Chapter 12

THE DISTRIBUTION OF PROPERTIES OF GALAXIES

12.1 Introduction

In this set of lectures the topic will be the distribution and statistics of various parameters that can be measured for spiral galaxies. The aim of this will be two-fold. In the first place this should lead to a better understanding of the origin of structure in the early universe and the subsequent formation and evolution of galaxies. Secondly, such information will enable us to put our Galaxy in context with other systems and through local studies we might then be able to better understand both its structure and that of spiral galaxies in general. The emphasis will be on properties that can be derived from observations as described in the two previous sets of lectures, namely photometry and kinematical studies. I will discuss in general only integral or global properties. A detailed discussion of Local Group galaxies will be given elsewhere in this course.

The classical publication in this area is Holmberg (1958). That comprehensive paper is based on a photometric study of 300 extragalactic nebulae and is the culmination of an investigation that took about ten years, starting with photographic studies of galaxies in the late forties. The basic data, painstakingly collected, consisted of a homogeneous set of integrated magnitudes, colors and diameters (and hence surface brightnesses) of the systems. Holmberg in particular discussed the dependence of color on the morphological type, which was found to become systematically bluer from elliptical towards irregular systems. Also, he was the first to investigate from a subsample in detail the statistical effects of internal dust absorption on the photometry and his results have guided those of more recent workers to a major extent. De Vaucouleurs (1959) also gave a review of integral properties at about the same time and his discussion in addition included the then available scarce information on mass distributions.

12.2 The distribution of disk surface brightness and scalelength

In a classical study, Freeman (1970) used published surface photometry of disk galaxies to demonstrate an unexpected effect. For the majority of disks it turned out that the extrapolated, face-on, central surface brightness μ_o fell in the surprisingly small range of 21.67±0.30 B-mag arcsec^{-2}. Exceptions were some S0's on the bright side and dwarf galaxies on the faint side. Immediately it was suspected that this constituted a selection effect, such as the one pointed out earlier by e.g. Arp (1965). The effect is in short terms that for galaxies to be selected as suitable candidates for surface photometry their central surface brightness had to fall within a limited range. Fainter surface brightnesses generally result in smaller angular diameters of the observable extent and such galaxies are difficult to see in the first place against the sky background. Brighter central surface brightnesses occur in systems with small luminosity scalelengths and such galaxies appear starlike.

The problem with Freeman's result is that its significance could not be evaluated well as a result of the fact that the sample is not a statistically complete one and indeed such a sample did not exist at that time. If a sample is complete with respect to known and well-defined selection criteria then it is in principle possible to correct for these effects. Before discussing that I will first see how a sample can be judged to be complete. There are various methods for this available, but the most useful one is the so-called $V/V_{\rm m}$-test, that was used first for studies of the distribution and evolution of quasars by Schmidt (1968). In this method one calculates for each object two volumes. The first volume V is the one that corresponds to a sphere with radius the distance to the object. Then the object is shifted to a larger distance until it drops out of the sample as a result of the selection criteria. Note that these can be any number (such as in Schmidt's study radio and optical brightness), but one should of course use the smallest distance at which it drops out of the sample. This distance is used to calculate the corresponding volume $V_{\rm m}$. For the sample one then calculates the mean of $V/V_{\rm m}$ of all the objects. For a uniform distribution in space (this is a fundamental assumption, that should always be kept in mind) this mean value $\langle V/V_{\rm m}\rangle$ should be equal to 0.5. If the objects are non-uniformly distributed, the calculation of the volumes proceeds as a weighted integral of the radius squared.

The test was originally designed to investigate space distributions of objects and in the case of quasars the cosmological evolution from samples that are presumed complete. The use of the test for statistical completeness of a sample is the reverse application. It is interesting to note that for a uniform distribution in space, one could be more demanding then requiring $\langle V/V_{\rm m}\rangle$ to be 0.5, since spacial uniformity of the objects predicts that the distribution of $V/V_{\rm m}$ is also uniform between 0 and 1. This property is seldom used in detail, usually because the sample is still too small. The uncertainty in $\langle V/V_{\rm m}\rangle$ can be calculated as follows. For a uniform distribution between 0 and 1, the r.m.s. value is $(12)^{-1/2}$ and for a large number of objects n, the probability distribution of $\langle V/V_{\rm m}\rangle$ becomes a Gaussian with a standard deviation equal to $n^{-1/2}$ times that of the uniform distribution. The uncertainty in $\langle V/V_{\rm m}\rangle$ then is $(12n)^{-1/2}$.

Statistically complete samples of galaxies can be drawn from catalogues such as the Uppsala General Catalogue (UGC; Nilson, 1973) in the north or the ESO/Uppsala Survey (EUC; Lauberts, 1982) in the south. These catalogues have been selected from respectively the Palomar Sky Survey prints and the ESO southern "quick blue survey" in the south and are designed to be complete to a diameter of 1 arcmin at a uniform isophote. Since these selections have been done by eye, the precise surface brightness of this isophote needs to be calibrated separately by using calibrated photometry of galaxies. For the UGC for example van der Kruit (1987) found that the isophote at which Nilson did the selection (and for which he gives diameters) is at 25.9 B-mag arcsec^{-2}. Application of the $V/V_{\rm m}$ test on these catalogues show that these are not entirely complete to diameters of 1 arcmin, but that complete samples down to a limiting diameter of 2 arcmin can be drawn. For this diameter the values of $\langle V/V_{\rm m}\rangle$ are 0.497 for the UGC and 0.485 for the EUC.

Disney (1976) was the first to investigate the effects of sample selection in a quantitative manner to see whether Freeman's result could follow from observational bias. He concluded that this could indeed be the case and for realistic values of the sky background he could even produce Freeman's number. Furthermore, his analysis also predicted the equivalent for elliptical galaxies ("Fish's law") and the difference of about 6 magnitudes between the mean central surface brightness for ellipticals with an $R^{1/4}$ luminosity distribution and exponential disks. This concept was extended in more detail by Disney and Phillipps (1983) and I will discuss this now in some detail, also because it can be used to extract information on distribution functions from complete samples.

Suppose that a galaxy has a luminosity law

$$L(R) = L(0)\ \exp\ -(R/h)^{1/\rm b}, \tag{12.1}$$

where $b = 1$ corresponds to the exponential disk and $b = 4$ to the $R^{1/4}$-law. The integrated luminosity then becomes

$$L_{\rm tot} = (2b)\,!\ \pi L(0) h^2. \tag{12.2}$$

If galaxies are selected to have a diameter at the isophote $\mu_{\rm lim}$ larger than $r_{\rm lim}$ on the sky, it is possible for each galaxy to calculate the maximum distance d out to which it will be included in the sample as

$$d = \frac{(0.4\ \ln 10)^{\mathrm{b}}}{\{\pi(2b)\ !\}^{1/2}} \frac{(\mu_{\mathrm{lim}} - \mu_{\mathrm{o}})^{\mathrm{b}}}{r_{\mathrm{lim}}}\ \mathrm{dex}\{0.2(\mu_{\mathrm{o}} - M + 5)\}. \tag{12.3}$$

Here again μ and M are surface brightness and luminosity expressed in magnitudes. This equation is evaluated for an face-on system, so that for an inclined disk for example μ_{o} is the observed central surface brightness (uncorrected for inclination). This distance d has a maximum as a function of μ_{o} that occurs for each value of r_{lim} or M at

$$\mu_{\mathrm{o}} = \mu_{\mathrm{lim}} - \frac{b}{0.2\ \ln 10}. \tag{12.4}$$

Similarly, Disney and Phillipps have derived an equation for the case that the selection is based on integrated magnitude m_{lim} within the limiting isophote μ_{lim}. In practice we may have to do here with a maximum observed surface brightness μ_{M}, for example in case of overexposure on photographic plates. Writing $s = 0.4\ \ln 10\ (\mu_{\mathrm{M}} - \mu_{\mathrm{o}})$ and $p = 0.4\ \ln 10\ (\mu_{\mathrm{lim}} - \mu_{\mathrm{o}})$ the limiting distance d becomes

$$d = \left[\left\{\sum_{n=0}^{n=2b} \frac{s^{\mathrm{n}}}{n!}\right\} \exp\,(-s) - \left\{\sum_{n=0}^{n=2b-1} \frac{p^{\mathrm{n}}}{n!}\right\} \exp\,(-p)\right]^{1/2} \mathrm{dex}\{0.2(m_{\mathrm{lim}} - M + 5)\}. \tag{12.5}$$

Again d has a maximum as a function of μ_{o}, that now also depends on μ_{M}. The peaks are now however much broader than in the previous case.

