# Properties of the Stellar Velocity Ellipsoid and Stability in Disks of Spiral Galaxies

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**Abstract.** Disks of spiral galaxies are characterized by effectively exponential brightness and presumably density distributions in both the radial and vertical directions. It is to be expected that the ratio between the scalelength and -height bears a relation to the axis ratio of the stellar velocity ellipsiod. Hydrostatic equilibrium connects the vertical velocity dispersion to the scaleheight. In the radial direction the velocity dispersion relates to the scalelength through conditions of local stability. Preliminary applications are presented.

## 1. Background.

The common stars in the solar neighbourhood have long been known to show an increase in their velocity dispersions with increasing age (e.g. Wielen, 1977). Random motions will increase as a result of scattering when stars encounter fluctuations in the galactic gravitational field. There are two general classes of models for this secular evolution.

The first is the Spitzer-Schwarzschild (1951, 1953: SS) mechanism. Stars are scattered by massive concentrations in the interstellar medium, later identified with Giant Molecular Clouds. The process stops effectively when the stars spent most of their time outside the gas layer. Secondly, Barbanis and Woltjer (1967; BW) proposed that the stars are being scattered by (transient) spiral features. This model was extended by Carlberg and Sellwood (1985). Because the frequency of encountering spiral arms and of the vertical motions are very different, the heating is not very effective in the z-direction.

Lacey (1984) and Villumsen (1985) concluded that the SS-mechanism was not able to reproduce the local age–velocity dispersion relation and the axis ratio of the velocity ellipsoid (cannot have $\sigma_z/\sigma_R$ less than 0.7, while locally it is 0.5 to 0.6). Jenkins and Binney (1990) argue that both mechanisms operate, where the BW-mechanism provides the velocity dispersions in the plane, while the SS-mechanism converts some of this energy into vertical motion.

Recently various workers (*e.g.* Walker *et al.*, 1996; Velázquez and White, 1999) have studied the possibility that infall of satellite galaxies (like the Sagittarius Dwarf for the Milky Way) may have resulted in the heating of the stellar disks and possibly the formation of "thick disks".

The fundamental criterion for *local* stability of galactic disks comes from Toomre (1964), where stability results from the stellar random motions (Jeans stability) at small lengthscales and from shear due to differential galactic rotation at longer lengthscales with no regime in between. The criterion thus contains the velocity dispersion $\sigma_{\rm R}$ and the epicyclic frequency $\kappa$ as a measure of the shear in the rotation. For stability then

$$Q = \frac{\sigma_{\rm R}\kappa}{3.36G\Sigma} > 1. \tag{1}$$

A second instability, also due to Toomre (1981) is swing amplification, shear amplifies disturbances and turns them into ingoing trailing waves. For a galaxy with a flat roration curve and $m$ spiral arms, swing amplification is prevented if (Sellwood, 1983)

$$X = \frac{R\kappa^2}{2\pi G m \Sigma} > 3. \tag{2}$$

It is generally accepted that disks are prevented from *global* instability by the presence of a dark halo with a high velocity dispersion or the higher velocity dispersion of the stars in the central parts of the disk. The former can be expressed by a semi-empirical criterion due to Efstathiou *et al.* (1982), which says that global stability requires

$$Y = V_{\rm rot}\left(\frac{h}{GM_{\rm disk}}\right)^{1/2} \gtrsim 1.1. \tag{3}$$

This criterion has been derived for galaxies with (1) a flat rotation curve with velocity $V_{\rm rot}$, and (2) an exponential disk surface density distribution $\Sigma(R) = \Sigma(0)\,\exp(-R/h)$. It is effectively a measure of the contribution of the disk mass to the amplitude of the rotation curve.

## 2. Measurements of the stellar velocity dispersion.

Stellar velocity dispersions have been measured in external spiral disks by van der Kruit and Freeman (1986) and Bottema (1993) and in the Galaxy by Lewis and Freeman (1989). In general terms, two possible radial dependences can be expected.

Disks have a thickness (or scaleheight) that is in good approximation constant with radius (van der Kruit and Searle, 1981). Then

$$\sigma_{\rm z} \propto {\rm e}^{-R/2h}.$$

A first assumption that can be made is that the anisotropy in the velocity ellipsoid is constant with radius, so that the radial velocity dispersion should have the same radial dependence as the vertical one.

