

# STRUCTURE OF GALAXIES

## 6. Dynamics of galaxies

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## Outline

### Rotation curves and mass distributions

- Exponential disk

- Dark matter halo

- Maximum disk hypothesis

- Independent checks on the maximum disk hypothesis

- Modified dynamics

### Dynamical relations

- Tully-Fisher relation for spirals

- Fundamental Plane for ellipticals

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# Rotation curves and mass distributions

## Exponential disk

The (infinitesimally thin) exponential disk, when self-gravitating, has a rotation curve of the following analytic form<sup>1</sup>:

$$V_{\text{rot}}^2(R) = \pi Gh\sigma_0 \left(\frac{R}{h}\right)^2 [I_0 K_0 - I_1 K_1]$$

with  $\sigma_0$  the central surface density and  $I$  and  $K$  modified Bessel functions *evaluated at*  $R/2h$ .

- ▶ This curve rises from the center to a **maximum** at  $R = 2.2h$  with

$$V_{\text{max}} = 0.8796(\pi Gh\sigma_0)^{1/2}$$

- ▶ and becomes **Keplerian** at large  $R$ .

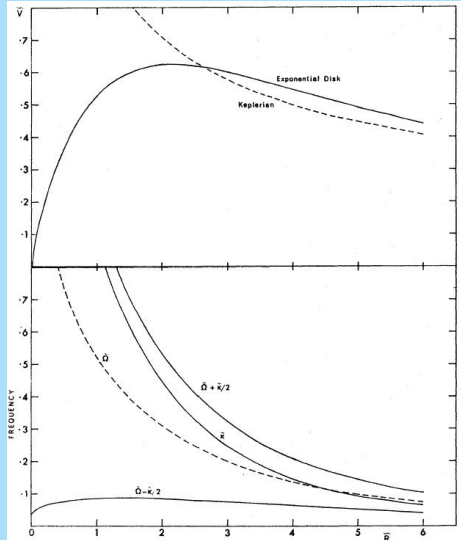
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<sup>1</sup>K.C. Freeman, Ap.J. 160, 811 (1970)

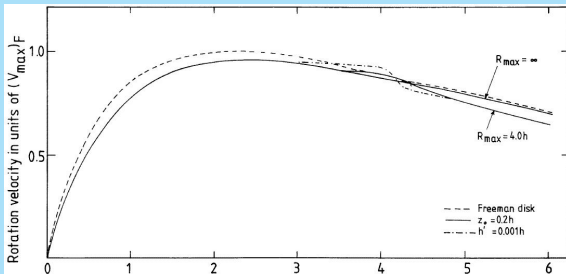
In the figure axes are dimensionless, so  $\tilde{R} = R/h$  and  $\tilde{V} = V\sqrt{h/GM}$  with  $M = 2\pi\sigma_0 h^2$ .

The lower half of the figure has the angular frequency  $\Omega$ , the epicyclic frequency  $\kappa$  and the Lindblad resonance frequencies  $\Omega \pm \kappa/2$ .

These frequencies is in units  $\sqrt{GMh^3}$ .



The rotation curve changes slightly when allowance is made for the **finite thickness** and the **truncation**<sup>2</sup>.



The dashed line has a **infinitely thin disk**, the full-drawn line has a **finite thickness** ( $z_0 = 0.2h$ ) without and with a shallow truncation (the scalelength changes by a factor 5 at  $R_{\max}$ ). The dot-dashed curve has a very sharp edge.

<sup>2</sup>P.C. van der Kruit & L. Searle, A.&A. 110, 61 (1982)

## Dark matter halo

Observations of spiral galaxies show **flat rotation curves** that do not show the Keplerian decline beyond the optical edge.

So add a dark halo with  $\rho \propto R^{-2}$  at large  $R$ .

This can be an **isothermal sphere**<sup>3</sup> or some other analytical function<sup>4</sup>.

In practice one may also directly infer a predicted rotation curve from the disk by calculated from the observed surface brightness profile.

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<sup>3</sup>e.g. C. Carignan & K.C. Freeman, Ap.J. 294, 494 (1985)

<sup>4</sup>K. Begeman, Ph.D. thesis (1987)

In the general case that the disk density distribution is  $\rho(R, z)$ , the rotation curve from the corresponding self-gravitating disk is

$$V_c^2(R) = -8GR \int_0^\infty r \int_0^\infty \frac{\partial \rho(r, z)}{\partial r} \frac{K(p) - E(p)}{(Rrp)^{1/2}} dz dr$$

with

$$p = x - (x^2 - 1)^{1/2} \quad \text{and} \quad x = \frac{R^2 + r^2 + z^2}{2Rr}$$

When the density distribution is separable in  $\sigma(R)$  and  $Z(z)$  this becomes

$$V_c^2 = -8GR \int_0^\infty r \sigma(r) \int_0^\infty \frac{\partial Z(z)}{\partial z} \frac{K(p) - E(p)}{(Rrp)^{1/2}} dz dr$$

The vertical distribution can for example be assumed to be the isothermal sheet.



We may in addition have a **bulge** with observed surface density  $\sigma(r)$ ; then for the self-gravitating case we have

$$V_c^2(R) = \frac{2\pi G}{R} \int_0^R r\sigma(r) dr + \frac{4G}{R} \int_R^\infty \left[ \arcsin\left(\frac{R}{r}\right) - \frac{R}{(r^2 - R^2)^{1/2}} \right] r\sigma(r) dr$$

For the **dark halo** the assumed the density law

$$\rho(R) = \rho_0 \left[ 1 + \left( \frac{R}{R_c} \right)^2 \right]^{-1}$$

results in

$$V_c^2(R) = 4\pi G \rho_0 R_c^2 \left[ 1 - \frac{R_c}{R} \arctan\left(\frac{R}{R_c}\right) \right]$$

To get the total rotation curve for a system consisting of three components add these circular velocities in quadrature:

$$V_{\text{circ}}(R) = [V_{\text{disk}}^2(R) + V_{\text{bulge}}^2(R) + V_{\text{halo}}^2(R)]^{1/2}$$

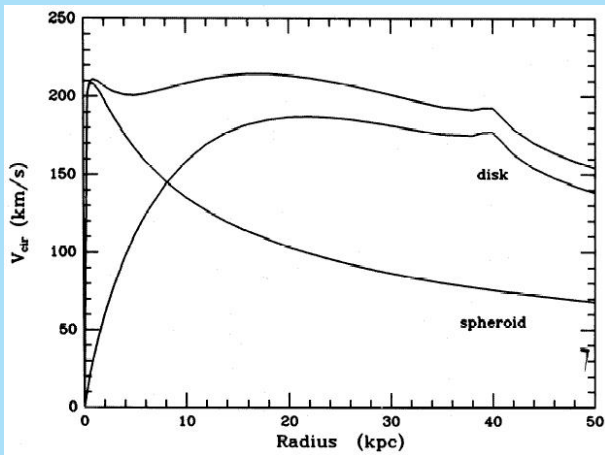
One can make things easier by fitting an exponential disk to the observations and use the analytic form of the corresponding rotation curve.