So for each galaxy we can calculate a volume $(4/3)\pi d^3$ within which it enters the catalogue or correspondingly a visibility d^3. In an unbiased sample and assuming uniform space densities the number of galaxies with a particular value of μ_{o} will be present proportional to this visibility. Disney's original argument then was the following. Samples such as were studied by Freeman have been selected according to angular diameter and possibly integrated magnitude. If we select galaxies exceeding a certain diameter limit at an isophote μ_{lim} of about 24 B-mag arcsec^{-2} (which is a likely value in practice), it them follows that d^3 has a peak at 21.8 and 15.3 B-mag arcsec^{-2} for $b = 1$ and $b = 4$ respectively. If the selection is done on integrated apparent magnitude a similar result obtains; if $\mu_{\mathrm{M}} = 19$ B-mag arcsec^{-2} the peak occurs at 18.5 and 12 B-mag arcsec^{-2} for $b = 1$ and $b = 4$ respectively. The values for the diameter selection now are very similar to those found by Freeman (21.6±0.3) and Fish (14.8±0.9) and consequently Disney has argued that this observational bias has actually produced these "laws".

The difference between the two values is also predictable in the following sense. An elliptical galaxy with an $R^{1/4}$ luminosity law and a disk with an exponential profile with both the same integrated luminosity and effective radius (which encloses half the total light) will have central surface brightnesses that differ by about 6 mag. This is only a result of the different central light concentrations of the two luminosity laws. So what these two empirical laws may be saying is that the majority of galaxies have the same length scale as a function of luminosity regardless of whether they are elliptical or disk.

Before discussing "Freeman's law" further two points should be made: (1) It has been known from the start that dwarfs with fainter values of $\mu_{\rm o}$ occur and their existence is therefore not an issue (what is an issue is whether or not these occur in such numbers so as to make a major contribution to the cosmic luminosity density). The early ones known were Local Group members recognized as conglomerations of faint stars and therefore selected in a completely independent way. (2) A number of studies have followed Freeman's, finding essentially the same result (although the dispersion tended to be larger). However, all of these were again based on samples that were not complete to well-defined selection criteria and consequently the resulting distributions could not be corrected for observational bias. The very extensive study by Grosbøl (1985) is also not complete in any statistical sense, because the selection was done on the basis of inclusion in the Reference Catalogue.

An attempt to provide information based on statistically complete samples was reported in van der Kruit (1987). Here galaxies were selected from background fields on deep IIIa-J Schmidt plates (providing J-magnitudes that are close to B) and after scanning the selection was made quantitative as follows: all disk galaxies (with some inclination and morphological type restrictions) which have a major axis diameter at the isophote of 26.5 J-mag arcsec^{-2} in excess of 2 arcmin were included. As it turned out —in agreement with the remarks above— this consisted of precisely all galaxies that would be selected with a similar angular size limit in the UGC. This dataset, consisting of 51 galaxies, confirmed Freeman's result for non-dwarf galaxies (Sc or earlier), namely $\mu_{\rm o} = 21.52 \pm 0.39$ mag arcsec^{-2} (roughly B-band). Dwarfs, which are of morphological type later than Sc and which turn out to be small in physical size as well when redshifts were available, are fainter and have $\mu_{\rm o} = 22.61 \pm 0.47$ mag arcsec^{-2}. Furthermore, selection as discussed above with the values for the limiting surface brightness $\mu_{\rm lim}$ appropriate to the present sample predicted that a peak should have occurred at a considerably fainter surface brightness than estimated above, since these deep plates provided a different value for $\mu_{\rm lim}$. So non-dwarf galaxies do have a relatively narrow distribution of $\mu_{\rm o}$ and this is not the result of selection effects. It is interesting to note that in terms of space densities dwarfs dominate by a large factor, but the dwarfs provide only about one-quarter of the cosmic luminosity density.

A major question is of course what happens at different wavelengths where absorption effects and contributions from young populations are entirely different. S. Westerhof and van der Kruit (unpublished) have repeated the analysis just described on IIIa-F plates of the same fields, as far as these existed. This gives a photometric band that is between standard V and R, such that $(J-F) = 1.25\ (B-V)$, where the J-band used above is very

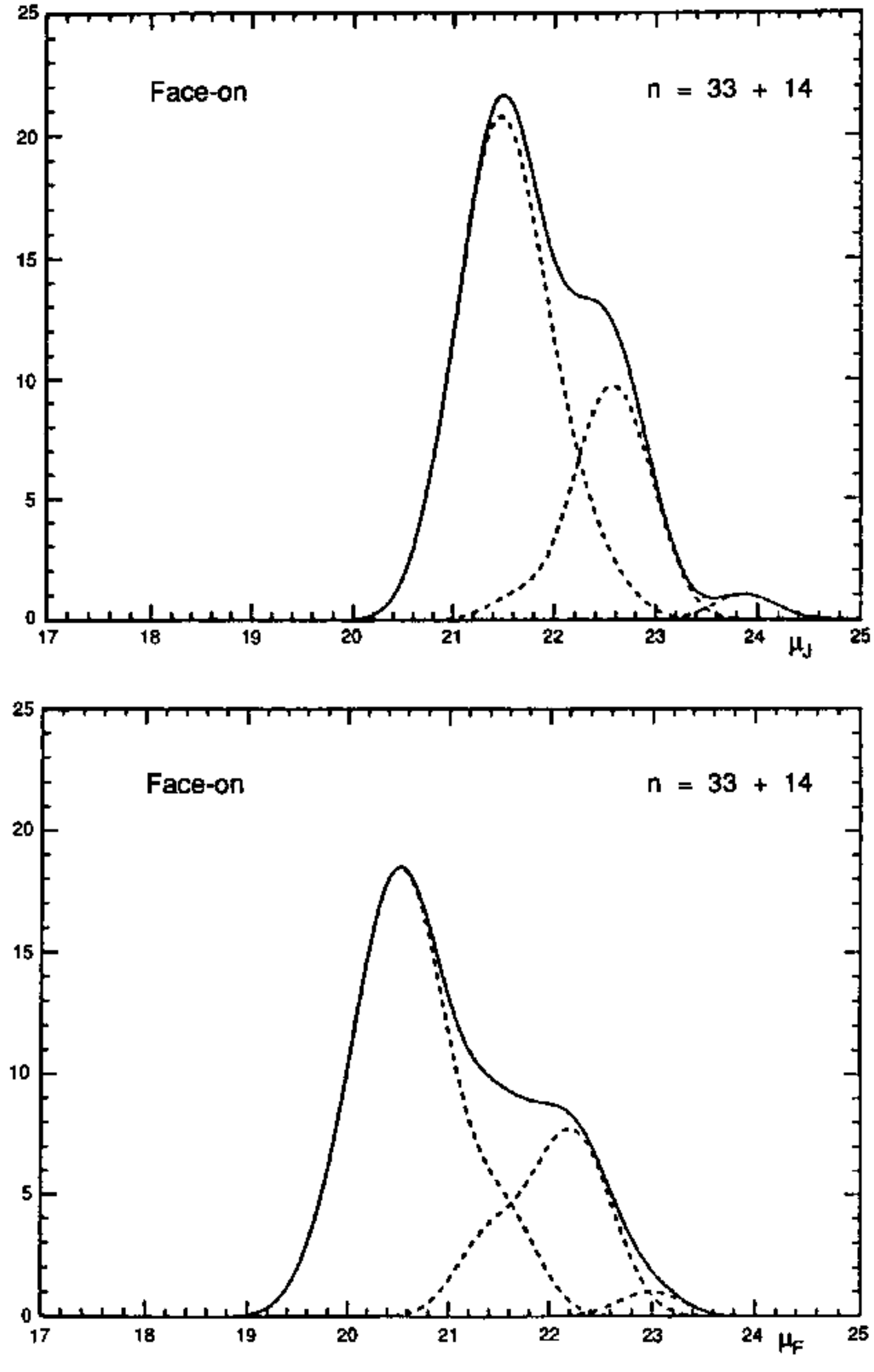

Face-on
n = 33 + 14
0
5
10
15
20
25
17
18
19
20
21
22
23
24
25
μJ
Face-on
n = 33 + 14
0
5
10
15
20
25
17
18
19
20
21
22
23
24
25
μF

