar "l,V-diagram".

To do this one needs an assumed radial variation of the velocity dispersion, the rotation curve (and from that the Oort constants) and the density distribution of the stars.

In the figure here we see a few such simulations. The three lines in each panel are form top to bottom: the circular motion from HI-observations, the stellar rotation velocity and peaks of Gaussians fitted to the resulting profiles.


NGC 5170
Model I
$<v_r^2>_0^{1/2} = 200$ km/s
Model I
$<v_r^2>_0^{1/2} = 80$ km/s
Model II
$<v_r^2>_0^{1/2} = 37$ km/s
Velocity (km/s)
400
200
0
-200
-400
-3 -2 -1 0 1 2 3
R (arcmin)

The second option is to measure the asymmetric drift.

The relevant equation was

$$V_{\rm rot}^2 - V_{\rm t}^2 = \langle V_{\rm R}^2 \rangle \left[\frac{R}{h} - R \frac{\partial}{\partial R} \ln \langle V_{\rm R}^2 \rangle - \left\{ 1 - \frac{B}{B-A} \right\} \right]$$

So we see that we need to measure:

- $V_{\rm rot}$, A and B from HI-synthesis or emission line spectroscopy.
- $V_{\rm t}$ from absorption line spectroscopy.
- h from surface photometry.

For a flat rotation curve:

$$\frac{B}{B-A} = 0.5 \text{ and } \kappa^2 = \frac{2V_{\rm rot}^2}{R^2}$$

For small asymmetric drift:

$$V_{\rm rot}^2 - V_{\rm t}^2 \approx 2V_{\rm rot}(V_{\rm rot} - V_{\rm t})$$

Consider two possibilities:

Model I with $\langle V_{\mathrm{R}}^2 \rangle / \langle V_{\mathrm{z}}^2 \rangle$ constant. Then

$$\langle V_{\mathrm{R}}^2 \rangle^{1/2} \propto \exp\left(-R/2h\right)$$

$$V_{\mathrm{rot}} - V_{\mathrm{t}} = \frac{\langle V_{\mathrm{R}}^2 \rangle}{2V_{\mathrm{rot}}} \left(\frac{2R}{h} - 0.5 \right)$$

- Model II with Q constant. Then

$$\langle V_{\mathrm{R}}^2 \rangle^{1/2} \propto R \exp\left(-R/h\right)$$

$$V_{\mathrm{rot}} - V_{\mathrm{t}} = \frac{\langle V_{\mathrm{R}}^2 \rangle}{2V_{\mathrm{rot}}} \left(\frac{3R}{h} - 2.5 \right)$$

How different are these models? For comparison calculate a Q (arbitrarily set to unity at one scalelength) for the first model:

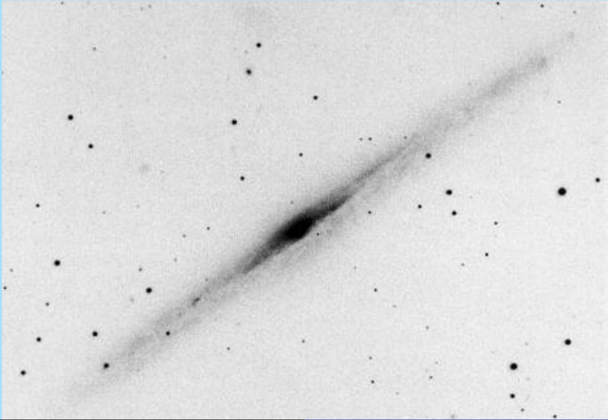
$R/h =$	$Q =$
1.0	1.17
1.5	**1.00**
2.0	0.96
3.0	1.06
4.0	1.31
5.0	1.73

We see that the models are really not different up to three or four scalelengths.

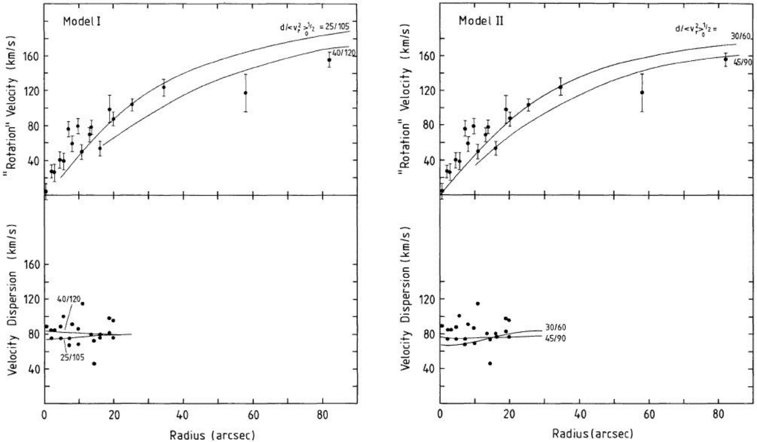
But at large R there is a contribution from the gas, which lowers the total velocity dispersion and increases the surface density and therefore lowers Q.

