STRUCTURE OF GALAXIES

4. Photometric parameters and evolution

Piet van der Kruit
Kapteyn Astronomical Institute
University of Groningen
the Netherlands

February 2010

Outline

Photometric parameters

Distribution of parameters

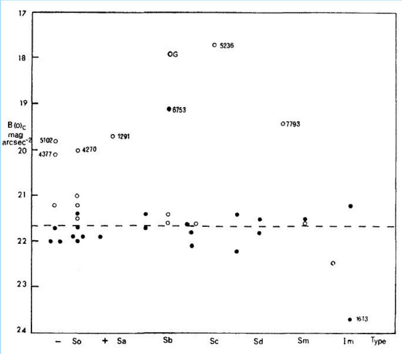
Ken Freeman[1] was the first to study the distribution of properties of exponential disks.

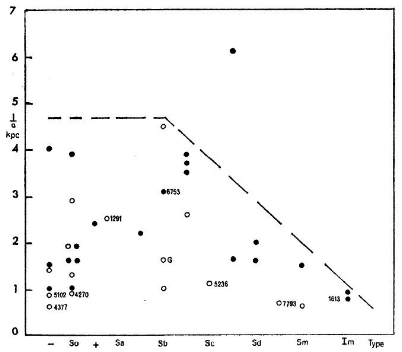
His results are in the following two figures; the small range of (extrapolated) face-on, central surface brightness is known as "Freeman's Law":

$$\mu_\circ = 21.67 \pm 0.30 \ \ \mathrm{B-mag\ arcsec}^{-2}$$

This has generated considerable discussion. The problem is that samples need to be statistically complete and Freeman's sample had serious selection effects.

[1] K.C. Freeman, Ap.J. 160, 811 (1970)


B(0)c
mag
arcsec-2
17
18
19
20
21
22
23
24
5102
4377
4270
1291
G
6753
5236
7793
1613
–
So
+
Sa
Sb
Sc
Sd
Sm
Im
Type

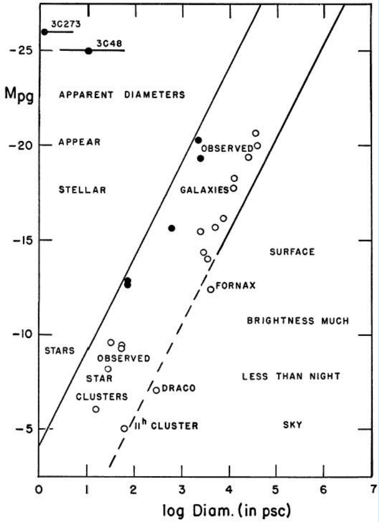

1/α
kpc
0
1
2
3
4
5
6
7
5102
4377
4270
1291
6753
G
5236
7793
1613
–
So
+
Sa
Sb
Sc
Sd
Sm
Im
Type

Selection effects

The selection was discussed first by Arp [a].

We see that there is a narrow band in this diagram,excluding objects that either are have surface brightnesses that are too faint or that appear stellar.

[a] H.C. Ap.J. 142, 402 (1965).



The selection effects operating here are:

- For a particular luminosity and a faint $\mu_\circ$ we get a large h, but for the most part the object is fainter than sky.
- For the same luminosity and a bright $\mu_\circ$ we get small h and the object will appear starlike.

We will quantify this below.

First we will consider the $V/V_{\rm max}$-test for completeness.

For this we need to know the selection criteria of the sample. These could be for example all objects down to a certain angular diameter (at some isophotal level) or integrated apparent magnitude.

Suppose that an object has a distance R. Now shift it in distance untill it drops out of the sample due to the completeness limit and call this distance $R_{\max}$.

Then we have V as the volume corresponding to R and $V_{\max}$ as the volume relating to $R_{\max}$.

Now, in case of a uniform space distribution each object has an uniform chance to be actually located throughout the volume $V_{\max}$.

In otherwords, the property $V/V_{\max}$ calculated for all objects in the sample should be distributed uniformly over the interval 0 to 1.

Note that $V/V_{\max}$ can usually be calculated without knowing the actual distance.

In practice the test is to calculate $\langle V/V_{\rm max}\rangle$. For a compete sample it is required that

$$\langle V/V_{\rm max}\rangle = 0.5.$$

The error in $\langle V/V_{\rm max}\rangle$ is $(12\ n)^{-1/2}$.

This is so, because all numbers between 0 and 1 have an average of 0.5 and a dispersion of $\sqrt{12}$.

Selection and Freeman's law

Mike Disney[2] suggested that Freeman's law is the result of sample selection (and not only of incompleteness).

In the process he also addressed the equivalent for elliptical galaxies, called Fish's law.

The analysis was later extended as in the following[3].

Assume luminosity-law (in linear units)

$$\sigma(R) = \sigma_\circ \exp\, -(R/h)^{1/b}$$

$b = 1$: exponential disk
$b = 4$: $R^{1/4}$ bulge or elliptical galaxy.

[2] M.Disney, Nature 263, 573 (1975)
[3] M. Disney & S. Phillipps, Mon.Not.R.A.S. 205, 1253 (1983); see also J.I. Davies, Mon.Not.R.A.S. 244, 8 (1990)

We then have for the integrated luminosity:

$$L_{\rm tot} = \int_0^\infty 2\pi R\sigma(R)dR = (2b)!\pi\sigma_\circ h^2$$

a. Diameter selection.