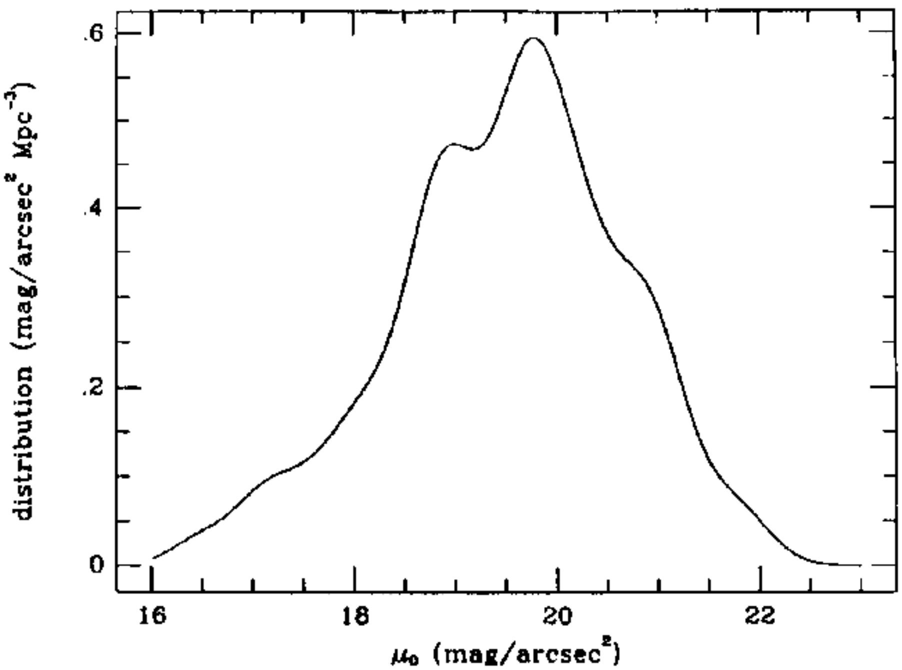
Figure 12.1: Generalized histograms of the face-on, extrapolated central surface brightness of a complete sample of disk galaxies (van der Kruit, 1987). The distributions have been corrected for sample selection effects. In both panels the brighter of the two distributions, indicated with dashed lines contains the systems with morphological type Sc or earlier ("non-dwarfs") and the fainter the remaining galaxies ("dwarfs"). The single faintest system is NGC 4392, which should probably be deleted from the sample. The top panel shows data from IIIa-J plates and correspond roughly to the B-band, while the lower panel comes from IIIa-F plates and is between the V- and R-band.

close to B. The face-on distributions of the central surface brightnesses of the systems in common is given in fig. 12.1, after correction for sample selection. This means that each galaxy has been weighed by the inverse of the volume that it is sampling calculated from the equations above. The means and the dispersions are as follows: for 33 non-dwarfs $J = 21.54 \pm 0.39$, $F = 20.63 \pm 0.49$; for 14 dwarfs $J = 22.52 \pm 0.32$, $F = 21.99 \pm 0.44$. The ratio of the J- and F-scalelengths is 1.07±0.13, so there is no significant change in h with wavelength and thus again no systematic evidence for radial color gradients in disks. The distributions just given are marginally narrower in J, but the effect is not significant. If absorption by dust and contributions of young populations are significant one would actually have expected the F-distributions to be narrower. In agreement with the bluer colors of the dwarfs the means separate indeed by going from J to F.

Elmegreen and Elmegreen (1984) have performed photographic surface photometry of a sample of 34 spirals both in B and in the I-band (0.83 μ). They do not list central surface brightnesses, but they do note that the B and I scalelengths are similar within each galaxy. The scatter is however large; from their published data the ratio of B- to I-scalelengths is 1.16±0.47. Leaving out the four most deviating galaxies the ratio still is 1.08±0.29. Although there is no systematic trend, the two scalelengths can sometimes be very different in one galaxy. I will comment on observations in the near-infrared below.

In the same way one can determine the distribution of scalelengths, again correcting for the known selection effects. These scalelengths exist from smaller than 1 kpc up to 7 kpc and the frequency distribution is roughly speaking exponential with an e-folding of about 1 kpc. This shows the overwhelming number density of dwarf systems (in a physical sense) and the rareness in space of large spirals like our own. On the other hand, it also follows from the data that still about 10% of all disk stars in the universe occur in disks with scalelengths larger than 4 kpc and this is not an improbably low number.

It is also possible to calculate the bi-variate distribution function of $\mu_{\rm o}$ and h from such samples, although one now does require large numbers. This was done in a preliminary way for the sample above in van der Kruit (1987), but has been improved significantly by R. de Jong and van der Kruit (unpublished) on a larger sample. This sample was drawn


distribution (mag/arcsec² Mpc⁻³)
μ₀ (mag/arcsec²)
.6
.4
.2
0
16
18
20
22

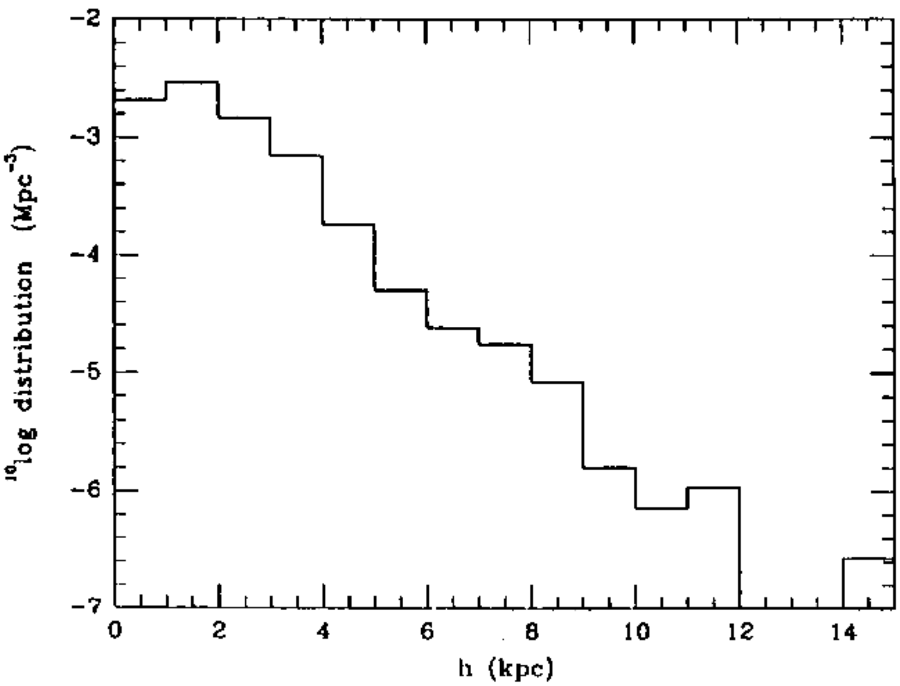
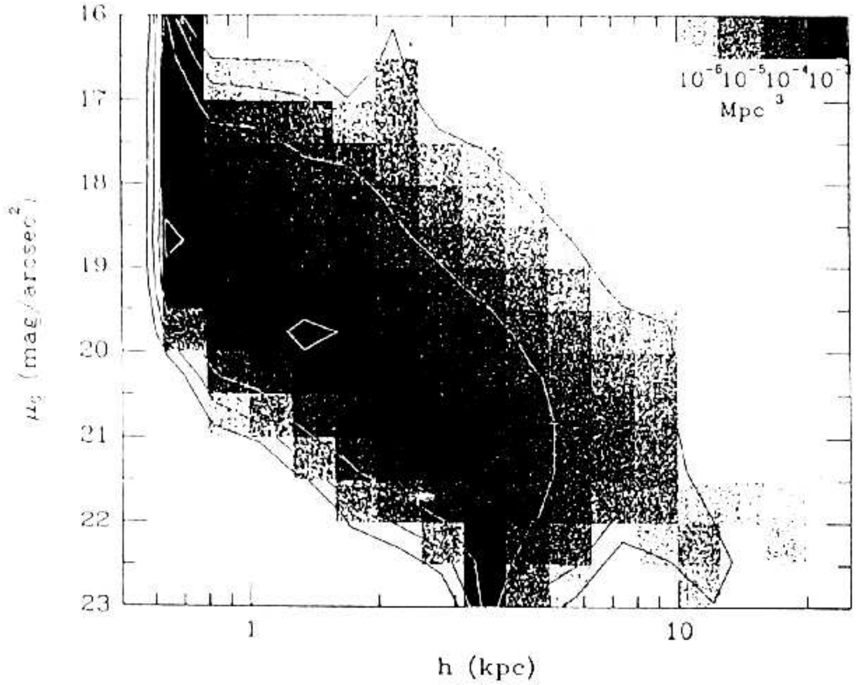

¹⁰log distribution (Mpc⁻³)
h (kpc)
−2
−3
−4
−5
−6
−7
0
2
4
6
8
10
12
14

Figure 12.2: Results for the distribution of central face-on surface brightness and radial scalelength from a statistically complete sample drawn from the study of Grosbøl (1985), consisting of almost 300 galaxies. Only galaxies with types Sc or earlier have been included. (a) The distribution of the central surface brightness after correction for the sample selection effects. (b) The distribution of radial scalelength for the same data. From de Jong and van der Kruit (unpublished).

from the study of Grosbøl (1985), that was based on galaxies in the Second Reference Catalogue and therefore not complete. However, since diameters and integrated magnitudes are known from Grosbøl's photometry we can use the $V/V_{\rm m}$ test to extract a sample that is complete. The photometry was performed on glass copies of the red Palomar Sky Survey and magnitudes are approximately R. The largest sample that could be extracted had a limiting diameter of 70 arcsec at the 23.5 R-mag arcsec^{-2} isophote and a limiting integrated magnitude of 11.6 and constituted 299 galaxies. Only 14 galaxies of type Scd or later occurred and therefore the main study concentrated on the sample without these (this means non-dwarfs in the terminology above).