Numerical experiments on dynamics of stellar disks give $Q \sim 1.5 - 2.0$ at all radii.

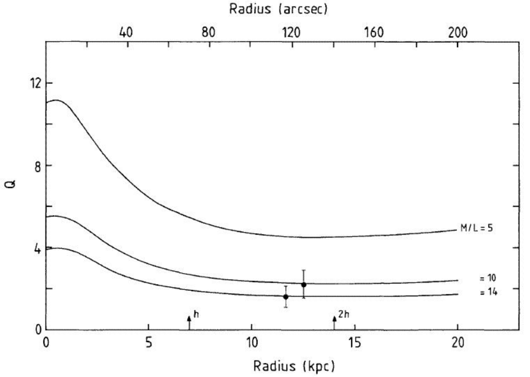
Back to the data on NGC 5170.



The fits to the data of NGC 5170 are as follows.

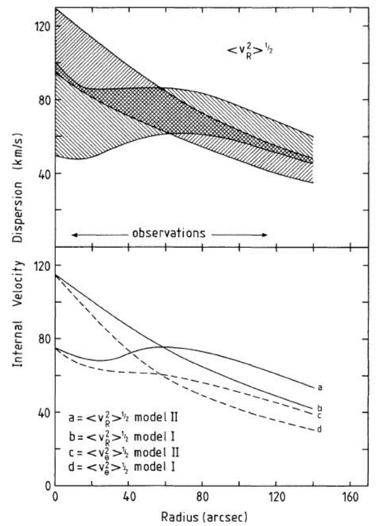


The resulting Q is as the lines in the figure below for Model I and the dots plus error bars in Model II for various assumed values for the disk M/L.



The velocity dispersions have the following radial distribution.

There is little difference between the two models.



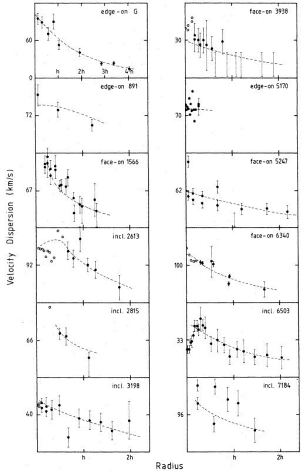
The Bottema relations

R. Bottema[a] observed stellar velocity dispersions in a set of 12 galaxies.

He then defined as fiducial values the radial velocity dispersion at one scalelength for inclined systems and the vertical velocity dispersion in the center for face-on systems.

This difference should roughly correct for the ratio between these dispersions.

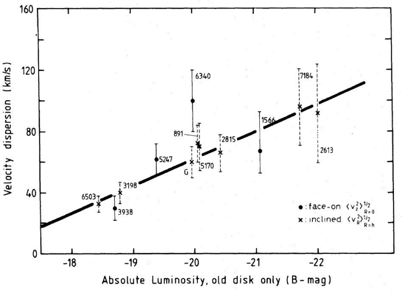
[a]Ph.D. thesis (1995); Bottema, A.&A. 275, 16 (1993)

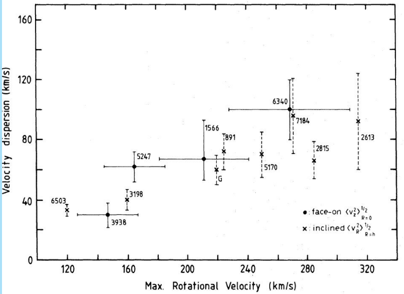


He then found the following relations

$$\langle V_{\mathrm{R}}^2 \rangle_{\mathrm{R=h}}^{1/2} = \langle V_{\mathrm{z}}^2 \rangle_{\mathrm{R=0}}^{1/2} = -17 \times M_{\mathrm{B}} - 279 \ \ \mathrm{km/s}$$

$$\langle V_{\mathrm{R}}^2 \rangle_{\mathrm{R=h}}^{1/2} = \langle V_{\mathrm{z}}^2 \rangle_{\mathrm{R=0}}^{1/2} = 0.29 V_{\mathrm{rot}} \ \ \mathrm{km/s}$$





Can we understand these relations?