If in addition there is gas, this should be treated in the same way.

In practice we have for the stars only surface *brightness* distributions, so we need an undetermined *mass-to-light ratio*  $M/L$  in order to turn this into a surface *density* distribution.

From the *solar neighborhood* we can only find that  $M/L$  is of order a few in solar units.

In principle one can make an approximately flat rotation curve by a careful tuning of the disk and bulge contributions, as here for the Galaxy.



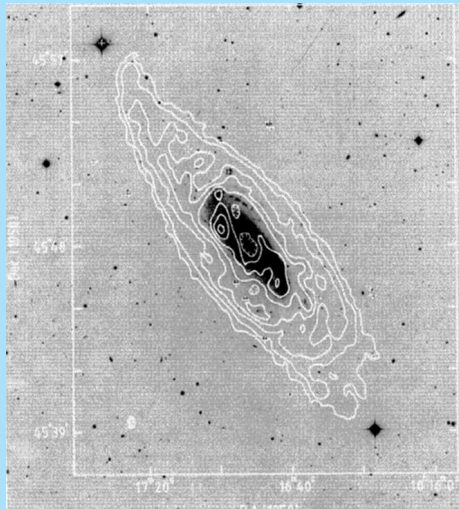
## Maximum disk hypothesis

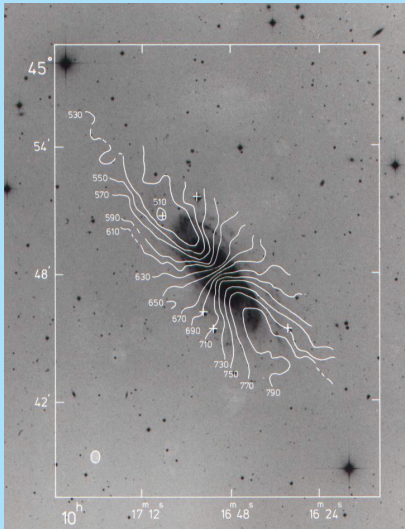
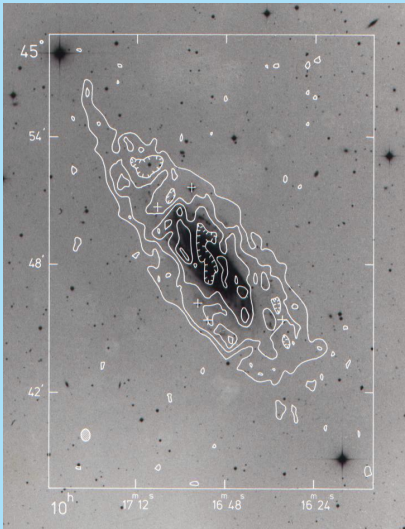
The following is from the analysis of the rotation curve of **NGC 3198<sup>a</sup>**, which has essentially no bulge.

The HI extends out to **11 scalelengths** on this long integration.

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<sup>a</sup>T.S. van Albada, J.N. Bahcall, K. Begeman & R. Sancisi, Ap.J. 295, 305 (1985)



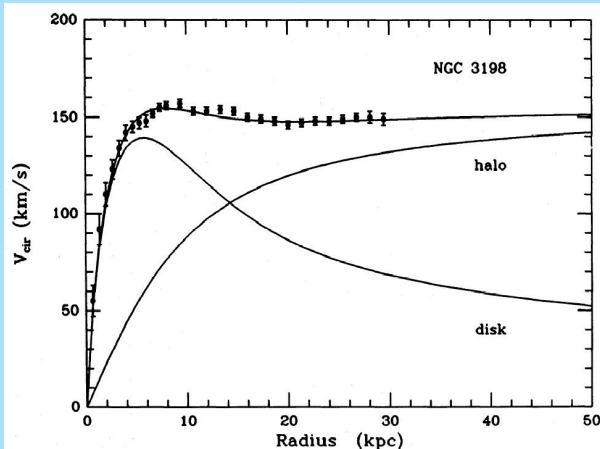


The procedure then is to choose an  $M/L$  of the disk that gives the **maximum amplitude of the disk rotation curve** that is allowed by the observations.

The two free parameters of the dark halo, **core radius  $R_c$**  and **central density  $\rho_o$**  are then used to fit the rotation curve.

This is called the **“maximum disk hypothesis”**, since it is a fit to the rotation curve with the largest amount of mass possible in the disk (and the largest  $M/L$ ).

The maximum disk solution to the rotation curve of NGC 3198 looks as follows.

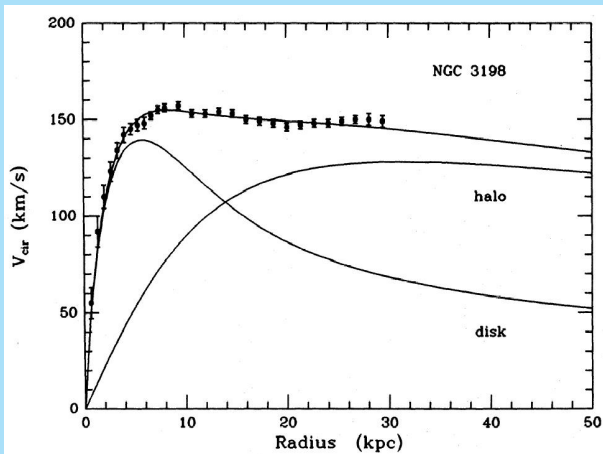


This particular model for NGC 3198 has a total mass of  $15 \times 10^{10} M_{\odot}$  within 30 kpc.