Suppose that a sample is complete for a radius larger than $\theta_{\rm lim}$ arcsec at an isophote of $\mu_{\rm lim}$ magnitudes arcsec^{-2}. For a radius R and a distance d the angular diameter is $\theta = R/d$ radians.

For clarity we now do the derivation only for an exponential disk.

The disk has an apparent radius

$$R_{\rm app} = h \ln\left(\frac{\sigma_\circ}{\sigma_{\rm lim}}\right)$$

In magnitudes arcsec^{-2} this is

$$R_{\rm app} = 0.4 \ln 10 \; h \left(\mu_{\rm lim} - \mu_\circ \right)$$

With $L = 2\pi\sigma_\circ h^2$ this becomes

$$R_{\rm app} = \frac{0.4 \ln 10}{\sqrt{2\pi}} \left(\frac{L}{\sigma_\circ} \right)^{-1/2} \left(\mu_{\rm lim} - \mu_\circ \right)$$

This can be rewritten as

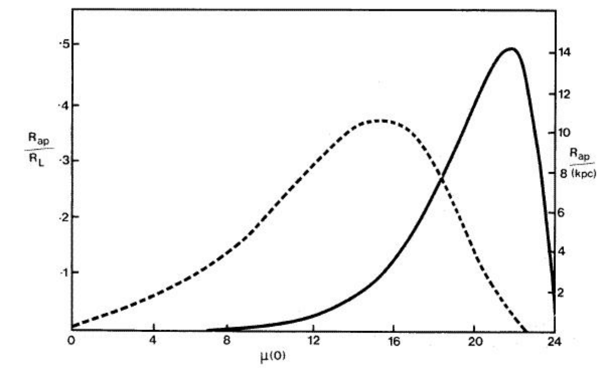
$$R_{\rm app} \sqrt{\frac{\pi\sigma_{\rm lim}}{L}} = \frac{0.4 \ln 10}{\sqrt{2}} 10^{-0.2(\mu_{\rm lim} - \mu_\circ)} \left(\mu_{\rm lim} - \mu_\circ \right)$$

The square-root term on the lefthand side is a kind of fiducial radius, that Disney and Phillipps write as $R_{\rm L}$.

The case with $\beta = 4$ for elliptical galaxies is

$$\frac{R_{\rm app}}{R_{\rm L}} = \frac{(0.4 \ln 10)^4}{\sqrt{8!}} 10^{-0.2(\mu_{\rm lim} - \mu_\circ)} (\mu_{\rm lim} - \mu_\circ)^4$$

In the following figure we see the behavior of $R_{\rm app}/R_{\rm L}$ as a function of the central surface brightness $\mu_\circ$ for the case of a diameter selection at an isophote of 24 (B-)magnitudes arcsec^{-2}.



The apparent diameter for exponential disks (full line) peaks at a central surface brightness of $(\mu_{\rm lim} - \mu_\circ) = 2.171$; for elliptical galaxies (dashed line) this occurs at $(\mu_{\rm lim} - \mu_\circ) = 8.686$.

Now when we express surface brightness μ in magnitudes arcsec^{-2} and distances (such as $\sqrt{\sigma/L}$) in parsec we can derive

$$\frac{L}{\sigma_{\rm lim}} = 10^{0.4(\mu_{\rm lim} - M + 5)}$$

Then for the maximum distance for a galaxy to remain in the sample d in parsec and angular radius limit $\theta_{\rm lim}$ in arcsec we get

$$d_{\rm size} = \frac{0.4 \ln 10}{\sqrt{2\pi}} \frac{\mu_{\rm lim} - \mu_{\circ}}{\theta_{\rm lim}} 10^{0.2(\mu_{\circ} - M + 5)}.$$

For the general case the result is

$$d_{\rm size} = \frac{(0.4 \ln 10)^b}{\sqrt{\pi (2b)!}} \frac{(\mu_{\rm lim} - \mu_{\circ})^b}{\theta_{\rm lim}} 10^{0.2(\mu_{\circ} - M + 5)}$$

The maximum of d occurs at

$$\mu_{\circ,\mathrm{max}} = \mu_{\mathrm{lim}} - \frac{b}{0.2 \ln 10}$$

b. Integrated magnitude selection

Now the sample is supposed complete up to a limiting integrated apparent magnitude m_{lim} within an isophote μ_{lim}.

Assume that the image is overexposed at isophote μ_{M} to allow for photographic surveys and define

$$s = 0.4 \ln 10(\mu_{\mathrm{M}} - \mu_{\circ}) \;\; ; \;\; p = 0.4 \ln 10(\mu_{\mathrm{lim}} - \mu_{\circ})$$