From the definition of Q we have

$$Q \propto \langle V_{\mathrm{R}}^2 \rangle^{1/2} \kappa \sigma^{-1}$$

For a flat rotation curve

$$\kappa \propto V_{\mathrm{rot}} R^{-1}$$

An exponential disk has

$$\sigma \propto \mu_{\circ}(M/L) \exp\left(-R/h\right)$$

Combining these equations gives

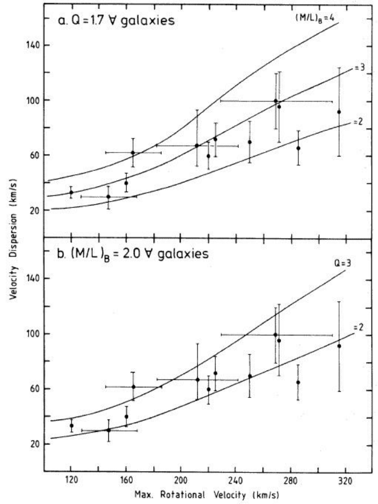
$$\langle V_{\mathrm{R}}^2 \rangle_{\mathrm{h}}^{1/2} \propto \mu_{\circ}(M/L) Q h V_{\mathrm{rot}}^{-1}$$

Now $L \propto \mu_\circ h^2$ and the Tully-Fisher relation gives $L \propto V_{\rm rot}^n$ with $n \approx 4$, so

$$\langle V_{\rm R}^2 \rangle_{\rm h}^{1/2} \propto \mu_\circ (M/L) Q V_{\rm rot} \propto \mu_\circ (M/L) Q L^{1/4}$$

So we expect that $\mu_\circ$, M/L and Q or at least their product are constant between disks.

With the actual observed central surface brightness the following curves result for either constant Toomre Q or constant M/L.



We had for hydrostatic equilibrium at the center

$$\langle V_{\mathrm{z}}^2 \rangle_{\mathrm{R=0}}^{1/2} = (2.3 \pm 0.1)\sqrt{G \sigma_\circ z_{\mathrm{e}}}$$

$\sigma_\circ$ is the central surface density and the range in the constant results from the choice of n.

The maximum rotation velocity of the exponential disk then is

$$_{\mathrm{disk}} = 0.88\sqrt{\pi G \sigma_\circ h} = (0.69 \pm 0.03)\langle V_{\mathrm{z}}^2 \rangle_{\mathrm{R=0}}^{1/2}\sqrt{\frac{h}{z_{\mathrm{e}}}}$$

With the Bottema relation between this central velocity dispersion and the maximum observed rotation velocity we get

$$\frac{V_{\mathrm{disk}}}{V_{\mathrm{rot}}} = (0.21 \pm 0.08)\sqrt{\frac{h}{z_{\mathrm{e}}}}$$

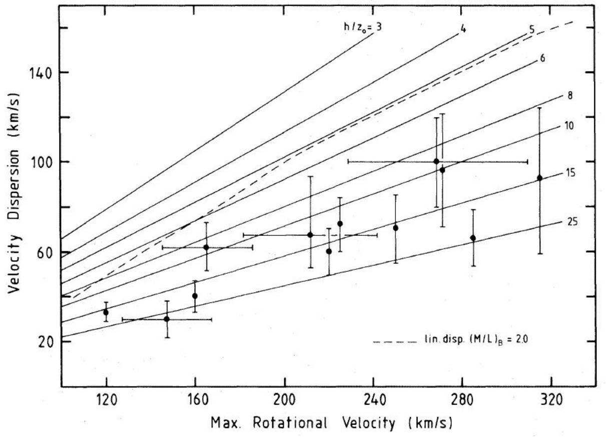
Analysis of a sample of edge-on galaxies gives for the ratio of scaleparameters 7.3 ± 2.2[13], so that

$$\frac{V_{\rm disk}}{V_{\rm rot}} = (0.57 \pm 0.22)$$

So disks in general are not maximum disk.

[13] M. Kregel, P.C. van der Kruit & R. de Grijs, Mon.Not.R.A.S. 334, 646 (2002)

Bottema[14] first showed with this argument that his relations implied that for maximum disk situations the stellar disks should be much flatter than observed.



For a flat rotation curve we have

$$\kappa = 2\sqrt{B(B-A)} = \sqrt{2}\frac{V_{\mathrm{rot}}}{R}$$

From the definition of Q and applying at $R = h$ we get

$$\langle V_{\mathrm{R}}^2 \rangle_{\mathrm{R=h}}^{1/2} = \frac{3.36G}{\sqrt{2}} Q \frac{\sigma(R=h)h}{V_{\mathrm{rot}}}$$

Using hydrostatic equilibrium (also at $R = h$) gives[15]

$$\frac{\langle V_{\mathrm{z}}^2 \rangle^{1/2}}{\langle V_{\mathrm{R}}^2 \rangle^{1/2}} = \sqrt{\frac{(7.2 \pm 2.5)}{Q} \frac{z_{\mathrm{e}}}{h}}$$

[15] P.C. van der Kruit & R. de Grijs, A.&A. 352, 129 (1999)

In the solar neighborhoud this axis ratio of the velocity ellisoid is ~ 0.5[16] and for the Galaxy we have $z_{\rm e} \sim 0.35$ kpc and $h \sim 4$ kpc, so that

$$Q \sim 2.5.$$

Taking all data and methods together it is found that this applies in all galaxies; disks are locally stable according to the Toomre criterion.

Numerical studies give such values for Q when disks are marginaly stable.