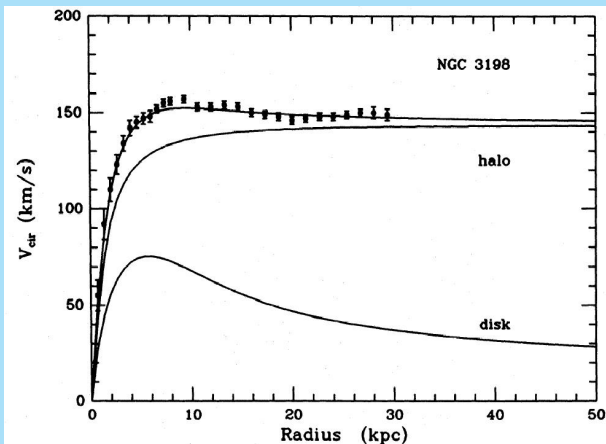
Within this radius the ratio of dark to visible matter is 3.9. At the optical edge this ratio is 1.5.

By adjusting the halo parameters one can minimize the dark halo mass by assuming that the rotation curve falls beyond the last measured point.

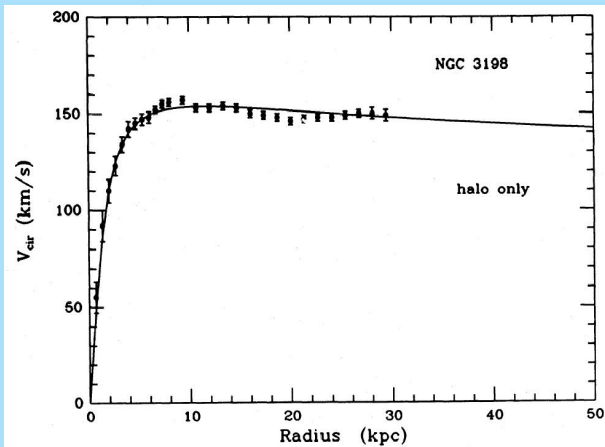




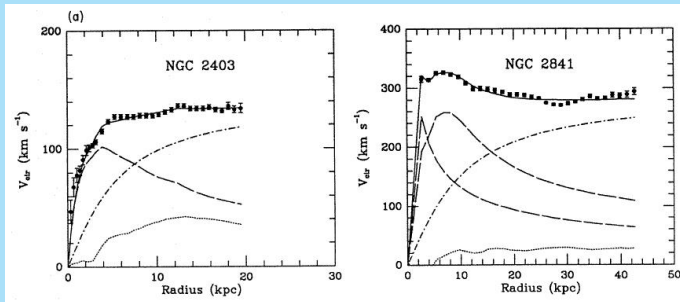
The difficulty with the maximum disk hypothesis is that it is possible to make similar good fits with **lower disk masses** ...



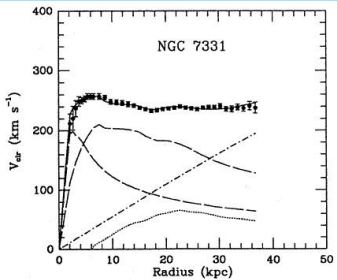
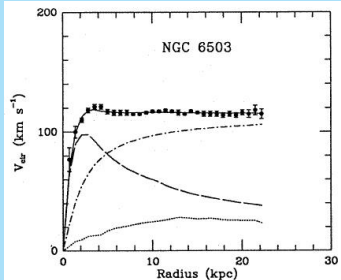
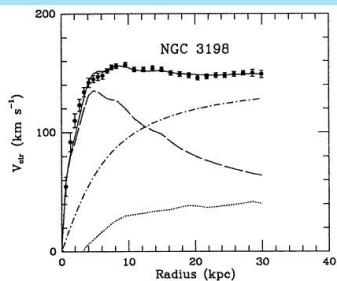
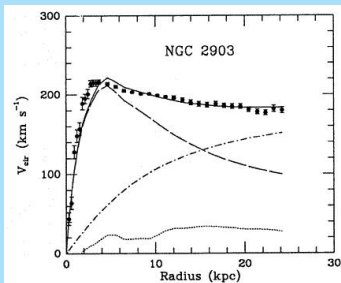
... and even **no disk mass at all!**



Begeman<sup>5</sup> observed 8 spirals, of which HI in **NGC 2841** goes out to **17.8 h (43 kpc)**.



<sup>5</sup>K. Begeman, Ph.D. thesis (1987); K. Begeman, A.H. Broeils & R.H. Sanders, Mon.Not.R.A.S. 249, 523 (1991)



Begeman's maximum disk fits have

- ▶  $(M/L)_{\text{disk}} = 3.1 \pm 1.2$  (9.4 for NGC 2841)
- ▶  $(M_{\text{halo}})_{R_{\text{opt}}} = 44 \pm 9\%$  (34 % for NGC 2841)

Broeils<sup>6</sup> made maximum disk fits to a sample of 23 galaxies with extended HI, accurate rotation curves and photometry. He studied the distribution of the parameters from the fits.

The **global mass-to-light ratio**  $M/L$  out to the maximum radius observed is in the range 10 to 20 (in B).

The **ratio of the dark to luminous matter** at some fiducial radius (either  $R_{25}$  or  $R = 7h$ ) correlates well with the maximum rotation velocity and reasonably well with integrated magnitude and morphological type.

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<sup>6</sup>A.H. Broeils, Ph.D. thesis (1992)

It is possible that these conclusions are influenced by the assumption of the maximum disk hypothesis.

The maximum disk hypothesis could lead to the following spurious results:

- ▶ Large  $V_{\max}$  results in disk surface density and therefore large  $(M/L)_{\text{disk}}$ .
- ▶ Large  $(M/L)_{\text{disk}}$  results in less dark matter.

Indeed:

$$\left(\frac{M}{L}\right)_{\text{disk}} = (0.014 \pm 0.003) V_{\max} + (0.72 \pm 0.60)$$

with  $r = 0.67$ .

$$\left(\frac{M_{\text{dark}}}{M_{\text{lum}}}\right)_{R_{25}} = (2.37 \pm 0.39) - (0.42 \pm 0.12) \left(\frac{M}{L}\right)_{\text{disk}}$$

with  $r = 0.62$ .