The second assumption could be that Toomre's parameter $Q$ for local stability is constant with radius. If we have a flat rotation curve, the radial dependence of the radial velocity dispersion would be

$$\sigma_{\rm R} \propto R\ {\rm e}^{-R/h}.$$

Over the range, where the velocity dispersion can be measured (at one or two exponential scalelengths), the difference is small and measurements are consistent with both equations. However, at larger $R$ the two would differ significantly. It is possible that the constant thickness of galactic disks results from marginal local stability at all $R$.

Bottema (1993) found that the radial velocity dispersion at one photometric scalelength (in $B$) correlates with both the absolute magnitude of the disk and the amplitude of the rotation curve. Especially the latter of these empirical relations will be used below.

$$\sigma_{\rm R,h} = 0.29\ V_{\rm rot}. \tag{4}$$

## 3. General considerations.

In this section I will describe the relationship between the stellar velocity ellipsoid and the disk scale parameters. Disks will be described by exponential surface density distributions

$$\Sigma(R) = \Sigma(0)\ {\rm e}^{-R/h}$$

(Freeman, 1970) and vertical density distributions (van der Kruit, 1988)

$$\rho(R,z) = \rho(R,0)\ {\rm sech}(z/h_{\rm z}).$$

The vertical velocity dispersion $\sigma_{\rm z}$ can be then calculated from

$$\sigma_{\rm z}^2 = 1.7051\pi G\Sigma(R)h_{\rm z}. \tag{5}$$

The mass-to-light ratio $M/L$ is constant as a function of radius. Further, I assume that the rotation curves are flat, so that

$$\kappa = 2\sqrt{B(B-A)} = \sqrt{2}\frac{V_{\rm rot}}{R}.$$

Finally, exclude for the moment Low Surface Brightness Galaxies, so that we have to do with disks following Freeman's (1970) law

$$\mu_{\rm o} = \Sigma(0)\left(\frac{M}{L}\right)^{-1} \approx 21.6\ \ B{-}{\rm mag\ arcsec}^{-2} = 142\ {\rm L}_{\odot}\ {\rm pc}^{-2}.$$

Evaluating Toomre's $Q$ at $R = 1h$ and using $\Sigma(0) = (M/L)\mu_{\rm o}$ and total disk luminosity $L_{\rm d} = 2\pi\mu_{\rm o}h^2$, we can write

$$\sigma_{\rm R,h} = \frac{1.68G}{{\rm e}\sqrt{\pi}} Q \left(\frac{M}{L}\right) \frac{\sqrt{\mu_{\rm o} L_{\rm d}}}{V_{\rm rot}}. \tag{6}$$

This can be reconciled with Bottema's relation (4) for constant $\mu_{\rm o}$, $M/L$ and $Q$ between galaxies, if

$$L_{\rm d} \propto V_{\rm rot}^4.$$

This is approximately the Tully-Fisher relation. We fix the proportionality constant using the Milky Way Galaxy and NGC 891 ($L_{\rm d} \sim 1.9 \times 10^{10}$ $L_\odot$ and $V_{\rm rot} \sim 220$ km s$^{-1}$), and then find

$$Q \left(\frac{M}{L}\right)_B \approx 5.7. \tag{7}$$

Evaluating hydrostatic equilibrium (5) at $R = 1h$ gives

$$\sigma_{\rm z,h} = \left\{ \frac{5.357}{\rm e} \, G \left(\frac{M}{L}\right) \mu_{\rm o} h_{\rm z} \right\}^{1/2}. \tag{8}$$

Eliminating $\mu_{\rm o}(M/L) = \Sigma(0)$ between eq.s (6) and (8) results in

$$\left(\frac{\sigma_{\rm z}}{\sigma_{\rm R}}\right)_{\rm h}^2 = \frac{7.77}{Q} \frac{h_{\rm z}}{h}. \tag{9}$$

So, we may use in principle the ratio of the disk's scale parameters to estimate the axis of the velocity ellipsoid independent of any assumption of $M/L$ or the distance scale. Note that at other radii we can make the same derivation (as long as the rotation curve is flat); then eq. (9) is multiplied on the righthand side by $h/R$. This would imply a substantial change in $\sigma_{\rm z}/\sigma_{\rm R}$ with radius; however, both eqs. (6) and (8) require corrections for the influence of the gas, which may be far from insignificant and radius dependent.