[16] W. Dehnen & J. Binney, Mon.Not.R.A.S. 298, 387 (1998)

Swing amplification and global stability

Swing amplification[17] of disturbances occurs as a result of the shear in rotating disks and turns these disturbances into growing trailing spiral waves.

It can be formulated in a criterion for prevention of this instability[18]

$$X = \frac{R\kappa^2}{2\pi G m \sigma(R)} \gtrsim 3$$

Here m is the number of spiral arms.

[17] A. Toomre, in a Cambridge conference on Structure and Evolution of Galaxies (1981)

[18] J.R. Sellwood, IAU Symp. 100, 197 (1983)

For a flat rotation curve this can be rewritten as

$$\frac{QV_{\rm rot}}{\langle V_{\rm R}^2\rangle^{1/2}} \gtrsim 3.97m$$

and with Bottema's relation it translates into

$$Q \gtrsim 1.1m$$

To prevent strong asymmetric $m = 1$ or bar-like $m = 2$ instabilities we require $Q \gtrsim 2$.

Numerical studies have indicated that disks with velocity dispersions as observed show global instabilities when evolving by themselves.

Disks can be stabilised by massive halos and therefore global stability requires that the disk mass has to be less than a certain fraction of the total mass, according to the criterion[19]

$$Y = V_{\rm rot} \left(\frac{h}{G M_{\rm disk}} \right)^{1/2} \gtrsim 1.1$$

This implies that within $R_{\rm max}$ the mass in the halo $M_{\rm halo} > 75\%$. This is also not true for maximum disk.

[19] G. Efstathiou, G. Lake & J. Negroponte, Mon.Not.R.A.S. 199, 1069 (1982)

The criterion can be rewritten as

$$Y = 0.615 \left[\frac{QRV_{\rm rot}}{h \langle V_{\rm R}^2 \rangle^{1/2}} \right]^{1/2} \exp \left(-\frac{R}{2h} \right) \gtrsim 1.1$$

Evaluating this at $R = h$ and using the Bottema relation gives

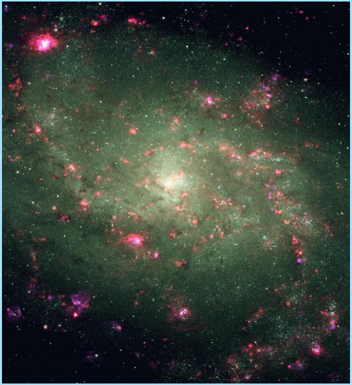
$$Q \gtrsim 2$$

Spiral structure

Density wave theory

We distinguish two types of spiral structure, grand design ...

and flocculent.



A comparative study of these two classes[20] suggests that in grand-design spiral strcuture there seems to be a strong underlying spiral wave in the stellar disk, while not in flocculent ones.

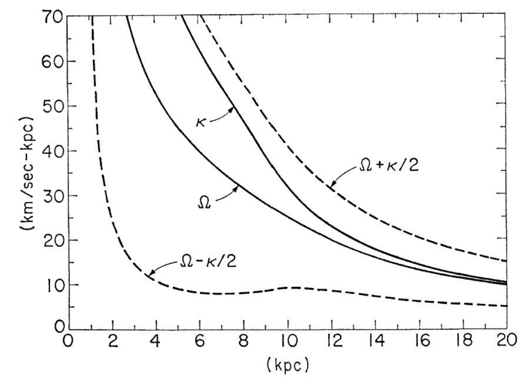
The density wave theory[21] was a response to the "winding dilemma", where material arms would wind up in a matter of 10^8 years or less.

The density wave is a spiral pattern, whose shape does not change with time, and which moves through the stellar and insterstellar disk.

[20] B.G. Elmegreen & D.B. Elmegreen, Ap.J.Suppl. 54, 127 (1984)
[21] C.C. Lin & F.H. Shu, Ap.J. 140,646 (1964), C.C. Lin, C. Yuan & F.H. Shu, Ap.J. 155, 721 (1969)

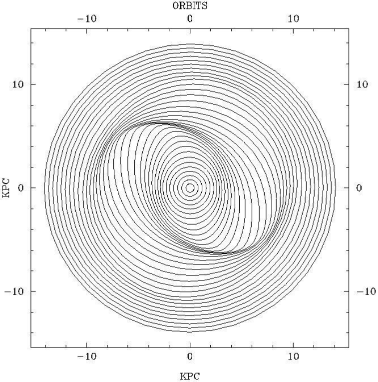
At the basis of a good description we can take the deduction that in the disk of our Galaxy (and in many others) the inner Lindblad resonance $\Omega - \kappa/2$ is fairly constant.

In this resonance a star goes through two epicycles during one revolution around the center. That means it describes a closed oval orbit in coordinate system moving with $\Omega - \kappa/2$.



In a disk where this property is constant over most radii we can get the following situation, where the stars are forced in orbits that line up as a spiral pattern.