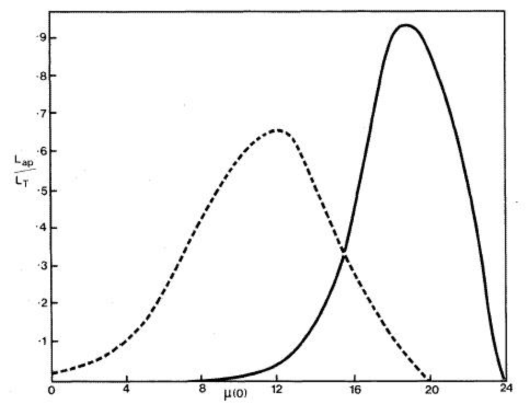
The maximum distance then comes out as

$$d_{\mathrm{magn}} = \left[A_{\mathrm{s}} \mathrm{e}^{-s} - A_{\mathrm{p}} \mathrm{e}^{-p}\right]^{1/2} 10^{0.2(m_{\mathrm{lim}} - M + 5)}$$

with

$$A_{\mathrm{s}} = \sum_{n=0}^{n=2b} \frac{s^n}{n!} \quad ; \quad A_{\mathrm{p}} = \sum_{n=0}^{n=2b-1} \frac{p^n}{n!}$$

The following figure below is for a limiting isophote of 24 magnitues arcsec^{-2} and a saturation isophote of 19 magnitudes arcsec^{-2}.



Again we see maxima as for diameter selection.

Note that both diameter and magnitude selection works in favor of disks around Freeman's surface brightness and elliptical systems near Fish's value.

Some actual values: For Palomar Sky Survey:

$\mu_{\rm lim} \approx 24$ B-mag arcsec^{-2}
$\mu_{\rm M} \approx 19$ B-mag arcsec^{-2}

Diameter selection: d^3 peaks at:
– 21.8 B-mag arcsec^{-2} for $b = 1$
– 15.3 B-mag arcsec^{-2} for $b = 4$

Magnitude selection: d^3 peaks at:
– 18.5 B-mag arcsec^{-2} for $b = 1$
– 12.0 B-mag arcsec^{-2} for $b = 4$

Observed:
$b = 1$: 21.6 ± 0.3 B-mag arcsec^{-2} (Freeman's law)
$b = 4$: 14.8 ± 0.9 B-mag arcsec^{-2} (Fish's law)

In any catalogue each galaxies has a value for d according to the selection criteria.

If both diameter and magnitude selection play a role the smalles of the two values is the appropriate one.

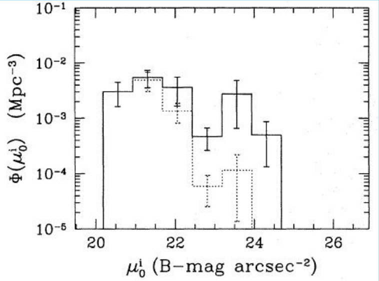
We can then define the visibility as the value for d^3 for each galaxy: in an unbiased sample and a uniform distribution a value of $\mu_\circ$ will occur at a frequency $\propto d^3$.

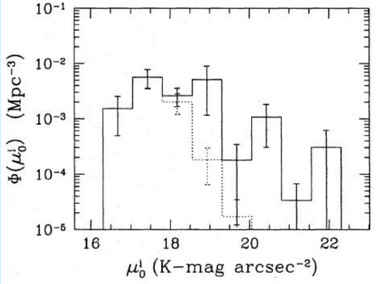
The equations for the visibility can of course also be used to correct complete sample for the volumes over which galaxies are sampled as a function of their properties in order te obtain space densities as a function of parameters.

This can be used to study the question of the origin of Freeman's law and whether it results from selection effects.

Allen & Shu[4] were the first to suggest that the selection only works at the faint level and that there is only a real upper limit to the central surface brightnesses.

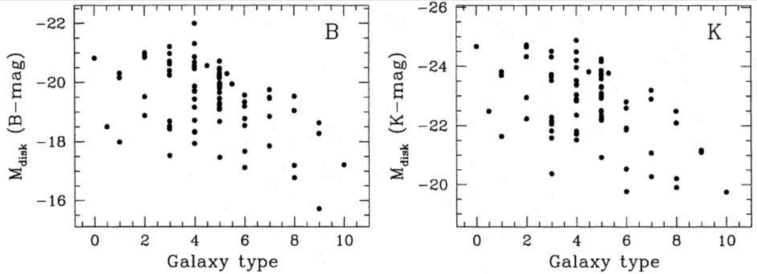
This is confirmed by Roelof de Jong[5], who also confirmed that the faint surface brightness disks are all of late type[6].



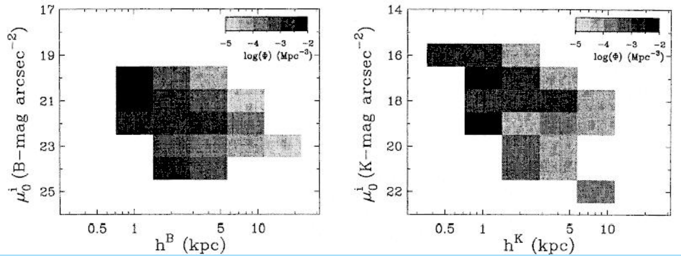


[4] R.J. Allen & F.H. Shu, Ap.J. 227, 67, (1979)
[5] R.S. de Jong, A.&A. 313, 45 (1996)
[6] P.C. van der Kruit, A.&A. 173, 59 (1987)

This is related to the fact that late type galaxies generally have fainter disks.



Data can be combined in bi-variate distribution functions.



From a weighing with the total luminosity it can be estimated that high surface brightness galaxies probably provide the majority of the luminosity density in the universe.