The distribution of central face-on surface brightness is now broader then for the sample above, namely 20.06$\pm$1.19 R-mag arcsec^{-2} after correction for selection effects using the prescripts given above (fig. 12.2a). So, each contribution from a galaxy has been weighed according to the volume it samples. One may suspect that the zero-point calibration of the photometry has an effect here, since Grosbøl use a single calibration curve from density to brightness (in mag arcsec^{-2}, including zero-point) for all plates, although his 14 calibrating plates gave consistent results. Grosbøl quotes an uncertainty of about 0.3 mag in $\mu_{\rm o}$. Comparison of his scalelengths with literature values show a scatter of about 20%. Since this slope is generally measured at a mean radius of 2–3 scalelengths, this introduces a further error of about 0.5 mag in $\mu_{\rm o}$. The effects together reduce the scatter in $\mu_{\rm o}$ considerably. The distribution of scalelengths again declines exponentially with an e-folding again of about 1 kpc (fig. 12.2b). Remember that no (morphological) dwarfs are included in the sample.

It is also possible to calculate the bivariate distribution function of $\mu_{\rm o}$ and h with these data. It is shown in fig. 12.3a, again after weighing each galaxy with the volume it samples. One can see a decrease in the density towards longer scalelengths for fainter central surface brightness; lines of equal density in the figure roughly follow those of constant integrated disk luminosity (or mass, if M/L is constant). This is remarkable, but it should be pointed out that there may be effects in the analysis of the luminosity profiles that could produce this in principle. Usually the fit is done in the outer parts, where the plates are not overexposed and the bulge contribution is probably small. Now if h is underestimated, one is overestimating the surface brightness (underestimating $\mu_{\rm o}$,


10^-6 10^-5 10^-4 10^-3
Mpc^-3
μ0 (mag/arcsec²)
h (kpc)

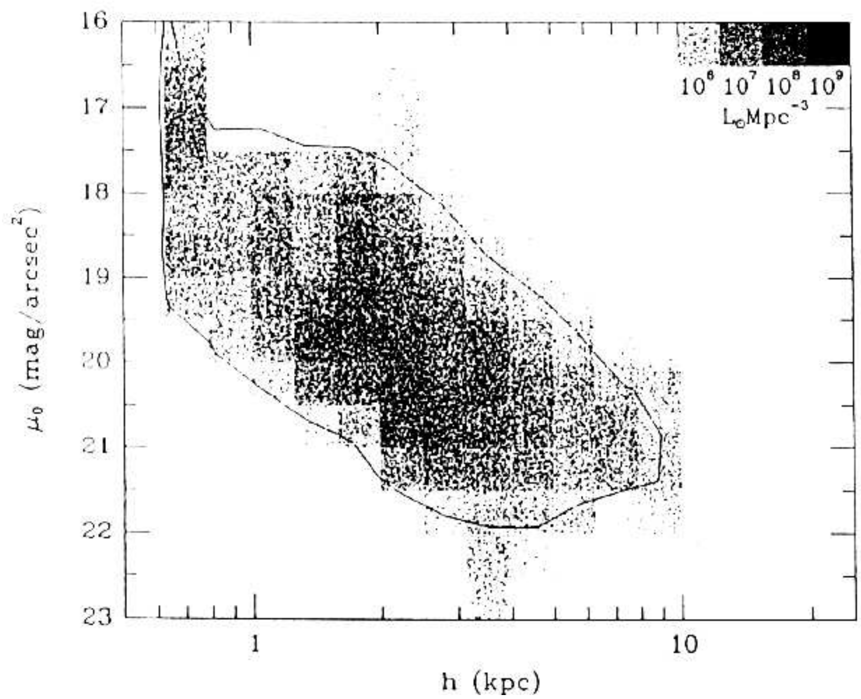

10^6 10^7 10^8 10^9
L⊙Mpc^-3
μ0 (mag/arcsec²)
h (kpc)

Figure 12.3: The bivariate distribution function of central face-on surface brightness and radial scalelength in spiral galaxies of type Sc or earlier. Each galaxy has been weighed again by the volume that it samples. (a) The number density distribution given at the top in Mpc^{-3}. (b) Each galaxy has been weighed further by its luminosity, so that this distribution measures the contribution to the cosmic luminosity distribution. From de Jong and van der Kruit (unpublished).

when expressed in magnitudes) and this coupling is in the same sense as observed in fig. 12.3a. Grosbøl notes in his paper that bulge light may have introduced a bias and that the effect is "stronger for galaxies with a short length scale of their disk". Although this effect appears unable to produce the relation in fig. 12.3a, it shows that the result does need confirmation from at least a study with a different procedure to fit the exponential disk profile. At this stage it is prudent to take the result therefore as preliminary and to attach not too much physical meaning to it yet. Furthermore, such an effect was not seen in the sample of van der Kruit (1987), although it was much smaller. The major conclusion then is that the most common spiral of type Sc or earlier has a central surface brightness of about 20 mag arcsec^{-2} in R (or 21.5 in B) and that the scalelength distribution drops off rapidly with increasing h.

A final exercise with these data is to weigh each galaxy further by its total disk luminosity, so that the diagram measures the relative contributions to the cosmic luminosity density. This is shown in fig. 12.3b. Galaxies that contribute most have a central surface density of about 20 R-mag arcsec^{-2} and a scalelength of about 4 kpc. It should thus be of no surprise that the sun is actually situated in a galaxy only a little bit larger than this.

Another approach has been followed by Phillips et al. (1987), who obtained a complete sample on the Fornax cluster. They digitized a complete Schmidt plate and selected objects automatically with a threshold of 25.5 B-mag arcsec^{-2} and then rejected the stellar images. This left them with 1550 galaxies which show a peak in the distribution of $\mu_{\rm o}$ of 21.8$\pm$0.9 B-mag arcsec^{-2}. The ones with central surface brightness brighter than 22.5 are uniformly distributed on the sky and after correction for the selection effects their distribution is somewhat skew, but has a mean and r.m.s. of 21.5$\pm$0.6 B-mag arcsec^{-2}. These authors then conclude that "the normal field galaxy population of disk galaxies does have a preferred value". The galaxies with fainter central surface brightness are on the sky strongly concentrated towards the Fornax cluster and then have scalelengths of 0.2 to 0.6 kpc. These are apparently common in clusters, but not in the field. Their distribution of $\mu_{\rm o}$ is essentially flat over the range 22.5 to 24.2 B-mag arcsec^{-2}, but their number increases sharply as a function of total luminosity (over the range M_B -14.4 to -11.9) for fainter systems.

12.3 Internal absorption in disk galaxies

Although not discussed yet in any detail, one must always in studies of surface brightness or integrated magnitudes be careful to correct observations for the internal absorption by dust in the galaxies. The first determination of these corrections is due to Holmberg (1958). Absorption may be determined from the systematic change of surface brightness (on top of the usual geometrical effect) and color as a function of inclination. However, the effect may be much smaller than expected intuitively. Take for example a disk with a very thin, completely opaque dust layer in its central plane. The absorption then is always half the light and the color is not affected. This means that there is no inclination dependency and we may be misled to think that disk are optically thin.