In a coordinate frame, rotating with the pattern speed $\Omega_{\rm p} = \Omega - \kappa/2$, the spiral pattern remains unchanged.



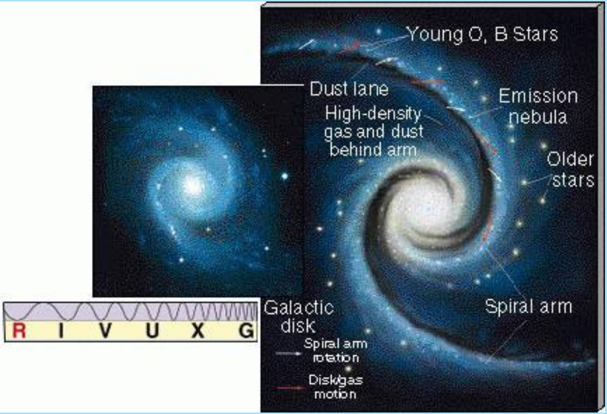
In the original density wave theory the density perturbations maintain themselves. The response of the stars to the perturbed gravitational field by the density concentrations in the arms results in a continuing pattern of density perturbations.

It was realized later by Toomre and others that the dissipation of energy in the waves is quick enough ($\sim 10^8$ years) that rejuvenation is required regularly.

It took until the first part of the seventies, before the underlying wave in the stellar disk was discovered in surface photometry[22].

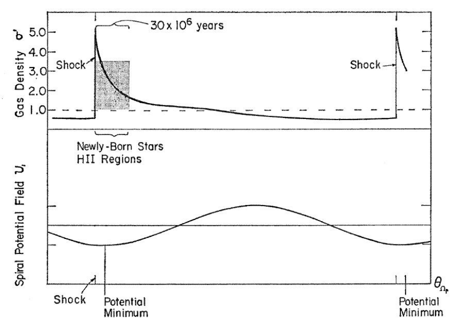
[22] F. Schweizer, Ap.J.Suppl. 31, 313 (1976)

The strongest confirmation came from studies of the interstellar medium.



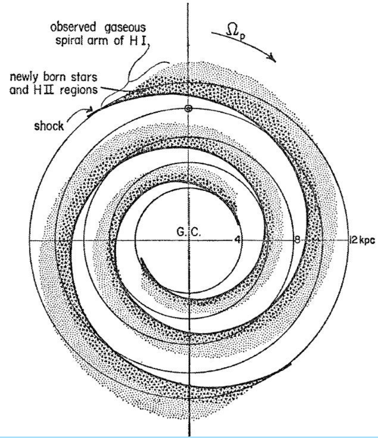
The reponse of the gas and dust is a-linear, since the relative velocities involved are supersonic[23]..

This gives shocks at the inner sides of the spiral arms and associated dustlanes and starformation.



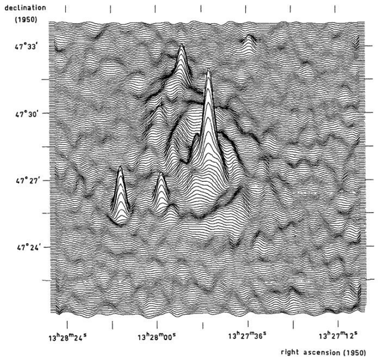
[23] W.W. Roberts, Ap.J. 158, 123 (1969)

The "delay" between dustlanes and HII-regions concerns the time between onset of gravitational instability and birth of MS-stars.

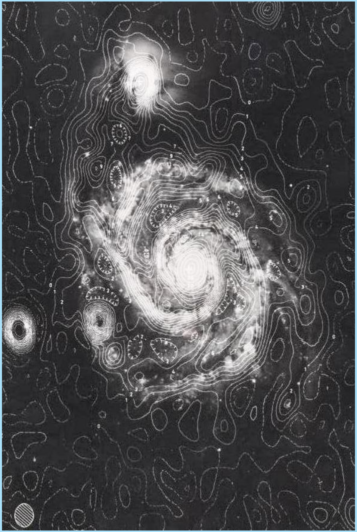


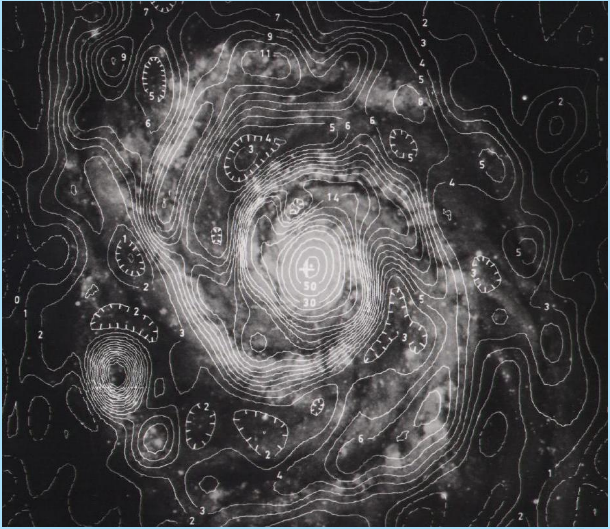
It was also confirmed by radio continuum studies with the new WSRT[24] in M51.