## Independent checks on the maximum disk hypothesis

There are independent ways in which the maximum disk hypothesis can be checked by independent measurement of  $M/L$ .

### a. The truncation feature in the rotation curve:

The truncation feature in the rotation curve can in principle be used to estimate the mass of the disk. It has been done in two cases where the mass of the halo within the truncation radius has been estimated:

- ▶ NGC 5907<sup>7</sup>:  $(M_{\text{halo}})_{R_{\text{opt}}} \approx 60\%$  (so not maximum disk)
- ▶ NGC 4013<sup>8</sup>:  $(M_{\text{halo}})_{R_{\text{opt}}} \approx 25\%$

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<sup>7</sup>S. Casertano, Mon.Not.R.A.S. 203, 735 (1983)

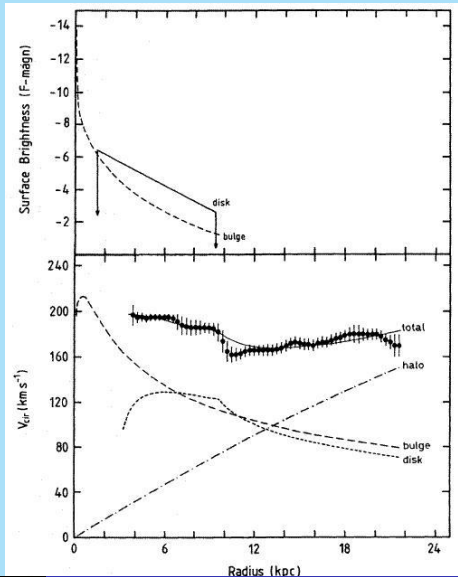
<sup>8</sup>R. Bottema, A.&A. 306, 345 (1996)



In NGC 4013 the disk and bulge must dominate dynamically in the inner regions.

The truncation feature is clearly visible.

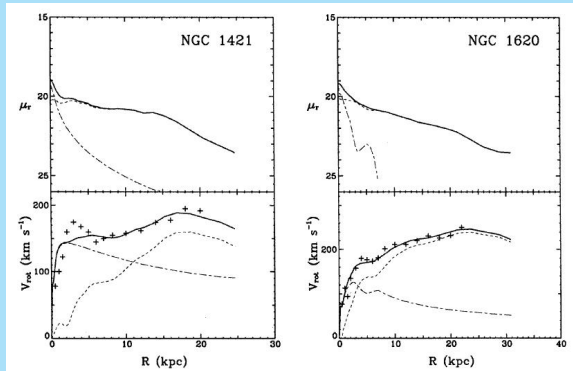
However, the fit to the rotation curve is not really maximum disk.



b. "Wiggles" in rotation curves The inner parts of rotation curves can often be fit without a dark halo and features in luminosity profiles seem to correspond to features in rotation curves<sup>9</sup>.

Top shows the light distributions of disk and bulge.

Bottom shows the rotation curve with constant  $M/L$  in both components.

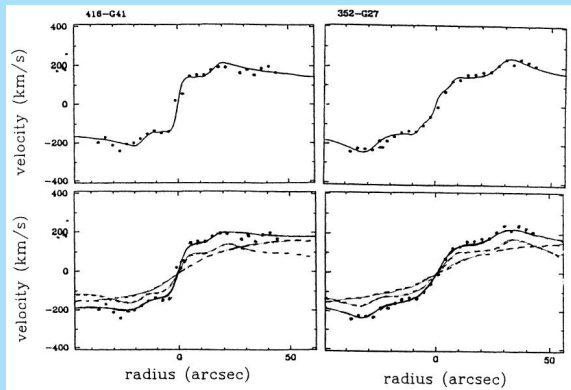


<sup>9</sup>E.g. S. Kent, A.J. 91, 1301 (1986)

This suggests **maximum disks**, but even if disks are not dynamically dominant in the inner parts the wiggles can still be reproduced.<sup>10</sup>

Top has the rotation curve from the photometry without a dark halo.

Bottom has reduced the disk mass by half and a dark halo added.



<sup>10</sup>P.C. van der Kruit, IAU Symp. 164, 227 (1995)

### c. Maximum rotation versus scalelength

Recently<sup>11</sup> the following argument has been used.

For a pure exponential disk the maximum in the rotation curve occurs at  $R = 2.2h$  with an amplitude of

$$V_{\max} \propto \sqrt{h\sigma_0} \propto \sqrt{\frac{M_{\text{disk}}}{h}}$$

For fixed disk-mass  $M_{\text{disk}}$  this gives

$$\frac{\partial \log V_{\max}}{\partial \log h} = -0.5$$

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<sup>11</sup>S. Courteau & H.-W. Rix, Ap.J. 513, 561 (1999)

So at a given absolute magnitude (or mass) lower scalelength disks should have higher rotation.

If galaxies are maximum disk (in practice  $V_{\text{disk}} \sim 0.85 V_{\text{total}}$ ) this should be seen in scatter of the Tully-Fisher relation<sup>12</sup>.

This is not observed and the estimate is that on average  $V_{\text{disk}} \sim 0.6 V_{\text{total}}$ .

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<sup>12</sup>The Tully-Fisher relation is a tight correlation between maximum rotation and total luminosity of disk galaxies; it will be discussed in more detail below

## d. Thickness of the HI-layer.

The thickness of the gas layer can be used to measure the surface density of the disk independent of the rotation curve.