Holmberg used a sample of 119 galaxies from his own photometry and calculated the apparent projected surface brightness $\mu'_{\rm obs}$ from the total magnitude m and the major axis diameter a from $\mu'_{\rm obs} = m + 5\log a$. The ratio of major to minor axis a/b was used to calculate the inclination and he then assumed that the absorption effect depended on inclination according to a sec-law, just as the Galactic absorption dependence inferred from galaxy counts as a function of latitude (Holmberg's article mentions a cosec-law, but his definition of inclination is the opposite of the usual one today, which is $i = 0°$ for face-on). This means that it is expected that

$$\mu'_{\rm obs} = \mu'_{\rm o} + A_B\{\sec(i) - 1\} = \mu'_{\rm o} + A_B\left(\frac{a}{b} - 1\right). \qquad (12.6)$$

From fits he then found that his data followed the observations (at least for $a/b < 3$) and that A_B is 0.4 mag for Sa-Sb galaxies and 0.28 for Sc. This shows that galaxy disks are not optically thick and this result has been used extensively since then. Although similar determinations (with somewhat different inclination dependencies, such as in particular $\log\{\sec(i)\}$ rather than $\sec(i) - 1$) have been published, most recently by Kodaira and Watanabe (1988), this result has not been changed significantly. It should be noted that exceptions exist; for example Jura (1980) has proposed that disks are optically thick and that absorption is actually the cause of the central surface brightness constancy that I just discussed extensively.

Holmberg also determined the systematic change of color index with inclination and also found a sec-law with inclination. Holmberg did note that his inclination dependency was that of "the ideal case when the obscuring matter is located as an absorbing screen entirely in front of the luminous matter". Clearly the actual arrangement in space is such that dust and stars are mixed and the situation more complicated. Holmberg discussed this in some detail and concluded that his empirical description may still apply to the real world.

The question has received renewed attention with the discovery that galaxies have rather high fluxes of far-infrared radiation as observed with the IRAS satellite. After all, the light absorbed by the dust must be reradiated in the infrared and many galaxies have IRAS fluxes similar to or even larger than in the optical (de Jong et al. 1984). A recent discussion of the absorption problem in the light of these observations has been given by Disney et al. (1989, but see also references therein) and I will follow that presentation here. The alternative to optical thickness of disks, as suggested by these IRAS fluxes, is that the dust heating comes from extra star formation in optically thick molecular clouds that cover only a small fraction of the area of the disk, leaving the disk in general optically thin.

Disney et al. discuss some alternative simple models. Take a uniform disk of stars with volume emissivity E^* and thickness T. First take a foreground screen of optical depth τ in the B-band. Then the optical surface brightness $L(i)$ is

$$L(i) = E^* T \sec(i) \ \exp\{-\tau \sec(i)\} \tag{12.7}$$

The Holmberg surface brightness (now not in magnitudes) is always $L'(i) = L(i)\cos(i)$ and it can be easily seen that this case corresponds to Holmberg's empirical law. The face-on extinction is

$$A_B = 1.086\tau. \tag{12.8}$$

The bolometric surface brightness $L_{\rm bol}$ is of course $E^* T \sec(i)$ and if the far infrared surface brightness $L_{\rm FIR} = L_{\rm bol} - L(i)$, then

$$\frac{L_{\rm FIR}}{L(i)} = \ \exp\{\tau \sec(i)\} - 1. \tag{12.9}$$

The ratio of integrated fluxes is of course the same.

Next take a uniform slab with stars and dust perfectly mixed and a mean free path for the optical radiation λ, so that $\tau = T/\lambda$. Then

$$L(i) = E^* \lambda \left[1 - \ \exp\left\{-\frac{T}{\lambda}\sec(i)\right\}\right]. \tag{12.10}$$

The face-on extinction is

$$A_B = -2.5 \log \left\{\frac{1 - \ \exp(-\tau)}{\tau}\right\}. \tag{12.11}$$

Now for the optically thick case $\tau \gg 1$ we have

$$L(i) = E^* \lambda = \text{constant} \tag{12.12}$$

and Holmberg's projected surface brightness varies as $\cos(i)$. The FIR surface brightness becomes

$$\frac{L_{\rm FIR}}{L(i)} = (\tau - 1)\sec(i). \tag{12.13}$$

For the optically thin case $\tau \ll 1$

$$L(i) = E^* T \sec(i) \tag{12.14}$$

and Holmberg's surface brightness becomes independent of i. Further

$$\frac{L_{\rm FIR}}{L(i)} = \frac{\tau}{2} \tag{12.15}$$

and is inclination independent, as expected intuitively.

An even more realistic case is a "sandwich model", in which the dust has a thickness pT, so that $\tau = pT/\lambda$. The solutions now are

$$L(i) = E^* T \sec(i) \left[\frac{1-p}{2} [1 + \exp\{-\tau(i)\}] + \frac{p}{\tau(i)} [1 - \exp\{-\tau(i)\}] \right], \tag{12.16}$$

$$A_B = -2.5 \log \left[\frac{1-p}{2} \{1 + \exp(-\tau)\} + \frac{p}{\tau} \{1 - \exp(-\tau)\} \right]. \tag{12.17}$$

The FIR to optical ratio can be found from this also using $L_{\rm bol} = E^* T$ as before and will not be written out here in full.

Let us consider the opaque case $\tau \gg 1$. Then

$$L(i) = E^* T \sec(i) \frac{1-p}{2}, \tag{12.18}$$

$$A_B = -2.5 \log \left\{ \frac{1-p}{2} \right\}, \tag{12.19}$$

$$\frac{L_{\rm FIR}}{L(i)} = \sec(i) \frac{(1+p)\tau - 2p}{(1-p)\tau + 2p}. \tag{12.20}$$

The surface brightness for the optically thick sandwich behaves just as in the case of an optically thin slab in its dependence upon i. Also the ratio of FIR to optical flux is independent of the inclination in both cases. This latter situation is roughly the observed case, but one may therefore not conclude from this that disks are indeed optically thin.

In the optical thin case $\tau \ll 1$ we have

$$L(i) = E^* T \sec(i) \left\{ 1 - \frac{1-p}{2\tau} \sec(i) \right\}, \tag{12.21}$$

which for small τ again approaches the thin slab and Holmberg's surface brightness is again independent of inclination. Also

$$A_B = -2.5 \log \left\{ 1 - \frac{1-p}{2} \tau \right\}, \tag{12.22}$$

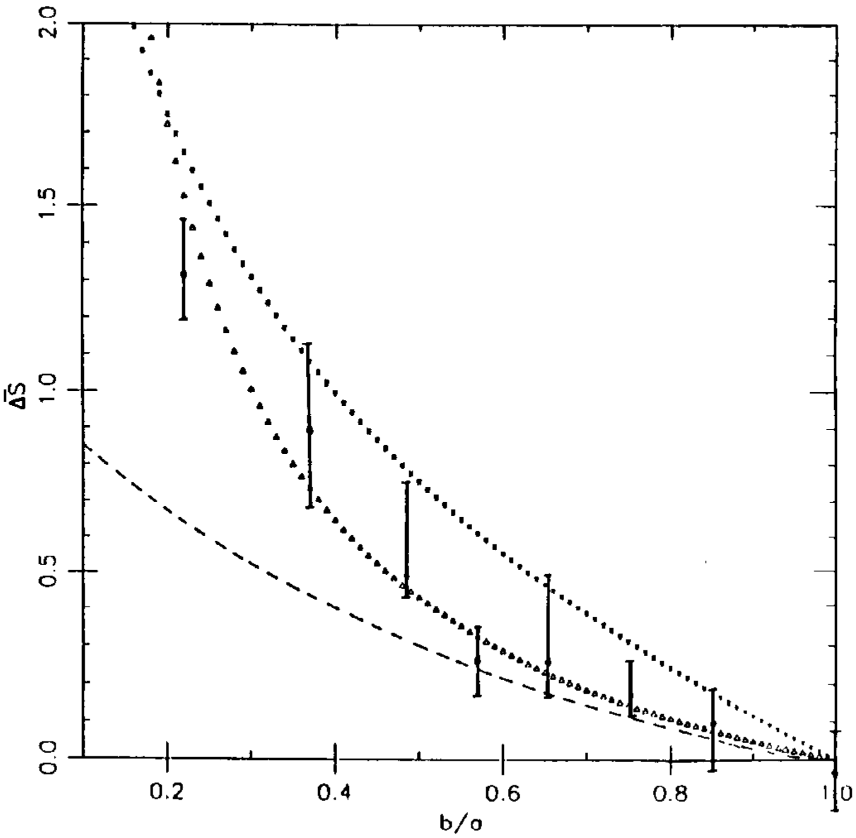


Figure 12.4: Effects of absorption on the surface brightness of disks in the analysis of Holmberg (1958). The vertical scale is the difference between observed reduced surface brightness (total luminosity divided by a circular area with radius the major axis) and that for face-on and the horizontal scale is the axis ratio. Data are for Sa-b galaxies from Holmberg. The lines are three models: The triangles are a uniform screen in front of the disk (Holmberg's sec-law), stars an optically thick slab model and the dashed line an optically thick sandwich model with $p = 0.5$. The models are described in the text. From Disney et al. (1989).

$$\frac{L_{\mathrm{FIR}}}{L(i)} = \frac{\tau}{2}. \tag{12.23}$$

These illustrative models show the strong dependence upon the precise model for the distributions. Disney et al. consider some more realistic models with exponential distributions, but the results so far suffice to show the main point, namely that an analysis as Holmberg's is not able to distinguish in general optically thick disks from optically thin ones. This is shown graphically in fig. 12.4, where some of Holmberg's data are compared to these schematic models. Here the difference $\mu'(i) - \mu_o$ is plotted against the axis ratio, which measures the inclination. The triangles show Holmberg's fit (or the screen model) with $A_B = 0.43$ mag, the stars an opaque slab and the dashed line an optically thick sandwich with $p = 0.5$. Optically thin slab and sandwich models predict no dependence upon i. It is clear from this that it is too early to conclude from this that disks are optically thin, and in agreement with the remarks above color information essentially adds nothing to this.