The compression holds at least for the magnetic field and possibly the relativistic electrons, so the synchrotron radiation will be enhanced at the inside of the arms and at the dustlanes.



[24] D.S. Mathewson, P.C. van der Kruit & W.N. Brouw, A.&A. 17, 468 (1972)

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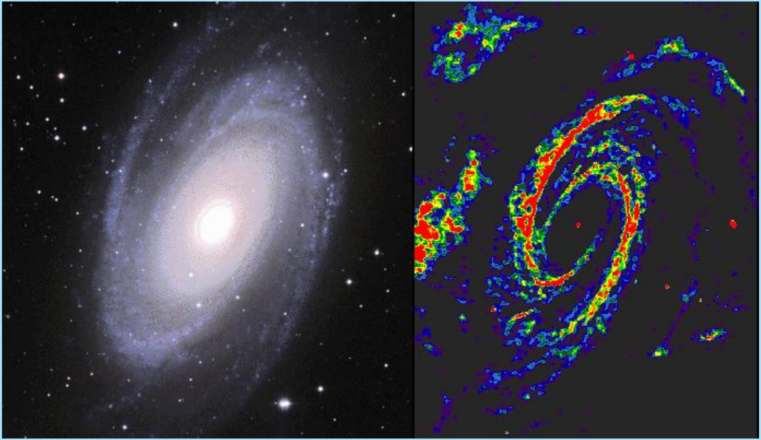
The next thing was to try and measure the streaming motions due to the density wave. This was tried in M81 using HI.

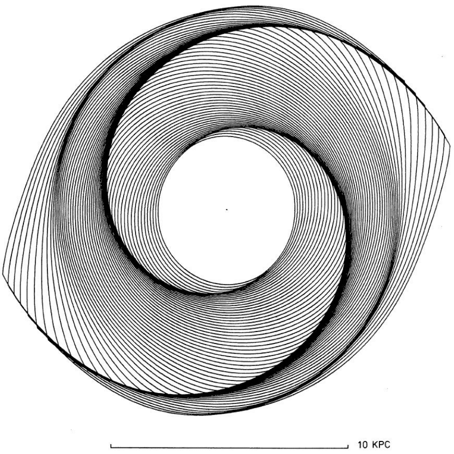


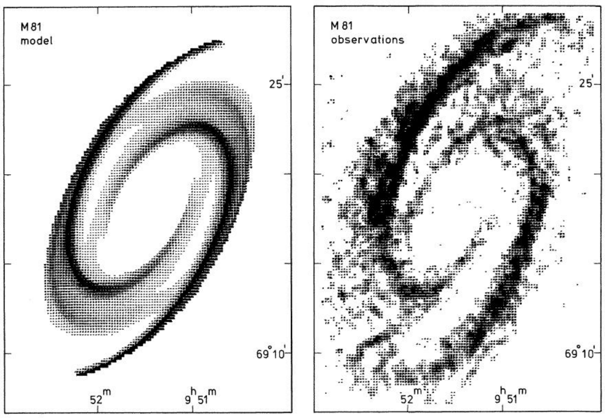
The Ph.D. thesis of H.C.D. Visser[25] analysed this in detail.

Using the surface photometry of Scheizer and HI-measurements at Westerbork, he was able to find an internally consistent representation of the observations of both the HI surface density distribution and the HI velocity field.

[25] 1978; see also A.&A. 88, 159 (1980)

Here are the (non-linear) streamlines of the gas.




M 81
model
25'
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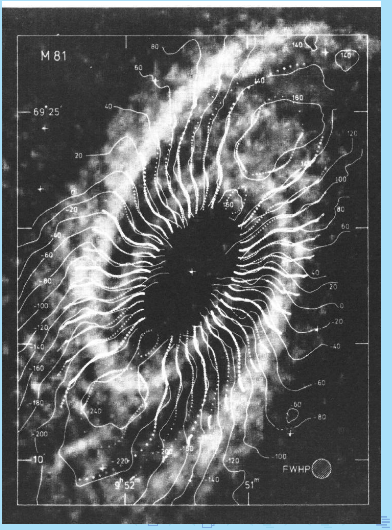
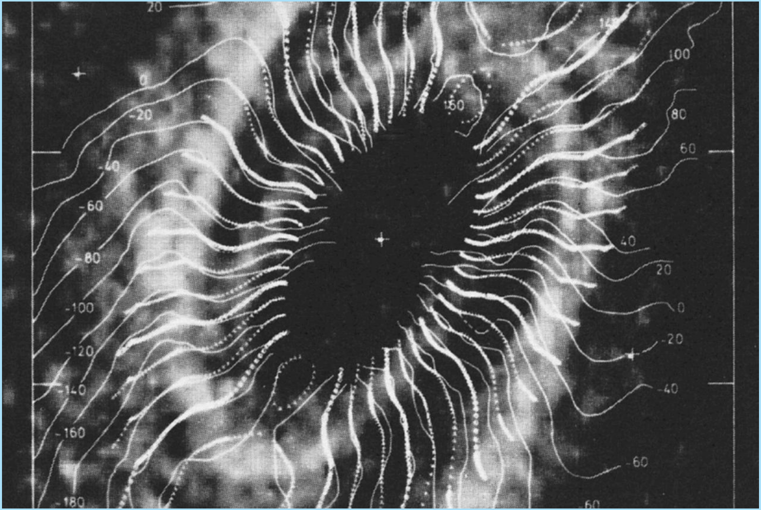
M 81
observations
25'
69°10'
52m
9h51m