Photometric evolution

Fundamentals

The fundamental discussion is by Tinsley[7].

The Initial Mass Function (IMF) is the distribution over stellar masses during star formation.

It is determined in the solar neighborhood independently for low and high mass stars:

- Low masses ($M < 1M_\odot$) from general distribution of masses of older stars in the disk, since these are all still present.
- High masses ($M > 1M_\odot$) from distribution of stellar masses in actual clusters and associations.

[7] B.M. Tinsley, Fund. Cosmic Physics 5, 287 (1980)

Normalisation is done such that the two parts join smoothly at $\approx 1 M_{\odot}$ (continuity constraint).

An usefull analytic form of IMF:

$$\phi(M) = x M_{\mathrm{L}}^{x} M^{-(1+x)} dM$$

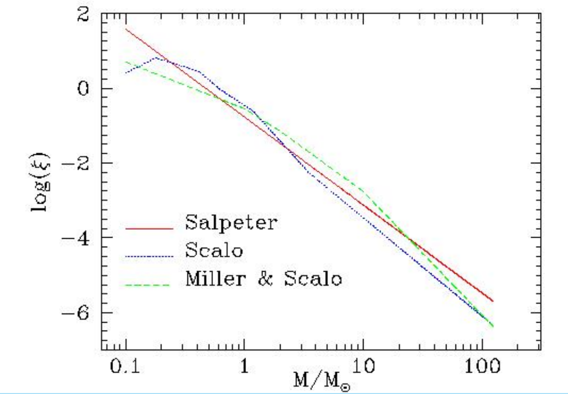
for

$$M_{\mathrm{L}} < M < M_{\mathrm{U}}$$

Usually $M_{\mathrm{L}} = 0.1 M_{\odot}$ and $M_{\mathrm{U}} = 50 M_{\odot}$.

The "Salpeter-function" has $x = 1.35$.

Here are some forms of the IMF often used.



Bi-model star formation was proposed by Larson[8]. It says that the two modes of star formation of high- and low-mass stars are independent and normalisation of the IMF must be done separately.

[8] R.B. Larson, Mon. Not. R.A.S. 218, 409 (1986)

The Star Formation Rate(SFR) is the total mass in newly formed stars as a function of time.

In the solar neighborhood it has been roughly constant with time.

It may vary between galaxies, but is usually taken independent of position in a galaxy.

With an IMF and a SFR it is possible to calculate the luminosity and colors of galaxies as a function of time.

This is done by first calculating the photometric evolution of a star clusters by assuming an IMF and using stellar evolution tracks.

In principle this needs to be done for different metal abundances.

These clusters can then be added according to the SFR (and the evolution of metal abundance with time).

Analytical models

Single burst.

First look at the Main Sequence; we have approximately:

$$L \propto M^{\alpha}$$

Rough values for α are 4.9 in U, 4.5 in B and 4.1 in V.

The main-sequence life-time is:

$$t_{\mathrm{MS}} = M^{-\gamma}$$

With M in $M_{\odot}$ the unit of time is $\approx 10^{10}$ years. A good value for γ is 3.

Assume that stars formed all at $t = 0$ and that the total mass is $\psi_\circ$. Then

$$\begin{aligned} L_{\rm MS}(t) &= \int_{M_{\rm L}}^{M_{\rm t}} \psi_\circ M^\alpha \phi(M) dM \\ &= \frac{x}{\alpha - x} M_{\rm L}^x \psi_\circ M_{\rm t}^{\alpha - x}, \end{aligned}$$

where

$$M_{\rm t} = t^{1/\gamma}$$

Now look at the giants. Assume all giants have a luminosity $L_{\rm G}$ and are in that stage for a time $t_{\rm G}$.

Reasonable values for $L_{\rm G}$ are 35 in U, 60 in B and 90 $L_\odot$ in V and 0.03 for $t_{\rm G}$.

The number of giants at time t is then

$$\begin{aligned} N_{\rm G}(t) &= \psi_\circ \phi(M_{\rm t}) \left| \frac{dM}{dt_{\rm MS}} \right|_{\rm M=M_t} t_{\rm G} \\ &= \psi_\circ \frac{x}{\gamma} M_{\rm L}^x M_{\rm t}^{\gamma - x} t_{\rm G} \end{aligned}$$

Now we can derive the Single Burst luminosity at time t:

$$L_{\rm SB}(t) = L_{\rm MS}(t) + N_{\rm G}(t) L_{\rm G}$$

Using $U_\odot = 5.40$, $B_\odot = 5.25$ and $V_\odot = 4.70$, and $M_{\rm L} = 0.1 M_\odot$, the following table can be calculated.

t	$(U-B)$	$(B-V)$	$(M/L)_{\mathrm{B}}$
0.01	-0.34	0.12	0.15
0.03	-0.06	0.45	0.38
0.1	0.18	0.64	1.12
0.3	0.38	0.79	2.79
1	0.56	0.90	6.95
3	0.66	0.96	14.9

Ongoing star formation.

Write the SFR as $\psi(t)$. Then

$$L(t) = \int_0^t \psi(t-t')L_{\mathrm{SB}}(t')dt'$$

For two extreme cases we get at $t = 1$:

Model	(U-B)	(B-V)	$(M/L)_{\rm B}$
Single burst	0.56	0.90	7.0
Constant SFR	-0.25	0.24	1.0

This spans the range of the observed two-color diagram with the single burst corresponding to elliptical and S0 galaxies and the constant SFR for Sc and later types.