It appears that consideration of IRAS fluxes compared to optical ones as a function of Hubble type can make a clearer case for either of the two possibilities. Certainly these are consistent with optically thick disks, but a more detailed discussion by Disney et al., that I will not repeat here, shows that effects of a non-uniform distribution of the dust and star formation in shielded molecular clouds make it difficult to find a unique resolution of this issue. All we can say is, that an effect as the one described above of a thin, opaque layer in the central plane is very well possible, in which case the surface brightness is decreased by a factor 2 or 0.75 mag. Disney et al. conclude that in special circumstances 2 mag is even possible, but certainly the case of optically thin disks is not disproved also.

12.4 Photometry in the near-infrared

It has become possible in recent times to extend mapping of luminosity distributions of galaxies to near-infrared wavelengths with the advent of the first two-dimensional detectors at these wavelengths. Before that it was only possible to obtain such information from a painstaking process of observing many individual positions, although raster scanning on large telescopes could be done with reasonable amounts of observing time and moderate resolutions since the advent of indium-antimonide (InSb) detectors. A special problem at near-infrared wavelengths is the fact that the thermal sky background surface brightness relative to that of the galaxies is much brighter than in the optical, while thermal emission from the telescope also poses special problems. At K (2.2μ) the photon shot noise form the thermal radiation of the telescope and the sky dominate the noise in the data; fortunately the number of photons received is so large that this shot noise can be suppressed with reasonable amounts of observing time to acceptable levels.

An important study in this rapidly developing field has been the thesis of Wainscoat (1986), who used the raster scanning technique on 4–meter class telescopes (AAT and UKIRT) to map edge-on galaxies at J (1.2μ), H (1.6μ) and K (2.2μ). He also compared the near-infrared photometry to optical work. An interesting feature is then found in early type galaxies (NGC 7814 and NGC 7123), where the luminosity is dominated by that of the bulge. In fact, these galaxies are classified as Sa, because in the optical a strong central dust lane is seen projected onto the bulge, although there is very little evidence indeed of a stellar disk. It then turns out that in the near-infrared one can observe a disk in emission just where in the optical the dust lane is seen. This emission clearly is from the stellar disk and it follows from this observation that the dust and the stars must have similar vertical scaleheights. This is completely different from the situation in our Galaxy and other later-type systems, where the stellar disk is always much thicker than the gas and dust layer. This means that in early-type galaxies the kinematics and dynamics of the dust and the stars of the disk are closely similar and no secular evolution of the stellar kinematics occurs. This is most likely related to the low gas density, which would make the typical timescale of the scattering of stellar orbits much longer then for example in the solar neighborhood. As Wainscoat points out, this may also open the possibility that dust is present throughout the bulges of early type galaxies, which would also have an effect on the observed color gradients in these components.

A further interesting observation by Wainscoat concerns the late-type edge-on IC 2531, which has very little bulge. In the K-band the observations of a vertical cross-cut through the disk appear to indicate that the vertical distribution of stars continues as an exponential all the way to the plane and therefore has a much sharper peak than would be predicted by the isothermal distribution with a sech^2-law. Unfortunately the observations as presented refer to a radial distance along the major axis that is not very far out in the disk and there possibly is at low z a contribution from bulge light. Furthermore there is always the possibility that there is recent star formation in the area of the dust lane, where also the gas resides and therefore significant contributions from red supergiants to the near-infrared luminosity can be expected. These effects need to be investigated further. However, as also discussed elsewhere in this course, the actual distribution of stars is in any case expected to deviate near the plane from the isothermal distribution, because younger populations must have smaller velocity dispersions and therefore are more confined to the plane of the disks.

It is now also possible to use two-dimensional array's in the near-infrared. The ones that are operational at this time are still limited in extent (roughly 60×60 pixels), but this is likely to change significantly very soon. The use of these devices considerably speeds up the collection of data on extended objects, although for larger galaxies a process of taking adjacent frames with a small overlap (so-called mosaicing) is still necessary to sample the whole image. As an example fig. 12.5 shows a preliminary K-band image taken at UKIRT of NGC 891, the edge-on galaxy that was mentioned extensively in the discussions on optical photometry in this course. Contour levels here are a factor 2 in intensity (about 0.75 mag) and the faintest contour is about 19.35 K-mag arcsec^{-2}. The sky surface brightness at these wavelengths is usually in the range 11 to 12 K-mag arcsec^{-2}, depending in particular on the level of thermal emission of the telescope and on the outside temperature. This particular observations constitutes about 3 hours on the telescope, including frequent blank sky frames, which are necessary in view of the rapid