The streaming motions are of the order of 10 km s^{-1}.

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40
20
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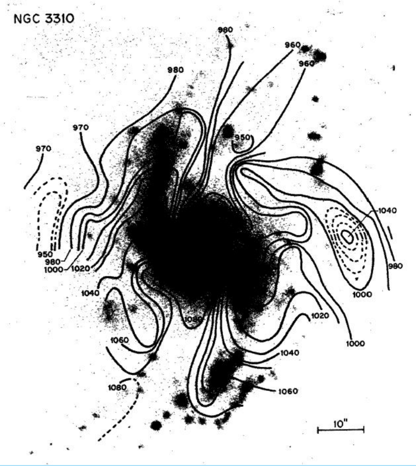
-20
-40
-60
-60

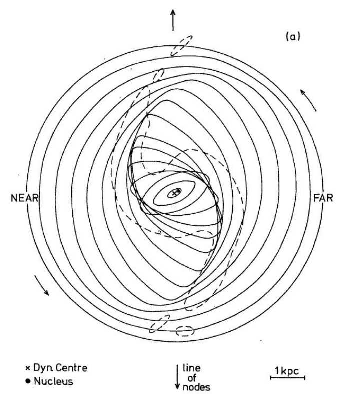
A very exceptional case is the disturbed, star burst galaxy NGC 3310, which is probably an example of a recent encounter[26].

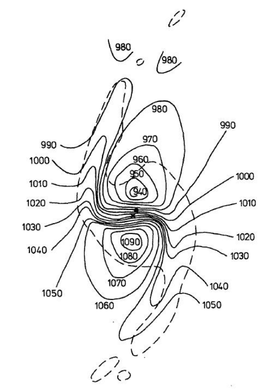
[26] P.C. van der Kruit & A.G. de Bruyn, A.&A. 48, 373 (1976); P.C. van der Kruit, A.&A. 49, 161 (1976)

The velocity field shows strong signs of streaming motions related to the spiral arms.



The streaming motions are here up to a third or so of the rotation velocity.





Stochastic star formation model

Density waves may be generated by tidal interactions, such as in M51 or in NGC 3310, or through Toomre's swing amplification.

The flocculent spiral stucture is probably the result of stochastic self-propagating star formation[27].

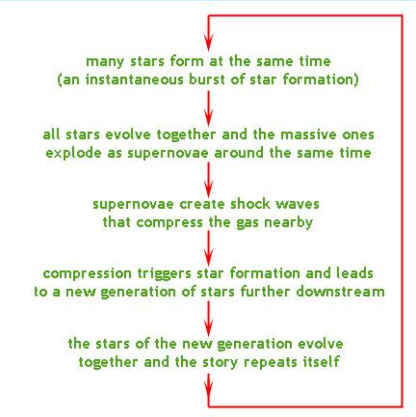
Since the propagation and indiced star formation is never 100%, also this will die out unless there is also spontaneous star formation.

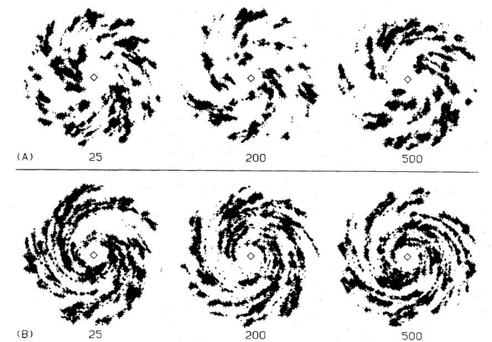
[27] H. Gerola & P.E. Seiden, Ap.J. 223, 129 (1978)

In this model star formation through supernova explosions is postulated to stimulate star formation in the neighborhood.

Such structures are then drawn out by differential rotation into arm-like features.

On the next page some simulations.