The density distribution of the exponential, locally isothermal disk was:

$$\rho_*(R, z) = \rho_*(0, 0) \exp(-R/h) \operatorname{sech}^2(z/z_0)$$

If the HI has a velocity dispersion  $\langle V_z^2 \rangle_{\text{HI}}^{1/2}$ , and if the stars dominate the gravitational field

$$\rho_{\text{HI}}(R, z) = \rho_{\text{HI}}(R, 0) \operatorname{sech}^{2p}(z/z_0)$$

$$p = \frac{\langle V_z^2 \rangle_*}{\langle V_z^2 \rangle_{\text{HI}}}$$

The full width at half maximum of this distribution is:

$$W_{\text{HI}} = 1.663p^{-1/2}z_0 \text{ for } p \gg 1$$

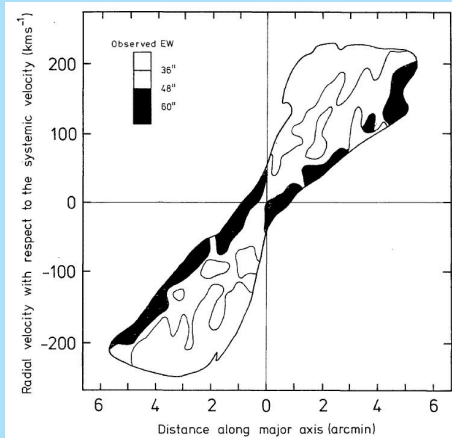
$$W_{\text{HI}} = 1.763p^{-1/2}z_0 \text{ for } p = 1$$

Then to within 3 %

$$W_{\text{HI}} = 1.7 \langle V_z^2 \rangle_{\text{HI}}^{1/2} \left[ \frac{\pi G(M/L)\mu_0}{z_0} \right]^{-1/2} \exp(R/2h)$$

So the gas layer **increases** exponentially in thickness with an e-folding of  $2h$ .

We look at the **equivalent width** of the H I-layer in **NGC 891**<sup>13</sup>.



<sup>13</sup>P.C. van der Kruit, A.&A. 99, 298 (1981)

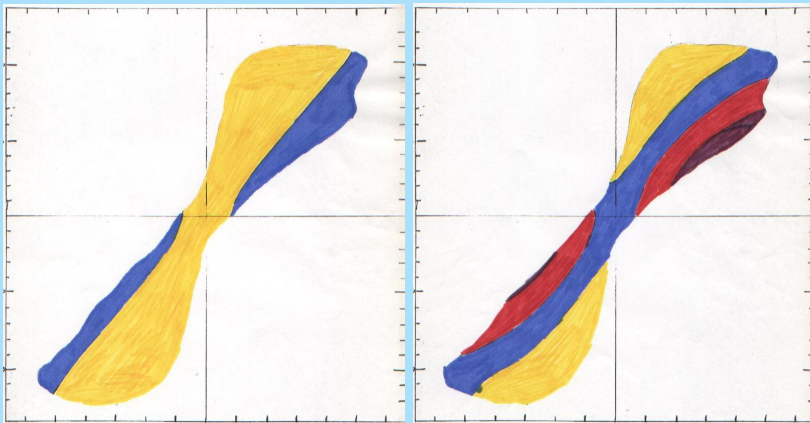


Three particular models were then calculated:

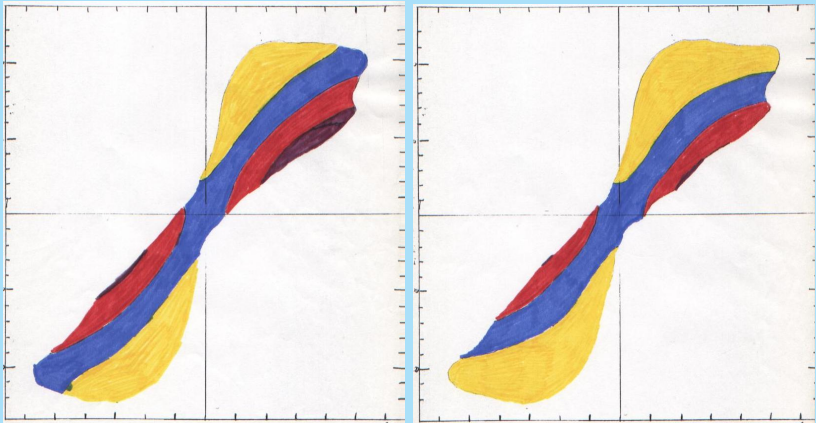
- ▶ **Model I** (40% of the mass within the optical radius in the disk),
- ▶ **Model II** with all the mass (including the dark mass) in the disk,
- ▶ **Model III** with a constant thickness of the HI-layer.

The  $W_{\text{HI}}$  in the observations were then calculated for disks with inclinations of  $87.5^\circ$  and  $90^\circ$ .

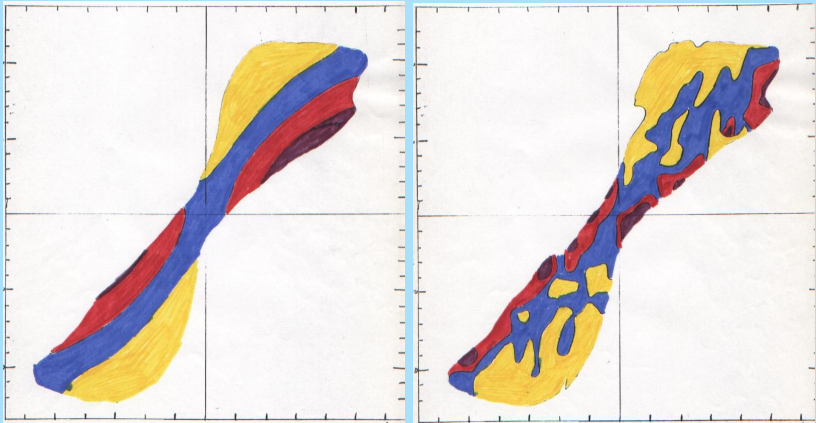
Here is the equivalent width in the  $(x, V)$ -diagram for Model I with inclinations of  $90^\circ$  (left) and  $87^\circ 5'$  (right).



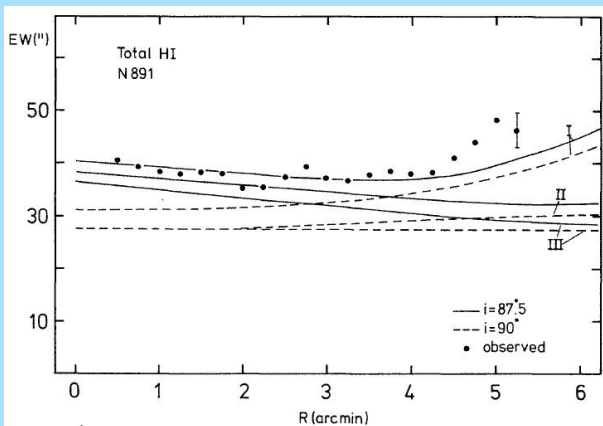
Here is the equivalent width in the  $(x, V)$ -diagram for **Model I** (left) and **Model II** (right) both at an inclination of  $87^\circ 5$ .