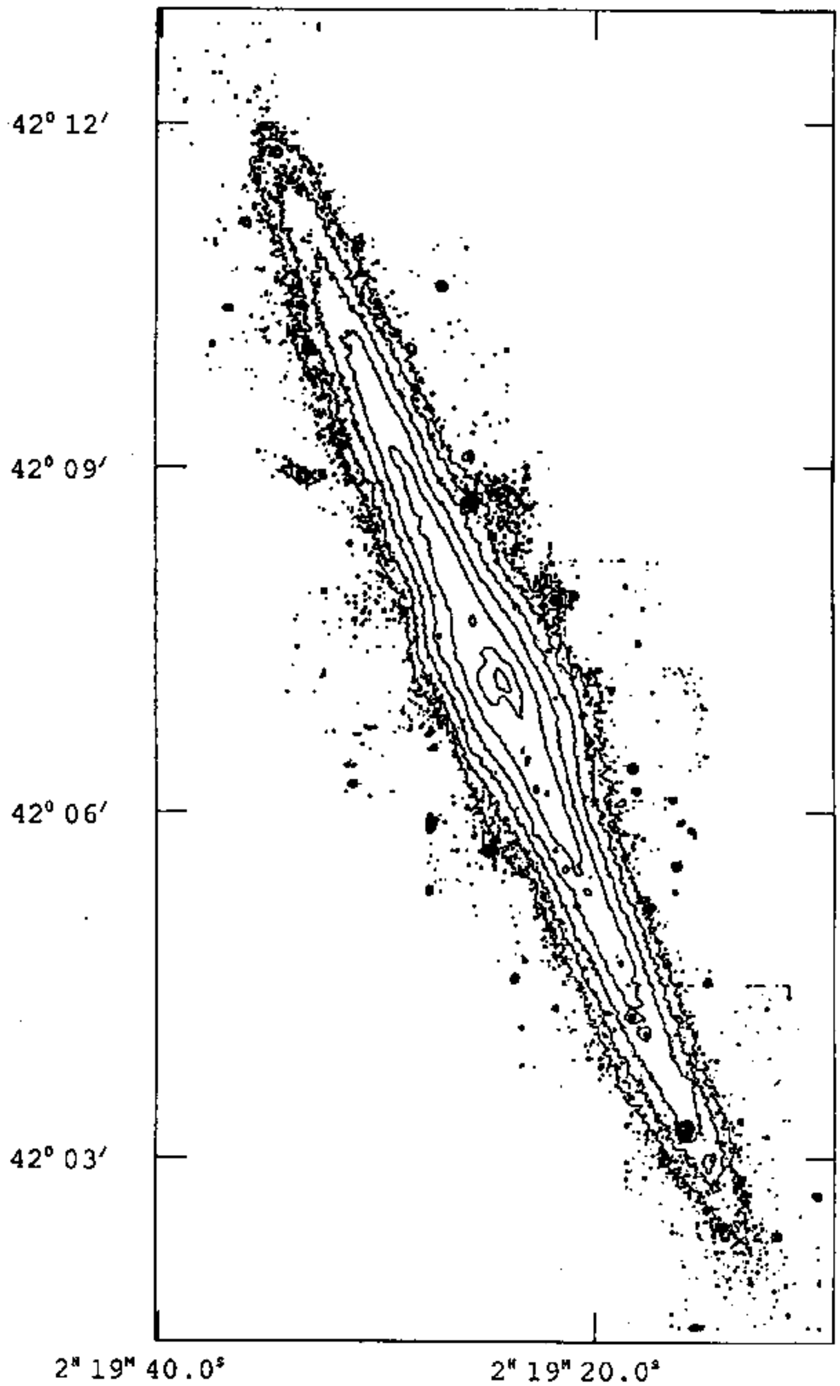

42° 12′
42° 09′
42° 06′
42° 03′
2ʰ 19ᴹ 40.0ˢ
2ʰ 19ᴹ 20.0ˢ

Figure 12.5: Preliminary K-band isophote map of NGC 891. The contours are spaced by 0.75 mag (a factor 2 in intensity) and the faintest one is at about 19.35 K-mag arcsec^{-2}. Note the much reduced of effect of the dust absorption. From Wainscoat and van der Kruit (unpublished).

variation of sky background levels. Note the similarity of the contours to a similar map of optical isophotes and the much reduced effect of the dust lane.

With these new detectors it will also be possible to perform more extensive surveys of the luminosity distribution in the near-infrared of more face-on systems. This would in particular provide important constraints on the range of variation of central extrapolated surface brightness independent of younger populations and would also be able to make more definite statements on the effects of dust absorption. No such surveys are available at this time, but these can be expected in the near future. Using single-point raster-scanning mapping techniques, Giovanardi and Hunt (1988) have performed near-infrared photometry of nine large Sc galaxies in J, H and K with a range of 3.5 mag in integrated luminosity. They have found that the dispersion in the extrapolated, face-on central surface brightness is comparable to that in B (namely 0.6 to 0.9 mag). Also the scalelengths are not statistically different from those determined in the B-band. However, this study is just a first attempt (the resolution is only 28 arcsec) and the sample is not complete in any statistical sense. Yet, the observed small spread in μ_o is interesting and, if it would hold up for larger and carefully selected samples, would indicate that the central surface brightness constancy is indeed a property of the old disk population (and therefore presumably of the total disk surface density) and that absorption and mix with younger populations have no appreciable effect on this. The similarity of optical and near-infrared scalelengths shows that there is no appreciable radial absorption effect on optically derived luminosity profiles and no radial effect of the mix with younger populations.

12.5 Integral luminosities, colors and total masses

In this section I will briefly discuss the integrated luminosities and the colors of spiral galaxies and the Tully-Fisher relation. An extensive review of most of this section has been given by Tinsley (1980) and little has changed fundamentally since then. I will not discuss the galaxy luminosity function here in detail; it has been reviewed recently by Binggeli et al. (1988).

It has been known for a long time that there is a correlation between the integrated color and the morphological type of a galaxy. In effect these colors follow a more or less straight line in the two-color diagram ranging roughly from $(U-B) = -0.3$, $(B-V) = 0.3$ for Sc and irregular galaxies to $(U-B) = 0.6$, $(B-V) = 1.0$ for ellipticals. This has been interpreted as a result of different histories of star formation, but similar ages for these various types, starting with the work of Tinsley (1968) and Searle et al. (1973). These methods are all in principle the same. First assume an initial mass function (IMF; the mean distribution over masses for star formation) and calculate what the change is in luminosity and colors of such a burst of star formation as a function of time. Then add such individual contributions with a weight as a function of age that follows the rate of star formation (SFR) over the life of a galaxy. This function ranges from a single initial burst of star formation for the early type galaxies to a constant SFR for late types. The basic result of these early studies was that the observed two-color diagram could be reproduced with a single IMF as inferred for the solar neighborhood and ages about 10^{10} years for all galaxies.

Following and extending Tinsley (1973, 1980), this result can be illustrated analytically in the following way. Let the IMF be a simple power law (masses in $M_\odot$)

$$\phi(M) = CM^{-(1+\mathrm{x})} dM \quad \text{for} \quad M_\mathrm{L} < M < M_\mathrm{U}. \tag{12.24}$$

For $M_\mathrm{L} \ll M_\mathrm{U}$ and $x > 0$ the constant C becomes after normalization of the IMF over all masses equal to xM_L^x. For the so-called Salpeter function (one of the earliest determinations of the IMF) we have $x = 1.35$. Now approximate the main sequence mass – luminosity relation with $L = M^\alpha$ (L and M in solar units) and the time that a star spends on the main sequence as $t_\mathrm{MS} = M^{-\gamma}$ (the unit of time is then the main-sequence lifetime of a one solar mass star or about 10^{10} years). Reasonable values for α are 4.9 in U, 4.5 in B and 4.1 in V and 3 for γ. Further we assume that each star becomes a giant for a period of time 0.03 with luminosities (in solar units) 35 (U), 60 (B) and 90 (V). The calculation of the luminosity evolution of a single burst of star formation with in total ψ_o stars at time $t = 0$ then proceeds as follows. At time t, the stars with mass $M > M_\mathrm{t} = t^{-1/\gamma}$ have evolved from the main sequence and the total light from main sequence stars therefore is

$$\begin{aligned} L_\mathrm{MS}(t) &= \int_{M_\mathrm{L}}^{M_\mathrm{t}} \psi_\mathrm{o}\, M^\alpha\, \phi(M)\, dM \\ &= \frac{x}{\alpha - x}\, M_\mathrm{L}^\mathrm{x}\, \psi_\mathrm{o}\, M_\mathrm{t}^{\alpha-\mathrm{x}}. \end{aligned} \tag{12.25}$$

The number of giants at time t (for $t_\mathrm{g} \ll t$) is

$$\begin{aligned} N_\mathrm{g}(t) &= \psi_\mathrm{o}\, \phi(M_\mathrm{t}) \left| \frac{dM}{dt_\mathrm{MS}} \right|_{\mathrm{M=M_t}} t_\mathrm{g} \\ &= \frac{\psi_\mathrm{o}\, x}{\gamma} M_\mathrm{L}^\mathrm{x}\, M_\mathrm{t}^{\gamma-\mathrm{x}}\, t_\mathrm{g}. \end{aligned} \tag{12.26}$$

Table 12.1: Colors and mass-to-light radios for a single burst model

t	$(U-B)$	$(B-V)$	$(M/L)_B$
0.01	−0.34	0.12	0.15
0.03	−0.06	0.45	0.38
0.1	0.18	0.64	1.12
0.3	0.38	0.79	2.79
1.0	0.56	0.90	6.95
3.0	0.66	0.96	14.9

The luminosity of the single burst then becomes

$$L_{\rm SB}(t) = L_{\rm MS}(t) + N_{\rm g}(t)L_{\rm g}. \qquad (12.27)$$

With $M_{\rm L} = 0.1$ (the low-mass end of the main sequence) and with (adapted) absolute magnitudes for the sun of $U_\odot = 5.40$, $B_\odot = 5.25$ and $V_\odot = 4.70$ we get results that are very close to more detailed calculations. For $t < 0.03$ we have to take $t_{\rm g} = t$. Some representative numbers are collected in table 12.1.

The single burst rapidly becomes redder with time, while the luminosity (see M/L) decreases. Of course in the calculation of M/L it is assumed that no stars form with mass below $M_{\rm L}$. For very short times t the actual value of $M_{\rm U}$ needs to be taken into account also.

We can then proceed to calculate the evolution of a galaxy by taking a SFR $\psi(t)$ from

$$L(t) = \int_0^{\rm t} \psi(t-t')\; L_{\rm SB}(t')\; dt'. \qquad (12.28)$$

For an initial burst (elliptical and S0 galaxies) we of course get back the values above. For a constant SFR (and taking $M_{\rm U} = 32$) we get at $t = 1$: $(U-B) = -0.25$, $(B-V) = 0.24$ and $(M/L)_B = 1.03$ and this corresponds to late-type galaxies. Actually, the precise form of the SFR is not important (see also Larson and Tinsley, 1978): The final colors and (M/L) depend only on the ratio of the present rate of star formation (over the last 10^8 years or so) to the mean SFR over the whole life of the galaxy.