Now let us look at some more detailed studies.

Detailed studies

Searle, Sargent & Bagnuolo[9] find the following luministies and colors for sinble bursts a number of slopes of the IMF.

TABLE 1

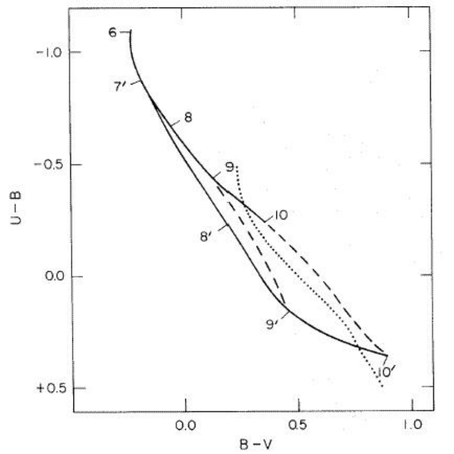
THE BRIGHTNESS AND COLORS OF MODEL STAR CLUSTERS AS A FUNCTION OF AGE

AGE	$\alpha = 2.1$			$\alpha = 2.45$			$\alpha = 3.2$		
(10^7 yrs)	M_V	$B-V$	$U-B$	M_V	$B-V$	$U-B$	M_V	$B-V$	$U-B$
0.1....	−7.2	−0.23	−1.18	−6.6	−0.22	−1.10	−5.6	−0.22	−1.10
0.3....	−7.6	−0.19	−0.96	−6.9	−0.21	−0.96	−5.9	−0.18	−0.96
1.0....	−7.4	−0.15	−0.83	−6.9	−0.18	−0.78	−6.0	−0.14	−0.78
3.0....	−6.2	−0.05	−0.60	−6.0	−0.03	−0.58	−5.6	−0.05	−0.58
10.0....	−5.0	+0.19	−0.22	−5.0	+0.19	−0.23	−5.0	+0.11	−0.23
30.0....	−3.8	+0.21	+0.03	−3.9	+0.34	0.00	−4.4	+0.27	+0.06
100.0....	−2.5	+0.44	+0.12	−2.8	+0.46	+0.16	−3.6	+0.47	+0.22
300.0....	−1.6	+0.66	+0.26	−1.9	+0.67	+0.28	−2.9	+0.68	+0.29
1000.0....	−0.9	+0.89	+0.38	−1.2	+0.90	+0.36	−2.3	+0.89	+0.36

Using this they get a predicted two-color diagram with the Salpeter IMF as in the following figure.

[9] L. Searle, W.L.W. Sargent & W.G. Bagnuolo, Ap.J. 179,427 (1973)

Here numbers x show the location of models of ages 10^x years old; with primes for SB models and unprimed for constant SF. All normal galaxies lie to the right of the dotted line.



Searle *et al.* conclude that the models and observations are consistent with:

- All galaxies $\approx 10^{10}$ years old.
- IMF everywhere similar to local IMF.
- Mean SFR averaged over sufficiently large area's and long times generally declines with time.
- Decay times vary among late-type galaxies; some show bursts, some show uniform SFR.

Larson & Tinsley[10] add:

- Precise form of SFR is not important. Important is only SFR over the last $\approx 10^8$ years to mean SFR over the life of the galaxy.
- Effects of different ages, metallicities and upper stellar masses are small.
- Interacting galaxies show more scatter in two-color diagram. This can be explained with bursts of 5% (fraction of mass to total stellar mass at time of burst; $b \sim 0.05$) and duration $\tau \approx 2 \times 10^7$ years.

[10] R.B. Larson & B.M. Tinsley, Ap.J. 219, 46 (1978)

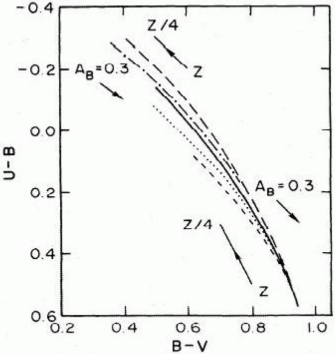
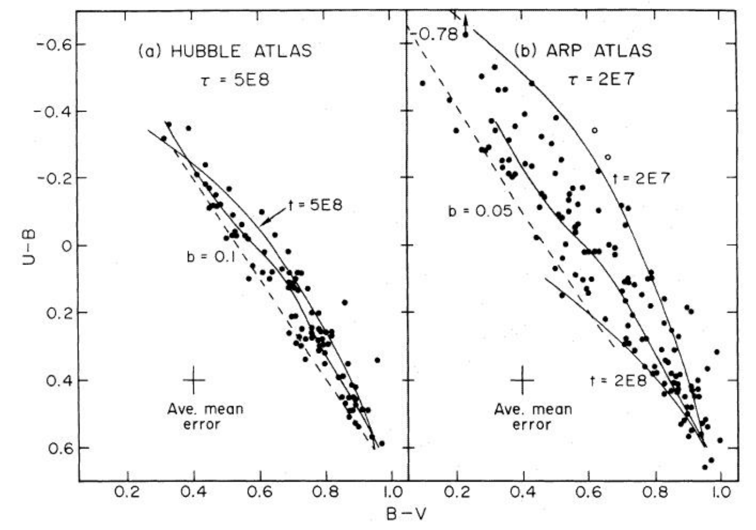