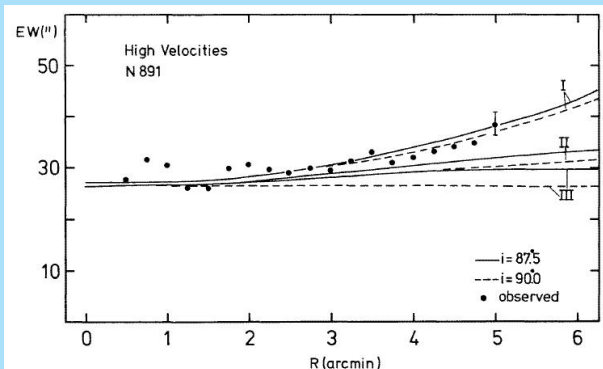


Here is the equivalent width in the observed  $(x, V)$ -diagram (left) and that for **Model I** with an inclination of  $87^\circ 5$  (right).



Also the thickness over all velocities and the “high” velocities (190 to 230 km/s) can be compared to observations.





NGC 891 is **not maximum disk**. Also this analysis shows that the dark matter cannot be in the disk.

## e. Thickness of the stellar disk

The vertical motions of the stars can be combined with the thickness of stellar disks to estimate of the **disk surface densities**  $\sigma$ .

For the isothermal sheet with space density  $\rho(z)$

$$\rho(z) = \rho(0) \operatorname{sech}^2(z/z_0)$$

we had for the velocity dispersion

$$\langle V_z^2 \rangle^{1/2} = \sqrt{2\pi G \rho(0)} z_0 = \sqrt{\pi G \sigma} z_0$$

Bottema<sup>14</sup> found that the stellar velocity dispersion at a fiducial radius correlates maximum in the rotation curve.

<sup>14</sup>R. Bottema, A.&A. 275, 16 (1993)

Using this relation we can estimate the disk surface density if we know  $z_o$  and the rotation curve.

Statistical analysis of samples of galaxies gives<sup>15</sup> then is

$$\frac{V_{\text{rot,disk}}}{V_{\text{rot,obs}}} = 0.56 \pm 0.06.$$

A working definition<sup>16</sup> of this ratio for a maximum disk is

$$\frac{V_{\text{rot,disk}}}{V_{\text{rot,obs}}} = 0.85 \pm 0.10.$$

So, in general galaxy disk appear to be **NOT** maximum disk.

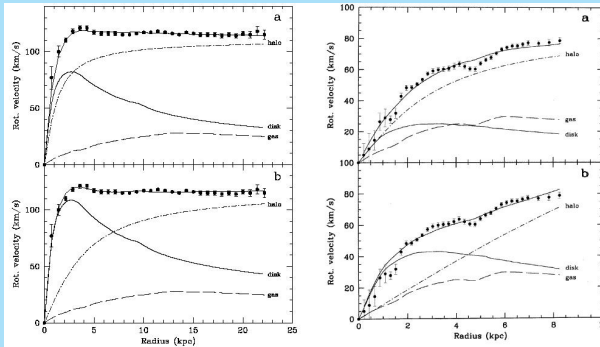
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<sup>15</sup>R. Bottema, A.&A. 275, 16 (1993); M. Kregel, P.C. van der Kruit & K.C. Freeman, Mon.Not.R.A.S. 358, 5003 (2004)

<sup>16</sup>P.D. Sackett, Ap.J. 483, 103 (1997)



Bottema's analysis<sup>17</sup> on a **high surface brightness** and a **low-surface brightness** galaxy gives a model according to the stellar velocity dispersion as at the top and the maximum disk hypothesis as at the bottom.



<sup>17</sup>R. Bottema, A.&A. 328, 517 (1997)

## f. Our Galaxy

The measured surface density<sup>18</sup> of the **stellar disk in the solar neighbourhood** is **50 to 80  $M_{\odot} \text{pc}^{-2}$**  and the **scalelength**<sup>19</sup> of the disk **4 to 5 kpc**.

With this it can be estimated that the luminous matter provides a maximum rotation velocity of  **$155 \pm 30 \text{ km/s}$** , while the observed value is  **$225 \pm 10 \text{ km/s}$** .

The Galaxy is then **not maximum disk**.

However, one can change the parameters within uncertainties to get different answers<sup>20</sup>.

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<sup>18</sup>K.H. Kuijken & G. Gilmore, M.N.R.A.S. 239, 605 (1989) find  **$46 \pm 9 M_{\odot} \text{pc}^{-2}$**  and J.N. Bahcall, Ap.J. 287, 926 (1984)  **$80 \pm 20 M_{\odot} \text{pc}^{-2}$**

<sup>19</sup>P.C. van der Kruit, A&A. 157,230 (1986)

<sup>20</sup>e.g. J.A. Sellwood & R.H. Sanders, Mon.Not.R.A.S. 233, 611 (1988); P.D. Sackett, Ap.J. 483, 103 (1997)

## Modified dynamics

Flat rotation curves may show that classic Newtonian gravity does not work at large distances<sup>21</sup>. For this purpose **Modified Newtonian Dynamics (MOND)**<sup>22</sup> was developed.

This has an acceleration  $\vec{g}$ , which is related to Newtonian acceleration  $\vec{g}_N$  as

$$\vec{g} \left( \frac{g}{a_0} \right) \left[ 1 + \left( \frac{g}{a_0} \right)^2 \right]^{-1/2} = \vec{g}_N$$

with  $a_0 \sim 1.2 \times 10^{-8} \text{ cm sec}^{-2}$ .