The approximation given is only indicative (the absolute magnitudes of the sun are incorrect, but are chosen to provide good answers), but suffices in practical cases to estimate effects. Two-color diagrams from observations and from more detailed models are compared in fig. 12.6. It can be seen there that the observations are reproduced very well by the calculations. In the figure the effects of making different assumptions (age, slope x of the IMF, $M_{\rm U}$ and metallicity Z) are also shown. It follows that these have

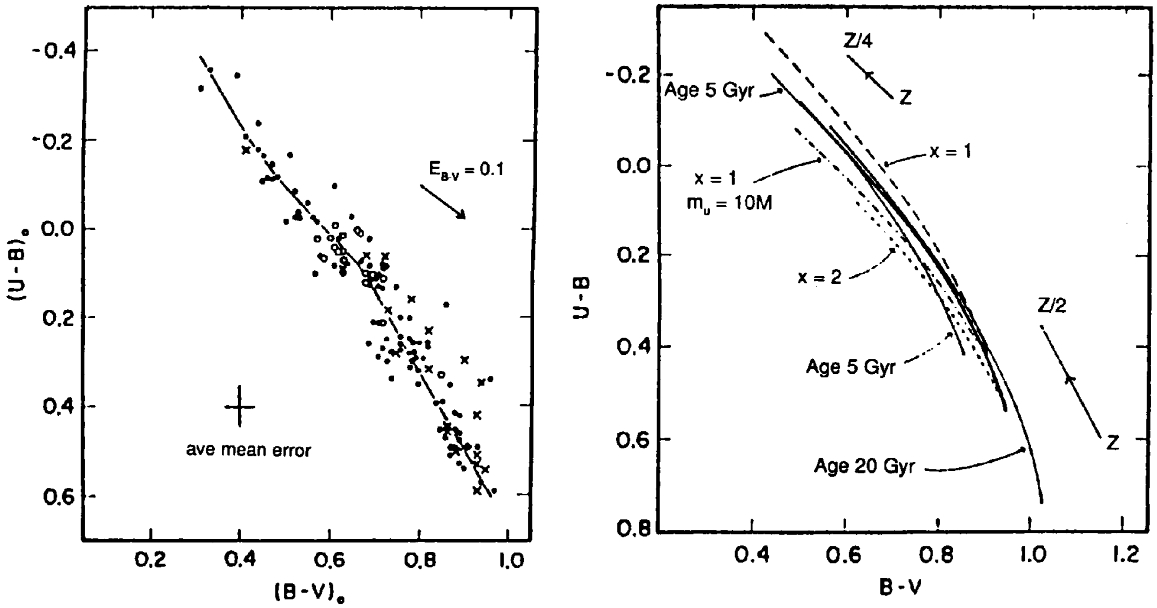


Figure 12.6: The two-color diagram of galaxies. (a) The observed distribution; filled circles are galaxies in the Hubble atlas, crosses E and S0 galaxies in the Virgo cluster and open circles Galactic open and globular clusters. (b) Model distributions. The solid line, which fits the data very well, has the IMF and the metallicity of the solar neighborhood and an age of 10 Gyrs. It ranges from an initial burst of star formation to a constant SFR. The effects of changing the age, the slope x of the IMF, the upper mass limit $M_{\rm U}$ and the metallicity Z have been indicated. From Tinsley (1980).

a very minor effect and the observed two-color diagram cannot be used to discriminate between these various possibilities.

Larson and Tinsley (1978) have also investigated the colors of peculiar galaxies. These turn out to have a much larger spread around the mean line of fig. 12.6. It turns out that this spread can easily be explained as the result of bursts of star formation of various strengths and ages in galaxies with various colors at the time of the burst. These bursts have strengths of up to 5% (ratio of mass in the burst to that of the stars already formed at that time) and durations of up to 2 10^7 years.

For spiral galaxies there is also an observed relation between the integrated luminosity and the amplitude of the rotation curve. The latter is usually measured from the width ΔV of the integrated HI-line profile at 21 cm (and corrected for inclination). Magnitudes need to be corrected statistically for absorption effects. Galaxies at moderate inclination are most suitable for this: for edge-ons the absorption correction to the magnitude is

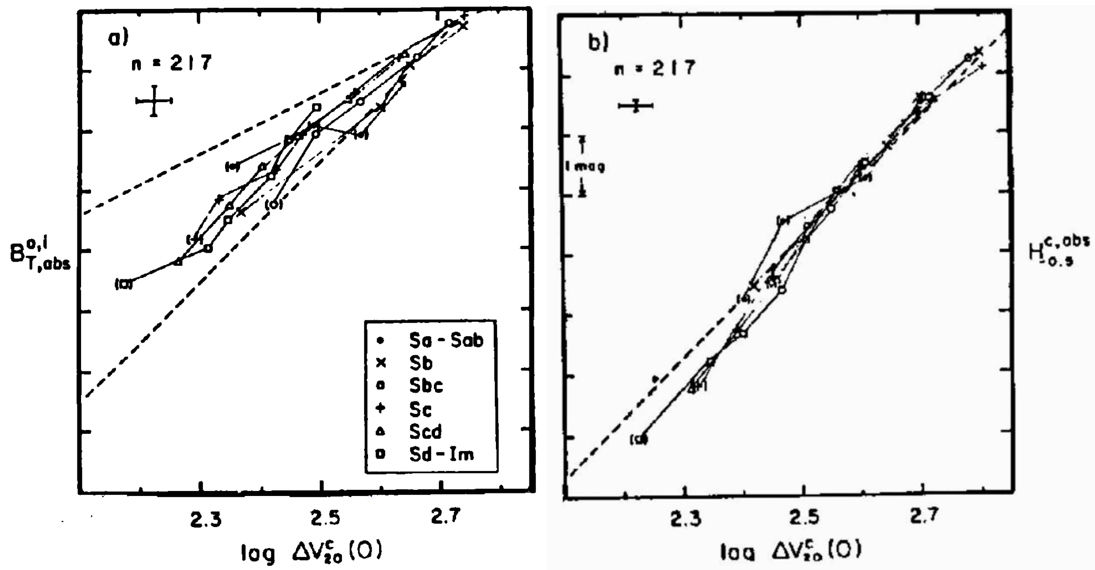


Figure 12.7: The Tully-Fisher relation in the optical (B at left) and the near-IR (H at right). The sample of 217 galaxies has been binned using regressions of the two variables and morphological types are distinguished by different symbols. At the right, the lines have slopes corresponding to $\alpha = 2$ and 4 in eq. Ł29), while at H only the latter is shown. The absolute magnitude scales are arbitrary (that is, independent of the actual Hubble constant) and distances have been calculated using a Virgocentric inflow model. From Aaronson and Mould (1983).

uncertain, while for face-on galaxies the inclination is difficult to determine. This relation, named after the discoverers (Tully and Fisher, 1977) has the approximate form

$$L \propto \Delta V^{\alpha}. \tag{12.29}$$

The value of α has been measured for various samples and at various wavelengths. A very extensive study is that presented by Aaronson and Mould (1983), who base their investigation on a sample of about 300 galaxies with radial velocities less than 3000 km s^{-1}, for which they have integrated magnitudes in both B and H (1.6μ). The result is shown in fig. 12.7. Distances have been derived from the redshifts, using a best-fit model for the pattern of streaming motions from infall towards the Virgo cluster (about 330 km s^{-1} at the Local Group). This excludes galaxies with redshifts less than 300 km s^{-1} and some in a conical shell around the Virgo cluster and leaves them with 217 systems. Three inferences can be made. The scatter is less in the near-IR than in the optical, probably due to unreliable absorption corrections and more severe effects from young stellar populations in the B-band. The slope is wavelength dependent, being steeper at H. For Sb and Sc galaxies α is about 3.5 in B and about 4.3 in H. A small type dependence appears in B, but no significant change in the relation is apparent with type at H, again probably due to effects from young stars at B.

For exponential disks the mass is proportional to $\sigma_o h^2$ and $V_{\rm max}$ to $(\sigma_o h)^{1/2}$, so that we expect that the mass is proportional to $V^4_{\rm max}/\sigma_o$. Now, taking Freeman's constancy of central surface brightness and a constant M/L between galaxies we expect for pure disks that $\alpha = 4$, roughly as observed. However, there are dark halos and this implies two possible things. If rotation curves are at all radii dominated by the dark matter, there should be a correlation not only between the amounts of dark and visible matter but also between their distributions. If on the other hand, the inner rotation curves are provided essentially by the disk alone, then the Tully – Fisher relation follows immediately, especially if a "disk – halo conspiracy" (see above) exists. I will return to this question later.

Finally, I note that the values given in this course for the disk color, luminosity and rotation speed of the Galaxy fit in very well with those of external galaxies in terms of the relations discussed here.

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