However, we need a value for $Q$. Numerical simulations by Efstathiou *et al.* (1982), Sellwood and Carlberg (1984) and others since then show that disks stabilize with $Q$ of the order 1.7 to 2. Using the stability criteria of of section 1, we set further limits when combining them with the Bottema relation (4). The relation for prevention of swing amplification (2) can be rewritten to give $Q \gtrsim 1.15m$. If we want to prevent disks from strongly forming bars (m=2), we need $Q$ to be at least about 2. Relation (3) for global stability translates for an exponential disk into

$$Y = 0.615 \left[\frac{QRV_{\rm rot}}{h\sigma_{\rm R}}\right]^{1/2} {\rm e}^{-R/2h} \gtrsim 1.1.$$

The lefthand side has a maximum at $R = 1h$ and with (5) this gives $0.69\sqrt{Q} \gtrsim 1.1$, or $Q$ has to be at least about 2. Note that a choice of $Q \sim 2$, gives through eq. (7), $(M/L)_B \approx 3$.

## 4. Application to the de Grijs sample.

De Grijs (1998; de Grijs & van der Kruit, 1996) has performed surface photometry on a statistically complete sample of edge-on disk galaxies and determined the scalelengths and scaleheights. This sample can be used to apply the method outlined above to (van der Kruit & de Grijs, submitted). At this stage many uncertainties remian –such as the influence of the gas and exact shape of the rotation curves–, while others can be circumvented –e.g. the constancy of central surface brightness needs not be assumed because the actually observed ones can be used–. However, from this dataset there is no evidence for a dependence of the axis ratio of the velocity ellipsoid on either rotation velocity or morphological type (see Fig. 1 and the table below), in spite of the fact that de Grijs found an incrase in $h_z/h$ towards later morphological types. We have started a program to observe the radial velocity dispersion in these edge-on galaxies.

| Type | Sb | Sbc | Sc | Scd | Sd |
|---|---|---|---|---|---|
| $(\sigma_z/\sigma_R)_h$ | 0.71 | 0.69 | 0.49 | 0.70 | 0.63 |
| | $\pm$ 0.14 | $\pm$ 0.16 | $\pm$ 0.17 | $\pm$ 0.20 | $\pm$ 0.22 |
| n | 11 | 7 | 6 | 11 | 5 |

## 5. IC 5249.

IC5249 is an extreme example of a “superthin” edge-on galaxy; its axis ratio on the sky is about 10 to 1. The radial velocity of $\approx$ 2360 km/s suggests a distance of 36 Mpc. The HI flux (28 Jy km $s^{-1}$; Matthewson *et al.*, 1992) corresponds to an HI mass of $\sim 8.5 \times 10^9$ $M_\odot$. The width of the HI-profile indicates a rotation velocity of only $\sim$ 100 km $s^{-1}$ in spite of the observed radius of $\sim$ 20 kpc. It also suggests that a significant fraction of the disk mass is in the form of HI-gas. The integrated magnitude in the $I$-band of 13.11 corresponds to $L_{tot} = 3 \times 10^9$ $L_\odot$, if there were no absorption.

The MOA Collaboration (Abe *et al.*, 1999) has recently published a paper in which they search for luminous matter in a “dark halo”. Abe *et al.* use unpublished 21-cm observations with the Australia Telescope.

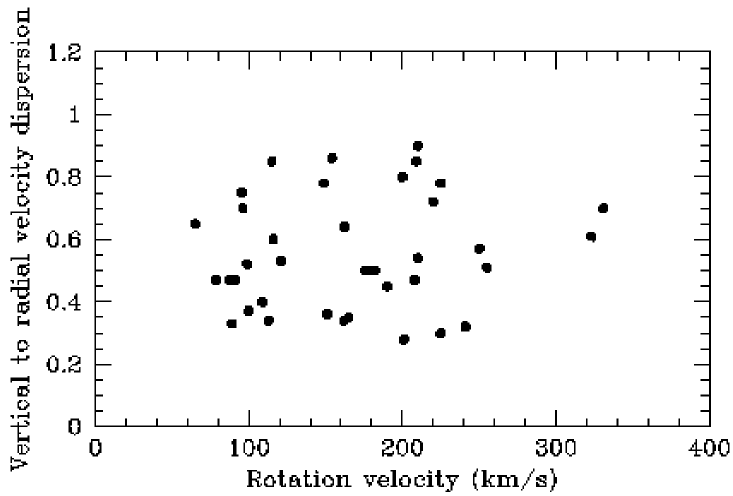


*Figure 1.* The ratio of the calculated vertical to radial velocity dispersions at one exponential scalelength from the center as a function of the rotation velocities.