FIG. 7.—Colors of models with monotonic SFRs and age 10^{10} yr. *Heavy line*, local IMF. *Long dashes*, IMF with slope $x = 1$. *Short dashes*, $x = 2$. The foregoing use case T supergiant colors and have an upper mass limit $m_U = 30\ M_\odot$. *Dot-dashes*, $x = 1$, $m_U = 30\ M_\odot$, and case C supergiant colors. *Dots*, $x = 1$, case T, and $m_U = 10\ M_\odot$. The reddening vectors for $A_B = 0.3$ show the RC2 formula for galactic reddening which depends on $B - V$. The other vectors indicate schematically how colors of red and blue galaxies, respectively, may change with a factor 4 reduction in metallicity.


(a) HUBBLE ATLAS
τ = 5E8
t = 5E8
b = 0.1
Ave. mean error
(b) ARP ATLAS
τ = 2E7
-0.78
t = 2E7
b = 0.05
t = 2E8
Ave. mean error
U−B
B−V
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.2
0.4
0.6
0.8
1.0

Rob Kennicutt[11] adds the integrated H_α fluxes (in the form of an equivalent width, providing independent information on recent formation of heavy stars.

Equivalent width is the wavelength interval in the continuum that corresponds to as much flux as the line.

His most important results are the following slides:

[11] R.C. Kennicutt, Ap.J. 272, 54 (1983)

- The slope of upper IMF is roughly that of the Salpeter function.

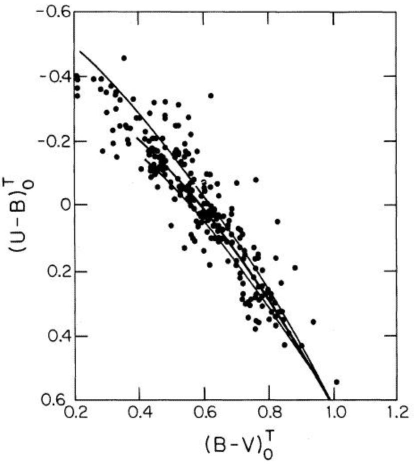


FIG. 3.—Two-color diagram from Shapley-Ames spiral galaxies, along with the model galaxy disk colors described in the text. The three curves correspond to the different mass functions adopted, the Miller and Scalo function (*lowest curve*), the extended Miller-Scalo (i.e., "Salpeter") function, and the shallow m^{-2} IMF (*top curve*).

- Galaxies have same upper mass limit in the IMF ($\approx 50 M_\odot$).

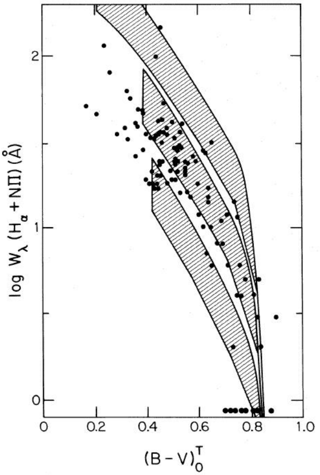


FIG. 4.—Observed emission line equivalent widths and corrected RC2 colors for observed galaxies, along with the evolutionary models. The effect of dust has been shown by plotting each model as an area, as described in the text. The IMFs corresponding to each model are the same as in Fig. 3.

Roelof de Jong[12] derives models to study the color gradients in disks and among different disks.

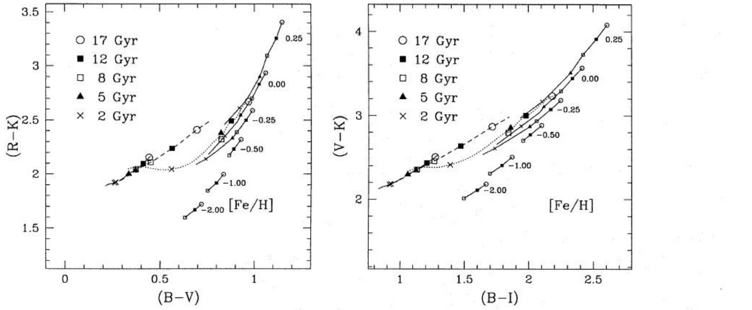


Fig. 6. Evolutionary color–color plots of stellar synthesis models. The symbols indicate the number of years after creation of this population. To the right in each panel, the different ages connected by solid lines, are the single burst models of Worthey (1994) for different metallicities. The corresponding [Fe/H] values are indicated next to them. To the left in each panel are the solar metallicity models of Bruzual & Charlot (1996). The dotted line indicates the single burst evolution. The dashed line is a model with an exponentially declining star formation rate. The leftmost dot-dashed line, overlapping the blue part of the exponentially declining SFR model, indicates a model with constant star formation. Bruzual & Charlot used the Johnson R and I passbands which were here converted to Kron-Cousins R and I passbands using the equations of Bessell (1979).

[12] R.S. de Joing, A.&A. 313, 377 (1996)

His conclusions are:

- Dust reddening plays a minor role.
- Outer parts have lower average ages and are more metal poor than inner parts of disks.
- Late type galaxies ($T \geq 6$)[13] have lower metallicites and younger average ages.