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<sup>21</sup>e.g. R.H. Sanders, Mon.Not.R.A.S. 223, 539 (1986); K. Begeman, A.H. Broeils & R.H. Sanders, Mon.Not.R.A.S. 249,523 (1991)

<sup>22</sup>e.g. M. Milgrom, Ap.J. 270, 365 (1983)

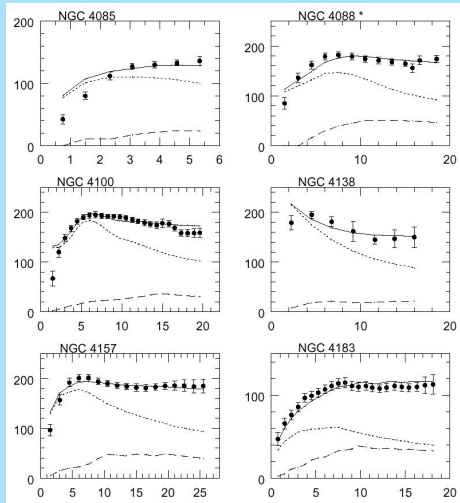
- ▶ For **large accelerations**  $g/a_0$  this reduces to Newtonian gravity. So on small scales (in the solar system or the inner parts of galaxies) we have  $g = g_N \propto R^{-2}$  and Keplerian rotation with  $V_{\text{rot}}^2 \propto R^{-1}$ .
- ▶ But at low accelerations it becomes  $g = (g_N a_0)^{1/2}$ . Since now  $g \propto R^{-1}$  this gives rise to  $V_{\text{rot}}^2 \propto R^0 = \text{constant}$ .

The result is that flat rotation curves can be produced **without** introducing a dark halo .

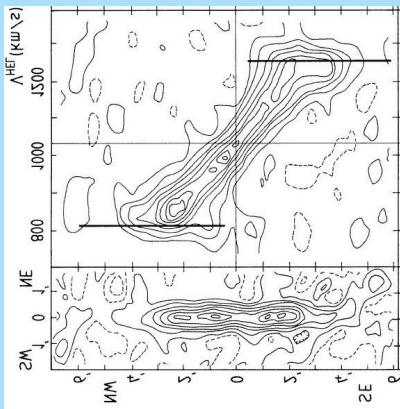
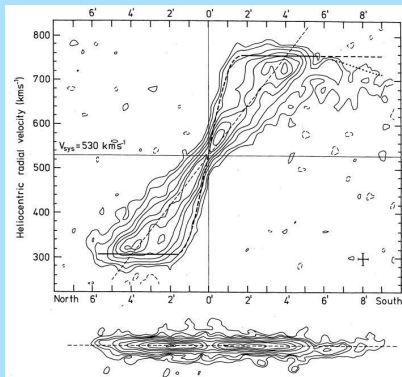
Here are some fits to actual rotation curves<sup>a</sup>. The full lines are the MOND-fits and the other lines show Newtonian curves for the stars and gas.

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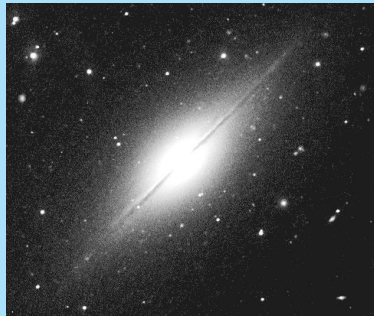
<sup>a</sup>R.H. Sanders & M.A.M. Verheijen, Ap.J. 503, 97 (1998)



NGC 891 and 7814 have the same rotation curves



but completely different light distributions.



This is inconsistent with MOND.

# Dynamical relations



## Tully-Fisher relation for spirals

For exponential disks:

$$M \propto \sigma_o h^2 \quad V_{\max} \propto (\sigma_o h)^{1/2}$$

Then

$$M \propto V_{\max}^4 \sigma_o^{-1}$$

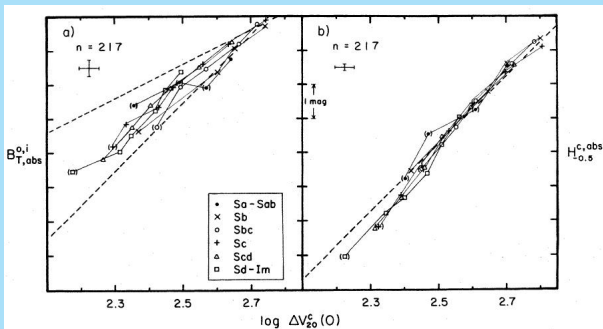
With Freeman's law and constant mass to light ratio  $M/L$ :

$$L \propto V_{\max}^4$$

This is the Tully-Fisher relation which has indeed been observed<sup>23</sup>. In practice  $V_{\max}$  is measured from the total width of the HI-profile, corrected for inclination, at a level 20 or 50% of the peak.

<sup>23</sup>R.B. Tully & J.R. Fisher, A.&A. 54, 661 (1977)

Aaronson & Mould<sup>24</sup> find exponents of 3.5 in B and 4.3 in H (1.6  $\mu$ ).



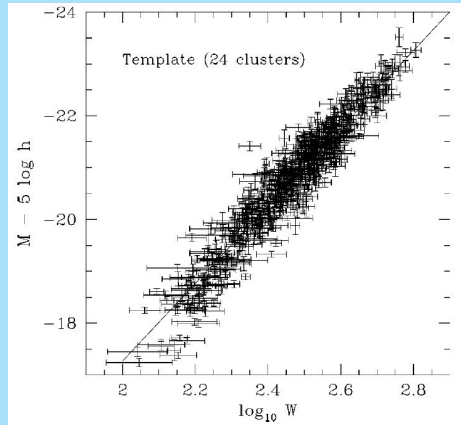
There is debate about the slope in observed relations.

<sup>24</sup>M. Aaronson & J.R. Mould, Ap.J. 265, 1 (1983)

In the I-band Giovanelli *et al.*<sup>a</sup> find from 555 galaxies in 24 clusters a slope of  $7.68 \pm 0.13$  (in magnitudes, which corresponds to exponent  $3.07 \pm 0.05$ ).

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<sup>a</sup>R. Giovanelli & 6 other authors, Ap.J. 477, L1 (1997)



A recent study of the Ursa Major Cluster<sup>25</sup> shows that the relation is tightest in the  $K'$ -band and there the slope is  $11.3 \pm 0.5$  (exponent  $4.5 \pm 0.2$ ).

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<sup>25</sup>M.A.M. Verheijen, Ph.D. thesis (1997) and Ap.J. 563, 694 (2001)

The following clever argument<sup>26</sup> makes use of the **scatter in the Tully-Fisher relation** to test if disks are **maximal**.