The projected surface density of HI is rather uniform with radius. They claim that the rotation curve is slowly rising out to the edge, but this is based on a first-moment analysis, although they know this to be incorrect for edge-on galaxies. Mathewson *et al.* (1992) observe for IC 5249 the standard double-horned profile, and this cannot be reconciled with both a uniform projected density distribution a slowly rising rotation curve. The rotation curve is probably rising quickly and remaining flat at $\sim$ 100 km s$^{-1}$.

Deep optical surface photometry shows less flattening than sky survey pictures. Actually, the vertical scaleheight $h_z$ is 0.66 kpc, which is twice as large as the Galaxy. Although Abe *et al.* fit the major axis profile with a bulge and an exponential disk, photographs nor deep isophotes give any impression of a bulge. I prefer a single exponential disk with $h$ = 7 kpc. The disk has sharp cut-off at about 17 kpc. The face-on surface brightness (not correcting for absorption) would be $\mu_o$ = 23.2 R-mag arcsec$^{-2}$.

Fig. 2 shows some well-known edge-on spirals. Compared to IC5249, the late types have higher $V_{rot}$ (typically 210 km s$^{-1}$), *smaller h* (4.5 kpc) and $h_z$ (0.4 kpc) and comparable $R_{max}$ (20 kpc).

We can use the data on IC5249 to estimate the stellar velocity disperions under two different assumptions:

*I. Assume "maximum disk".* This would mean that the exponential disk is fitted to the rotation curve with the maximum amplitude possible: 100 km s$^{-1}$ at 17 kpc. This is slowly rising and is almost certainly incon-

*Figure 2.* Some well-known edge-on spirals, taken from the Digitized Sky Survey, compared to the superthin galaxy IC5249 (top-center). The others are on the left NGC5907 and 891, in the middle NGC4565 and 7814 and on the right NGC4244, 5170 and 4594.

sistent with the integrated HI-profile. Then the central surface density $\Sigma(0) = 137$ $M_\odot$ $pc^{-2}$, and the disk mass is $2.9 \times 10^{10}$ $M_\odot$. Assume that the gas is also in an exponential disk with same scalelength. Then about 1/3 of the disk surface density is everywhere in the form of HI. Assume that the HI has a (one-dimensional) velocity dispersion of 7 km $s^{-1}$ (e.g. van der Kruit & Shostak, 1984) and the Toomre stability parameter $Q$ equal to 2 at all radii. Then we can calculate the stellar velocity dispersions after correction for the influence of the HI. For $\kappa$, I have taken the average for a flat and a solid body rotation curve.
*II. Assume global stabilisation by a dark halo.* We then can use criterion (3). The disk mass then is $1.3 \times 10^{10}$ $M_\odot$ and the central surface density $\Sigma(0) = 63$ $M_\odot$ $pc^{-2}$. Making the same assumptions as above, it follws that now about 2/3 of the disk surface density is in the form of HI. In this case $\kappa$ has been calculated only for a flat rotation curve.
Application at two galacticentric radii gives the stellar velocity dispersionsin the following table. We see that the axis ratio of the velocity ellipsoid is small, but not extremely so.

| R (kpc) | 10 | | 15 | |
|---|---|---|---|---|
| | I | II | I | II |
| $\sigma_z$ (km s$^{-1}$) | 27 | 24 | 18 | 15 |
| $\sigma_R$ (km s$^{-1}$) | 73 | 52 | 42 | 38 |
| $\sigma_z/\sigma_R$ | 0.37 | 0.46 | 0.43 | 0.39 |

The rotation velocity is rather low and the scalelength large. This implies a low surface density and much of the disk then is in the form of HI. The conclusion is that IC5249 is a Low Surface Brightness Galaxy. The scaleheight is relatively large (0.66 kpc), but we see only the peak near the plane in pictures as Fig. 2. The super-thin appearance on the sky results from the combination of low intrinsic surface brightness and large scalelength.

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