[13] Following de Vaucouleurs himself the de Vaucouleurs types are given numerical values, e.g. T=1 → Sa, T=3 → Sb, etc. So here is meant types later than Sc.

Schmidt's law for star formation

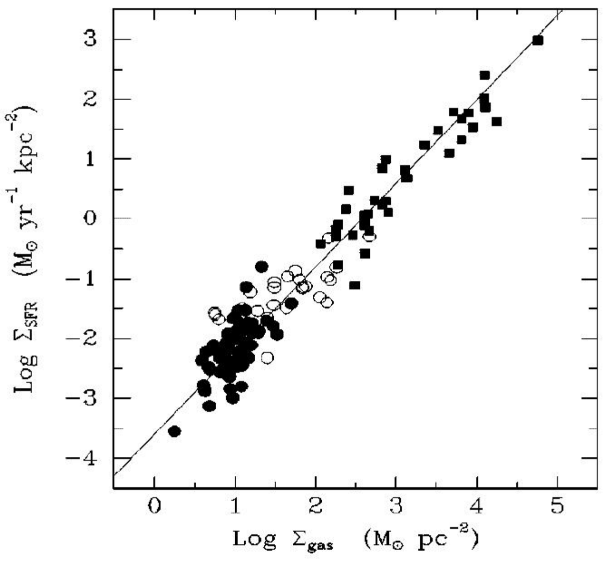
Maarten Schmidt[14] proposed that the star formation rate relates to the gas density as

$$\mathrm{SFR} \propto \rho^2$$

Often this was immediately translated in (observable) surface properties.
The latest result[15] gives

$$\Sigma_{\mathrm{SFR}} = (2.5 \pm 0.7) \times 1^{-4} \left(\frac{\Sigma_{\mathrm{gas}}}{1 M_\odot \mathrm{pc}^{-2}} \right)^{1.4 \pm 0.15} \quad M_\odot \mathrm{year}^{-1} \mathrm{kpc}^{-2}$$

[14] M. Schmidt, Ap.J. 129, 243 (1959)
[15] R.C. Kennicutt, Ann.Rev.Astron.Astrophys. 36, 189 (1998)


Log Σ_{SFR} ($M_\odot$ yr^{-1} kpc^{-2})
Log Σ_{gas} ($M_\odot$ pc^{-2})
3
2
1
0
−1
−2
−3
−4
0
1
2
3
4
5

Population synthesis

Her one attempts to fit the intermediate resolution spectra with those of observed stars.

Best method now is by fitting integrated spectra of generations of particular age and metallicity[16].

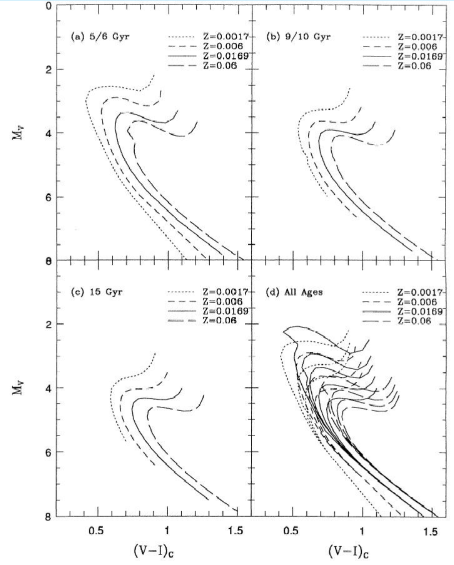
The steps are the following.:

- Measure spectra of stars of various ages and metallicity.
- Synthesize integrated spectra of generations from a set of isochrones.
- Fit using least-squares techniques to galaxy spectra.

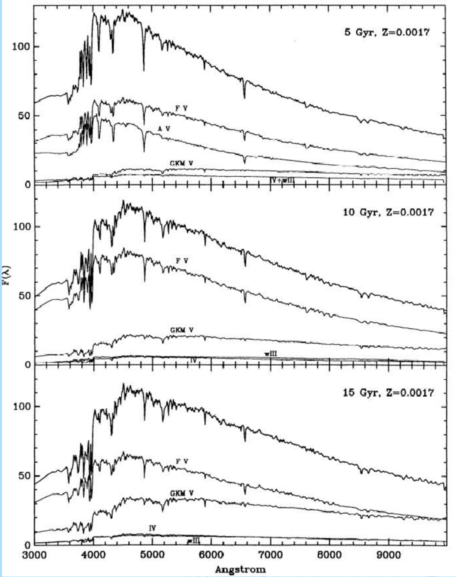
---

[16] For example A.J. Pickles, Ap.J. 296, 340 (1985); Ap.J.Suppl. 59, 33 (1985) or A.J. Pickles & P.C. van der Kruit A.&A.Suppl, 84, 421 (1990) and 91, 1 (1991)

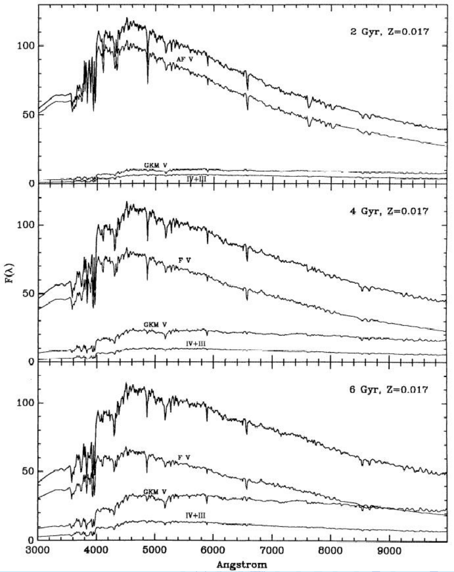
Here is a set of isochrones used by Pickles & van der Kruit.