For a pure exponential disk the maximum in the rotation curve occurs at  $R = 2.2h$  with an amplitude of

$$V_{\max} \propto \sqrt{h\Sigma_0} \propto \sqrt{\frac{M_{\text{disk}}}{h}}$$

For fixed disk-mass  $M_{\text{disk}}$  this gives by differentiation

$$\frac{\partial \log V_{\max}}{\partial \log h} = -0.5$$

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<sup>26</sup>S. Courteau & H.-W. Rix, Ap.J. 513, 561 (1999)

So at a given absolute magnitude (or mass) **lower scalelength** disks should have **higher rotation**.

If all galaxies are maximum disk this effect should be visible in the **scatter of the Tully-Fisher relation**.

But it is **not** observed.

The estimate can be made that on average  $V_{\text{disk}} \sim 0.6V_{\text{total}}$ .

## Fundamental Plane for ellipticals

We can do an equivalent thing for **elliptical galaxies**.

The **isothermal sphere** has at large  $R$

$$\rho(R) = \frac{\langle V^2 \rangle}{2\pi G} R^{-2}$$

The **core radius** is

$$r_o = \left( \frac{4\pi G \rho_o}{9 \langle V^2 \rangle} \right)^{-1/2}$$

For a better description we need **King-models**<sup>27</sup>.

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<sup>27</sup>I. King, A.J. 67, 471 (1962)

For these he introduced a tidal radius  $R_t$ , then

$$\langle V^2 \rangle^{1/2} \propto \rho_o M(R_t) f \left( \frac{R_t}{r_o} \right)$$

with  $f(R_t/r_o)$  some numerical function.

The central surface density is

$$\sigma_o = \rho_o r_o g \left( \frac{R_t}{r_o} \right)$$

with  $g(R_t/r_o)$  another numerical function.

For ellipticals  $\log(R_t/r_o) \approx 2.2$ .

With Fish's law (constant central surface brightness) and constant  $M/L$  it then follows that

$$L \propto \langle V^2 \rangle^2$$

This is what has been observed. The velocity dispersion then in practice is the one observed at the position of the center.

The scatter can even be reduced with a related three-parameter relation, that is called the **Fundamental Plane**.

It is a relation between some consistently defined **radius** (core or effective radius)  $R$ , the observed **central velocity dispersion**  $\sigma$  and a consistently defined **surface brightness** (central or at the effective radius)  $I$ <sup>28</sup>:

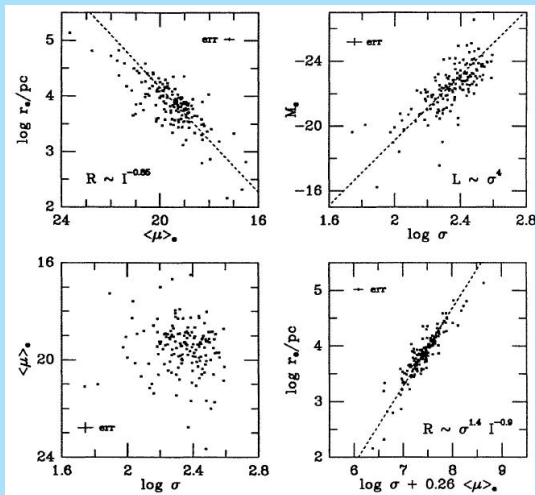
$$R \propto \sigma^{1.4 \pm 0.15} I^{-0.9 \pm 0.1}$$

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<sup>28</sup>J. Kormendy & S.G. Djorgovski, Ann.Rev.Astron.Astrophys. 27, 235 (1989)



In various projections the Fundamental Plane looks as follows.



## Distance determinations

Both the Tully-Fisher relation and the Fundamental Plane are powerful means for the determination of distances.

The most powerful methods are the following:

- ▶ **Type Ia Supernovae**. The peak brightness of these objects has a small intrinsic scatter.
- ▶ **The Tully-Fisher relation** for spiral galaxies.
- ▶ **The Fundamental Plane** for elliptical galaxies.
- ▶ **Surface brightness fluctuations**. Due to the fact that the surface brightness results from individual stars, there are statistical fluctuations. The resolution of these depends on the distance. It is applicable to ellipticals and bulges of spirals.
- ▶ **Type II Supernovae**. These result from massive stars and can be used through the Baade-Wesselink method.

The first three of these need calibration and the others checks with distances obtained from **Cepheids**.

This was done in the **Key Project to measure the Hubble constant** with the **Hubble Space Telescope**.

In this project (and related ones) direct Cepheid distances were determined for in total 31 galaxies.

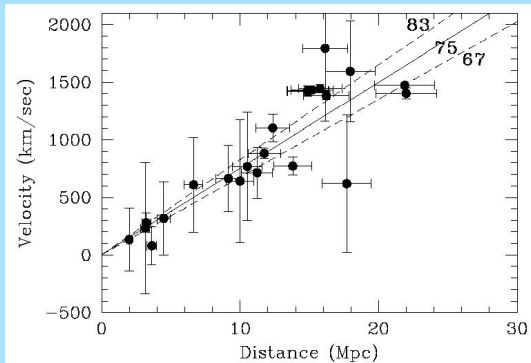
The project was concluded in the final one of 28 papers<sup>29</sup>.

The galaxies with Cepheid distances give their own distance scale. The resulting value for the **Hubble constant** (including only random errors) is

$$H_0 = 71 \pm 6 \text{ km sec}^{-1} \text{Mpc}^{-1}$$

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<sup>29</sup>J.Mould *et al.* Ap.J. 529, 786 (2000)



The Cepheid distances calibrate the secondary methods listed above.

The final result (including as much as possible systematic errors)<sup>30</sup>  
is

$$H_0 = 72 \pm 8 \text{ km sec}^{-1} \text{Mpc}^{-1}$$

However, using the Cepheids to directly calibrate Supernovae Ia  
peak absolute magnitudes leads to<sup>31</sup>

$$H_0 = 62 \pm 6 \text{ km sec}^{-1} \text{Mpc}^{-1}$$

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<sup>30</sup>W. Freedman et al., Ap.J. 553, 47 (2001)

<sup>31</sup>A. Sandage et al., Ap.J. 653, 843 (2006)