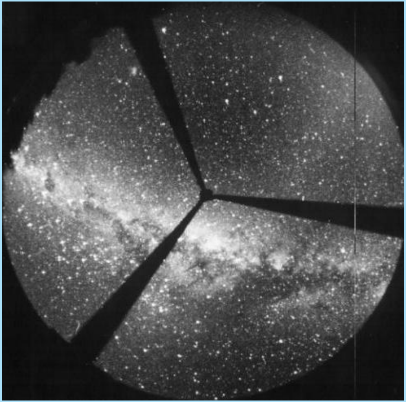


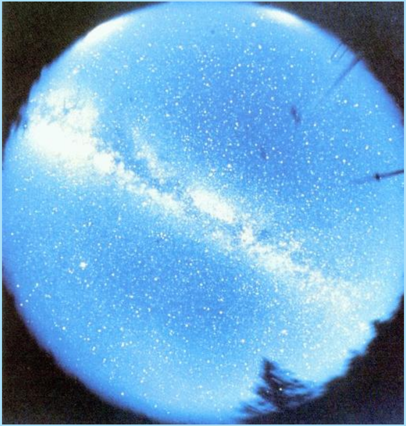


It has been suggested[28] that grand-design spiral structure is produced by bars or tidal encounters, while flocculent spiral structure results if the disk is left by itself.

[28] Kormendy & Norman, Ap.J. 233, 539 (1979)

The Hubble type of the Galaxy





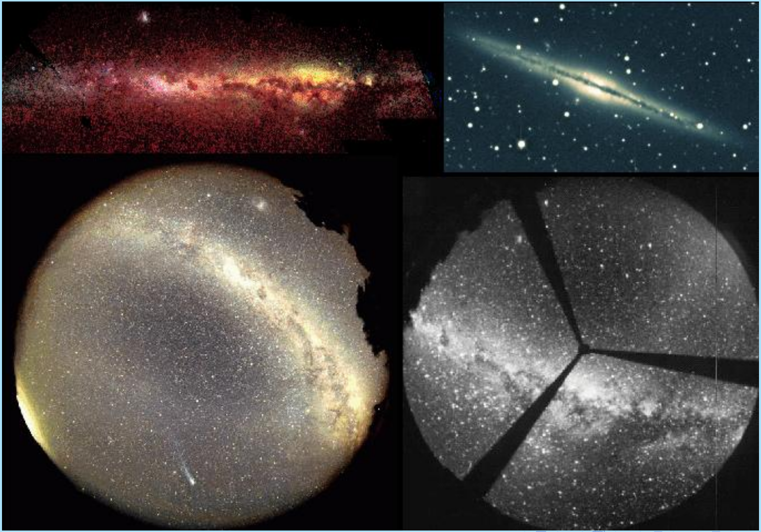
Various ways have been devised in the past:

- From the scalelength of the HII-regions: Sb or Sc
- From the disk color index $(B - V) = 0.85 \pm 0.15$: Sb?
- The HI-surface density – surface brightness ratio at $3h$; the log of that value is -0.5 ± 0.3. Compare with the Wevers sample: Sb
- The bulge-to-disk ratio: Sb or Sc
- The CO-distribution: Sb
- The similarity to NGC 891: Sb

In the Virgo 6°-core:

	$h < 4$ kpc	$h > 4$ kpc
I, I-II	0	6
II	20	5

Most likely Hubble type: SbI-II



Galaxies similar to our own.

Criteria:

- Disk scalelength h = 4 - 6 kpc
- Disk color $(B - V)$ = 0.6 - 0.8
- Bulge luminosity / total luminosity $L/L_{\rm tot}$ = 0.10 - 0.20
- Bulge effective radius $R_{\rm e}$ = 2 - 3 kpc
- Rotation velocity $V_{\rm rot}$ = 210 - 230 km/s
- HI-mass $M_{\rm HI} = (4 - 10) \times 10^9 M_\odot$

Closest in these parameters[29] (for NGC 891 disk luminosity parameters refer only to the old disk).

	The Galaxy	NGC 891	NGC 5033
Type	SbI-II	Sb	SbcI-II
Bulge R_e (kpc)	2.7	2.3	2.9
Bulge b/a	~ 0.7	0.7	?
$L_{\rm bulge}(L_\odot)$	2×10^9	1.5×10^9	4×10^9
Disk $\mu_{0,\rm B}$	22.1	22.9	22.0
Disk h (kpc)	5	4.9	5
$L_{\rm disk}(L_\odot)$	1.7×10^{10}	6.9×10^9	1.7×10^{10}
Disk $(B-V)$	0.8	0.9	0.6
Disk $R_{\rm max}$ (kpc)	20 - 25	21	22
$L_{\rm bulge}/L_{\rm tot}$	0.12	0.07	0.19
$V_{\rm rot}$ (km s^{-1})	220	225	215
$M_{\rm HI}(M_\odot)$	8×10^9	4×10^9	4×10^9

[29] using $H = 75$ km s^{-1} Mpc^{-1}