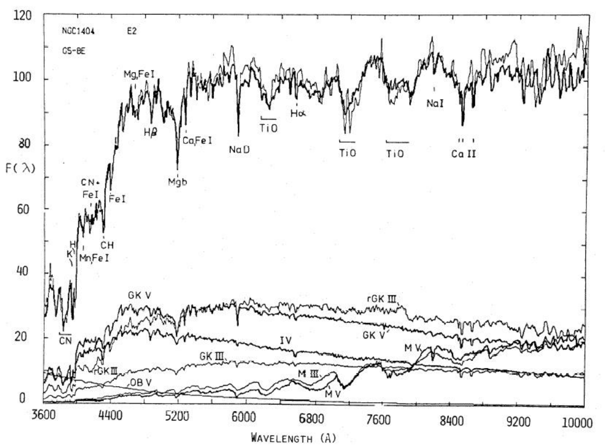
These are synthesized spectra of a metal poor cluster at three ages.



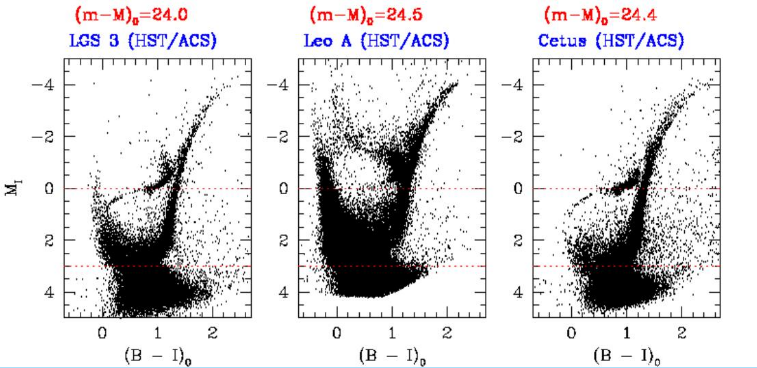
These are synthesized spectra of a metal rich cluster at three ages.



This a an example of a spectrum of an elliptical galaxy fitted by a set of stellar spectra.



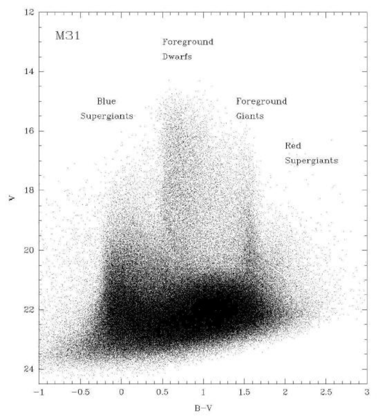
It is now possible to directly observe colour-magnitude diagrams in dwarfs galaxies in the Local Group[17].

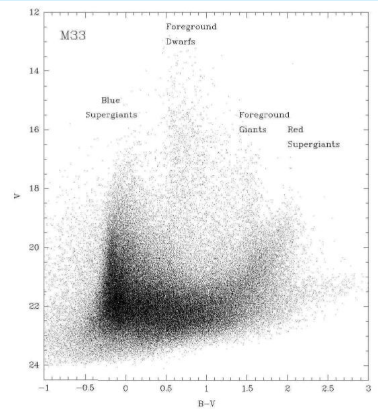


---

[17] See E. Tolstoy, V. Hill & M. Tosi, Ann.Rev.A.&A. 47, 371 (2009) for a review.

Even in M31 and M33 it has been possible now[18].

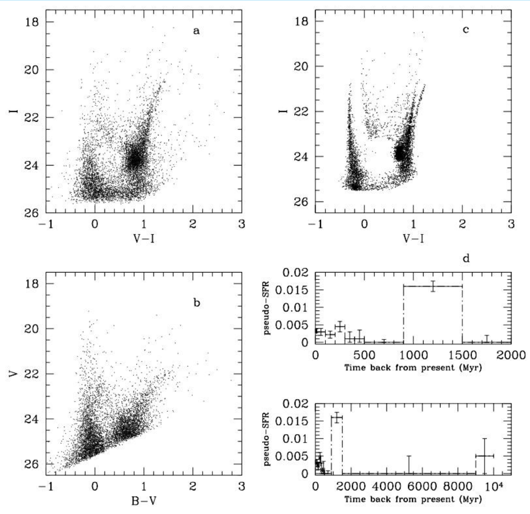




[18] P. Massay et al., A.J. 131, 2486 (2006)

One can use such data to derive star formation histories (SFH) and for studies of abundance distributions.

Here is the SFH for Leo I based on HST data[a] (left data, right convolved model and SFH).



[a]E. Tolstoy et al., A.J., 116, 1244 (1998)